Multiscale Design Optimization of Hopper Cars Employing Functionally Graded Honeycomb Sandwich Composites

by

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Abstract

A novel multi-stage structural design optimization procedure has been developed for the weight minimization of hopper cars. The procedure has been tested under various loading conditions according to guidelines established by regulating bodies, as well as a novel load case which considers fluid-structure interaction by means of explicit finite elements employing Smoothed Particle Hydrodynamics. The first stage in the design procedure involves topology optimization whereby optimal beam locations are determined within the hopper car wall structure design space. Through determination of optimal beam locations, a novel frame was designed which concentrates mass in critical regions of the hopper car. In the second stage, hexagonal honeycomb sandwich panels have been implemented in regions of low criticality and are optimized by means of a multiscale design optimization (MSDO). The MSDO drew upon the Kreisselmeier-Steinhausser equations to calculate a penalized cost function for the mass and compliance of a hopper car Finite Element Model (FEM) at the mesoscale. For each iteration in the MSDO, the FEM was updated with homogenized sandwich composite properties according to four design variables of interest at the microscale, and a cost penalty is summed with the base cost by comparing results of the FEM with the imposed constraints. Stochastic search algorithms were employed in the MSDO to optimize for minimal mass and compliance simultaneously, and to generate a Pareto front using a multi-objective approach. Efficacy of the novel design methodology has been compared according to a baseline employing conventional materials and methods. By invoking the methodology in a case study, it is demonstrated that a mass savings as high as 16.36% can be yielded for a single hopper car without sacrificing rigidity, which can be translated into a reduction in greenhouse gas emissions of 13.09% per car based on available literature.

In memory of Almir, whom I had the privilege of calling friend.

“And perhaps, if only the tale told by wise men is true and there is a bourne to welcome us, then he whom we think we have lost has only been sent on ahead. Farewell.”

- Seneca the Younger

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Foreword

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- [1] **A. Al-Sukhon** and M.S.A. ElSayed, “Fluid-Structure Interaction for the Multidisciplinary Design Optimization of Hopper Cars Employing Honeycomb Sandwich Composites”, *AIAA Scitech 2020 Forum*, Jan 6-11, 2020, p. 1633, 2020, Orlando, FL. doi: 10.2514/6.2020-1633
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By reading or printing this thesis, overall entropy in the universe has been increased. The author accepts no liability for damages incurred as a result of this universal increase in chaos, nor its contribution towards heat death.

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Chapter 1

Introduction

1.1 Background and Motivation

The proliferation of environmental concerns has resulted in the necessity for improvements focused on the reduction of Greenhouse Gas (GHG) emissions in contemporary technology. Despite its relatively small population, Canada is one of the top GHG emitters per capita in the developed world, and its contribution to climate change cannot be dismissed [1]. Reduction of emissions can be accomplished through various means, from high level policy decisions encouraging responsible transitions to cleaner energy sources all the way down to specific improvements in technology which enhance energy efficiency and reduce burning of fossil fuels. In Canada, 24% of emissions come from the transportation sector, to which rail contributes 1% [2]. While this number may seem small, it amounts to a total of 6.6 Mt of GHG emissions per annum, which amounts to just under the amount released by domestic aviation [3]. Many avenues of research and technological development are available to reduce emissions and cost from this sector, from systemic overhauls leading to improved operational efficiency, advancements in aerodynamics, and improved suspension [4–7]. In order to address rising climate concerns, it will be necessary to compound every measure which can be taken to reduce GHG emissions and reduce consumption of non-renewable energy sources. The present work seeks to employ design techniques and high-performance materials normally confined to aerospace to address the problem by reducing the structural weight of railcars.

Figure 1.1: Image of a hopper car as manufactured by *National Steel Car* [15].

Hopper cars, used for carrying bulk granular cargo on railways, are the most common car type in the CN rail fleet as of 2018 [8]. In Figure 1.1, a hopper car manufactured by *National Steel Car* is depicted. Therefore, thesis focuses on the minimization of structural weight of hopper cars. It has been found that, in general, reducing the weight of any vehicle by 10% can increase relative energy savings by as much as 5-8% [9,10]. In addition to energy savings and GHG reduction, it has further been found that structural weight reduction in rail vehicles can improve life-cycle metrics on tracks resulting in less frequent maintenance, which can further reduce costs and emissions [11]. While many efforts have been made to reduce car-body weight in passenger rail, freight rail remains scarcely investigated [12–14]. In commercial freight rail, reduced weight of the car-body can potentially result in a greater yield of transported cargo, thereby significantly reducing costs or bolstering profits. The increase in cargo capacity can also reduce the number of cars required, therefore reducing potential harm to the environment through the manufacturing process.

1.2 Literature Review

Mass reduction in vehicles has generally been attained in one of the following two methods: substitution of lightweight materials and structures, or numerical optimization. In this chapter, a brief review of both major methods have been investigated. First, potential materials and other lightweight structural candidates are discussed for substitution. Then, an overview of the optimization theory employed for structural optimization has been discussed in this thesis.

1.2.1 Weight Minimization Methodology: Lightweight Materials

A High-Performance Composites

Substitution of novel high performance materials and structures in vehicles has largely taken the form of *Fiber Reinforced Polymers (FRP)* or high strength alloys. Magnesium alloy corrugated sandwich panel extrusions have been proposed by Lee *et al.*, where the extrusion dimensions of each panel have been optimized *a priori* [14]. Cho *et al.* employed CFRP-Aluminum honeycomb sandwich composites to the under-frame and roof of a K-EMU car-body, resulting in a weight reduction as high as 29% [16]. Using a multiscale approach where the material data for composite sandwich structures was characterized at the coupon level and then gradually extrapolated to the component level, Zinno *et al.* developed a novel lightweight roof for a rail car which abides by European Code [17]. As part of a joint venture to counteract a shortage of covered hopper cars, researchers at *Cargill, Inc.*, *Southern Pacific Transportation Co.* and *ACF Industries, Inc.* introduced a fully functional fiberglass covered hopper car which offered a significantly lighter car-body structure [18].

B Stiffened Panels

Stiffened panels are also a common in lightweight structural design. In aerospace, Chintapalli *et al.* used size optimization to design stiffened skin-stringer tension and compression panels for wingbox applications, achieving lower mass cross sections than well recognized textbook methods [19]. Lamberti *et al.* employed NASA's *PANDA-II* algorithm to determine optimal panel cross-sectional sizes for a lifting body fuselage [20]. Butler *et al.*

have determined the optimal design of stiffened panels through parametric and numerical studies, with results showing 20% mass savings [21]. While they have largely been confined to aerospace applications, stiffened panels have demonstrated the ability to provide resilient flexural rigidity comparable to sandwich panels, and their potential application should therefore be investigated in the present work [22].

C Sandwich Panels

Sandwich panels have been lauded for their high strength-to-weight ratios, resistance to impact, and resistance to buckling [22–24]. Light-weighting through the material substitution of typical airframe materials with composites such as honeycomb core sandwich structures have been used in the design of the Airbus A380 and the Boeing 777 to improve efficiency. Traditionally, the use of these composites has been limited to aerospace, however due to improvements in their manufacturability, sandwich composites have seen an increase in their use across many novel applications [25, 26]. Hudson *et al.* have determined an optimal plywood-based sandwich panel for implementation in passenger rail which is 60% lighter than a benchmark structure [27]. Xu *et al.* have optimized sandwich panels using a multi-objective evolutionary approach to design lightweight sandwich panels providing superior noise damping for improved passenger comfort [28]. Seo *et al.* developed a hybrid composite bodyshell for the Korean Tilting Train Express which met all the safety requirements and reduced the car-body mass by 30% [29].

Often, sandwich panels are compared to stiffened panels in cases of compression and bending [22, 30]. Sandwich panels have long been favoured in the aerospace industry for use in locations of aircraft where high loads are not expected, such as in the leading and trailing edge, the ailerons and in flooring or internal walls [31]. Many recent studies have also examined the feasibility of sandwich panels for aircraft in locations of higher structural importance, such as the fuselage [32, 33]. The strength of sandwich panels is explained through simple beam theory, which gives the flexural rigidity of a sandwich panel as [34]:

$$D = \frac{E_f w t_f^3}{6} + \frac{E_f w t_f (H_c + t_f)^2}{2} + \frac{E_c w H_c^3}{12} \quad (1.1)$$

where D is the flexural rigidity of the panel, E_f and E_c are the elastic moduli of the face and core respectively, w is sandwich width, t_f is the thickness of the face sheets and H_c is the thickness of the core. Should the following restrictions apply:

$$\frac{d}{t_f} > 5.77 \quad (1.2a)$$

$$\frac{E_f}{E_c} \frac{t_f}{H_c} \left(\frac{H_c + t_f}{H_c} \right)^2 > 16.7 \quad (1.2b)$$

then the first and third terms reduce to a negligible value, and the sandwich panel is considered to have *very thin faces*. The rigidity of the sandwich panel is then reduced to:

$$D = \frac{E_f w t_f (H_c + t_f)^2}{2} \quad (1.3)$$

Equation (1.3) implies that the sandwich rigidity is dictated primarily by the face material and core thickness. Considering that solid materials are used in the faces for their normally superior strength, the implication is that a sandwich with very thin face sheets is then clearly mass efficient. Another implication of significance is that when the faces are very thin, shear stress is confined exclusively to the core, while flexural stresses are of significance only in the faces [34].

D Material Candidate Conclusions

While FRP have garnered much attention in the previous decades, their use in freight rail can raise many questions on concerns not yet investigated. These concerns include resistance to the elements, to ultraviolet damage, and to high-cycle fatigue. This thesis shall focus its efforts on attaining superior mass reduction with conventional materials such as Steel and Aluminum. As such, sandwich panels and stiffened panels are proposed for further consideration.

1.2.2 Weight Minimization Methodology: Structural Optimization

The second approach to mass reduction in vehicles is to perform numerical optimization and obtain structures of high strength-to-weight or rigidity-to-weight ratios. Structural optimization often employs numerical optimization theory and algorithms with the objective of minimizing a *cost function*, which mathematically represents the design characteristic of interest. In structural optimization, this cost is often associated with either mass or compliance. In this section, various numerical strategies are discussed to elaborate upon these numerical optimization techniques.

A Basic Optimization Theory

In structural design, it is often necessary to employ numerical optimization techniques in order to best determine where lightweight material might be used, or perhaps even removed altogether. In general, optimization techniques have four attributes: a design space, a cost function, a set of constraints, and bounds on the variables. The general problem statement is written as:

$$\begin{aligned} \min_{\mathbf{x}} \quad & C(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{A}_{eq}\mathbf{x} = \mathbf{b}_{eq} \\ & \mathbf{Ax} < \mathbf{b} \\ & c_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, n \\ \text{where } \mathbf{x} \in & [\mathbf{x}_{LB}, \mathbf{x}_{UB}] \end{aligned} \tag{1.4}$$

where C is the cost function calculated according to the design variables in vector $\mathbf{x}$. The design variables are bounded according to the constraint matrices $\mathbf{A}_{eq}$ and $\mathbf{b}_{eq}$ for equality constraints, and $\mathbf{A}$ and $\mathbf{b}$ for inequality constraints. For non-linear constraints, a set of functions $c_i(\mathbf{x})$ are introduced where c_i is the i^{th} function calculated according to the design variables. Usually, the solution space is bounded by the lower bounds in vector $\mathbf{x}_{LB}$ and upper bounds in vector $\mathbf{x}_{UB}$.

B Multi-Objective Optimization

Often in structural design, there can be more than one design objective. For a problem of the form:

$$\begin{aligned} \min_{\mathbf{x}} \quad & (f_1(\mathbf{x}), f_2(\mathbf{x})) \\ \text{s.t.} \quad & \mathbf{A}_{eq}\mathbf{x} = \mathbf{b}_{eq} \\ & \mathbf{Ax} < \mathbf{b} \\ & c_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, n \\ & \text{where } \mathbf{x} \in [\mathbf{x}_{LB}, \mathbf{x}_{UB}] \end{aligned} \tag{1.5}$$

where f_1 and f_2 are two competing objectives, there does not exist one single optimum, but an entire set of optima. Such a set is called a *Pareto Front*, which, when plotted with each objective on one axis, produces a curve displaying the trade-off between the two functions. This curve can then be used for determination of the design which provides an acceptable compromise between each objective. Techniques for selecting the Pareto-Optimal point are problem dependent and are therefore not discussed here. Upon establishing the multi-objective problem specific to the design of a hopper car employing sandwich panels in Section 3.3.2.A, a methodology for determining the Pareto-Optimal point is discussed.

In addition to determination of the structural candidate of choice, this chapter has also reviewed and investigated the theory behind optimization methodologies for the purpose of reducing the mass of a hopper car. Implementation of topology optimization and multi-objective optimization for the optimization of a hopper car shall be discussed in the subsequent chapter upon fully defining the numerical model.

C Topology Optimization

Topology Optimization (TO) is a numerical design methodology wherein the optimal material layout can be determined in a given design space, often a *Finite Element Model (FEM)* or any other numerical model. In determining optimal material layout, it is then possible to determine where material can be removed within a given structure, and a redesign can be achieved which employs as much as 40% less mass [35]. Although results are often

promising, TO often generates intricate structures which are not possible to produce without employing additive manufacturing or other advanced manufacturing techniques [35,36]. Alternatively, the design must be altered such that it is manufacturable using more readily available techniques, resulting in a potentially less-than-optimal design due to the inability to faithfully reproduce computational results [37].

Large scale structures have frequently been optimized in aerospace using topology optimization to determine ideal frame layouts for stiffeners or beams [38,39]. Within the context of land vehicles, Zhong *et al.* optimized the structure of a bus body frame by implementing topology optimization to determine ideal beam locations and subsequently sizing them [40]. A similar procedure has been demonstrated by Bendsøe *et al.* in [41], taken from a larger study by Thomas *et al.*, wherein the structural frame for a hybrid electric bus is optimized first using TO, then further refinement techniques such as size optimization are employed to remove stress concentrations [42]. In rail, TO has been used by Wennberg *et al.* to determine where the shape of a novel car-body shell [13,43]. Lee *et al.* performed topology optimization on the inner corrugated wall structure of a passenger car in order to determine where corrugation should be concentrated [44]. Liu and Wakeland developed a multi-level optimization algorithm which employed topology optimization to determine the number of channels required and their respective layouts as part of the larger algorithm. The methodology showed a promising mass reduction as high as 36% [45]. Kuczek and Szachniewicz presented topology optimization for the determination of optimal beam layouts in a passenger railcar employing fiber reinforced composites [46]. Kuczek has also presented a similar methodology for a manufacturable hopper car, stopping short of performing any sizing [47].

Many different implementations of TO exist, however the methodology most widely implemented is the *Solid Isometric Material with Penalization (SIMP)* method by Bendsøe, and thus it is the choice program in this thesis [48,49]. The SIMP Topology Optimization problem can be written as:

$$\begin{aligned}
\min_{\boldsymbol{\rho}} \quad & C(\boldsymbol{\rho}) = \mathbf{f}^T \mathbf{u} \\
\text{s.t.} \quad & \mathbf{K}(\boldsymbol{\rho}) \mathbf{u} = \mathbf{f} \\
& V(\boldsymbol{\rho}) \leq V_{frac} \\
\text{where} \quad & \mathbf{K}(\boldsymbol{\rho}) = \sum_{e=1}^{n_{elements}} (\boldsymbol{\rho}_e^p \mathbf{K}_e^0)
\end{aligned} \tag{1.6}$$

where C is the compliance of the design space under the loading conditions as a function of the relative element density matrix $\boldsymbol{\rho}$. The problem is constrained by equilibrium, where $\mathbf{K}$ is the stiffness matrix, $\mathbf{u}$ is displacement, and $\mathbf{f}$ is the force applied on all elements. Normally, a volume fraction constraint V_{frac} is applied, where V is the volume as a function of the density. When a discrete optimization is used such that $\rho(e) = 0$ or $\rho(e) = 1$, solution time has been found to be very high and can cause singularities. The SIMP method corrects this by introducing a penalty factor [50]:

$$\mathbf{E} = \boldsymbol{\rho}^p \mathbf{E}^0, \quad \boldsymbol{\rho} \in [\rho_{min}, 1], \quad p > 1 \tag{1.7}$$

where $\mathbf{E}$ is the penalized stiffness tensor, p is a penalty factor, and $\mathbf{E}^0$ is the original stiffness tensor. As the relative density is reduced, the stiffness of each corresponding element is subsequently penalized according to p thereby giving a distribution of elements of varying densities between ρ_{min} and unity. The ρ_{min} value is introduced in order to further prevent any introduction of singularities.

In this thesis, use of topology optimization is proposed to determine the ideal beam layout for the hopper car. First, an element density contour plot shall be generated using the SIMP algorithm. Next, the contour plot is used as a map to determine where new beams for the structure should be placed. Where beams overlap areas of low density, the beams are instead removed. While most design procedures at this time have employed TO for one iteration, the implementation in this thesis has been performed in an iterative fashion until no change was observed in the relative density contour plot and all beams are overlapping with the areas of high element density. The full procedure has been outlined and performed in Section 4.2.

D Size Optimization

A common drawback of TO is that since material is removed, the geometry might be altered in such a way that stress concentrations and singularities can arise. Applying stress constraints within a TO poses a challenge since stress constraints can be highly localized and non-linear, therefore making convergence to an optimum difficult [51, 52]. A further difficulty encountered in the implementation of TO in the context of the present work is that it largely deals with the removal of material, which can be useful in the design of the frame structure, however it cannot deal with the determination of optimal honeycomb sandwich panel sizes. Therefore, in the application of TO, it will be necessary to couple the results with a *Size Optimization (SO)* in order to refine the results for maximum approach to optimality. Size optimization is diverse in its applications and has been used in several studies to refine designs of optimal topology, resize mechanical elements, and reduce stress concentrations [35, 40, 43, 44]. Structural sizing of railcars has been investigated without intervention from topology optimization. Harte *et al.* employed a multi-level approach to determine the ideal shape and size of a passenger LRV [12]. Kuczek and Mrzygłód have combined topology optimization with size optimization to perform a multi-stage optimization of an occupant survival space in a passenger car by employing evolutionary algorithms. By employing both methodologies in tandem, a novel car frame design with improved passenger safety and reduced weight was generated.

E Multiscale Design Optimization

Multiscale Design Optimization (MSDO) has garnered much attention across many disciplines in recent years [53–56]. A MSDO involves the optimization of material and structural characteristics at the micro-scale for translation into improved performance at the macro-scale. In addition to topology optimization, Wennberg and Stichel also investigated MSDO for the application multi-functional sandwich panels in a passenger car [13]. By optimizing a characteristic panel at the micro-scale for maximum performance in several objectives, the macro-scale structure was improved by all metrics at a mass reduction of 40%. On the component level, Montemurro *et al.* have employed multiscale design optimization for

sizing of sandwich panel geometric properties for resistance to buckling [57, 58]. Molavitarabi and Laliberte have developed an MSDO algorithm for the implementation of a lattice core sandwich panel in the flooring of a hopper car, with results suggesting a 12.5% mass reduction [59].

F Other Optimization Techniques

Many other composite optimization techniques exist, including *Shape Optimization*, *Composite Optimization*, and *Lattice Optimization* [50]. Although future work might consider such techniques, they have not been evaluated presently. Shape optimization is not used because altering the overall form of the hopper car can introduce different aerodynamic loads and can drastically change the flexibility of the car, which can drastically affect the aerodynamic loads and dynamic stability of the hopper car. Although Composite Optimization can produce valuable ply sizing and lay-up results for a *Fiber Reinforced Polymer* (FRP) or for any other layered composite, these materials are currently out of scope of the present work and have not been included for consideration in the design optimization. Lattice Optimization has also shown much promise in the design of additively manufactured components, however the manufacturability of a hopper car comprised of additively manufactured alloys is not feasible at the time of writing [35, 60]. Therefore, until such time that additive manufacturing shows promise for large scale and large structure production, this form of optimization is left for future work.

G Structural Optimization Conclusions

In the literature studied, it is evident that while any technique correctly applied can significantly minimize the mass of vehicles, a combination of theories can offer further benefits. In this thesis, a multi-stage and multi-objective approach shall be used for the multiscale design optimization of a hopper car structure. The first stage shall employ topology optimization in order to minimize the weight of the overall structure, while the second stage shall employ a multi-objective MSDO to optimize the structure at the component level.

1.3 Thesis Objectives

- Structures comprised of metallic stiffened panels and sandwich panels remain scarcely investigated for rail applications. This thesis shall conduct a preliminary investigation to explore two potential metallic structural candidates and isolate that which provides the highest strength-to-weight ratio. The structural candidate showing the most promise shall then be investigated further.
- For a robust design, load cases according to regulatory bodies shall be reviewed and implemented into a high-fidelity *Finite Element Model (FEM)*. The FEM will additionally consider a novel *Fluid-Structure Interaction (FSI)* employing *Smoothed Particle Hydrodynamics (SPH)*.
- The developed FEM of the hopper car shall be employed in a novel multi-stage design optimization procedure. In the first stage, topology optimization is used to determine the areas of high criticality in the structure and deliver a relative density contour plot. The density plot shall then be employed to attain the ideal layout of the structural frame of the hopper car. In the second stage, sandwich panels are inserted into areas of low criticality, and a MSDO shall be employed to determine the ideal micro-structural characteristics of core.
- It is expected that these efforts applied to a case study shall yield a novel lightweight hopper car design employing metallic honeycomb sandwich panels without compromising on strength or rigidity.

1.4 Thesis Overview

This thesis has been divided into six chapters, each building on the last. This section will give an overview of each chapter. Chapter 1 gives an overview of the motivations and objectives of the present work. In Chapter 2, different configurations of sandwich panels are investigated to determine which core topology theoretically provides the superior strength-to-weight ratio. Chapter 3 discusses all numerical methodology deployed for physics simulation and structural optimization. Both the explicit FEM, which models the FSI, and the

implicit model, which models all other load cases laid out according to regulation, are developed. Additionally, the numerical optimization strategies for sizing of mechanical elements have been elaborated upon in the second half of the chapter including the discussions of both the analytical approach and the MSDO. In Chapter 4, a multi-stage design optimization procedure has been developed using the established knowledge of previous chapters and was applied to the hopper car structure to design a novel lightweight car-body employing hexagonal honeycomb sandwich panel. Results have been tabulated and discussed from the application of the multi-stage optimization to the hopper car-body in Chapter 5. Useful implications drawn from the optimization have been recorded for future designers. Among the results are the recommended final design for a single characteristic panel in the hopper car, along with an extrapolation to full structure. A comparison between the advantages of the analytical approach and the MSDO have been discussed in this chapter as well. Conclusions to the research project have been drawn in Chapter 6, along with broader implications to industrial applications. Contributions to the state of the art can also be found in this chapter, along with suggestions for future work from the author.

Chapter 2

Theoretical Framework: Analytical Study

In this chapter, underlying theoretical framework for the materials and methods considered in this thesis are discussed. The first section begins with an investigation into the parametric optimization of two candidates of structural configuration, namely, a novel semi-monocoque wall comprised of stiffened panels, and sandwich panels of varying core topologies. The stiffened panel configuration has capitalized upon the strength fostered by a compression and tension panel working in unison to provide a structure with high specific flexural rigidity. Both the compression and tension panels are designed separately under the expected hydrostatic loading endured by a hopper car wall, and their results combined to formulate the final structure. The strength-to-weight characteristics of this novel configuration are contrasted against that of sandwich panels, for which five core topologies have been considered. The two structural candidates have been compared in a theoretical load case concerning a hopper car wall which has been divided into five panels. Upon completion of the study, all candidates have been compared, and that which provides the largest strength-to-weight ratio has been considered for the remainder of this thesis.

2.1 Candidate Structure Theory

This section begins its investigation by synthesizing a novel semi-monocoque wall configuration which employs a set of compression and tension panels working in tandem to provide a bending resistant wall structure. Then, sandwich panels of varying core topologies have been investigated for use in hopper cars under the same loading conditions. The section shall conclude with a final comparison of all candidates and determination of the recommended structural candidate of choice for further study in the remainder of the present work.

2.1.1 Semi-Monocoque Panels

Stiffened panels are thin sheets of material supported by stringers, which serve to enhance the compressive and tensile strength of the panel. In aircraft, the skin of the panel resists shear and torsional loads while offering a surface upon which lift forces act, as well as protection from environmental conditions outside the structure. Working in tandem with the skin, stringers offer tensile and compressive strength to prevent buckling and fatigue [31]. Stiffened panels are most commonly used in the load bearing regions of aircraft, namely the wingbox and fuselage. In the wings of an aircraft, the wingbox is constantly in bending due to aerodynamic loads; the top of the wing is in compression, while the bottom is in tension. The loading can also alternate depending on the direction of bending in the wing, therefore panels on both sides are often designed to resist both tension and compression. As a result of their design, the wings of an aircraft can be said to behave in a similar manner to an I-beam or sandwich panel; the extreme edges of the structure resist tension and compression to create a structure which provides rigidity against bending, while at the same time offering a lightweight design due to the empty space between faces.

It is hypothesized that due to the similarities of a sandwich panel and a wingbox structure, a semi-monocoque wall, for which a cross section has been depicted in Figure 2.1, employing a tension and compression panel might be an ideal candidate for a hopper car wall. In this section, a preliminary design methodology investigating the potential use of stiffened panel constructions in the wall structure of a hopper car wall carrying hydrostatic

loading has been developed. The wall panels of the hopper car have been treated as though they are the wings of an aircraft with two wall layers: an outer wall in tension and an inner wall in compression. It is anticipated that since the bulk of the material is distributed further from the neutral axis, an increase in flexural rigidity of the overall structure will be observed for a lower material content thereby resulting in a lower mass than the standard thick wall.

Figure 2.1: Schematic of the hopper car wall structure depicting layout of stiffened panels.

The hopper car wall loading diagram, modeled conceptually as a beam, is shown in Figure 2.2. At the base of the structure, the hydrostatic pressure due to the cargo is given by the equation [61]:

$$(w_o)_{wall} = 1.25W h_w \left(\frac{1 - \sin(\phi)}{1 + \sin(\phi)} \right) \quad (2.1)$$

where W is the specific weight of the cargo, h_w is the total depth of the cargo, and ϕ is the static angle of repose. In this thesis, the static angle of repose is always considered as $\phi = 25^\circ$. In the interest of a conservative design, a safety factor of 1.25 is applied and the hopper car is considered fully laden. The hydrostatic pressure has been distributed such that the pressure is at a maximum equal to $(w_o)_{wall}$ at the base of the wall, and decreases linearly so that the pressure reduces to zero at the top of the wall. The wall has been

divided vertically into $n_{pan} = 5$ panels as shown in Figure 2.2. The choice of number of panels is arbitrary; it is not within the scope of this study to isolate the optimum number of panels. Rather, the objective of the study is to investigate how given a number of panels the structural candidates perform. Each panel is then treated as a pair of skin-stringer panels, with one in compression and the other in tension and spaced apart by spacing s_p as shown in Figure 2.1.

In order to characterize the loads on the wall structure, it is necessary to determine the maximum bending moment experienced in each of the panels. This can be done using simple beam theory. The maximum bending moment per unit width experienced by one of n_{pan} panels is expressed by the equation:

$$M_{max} = \left(\frac{w_A - w_B}{6L} \right) L_{max}^3 - \frac{w_A}{2} L_{max}^2 + \left(\frac{w_A}{3} + \frac{w_B}{6} \right) L_{max} L \quad (2.2)$$

where w_A and w_B are the distributed highest and lowest load intensity at either end of the panel respectively. The term L , the total length, is not to be confused with the variable L_{max} , which denotes the local length along each panel where the maximum bending moment occurs. This location is given by the equation:

$$L_{max} = L \frac{(\beta_1 - (\beta_1^2 - \frac{1}{3}(\beta_1 - \beta_2)(2\beta_1 + \beta_2)))^{1/2}}{\beta_1 - \beta_2} \quad (2.3)$$

where β_1 and β_2 are loading factors such that the relationship $w_A = \beta_1 (w_o)_{wall}$ and $w_B = \beta_2 (w_o)_{wall}$ holds. The assignment of these factors is dependent on the number of panels and which of such panels is under analysis. For instance, for the first of five panels, $\beta_1 = 1.0$ and $\beta_2 = 0.8$ since the load linearly decreases from a maximum of $(w_o)_{wall}$ at the lower end of the panel to a pressure of $0.8(w_o)_{wall}$ at the higher end of the panel. Then, for the second panel $\beta_1 = 0.8$ and $\beta_2 = 0.6$ as the load continues to decrease linearly. It is assumed that each panel is simply supported on its own due to the rigidity of any supporting structures around it, such as the ribs and spars which divide them from their other surrounding panels. The panel loading schematic in Figure 2.1 has been produced to demonstrate the role of each of these variables in determining the maximum bending moment.

Figure 2.2: Free body diagram of the hopper car wall structure modeled as a beam.

Now that the load has been characterized on the semi-monocoque wall, it is possible to proceed with designing the two faces: one which carries compression, and another which carries tension. In the following sections a parametric approach has been employed to manage the design of each panel, beginning with the compressive face.

A Design of Compression Panels

In design of aircraft, compression panels are constructed to resist buckling on the top of the wingbox when the wing bends up and on the bottom of the wingbox when the wing bends down. Optimization of these panels for maximum strength-to-weight ratios has garnered much attention over the span of decades, specifically those employing numerical approaches [19–21]. Although such numerical methods are excellent tools for refinement of

potential designs, it is the objective of this section to analytically compare the mass efficiency of a semi-monocoque wall to other sandwich panels. As such, an analytical approach is proposed for use in the determination of the superior material candidate.

An approach by Budiansky [62] has been developed to consider a parametric optimization of compression panels using hat-shaped stiffeners. Within this parametric approach, hat-shaped stiffeners have been considered, as they are among the most efficient of cross-sectional shapes. By using the most efficient cross section available, the evaluation of the candidate of highest mass efficiency is facilitated [31]. In Figure 2.1, the design variables for the stiffened compression panel have been depicted. From Budiansky, the weight index of a compression panel under a specified load considering plasticity is given by the equation [62]:

$$\Psi_c = \frac{W}{\rho L^2} = \left(\frac{P_i}{\varepsilon_y E L} \right) \left(\frac{\sigma_o}{\sigma_y} \right) \quad (2.4)$$

where the weight index, Ψ_c , is a unitless measure of the mass of a panel given by the division of the mass per unit length, W , divided by the density of the material, ρ , and the length of the panel, L , squared. The minimum weight index indicates the amount of material which will be required to prevent failure according to the distributed compressive edge loading per unit width, P_i , the length of the panel, L , the elastic modulus of the material, E , and the yield strain ε_y . The ratio of the optimum, or *naive optimum*, stress and the yield stress is given by the equation [62]:

$$\frac{\sigma_o}{\sigma_y} = \frac{\pi}{5\varepsilon_y} \left(\frac{2}{3} \right)^{1/2} \left(\frac{13\eta_t\eta_p}{1 - \nu_p^2} \right)^{1/4} \left(\frac{P_i}{EL} \right)^{1/2} \quad (2.5)$$

Now, a unitless load index can also be defined as:

$$\Pi_c = \frac{P_i}{\lambda EL} \quad (2.6)$$

where λ is a knockdown factor set to 0.9 for most cases [62]. Assuming that the load applied induces a perfectly distributed stress throughout the cross section, the modified deformation theory formulas defined by Stowell and Pride, which in turn employ the Ramberg-Osgood equations for plasticity, can be used to determine the factors η_t , η_s , η_p , and ν_p [63, 64]:

$$\frac{E_{tan}}{E} \equiv \eta_t = \left(\left(1 + \frac{3n}{7} \right) \left(\frac{\sigma}{\sigma_y} \right)^{n-1} \right)^{-1} \quad (2.7a)$$

$$\frac{E_{sec}}{E} \equiv \eta_s = \left(\left(1 + \frac{3}{7} \right) \left(\frac{\sigma}{\sigma_y} \right)^{n-1} \right)^{-1} \quad (2.7b)$$

$$\frac{E_p}{E} \equiv \eta_p = \eta_s \left(\frac{1}{2} + \frac{1}{4} \left(1 + \frac{3\eta_t}{\eta_s} \right) \right)^{1/2} \quad (2.7c)$$

$$\nu_p = \frac{1}{2} - \left(\frac{1}{2} - nu_f \right) \eta_s \quad (2.7d)$$

where σ is the applied stress, while E_{tan} , E_{sec} , and E_p are the secant, tangent and plastic moduli respectively. Budiansky concluded through investigation of analytical results from Farrar [65] that the optimal stress wherein all failure modes of the compression panels occur simultaneously is given as [62]:

$$\sigma_{opt} = \alpha E \sqrt{\frac{P_i}{EL}} \quad (2.8)$$

where $\alpha = 0.96$ for a hat stiffened panel according to Farrar's findings [65]. By substituting Equation 2.8 into Equations 2.7a and 2.7b, the Ramberg-Osgood relations can be re-written as:

$$\eta_t = \left(\left(1 + \frac{3n}{7} \right) \left(\frac{0.96E_f \Pi^{1/2}}{\sigma_y} \right)^{n-1} \right)^{-1} \quad (2.9a)$$

$$\eta_s = \left(\left(1 + \frac{3}{7} \right) \left(\frac{0.96E_f \Pi^{1/2}}{\sigma_y} \right)^{n-1} \right)^{-1} \quad (2.9b)$$

For consideration of the compression panels, Aluminum has been assumed as the material of choice with a material number of $n = 21$, a yield stress of $\sigma_y = 517MPa$, an elastic modulus of $E = 72GPa$, and a Poisson ratio $\nu = 1/3$. The choice of Aluminum is based primarily on the availability of literature considering its alloys in stiffened panels. For Aluminum, the material number of n has been determined by Budiansky [62] and is employed in the Ramberg-Osgood equations. Choice of material is not in the scope of this analysis, therefore

Figure 2.3: Schematic of the hopper car semi-monocoque panel depicting layout of stiffened panels and resulting loading.

optimum material choice has not been considered. For this parametric study the material choice poses no issue since the focus is to determine optimum geometry irrespective of material, however for the design of a hopper car steel shall eventually be considered.

The challenge is then to determine the load P_i on the panel and to then leverage it to determine the appropriate weight index. Leading up to the current section, the bending moment per unit length in the panel is determined according to beam theory. If the thickness of each compression panel is assumed small relative to their corresponding widths, the load can be assumed as one dimensional along the edge of the panel. The bending load then generates a force couple which compresses the inner compression panel, while inducing tension in the outer tension panel with the load per unit width given according to the equation:

$$P_i = \frac{M_{max}}{s_p} \quad (2.10)$$

where s_p is the spacing between the tension and compression panel according to Figure 2.1, b_{comp} is the width and depth of the hat stiffener, and t_{comp} is the material thickness for both the skin and stiffener. Here, the spacing must be chosen appropriately such that the thickness is not too great, while still offering a large enough space between panels to prevent interference and increase strength. Figure 2.3 demonstrates the resulting P_i due to M_{max} for one of the panels from Figure 2.2.

To determine an appropriate spacing value, the dimensions of b and t must be calculated. For this purpose, the functions derived by Budiansky are employed [62]:

$$\frac{b_{comp}}{L} = \frac{1}{\pi} \sqrt{\frac{(75/13) (\sigma/\sigma_y) \varepsilon_y}{\eta_t}} \quad (2.11a)$$

$$\frac{t_{comp}}{L} = \frac{1}{\pi} \sqrt{\frac{3 (1 - \nu_p^2) \varepsilon_y \sigma}{\eta_p \sigma_y}} \quad (2.11b)$$

By substituting Equations 2.9a and 2.9b along with Equations 2.7c and 2.7d into the equations above, a relationship for the optimal width and thickness of the stiffener and skin can be deduced. These relationships have been plotted below in Figure 2.4 for all values of P_i in the form of the load index Π_c . By observing the limits of Figure 2.4, the largest ratio of width to length is $b/L = 0.0695$, and that for width to thickness is $t/b = 0.0465$. For demonstrative purposes, a panel length of $L = 660.4$ mm has been assumed. Given the maximum ratio in the feasible load space, the largest stiffener width possible is $b = 46.23$ mm and $t = 2.15$ mm, which in turn implies that if a spacing of $s_p = 101.6$ mm is used the spacing is then adequate to allow for room between the two panels regardless of required sizing. This information has been employed as a starting point to determine the load expected in Equation 2.10 for the time being. The thickness of the panel using such spacing is quite large, therefore once a useful design range for the variables of interest are determined, it will be possible to reduce the spacing and determine an appropriate design.

(a) Compression panel design variables with $s_p = 101.6$ mm

(b) Compression panel design variables with $s_p = 57.2$ mm

Figure 2.4: Compression panel design variables with respect to the loading index. Note that for Figure 2.4a, the load space has been compressed for better visual representation of the results. In Figure 2.4b, the full load space is shown with respect to Π_c

The expected loads P_i have been calculated for each panel and are shown in Figure 2.4a as vertical lines indicating the locations of their corresponding load index. The expected width required for the compression panel undergoing the highest load, Panel 1, is determined to be $b_{comp} = 15.85$ mm and the thickness is determined to be $t_{comp} = 0.40$ mm. It is now evident that, while a spacing of $s_p = 101.6$ mm can guarantee enough room for the panels, it is far too large for the required design. From here, the allowable spacing is iteratively scaled down to determine a feasible value. With a spacing of $s_p = 57.2$ mm, the required width would be $b_{comp} = 26.42$ mm, and the required thickness would be $t_{comp} = 0.735$ mm as shown in Figure 2.4b. Assuming that the outer tension panel will be of equal or lesser sizing, the minimum spacing between each panel will be 2.85 mm which should give adequate room between both panels to prevent interference. By enforcing this spacing for all panels, adequate spacing is guaranteed since their cross sections will be of lesser values than Panel 1.

B Design of Tension Panels

In the previous section, design of compression panels is carried out for the inner portion of the semi-monocoque wall configuration. In this section, the design of the outer wall has been discussed. Due to the cargo loading applying stress from the inside, the outer portion of the semi-monocoque wall is under tension, and as such its design has been treated as a *tension panel*. In the design of a tension panel, damage tolerance to material imperfections are normally considered, however for the present work only static strength has been assessed. The stress in the panel under static tensile loading is given by the equation:

$$\sigma = \frac{2P_i b}{A} \quad (2.12)$$

where b is the width of the hat stiffener, A is the panel area, and P_i is the tensile load per unit width. Once again, the width of the panel is assumed to be twice the width of the stiffener. For optimal sizing, it is a condition that $\sigma = \sigma_y$, where σ_y is the yield stress. A function for the minimum area required is consequentially determined to be:

$$A = \frac{2P_i}{\sigma_y} b_{tens} \quad (2.13)$$

Now, the weight index from Equation 2.4 is recalled and re-written for the tension panel:

$$\Psi_t = \frac{W_i}{\rho L^2} = \frac{A}{L^2} \quad (2.14)$$

By substituting Equation 2.13 into the above, the optimal weight index has been determined to be:

$$\Psi_t = \frac{2P_i}{\sigma_y} \frac{b}{L^2} \quad (2.15)$$

In the interest of ensuring spacing between the tension and compression panel is conserved, it should be enforced that the width-to-length ratio for both panels are the same. By enforcing this condition, along with the substitution of $\sigma_y = \varepsilon_y E$, the weight index of the tension panel is re-written as:

$$\Psi = \frac{2}{\varepsilon_y} \Pi_t H \quad (2.16)$$

where H is the enforced panel depth of b/L for the corresponding compression panel. Note that the tensile load index $\Pi_t = P_i/EL$ has been substituted in the above equation as well to determine a weight index with respect to the tensile load index.

Now that two fully defined expressions for calculating the weight indices of a compression and tension panel have been determined, the total weight index can be written as:

$$\Psi_{tot} = \Psi_c + \Psi_t \quad (2.17)$$

where the subscripts for c and t correspond to the weight indices for the compression and tension panel respectively. Now that a total weight index for the semi-monocoque wall has been defined, a design of sandwich panels is performed in the next section in preparation for the final comparison.

2.1.2 Sandwich Panels

In the previous section, a semi-monocoque wall configuration was investigated. In this section, a parametric study on sandwich panels as potential structural candidates are discussed. Although the objective in the design of sandwich panels should be to attain thin faces and the lightest possible core, it is unfortunately not always possible because high stiffness to weight ratios cannot guarantee structural integrity. Although core properties have little effect on panel rigidity, they do have significance in prevention of failure due to loading. Sandwich panels under loading are prone to undergo several failure modes, which have been studied extensively [66–68]. As a result, it is also of paramount importance to consider the loading expected conditions on the panel and, consequently, the expected failure criteria. As discovered from Equation (1.3), sandwich rigidity is primarily affected by face material and core geometry. Assuming the panel is comprised of the same material, the geometry of the sandwich becomes the key factor and it is now necessary to examine which core type provides the lowest mass under the loading conditions. Four design variables have been considered for optimization in the form given by Rathbun *et al.* [23]:

$$\lambda_f = \frac{t_f}{\ell}, \quad \lambda_c = \frac{t_c}{\ell}, \quad \Lambda_c = \frac{H_c}{\ell}, \quad \xi = \frac{L_c}{H_c} \quad (2.18)$$

where t_f is the face thickness, t_c is core member thickness, L_c is core member length, H_c is core thickness, and ℓ is a characteristic length defined as $\ell = M/V$, where M is the induced bending moment and V is the shear. Sandwich panels are then divided into two subgroups: prismatic type and truss type. Truss type cores include the pyramidal cores, while the prismatic grouping includes the square and hexagonal honeycombs, as well as the corrugated core topologies. For all truss type cores, a fifth dependent variable, θ , has been defined as:

$$\theta = \frac{1}{\xi} \quad (2.19)$$

The geometry of each core and its corresponding design variables are depicted in Figure 2.5.

The unitless weight index of a sandwich panel is then again given by the equation from

Figure 2.5: Sandwich panel geometry according to the design variables in Equation (2.18) [23].

Rathbun *et al.* [23]:

$$\Psi = 2\lambda_f + \lambda_c^n \Lambda_c^{1-n} m(\xi, \nu) \quad (2.20)$$

where n is an integer and $m(\xi, \nu)$ is a function dependent on ξ and ν which changes with core geometry. The most common failure modes associated with sandwich panels are face yielding, face or intracellular buckling, core yielding or shearing, and core buckling [23,66]. In certain cases, a fifth failure mode for indentation is considered, however it occurs in cases where the load is specifically concentrated. Considering that, in general, a hopper car is not expected to bear such concentrated loads, it is ignored in this analysis. The failure modes are generalized by Rathbun *et al.* in the context of pure bending and given by equations [23]:

$$\Pi \frac{\Lambda_c^{-1} \lambda_f^{-1}}{\varepsilon_y} \leq 1 \quad (\text{Face Yielding}) \quad (2.21a)$$

$$\Pi h(\xi, \nu) \Lambda_c \lambda_f^{-3} \leq 1 \quad (\text{Face Buckling}) \quad (2.21b)$$

$$\frac{\Pi f(\xi) \Lambda_c^c \lambda_c^{-b}}{\varepsilon_y} \leq 1 \quad (\text{Core Yielding}) \quad (2.21c)$$

$$\Pi g(\xi, \nu) \Lambda_c^l \lambda_c^{-k} \leq 1 \quad (\text{Core Buckling}) \quad (2.21d)$$

where once again, b , c , k , and l are integers and $f(\xi)$, $g(\xi, \nu)$, and $h(\xi, \nu)$ are non-dimensional functions which are all dependent on core topology. By combining the equations from the work of Wicks and Hutchinson in [30] with the three-parameter generalization methodology from Rathbun *et al.* in [23], the following equations for hexagonal honeycomb cores have been developed:

$$f(\xi) = 3\xi \quad (2.22a)$$

$$g(\xi, \nu) = \begin{cases} \frac{12\sqrt{3}(1-\nu^2)\xi^3}{\pi^2(5.35+4\xi^2)}, & \text{if } \xi \leq 1 \\ \frac{12\sqrt{3}(1-\nu^2)\xi^3}{\pi^2(5.35\xi^2+4)}, & \text{if } \xi > 1, \end{cases} \quad (2.22b)$$

$$h(\xi, \nu) = \frac{1728(1-\nu^2)\xi^2}{379\pi^2} \quad (2.22c)$$

$$m(\xi) = \frac{2}{\xi\sqrt{3}} \quad (2.22d)$$

Additionally, it was found that $b = 1$, $c = 0$, $k = 3$, $l = 2$, and $n = 1$. The developed equations for the hexagonal honeycomb core can now be used in the three-parameter optimization methodology by Rathbun *et al.* in [23].

Now that the non-dimensional functions have been determined for a hexagonal honeycomb core topology, it is necessary to characterize the load index, Π , on the panels. Once again, the wall has been divided into $n_{pan} = 5$ panels, with all accompanying assumptions carrying over from Section 2.1.1.A. When the panels are under bending, the non-dimensional load index, Π , is defined as [23]:

$$\Pi = \frac{V^2}{EM} \quad (2.23)$$

where M is the maximum bending moment, V is the maximum shear, and E is the material elastic modulus. The maximum bending and shear are derived again in accordance with the methodology used to determine the moment in Equation 2.2, leading to the equations:

$$M_{max} = \left(\frac{w_A - w_B}{6L} \right) L_{max}^3 - \frac{w_A}{2} L_{max}^2 + \left(\frac{w_A}{3} + \frac{w_B}{6} \right) L_{max} L \quad (2.24a)$$

$$V_{max} = - \left(\frac{w_A L}{3} + \frac{w_B L}{6} \right) + w_A L + \frac{1}{2} (w_B - w_A) L \quad (2.24b)$$

where L_{max} remains defined as it was in Equation 2.3, $w_A = \beta_1 (w_o)_{wall}$ and $w_B = \beta_2 (w_o)_{wall}$. Now, Equations 2.24a and 2.24b are substituted into Equation 2.23 to determine the load index for each panel. The optimum sandwich panel weight index, Ψ , has been determined using the three-parameter optimization from [23] for all core topologies, with the hexagonal honeycomb core employing Equations 2.22a through 2.22d. For the sake of brevity, this three-parameter procedure is not shown in the present work, and the reader looking for more detail should refer to the work of Rathbun *et al.* [23].

The weight-to-load index relationships are shown for all core topologies of interest in Figure 2.6. To determine $(w_o)_{wall}$, a wall height of $h_w = 3.3$ m and a static angle of repose of 25° are used. For the cargo, the density of bulk coal ($\rho = 850$ kg/m³) multiplied by the gravitational constant is used to determine W . A failure strain of $\varepsilon_y = 0.007$ is employed to maintain consistency with the stiffened panel optimizations in sections prior. The expected load index for each panel based on the maximum bending moment and shear are shown by the vertical lines in Figure 2.6.

The semi-monocoque wall structure designed in Section 2.1.1 has been compared to the various sandwich panel topologies for each panel as well. In Figure 2.6, the asterisks denote the weight index for the semi-monocoque wall structure in bending at the corresponding load indices for each panel. The conversion from the compression and tension weight indices to the bending weight index is performed by first determining the mass of the optimum

Figure 2.6: Weight-to-Load Index of sandwich panels with varying core topologies on a logarithmic scale for Aluminum

structure according to the equation:

$$W_{ct} = (\Psi_t + \Psi_c) \rho L^3 \quad (2.25)$$

where W_{ct} is the weight of the semi-monocoque panel structure, and Ψ_t and Ψ_c are the weight indices of the tension and compression panel respectively. The weight index for bending is then given by the equation:

$$\Psi = \frac{W_i}{\rho L} \quad (2.26)$$

where W_i denotes the weight per unit area. By dividing W_{ct} by the area of the panel, $A = 2bL$, the weight index then becomes:

$$\Psi = \frac{(\Psi_t + \Psi_c) L}{2 \left(\frac{b}{L}\right) \ell} \quad (2.27)$$

Using the above equation, the corresponding weight indices are plotted in Figure 2.6 using the corresponding value of b/L determined for each panel.

From Figure 2.6, it is evident that hexagonal honeycomb sandwich panels are the superior choice of sandwich core topologies, although as observed in [23], the difference is not significant from the square honeycomb core.

2.2 Summary of the Theoretical Framework

In this chapter, a parametric study investigating a novel semi-monocoque wall structure is carried out. Results of the parametric study were compared against the study performed in Rathbun *et al.* in [23]. By performing the parametric optimization, the weight-to-load indices for each structural candidate have been compared and investigated for varying load. It has been determined that, for the expected range of loading, the hexagonal honeycomb sandwich panel is the structure of highest strength-to-weight ratio and is therefore the subject of study for the remainder of this thesis.

Chapter 3

Theoretical Framework: Numerical Study

In the previous chapter, a series of parametric studies were conducted, culminating in the decision to employ hexagonal honeycomb sandwich panels in the weight minimization of a hopper car. While these studies were able to ascertain the ideal candidate sub-structure of all those under consideration in the anticipated loading conditions, it is necessary to develop a numerical model to analyze the behaviour of a hopper car under the expected loading conditions with high-fidelity for a robust design. The *Finite Element Method (FEM)* is proposed for use in this thesis to model the loads on a hopper car and the ensuing stresses in the structure. Details of the development of a FEM for a hopper car have been laid out in this chapter. In addition to the traditional implicit methods of modeling through linear static structural analysis, an explicit model for *Fluid Structure Interaction (FSI)* is introduced. The explicit model is used to explore the interaction of cargo sloshing within the hopper car under an inertial load, and the equivalent static loads are calculated and extracted for use in the implicit model. After establishing the FEM of the hopper car, framework for an analytical approach and an MSDO approach for the numerical optimization of a hopper car wall is developed for later implementation in a multi-stage design optimization.

3.1 Requirements from the Manual of Standards and Recommended Practices

The *Association of American Railroads (AAR)* is an industry trade group representing the interests of the vast majority of major freight railroad companies in North America. In the design of hopper cars, the majority of guidelines are found in the AAR *Manual of Standards and Recommended Practices (MSRP)*, which highlight the agreed upon principles for regulation, manufacturing and design of freight rail vehicles. In the present work, the primary subject of consideration for the design optimization procedure has been *Section C, Part II – Design, Fabrication, and Construction of Freight Cars, M-1001* [61]. For the purpose of brevity, when the MSRP is referenced in this work it should be assumed that, unless otherwise stated, the subject of reference is the aforementioned section. From this section, four cases of import have been identified, along with a fifth novel case: a Fluid Structure Interaction (FSI). The details for each load case are as follows:

- **Live Vertical Loading (LVL):** The LVL is meant to test the structural integrity of the hopper car when fully laden. The load case comprises a floor loading with a pressure $P = Wh_w$ to simulate the cargo, a side wall lateral hydrostatic load according to Equation (2.1), and a roof pressure of 720 Pa along with a concentrated load of 1335 N on the middle of the roof. Finally, to simulate the inertial loads resulting from a sharp turn, a lateral acceleration of 0.3 g is considered [61]. All loads are multiplied by a safety factor of 1.25 per requirements of the MSRP.
- **Draft Tensile Load (DTL):** The draft tensile load simulates the hopper car under conditions where excessive pulling might occur. The same loads as the LVL are applied without the factor of safety, along with a tensile load of 2,805,660 N on the coupler [61].
- **End Compressive Load (ECL):** The compressive load test simulates a situation in which one hopper car crashes into another, resulting in excessive compressive loading on the underframe. Here, it is assumed that the hopper car is unladen and a 4,448,000 N compressive load distributed along the under frame is applied.

- **Modal Analysis (MA):** The MSRP mandates that damping be applied for natural flexible body modes with a frequency under 12 Hz [61]. A modal load case has been considered to evaluate and control the natural frequencies of the structure over the course of the design optimization.
- **Fluid Structure Interaction (FSI):** A fluid structure interaction employing a fluid as cargo is considered under a sloshing scenario in an attempt to emulate the LVL under a dynamic load. The entire structure and its contents have been excited by a 0.3g acceleration with a load factor of 1.5 over a time of 1.85 seconds to simulate a sharp turn, which causes the cargo to slosh into the side walls.

It should be noted that in the present work this list of load cases is not comprehensive as there are loads which must be considered on specific mechanical elements of the hopper car. The load cases surmised above, however, adequately characterize the of expected loading on the overall hopper car structure for the purposes of preliminary design of a car-body. Furthermore, other considerations such as potential global structural buckling, impact loading and fatigue have not been treated as factors for concern at the time of writing. While such matters are significant in the design of the hopper car, they require comprehensive optimization procedures on their own and thus have been excluded from this study. It is, however, worth noting that honeycomb sandwich composites have demonstrated high resistance to buckling [62] and impact loading [24, 69], therefore it remains largely feasible to employ metallic hexagonal honeycomb sandwich panels for the application of hopper cars.

3.2 Hopper Car Finite Element Model

Two finite element models have been developed to simulate the hopper car structure: the first employing *Explicit Analysis* to simulate FSI, and a second employing *Implicit Analysis* to simulate linear static structural and modal analyses. In this section, each model is established in preparation for optimization. Beginning with the explicit model, the FSI load case is considered. Then, a linear static implicit model is produced which accounts for

the LVL, DTL, ECL and MA in tandem with the dynamic loads of the FSI translated to equivalent static loads, which has culminated in a robust and high-fidelity FEM of a Hopper Car for optimization.

3.2.1 Explicit Finite Element Model

Explicit FEM deals with the simulation of time-dependent physics where the nodal acceleration is relevant [70]. Generally, the physics simulated where such time-dependent considerations are relevant are in situations of high strain rates, non-linear boundary conditions, or large non-linear displacement. For instance, explicit analysis is used frequently to simulate forming of metals, crash physics and bird strike on aircraft [71–73]. With the development of explicit analysis, there has been an introduction of FSI for consideration of fluid sloshing in tanks [70]. Although it is often the case that CFD is used for the modeling of fluids, if CFD is used for FSI, it is then necessary to create two different models employing very different physics and mesh requirements followed by a data transfer from each model to describe the interfaces. Furthermore, the greatest advantage provided in the use of explicit analysis for FSI is the capability to handle any form of fluid motion without selection of a solver. Through explicit analysis, these requirements are circumvented since the fluid can be represented using an *Arbitrary Lagrangian Eulerian (ALE)* model, which allows for relative motion of both nodes and elements to reduce distortion, or *Smoothed Particle Hydrodynamics (SPH)*, which is entirely mesh-free and is based on interactions between individual elements based on a kernel function [74, 75].

In the present work, only the SPH methodology has been considered, while ALE has been left for future study. From the background literature SPH is advantageous over ALE due to its mesh-free nature, which allows for less concern on the quality of the mesh during simulation. The drawback with SPH, however, is that in general a finer mesh is required for equivalent accuracy to ALE [75]. Xin *et al.* have SPH used to model the cargo of a hopper car in the context of a crash simulation and have demonstrated its effectiveness in modeling the cargo motion, but have not made conclusions on its ability to model its induction of stress in the structure [76]. In the coming sections, a novel FSI model employing explicit analysis is developed for consideration of sloshing.

A The Base Model

Due to confidentiality concerns, it was not possible to obtain an industry standard model for analysis. Instead, a basic model has been generated for demonstration of the methodology, and the implementation on a full industrial model has been left for designers. The finite element mesh is modeled in *Hypermesh* and solved using *RADIOSS* based on a simplified hopper car geometry employing 42,875 quadratic and triangular P1_SHELL elements. An average edge length of 5 cm employing the QEPH element formulation has been used, and a shell element thickness of 6.35 mm has been used based on the findings of Attar in [77]. Since a linear elastic material law (M1_ELAST) has been used, QEPH elements are recommended due to their superior element solve times while being able to handle both elastic and elasto-plastic material laws [70]. Quadratic (QUAD4N) shell elements are used wherever possible as per best FEM practices, however due to the complicated nature of the geometry, 1.26% of the model is comprised of triangular (TRIA3N) elements. To simulate the frame of the car, 930 P3_BEAM elements are used with $75 \times 75 \times 6.35$ mm square-shaped cross sections for simplicity. In addition, an undercarriage of 36 elements using an I-Beam geometry of $W200 \times 300 \times 0.635$ have been used, and are connected to the model using RBE3 elements to ensure no artificial stress concentrations are observed to due to single-node connections. For the cargo, 145,655 SPH elements are used to model the granular cargo sloshing in the car-body. For the material law, MLAW10 is used, which employs the Drucker-Prager failure criterion to model granular materials and rock-concrete. The Drucker-Prager yield criterion uses a modified von Mises failure criteria to incorporate the effects of pressure and large loading according to the equation [70, 78]:

$$F = J_2 - (A_0 + A_1 P + A_2 P^2) \quad (3.1)$$

where F is failure stress, J_2 is the second invariant of the deviatoric stress, and P is the pressure in the cargo. The material coefficients A_0 , A_1 , and A_2 are calculated according to the equations [70]:

$$A_0 = k^2 \quad (3.2a)$$

$$A_1 = 6k\alpha_{carg} \quad (3.2b)$$

$$A_2 = 9\alpha_{carg}^2 \quad (3.2c)$$

with k and α_{carg} given as:

$$k = \frac{6c_{carg} \cos(\phi)}{\sqrt{3}(3 - \sin(\phi))} \quad (3.3a)$$

$$\alpha_{carg} = \frac{2 \sin(\phi)}{\sqrt{3} * (3 - \sin(\phi))} \quad (3.3b)$$

where ϕ is the static angle of repose given as 25° , and c_{carg} is the cohesion parameter for the cargo. Fine tuning of these parameters for modeling of the cargo is non-trivial and requires empirical data and comprehensive experimentation. Since the focus of this thesis is to perform a multi-stage design optimization to minimize the weight of a hopper car, it is out of scope to perform testing at this time. The properties for steel, which have been determined according to the findings in [77], are shown in Table 3.1 along with the estimated parameters used to model the SPH cargo using the MLAW10 card in *Altair RADIOSS* [79].

Table 3.1: Material Parameters for the Steel and SPH Cargo

Material	Parameter	Value	Units
Steel	Elastic Modulus, E_s	210	GPa
	Poisson Ratio, ν_s	0.3	-
	Density, ρ_s	7800	kg/m ³
Cargo	Initial Density, ρ_{cargo}	850	kg/m ³
	Cargo Elasticity Parameter, E_{cargo}	26.813	MPa
	Poisson Ratio, ν_{cargo}	0.25	-
	First Material Parameter, A_0	3338.5×10^9	Pa ²
	Second Material Parameter, A_1	2075700	Pa
	Third Material Parameter, A_2	3338.5	-
	Bulk Unloading Modulus, B_{UNL}	144	GPa
	Static Angle of Repose, ϕ	25°	-
	Soil Cohesion Parameter, c_{carg}	1.5×10^6	Pa

The lateral 0.3g acceleration with a 1.25 safety factor has been applied to both the car structure and the cargo. The car has been constrained using **SPC** boundary conditions at locations where the car bogies would mate with the hopper car-body in the X and Z directions to prevent motion upwards and to the side. Normally, one node is constrained in the Y direction to prevent rigid body motion, however it will be seen in the next section where model symmetry is discussed that this is not necessary. Since the car-body is constrained at various regions, rigid body rotation is prevented, therefore all rotations remained unconstrained to prevent any excess rigidity. The base simulation took 129 hours to completion on a 4 core CPU using parallelization. Explicit analysis for a model of this size is computationally costly, and results generation would be too long to provide meaningful results in an optimization loop where the simulation might be expected to run hundreds of times. In the next subsection, simulation time reduction techniques employed in an effort to provide a less cumbersome model have been discussed.

B Computational Efficiency

Although explicit analysis can provide useful solutions for complicated dynamic problems, it often comes at a heavy computational cost. It is therefore necessary to deploy all available measures with regards to computational speed-up . The trivial solution in improving the simulation solve time is to enforce model symmetry. Although model symmetry does not affect the solve time for each element, it does reduce the number of nodal degrees of freedom by half, thereby offering a drastic improvement for no cost in solution accuracy. Model symmetry is only enforceable along the plane cutting across the length of the hopper car in the center, where **SPC** boundary conditions restricting translational motion in the Y direction and rotation in the X and Z directions have been applied to prevent motion of the hopper car in the longitudinal direction, and to prevent rotation about directions lying on the cutting plane. Since the longitudinal rigid body motion is constrained at the plane of symmetry, it prevents the final rigid body motion in that direction and so the singularly constrained node preventing rigid body motion is not necessary. Solution time of the half model is 93 hours on a 4 core CPU.

In explicit analysis future states of a system are calculated based on the current state,

and so the key factor in determination of solve time is the *time step* of the simulation. The time step is how far of a period forward these future states can be predicted. The physical explanation for this limitation is based on how fast information can travel through each element, which in turn is dictated by the size of the elements and the speed of sound within them. To explain this concept, it is convenient to envision a beam with an applied vibration at one end; it is not possible to determine the state of vibration at the other end of the beam prior to the arrival of the shockwave resulting from the vibration traveling at the speed of sound. The critical time step is the smallest possible time step such that a future state is never calculated further ahead than the time it takes for information to reach that location, and is therefore given by the equation:

$$\Delta t = \Delta t_{sca} \frac{l_c}{c} \quad (3.4)$$

where Δt is the time step, Δt_{sca} is a scaling factor and l_c is the critical element length. A scaling factor of $\Delta t_{sca} = 0.9$ has been used to ensure that the time step is conservatively calculated. The speed of sound within the element, c , is given by:

$$c = \sqrt{\frac{E}{\rho}} \quad (3.5)$$

It is important to note that the critical element is the smallest element edge length, meaning that in any model in which there is a refined region, that region will slow the rest of the simulation. A further consideration is that high element distortions can shrink critical element lengths, which in turn exponentially reduces the time step and drives the simulation time to infinity. It is therefore necessary to reduce high element distortions where possible by accurately modeling element failure when it is expected, or to mitigate errors in the simulation which can produce unrealistic deformations. In the presented simulation it is a feature that in order to prevent initial penetrations between SPH elements and their interfacing with the Lagrangian element structure, the SPH elements are initially suspended in the air. Considering that downwards gravity has been applied in the model, these suspended elements are simultaneously dropped on the structure over the course of the first 0.9 seconds of the simulation. This produced an unrealistic load case for the hopper car;

the total mass of the cargo in the half model is 58,879.84 kg, and the result of it falling on the structure results in heavy element distortion, which in turn drastically reduces the time step for this settling period. For this reason, it is necessary to use the /KEREL card which dampens kinetic energy of the system over the first 0.9 seconds and allowing the particles to drop slowly and settle at near zero velocity. By allowing settling of the fluid and dampening their unrealistic impact on the structure, solve time is increased. This phase of the simulation is referred to in this thesis as the *settling phase*, and is largely ignored due to its inability to produce meaningful results.

The relevant portion of the simulation takes place from a time of 0.9 seconds onward, and is referred to as the *sloshing phase*, which is the time at which all particles had settled. At this point, the 0.3g lateral acceleration with a 1.5 safety factor has been applied in the lateral direction of the hopper car and the /KEREL function is terminated so that the particle motion is no longer interrupted. Unfortunately, the simulation time in running this analysis was found to still be lengthy and so it was necessary to consider direct methods of improving the time step.

Due to the costly nature of explicit analysis, it is often convenient to trade accuracy of a simulation for faster solve time. Although this can be accomplished through the use of larger elements, it is recommended in explicit that element size not exceed 5-10 mm in the regions of interest, as coarse elements can result in artificial rigidity and low solution accuracy [70]. Instead, it is possible to enforce a higher time step by imposing nodal time step control, for which time steps are calculated according to the equation:

$$\Delta t_{nodal} = \Delta t_{sca} \sqrt{\frac{2m}{k}} \quad (3.6)$$

where m is the nodal mass and k is the corresponding stiffness, and the node which produces the lowest time step using these values is the relevant time step for the simulation. By enforcing nodal time step control, it is then possible to apply *mass scaling*, where mass is artificially added to the nodes resulting in an increased overall time step at the cost of an introduction of artificial translational kinetic energy. Through proper control of the amount of mass error introduced, it is possible to increase the time step by an order of

Figure 3.1: Mass scaling implementation decision tree.

magnitude without resulting in major loss of accuracy or introduction of excessive energy. An incremental sensitivity study into the mass error introduced by imposing a higher time step has been conducted according to the process flow diagram in Figure 3.1. A higher time step has been incrementally imposed until the introduced mass error was so high that it had compromised result accuracy.

A scaling factor of $\Delta t_{sca} = 0.9$ is used for the imposed time step, as per software recommendations, in order to guarantee simulation stability by reducing the imposed time step [70]. An initial time step of $\Delta t = 2.017 \times 10^{-6}$ is enforced. The simulation was run for the first few time steps whereupon the mass error was found to be only $\frac{dM}{M} = 0.0013 =$

Figure 3.2: Time history plot of mass scaling error.

0.13%. Given that $\frac{dM}{M} < 0.02$, the next imposed time step is multiplied by a factor of 1.2. This process was repeated until a time step of $\Delta t = 4.2 \times 10^{-6}$ was achieved. At this point, it was found that the mass error began at $\frac{dM}{M} = 0.05874$ and was found to increase rapidly in the first 0.5 seconds of simulation, then finally stabilizing to a slower increase as can be seen in Figure 3.2. At 1.85 seconds of in-simulation time, it was found that the mass error converged to 1.62% and, a further factoring of the time step by a value of 1.2 eventually yielded greater than 2.0% mass error. Consequently, the time step of $\Delta t = 4.2 \times 10^{-6}$ is deemed to be the critical mass scaling time step and is used for the final analysis. Recommendations from the *RADIOSS* user manual for mass scaling cite a mass error of no more than 2% [70], however engineering judgment has been exercised in this thesis to ensure accuracy is not affected significantly. An introduction of a mass error of $\frac{dM}{M} = 0.02$ on the cargo weighing 58,879.84 kg and an initial car structure of 6827.88 kg would introduce only 1314.15 kg of mass error. This mass error is equivalent to 1.54m^3 of coal as cargo, which is a statistically insignificant introduction of mass.

It is apparent from Figure 3.3 that in the first 0.5 seconds of the simulation, kinetic energy rapidly spiked to a value an order of magnitude higher than expected as a result of the particles dropping from their initial positions suspended in the air. The kinetic energy relaxation then suspended their motion, resetting the kinetic energy to a near-zero value and repeatedly did so such that the SPH particles did not impact the structure at

Figure 3.3: Time history plot of simulation energy.

high velocity. Note that internal strain energy remained at a value of zero until after the kinetic energy relaxation had been applied. In the sloshing phase up to 0.9 seconds, it is shown that the total and internal energies are effectively equal, implying that the majority of the energy in the system has been translated into strain energy in the structure, while only a fraction has been converted to kinetic energy and implying that the system had settled. This remained true through the sloshing phase, meaning that the majority of the energy worked to deform the structure. A final note of importance is that Hourglass energy remained virtually zero, implying that the elements in the structure did not undergo artificial hourglass deformation. The final simulation run time is 36 hours on a 4 core CPU, which is significantly more manageable than the original run time of 129 hours.

C The Equivalent Static Loads Method and its Application

Despite the success in producing an explicit model for FSI with solve time 72% faster than the baseline, the simulation time was still 36 hours and could not be readily or meaningfully integrated into any form of iterative design loop. To solve this problem, an equivalent static model is proposed. Optimization in explicit analysis is usually performed according to the *Equivalent Static Loads Method of Optimization (ESLMO)* [80, 81]. For instance, the ESLMO has been used for the optimal design of a crash box for an auto-mobile, an aircraft wing undergoing coupled dynamic bending and twist loading, and in component for

a robot undergoing dynamic motion [82–84]. The ESLMO uses the dynamic displacements and accelerations of the time dependent explicit model in the following equation:

$$\mathbf{F}_{eq} = a_d (\mathbf{M}\ddot{\mathbf{x}}_d + \mathbf{K}\mathbf{d}_a) \quad (3.7)$$

where $\mathbf{F}_{eq}$ is the equivalent static load vector for each nodal, $\mathbf{M}$ and $\mathbf{K}$ are the mass and stiffness matrices respectively, and $\ddot{\mathbf{x}}_d$ and $\mathbf{d}_a$ are the nodal displacements and accelerations experienced by the structure respectively. Here, a dynamic loading factor, $a_d = 1.25$, has been implemented.

The necessity of the ESLM lies in the concern that dynamic loading on a structure is erratic and variable, and therefore all time steps must be considered. It is theorized that, in the case of sloshing in a hopper car, the dynamic loads are not erratic as the previously cited cases – one or two critical load cases can be presumed to be of importance in characterizing the equivalent static loads. In sloshing, the loading occurs over a relatively long period of time of 1.85 seconds and is applied slower than the case of a crash or gust. This is made evident by Figure 3.4, which shows the time history plot for the total contact interface on the hopper car. Over the course of the entire FSI, the loading remained relatively constant on the structure, save for a small amount of fluctuation of no more $\pm 50,000N$. This fluctuation occurred because no damping is applied; upon SPH collision with the wall, the bending mode of the wall is excited thereby producing a small amount of vibration. Due to the vibration, the contact force, which is calculated based on proximity of the contact interfaces, is subsequently removed and reapplied to the wall, creating an unrealistic fluctuation in loading. This problem can be resolved by applying appropriate damping on the structure, however it is non-trivial to determine a realistic damping ratio for Rayleigh damping without empirical data. The most conservative critical time steps in the fluctuation are taken to be the true loads for consideration at $T = 1.35$ s, and $T = 1.5$ s. Finally, by applying an additional dynamics safety factor of $a_d = 1.25$ the interface forces for all times in the sloshing portion of the analysis predict a higher load than all times without any mass scaling. This indicates any time step in the sloshing phase of the mass scaled simulation with the applied safety likely provides a conservative load case for an equivalent static

Figure 3.4: Time history plot of total interface contact force.

model. Knowing that a conservative load has been identified on the hopper car for FSI, the maximum accelerations and displacements for each node at times $T = 1.35$ s and $T = 1.5$ s in the sloshing phase are extracted and applied in Equation (3.7) to produce a maximum equivalent static force for each node. These nodes are then used in the implicit linear static structural FEM, which has been discussed in the next section.

3.2.2 Implicit Finite Element Model

As previously explained, the explicit model employing FSI was found to be costly in computational time, therefore it is not possible to integrate it directly into a design optimization loop. The ESLM is applied using Equation (3.7) and the maximum nodal accelerations and displacements on the structure to produce an equivalent static load case according to the critical time steps. For the FSI load case, the model has been constrained exactly as the explicit model was, and the ESL are translated using a *MATLAB* script to process the nodal motion into static forces and moments [85]. In addition to the FSI load case, four other load cases have been implemented into the model according to Section 3.1.

Maximum solve efficiency is required in order to obtain results in a reasonable time frame from a design loop which directly incorporates FEM into the design procedure. It was observed in the explicit model that in the case of sloshing, only one side of the car – the wall resisting the sloshing of the fluid – experienced significant loading. Through

Figure 3.5: FEM of the hopper car equivalent static model.

this observation, it has been determined that a quarter model is feasible, as one side can accurately represent the design requirements for both sides of the car. The quarter model is produced by employing SPC boundary conditions in the 1-5-6 directions on the nodes intersecting the plane of symmetry cutting directly through the longitudinal center of the car. In the explicit model, elements of 5 cm in size were used due to the requirement of highly refined elements. In the static model, however, such refinement is not required in many locations of low stress such as on the walls closest to the middle of the car, and so the mesh has been coarsened in these areas to further reduce simulation time. The number of elements was reduced to 14,257 CQUAD4 quadratic and CTRIA3 triangular shell elements in total, while the number of beam elements (CBEAM) remained the same. Element transitions are used appropriately such that no element had a Jacobian of less than 0.5, and only 4.1% of elements had a Jacobian of less than 0.7 thereby implying that not major computational error has been introduced. The mesh of the implicit model is depicted in Figure 3.5.

A Super-element Method

In the optimization of large structures such as a hopper car, it is advantageous to reduce design variables as much as possible while maintaining a thorough enough search for a robust solution. As discussed in Chapter 2, a honeycomb sandwich panel has four design variables of importance. Should these four design variables be employed to describe every hopper car wall simultaneously, the results might be easily and rapidly attained for the whole structure and produce a highly manufacturable result due to simplicity, however they would also be far from the global optimum as well. At the same time, should the structure be broken into smaller components, the results might be closer to the global optimum, however the results would not be manufacturable and by extension effectively useless. A reasonable solution to this problem is to divide the structure into smaller parts which adequately provide a robust problem while mitigating concerns for manufacturability. In the present work, it is out of scope to investigate in detail the manufacturability and cost trade-off between smaller versus larger panel sizes. The hopper car has instead been divided into 5 side panels, 5 roof panels, and 10 floor panels of varying sizes such that it is not unreasonable to assume they are manufacturable while also providing a through set of design variables.

The division of the hopper car into smaller components has introduced a further complication. With a total of 19 panels to account for, each with four design parameters, the number of parameters has increased to a total of 76 not including any of those which might be associated with the beams. While simultaneously accounting for all design variables is advantageous in obtaining a closer-to-global optimum, it is unlikely to be a successful endeavour as convergence is expected to be poor and computational cost high. The super elements method is proposed as a solution; the hopper car is divided into smaller sections which are optimized separately. This subdivision is done by using the *Guyan Reduction* to generate a *Direct Matrix Input at Grid points* (DMIG) file [86]. By using a DMIG, the majority of the model is converted into a reduced stiffness matrix, and is associated to the detailed sub-model with ASPC enforced displacement boundary conditions along the interface. By using the Super-Element method, the number of *Degrees of Freedom (DOF)* have been reduced to those present in the sub-model, thereby condensing solve times.

Some challenges are posed by the Super-Element method as defined here. First, selection of the nodal interface set which couples the sub-model to the DMIG, known as the ASET, must be as small as possible for maximum solution condensation. Second, if the ASET lies in regions of the model which undergo large displacement the ASET will lose validity if rigidity is changed. This occurs because at the nodes where the ASET is invoked, displacements are enforced at the coupling interface. This in turn implies that by increasing rigidity near the ASET, stress will increase as well, which is contrary to reality. Therefore, a compromise must be found by selecting an ASET which is relatively distant from the panel of interest such that the enforced displacements do not invalidate the due to changing rigidity, while also selecting the set such that a maximum number of DOF are reduced. This selection is expected to vary depending on the displacement field experienced by the hopper car, and therefore is performed in Section 4.3.1 after an optimized global model has been established.

3.2.3 Homogenization of Sandwich Composites - Verification and Validation

Since hexagonal honeycomb sandwich panels have been proposed for use in the present work, it will eventually be necessary to perform numerical modeling of the core. modeling of the cores of sandwich panels, however, can be an incredibly taxing process depending on core topology and can result in highly refined finite element meshes with long solve times. To circumvent this issue, homogenization of core properties is proposed such that the sandwich panels might be modeled as composite elements with isotropic faces and an orthotropic and homogenized core. Homogenization of sandwich panel cores has been successfully implemented in various work for decades, however many of these studies have been performed for polymer sandwich panels [87–89]. Here, the validity of homogenization techniques from Burton and Noor in [90] for the purposes of modeling metallic hexagonal honeycomb panels in FEM are investigated. The resultant deflection from the homogenized FEM is then compared according to the theory given by Allen [34].

For a simply supported sandwich beam with an evenly distributed load, the deflection is given as:

$$\delta = \frac{5qL^4}{384D} \quad (3.8)$$

where q is the distributed load, L is the length of the panel, and D is the flexural rigidity as defined in Equation 1.3. With the theoretical benchmark determined for deflection, it is then possible to proceed with numerical modeling. For core homogenization, the equations in [90] are used with cell sizes of $s = 3.175$ mm and $s = 6.35$ mm. The cell foil thickness is set to be $t_c = 0.508$ mm, and the cell wall size is $L_c = s(\sin(\pi/3))$. The properties of steel are used so that the Young's Modulus $E_f = 210$ GPa, the Poisson Ratio is $\nu_f = 0.3$, the density is $\rho_f = 7850$ kg/m³, and the shear modulus is given by the relationship:

$$G_s = \frac{E_f}{2(1 + \nu_f)} \quad (3.9)$$

Finite Element modeling has been performed using *Altair OptiStruct* [91]. The simply supported sandwich panel has been modelled using 2415 first order PCOMPP quadratic elements with steel face sheets as the top and bottom plies, while the homogenized core has been modeled using MAT8 to simulate an orthotropic material in the middle ply. SPC constraints are applied in the 1, 2 and 3 directions at one end and the 1 and 3 directions at the other. Finally, a pressure load of $P = q/w$ is applied over one face of the panel, where w is the width of the panel, so that the total applied load is equivalent to that on the theoretical simply supported panel. For all analyses, $w = 103.8$ mm, $L = 575.3$ mm, $t_f = 1.27$ mm and $H_c = 6.35$ mm. Results of the simulation are shown in Table 3.2, while the homogenized core properties are shown in Table 3.3. Both cell sizes have been tested with a range of loads to ensure validity in general. Results from the 6.35 mm cell with a load of 1000 N/m is shown in Figure 3.6.

From the results shown, it is evident that homogenized core properties provide an accurate representation of panel behaviour under loading according to the accepted sandwich theory. As a further conclusion, the results show a clear indication that core density and stiffness have no effect of significance on the rigidity of the panel, just as predicted by theory. Any differences between theory and simulation can be attributed to the lack of consideration of shear stiffness of the core, the fact that the panel is modeled in two dimensions

Table 3.2: Comparison of theoretical deflection to homogenized FEM panels.

Cell Size	Load	Theoretical Deflection	FEM Deflection	Error %
[mm]	[N/m]	[mm]	[mm]	
1/8	1000	1.78	1.78	0.0
	1500	2.66	2.64	-1.2
	2000	3.55	3.52	-0.8
	2500	4.44	4.40	-0.9
1/4	1000	1.78	1.76	1.1
	1500	2.66	2.67	0.4
	2000	3.55	3.57	0.5
	2500	4.44	4.46	0.4

as opposed to one as was done in the theoretical model, or simply due to compounding of rounding errors.

Table 3.3: Homogenized core properties for use in OptiStruct simulation using **MAT8**.

Cell Size	ρ_c	$\mathbf{E}_{1c}$	$\mathbf{E}_{2c}$	ν_{12c}	$\mathbf{G}_{12c}$	$\mathbf{G}_{13c}$	$\mathbf{G}_{23c}$
[mm]	[kg/m ³]	[MPa]	[MPa]	[-]	[MPa]	[MPa]	[MPa]
3.175	2219	2905	2863	0.44	1835	5170	8615
6.35	1109	377.3	375.9	0.44	229.4	2585	4308

3.2.4 Model Summary

An explicit model for FSI has been developed to examine the interaction between a hopper car and its fluid contents during a lateral acceleration case in which the contents slosh and push onto the side walls. The displacement contour plot of the interaction at the time of highest interface loading ($t = 1.5s$) is shown in Figure 3.7a, while the stress contour plot for the ESL using the maximum nodal accelerations and displacements in conjunction with Equation 3.7 is shown in Figure 3.7b. The maximum displacement in the FSI model is 2.589 cm, while in the ESM it is 3.594 cm, showing that the ESM over-predicted the maximum displacement by a factor of 1.38 which is approximate to the applied dynamic load factor of $a_d = 1.25$, consequently indicating the generation of the ESM is successful. It is clear from these results that the ESL provided a conservative load case in the equivalent static model and can therefore be used to represent the FSI in implicit analysis.

Figure 3.6: Deflection of a sandwich panel with a homogenize 6.35 mm hex cell core and a distributed load of 1000 N/m.

By developing a model employing FSI and validating that it can provide a conservative loading estimate, it will be possible in future work to employ the model to study load cases beyond those available in the MSRP for safe measure. For instance, it might be possible to investigate a crash situation wherein and evaluate its performance using other materials. For the current line of study, however, the FSI has been used solely in providing a safe estimate of loading on the hopper car for the optimization procedures to come. Now that a FEM has been developed, a deeper understanding of the structure is available and it is now possible to discuss how such a structure might be structurally optimized. In the next section, several numerical optimization methods have been discussed for such an end.

3.3 Numerical Size Optimization Methods

In Chapter 2, a parametric methodology based on the literature from Rathbun *et. al* was developed to investigate the feasibility of sandwich panels under bending loading [23]. The drawback of the aforementioned method is its inability to simultaneously optimize all four relevant sandwich dimensions simultaneously due to its parametric approach. Now that

(a) The displacement contour plot car at $t = 1.5s$, where interface loads are observed to be the highest.

(b) The displacement contour plot for the equivalent static model based on the maximum nodal accelerations and displacements.

Figure 3.7: Comparison of explicit fluid-structure interaction displacement contour plot to that of the displacement plot of the equivalent static model under implicit loading.

a numerical model has been developed, it is possible to discuss in further detail how size optimization might be used in the multiscale design optimization of a hopper car.

Selection of the appropriate numerical optimization algorithm is a critical task in the design optimization of any system. An incorrect algorithm selection can produce inaccurate results, poor convergence to optimality, or long solve times. In general, optimization algorithms can be divided into two major groups: *Gradient Descent Algorithms* and *Stochastic Algorithms*. Gradient Descent Algorithms are the preferred method of optimization; however their implementation requires that the cost function under consideration is smooth and continuous such that a Gradient and Hessian can be calculated for all points in the design space. When it is not possible to model the system as smooth and continuous, it is then necessary to use Stochastic methods for optimization, wherein smoothness and continuity are not a necessity. Another advantage of stochastic algorithms is their ability to consider multiple objectives with ease through the generation of a *Pareto Front*, which plots a curve relating the two competing objectives to one another. In this section, two approaches for numerical optimization will be introduced: an analytical approach for rapid generation of results and an MSDO employing FEM directly in the design loop.

3.3.1 Analytical Approach: Gradient Descent Algorithm

Gradient descent algorithms are optimization algorithms which leverage the gradient and Hessian of a cost function to obtain results. Therefore, as previously mentioned, Gradient Descent Algorithms require a smooth and continuous cost function. Consequentially, it is necessary to attempt to simplify the problem as much as possible such that the problem can have a solvable gradient and Hessian. The drawback of such a methodology is that it can severely limit the potential use of cost functions of higher fidelity, as more often than not comprehensive structural engineering problems cannot be idealized as smooth nor continuous [92]. It is therefore necessary to determine in the search for an algorithm whether such a simplification provides close to agreeable results which meet all the constraints. To develop an appropriate cost function, the following assumptions have been made:

- The hopper car wall is assumed simply supported at all edges and does not have any

curvature in order to simplify calculations. It is also assumed that the panels are loaded in bending or compression only.

- If a sandwich panel can be designed with a superior rigidity than a plate of equivalent material and a baseline thickness, it follows that the panel likely outperforms the plate.
- The dimensions from the FEM can be used to approximate wall panel dimensions in the theoretical model, and the radius of curvature is large enough that the wall can be considered nearly flat.

A Defining the Cost Function

The first key component of a size optimization problem is an objective or cost function relating the design variables. Here, the objective is to reduce the mass of a sandwich panel for implementation in a hopper car wall. The mass of a sandwich panel is given by the function:

$$m(t_f, H_c) = (2\rho_f t_f + \rho_c H_c) wL \quad (3.10)$$

where the design variables are the same as those from Equation 2.18. The density of the core is given by Burton and Noor in [90] as:

$$\rho_c = \rho_f \left(\frac{2t_c}{L_c(\eta + \cos(\theta)) \sin(\theta)} \right) \quad (3.11)$$

where η is the ratio between the cell wall widths given as $\eta = 2L_w/L_c$, and θ is the angle of the between them as shown in Figure 3.8. Since bidirectional strength is preferred, $L_w = 0.5L_c$ so that the wall width ratio $\eta = 1$ and the angle between them is set to $\theta = 60^\circ$. It is important to note that [90] gives all relevant core properties as a function of the four design variables under study here. Substituting Equation 3.11 into Equation 3.10 along with these constraints, a mass function for all design variables that is independent of global wall dimensions and material properties has been developed for use in an optimization problem:

(a) Hexagonal honeycomb sandwich core cell. (b) Hexagonal honeycomb core overall layout.

Figure 3.8: Hexagonal honeycomb core design variables.

$$m(L_c, t_c, t_f, H_c) = \left(\frac{2t_c}{1.5 \left(\frac{\sqrt{3}}{2} \right) L_c} \right) H_c + 2t_f \quad (3.12)$$

B Defining Constraints

With a mass function and a design space defined, all that remains is to define any relevant constraints. First, lower and upper bounds have been defined for each design variable according to Table 3.4. It is also advantageous to ensure that constraints abiding by Equations 1.2a and 1.2b are enforced. Finally, the sandwich panel has been compared to the baseline which it is attempting to replace: a 6.35 mm steel plate with the properties in Table 3.1. The rigidity of a plate is defined as:

$$D_p = \frac{Et_p^3}{12(1 - \nu^2)} \quad (\text{Rigidity}) \quad (3.13)$$

where t_p is the plate thickness. To ensure a superior rigidity, a constraint function has been defined as:

$$c(L_c, t_c, t_f, H_c) = D - D_p \leq 0 \quad (3.14)$$

Now that all constraints have been identified, the final optimization problem can be written as:

$$\begin{aligned}
\min_{\mathbf{x}} \quad & m(\mathbf{x}) = \left(\frac{2x_2}{1.5 \left(\frac{\sqrt{3}}{2} \right) x_1} \right) x_4 + 2x_3 \\
\text{s.t.} \quad & D(\mathbf{x}) - D_p \leq 0 \\
& 5.77 - \frac{x_3 + x_4}{x_3} \leq 0 \\
& 16.7 - \frac{E_f}{E_c(\mathbf{x})} \frac{x_3}{x_4} \left(\frac{x_4 + x_3}{x_4} \right)^2 \leq 0
\end{aligned} \tag{3.15}$$

where $\mathbf{x}_{LB} \leq \mathbf{x} \leq \mathbf{x}_{UB}$

and $\mathbf{x} = [L_c, t_c, t_f, H_c]$

A *MATLAB* script employing *Sequential Quadratic Programming (SQP)* through the `fmincon` function is used to solve the problem [85]. Since SQP is only capable of local optimum searches, a multi-start solution has been implemented with 20 pseudo-randomly selected starting points. The solution converged rapidly to the point $\mathbf{x} = [9.525, 0.0178, 0.3175, 3.9257]$ for the final variables. It is apparent that in seeking to minimize mass the algorithm approached the upper bound for L_c and the lower bounds for t_c, t_f and H_c , which indicated that the solution might not be adequately constrained. While it might certainly have been possible that the proposed optimal design might be adequate, a more rigorous approach has been taken for an analytical optimization. In the next section, an approach using a multi-objective optimization employing stochastic algorithms is discussed.

Table 3.4: Upper and Lower Bounds for Design Variables L_c , t_c , t_f , and H_c .

Design Variable	Lower Bound	Upper Bound
[-]	[mm]	[mm]
Core wall width, L_c	3.175	9.525
Core foil thickness, t_c	0.01778	1.27
Face Thickness, t_f	0.3175	6.35
Core Thickness, H_c	3.175	25.4

3.3.2 Stochastic Algorithms

Because they are often costlier in solve time, stochastic algorithms are best avoided in favour of gradient descent algorithms. However, it is seen in Section 3.3.1 that a gradient descent is insufficient in providing a certain solution. In this section, options for stochastic optimization methodologies are discussed as potential candidates as part of a larger optimization process. These algorithms are capable of isolating global minima without the use of a gradient, and are therefore capable of performing optimization on non-deterministic functions where the design variables cannot easily or possibly be related to the cost function [93]. Another feature of stochastic methods is their ability to perform a global search in the design space, thereby avoiding local optima. Finally, stochastic approaches are also capable of multi-objective optimization, which allows for the generation of Pareto fronts to determine ideal solutions satisfying multiple objectives. At the same time, due to the pseudo-random approach used by the stochastic approach, a great deal of function evaluations are required, resulting in long solve times to convergence. By understanding the nature of the design problem well, it is possible to bypass this complication by guiding the algorithm using near-optimum seed points. In this section, implementations of different algorithms in the context of hopper car structural optimization, and combinations thereof, have been discussed.

A Analytical Approach: Multi-Objective Optimization

A major shortcoming of the previously used gradient descent method was the inability to account for and optimize multiple different aspects of the sandwich panel requirements, and as a result it was not immediately clear whether sandwich strength is adequate. Optimizing for mass alone resulted a tendency towards the bounds of the design space and there is reason to believe the results will be inadmissible. Due to these concerns, a more rigorous approach is proposed to ensure the true accuracy of GDA. As seen from the FEM, the hopper car loads are difficult to characterize into simple theoretical loading conditions on the panels individually and by extension making it difficult to calculate any meaningfully accurate stresses to test against the loading criteria for constraint applications. Instead,

creation of a stress function which accounts for the various failure modes of the panel has been proposed. From [66] and [94], failure modes of a honeycomb sandwich panel assuming equal biaxial loading are written as:

$$\sigma_{fY} = \sigma_Y \quad (\text{Face Yielding}) \quad (3.16a)$$

$$\sigma_{icb} = \frac{E_f}{1 - \nu_f^2} \left(\frac{t_f}{s} \right)^2 \quad (\text{Intracellular Buckling}) \quad (3.16b)$$

$$\sigma_{fw} = 0.33 \left(\frac{E_c t_f}{E_f H_c} \right)^{\frac{1}{2}} E_f \quad (\text{Face Wrinkling – Preliminary Designs}) \quad (3.16c)$$

$$\sigma_{cr} = \frac{(H_c + t_f)^2}{2t_f H_c} \sqrt{G_{c,xz} G_{c,yz}} \quad (\text{Shear Crimping – Compressive}) \quad (3.16d)$$

$$\tau_{cy} = 1.7 E_s \left(\frac{\rho_c}{\rho_f} \right)^3 \quad (\text{Shear Crimping – Shear}) \quad (3.16e)$$

where $s = L_c / \sin(\theta)$ and subscripts c and f denote core and face properties respectively while subscripts x , y , and z denote the two in-plane directions and the out-of-plane directions, respectively. Note that due to biaxial loading, the factor of 2 has been divided out from Equation 3.16b since it is necessary that:

$$\frac{\sigma_x}{\sigma_{icb}} + \frac{\sigma_y}{\sigma_{icb}} = 1 \quad (3.17)$$

With these failure modes defined, it is now possible to define a stress function as the sum of all yield stresses as:

$$\sigma(L_c, t_c, t_f, H_c) = \sigma_{fY} + \min(\sigma_{icb}, \sigma_{fw}, \sigma_{cr}) + \tau_{cy} \quad (3.18)$$

In the stress equation, σ_{fY} is constant, which makes its sensitivity to the function effectively zero. This can cause problems since face yielding is a relevant failure mode. To circumvent this issue, the whole equation is multiplied by the effective thickness of the sandwich panel under the assumption that the loading in the plane of failure is constant throughout:

$$F(L_c, t_c, t_f, H_c) = (\sigma_{fY} + \min(\sigma_{icb}, \sigma_{fw}, \sigma_{cr}) + \tau_{cy}) (t_f + H_c) \quad (3.19)$$

Since all stresses are analyzed under the same cross-sectional area, the multiplication of Equation 3.18 to get Equation 3.19 will only add sensitivity to σ_{fY} while still maintaining viability as a function to accurately represent failure limits. Now it is possible to construct a multi-objective optimization problem where the mass function, Equation 3.12, and the failure load function, Equation 3.19, are simultaneously optimized:

$$\begin{aligned}
& \min_{\mathbf{x}} && (m(\mathbf{x}), F^{-1}(\mathbf{x})) \\
& \text{s.t.} && D(\mathbf{x}) - D_p \leq 0 \\
& && 5.77 - \frac{x_3 + x_4}{x_3} \leq 0 \\
& && 16.7 - \frac{E_f}{E_c(\mathbf{x})} \frac{x_3}{x_4} \left(\frac{x_4 + x_3}{x_4} \right)^2 \leq 0
\end{aligned} \tag{3.20}$$

where $\mathbf{x}_{LB} \leq \mathbf{x} \leq \mathbf{x}_{UB}$

and $\mathbf{x} = [L_c, t_c, t_f, H_c]$

Here, the failure load function has been inverted so that its minimization results in a superior value.

Although solve times are fast for the analytical solution, it is advantageous to reinforce the procedure through the implementation of close-to-optimum starting points. The SQP methodology used prior is implemented once more to individually determine the optimal point for Equations 3.19 and 3.12. The individual optimal points are then used as seeding points for the `paretosearch` function in *MATLAB*, which uses the mesh-based *Pattern Search (PS)* local optimization algorithm to produce the Pareto front shown in Figure 3.9. The optimization procedure is then repeated using the `gamultiobj` algorithm, which produced a Pareto front using a *Genetic Algorithm (GA)* as the optimizer. The Pareto fronts are further refined using the `fgoalattain` algorithm, which optimizes an objective in an effort to find a better solution than a weighted “goal” baseline. Here, the goal is set as the optimal point from the corresponding solver such that `fgoalattain` produces a better or equivalent resulting cost.

Despite the GA’s ability to perform a global search, the PS algorithm proved to be a more effective tool when given close-to-optimum starting points as the Pareto front discovered by

Figure 3.9: Pareto fronts for the optimization problem in Equation 3.20 obtained using **paretosearch** and **gamultiobj** with and without supplementation of **fgoalattain** to refine the solution. Theoretical optimum is plotted for reference based on the solutions from individually optimized functions.

the PS based algorithms produced a smoother front with lesser objective values. To select the optimal solutions from each front, the theoretical optimum, also known as a *Utopia Point*, has been leveraged. A Utopia point is defined as [95]:

$$\mathbf{F}^\circ \iff F_i^\circ = \min \{F_i(\mathbf{x}) | \mathbf{x} \in \mathbb{X}\} \forall i \in \{1, 2, \dots, k\} \quad (3.21)$$

where $\mathbf{F}^\circ$ denotes the Utopia point, F_i° denotes the i^{th} of k points in the Pareto front with corresponding $\mathbf{x}$ in the Pareto design space $\mathbb{X}$. Here, the Utopia point is defined based on the optimal function evaluations for each individual cost function when optimized exclusively. The Utopia point, in general, is not attainable as a multi-objective optimization implies a trade between objectives by nature. Therefore, the solution has been defined as the point

on each Pareto front which is closest to the Utopia Point. The points nearest to the Utopia for each search algorithm are recorded in Table 3.5, along with their associated costs for each function, and the total number of function evaluations required for the Pareto front generation as a metric to describe the efficiency of each algorithm.

For this application, it is clear that the PS algorithm is the superior methodology. Despite the rigorous search performed by employing GA, the PS produced the results closes to the Utopia. This is attributable to the fact that seed points have been given using the SQP search algorithm, which allowed for significant refinement of the search space and consequently leading to a more concentrated search within the locally optimal design space. Despite the superiority of the PS in finding a solution, all algorithms appeared to converge to approximately the same points in the design space. In Chapter 5, it shall be determined if the analytical approach is rigorous enough in the context of the hopper car FEM by plugging each result into the sub-model to determine its suitability.

B Finite Element Integrated Multiscale Design Optimization

So far, all routines for the design optimization of a hopper car have been parametric or analytical in nature, and not representative of reality with certainty. While the parametric approach has provided useful design intuition and the analytical numerical approach has provided valuable starting design variables, these approaches are not comprehensive. To refine the solutions found by a parametric and analytical approach, a *Multiscale Design*

Table 3.5: Best points from each Pareto front in Figure 3.9 for each search algorithm.

Search Algorithm	L_c	t_c	t_f	H_c	$m(x)$	$F^{-1}(x)$	Function Evaluations
	$[m \times 10^3]$				$[m \times 10^3]$	$[m/N \times 10^8]$	
Pattern Search	6.6477	0.5789	0.3175	7.9015	1.6944	1.1162	2,050
Genetic Algorithm	7.7038	0.6100	0.3278	10.0285	1.8782	0.9051	41,281
Pattern Search + Goal Attain	4.0378	0.4247	0.3175	7.8080	1.8994	0.7944	4,480
Genetic Algorithm + Goal Attain	7.6813	0.5584	0.3308	10.0550	1.7870	1.0664	37,466

Optimization (MSDO) routine integrating FEM is proposed. In the MSDO, the design variables are used at the microscale to acquire the homogenized core properties according to Burton and Noor [90]. The homogenized core properties are then brought to the meso-scale and implemented into the FEM. Upon analysis in FEM the results are extracted and compared to the failure criteria in Equations 3.16, which are in turn determined by the design variables at the microscale. A process diagram for the FEM-integrated MSDO is shown in Figure 3.10.

Since the proposed MSDO procedure integrates FEM into the design loop directly, gradient methods are not applicable and stochastic methods must be used. Due to the randomized nature of gradient-free stochastic algorithms, a starting point in the feasible design space is unlikely, which can result in a large number of costly function evaluations yielding no results. A penalty method for optimization employing the *Kreisselmeier-Steinhausser (KS)* equation has been found to be an ideal methodology to circumvent this issue and is proposed for use [96–98]. Through this approach, the constraints are calculated as a secondary term in the cost function according to the equation:

$$\Phi(\mathbf{x}) = f(\mathbf{x}) + \sum_{m \in \mathbb{I}} W_m h_m(KS(\mathbf{x})) \quad (3.22)$$

where m is the number of constraints, $f(\mathbf{x})$ is the base cost function, and W_m is a penalty coefficient so that the m^{th} exterior penalty function, $h(KS(\mathbf{x}))$, is appropriately proportionate to the cost function. The exterior penalty function is then calculated as:

$$h_m(\mathbf{x}) = \max(0, (1 - KS(\mathbf{x})))^2 \quad (3.23)$$

for constraints of the form:

$$1 - c_m(\mathbf{x}) \leq 0 \quad (3.24)$$

where c_m represents the m^{th} constraint, and $KS(\mathbf{x})$ is the Kreisselmeier-Steinhausser equation given as:

Figure 3.10: Process diagram for FEM-Integrated MSDO.

$$KS(\mathbf{x}) = g_{\max} + \frac{1}{\rho} \ln \left(\sum_{i=1}^N e^{(\rho g_i(\mathbf{x}) - g_{\max})} \right) \quad (3.25)$$

where g_i is the constraint violation of the i^{th} of N elements or nodes (depending on the constraint type) in a design space, with the highest constraint violation being given by $g_{\max}$. The constraint violation is calculated as:

$$g_m = \frac{|C_{\text{calc}}(\mathbf{x})|}{C_{\text{allow}}(\mathbf{x})} \quad (3.26)$$

where the $C_{\text{calc}}(\mathbf{x})$ and $C_{\text{allow}}(\mathbf{x})$ indicate the calculated and allowable value under consideration respectively.

Now, a cost function has been produced which is capable of considering constraints without the concern for the feasibility of any points in the design space. Should a point be infeasible, it is heavily penalized by Equation 3.25. Meanwhile, Equation 3.23 ensures that all feasible points do not have their associated cost penalized and calculated only by the base cost. The advantages are two-fold: the need for a stochastic optimizer to find a feasible starting point is negated, and the optimizer is able to consider constraints *a posteriori*, which require a function evaluation prior to checking of feasibility. Although this MSDO methodology is capable of producing results for the hopper car in a stand-alone analysis, in the next chapter it will be employed as part of a full multi-stage design procedure in a case study for the determination of the optimal design variables for a sandwich panel in the context of a hopper car. Results of the MSDO will then be compared to the analytical approach discussed earlier in this section in Chapter 5 to determine whether this rigorous approach is necessary.

3.4 Summary of Numerical Methods

In this chapter, a FEM for the hopper car was developed using an explicit approach to calculate the FSI of the structure with its contents as well as an implicit approach in order to implement a comprehensive set of load cases. After developing an understanding of the structure through FEM, it was then possible to begin approaching the optimization of the

hopper car through numerical methods. An analytical approach employing multi-objective optimization was synthesized to generate rapid results. It was found that while these results might produce useful initial designs, it was also determined that *a priori* confirmation of the results was not possible. A full MSDO was proposed as a solution, wherein the optimization calls upon the FEM within the design loop to check results each iteration and calculate a penalized cost function. With these concepts fully established, a design procedure for the hopper car can be discussed in the proceeding chapter.

Chapter 4

Case Study: Multi-Stage Design Optimization of a Hopper Car

In the previous chapter, a FEM was developed for use in a design optimization procedure for hopper cars employing hexagonal honeycomb sandwich panels. Next, various numerical methodologies for performing an optimization of a sandwich panel applied to a hopper car were investigated and defined. In this chapter, a full MSDO procedure for a hopper car is carried out using all the accumulated principles discussed so far in the present work. First, a baseline model is produced using current commercial materials and using commercially available software for optimization. Upon establishing a baseline, the first stage of the optimization procedure is executed where a Topology optimization for determining ideal beam layouts in the overall structure is used to generate a novel structure which concentrates the mass of the structure in critical areas. By concentrating the mass in critical areas, the overall mass is reduced, while opening a pathway for the implementation of lightweight hexagonal honeycomb sandwich panels. Then, the MSDO synthesized in Section 3.3.2.B is implemented on the novel structure as the second stage of the optimization to size the sandwich panels at the component level. It is expected that the multi-stage optimization in this chapter will produce results which are superior to the baseline produced using commercial methods, and a weight reduction which is competitive with the optimized baseline.

Figure 4.1: Size optimization design variables for the baseline hopper car employing conventional materials.

4.1 Creation of a Baseline Model

For the optimization of the structural baseline, the beams have been divided into 22 sets, the walls and roofs have been divided into 5 panels each, and the floor has been divided into 9 panels. Each beam set is given 4 design variables, and each panel one design variable as shown in Figure 4.1 for a total of 110 design variables: 22 beams with beam heights H , beam widths B , beam thicknesses T_1 and T_2 , as well as 19 plate panels with thickness T_p . *Altair OptiStruct* is used for the optimization by means of the *Global Response Surface Method (GRSM)*, which employs a DOE based approach construct a rough response surface for the design space and conduct a search for the point of lowest cost [91]. All loads defined according to Section 3.1 have been weighted as equally important for the design optimization so that the design produced that is satisfactory for all load cases. For modal analysis, a natural frequency of 12 Hz for the whole car has been enforced so that no modal damping is required for all resultant designs [61]. As previously mentioned, buckling load cases have not been considered for this optimization as consideration of buckling will require a full optimization on its own for each load case in specific regions of the hopper car and is therefore out of scope for the present work.

With a sizing procedure, it is necessary to ensure excessive stress or displacement does not occur. The optimization has been constrained such that the von Mises stress in all elements does not exceed 423.3 MPa. Furthermore, since buckling has not been considered, it

is necessary to constrain the compressive stresses. While buckling is a global criterion, beam crippling stress is a local failure mode and can be accounted for as part of the optimization of the beams. For a square cross-sectional tube, the crippling stress is given as [31]:

$$\sigma_{cc} = B \left(\frac{gt^2}{A} \right)^m \sqrt{E^m \sigma_{cy}^{2-m}} \quad (4.1)$$

where $B = 0.58$, $m = 0.8$, and $g = 12$ for a square cross section [31], while E and σ_{cy} are the elastic modulus and yield stress respectively. The cross-sectional area, A , is recalculated each iteration according to the design variables. The thickness, t , is the smallest of the two thickness variables T_1 and T_2 . Because there are no local buckling criteria for the plate elements, no lower bound has been enforced for the them. It is worth noting, however, that since the von Mises stress has been used as the failure limit, compressive stresses have been limited in some manner.

By using the ESLM to extract the loads from the explicit model, it has been assumed that the implicit model has a similar stiffness matrix. By changing the size of the mechanical elements, however, this assumption will be invalidated. Due to the costly nature of explicit analysis, it is not viable to recalculate the equivalent static loads at each iteration, therefore rigidity must be preserved through displacement constraints. From Equation 3.7, the force due to the explicit loads are directly proportional to the displacement and acceleration. It is not possible to control for the acceleration in implicit FEM, however by enforcing the displacement constraints it is possible to ensure that the loads provide a conservative estimation. By ensuring the displacement in the implicit model does not exceed that of the explicit, it is implied that the forces shall not be increased despite the reduction in rigidity. Although this is not true for all nodes, it is assumed that the dynamic factor applied during the conversion to ESL will provide a conservative estimate of loading. This displacement constraint has been extended to displacements in all load cases of the original model as a final precaution. A summary of the constraints used in the optimization are shown in Table 4.1.

The optimization converged in 20 iterations to a mass of 4,259.4 kg for the quarter model, implying a total car-body mass of 17,037.6 kg. The resulting mass is on the same

Figure 4.2: Convergence plot for the baseline sizing optimization

order of magnitude of mass with the car-body weight in [99] which indicates the baseline is relatively realistic in its representation of the structural characteristics of a standard covered hopper car. Results from the optimization for each design variable are shown in Appendix A.1. In observing the optimization convergence plot in Figure 4.2, it is important to note that the converged baseline mass resulted in a greater mass than the starting point. This indicates that the original structure is out of compliance, and therefore required re-sizing for structural integrity specifically in the ECL and DTL.

Table 4.1: Constraints for GRSM Optimization using OptiStruct.

Constraint	Upper Bound	Lower Bound
Shell Von Mises Stress (MPa)	423.3	-
Beam Von Mises Stress (MPa)	423.3	-
Beam Minimum Stress (MPa)	-	Equation 4.1
Displacement LVL (cm)	3.1	-3.1
Displacement DTL (cm)	5.0	-5.0
Displacement ECL (cm)	8.1	-8.1
Displacement FSI (cm)	6.5	-6.5
Natural Frequency (Hz)	-	12.0

4.2 Stage I: Topology Optimization

In Chapter 2, a topology optimization methodology was proposed for use in determining the optimal beam structure layout of the hopper car. By determining the optimal beam layout, the structural rigidity will be provided by the frame comprised of load bearing beams. By concentrating the mass in the beams, a pathway for implementation of hexagonal honeycomb sandwich panels has been opened, which will behave as walls to contain the cargo. A process flow diagram summarizing the TO design procedure is shown in Figure 4.3. The TO is carried out in an iterative manner where one iteration will perform a topology optimization, then replace beams as necessary. The topology optimization is then run once more to determine if the optimal beam layouts have been determined. If a change is observed in the results of the TO which calls for high-density elements in areas not supported by beams, new beams are added to strengthen the structure until the TO no longer produces such a change.

The TO has been performed by minimizing the compliance of the structure as the cost function, and with a volume fraction limit of one quarter. The widths of the high-density regions have been constrained between 1.5 and 3.5 times the element size so that the high-density regions are not excessively dispersed. Results for each iteration of the TO are shown in Figure 4.4, with the initial models for each iteration on the left and the element density contour plots on the right. Between each iteration, any beams shown to be unnecessary by the element density contour plot are removed. Whether a beam is determined to be unnecessary based on whether it lies in an area of an element density of 0.67 or higher.

From the element density contour plot shown in Figure 4.4f, it is apparent that the majority of the wall structure is of low relative density, indicating that the beams are providing the majority of the rigidity in the design shown in Figure 4.4e. Since there are no clear indications of where further beams might be added, the design in Figure 4.4e is selected as the final design for this step in the MSDO. In the areas of low density, hexagonal honeycomb sandwich panels replaced the solid steel walls in an attempt to reduce the presence of solid material as much as possible. In certain areas, there are large swathes of high-density elements which cannot be addressed through the implementation of beams. Despite this

Figure 4.3: Topology Optimization process flow diagram.

shortcoming, it is expected that these sandwich panels will provide the rigidity necessary to prevent excessive deformation or failure, specifically after the panels are optimized in the later steps of this MSDO procedure.

With the TO complete, the same sizing procedure from Section 4.1 is employed for the design in Figure 4.4e. The topology optimized design has 49 beams with 4 design variables each, as well as 22 panels with one design variable each for a total of 218 design variables. The same constraints from Table 4.1 are applied once more, and the GRSM is used to size the beams appropriately. The mass of the quarter model converges to a final value of 3,322.1 kg, implying a total car-body mass of 13,288.4 kg. Individual results for each design variable are shown in Appendix A.1. The total mass of the hopper car has been reduced by 3,749.2 kg or 22.0% as compared to the baseline model, despite a twofold increase in the number of beams. This indicates that the strategy of concentrating the structural mass of the hopper

Figure 4.4: Element density contour plots from the different topology optimization iterations in ascending order. Shown on the left are the beam layouts used in each iteration. Red indicates beams which have been added for the present iteration, while blue indicates beams which were initially present or present in previous iterations.

car within beams through the use of topology optimization is a viable methodology for the structural optimization of the hopper car.

Now that a car-body with optimal beam placement has been designed, it is possible to apply sandwich panels in the car walls and roofs with less concern for major changes in rigidity and stresses. Although it could be sufficient to replace the walls with a generic hexagonal honeycomb panel, a panel-by-panel optimization procedure will be discussed in the subsequent section. By using the panel-by-panel design procedure, an optimal hexagonal

Figure 4.5: Convergence plot for the sizing of the topology optimized model.

honeycomb panel has been designed for minimum mass and maximum rigidity.

4.3 Stage IIA: Characteristic Panel Multiscale Design Optimization Procedure

In the first part of this chapter, a baseline using industrial methods was established. Then, the baseline underwent a series of topology optimizations to synthesize a novel structure with ideal beam layouts to make up the structural frame. While the beams are expected to carry the majority of the loading, the walls cannot be removed entirely. As has been demonstrated thus far in the present work, sandwich panels can offer superior rigidity at a lighter weight than a solid steel panel of equivalent area. Therefore, in the areas of low criticality sandwich panels will replace these solid panels.

In Section 3.3.2.B, an MSDO procedure is defined for optimizing a sandwich panel using a penalty-based optimization method employing the Kreisselmeier-Steinhausser equations to calculate a penalized cost function based on FEM results. In this section, the MSDO is applied to the design of a single wall panel on the hopper car to determine the optimal design parameters of the faces and core. The procedure shown in Figure 3.10 is followed, where a sub-model is prepared and optimized using a cocktail of algorithms to first understand the design problem, and then to determine the optimal sandwich panel properties. First,

a sub-model will be defined and validated in an effort to reduce cost function evaluation time. After an appropriate sub-model is selected, two cost functions are defined using Equation 3.22; one for a penalized mass, and another for penalized compliance. To gain a better understanding of the design problem and appropriately apply penalty weightings, a response surface is created using each penalized cost function with constraints enforced in the form of the Kreisselmeier-Steinhausser equations. Next, a pattern search is run for each cost function using the points resulting in lowest cost from each corresponding response surface for the optimization of the penalized functions. Finally, a Pareto front is found for the design space to determine the ideal mass-to-compliance ratio for the panel. An overview of the full design procedure is shown in Figure 4.6.

4.3.1 Sub-Model Creation

Since Section 3.3.2.A demonstrated that thousands of function evaluations might be required for an optimal point to be found in an optimization procedure, it is necessary to reduce the calculation time of the objective function as much as possible. In the MSDO procedure, the obvious time limiting step is the FEA of the hopper car model, which took an average of 53 seconds to complete. In Section 3.2.2.A, the Super-Elements method of sub-modeling was discussed as a potential methodology for addressing computational efficiency in FEM. In this section, the implementation of this method and its validity are explored for use in the MSDO.

As previously mentioned, two challenges must be addressed in the development of a sub-model: the ASET must be selected in close enough proximity to the region of interest so that solve time is adequately condensed, while ensuring at the same time it is far enough from the region of interest such that the enforced displacements do not produce counter-intuitive or inaccurate results in the sub-model. In Figure 4.7, the wall panel of interest for this case study has been highlighted in red. The proposed sub-model is highlighted in dark grey, while the portion of the model which will be reduced is highlighted in light grey. By using the Super-Elements method discussed previously, the solve time has been reduced to an average of 18 seconds. To ensure the selection of the ASET is appropriate, a sensitivity study has been performed on the sub-model with changing core thickness for which sample

Figure 4.6: Overview of the Panel-By-Panel MSDO procedure.

Figure 4.7: Depiction of the sub-model of interest. Light grey represents the portion of the model in the DMI, while dark gray represents the sub-model under consideration. Inside the sub-model, the highlighted red panel is the design space.

results are shown in Figure 4.8. It is demonstrated in the figure that both compliance and face stress are inversely proportional to the core thickness as predicted by theory. It is worth noting that the overall compliance of the model is not significantly changed since only one panel has been considered. The figure also compares the von Mises face stress in the sub-model to the global model and shows that over the design space, there is no meaningful error in stress between the sub-model and global model.

In this section, an appropriate ASET is determined for the implementation of the Superelement method. The implementation reduces the CPU time of the simulation by 66% without introduction of any meaningful error. The final mass of the sub-model is 983.634 kg out of the original mass of 3,322.1 kg and is used as a measure of the baseline for the panel optimization. Considering that a Pareto Search algorithm might require 2,000 function evaluations, the anticipated time savings to determine the Pareto front is estimated to be 19.44 hours. Now that confidence in the sub-model has been established, it is possible to carry on with the optimization of the design space of interest.

Figure 4.8: Sensitivity study to test the validity of the ASET using the LVL as the example load case. It is demonstrated that with increasing core thickness, the overall compliance and von Mises stress in the faces are reduced in the sub-model. Furthermore, the von Mises stress in the faces are compared and found to be within 1% of one another.

4.3.2 Implementation of Optimization Algorithms

In Section 3.3.2.B, a cost function integrating FEM directly into the optimization procedure was developed which makes use of the Kreisselmeier-Steinhausser equations to employ a penalty-based approach for optimization. In this section, the application of this cost function to the optimization procedure as previously devised is studied. In Section 3.3.2.A, two cost functions were derived; one representative of mass, and another considering the inverse of the distributed failure load. The inverse failure load had been considered due to the necessity to ensure the designed sandwich panels are not susceptible to failure. The disadvantage of this methodology was that it was not possible to directly optimize the sandwich panels for a specific load case and therefore is not necessarily optimal. Now that a methodology integrating FEM has been conceived, it is not necessary to maximize the allowable failure loads of the panel. Instead, ensuring the objective has become to ensure a structure with the highest possible rigidity with the lowest possible mass is found. Therefore, two cost functions of interest have been formulated for minimization: one for mass, and another for compliance, thereby indirectly improving rigidity.

As previously mentioned, checking for constraints *a priori* in the optimization – that

is, checking prior to evaluation of the cost function – is not advantageous and can lead to long convergence time. By checking constraints *a posteriori* and employing the penalty methods discussed, any infeasible design points are penalized instead of discarded, allowing the optimization algorithm to use the nearby points to approach the feasible design space by searching for a lower penalty. In the following sub-sections, the penalized mass and penalized compliance functions will be defined in detail according to their assigned constraints.

A The Penalized Mass Function

The penalized mass function considers the base cost as the mass of the FEM, while penalties are applied for excessive displacement and stresses. For displacements, all load cases are constrained just as they are in Table 4.1 in order to preserve the rigidity of the structure as much as possible. The stresses in the sandwich panels are constrained according to the failure criterion noted in 3.16, while stresses in beams are constrained to prevent tensile failure using the von Mises criterion and for crippling according to Equation 4.1. It is also important to ensure a design produced does not change the structural integrity of the walls surrounding it, therefore all shell elements in the sub-model are constrained according to the von Mises criterion as well. All other constraints from prior optimizations have been retained, including the modal lower limit of 12 Hz and the face-to-core ratio requirement of Equations 1.2a and 1.2b.

With all constraints mathematically defined, the penalized mass cost function can be written as:

$$\begin{aligned} \Phi_{mass}(\mathbf{x}) = M(\mathbf{x}) + W_{stress} \left(\sum_{m=1}^{32} h_m(KS(g_m(\mathbf{x}))) \right) + W_{disp} \left(\sum_{m=32}^{36} h_m(KS(g_m(\mathbf{x}))) \right) \\ + W_{modal} h_{37}(g_{37}(\mathbf{x})) + W_{geom} h_{38}(g_{38}(\mathbf{x})) \quad (4.2) \end{aligned}$$

where $\Phi(\mathbf{x})_{mass}$ is the penalized mass function, $M(\mathbf{x})$ is the base mass, h_m is the m^{th} constraint penalty function calculated according to Equation 3.23, which in turn is a function of the KS equations. For the KS equations, constraint violation factors of g_m are deter-

mined according to the equations in Table 4.2. It should be noted that the KS equations are useful only for constraints which vary by element or node. For the geometric and modal constraint, the penalty function has been simply calculated as:

$$h_m(\mathbf{x}) = \max(0, (1 - g_m(\mathbf{x})))^2 \quad (4.3)$$

The weightings of each penalty function, denoted by W with a subscript corresponding to the constraint type, have been determined in the next section by plotting the response surface and determining an appropriate value. The weightings are valued such that the penalties are of a similar order of magnitude to the mass function and produce a response surface which effectively penalizes constrain violations.

B The Penalized Compliance Function

The penalized compliance function has been defined in a similar manner to the mass function. Compliance is calculated by the FE solver for each load case. These compliances have been summed to produce the base compliance function. While each compliance are not controlled individually through a sum of compliance, emphasis on critical load cases can be placed depending on the design situation. Only two constraints have been applied to the penalized compliance function: a mass fraction of 0.85 to guarantee a minimum mass reduction of 15%, and the same thin face criteria according to Equation 1.2a. The penalized compliance function is then written as:

$$\Phi_{comp}(\mathbf{x}) = \sum_{j=1}^4 C_j(\mathbf{x}) + W_{geom} h_{geom}(g_{geom}(\mathbf{x})) + W_{mfrac} h_{mfrac} \left(\frac{M(\mathbf{x})}{M_{baseline}} \right) \quad (4.4)$$

where C_j is the compliance for the j^{th} load case as a function of the design variables, $M(\mathbf{x})$ is the calculated mass, $M_{baseline}$ is the baseline mass of 983.634 kg, and W is once again a weighting value assigned to appropriately penalize infeasible design points, where its subscript indicates the affiliated constraint. Because all constraints in this function are global, the KS equations have not been employed, and penalties are applied using

Table 4.2: Constraints for the penalized mass function. Note that for upper bound limits, the constraint is C_{calc}/C_{allow} , while the inverse is true for lower bound constraints. All constraints use absolute values.

Constraint	Number	Equation	Description
Face Yielding	$m = 1, \dots, 4$	$(g_{fY})_m = \frac{\sigma_f}{\sigma_Y}$	Face yielding constraint for all four load cases. Enforced using element von Mises stress in the faces at the corners.
Intracellular Buckling	$m = 5, \dots, 8$	$(g_{icb})_m = \frac{\sigma_3}{\sigma_{icb}}$	Intracellular Buckling constraint for all four load cases. Enforced using minor principal stress in the faces at the corners and Equation 3.16b.
Face Wrinkling	$m = 9, \dots, 12$	$(g_{wr})_m = \frac{\sigma_3}{\sigma_{wr}}$	Face wrinkling constraint for all four load cases. Enforced using minor principal stress in the faces at the corners and Equation 3.16c.
Compressive Crimping	$m = 13, \dots, 16$	$(g_{cr})_m = \frac{\sigma_3}{\sigma_{cr}}$	Shear crimping constraint due to excessive compression. Enforced using minor principal stress in the faces at the corners and Equation 3.16d
Shear Crimping	$m = 17, \dots, 20$	$(g_{scr})_m = \frac{\tau_{max}}{\tau_{scr}}$	Shear crimping constraint due to excessive shear. Enforced using shear in the faces and cores at the corners and Equation 3.16e
Shell Yielding	$m = 21, \dots, 24$	$(g_{shells})_m = \frac{\sigma_{V Mmax}}{\sigma_Y}$	Constraint to ensure no failure occurs in nearby shell elements using the von Mises failure criterion. Enforced using stress in default Z-offsets in shell element corners.
Beam Yielding	$m = 25, \dots, 28$	$(g_{beams})_m = \frac{\sigma_{V Mmax}}{\sigma_Y}$	Constraint to ensure no failure due to excessive stress in beams using the von Mises failure criterion. Enforced using maximum beam stress in beam elements at all points.
Crippling	$m = 29, \dots, 32$	$(g_{cc})_m = \frac{\sigma_{min}}{\sigma_{cc}}$	Constraint to ensure crippling does not occur in beams. Enforced using minimum beam stress in beam elements at all points and Equation 4.1
Displacement	$m = 33, \dots, 36$	$(g_{disp})_m = \frac{d_{max}}{d_{lim}}$	Displacement constraint to preserve rigidity in all load cases. Enforced using maximum norm of nodal displacement vector and displacements in ESM.
Natural Frequency	$m = 37$	$g_{modal} = \frac{f_{lim}}{f_{nat}}$	Lower limit on natural frequencies in the structure per MSRP regulations [61].
Thin Faces	$m = 38$	$g_{geom} = \frac{t_f + H_c}{5.77t_f}$	Constraint to ensure <i>thin face</i> criteria is kept based on Equation 1.2a.

Equation 3.23 exclusively as a function of g .

4.3.3 Design of Response Surfaces and Selection Penalty Factors

In the previous section, two penalized cost functions were developed for mass and compliance, however appropriate weightings had not been defined for the penalty terms. In this section, the response surfaces calculated from sample points in the design space for two variables at a time are plotted. Using the response surfaces, appropriate weighting factors can be determined for the penalty terms by modeling the behaviour of the penalized cost function in infeasible regions of the design space. The weightings are selected by qualitatively comparing the response surface of the base cost function to that of the penalized cost function. If the response surfaces are similar to one another, it is clear that any applied penalty factor is not large enough to meaningfully punish the cost. Although it is possible to apply a very large weighting factor, this presents two challenges: first, it is unclear how large a number should be used initially and whether such a number would be large enough to begin with, and second, an excessively large penalty could result in an enormously steep valley at the barrier of the feasible design space. Should a steep cliff form at the barrier of the feasible design space, poor convergence is likely in such regions [100].

Table 4.3: Penalized mass and compliance weightings.

Constraint	Weighting
Stress	100
Displacement	100
Modal	200
Mass Fraction	5000
Geometric	1000

To determine the weighting factors, an iterative approach has been taken. The base cost function was plotted next to the penalized cost function for mass and compliance and the penalty factor was slowly increased. Eventually, the penalty was large enough such that the feasible region could be easily distinguishable from the infeasible region. An obviously infeasible point was then selected, and the cost function was repeatedly assessed with varying weighting factors. The weighting was increased until it was determined qualitatively that the

applied penalty for infeasibility was meaningfully penalizing the cost, while still resulting in a reasonably smooth behaviour near the barriers of the feasible design space. The weightings used for creation of the response surfaces are shown in Table 4.3. Response surfaces have been created by bringing into play the listed weighting factors and running the cost function in a sample set of 625 equidistant points for two of the four variables at a time, while fixing the other two. Linear interpolation between points has been used to create the resultant response surface. In Figure 4.9, the response surfaces with respect to the face thickness, t_f , and core thickness, H_c , are shown. In Figure 4.10, the response surfaces are shown instead for the core wall width, L_c , and the core foil thickness, t_c .

In Figures 4.9a and 4.10a, the response surfaces for mass and penalized mass as a function of the face and core thickness are shown. A full range of realistic values have been used to show the full design space, however the baseline thickness for the panel of interest is 3.372 mm, therefore any face thickness larger than 1.686 mm becomes rapidly infeasible for a superior design. Considering such a small baseline thickness, the initial design intuition of taking the thinnest possible face and thickest possible core was also infeasible, since a core thickness larger than 1.50 cm produces a mass higher than the baseline as well. Another note of importance is that a thick core coupled with a thin face is infeasible given the large penalty at the top left of the design space; Equation 3.16c shows that for such a ratio, the failure limit for wrinkling decreases rapidly. As a result, despite the reduced stress from increased core thickness, the design is consequently more susceptible to wrinkling if the core-to-face thickness ratio is too large given that core-to-face elasticity ratio remains the same. Marked on both plots with an asterisk is the point of lowest cost on the response surface with both the lowest penalty and mass at $t_f = 0.5689$ mm and $H_c = 3.175$ mm.

The compliance and penalized compliance plots with respect to face and core thickness are shown in Figures 4.9b and 4.9d respectively. By comparing the two plots against one another, it is re-affirmed that compliance has very little sensitivity to changes the core wall width and core wall thickness, as supported by sandwich theory. This does, however, change when the core wall thickness large and the core wall width is small, at which point the core begins to behave as a solid material.

In Figures 4.10a and 4.10c, the mass and penalized mass response surfaces are shown

(a) Mass of Side Panel 4 as a function of t_f and H_c .

(b) Penalized Mass of Side Panel 4 as a function of t_f and H_c .

(c) Compliance of Side Panel 4 as a function of t_f and H_c .

(d) Penalized Compliance of Side Panel 4 as a function of t_f and H_c .

Figure 4.9: Objective contour plots as a function of t_f and H_c to test sensitivity and determine close-to-optimum starting points. For this plot, $L_c = 6.35 \times 10^{-3}$ m and $t_c = 5 \times 10^{-4}$ m

with respect to core wall width and core foil thickness. The optimal point appears to have been found near the barrier of the feasible region, which is to be expected; a large core wall width and small foil thickness can greatly reduce mass as they are the sole factors in core density, but can result in high susceptibility to failure of all forms. Therefore, the optimal point would be close to the barrier where mass is the lowest it could be without initiating failure. It is important to note that although the response is relatively smooth near the barrier, there is a sharp increase in penalty as the core wall width increases and

(a) Mass of Side Panel 4 as a function of L_c and t_c .

(b) Penalized Mass of Side Panel 4 as a function of L_c and t_c .

(c) Compliance of Side Panel 4 as a function of L_c and t_c .

(d) Penalized Compliance of Side Panel 4 as a function of L_c and t_c .

Figure 4.10: Objective contour plots as a function of L_c and t_c to test sensitivity and determine close-to-optimum starting points. For this plot, $t_f = 5 \times 10^{-4}$ m and $H_c = 6.35 \times 10^{-3}$ m

foil thickness decreases onwards. This is because at these points, the allowable stresses preventing failure simultaneously become so low while induced stress increases. As a result, the penalty applied by the KS equations becomes so high that the cost goes to infinity, resulting in the sharp cut-off shown on the bottom right.

Finally, Figures 4.10b and 4.10d show the compliance and penalized compliance with respect to the core wall width and core foil thickness. It is demonstrated that, as predicted by theory, the compliance has virtually no sensitivity to these variables since core density

and stiffness is not a factor of importance unless it approaches the stiffness of the faces. At the top left of the plots, where wall thickness is high and wall width is low, the core effectively behaves more as a solid material, and so an improvement in compliance is observed. Based on the the penalties encroached in these areas from the KS equations due to unreasonably high mass, it is demonstrated that modification of these variables for the purpose of improving compliance is highly inefficient.

The points of lowest cost from all penalized cost response surface plots have been extracted and are shown in Table 4.4. It is important to note that these points are not necessarily optimal, despite their high potential. Since two of the four design variables are fixed for the creation of the response surface, the allotted material for the design of the sandwich panels might not be optimally distributed. By plotting the response surfaces, only near optimal points have been found which accordingly give useful design intuitions. Many of the conclusions in this section are not new and have been discussed extensively in literature, however by observing these design intuitions in the response surfaces confirmation has been attained that the FEM and its integration into the MSDO largely follow the expected design intuition predicted by theory. Given that trust has been established in the procedure and regions of local optimality have been discovered, it is possible to proceed with a full optimization using the previously discussed strategies in Section 3.3.2.

Table 4.4: Points of lowest cost from the penalized response surfaces in Figures 4.9 and 4.10. Note that because penalized costs are used, the calculated cost is not necessarily representative of the actual mass or compliance.

Cost Function of Interest	L_c	t_c	t_f	H_c	$\Phi_{\text{mass}}(\mathbf{x})$	$\Phi_{\text{comp}}(\mathbf{x})$
	$[\mathbf{m} \times 10^3]$				$[\mathbf{kg}]$	$[\mathbf{m/N}]$
Penalized Mass	6.350	0.500	0.569	3.175	847.62	12,992
	7.144	0.287	0.500	6.350	1,121.1	13,278
Penalized Compliance	6.350	0.500	1.323	3.175	925.39	12,241
	3.175	0.07	0.500	6.350	3,651.9	13,281

4.3.4 Individual Cost Function Optimizations

In the previous section, response surfaces were created for the purpose of visualizing the design space and determining appropriate weightings for the penalty terms in the penalized cost functions. A secondary result from these response surfaces is the ability to obtain near-optimal points for two of the four design variables at a time for both mass and compliance. By obtaining near optimal points, the design space has been refined to a smaller region of interest. This is highly advantageous; in Section 3.3.2.A, it is determined that a global search algorithm, such as the GA, can take ten to twenty times as many function evaluations as a local search algorithm. By understanding the design space through visualization of the response surfaces using only 625 function evaluations for sampling points, a local region of interest containing the minimum local optima has been discovered, negating the need for a global search. In this section, individual optimizations will be carried out near these local optima in order to determine the design variables providing the ideal mass and compliance for the side panel of interest. By isolating the designs offering the least mass and compliance individually, it is possible to repeat the same process for the analytical optimization in Section 3.3.2.A wherein a Utopia point is determined and eventually used in the Pareto Front to determine the design which simultaneously provides the greatest mass and compliance reduction.

The points resulting in lowest cost from the response surface and the design intuitions for shrinking the design space are now leveraged as seed points to perform individual local optimizations of penalized mass and compliance. The seed point for the penalized mass is a combination of the best t_f and H_c values found from the response surface in Figure 4.9a, and the L_c and t_c values from Figure 4.10a to create a unified near-optimum vector. A similar seed point is generated for penalized compliance using the points of lowest cost from Figure 4.9b and Figure 4.10b. A *MATLAB* algorithm employing these seed points with the `patternsearch` subroutine has been used to obtain the convergence plots shown in Figure 4.11, which show the lowest evaluated cost for each iteration of a Pattern Search algorithm. Because of the proximity to the response surface optimum, convergence is attained in 145 function evaluations over 34 iterations for penalized mass, and 305 function

evaluations over 76 iterations for penalized compliance.

(a) Pattern Search convergence plot for the refined optimization of penalized mass.

(b) Pattern Search convergence plot for the refined optimization of penalized compliance.

Figure 4.11: Pattern search convergence for individual optimizations.

Although no analytical solutions exist which can prove global optimality for a non-smooth and non-continuous cost function, a rigorous approach has been taken to search the design space thus far in search of the optimum in the design space which satisfies all constraints. Optimal points have been found for penalized mass and penalized compliance individually, thereby giving the design of a side panel with superior mass savings, and another for superior rigidity. The converged design variables are shown in Table 4.5 under the row labeled *Individual Optimizations*. A notable difference in results for optimal compliance is observed when compared to the results from the response surfaces. This is because when the response surfaces were created, mid-range values had been enforced for two of the four design variables at all times. Given the freedom to optimize all design variable simultaneously, the PS distributes the material such that the core thickness is maximized, while reducing the mass fraction by removing material from the core cells. This shows once again that, as predicted by theory, rigidity is best increased in the sandwich panel by maximizing core thickness. Although this intuition is certainly useful, it is worth noting that the penalized compliance function does not account for failure due to loading. For this reason, it is once more emphasized that the design employing maximum rigidity is not necessarily the optimal design.

4.3.5 Pareto Front Generation

So far, close-to-optimum points have been identified exclusively for minimal mass with restricted stresses and displacements, or minimal compliance with restricted mass. Though it can be left to a potential designer to determine from intuition or industrial requirements whether mass savings or rigidity should be prioritized in a final design, a method should be discussed to determine a lightweight design with superior rigidity. In order to do so, a Pareto front is generated in following the same procedure deliberated in Section 3.3.2.A. All accumulated design intuition so far is to be used in the generation of such a front; the success in producing a smooth Pareto front which gives meaningful results is highly dependent on the design of the problem and the seed points given for initialization. As was done in Section 3.3.2.A for the analytical optimization, the seed points are chosen based on the individual optimizations from the previous section. In addition to these points, the response surface points of lowest cost from Table 4.4 are used as well.

Table 4.5: Seed points for the Pareto search algorithms.

Source	L_c	t_c	t_f	H_c
	$[\mathbf{m} \times 10^3]$			
Response Surface	7.144	0.287	0.569	3.175
Individual Optimizations	8.275	0.378	0.557	3.176
	9.523	0.024	1.386	12.160

The `paretosearch` and `gamultiobj` subroutines in *MATLAB* have been employed using the FEM embedded MSDO procedure to calculate the cost for each iteration to generate the Pareto front shown in Figure 4.12. Because a multi-objective methodology is used here, a constraint for mass in the penalized compliance function would result in the mass being simultaneously an objective and a constraint, which will result in poor convergence. Additionally, if the geometry constraints are accounted for in the penalized compliance function, the geometry will be doubly penalized without reason. Therefore, for the multi-objective optimization using a Pareto search, the first cost function is the penalized mass according to Equation 4.2, while the second cost function is simply the total compliance. Once again, the Utopia point is defined as the optimal cost for each cost function when optimized in-

dividually according to Equation 3.21. Note that the Utopia ordinate for compliance has not changed because the individually optimized compliance is in the feasible region and therefore not penalized. Final results are shown in Table 4.6 for each algorithm, along with the number of function evaluations required for the front generation.

Table 4.6: Best points from each Pareto front in Figure 4.12 for each search algorithm.

Search Algorithm	L_c	t_c	t_f	H_c	$m(\mathbf{x})$	$C(\mathbf{x})$	Function Evaluations
	$[m \times 10^3]$				$[kg]$	$[m/N]$	
Pattern Search	9.5250	0.4874	1.0716	5.9531	903.43	11,932	2,355
Genetic Algorithm	8.4053	0.3344	1.1823	5.3581	907.79	11,801	16,453

Figure 4.12: The Pareto front resulting from the multi-objective optimization of penalized mass and total unpenalized compliance. The Utopia point has a mass of $M(\mathbf{x}) = 833$ kg and a total compliance of $C(\mathbf{x}) = 11,910$ m/N.

Using both a PS and a GA for the Pareto front generation produces similar results. Once again, the pattern search has produced superior results, which can be attributed to a close-to-optimum selection of starting points. In this case the implementation of starting points has allowed for more points in the Pareto set have been found using the PS algorithm, however it can be seen in Figure 4.12 that both fronts are approximately the same. Hybridization with `fgoalattain` has not been implemented for search refinement since there is no reason to expect a superior solution based on the findings in Section 3.3.2.A.

4.4 Stage IIB: Functionally Graded Multiscale Design Procedure

Functionally graded lightweight structures have been created using topology optimization which offer high resistance to buckling, as well as lightweight cellular composites with high stiffness-to-weight ratios [101, 102]. In this section, an alternative novel approach is briefly discussed as a substitute for the second stage: a functionally graded MSDO. By employing the same SIMP density algorithm, a relative element density plot has been generated which offers a map to determine where high-density hexagonal honeycombs should be used, and where low-density ones can be substituted instead. The objective of this procedure is not necessarily to produce a finalized and manufacturable design, but to approach global optimality using functionally graded hexagonal sandwich panels to demonstrate the potential of such materials.

Employing the SIMP algorithm in *OptiStruct*, a topology optimization is executed to minimize compliance with a constraint enforcing a volume fraction no larger than one-third the original mass. Additionally, a minimum dimension constraint of 2 m has been applied. The application of the minimum dimension constraint encouraged the dispersal of element densities over a larger area on the relative density contour plot. The algorithm produced the density contour plot shown in Figure 4.13a. The element densities are then extracted and sorted according to Table 4.7 using a *MATLAB* script to produce the resulting functionally graded car-body in Figure 4.13b. A fixed face thickness of 0.3175 mm and a fixed core thickness of 7.9 mm are assumed for the whole car in accordance with the findings of the

Pattern Search in Table 3.5. The core wall width, L_c , and core wall thickness, t_c , are determined using a *MATLAB* script performing an optimization according to the problem:

$$\begin{aligned}
& \min_{\mathbf{x}} && F^{-1}(\mathbf{x}) \\
& \text{s.t.} && D(\mathbf{x}) - D_p \leq 0 \\
& && 5.77 - \frac{t_f + H_c}{t_f} \leq 0 \\
& && 16.7 - \frac{E_f}{E_c(\mathbf{x})} \frac{x_3}{x_4} \left(\frac{H_c + t_f}{H_c} \right)^2 \leq 0 \\
& && H_c \rho_f \left(\frac{2x_2}{1.5 \left(\frac{\sqrt{3}}{2} \right) x_1} \right) - \rho_r (0.00635 - 2t_f) \leq 0
\end{aligned} \tag{4.5}$$

where $\mathbf{x}_{LB} \leq \mathbf{x} \leq \mathbf{x}_{UB}$

and $\mathbf{x} = [L_c, t_c]$

where $F^{-1}(\mathbf{x})$ has been defined according to Equation 3.19. The problem has also been constrained such that the resulting core mass per unit area shall not exceed the mass per unit area of the baseline plate thickness of 6.35 mm minus the face mass per unit area multiplied by a relative density assigned according to Table 4.7. The solution to this problem has produced the core wall width, L_c , and thickness, t_c , providing the maximum possible strength for each functionally graded group, along with their resulting properties shown in Table 4.8.

Table 4.7: Relative Density Sorting Criteria

Group	Solid	Very High Density ($\rho_r = 0.5$)	High Density ($\rho_r = 0.3$)	Medium Density ($\rho_r = 0.25$)	Low Density ($\rho_r = 0.1$)	Very Low Density ($\rho_r = 0.05$)
Relative Density Range	$\rho \geq 0.7$	$0.5 \leq \rho < 0.7$	$0.3 \leq \rho < 0.5$	$0.2 \leq \rho < 0.3$	$0.1 \leq \rho < 0.2$	$\rho < 0.1$

As opposed to the panel-by-panel MSDO, the functionally graded MSDO has allowed for the further concentration of mass to areas of high structural criticality, which, by extension,

this has allowed for further mass reduction in areas of low load. The functionally graded quarter-model has a total mass of 2335.96 kg, which is the lightest of all designs produced thus far. Deeper investigation into the functionally graded model has not been performed in the present work, as at the time of writing such a design is not feasible for manufacturing. Instead, the functionally graded design is meant to establish a theoretical optimum for comparison to the panel-by-panel MSDO demonstrated in Stage-IIA. Further details of the work done to produce a functionally graded design have been published by Al-Sukhon and ElSayed in [103].

Table 4.8: Functionally Graded Core Properties

Group	Very High Density	High Density	Medium Density	Low Density	Very Low Density
Core Wall Length (L_c) (mm)	7.89	8.78	3.18	3.18	9.51
Core Wall Sheet Thickness (t_c) (mm)	0.90	0.91	0.28	0.11	0.17
Core Density (ρ_c) (kg/m ³)	1371	1259	1049	420.0	210.0
Elastic Modulus Long. (E_{1c}) (MPa)	694.58	540.35	314.30	20.31	2.54
Elastic Modulus Trans. (E_{2c}) (MPa)	690.63	537.75	313.23	20.30	2.54
In-Plane Shear Modulus (G_{12c}) (MPa)	425.04	329.66	190.78	12.21	1.53
Out-of-Plane Shear Modulus Long. (G_{13c}) (MPa)	3174.62	2916.78	2430.65	972.26	486.13
Out-of-Plane Shear Modulus Trans. (G_{23c}) (MPa)	5291.03	4861.30	4051.08	1620.43	810.22
Poisson Ratio (ν_{12c})	0.44	0.44	0.44	0.44	0.44

(a) Element density distribution plot used for functional grading

(b) Functionally graded sandwich panels as applied to the hopper car, where warmer colours indicate higher density elements

Figure 4.13: Resulting element density contour plot and corresponding functionally graded model for Stage IIB

Chapter 5

Results and Discussion

Over the course of the present work, several methodologies for optimization have been discussed. In Section 3.3.2.A, an analytical multi-objective optimization was investigated as a potential solution for deducing the optimal hexagonal honeycomb sandwich panels. While this methodology allowed for rapid generation of results, many assumptions had been made about the nature of the structure, such that it is simply supported and flat. Furthermore, the method used analytical cost functions of arbitrary form to perform the optimization, and therefore do not give the true final mass of the panel. Therefore, in Section 3.3.2.B, a MSDO was proposed which integrated FEM to provide a high-fidelity load case to generate a penalized cost function that is checked against the theoretical limits directly. In the previous chapter, a multi-stage structural optimization was carried out employing Topology Optimization to achieve a globally optimal structure. The TO was then coupled with an MSDO at the component level for determination of the design with the highest strength-to-rigidity ratio.

In this chapter, the results produced by the all optimization procedures are discussed. The chapter begins with a summary of the final candidate designs based on the analytical optimization in Section 3.3.2.A and those resulting from Section 4.3. Then, the success of the full MSDO optimization procedure is confirmed by checking all load cases and constraints for all produced candidate designs.

5.1 Candidate Designs

The resulting design variables from Table 3.5 have been implemented into the sub-model to determine their corresponding masses and compliances in the context of the hopper car. In Table 5.1, these final results from the analytical multi-objective optimization for all algorithms used along with the MSDO employing both the PS and GA algorithms. Alongside each result set, a summary of which dominant failure modes occurred if any constraint violations are observed.

5.2 Comparison of Algorithms

Based on the candidate designs, the MSDO employing the PS algorithm produces the design closest to the Utopia at the lowest computational cost. The generation of a Pareto front using the PS takes a mere 2,355 function evaluations, whereas the GA takes 16,453 while providing results within 4% of the PS. The computational speed and closeness of final results demonstrates the importance of good seed points for a local search algorithm. By providing good seed points, a local search algorithm produces similar results to a global search algorithm in a fraction of the time.

The analytical approach produces results similar to the MSDO, however it falls short of ensuring failure does not occur in practice. This is attributable to the fact that the analytical approach optimizes the sandwich panels without considering any actual loading and instead aims to maximize sandwich strength. As a result, a lightweight panel of high strength has been developed, however not one which has adequate strength under the actual loading conditions. It is therefore recommended that the analytical method be used in support of a design rather than on its own.

5.3 Full Scale Application of Results

So far, only one local optimization has been carried out on a single wall section in the hopper car. Two options are available for a potential designer to determine how to proceed from here: either the procedure can be repeated for all panels simultaneously, or each panel

Table 5.1: Final results from all methodologies, along with a commentary on which constraints are violated in the *Summary* column.

Methodology	Mass [kg]	Compliance [N/m]	Mass Savings [%]	Max Constraint Violation [%]	Summary
Baseline	983.63	20,678	-	-	Statistics of the baseline from Section 4.1 are shown here for reference. Only the mass of the sub-model is considered.
Analytical MSDO	PS	13,717	12.23	26.33	Resulting structure fails due to face yielding and excessive stresses in the beams. Maximum constraint violation due to face yielding.
	GA	13,660	12.19	30.08	Resulting structure fails due to face yielding, face wrinkling, and excessive stresses in the beams. Maximum constraint violation due to wrinkling.
	PS +FGA	13,755	13.05	26.55	Resulting structure fails due to face yielding and excessive stresses in the beams. Maximum constraint violation due to face yielding.
	GA +FGA	13,549	11.47	32.46	Resulting structure fails due to face yielding and excessive stresses in the beams. Maximum constraint violation due to wrinkling.
FEM-Integrated MSDO	PS	11,932	8.15	0.00	Resulting structure fully compliant.
	GA	11,801	7.71	0.00	Resulting structure fully compliant.

can be optimized individually as originally intended. It is not advantageous to perform a single optimization for all panels simultaneously treated as one; the result would simply be those results for the most conservatively loaded panel and would result in oversized panels in less critical regions. It is instead suggested that each panel be optimized one by one from this point onwards. Although a comprehensive design optimization would employ the local MSDO procedure for each panel one after the other, for the sake of brevity and to determine the effectiveness of the proposed design procedure the results of the only panel optimized will be extrapolated for the all side walls and roofs in the hopper car. The design variables determined by the MSDO approach employing the PS algorithm is used since it was determined to have produced the lowest mass for the least amount of function evaluations, while also ensuring failure does not occur.

The final extrapolated model has been re-run under the loading conditions used for optimization. Maximum stresses observed for each mechanical element are shown in Table 5.2, and sample von Mises stress contour plots in the faces are shown in Figure 5.1. For 2D elements, center stresses with stress averaging applied are displayed for maximums, and minimum principal stresses at the centers for the minimums. For beams, the maximum von Mises and minimum normal stresses are displayed. For the allowable stresses in the sandwich panels Equations 3.16a through 3.16e are used. In this case, wrinkling is found to be the dominant local buckling case, therefore all minimum principal stresses must adhere to its failure stress.

Table 5.2: Stress results from extrapolation of optimized side panel to all wall and roof components of the structure based on results from Stage-IIA.

Component	Stress Type	Observed Stress [MPa]				Allowable Stress [MPa]
		LVL	DTL	ECL	FSI	
Faces	Max VM	169.9	373.7	367.0	396.6	423.3
	Min P3	-173.7	-265.6	-322.8	-329.9	-516.6
Core	Shear XZ, YZ	37.9	131.6	29.7	126.2	174.56
Shells	Max VM	414.5	410.7	415.1	457.6	423.3
Beams	Max VM	173.4	491.6	429.5	416.7	423.3
	Min Norm	-89.25	-131.1	-288.8	-216.7	-185.8

Figure 5.1: Element stress contour plots for maximum von Mises face stresses in element centers. Stress averaging has been applied for post-processing.

The lowest natural frequency exhibited by the extrapolated model is 12.08 Hz, which is just above the allowable minimum of 12.0 Hz. It should be noted that for the compressive beams, only the minimum principal stresses in the third principal direction are shown and only the minimum allowable stress among all beams is shown. For this reason, it is not necessarily true that by virtue of having a lower than acceptable stress there is crippling failure observed. For instance, the beam which gives the allowable stress in the table above undergoes a stress of -32.7 MPa in its worst compressive load case. The inclusion of the minimums is merely to show the proximity to which the final extrapolated result is able to produce a functional design. One failure in the structure does occur however; there is excessive von Mises stress in the shells under the FSI load case. The constraint violation here, however, is a mere 8.1% and can be quickly resolved through refinement of the final design and does not imply major infeasibility of the structure over all.

5.4 Mass Savings

The ultimate purpose of this thesis has been to develop an optimization procedure which will implement sandwich panels into a covered hopper car to reduce the weight of the structure. In this endeavour the present work has been successful; the full scale application of the results from the PS based MSDO in Table 4.6 produce a quarter model with a mass of 2,865.73 kg, whereas the baseline quarter model has a mass of 4,259.4 kg, and the topology optimized baseline has a mass of 3,322.1 kg. This implies that an average mass savings of 32.72% can be anticipated by employing all optimization techniques discussed in Chapter 4, while an average mass reduction of 13.74% can be expected over the use of conventional materials. The functionally graded model had a final mass of 2335.96 kg, offering a structure which is 18.5% lighter than that produced by the Stage-IIA MSDO and 45.2% lighter than the baseline. Through this finding, it has been demonstrated that while the Stage-IIA MSDO can provide a lightweight design, further reduction in mass can be achieved through functional grading. Unfortunately, at the time of writing, manufacturing techniques for a functionally graded hopper car have not reached readiness for full production. Future work should, however, investigate functionally graded hopper car structural designs for the time when such manufacturing techniques have been brought forward.

By virtue of using sandwich panels, the mass of a hopper car can be significantly reduced, and by extension GHG emissions. Assuming that half the mass of a hopper car is located in the car-body and that the extrapolation discussed in Section 5.3 holds true, a mass reduction of 16.36% can be expected over the whole hopper car. As a result, based on the findings in [10] a reduction in GHG emissions of between 8.18% and 13.09% per car is possible by employing the structural weight reduction techniques in this thesis. While such savings can be statistically insignificant on the national level, extension of this design methodology to further vehicles such as passenger rail, automotive and aerospace can result in potentially similar GHG reduction.

5.5 Improvements in Rigidity

The sum of all compliances by load case for the full structure is 20,007 m/N, which can be explained by the fact that the majority of the solid structural material has been removed therefore increasing compliance significantly. The displacements in the full model are 3.44 cm for the LVL, 4.03 cm for the DTL, 1.80 cm for the ECL, and 3.75 cm for the FSI, all of which are compliant with the displacement constraints in Table 4.1 save for the LVL. The resulting compliance of the final design does still exhibit a compliance which is 3.24% lower than the baseline model, indicating that by employing sandwich panels and re-locating and re-sizing the beams the mass has been reduced without compromising on rigidity.

Chapter 6

Conclusions and Future Work

In the present work, parametric studies have been conducted to investigate two candidate structural sub-assemblies: a semi-monocoque wall, and a sandwich panel with varying core topologies. The double wall stiffened panel has been demonstrated to be viable under bending loads due to the ability for one side of the wall being designed as a compression panel, while the other was designed as a tension panel. The proposed new construction was compared analytically to the performance of sandwich panels under bending and, while it was found that sandwich panels largely reign superior in such a loading regime, the viability of a double-walled stiffened panel has been proven. It was also demonstrated in the parametric study that of all sandwich core topologies that hexagonal honeycomb cores provide the highest strength-to-weight ratio in the relevant load regimes. Coupled with the added benefits of bi-directional rigidity of hexagonal honeycomb cores, the core topology is the clear choice for the majority of applications.

The information learned from parametric studies was drawn upon for the numerical optimization of the sandwich panels for use in the car-body of a hopper car. Two main numerical approaches were considered: an analytical approach for rapid design generation and a MSDO employing FEM. The analytical approach employed equations from sandwich beam theory, sandwich failure theory and core homogenization theory to determine the ideal design variables using a multi-objective search algorithm without the need for considering the load. The MSDO employed the same theories, but did consider the load on the sandwich panel through the use of FEM with loading applied according to the AAR MSRP and

determined the optimal solution by finding the Utopia point on a Pareto front shared by the mass and compliance of the hopper car. In addition to the loads promulgated by the regulatory bodies, a fluid-structure interaction was considered using SPH in an explicit FEM simulation. Equivalent static loads were transferred from the explicit model using a simplified version of the equivalent static loads method according to [81] for critical time steps only to reduce simulation time. It was found that ESLM produces approximate loading conditions to the dynamic loads at the critical time step, with maximum stress within 5% of one another in both cases. It was also found that the ESLM results in a much more conservative design case as compared to its implicit counterpart in the MSRP employing traditional linear static loading, and therefore can produce a reliable design.

Both the analytical and MSDO procedures provided useful designs, however the analytical approach resulted in designs of inferior strength which could not bear the imposed load. This result is to be expected, since the analytical approach cannot consider loading directly and ensures structural strength by employing a generalized failure load function. The analytical approach, however, has the advantage of rapid results generation due to the swiftness of the function evaluations. The MSDO, on the other hand, produces fully compliant designs, despite coming at a large computational cost due to the FEM being re-evaluated for each function evaluation. All solutions employed the Pattern Search and Genetic Algorithms to generate their Pareto fronts. It was found during the analytical optimization that hybridizing these algorithms with `fgoalattain` does not provide meaningful improvement in the results for the added computational cost and was therefore not employed in the MSDO. A Utopia point is determined by optimizing purely for mass and compliance separately, then the point in closest proximity to it was selected for the final design for both approaches. For the MSDO, the PS and GA provided results within 4% of one another, implying that there is no benefit to a global search if the search space can be narrowed. As a result, for both approaches implementation of seeding points was critical; by implementing seeding points, results of the PS are similar to the GA since the search is confined to the region of lowest cost. Seeding points can be generated by individual optimizations while determining the Utopia point, as well as by generating response surfaces of the cost functions and determining the designs of minimum cost therein. An additional

method of generating good design points can be to first use the analytical approach for rapid design generation under various circumstances, and to then implement its results as seeding points in the MSDO.

Of the two approaches, the MSDO is recommended for future work since fully compliant designs are produced by it. Under the MSDO, the PS algorithm is the superior algorithm of choice due to its low cost, yet only provided that good seeding points can be established. Selection of the best point by means of proximity to the Utopia point proved successful. By extrapolating the results of the characteristic panel optimized by the MSDO to the full structure, mass savings in the car-body of 32.72% can be expected by coupling the MSDO in a multi-stage design optimization where Topology Optimization is used to determine the optimal layout of the frame. Assuming that the car-body comprises half the structural material in a hopper car, 16.36% mass savings can be expected for each hopper car. According to the findings of Helms *et al.*, the resulting reduction in GHG emissions can be somewhere between 8.18% and 13.09%. Further investigation of hexagonal honeycomb sandwich panels for implementation in hopper cars is encouraged; studies considering other design factors are suggested in Section 6.2, where future work is discussed.

6.1 Contributions to the State of the Art

This thesis has made the following contributions to the state of the art:

- A fluid structure interaction between a hopper car and its fluid contents has been successfully modeled. While the model requires refinement through validation and further study of the dynamics of its granular contents, the model is capable of predicting the stresses induced on a hopper car under dynamic loading situations.
- The equivalent static loads due to the FSI were effectively determined for the creation of an implicit model, thereby reducing the solve time significantly albeit providing conservative results.
- A numerical multi-objective optimization has been synthesized according to well established theories concerning sandwich core homogenization according to Burton and

Noor [90], sandwich beam theory according to Allen [34], and sandwich failure theory according to [66]. The analytical approach rapidly provides starting points for early design iterations which produce lightweight and strong designs. The generation of a Pareto front allows for a designer to visualize the trade-off between mass and strength.

- Topology optimization has been successfully implemented for determination of a novel frame layout for a lighter hopper car using conventional materials. The novel hopper car structure concentrates the mass of the car-body towards critical regions undergoing relatively high loads. Since the mass is re-prioritized, an avenue has been opened up for implementation of novel lightweight materials in place of heavy conventional steel.
- The topology optimization has been coupled with a novel multi-scale design optimization procedure for hexagonal honeycomb sandwich panels. The algorithm homogenizes the core properties according to the design variables at the micro-scale and checks for failure at the macro-scale. Through this coupled procedure, a preliminary design for a novel lightweight hopper car employing optimized honeycomb sandwich panels has been produced.
- A theoretical optimum has been established by devising a density-based approach to establishing a functionally graded hopper car-body structure. By employing functionally graded sandwich panels, high density hexagonal honeycomb sandwich panels can bear load in critical regions of the hopper car, while low density panels are substituted in regions of low criticality.
- Thus far, the design procedure has been tested for only a select set of load cases, however the addition of other load cases can result in a change in design results. Though the load cases considered are not comprehensive, it is expected that those tested represent the majority of the expected loading on a hopper car and will not invalidate the design procedure.

6.2 Future Work

The following items should be addressed in future work for the full development of a hopper car employing hexagonal honeycomb sandwich panels:

- In the present work, the FSI load case has not been validated based on experimental data. Studies should be performed on the contents of the hopper car to ensure the accuracy of the cargo fluid properties using SPH. Furthermore, a complete battery of loading tests should be performed on a fully laden hopper car to determine the real stresses induced in the hopper car structure as a result of the motion of the fluid.
- Structural damping was not considered for the explicit model, which results in a fluctuation in the loads experienced by the structure. A sensitivity study coupled with experiments should be performed to determine realistic Rayleigh damping coefficients for the explicit model.
- This thesis has not considered impact loading in its optimization processes. While honeycomb sandwich panels have demonstrated good resistance to low and high velocity impact, it is not certainly true that this translates to the hopper car designed at this time. A suggested course of action would be to repeat the entire proposed design procedure for impact loading and then overlap the two designs. Impact loading can be estimated in an implicit fashion using dynamic loading factors, or explicitly through investigation of crash mechanics.
- Global buckling of the structure has not been considered in the present work. Buckling presents a challenge on its own since it is non-trivial to determine which load case presents concerns for buckling of the structure. Performing an investigation into areas of the hopper car which are susceptible to buckling followed by employing an MSDO explicitly for prevention of buckling is proposed for future work. Resulting designs can then be overlapped with the current design to provide a buckling resistant structure.
- At the time of writing, no consideration of fatigue has been undertaken in the numerical optimization of the novel hopper car structure. Testing should be performed on all

final designs to ensure the novel hopper car structure adheres to Chapter 7 of Section C-II in the MSRP, which outlines conditions for failure prevention in the context of fatigue.

- At this time, only preliminary implementation of hexagonal honeycomb sandwich panels has been considered. Manufacturability of the final design is not investigated, nor are any other practicalities which designers commonly face such as fastening, corrosion, resistance to the elements or financial viability. These factors should be considered prior to full implementation.
- A functionally graded hopper car structure has been proposed and briefly investigated in the later chapters of this thesis. At the time of writing, functionally graded hopper cars are of low technology level readiness, however with the advancement of large-scale additive manufacturing, such a hopper car structure is proposed for further investigation for a time when the means of production are available.

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Appendix A

Sizing Results

A.1 Baseline Model Sizing Results

Table A.1: Baseline Model Beam Sizing Results

Beam Number	Base (B) [mm]	Height (H) [mm]	Thickness 1 (T_1) [mm]	Thickness 2 (T_2) [mm]
1	99.95	100.00	3.70	3.18
2	64.09	65.97	5.54	5.59
3	100.00	100.00	15.88	15.88
4	85.14	100.00	6.15	3.43
5	82.06	81.43	4.00	4.02
6	88.89	89.05	6.07	9.89
7	50.70	50.00	3.18	3.18
8	70.20	100.00	5.20	3.31
9	65.88	66.61	3.74	3.72
10	57.85	100	15.88	15.88
11	98.28	99.96	3.42	3.18
12	58.56	100	4.87	3.31
13	50.00	50.00	3.32	3.32
14	100.00	100.00	15.88	15.88
15	50	65.82	4.13	3.61
16	50	50.00	3.18	3.18
17	50	50.00	3.33	3.32
18	84.12	83.74	6.53	6.54
19	50.00	50.00	3.79	3.79
20	57.97	54.99	4.80	4.78
21	90.13	76.48	5.08	5.42
22	50.00	50.00	3.25	3.25

Table A.2: Baseline Model Shell Sizing Results

Panel	Thickness (T_P)
	[mm]
Side Panel 1	3.079
Side Panel 2	3.302
Side Panel 3	7.627
Side Panel 4	5.629
Side Panel 5	7.991
Back Panel 1	11.74
Back Panel 2	1.079
Roof Panel 1	3.196
Roof Panel 2	4.029
Roof Panel 3	4.266
Roof Panel 4	5.078
Roof Panel 5	3.274
Floor Panel 1	1.552
Floor Panel 2	1.442
Floor Panel 3	12.57
Floor Panel 4	12.09
Floor Panel 5	8.756
Floor Panel 6	12.70
Floor Panel 7	8.640
Floor Panel 8	8.230
Floor Panel 9	6.634
Floor Panel 10	11.44

A.2 Topology-Optimized Model Beam Sizing Results

Table A.3: Topology-Optimized Model Beam Sizing Results

Beam Number	Base (B) [mm]	Height (H) [mm]	Thickness 1 (T_1) [mm]	Thickness 2 (T_2) [mm]
1	87.67	99.40	3.38	3.18
2	69.15	68.86	4.70	4.71
3	99.98	96.49	5.43	6.22
4	96.29	99.13	3.18	3.18
5	80.04	62.74	3.18	3.18
6	98.58	98.44	5.56	6.97
7	66.90	80.31	4.68	3.72
8	60.74	97.21	3.82	3.18
9	77.51	81.51	3.59	3.75
10	99.96	99.96	15.86	15.87
11	92.82	99.99	8.07	3.18
12	96.51	99.98	11.2	5.24
13	85.29	85.87	4.83	4.83
14	99.69	99.70	15.84	15.84
15	98.94	99.99	9.02	6.34
16	96.51	99.99	5.06	3.48
17	57.66	70.36	3.38	3.18
18	51.71	51.51	4.51	4.87
19	79.92	97.50	9.27	3.18
20	95.81	99.98	7.95	5.25
21	75.72	54.44	3.96	4.51
22	55.15	68.23	3.65	3.18
23	98.07	99.33	6.77	6.15
24	88.87	99.45	7.02	4.35
25	95.14	97.75	7.32	5.45
26	60.35	93.49	3.18	3.18
27	99.96	63.55	3.64	4.16
28	73.73	65.19	3.57	5.07
29	68.41	54.44	3.18	4.21
30	99.99	99.95	5.38	8.20
31	74.54	99.83	3.18	3.18
32	50.00	50.00	3.18	3.2
33	50.00	50.00	3.18	3.18
34	50.00	50.00	3.18	3.18
35	96.34	99.93	4.72	6.79
36	88.77	99.01	4.77	4.28
37	50.86	50.85	3.18	3.18
38	51.19	50.32	4.01	4.06
39	55.18	53.26	4.05	4.10
40	89.49	81.99	6.61	6.52
41	95.71	86.78	6.63	7.22
42	93.77	93.92	5.85	5.79
43	95.91	99.99	8.90	6.64
44	93.41	99.99	14.25	5.28
45	99.96	100.00	9.66	3.50
46	82.35	100.00	6.19	3.21
47	99.99	96.23	3.77	6.77
48	99.99	97.69	5.14	8.85
49	99.99	95.81	3.18	6.35

Appendix B

RADIOSS Deck

In this appendix, the cards employed in the *RADIOSS* Deck are shown for the reader. Note that due to the size of the model, the mesh geometry has not been included here. Rather, cards for material data, properties, and contact interfaces have been listed in the hopes of assisting future researchers. All data on these cards are in standard SI units (kg, m, s). In *RADIOSS*, data entries in the cards are organized into numbered columns of 10 characters. Ensure when entering any of the data below that all numbers are within their required columns.

B.1 Starter Deck (SPH_HopperCar_0000.rad)

```
#-----|
#--1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
##
/BEGIN
Test1
      2019          0
                kg          m          s
                kg          m          s
##
##-----|
## Title
##-----|
/TITLE
SPH_HopperCar
##-----|
## Input-Output Flags
##-----|
/IOFLAG
                                     1
/SPHGLO
#      Alpha_sort  Nghost  Lneigh  Nneigh
                0.25      0      160      0
/AMS
      0
##-----|
## Material Law No 1. ELASTIC
##-----|
#--1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
#- 2. MATERIALS:
#--1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
/MAT/ELAST/1
Steel
#      RHO_I
                7700.0          0.0
#      E          nu
      210000000000.0          0.28
##-----|
## Material Law No 10. ROCK-CONCRETE
##-----|
#--1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
/MAT/LAW10/2
Soil
#      RHO_I
                850.0
```

```

#          E          Nu
#      26812945.0      0.25
#          A0          A1          A2          Amax
#      3338500000000.0      2075700.0      0.32261.300000000000000E+20
#          C0          C1          C2          C3
#          0.0      25600000000.0      25600000000.0      100000000000.0
#          P_min      PSH
#      -1.0000000000000E+30      101300.0
#          BUNL      MUMAX
#      144000000000.0      0.44

##-----
## Shell Property Set (pid 1)
##-----
#---1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
# SPHCEL elements of /PART/17/Cargo:
#---1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
#---1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
#- 6. GEOMETRICAL SETS:
#---1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
#          hm          hf          hr          dm          dn
#          N      Istrain      Thick      Ashear      Ithick      Iplas
# /PROP/SHELL/1
# Plate
#      Ishell      Ismstr      Ish3n      Idrill
#          24          0          0          0
#          5          1          0.00635          0.0          0.0          0.0
#---1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
# /PROP/SHELL/3
# ThickPlate
#      Ishell      Ismstr      Ish3n      Idrill
#          24          0          0          0
#          hm          hf          hr          dm          dn
#          0.0          0.0          0.0          0.0          0.0
#          N      Istrain      Thick      Ashear      Ithick      Iplas
#          5          1          0.016          0.0          0          0
##-----
## Beam Property Set (pid 3)
##-----
#---1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
##HMBEAMSECASSOC      PROPASSOC      1
# OmegaDof      Ishear
# /PROP/BEAM/6
# CBeam
#          Ismstr
#          4
#          dm          df
#          0.0          0.01
#          Area          Iyy          Izz          Ixx
#          0.001743711.38135107799167E-061.38135107799167E-062.76270215598334E-06
#          000 000          0
#---1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
##HMBEAMSECASSOC      PROPASSOC      2
# OmegaDof      Ishear
# /PROP/BEAM/7
# IBeam
#          Ismstr
#          4
#          dm          df
#          0.0          0.01
#          Area          Iyy          Izz          Ixx
#          0.009202421.80754437973933E-041.69887420934833E-051.97743180067416E-04
#          000 000          0
##-----
## SPH Property Set
##-----
#---1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
# /PROP/TYPE34/8
# sph_6
#          mp          qa          qb          alpha_cs
#          0.404241795145725          2.0          1.0          0.0          0
#          order          h          xi_stab
#          0          0.075          0.3
##-----
## Functions
##-----
#---1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
#- 7. FUNCTIONS:
#---1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|

```



```

##HWCOLOR curves 1 4
/FUNCT/1
Gravity
#
      X      Y
      0.0    9.81
      4.0    9.81
#---1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
##HWCOLOR curves 2 5
/FUNCT/2
Lateral
#
      X      Y
      0.0    0.0
      0.9    0.0
      1.2    5.0
      2.0    5.0
      2.5    0.0

##-----
## Gravity Loads
##-----
#---1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
#- 8. GRAVITIES:
#---1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
##HWCOLOR loadcollectors 3 63
/GRV/3
Gravity
##funct_IDT      DIR      skew_ID sensor_ID      grnod_ID      Ascale_x
      1          Z          0          0          5          1.0000000000000000 -1.0000000000000000
/GRNOD/PART/5
Loads
      1          2          3          4          5          6          7          8          9          10
      11         14         15         17
#---1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
#---1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
#- 9. INTERFACES:
#---1---|---2---|---3---|---4---|---5---|---6---|---7---|---8---|---9---|---10---|
##HWCOLOR loadcollectors 4 63
/GRV/4
Lateral
##funct_IDT      DIR      skew_ID sensor_ID      grnod_ID      Ascale_x
      2          X          0          0          5          1.0000000000000000 1.0000000000000000
##-----
## Type 7 Interfaces
##-----
##HMGROUP 1 4
/INTER/TYPE7/1
ShellSPH
## Slav_id Mast_id Istf Ithe Igap Ibag Idel
      13      1      0      0      0      0      0      0
#      Fscalegap Gap_max Fpenmax
      0.0      0.0      0.8
#      Stmin Stmax %mesh_size dtmin Irem_gap Irem_i2
      0.0      0.0      0.0      0.0      0
#      Stfac Frac Gapmin Tstart Tstop
      0.0      0.0      0.0      0.0      0.0
#      IBC Inacti VisS VisF Bumult
      000      5      0.05      1.0      0.2
# Ifric Ifiltr Xfreq Iform sens_ID fct_IDf AscaleF
      0      0      0.0      0      0      0      0.0

```

B.2 Engine Deck – Settling Phase (SPH_HopperCar_0001.rad)

```

/VERS/2017
/RUN/Test1/0/
0.9000000000000000
/ANIM/VECT/ACC
/ANIM/VECT/CONT
/ANIM/VECT/DISP
/ANIM/VECT/VEL
/ANIM/VECT/VROT
/ANIM/VECT/DROT
/ANIM/SHELL/TENS/STRESS/UPPER
/ANIM/SHELL/TENS/STRESS/LOWER

```

```

/ANIM/SHELL/TENS/STRESS/ALL
/ANIM/SHELL/TENS/STRAIN/UPPER
/ANIM/SHELL/TENS/STRAIN/LOWER
/ANIM/SHELL/TENS/STRAIN/ALL
/ANIM/ELEM/ENER
/ANIM/ELEM/VONM
/ANIM/ELEM/HOURG
/ANIM/DT
0.0000000000000000 0.0500000000000000
/PRINT/-1000
/TFILE/O
0.0100000000000000
/MON/ON
/ANIM/NODA/DMA5
/KEREL

```

B.3 Engine Deck – Sloshing Phase (SPH_HopperCar_0002.rad)

```

/VERS/2017
/RUN/Test1/0/
2.5000000000000000
/ANIM/VECT/ACC
/ANIM/VECT/CONT
/ANIM/VECT/DISP
/ANIM/VECT/VEL
/ANIM/VECT/VR0T
/ANIM/VECT/DR0T
/ANIM/SHELL/TENS/STRESS/UPPER
/ANIM/SHELL/TENS/STRESS/LOWER
/ANIM/SHELL/TENS/STRESS/ALL
/ANIM/SHELL/TENS/STRAIN/UPPER
/ANIM/SHELL/TENS/STRAIN/LOWER
/ANIM/SHELL/TENS/STRAIN/ALL
/ANIM/ELEM/ENER
/ANIM/ELEM/VONM
/ANIM/ELEM/HOURG
/ANIM/DT
0.0000000000000000 0.0500000000000000
/PRINT/-1000
/TFILE/O
0.0100000000000000
/MON/ON
/ANIM/NODA/DMA5
/DT/NODA/CST/O
0.9000000000000000 4.200000000000000E-06

```

Appendix C

Derivation of Hexagonal Honeycomb Core Three-Parameter Optimization Equations

In Section 2.1.2, the three-parameter optimization according to Rathbun *et al.* was performed for several core topologies [23]. Among those core topologies was the hexagonal honeycomb sandwich panels, which were derived according to the proceeding methodology.

From Wicks and Hutchinson, the weight per unit area of a sandwich panel is given by the equation [30]:

$$W = \rho \left(\frac{2H_c t_c}{\sqrt{3}L_c} + 2t_f \right) \quad (\text{C.1})$$

Then, by dividing the equation by ρ and ℓ , the following is obtained:

$$\frac{W}{\rho \ell} = \Psi = \frac{2H_c t_c}{\sqrt{3}L_c \ell} \quad (\text{C.2})$$

which then gives the unitless weight index for the sandwich panel having a hexagonal honeycomb sandwich panel as:

$$\Psi_{HC} = \frac{2\lambda_c}{\xi\sqrt{3}} + 2\lambda_f \quad (\text{C.3})$$

Next, from Wicks and Hutchinson, the four constraints of relevance for the hexagonal honeycomb sandwich panel are [30]:

$$\frac{M}{t_f H_c \sigma_y} \leq 1 \quad (\text{Face Yielding}) \quad (\text{C.4a})$$

$$\frac{1728(1 - \nu^2)L_c^2 M}{379\pi^2 E_f t_f^3 H_c} \leq 1 \quad (\text{Face Buckling}) \quad (\text{C.4b})$$

$$\frac{\sqrt{3}V L_c}{t_c H_c \tau_y} \leq 1 \quad (\text{Core Yielding}) \quad (\text{C.4c})$$

$$\frac{\sqrt{3}V L_c}{t_c^3 H_c g(L_c, t_c, H_c)} \leq 1 \quad (\text{Core Buckling}) \quad (\text{C.4d})$$

where:

$$g(L_c, t_c, H_c) = \begin{cases} \frac{\pi^2 E}{12(1-\nu^2)} \left[5.35 \frac{t_c^2}{H_c^2} + 4 \frac{t_c^2}{L_c^2} \right], & \text{if } L_c > H_c \\ \frac{\pi^2 E}{12(1-\nu^2)} \left[5.35 \frac{t_c^2}{L_c^2} + 4 \frac{t_c^2}{H_c^2} \right], & \text{if } H_c > L_c \end{cases} \quad (\text{C.5})$$

In the following, the constraints defined above are then converted into a dimensionless format in line with those in the three-parameter optimization procedure according to Rathbun *et al.* by employing the relationships $\sigma_y = \varepsilon_y E$, $\ell = M/V$ and $\Pi = V^2/EM$ [23]. Beginning with face yielding:

$$\begin{aligned} \frac{M}{t_f H_c \sigma_y} \left(\frac{1/\ell^2}{1/\ell^2} \right) &\leq 1 \\ \frac{\Lambda_c^{-1} \lambda_f^{-1}}{\varepsilon_y} \Pi &\leq 1 \end{aligned} \quad (\text{C.6})$$

Then, for the face buckling, assuming $L_c > H_c$, the constraint inequality becomes:

$$\begin{aligned} \frac{1728(1-\nu^2)L_c^2 M}{379\pi^2 E_f t_f^3 H_c} \left(\frac{H_c}{H_c} \right) \left(\frac{1/\ell^3}{1/\ell^3} \right) &\leq 1 \\ \frac{1728(1-\nu^2)\xi^2 \Lambda_c V^2}{379\pi^2 E \lambda_f^3 M} &\leq 1 \\ \frac{1728(1-\nu^2)}{379\pi^2} \xi^2 \Lambda_c \lambda_f^{-3} \Pi &\leq 1 \end{aligned} \quad (\text{C.7})$$

For core yielding, using the von Mises stress criterion that:

$$\tau_y = \frac{\sigma_y}{\sqrt{3}} \quad (\text{C.8})$$

the failure limit for core yielding then becomes:

$$\frac{3VL_c}{t_c H_c E \varepsilon_y} \leq 1 \quad (\text{C.9})$$

Now, the equation is modified according to:

$$\begin{aligned} \frac{3VL_c}{t_c H_c E \varepsilon_y} \left(\frac{1/\ell}{1/\ell} \right) &\leq 1 \\ \frac{3V^2 \xi}{\lambda_c E M \varepsilon_y} &\leq 1 \\ \frac{3\xi}{\lambda_c \varepsilon_y} \Pi &\leq 1 \end{aligned} \quad (\text{C.10})$$

Finally, for core buckling:

$$\begin{aligned} \frac{\sqrt{3}VL_c}{t_c H_c g(L_c, t_c, H_c)} \frac{1/\ell}{1/\ell} &\leq 1 \\ \frac{\sqrt{3}V^2 \xi}{\lambda_c M} &\leq 1 \end{aligned} \quad (\text{C.11})$$

Now, observing Equation C.5 isolated from the remainder of the inequality:

$$\frac{12(1-\nu^2)}{\pi^2 E \left[5.35 \frac{t_c^2}{H_c^2} + 4 \frac{t_c^2}{L_c^2} \right]} = \frac{12(1-\nu^2)}{\pi^2 E \left[5.35 \frac{t_c^2}{H_c^2} + 4 \frac{t_c^2}{L_c^2} \right]} \frac{L_c^2}{L_c^2} = \frac{12(1-\nu^2)(1/\ell^2)}{\pi^2 E_f \lambda_c^2 (5.35\xi^2 + 4)} = \frac{12(1-\nu^2)\Lambda_{cc}^2 \xi^2}{\pi^2 E_f \lambda_c^2 (5.35\xi^2 + 4)} \quad (\text{C.12})$$

Then, taking the resulting solution above and substituting into Equation C.11 and applying the same method for $H_c > L_c$, the final equation shown is achieved:

$$\Pi \Lambda_c^2 \lambda_c^{-3} g(\xi, \nu) \leq 1 \quad (\text{C.13})$$

with $g(\xi, \nu)$ defined as:

$$g(\xi, \nu) = \begin{cases} \frac{12\sqrt{3}(1-\nu^2)\xi^3}{\pi^2(5.35+4\xi^2)}, & \text{if } \xi \leq 1 \\ \frac{12\sqrt{3}(1-\nu^2)\xi^3}{\pi^2(5.35\xi^2+4)}, & \text{if } \xi > 1, \end{cases} \quad (\text{C.14})$$

In summary, the non-dimensional weight index and constraint equations for the hexagonal honeycomb sandwich panel are defined as:

$$\frac{2\lambda_c}{\xi\sqrt{3}} + 2\lambda_f \quad (\text{Weight index}) \quad (\text{C.15a})$$

$$\Pi \frac{\Lambda_c^{-1} \lambda_f^{-1}}{\varepsilon_y} \leq 1 \quad (\text{Face Yielding}) \quad (\text{C.15b})$$

$$\Pi \frac{1728(1-\nu^2)}{379\pi^2} \xi^2 \Lambda_c \lambda_f^{-3} \leq 1 \quad (\text{Face Buckling}) \quad (\text{C.15c})$$

$$\Pi \frac{3\xi \lambda_c^{-1}}{\varepsilon_y} \leq 1 \quad (\text{Core Yielding}) \quad (\text{C.15d})$$

$$\Pi g(\xi, \nu) \Lambda_c^2 \lambda_c^{-3} \leq 1 \quad (\text{Core Buckling}) \quad (\text{C.15e})$$

From these equations, Equations 2.21a through 2.21d are realized, along with the associated constants, by comparison to the generalized three-parameter equations from Rathbun *et al.* [23].