Air-Borne Sound Transmission through Triple-Leaf Walls

Modelling, Measurement and Optimization

by

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Abstract

The Midply™ triple-leaf shear resistive wall is designed by FPInnovations and UBC to be employed in mid-rise wood building. Compared to double-leaf structures, this wall has a weaker low-frequency sound insulation due to the additional resonance created by the middle-leaf. The original contribution of this thesis is developing a method to predict the air-borne sound transmission through triple-leaf walls, which can incorporate perforated plates. The model is based on a modified Transfer Matrix Method (TMM) that accounts for the losses at the perimeter of the finite cavity. The air-borne sound transmission tests performed on simplified small-scale structures showed that the modified TMM model has acceptable predictions in most frequencies, although Statistical Energy Analysis (SEA) was superior for high-frequency predictions. The research suggests that the sound insulation in triple-leaf structures could be improved through careful perforation of the middle-leaf, which is suggested for future work.

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NOMENCLATURE

Roman Symbols

A	Equivalent absorption in room [m ²]
c_o	Speed of sound in air [m/s]
D	Bending stiffness of plate [Pa.m ³], Directional energy density [m]
d	Hole diameter [m]
E	Energy [J], Modulus of elasticity [Pa]
e	Energy density [W]
f_{oi}	Resonance frequency of multilayered system , i-th [Hz]
f_s	Schroeder frequency [Hz]
j	Imaginary unit

k'_{cav}	Cavity stiffness [Pa/m]
k'_o	Complex wavenumber [rad/m]
k_b	Wavenumber in plate, flexural [rad/m]
$k_{o,z}$	Wavenumber in air, z-direction [rad/m]
k_o	Wavenumber in air [rad/m]
k_r	Wavenumber amplitude of radiated power [rad/m]
L	Depth of cavity [m]
l	Dimension of finite structure (height and width) [m]
L_E	Energy level [dB]
L_p	Sound pressure level [dB]
M	Modal overlap
m'	Surface density [kg/m ²]
N	Number of modes
n	Modal density
p	Sound pressure amplitude [Pa]
r	Velocity ratio, air to plate [–]
S	Surface area of structure [m ²]

s	Porosity ratio $[-]$
T	Reverberation time $[s]$
t	Thickness $[m]$
TL	Sound transmission loss $[dB]$
TL_o	Sound transmission loss, normal incidence $[dB]$
U	Perimeter length $[m]$
V	Volume of room $[m^3]$
v	Normal velocity amplitude $[m/s]$
v_h	Air velocity in holes of perforated plate $[m/s]$
W	Power $[W]$
Z	Wave impedance $[Pa.s/m]$
Z_h	Air impedance inside hole $[Pa.s/m]$
Z_o	Characteristic impedance of air $[Pa.s/m]$

Greek Symbols

η	Loss factor $[-]$
η_o	Viscosity coefficient of air $[kg/m.s]$
λ	Wavelength $[m]$

ν	Poisson's ratio $[-]$
ω	Angular frequency $[\text{rad/s}]$
ρ_o	Density of air $[\text{kg/m}^3]$
σ	Radiation efficiency $[-]$
τ	Transmission coefficient $[-]$
θ	Angle of incidence, normal $[\text{rad/s}]$

Abbreviations

SEA Statistical Energy Analysis

TMM Transfer Matrix Method

Subscripts

c	Critical
cav	Cavity
co	Coincidence
$double$	Double-leaf structure
i	Index of subsystem
ij	From subsystem i to j
inc	Incident

p Solid plate

pp Perforated plate

r Reflected

$react$ Reactive, imaginary part

$resist$ Resistive, real part

t Transmitted

$triple$ Triple-leaf structure

Superscripts

d Dissipated

in Input

t Total

Chapter 1

Introduction

1.1 Preface

Double-leaf studded wood walls are commonly used assemblies in low-rise lightweight buildings. However this type of wall is less suitable for mid-rise buildings, since it performs poorly under shear loads. Main sources of this type of load are wind and earthquake loads. One of the structures designed and developed by FPInnovations¹ and the UBC Civil Engineering Department² that can effectively withstand shear loads is the Midply™ triple-leaf studded wood shear wall [1, 2], shown in Figure 1.1. This symmetric structure consists of a wood framed double panel wall and a shear resistive layer in the middle, which is attached on both sides to the outer

¹Private, non-profit scientific research centre, specializes in the creation of scientific solutions in support of the Canadian forest sector, <http://fpinnovations.ca/>

²Visit website: <http://www.civil.ubc.ca>

panels through studs and resilient channels. The composition of the wall (from the top layer to the bottom layer) is given in Table 1.1 (left to right) [1, 2, 3, 4].

Figure 1.1: The triple-leaf wood wall

Element	Gypsum board	Resilient channels	Wood Studs	Plywood shear layer	Wood Studs	Resilient channels	Gypsum board
Depth [mm]	26	26	76	13	76	26	26
Mass [kg]	176.2	8.0	29/19.6	66.4	29/19.6	8.0	176.2

Table 1.1: Composition of the Midply™ wall

1.2 Motivations for Research

There are two sound transmission paths across the triple-leaf structure: 1- structure-borne and 2- air-borne, shown in Figure 1.2. In structure-borne sound transmission the sound travels through the studs and the connections, whereas in air-borne transmission the sound is transmitted through the air cavities.

Figure 1.2: Sound transmission paths in the triple-leaf wood wall

The resilient channel are metal channels that are directly attached to the studs from one side and to the gypsum board from the other side. They minimize the structure-borne sound transmission by decoupling the gypsum boards from the studs by introducing a spring-like element [5, 6]. They are installed horizontally and provide little or no shear resistance. In Figure 1.3 a typical resilient channel is illustrated.

Figure 1.3: Resilient channels

This research is focused on the air-borne sound transmission path, because with

the existence of resilient mountings on both sides of the wall the structure-borne path is significantly suppressed. Hence, with structure-borne path treated, the main sound transmission mechanism is air-borne. Figure 1.4 illustrates the sound transmission loss predicted by a simple theoretical model of the air-borne sound transmission [7, 8, 9].³

Figure 1.4: Adding a third leaf in the middle of a symmetric double-leaf wall: the effect on transmission loss (TL). Double-leaf wall: — (black solid line), triple-leaf wall: — — (grey dashed line).

In this figure the vertical axis “TL” (sound transmission loss) is a logarithmic value

³The multi-layered structure is modelled using using a one-dimensional mass-spring approach for normal sound transmission, discussed in the following chapters.

representing the sound insulating quality of the structure, expressed as

$$TL = 10 \log\left(\frac{\text{Incident sound power}}{\text{Transmitted sound power}}\right)$$

The model predicts that adding a panel in the middle of a double-leaf wall creates an extra resonant mode at frequency f_{o2} , higher than the resonant frequency for the double-leaf wall system at f_{o1} . This extra mode diminishes the sound insulation of the system in low and mid frequencies. These mode shapes are demonstrated in Figure 1.5.

Figure 1.5: Left: Resonance mode shape of the double-leaf wall at f_{o1} , Right: Resonance mode shape of the triple-leaf wall at f_{o2}

Sound transmission tests performed on the MidplyTM wood shear wall in the Acoustics Group, NRC, Ottawa confirmed the predictions. In Figure 1.6 the sound insulating capability of the double- and the triple-leaf wood walls are compared [3, 4].

Figure 1.6: Sound transmission loss through the Midply™ triple-leaf wood wall: $\times$ — (red solid line), double-leaf wood wall: $\square$ — — (blue dashed line).

Although the measurement sequence in Figure 1.6 (adding the third layer on top) is not the same as the one in Figure 1.4 (adding the third layer in the middle), the test results in Figure 1.6 confirm the negative effect the additional resonance (f_{o2}) has on the sound insulation of triple-leaf walls. Adding an extra leaf reduces the TL between 50 and 80 Hz while increasing the TL for frequencies greater than 100 Hz.

This thesis aims to develop a model for air-borne sound transmission and to use this model to suggest methods to improve the adverse effects of the second mass-spring mode on sound insulation.

1.3 Hypotheses

It is believed that air-borne sound transmission is the main sound transmission mechanism for this type of wall, once structure-borne paths are treated. It is also speculated (previous studies, e.g. [10]) that perforating the middle layer (shear resistive panel in the wood wall) can improve the air-borne sound insulation of the triple-leaf (at f_{o2} in Figure 1.4), by creating only one wider cavity instead of two narrower ones, and the perforations may introduce a new damping mechanism.

1.4 Objective

The objective of the thesis is to develop a simple analytical model that accurately predicts the air-borne sound transmission through triple-leaf walls that may or may not have a perforated mid-leaf and can account for potential losses in the cavities.

The model will be validated using sound transmission tests on small-scale single- and multi-layered structures consisting of Acrylic and Aluminum plates. The reason for choosing small-scale test specimens is that compared to full-scale walls they are more manageable, hence more time, cost and labour could be saved. Moreover, it is preferred to perform measurements on structures that consist of plates that are homogeneous and isotropic and their material properties are already available, e.g. Aluminum and Acrylic plates. In fact perforated Aluminum (and Acrylic)

plates with a wide range of hole sizes and spacing are available in the market for purchase, whereas there are no commercially available perforated wood panels.

The effect of second mass-spring resonance introduced by the middle leaf of the triple-leaf structure on TL will be shown. It is shown using this model that the TL can be improved using a perforated middle-leaf. Finding the specifications of the acoustically optimum structure, using the parametric study is also important.

1.5 Layout of the Chapters

In Chapter 2 the most important theoretical models for air-borne sound transmission through multi-layered structures, available in the literature are summarized, and their relevance to the thesis is discussed.

In Chapter 3 the vibro-acoustic models used for prediction of air-borne sound transmission through multi-layered structures for different frequency regions are introduced. The measurement method and the theory behind it is presented, and the test setup as well as the test plan and procedure are shown.

In Chapter 4 the test results of different single-, double-, triple-leaf structures are presented and compared to the prediction models. After refining the prediction models, they are used in parametric studies to identify the optimum structures.

In Chapter 5 the work done in the thesis and its significance are summarized and the suggested further research is discussed.

1.6 Summary

The inspiration and motivation for the research were discussed. The hypotheses to be tested were stated, also the objectives to be achieved was presented, and finally the layout of the following chapters was given. The dominant sound transmission path through the triple-leaf wood shear wall having resilient channels is believed to be air-borne [5]. The simple predictions [7, 8, 9] and measurements [3] show that in high frequency region the triple-leaf structure has better sound insulation compared to the double-leaf walls. Nevertheless, the additional mass-spring resonance in low frequencies weakens the TL, which is believed that can be avoided by introducing perforations in the middle leaf. The objective of this thesis is: to develop a theoretical model that can predict the air-borne sound transmission through perforated and non-perforated triple-leaf walls, to validate the models by performing air-borne sound transmission measurements on different small-scale single- and multi-layered structures and to find the acoustically optimum structure using a parametric study.

Chapter 2

Literature Review

The urge to design low noise buildings and machines (such as aircrafts, automobiles, etc.) inspired many researchers to study the sound insulation in different structures, experimentally and theoretically. In this chapter, an overview of the methods and techniques available for modelling air-borne sound transmission through multilayered structures is presented. In this chapter and subsequent chapters, “air-borne sound transmission” is shortened to “sound transmission”.

2.1 Transfer Matrix Method (TMM)

Beranek et al., 1949 [11] developed a one-dimensional model for sound transmission of normal incidence acoustic waves across infinite multilayered structures including fluid cavities. The layers and cavities were related to each other by an impedance

transfer method. Their model was based on an acoustic wave solution for cavities and assuming plates as rigid masses in the low frequency region. When the depth of the cavity is much smaller than the acoustic wavelength, the impedance of the cavity will resemble that of a spring. In that case, a double layer structure with air cavity in between, is modelled as a mass-spring-mass system. Although the model is a good first estimate of the sound transmission across multilayered walls, it only considers normal incidence, which is far from the reality.

London, 1949 [12] and 1950 [13] developed an oblique incidence sound transmission model for double-leaf walls, built on the progressive wave method and the impedance of the single-leaf wall that in addition to its mass, incorporated damping and flexural behaviour of the structure as well. He calculated the mass-spring-mass resonance frequency of the double panel system. In addition, he showed that cavity resonance (standing waves) take place at frequencies where acoustic half wavelength and its harmonics fit the cavity depth. Because of the nature of the model several equations had to be solved simultaneously for each angle of incidence, therefore the solution was time consuming. He integrated the low frequency approximation of oblique transmission coefficient over all angles of incidence to calculate the reverberant sound transmission. The results were satisfactory for low frequencies, but failed to capture the high frequency behaviour. As it seemed that the reverberant field assumption (i.e. uniform angular distribution of energy) was not accurately modelling the energy distribution on the boundaries of the chambers, he added a correction term to the plate impedance to make up for the difference between the measured and predicted data. However no physical

interpretation was given.

Because of the inaccuracy in assuming random energy distribution on the boundaries of the chamber (including test structures), Beranek, 1971 [14] and Jones, 1979 [15] attempted to deal with this problem by using the “Field incidence” method. This method introduced an upper limit to the angle over which the integral for random sound transmission was calculated. The upper limits were adjusted for single plates modelled as rigid mass (hence insensitive to angle of incidence) to give good agreement with measured data. For that reason, it was inappropriate for multilayered structures with cavities, especially at mass-spring-mass resonance. This model was highly sensitive to the angle of incidence, because of a singularity at the grazing incidence angle (parallel to the plates).

Mulholland et al., 1967 [16], developed the multiple reflection method for double-leaf structures. They added cavity absorption to the model in order to be able to obtain better agreement between the field incidence prediction and the measurement at the mass-spring-mass resonance. However the absorption coefficient was too high to be realistic. Cummings et al., 1968 [17] corrected the model by including edge absorption, but the absorption coefficient was still too high.

Mulholland et al., 1968 [18] (and similarly later Au et al., 1987 [19]) extended Beranek’s model [11] to the oblique incidence sound transmission and derived an explicit angle dependent formula for TL of the double-leaf wall, ready to be integrated over all angles of incidence to find the TL in the reverberant field. This model was less computationally demanding than the previous models, which re-

quired simultaneous solution of the equations, yet it was specific to double-leaf walls and no general formula was presented for more complex structures.

Munjal, 1973 [7] and 1987 [8] described a general method called the transfer matrix method (TMM) to determine the sound transmission across infinitely extended multilayered systems, based on the classic wave approach. This method was not limited to double-leaf wall systems, since it was possible to calculate the sound transmission of any multilayered system just by the chain multiplication of certain low rank matrices¹.

Lauriks et al., 1992 [20] developed a matrix approach to model the sound transmission through an infinitely extended double panel structure, consisting of an elastic solid plate, multiple layers of porous material and another elastic solid plate. They employed the Biot theory [21] to model the porous material. The incorporation of porous layers in the TMM model was a step forward in further development of the model.

As the results from the random and field incidence prediction methods were unsatisfactory, Kang et al. [22] performed numerical simulations on the directional distribution of the energy incident on the boundaries of the chamber. They noticed that the exact angular weighting is dependent on the geometry of the chamber. They concluded that the angular distribution of energy is not uniform, but it follows the Gaussian distribution. Thus, all previous work that assumed a uniform distribution could benefit from this finding.

¹More details in Chapter 3

The sound transmission through finite multilayered structures are different from the infinite ones [9], because in finite structures there are structural modes that can radiate and transmit energy and there is diffraction by the test opening (aperture) and energy loss through the perimeter of the structure, all of which affect the sound transmission.

Many researchers tried to consider the effect of finiteness of the structures on sound transmission. Sewell, 1970 [23] calculated the forced (non-resonant) sound transmission² of a finite single-leaf plate in an infinitely extended rigid baffle by solving the differential equation of the plate. The low frequency TL prediction was improved, compared to the infinite plate solution. However for multilayered structures, solving the differential equations simultaneously becomes complicated.

Villot et al. [24] proposed applying spatial windowing on infinite multilayered structures, to calculate the sound radiation efficiency of finite structures. The method was based on the wave approach and was applicable to multilayered structures. The predictions showed good agreement with the measured data in the low frequency region.

²A type of sound transmission for which the wavelength of the plate is the same as the wavelength of the incoming wave and is not controlled by the properties of the plate. Since the plate is non-resonant, the response is not proportional to its damping.

2.2 Statistical Energy Analysis (SEA)

Lyon et al., 1965 [25] developed the “Statistical Energy Analysis” framework to estimate the frequency band-averaged, steady-state vibro-acoustical energy flow within resonant systems. They indicated that the power flow between two linearly coupled subsystems is directly proportional to the coupling loss factor and the difference between the average modal energies of subsystems. In SEA it is required that there are many resonant modes in a frequency band of interest and the energy is carried in resonance modes [26].

Maidanik, 1962 [27] found a relation between the vibration velocity of a finite, simply supported plate and the acoustic pressure in the adjacent room, in terms of radiation resistance. Crocker et al., 1969 [28] further developed this concept to calculate the resonant transmission in terms of SEA. They found the resonant transmission to be significant in the frequency region close to the coincidence, where the trace wavelength in air matches the bending wavelength on the plate and the vibration modes of the plate radiate sound efficiently. Price et al., 1970 [29] extended the work of Crocker et al. to a model for resonant and non-resonant sound transmission of double panels, which incorporated the cavity and panel damping.

Brekke, 1981 [30] presented a simple formula for resonant and non-resonant sound transmission loss of the single-, double- and triple-leaf walls. The resonant transmission model was dependent on sound radiation efficiency of panels and for non-resonant transmission he extended the single panel formula by Sewell [23] to model

double and triple leaf walls. He supposed that the resonant and non-resonant sound transmission paths could be super-imposed. He found that below the critical frequency the sound transmission is dominated by non-resonant transmission and above that it is dominated by resonant transmission. He also modelled the sound transmission loss of triple-leaf walls and realized that below the first cavity resonance, the middle panel has no influence on the TL. For double- and triple-leaf walls above the first cavity resonance, the cavity is modelled as a reverberant subsystem, which means that the finite dimensions are explicitly considered in the formulation.

Craik, 2000 [31] employed SEA to model the sound transmission through framed lightweight double-leaf walls, where structural coupling between partitions and studs affects the sound insulation. He incorporated the structural connection in the model of Price et al. [29] and showed that SEA model is appropriate for the frequency range above the mass-spring-mass resonance. He also noticed that at low frequencies the double-leaf wall behaves as a single-leaf. The connection of the frame to the panels by means of the nails can either be modelled as line or point, depending on the wavelength of the excitation.

Craik, 2003 [32] revised the model for non-resonance coupling loss factor from room to cavity given by Price et al. [29]. This led to a better agreement between predicted and measured results of sound transmission loss of double-leaf walls in some cases, but not all.

2.3 Modal Decomposition Method

Osipov et al., 1997 [33] studied the effect of room-plate-room coupling on sound transmission of a single panel in the low frequency range. Using modal analysis and parametric study they discovered that the sound transmission loss in low frequency is highly dependent on the geometry and the dimensions of the rooms and the plate.

Dijckmans et al., 2009 [34] and 2010 [35] extended the single wall direct (non-resonant) sound transmission model to the double wall, using the wave based method originated by Desmet et al., 2001 [36] and [33]. The fully coupled three dimensional vibro-acoustic model of room - double wall - room was solved and compared to measurement data. Although the measured data were predicted more accurately by their model (when compared to TMM predictions) it still under-predicted the measurement.

Xin et al., 2011 [37] modelled a clamped triple-leaf wall system using modal functions. It enabled them to calculate the TL of the triple-leaf system as well as the mass-spring, cavity and plate resonance frequencies. The predictions agreed well with the measurements, but the quantity of the matrices to be calculated was significantly high and the rank of the matrices grew with the number of modes included in the model.

2.4 Walls including Perforated Panels

Maa, 1975 [38], 1985 [39] and 1998 [40] formulated the impedance of microperforated rigid plates. In his work, the perforated plate was modelled as a grid of small tubes. The impedance consisted of a dissipative and a reactive (mass-like) part and was found to be a function of frequency, the size of the holes and the thickness of the plate. The impedance model made possible the theoretical prediction of the sound absorption and transmission of rigid perforated plates.

Dupont et al., 2003 [41] extended Maa's work to multilayered structures. They modelled an infinite double-leaf structure, consisting of one microperforated plate and one rigid plate coupled through a cavity. The impedance model of the perforated plate was a combination of a rigid perforated plate (as in [38]) and a flexible infinite plate in parallel. They solved the sound pressure distribution across the multilayered structure simultaneously in a matrix system and calculated the sound absorption and the sound transmission of the structure. The measurements showed that the perforated double-leaf system was a good sound absorber, even without placing any conventional absorptive material (e.g. fibrous material with a significant flow resistivity).

Takahashi et al., 2002 [42] presented a model for infinite flexible perforated plates, enhancing the previous models by considering the interaction between holes and the solid material. Ismail et al., 2011 [43] predicted the normal incidence TL of an infinite perforated double-leaf structure similar to the one in [41], based on the model of Takahashi et al. [42]. The theoretical model predicted that although

perforating one of the plates of the double-leaf system improves the TL at the double-leaf resonance, it worsens the sound transmission at higher frequencies. However they did not provide measurement results for comparison.

Putra et al. [44], 2010 developed a wave based model for the radiation efficiency of simply supported unbaffled perforated plates based on the superposition of the plate modes. They found that the plate perforations decrease the radiation efficiency. Although the prediction results were close to the measured data, the procedure was computationally rigorous and the proposed simplified formula was only applicable to low frequencies.

2.5 Conclusion

In the literature, three distinct types of vibro-acoustic modelling methods were found to be useful for modelling air-borne transmission through a triple-leaf wall. They are Transfer Matrix Method (TMM), Statistical Energy Analysis (SEA) and various modal decomposition (wave based) methods. Each method has its own advantages and drawbacks, which are discussed here.

There are many studies available that have used TMM to model the forced (non-resonant) TL of single- and double-leaf structures. The original TMM model is designed to model infinitely extended structures. This is usually the case in high frequency region where the structure is much larger than the wavelength. Nevertheless, it is possible to simulate the finiteness of the system by applying the spatial

windowing [24] and to obtain more accurate predictions in the low frequency region. The advantage of this method is that it is easy to set up for multilayered structures. The weakness of the TMM model comes into sight in higher frequency range, where the resonant transmission of the structures becomes important.

The other widely used method is the SEA. Predictions for simple multilayered structures are available in the literature. This method is suitable for calculating the space-averaged sound transmission in complex structures. It can inherently deal with finite structures without the difficulties associated with the other methods. Since the method requires the subsystems to be resonant, it becomes incapable to accurately describe the acoustical response in low frequency regions, where the modal density of the subsystems are low and the assumptions of SEA are not satisfied.

Modal decomposition methods are the third prediction method that has been considered. The method works well in the low frequency region, where the multilayered structure couples with the adjacent rooms and forms a larger vibro-acoustical system. The difficulty linked with this method is that the number of matrices to be calculated is high and the rank of the matrices grow with the number of modes included in the model. Therefore the computational load associated with this method can be large. Moreover, in high frequencies where the modal density is high, a frequency band average method, namely SEA, can be more useful.

2.6 Summary

The methods for modelling the air-borne sound transmission through multi-layered structures and perforated plates, which are available in the literature were discussed. The TMM and SEA models can be employed to model the sound transmission through the triple-leaf walls. However they need to be developed beyond the basic models given in the literature in order to incorporate the physical complexities associated with the test setup (and a perforated mid-leaf). Also the available impedance model of the perforated plates has the capability of being built into the TMM and SEA models. As discussed earlier, the SEA model is more appropriate for higher frequency range, where the subsystems are resonant. Therefore it may be necessary to complement the SEA predictions by providing the results from the TMM model for the mid and low frequency range.

Chapter 3

Methods

3.1 Theoretical modelling of the walls

In this section various theoretical models for the air-borne sound transmission through single- and multilayered walls are presented to determine the most suitable approach for modelling a triple-leaf wall. The frequency range of interest in building acoustics is roughly between 50 Hz and 6300 Hz. In this thesis the frequency range is divided into three regions: low frequencies (50 – 800 Hz), where the modelled structure can be described as a one-dimensional system; mid frequencies (1000 – 2500 Hz), where the system is acoustically two-dimensional and the acoustic wavelength matches the larger dimensions of the structure, e.g. length and width; and high frequencies (> 2500 Hz), where the structure is acoustically three-dimensional.

First the simple one-dimensional mass-spring models of infinitely extended structures, suitable for low frequency approximation are discussed. Then the more complex transfer matrix method is introduced. This method works for TL modelling in all frequency ranges. Afterwards theoretical models for sound transmission through finite systems and structures in reverberant fields are presented. Ultimately the statistical energy analysis for modelling in higher frequency range is studied.

3.1.1 Mass-Spring Model: Low frequency approximation

3.1.1.1 Single infinite plate

The bending wave field in a solid plate is coupled to the acoustic wave in the surrounding fluid (e.g. air) using the concept of wave impedance [9][45]. A simple vibro-acoustic model of a single-leaf partition can be obtained by assuming the plate to be an infinitely extended, thin, elastic, homogeneous, isotropic plate, shown in Figure 3.1.

Figure 3.1: Velocity of a vibrating plate and the air pressure on the surface.

The wave impedance, $Z_p = \frac{p}{v_p}$, which is the ratio of the complex amplitude of the sound pressure p at a point on the plate to the complex amplitude of the normal velocity of the plate at the same point v_p , is given in [12] as

$$Z_p = \frac{(-\omega^2 m'_p) + [D_p (1 + j\eta_p^t) (k_o \sin \theta)^4]}{j\omega} \quad (3.1)$$

where ω [rad/s] is the angular frequency, $k_o = \frac{\omega}{c_o}$ [rad/m] the wavenumber of the air (c_o [m/s] the speed of sound in the air) and θ the angle of incidence of the acoustic wave on the plate, $m'_p = \frac{\text{mass}}{\text{surface area}}$ [kg/m²] the surface density and η_p^t , the total loss factor of the plate, is the fraction of the energy lost in one radian cycle through transmission and dissipation (heat). The bending stiffness of the plate is $D_p = \frac{E_p t_p^3}{12(1 - \nu_p^2)}$ [Pa.m³], where E_p [Pa] is the modulus of elasticity, t_p [m] the thickness and ν_p the Poisson's ratio of the plate.

For this plate, the coincidence frequency for θ is defined as the frequency at which

the trace wavenumber of the incident acoustic wave, $k_o \sin \theta$ coincides with the flexural wavenumber of the plate, $k_b^4 = \omega^2 m'_p / D_p$. Hence the coincidence frequency ω_{co} is calculated as

$$\omega_{co} = (m'_p / D_p)^{1/2} (c_o / \sin \theta)^2 \quad (3.2)$$

which has a minimum value ω_c called the critical frequency given as

$$\omega_c = c_o^2 (m'_p / D_p)^{1/2} \quad (3.3)$$

Using Equations 3.2 and 3.3 the wave impedance of the plate in Equation 3.1 is rewritten as

$$Z_p = j\omega m'_p \left(1 - \left[\left(\frac{\omega}{\omega_c} \right)^2 (1 + j\eta_p^t) \sin^4 \theta \right] \right) \quad (3.4)$$

For the frequency range well below the critical frequency, the ω^2 dependent term becomes negligible. Therefore the wave impedance of the plate is simplified as [12]

$$Z_p = j\omega m' \quad (3.5)$$

which is in the form of the impedance of a rigid mass (“mass law”).

The sound transmission coefficient τ is defined as

$$\tau = \frac{\text{Transmitted sound power}}{\text{Incident sound power}}$$

For a single-leaf plate the sound transmission coefficient is calculated as [14]

$$\tau(\theta, \omega) = \frac{1}{1 + \left(\frac{Z_p \cos \theta}{2Z_o} \right)^2} \quad (3.6)$$

where $Z_o = \rho_o c_o$, is the characteristic impedance of the air, where ρ_o [kg/m³] is the density of air.

A logarithmic level known as “sound transmission loss” (TL) is often used instead of τ , and is defined as

$$TL = 10 \log_{10} (1/\tau) \text{ [dB]} \quad (3.7)$$

The TL for the single leaf plate will be

$$TL_p = 10 \log_{10} \left[1 + \left(\frac{Z_p \cos \theta}{2Z_o} \right)^2 \right] \quad (3.8)$$

where Z_p is calculated from Equation 3.1 or 3.4.

According to the “mass law” (Equation 3.5), the single-leaf plate can be approximated as a rigid mass. Therefore Equation 3.8 becomes [14]

$$TL_{p, mass} = 10 \log_{10} \left[1 + \left(\frac{\omega m' \cos \theta}{2Z_o} \right)^2 \right] \quad (3.9)$$

Figure 3.2 illustrates the predicted TL of a typical single plate using the complete impedance of infinite flexible plate (Equations 3.1 and 3.8) and rigid mass model (“Mass law”: Equations 3.9).

Figure 3.2: TL of a single plate, modelled as: infinite flexible plate — (black solid line), infinite rigid mass — — (grey dashed line)

Because of the second term in the Z_p formula (Equation 3.4), around the critical frequency the TL drops significantly due to the decrease in the impedance, and rises quickly above that frequency.

3.1.1.2 Double-leaf infinite wall

In this section the vibro-acoustic model for the infinitely extended double-leaf structure is discussed and the mass-spring-mass approximation for low frequencies is derived.

Figure 3.3: Sound propagation inside the cavity

Similarly, for an infinitely extended double-leaf wall separated by a cavity with the depth L , the impedance values of two ends of a cavity at $z = 0$ and $z = L$ (Figure 3.3 subscript $cav, 1$ and $cav, 2$ respectively) are related by [46]

$$\frac{Z_{cav,1}}{Z_o} = \frac{Z_{cav,2} + j \frac{Z_o}{\cos \theta} \tan(k_{o,z} L)}{j \cos \theta Z_{cav,2} \tan(k_{o,z} L) + Z_o} \quad (3.10)$$

Assuming that at $z = L$ the termination is rigid and non moving, $Z_{cav,2} \rightarrow \infty$, therefore the impedance of the cavity is written as

$$Z_{cav} = \frac{Z_o / \cos \theta}{j \tan(k_{o,z} L)} \quad (3.11)$$

At $k_{o,z} L = n\pi$ or equally $L \cos \theta = n\lambda/2$, $n = 1, 2, 3, \dots$, where acoustic half wavelength $\lambda/2$ and its harmonics fit the cavity depth L , the energy is completely transmitted through the cavity, therefore the TL drops significantly. The cavity resonance frequency is calculated as

$$f_{cav,n} = \frac{nc_o}{2L \cos \theta}, \quad n = 1, 2, 3, \dots \quad (3.12)$$

For $k_{o,z} L \ll 1$: $\tan(k_{o,z} L) \sim k_{o,z} L$, the air in the cavity behaves as a spring and the impedance is approximated as

$$Z_{cav} = \frac{\rho_o c_o^2 / \cos^2 \theta}{j\omega L} \quad (3.13)$$

The stiffness of the cavity is defined as

$$k'_{cav} = \frac{\rho_o c_o^2}{L} \quad (3.14)$$

and the Equation 3.13 is rewritten as $Z_{cav} = \frac{k'_{cav} / \cos^2 \theta}{j\omega}$. Since the plates are infinitely extended and rigid in low frequencies, the cavity will be one-dimensional and a single spring can adequately represent the cavity.

Therefore a mass-spring-mass model can simulate the vibro-acoustic performance of the double-leaf wall in low frequencies.

The resonance frequency of the mass-spring-mass system shown in Figure 3.4 is calculated as [13]

$$f_{\text{double}} = \frac{1}{2\pi \cos \theta} \left[k'_{\text{cav}} \left(\frac{m'_1 + m'_2}{m'_1 m'_2} \right) \right]^{1/2} \quad (3.15)$$

Figure 3.4: The infinite double-leaf structure modelled as mass-spring system.

Figure 3.5 sketches the TL of a typical double-leaf wall, predicted using impedance of flexible infinite plates (Equation 3.1) and travelling waves in the cavity (Equation 3.10) compared to the simplified mass-spring model (Equations 3.5 and 3.13).

Figure 3.5: Predicted TL of a double-leaf wall, modelled as: infinite flexible plates with travelling waves in the cavity — (black solid line), mass-spring-mass — — (grey dashed line)

The resonance associated with the mass-spring-mass system is located at f_{double} . It appears as a dip in the TL curve which is the indication of a high sound transmission. The standing waves in the cavity occurs at f_{cav} and result in low sound insulation. This behaviour is not captured by the simple spring model for the cavity.

3.1.1.3 Triple-leaf infinite wall

Figure 3.6: The infinite triple-leaf structure modelled as mass-spring system.

Similar to section 3.1.1.2 a triple-leaf wall can be modelled as a mass-spring-mass-spring-mass system in low frequencies. In addition to cavity resonances at frequencies given by Equation 3.12, the system will have two system resonance frequencies calculated as [37]

$$f_{\text{triple},1} = \frac{\sqrt{2}}{4\pi} \sqrt{\frac{\Lambda_1 + \Lambda_2 - \sqrt{(\Lambda_1 - \Lambda_2)^2 + 4k'_1 k'_2 m'^2_1 m'^2_3}}{m'_1 m'_2 m'_3}} \quad (3.16)$$

$$f_{\text{triple},2} = \frac{\sqrt{2}}{4\pi} \sqrt{\frac{\Lambda_1 + \Lambda_2 + \sqrt{(\Lambda_1 - \Lambda_2)^2 + 4k'_1 k'_2 m'^2_1 m'^2_3}}{m'_1 m'_2 m'_3}} \quad (3.17)$$

where $\Lambda_1 = k'_1 m'_3 (m'_1 + m'_2)$ and $\Lambda_2 = k'_2 m'_1 (m'_2 + m'_3)$. Here k'_1 and k'_2 are the stiffness of the cavities, obtained from Equation 3.14.

Figure 3.7: Predicted TL of a triple-leaf wall, modelled as: infinite flexible plates with travelling waves in the cavity — (black solid line), mass-spring-mass-spring-mass — — (grey dashed line)

The effect of placing a third leaf, right in the middle of a symmetric double-leaf ($m'_1 = m'_2$) can be seen in Figure 3.8. For this triple-leaf system, the lowest mass-spring resonance is identical to the system resonance of the double-leaf ($f_{\text{triple},1} = \frac{1}{2\pi \cos \theta} \left(\frac{2\rho_o c_o^2}{m' L} \right)^{1/2}$).

Figure 3.8: Adding a third leaf in the middle of a symmetric double-leaf wall: the effect on TL. Double-leaf: — (black solid line), triple-leaf: - - (grey dashed line)

The predictions here show that the existence of second system resonance in the triple-leaf wall reduces the TL around that mode. Aside from that, the overall performance of the triple-leaf wall is better than the double-leaf.

3.1.2 Transfer Matrix Modelling of Multilayered Structures

3.1.2.1 Introduction

The transfer matrix method (TMM) is a computationally easy procedure for analysis of the sound transmission through linear, one-dimensional dynamic multilayered systems that consist of lumped or distributed mechanical or acoustical elements,

such as fluid, elastic or poro-elastic materials. This method relates the acoustic pressure and acoustic or vibration velocity at an interface between two elements in the system to another interface by means of a 2×2 matrix system [7, 8]. Therefore it is possible to relate the acoustic properties of the receiving side of a multilayered wall to those of the source side by successive multiplication of the transfer matrices of the constituent elements.

$$\begin{bmatrix} p_n \\ v_n \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \cdot \begin{bmatrix} p_{n+1} \\ v_{n+1} \end{bmatrix} \quad (3.18)$$

Figure 3.9: Transfer matrix formulation of a vibro-acoustic element

Since TMM is one-dimensional the elements are assumed to have finite thickness but extend infinitely in other dimensions. Further it is assumed that only plane waves travel through the multilayered system. The elements of the transfer matrix are frequency dependent terms obtained by solving dynamical and continuity equations at the interfaces.

3.1.2.2 Derivation of transfer matrix system for multilayered walls

Assuming that an acoustic wave with pressure p_{inc} and particle velocity v_{inc} is incident on the subsystem that is facing the source side at the angle θ . The wave reflected from the surface will have the pressure p_r and the particle velocity v_r . As air is surrounding the whole system, the reflected and transmitted wave will travel with angle θ . By using force balance equation and velocity continuity at interfaces, the elements of the transfer matrices for double- and triple-leaf systems are obtained.

Plate facing the source side, Figure 3.10:

Figure 3.10: Sound transmission through the plate facing the source side

Force balance:

$$p_{inc} + p_r - p_{n+1} = Z_p v_p \quad (3.19)$$

Continuity:

$$v_{inc,z} - v_{r,z} = v_{plate} = v_{n+1} \quad (3.20)$$

where $v_z = \frac{p}{Z_o} \cos \theta$ is the velocity in the direction normal to the plate. After eliminating p_r from Equations 3.19 and 3.20 the transfer matrix system for the plate facing the source is written as

$$\begin{bmatrix} p_{inc} \\ v_{inc} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \left(Z_{plate} + \frac{Z_o}{\cos \theta} \right) \\ \frac{1}{2Z_o} & \frac{1}{2} \left(\frac{1}{\cos \theta} + \frac{Z_{plate}}{Z_o} \right) \end{bmatrix} \cdot \begin{bmatrix} p_{n+1} \\ v_{n+1} \end{bmatrix} \quad (3.21)$$

Cavity, Figure 3.11:

Figure 3.11: Sound propagation inside the cavity

At $z = 0$

$$p_n = p_+ + p_- \quad (3.22)$$

$$v_n = v_{+,z} - v_{-,z} \quad (3.23)$$

At $z = L$

$$p_{n+1} = p_+ \exp(-jk_{o,z}L) + p_- \exp(jk_{o,z}L) \quad (3.24)$$

$$v_{n+1} = [p_+ \exp(-jk_{o,z}L) - p_- \exp(jk_{o,z}L)] \frac{\cos \theta}{Z_o} \quad (3.25)$$

where $k_{o,z} = k_o \cos \theta$ is the wavenumber in air in z direction.

By solving four equations simultaneously for p_+ , v_+ , p_- and v_- the transfer matrix system for the cavity becomes

$$\begin{bmatrix} p_n \\ v_n \end{bmatrix} = \begin{bmatrix} \cos(k_{o,z}L) & j \frac{Z_o}{\cos \theta} \sin(k_{o,z}L) \\ j \frac{\cos \theta}{Z_o} \sin(k_{o,z}L) & \cos(k_{o,z}L) \end{bmatrix} \cdot \begin{bmatrix} p_{n+1} \\ v_{n+1} \end{bmatrix} \quad (3.26)$$

Plate inside a cavity, Figure 3.12:

Figure 3.12: Sound transmission through the plate, placed inside the cavities

Force balance:

$$p_n - p_{n+1} = Z_p v_p \quad (3.27)$$

Continuity:

$$v_n = v_p = v_{n+1} \quad (3.28)$$

The transfer matrix system for the plate placed inside the cavities is obtained as

$$\begin{bmatrix} p_n \\ v_n \end{bmatrix} = \begin{bmatrix} 1 & Z_p \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} p_{n+1} \\ v_{n+1} \end{bmatrix} \quad (3.29)$$

Plate facing the source side, Figure 3.13:

Figure 3.13: Sound transmission through the plate facing the receiving side

Force balance:

$$p_n - p_t = Z_p v_p \quad (3.30)$$

Continuity:

$$v_n = v_{plate} = v_{t,z} \quad (3.31)$$

where p_t and v_t are the transmitted sound pressure and particle velocity respectively, and $v_{t,z} = v_t \cos \theta = \frac{p_t}{Z_o} \cos \theta$. So the transfer matrix system for the plate facing the receiving is given as

$$\begin{bmatrix} p_n \\ v_n \end{bmatrix} = \begin{bmatrix} 1 & Z_p \cos \theta \\ 0 & \cos \theta \end{bmatrix} \cdot \begin{bmatrix} p_t \\ v_t \end{bmatrix} \quad (3.32)$$

The sound transmission through a multilayered system shown in Figure 3.14 is calculated by successive multiplication of transfer matrices relating different interfaces.

Figure 3.14: Sound transmission through the infinitely extended multi-layered structure

$$\begin{bmatrix} p_{inc} \\ v_{inc} \end{bmatrix} = \begin{matrix} i \\ \cdot \end{matrix} \begin{bmatrix} \cdot \\ \cdot \end{bmatrix} \begin{matrix} 1 & n \\ \cdot & \dots & \cdot \end{matrix} \begin{bmatrix} \cdot \\ \cdot \end{bmatrix} \begin{matrix} n+1 & t-1 \\ \cdot & \dots & \cdot \end{matrix} \begin{bmatrix} \cdot \\ \cdot \end{bmatrix} \begin{matrix} t \\ \cdot \end{matrix} \begin{bmatrix} p_t \\ v_t \end{bmatrix} \quad (3.33)$$

Using the equation above, the TL of the multilayered structure is obtained by

$$TL = 10 \log_{10} \left(\frac{p_{inc}}{p_t} \right)^2 \quad (3.34)$$

or the transmission coefficient

$$\tau = \frac{p_t^2}{p_{inc}^2} \quad (3.35)$$

3.1.2.3 Impedance of a perforated plate

Figure 3.15: The cross-section of the perforated plate

The impedance of an infinitely extended perforated plate is defined as

$$Z_{pp} = \frac{p_1 - p_2}{\bar{v}} \quad (3.36)$$

where

$$\bar{v} = s v_h + (1 - s) v_p \quad (3.37)$$

is the average velocity of the plate, v_h the air velocity in the holes and v_p is the velocity of the solid portion of the plate. The porosity ratio s is the ratio between the surface area of the holes to the total area of the plate. According to [42] the equivalent force acting on the holes, $p_1 - p_2$ is balanced with the reactive force proportional to the air velocity in the holes, and the friction force caused by the relative movement between the air in the holes and the plate.

$$p_1 - p_2 = Z_{h,react}v_h + Z_{h,resist}(v_h - v_p) \quad (3.38)$$

Based on Maa's approximation [38, 39, 40] the impedance of air inside the hole, Z_h , is expressed as

$$Z_h = Re(Z_h) + j Im(Z_h) = Z_{h,resist} + Z_{h,react}$$

$$Z_{h,resist} = Re(Z_h) = \frac{8\eta_o t}{(d/2)^2} \left(\sqrt{1 + \frac{X^2}{32}} + \frac{\sqrt{2}dX}{8t} \right) \quad (3.39)$$

and

$$Z_{h,react} = j Im(Z_h) = -j\rho_o\omega t \left(1 + \frac{1}{\sqrt{9 + (X^2/2)}} + \frac{0.85d}{t} \right) \quad (3.40)$$

where η_o is the viscosity coefficient of air which is equal to $1.8362 \times 10^{-5} \left[\frac{\text{kg}}{\text{ms}} \right]$, t [m] is the thickness of the perforated plate, d [m] is the diameter of each hole and

$X = (d/2) \sqrt{\omega \rho_o / \eta_o}$. From Equations 3.36, 3.37 and 3.38 (inspired by [42]) the impedance of the plate is obtained as

$$Z_{pp} = \frac{(r-1) \cdot Z_{h,resist} + r \cdot Z_{h,react}}{1 + (r-1) \cdot s} \quad (3.41)$$

where $r = \frac{v_h}{v_p}$ is the ratio of the velocity of air inside the holes to the velocity of the solid part of the plate and is calculated as

$$r = \frac{Z_{h,resist} + (1-s)Z_p}{Z_{h,resist} + (1-s)Z_{h,react}} \quad (3.42)$$

where Z_p is the impedance of the corresponding non-perforated plate given in Equation 3.4.

The impedance of a perforated plate with density much higher than the air is approximated as $Z_{pp} \approx \frac{Z_h}{s}$. The TL of a multi-layered perforated plate can be calculated by inserting the complex impedance of the perforated plate into TMM model (Z_{pp} instead of Z_p).

3.1.3 Sound transmission of finite walls

In lower frequency range, where the wavelength is not short compared to the dimensions of the structure, a structure is not accurately represented by an infinite model. Finiteness of a structure affects its radiation efficiency, hence affects the

sound insulation. Therefore it will be beneficial if the prediction models can account for the finiteness of the structure.

An estimate of the sound transmission coefficient for a finite system could be obtained by applying the spatial windowing technique over the infinite system. The average sound transmission coefficient of the finite system τ_{finite} , originally developed by Villot [24] and later modified by Villot et al [47] is given as

$$\tau_{finite}(f, \theta) = \tau_{infinite}(f, \theta) \sigma(k_p) \cos \theta \quad (3.43)$$

where σ the radiation efficiency and k_p is the trace wavenumber which is the wavenumber in the plane of the structure and is given by $k_p = k_o \sin \theta$. The modified equation helps with a better prediction of TL measurements, particularly at lower frequencies.

The radiation efficiency $\sigma(k_p)$ (averaged over all angles of wave propagation in the plane of the structure) is expressed in [47]. A simplified one-dimensional formula for radiation efficiency of multilayered structures is given by Vigran [48] assuming isotropic materials with aspect ratio greater than 1:2. This equation is given as

$$\sigma(k_p)_{1D} = \frac{lk_o}{2\pi} \int_0^{k_o} \frac{\sin^2 \left[(k_r - k_p) \frac{l}{2} \right]}{\left[(k_r - k_p) \frac{l}{2} \right]^2 \sqrt{k_o^2 - k_r^2}} dk_r \quad (3.44)$$

where $l = (L_x L_y)^{1/2}$ is the typical (characteristic) dimension of the finite structure with L_x and L_y as dimensions of the structure and k_r is the wavenumber amplitude

of the radiated power.

3.1.4 Sound transmission in a diffuse field

The TMM model can predicting TL for a specific angle of incidence (or a range of angles), whereas theoretically in reverberant test chambers acoustic waves are incident on the room boundaries with equal probability from all angles (uniform angular distribution, Figure 3.16).

Figure 3.16: Uniform distribution of acoustic waves on boundaries of a reverberant chamber

There are a number of ways to calculate the random incidence TL from the angle dependent TMM predictions. Here, a few of the most important ones are presented, and compared later in Chapter 4.

3.1.4.1 Simple model

In [9] a simple method to calculate the random incidence TL was given in terms of normal incidence TL.

$$TL_{\text{random}} = TL_0 + 10 \log (0.23TL_0) \quad (3.45)$$

where TL_0 is the normal incidence TL.

3.1.4.2 Random incidence integration

The transmission coefficient for a structure that is located between two reverberant rooms is obtained by the angular averaged integration [49][50][51][52], as

$$\tau_{\text{diffuse}}(f) = \frac{\int_0^{\pi/2} \tau(f, \theta) D(\theta) \sin(\theta) \cos(\theta) d\theta}{\int_0^{\pi/2} D(\theta) \sin(\theta) \cos(\theta) d\theta} \quad (3.46)$$

where the directional energy density of the incident waves $D(\theta)$ is equal to 1 for the diffuse field. This integral calculates the diffuse field sound transmission coefficient $\tau_{\text{diffuse}}(f)$ by averaging the angular dependent transmission coefficients $\tau(f, \theta)$ over the incidence angles ranging from 0 to $\pi/2$.

3.1.4.3 Gaussian weighted integration

It was found by the numerical study in [22] that the angular distribution of the incident energy agrees better with the Gaussian weighting function $D(\theta) = \exp(-\beta\theta^2)$, where β is determined by the geometry of the room and is in the range of 1 to 2. This term has a maximum value at zero degrees, shown in Figure 3.17,

which indicates that the acoustic waves in reverberant chambers have a greater probability of incidence around the angle normal to the boundaries.

Figure 3.17: Gaussian weighting function $\exp(-\beta\theta^2)$, with $\beta = 2$

The Gaussian weighted average sound transmission coefficient is written as

$$\tau_{\text{Gauss.}}(f) = \frac{\int_0^{\frac{\pi}{2}} \tau(f, \theta) \exp(-\beta\theta^2) \sin(\theta) \cos(\theta) d\theta}{\int_0^{\frac{\pi}{2}} \exp(-\beta\theta^2) \sin(\theta) \cos(\theta) d\theta} \quad (3.47)$$

It is best to replace the $\tau(f, \theta)$ with the value obtained for finite plates in Section 3.1.3 in order to get a more accurate estimate (compared to that for infinite plates).

3.1.5 Statistical Energy Analysis (SEA)

Statistical Energy Analysis is a framework to estimate the frequency band-averaged, steady-state vibro-acoustical energy flow within resonant systems. It can deal with

finite sized subsystems, without the need for modal analysis or other indirect techniques. SEA is used in the prediction of TL and the calculation of TL from sound transmission tests.

According to Lyon et al. [25] the power flow between two linearly coupled subsystems is directly proportional to the coupling loss factor and the difference between the average modal energies of subsystems. Considering a vibro-acoustical system, having three elements (such as plate, room, cavity, etc.) as in [28]

Figure 3.18: SEA power flow for a 3-element system

The generalized power balance is written as [28]

$$\begin{aligned}
W_1^{in} + W_{21} + W_{31} &= W_{12} + W_{13} + W_1^d \\
W_2^{in} + W_{12} + W_{32} &= W_{21} + W_{23} + W_2^d \\
W_3^{in} + W_{13} + W_{23} &= W_{31} + W_{32} + W_3^d
\end{aligned} \tag{3.48}$$

In SEA terms

$$\begin{aligned}
W_1^{in} + \omega \eta_{21} E_2 + \omega \eta_{31} E_3 &= \omega \eta_{12} E_1 + \omega \eta_{13} E_1 + \omega \eta_1^d E_1 \\
W_2^{in} + \omega \eta_{12} E_1 + \omega \eta_{32} E_3 &= \omega \eta_{21} E_2 + \omega \eta_{23} E_2 + \omega \eta_2^d E_2 \\
W_3^{in} + \omega \eta_{13} E_1 + \omega \eta_{23} E_2 &= \omega \eta_{31} E_3 + \omega \eta_{32} E_3 + \omega \eta_3^d E_3
\end{aligned} \tag{3.49}$$

where η_i^d is the damping loss factor for subsystem i , E_i is the time and space averaged energy, and η_{ij} is the coupling loss factor between subsystems i and j .

The consistency relationship for coupled subsystems is given in [25] as

$$n_i \eta_{ij} = n_j \eta_{ji} \tag{3.50}$$

where modal density of each subsystem $n = \frac{\Delta N}{\Delta \omega}$ is the number of modes ΔN in the frequency band $\Delta \omega$. Using Equation 3.50 the equation set 3.49 is rewritten as

$$\begin{aligned}
W_1^{in} &= \omega \eta_{12} n_1 \left(\frac{E_1}{n_1} - \frac{E_2}{n_2} \right) + \omega \eta_{13} n_1 \left(\frac{E_1}{n_1} - \frac{E_3}{n_3} \right) + \omega \eta_1^d E_1 \\
W_2^{in} &= \omega \eta_{21} n_2 \left(\frac{E_2}{n_2} - \frac{E_1}{n_1} \right) + \omega \eta_{23} n_2 \left(\frac{E_2}{n_2} - \frac{E_3}{n_3} \right) + \omega \eta_2^d E_2 \\
W_3^{in} &= \omega \eta_{31} n_3 \left(\frac{E_3}{n_3} - \frac{E_1}{n_1} \right) + \omega \eta_{32} n_3 \left(\frac{E_3}{n_3} - \frac{E_2}{n_2} \right) + \omega \eta_3^d E_3
\end{aligned} \tag{3.51}$$

In Equation 3.51, the terms inside the brackets represent the difference between the modal energy of coupled subsystems. Although the unknowns in Equation 3.51 such as loss factors and modal density can be measured, there are theoretical values in the literature that assist in the calculation of power flow. In this thesis, statistical energy analysis is employed: 1- to extract the TL values from sound transmission tests (made between two reverberation chambers coupled by the test specimen), 2- to estimate the power dissipated from the multilayered walls using the approach developed later in this thesis and 3- to predict the TL of multilayered walls.

In order for SEA to work accurately, the subsystems need to be diffuse. The total damping and the number of modes in a frequency band, n , together determine the quality of the diffuse field. In terms of statistical energy analysis: in a diffuse field the modal overlap $M > 1$, and M is defined as

$$M = \omega \eta^t n \tag{3.52}$$

where n is the modal density and η^t is the total loss factor.

As stated in [26]

$$M = \frac{\text{Modal bandwidth}}{\text{Modal spacing}} \quad (3.53)$$

The condition $M > 1$ requires that at least three modes fit in the average modal bandwidth (double the half-bandwidth). A large modal overlap in a frequency band indicates that a large number of damped modes exist in that band and a greater probability the SEA prediction will be a good estimate of the mean energy in the subsystem.

3.2 Measurement Method

In this section the standard procedure for the measurement of air-borne sound transmission is introduced. Also the theory behind the standard and its limitations are discussed. The formula needed for TL calculation is derived using the classic diffuse field theory and the SEA method. Finally the test setup is presented in detail.

3.2.1 Introduction

The air-borne sound transmission across multilayered walls can be measured using reverberation chambers, following standard procedures. Instead of full scale

measurements on wood walls, a number of small, lightweight test plates have been carefully chosen to simulate the characteristics of the real wall. This decision has been made to improve test reproducibility and to reduce the required labour and time. The tests are performed on Acrylic and Aluminum plates, which are homogeneous and isotropic and their material properties are already known. The test specimens have been designed in the way that the structural coupling between the plates are minimum, so that it will be possible to measure the air-borne sound transmission exclusively.

3.2.2 Theory

The measurement of air-borne sound insulation is performed in accordance with the American Society for Testing and Materials (ASTM) E90-09 [53]. This standard test method provides a procedure for calculating the air-borne sound transmission through a wall separating two adjacent reverberant test chambers, as illustrated in Figure 3.19.

Figure 3.19: The setup for air-borne sound transmission test (sketch for theoretical explanation).

3.2.2.1 Diffuse field method

The test is based on the classic method explained below [46]:

Suppose: 1- the sound field in rooms are diffuse (random incidence on the wall),
 2- the only sound transmission path from the source room to the receiving room is through the specimen. Then the TL is written as

$$TL = 10 \log \left(\frac{W_1}{W_2} \right) \quad (3.54)$$

where W_1 is the sound power incident on the wall in the source room and W_2 is the sound power that is transmitted to the receiving room. These sound powers

are written as

$$\begin{aligned} W_1 &= \frac{c_o S}{4} e_1 = \frac{c_o S}{4 Z_o} p_1^2 \\ W_2 &= \frac{c_o A}{4} e_2 = \frac{c_o A}{4 Z_o} p_2^2 \end{aligned} \quad (3.55)$$

where e is the energy density in the rooms, S is the surface area of the test element and A is the equivalent sound absorption area in the receiving room. The equivalent absorption A is related to the decay time T through Sabine formula [54]

$$A = 55.26 \frac{V}{c_o T} \quad (3.56)$$

where V is the volume of the receiving room. Reverberation time T is the time that takes for the energy level in a subsystem to decay 60 dB, after the excitation source is turned off.

Substituting 3.55 into 3.54

$$TL = L_{p,1} - L_{p,2} + 10 \log \frac{S}{A} \quad (3.57)$$

where $L_{p,1}$ is the space and time averaged sound pressure level in the source room and $L_{p,2}$ is the sound pressure level in the receiving room (usually both in third octave bands).

The Equation 3.57 is rewritten as

$$TL = L_{p,1} - L_{p,2} + 10 \log \left(\frac{ST_2}{0.161V_2} \right) \quad (3.58)$$

3.2.2.2 Alternative SEA formulation

It is possible to derive the TL formula in Equation 3.57 using the SEA approach. Considering the three-subsystem model in Figure 3.20 consisting of two resonant chambers and the non-resonant test specimen in the middle, with the sound source existing only in the source room:

Figure 3.20: The three element SEA subsystems consisting of two rooms and a solid specimen

The power balance for the source room (subsystem 1) is

$$W_1 + W_{31} = W_1^d + W_{13} \quad (3.59)$$

and for the receiving room

$$W_{13} = W_{31} + W_3^d \quad (3.60)$$

In SEA terms the Equation 3.59 and 3.60 are rewritten as

$$\begin{aligned} W_1 + \omega \eta_{31} E_3 &= \omega (\eta_1^d + \eta_{13}) E_1 \\ \omega \eta_{13} E_1 &= \omega (\eta_{31} + \eta_3^d) E_3 \end{aligned} \quad (3.61)$$

The sum of loss factors inside each bracket in Equation 3.61 is known as η_i^t , the total loss factor of subsystem i . The reason for this definition is that this parameter is calculated for both test chambers from reverberation time measurements, as a part of the TL tests.

Rearranging the second row of Equation 3.61

$$\eta_{13} = \eta_3^t \frac{E_3}{E_1} \quad (3.62)$$

In [26] the total loss factor η^t is related to reverberation time T by

$$\eta^t = \frac{2.2}{fT} \quad (3.63)$$

Similarly, E_i is calculated from the measured sound pressure level, L_p , [26] as below

$$L_E = L_p + 10 \log V - 25.4 \quad (3.64)$$

where L_E is the energy level in dB related to E as

$$L_E = 10 \log \frac{E}{10^{-12}} \quad (3.65)$$

The non-resonant room-to-room coupling loss factor, η_{ij} , is given by [29]

$$\eta_{ij} = \frac{13.7S\tau_{ij}}{fV_i} \quad (3.66)$$

where S is the surface area of the specimen. From Equations 3.62, 3.63 and 3.66 the transmission coefficient of that specimen τ is written as

$$\tau = \frac{V_3}{0.161ST_3} \frac{E_3}{E_1} \quad (3.67)$$

In form of TL and using Equation 3.64

$$TL = 10 \log(1/\tau) = L_{p,1} - L_{p,3} + 10 \log\left(\frac{ST_3}{0.161V_3}\right) \quad (3.68)$$

which is the same as Equation 3.58, and Equation 3.57 because Equation 3.56 can be written as

$$T_3 = \frac{0.161V_3}{A_3}$$

3.2.3 Deviation from diffuse field assumption

The diffuseness of the test chambers are estimated by calculating the modal overlap M . Alternatively for an acoustic volume, there is a frequency called Schroeder frequency f_s that separates the frequency region dominated by individual modes from the region dominated by a reverberant sound field with high modal overlap. The condition $M > 1$ is directly related to the Schroeder frequency limit, f_s [55] as:

$$f > f_s = 2000 \sqrt{\frac{T}{V}} \quad (3.69)$$

In a perfectly diffuse field the sound pressure level is constant throughout the enclosure. Whereas in a sound field dominated by individual modes the sound pressure tends to have large spatial variation. The accuracy of the measured TL is calculated by the “95% confidence interval” which is a function of the standard deviation of the sound pressures and absorptions inside the chambers (Appendix

B).

3.2.4 Setup and procedure

The sound transmission loss tests were performed in the reverberant chambers of the Acoustics group at the National Research Council Canada, NRC (Figure 3.21) which are compliant with the ASTM E90 standard. There are two horizontally adjacent rooms which have low flanking sound transmission, so the only sound transmission path between the chambers is directly through the specimen. The large chamber is 7.96 m long, 6.48 m, wide 4.88 m high and the small chamber is 8.31 m long, 5.09 m, wide 3.48 m high. The plan view of the test chambers is shown in the Figure 3.21.

Figure 3.21: Plan view of the test chambers at NRC: Large chamber (left), Small chamber (right)

A 3.658 m wide 2.438 m high test frame is enclosing the test structure and is placed in the gap between the two rooms. The test structure consists of the filler wall and a 1.219 m wide $\times$ 1.219 m high (4 ft. $\times$ 4 ft.) aperture for the specimens with the same surface area, as shown in Figure 3.22.

Figure 3.22: Test wall in the test frame, the specimen positioned in the aperture

The filler wall consists of a double steel stud framing with two layers of gypsum board Firecode Type X directly attached on each side and an air gap. Cavities between studs are filled with insulation (Detailed description in Appendix A.2). The filler wall was designed to have a high sound insulating properties that was confirmed by measuring the TL of the filler wall before cutting an aperture for the test specimen.

Each plate is tightly supported on both sides by wood strips along the perimeter of the plate, as shown in Figure 3.23. Each wood strip is covered by a closed-cell foam on its surface and is attached to the filler wall by screws.

Figure 3.23: Test specimen installed in the aperture in the filler wall

The source sound is generated in either rooms by two loudspeakers sitting in the bottom corners of the test chambers away from the test specimen. The signal generated by the loudspeakers is close to random white noise, with more power in high frequencies to ensure the sound pressure levels in the receiving room is higher than the background noise in all frequency bands. The sound pressure levels produced by the loudspeakers are low enough to guarantee the linearity of the vibration in test specimens. The average sound pressure level inside both chambers is calculated from the sound pressure recorded at 9 different positions in the chambers. For the forward measurement, the small chamber is the source room and is excited by the loudspeakers. Then the reverberation time in the large chamber is calculated using sweep sine signal¹. The time domain data is converted into frequency domain data using the Fourier transformation. The sound pressure

¹A tonal sound with a frequency that increases with time, covering the whole frequency range of interest

levels and the reverberation time data are calculated for third-octave bands 50 Hz to 6300 Hz. This range is the usual frequency range of interest in building acoustic problems. The measurement process is repeated for the reverse sound transmission path by exciting the large chamber and measuring the average sound pressures, then calculating the reverberation time for the small chamber.

3.3 Plan and procedures

In this section an outline of the measurement procedure and the plan for the analysis of the results are given by the following steps:

- 1- Test "TLA-13-112"²: measuring the TL of the single-leaf plate shown in Figure 3.24, with the composition given in Table 3.1.

Figure 3.24: Test "TLA-13-112": single-leaf specimen installed in the filler wall

²This is the number that is associated with a test on a specific test specimen. The material properties of all plates are given in Appendix A.1.

Specimen	Plate, small chamber side	Cavity depth[mm]	Plate middle	Cavity depth[mm]	Plate, large chamber side
Single-leaf “TLA-13-112”	-	-	AL-1	-	-

Table 3.1: Composition of the single-leaf test specimen “TLA-13-112”

2- Predicting the TL of the specimen in step 1 using the following models: a) Normal incidence on an infinite structure (Section 3.1.1), b) Simple diffuse estimate (Section 3.1.4.1), c) TMM with random incidence integration, for both infinite and finite structures (Section 3.1.4.2) and d) TMM with Gaussian weighted incidence integration (Section 3.1.4.3).

3- Comparing the results in steps 1 and 2 to see which model fits the test result the best. If not satisfactory, the prediction model is refined (e.g. β in Gaussian weighting is modified).

The steps 1 to 3 have been schematically shown in Flowchart “F1” (Figure 3.30).

4- Test “TLA-13-131”: measuring the TL of the double-leaf wall shown in Figure 3.25, with the composition given in Table 3.2.

Figure 3.25: Test “TLA-13-131”: double-leaf specimen installed in the filler wall

Specimen	Plate, small chamber side	Cavity depth[mm]	Plate middle	Cavity depth[mm]	Plate, large chamber side
Double-leaf “TLA-13-131”	AC-1	93	-	93	AC-1

Table 3.2: Composition of the double-leaf test specimen “TLA-13-131”

- 5- Predicting the TL of the double-leaf wall using the best method in step 2.
- 6- Comparing the results in steps 4 and 5. If the prediction results are not satisfactory the cavity model is updated.
- 7- Test “TLA-13-139”: measuring and predicting the TL of another double-leaf wall shown in Figure 3.26 (similar to steps 4 and 5), with the composition given in Table 3.2, in order to test the cavity model.

Figure 3.26: Test “TLA-13-139”: double-leaf specimen installed in the filler wall

Specimen	Plate, small chamber side	Cavity depth[mm]	Plate middle	Cavity depth[mm]	Plate, large chamber side
Double-leaf (“TLA-13-139”)	AC-1	93	AL-1	-	-

Table 3.3: Composition of the double-leaf test specimen “TLA-13-139”

The steps 4 to 7 have been schematically shown in Flowcharts “F2” and “F3” (Figure 3.31 and 3.32).

8- Test “TLA-13-124”: measuring the TL of the triple-leaf wall shown in Figure 3.27, with the composition given in Table 3.4.

Figure 3.27: Test “TLA-13-124”: triple-leaf specimen installed in the filler wall

Specimen	Plate, small chamber side	Cavity depth[mm]	Plate middle	Cavity depth[mm]	Plate, large chamber side
Triple-leaf (“TLA-13-124”)	AC-1	93	AL-1	93	AC-1

Table 3.4: Composition of the triple-leaf test specimen “TLA-13-124”

9- Predicting the TL of the triple-leaf structure using the TMM model and the SEA model, to compare the applicability and accuracy of these methods.

10- The measured TL of the double-leaf (“TLA-13-131”) and the triple-leaf (“TLA-13-124”) walls are compared to investigate the effect of the middle panel on the sound insulation. In case the middle panel has an adverse effect on the sound insulation capability of the wall, the panel is replaced by a perforated one and the effectiveness of introducing perforation to the mid panel is investigated. Flowchart “F4” (Figure 3.33) shows this step schematically.

11- Test “TLA-13-117”: measuring the TL of the perforated single-leaf wall shown

in Figure 3.28, with the composition given in Table 3.5, and validating the prediction model.

Figure 3.28: Test “TLA-13-117”: perforated single-leaf specimen installed in the filler wall

Specimen	Plate, small chamber side	Cavity depth[mm]	Plate middle	Cavity depth[mm]	Plate, large chamber side
Perf. single-leaf “TLA-13-117”	-	-	AL-P-1	-	-

Table 3.5: Composition of the perforated single-leaf test specimen “TLA-13-117”

12- Test “TLA-13-125”: measuring the TL of the perforated triple-leaf wall shown in Figure 3.29 with the composition given in Table 3.6, and validating the prediction model. The hypothesis of the effectiveness of perforation is tested.

Figure 3.29: Test “TLA-13-125”: perforated triple-leaf specimen installed in the filler wall

Specimen	Plate, small chamber side	Cavity depth[mm]	Plate middle	Cavity depth[mm]	Plate, large chamber side
Perf. triple-leaf “TLA-13-125”	AC-1	93	AL-P-1	93	AC-1

Table 3.6: Composition of the perforated single-leaf test specimen “TLA-13-125”

13- Parametric study is performed on the triple-leaf walls and the structure is optimized.

The procedure above is schematically shown in the following flowcharts.

Figure 3.30: Single-leaf wall: flowchart “F1”, perforated or regular

Figure 3.31: Double-leaf wall: flowchart "F2"

Figure 3.32: Calculating cavity damping of the double-leaf wall using SEA modelling: Flowchart “F3”

Figure 3.33: Flowchart: Testing the hypothesis: measuring TL of regular and perforated triple-leaf wall (F4)

3.4 Summary

In this chapter different theoretical modelling methods used in the thesis for the prediction of air-borne sound transmission through multi-layered structures were presented. The method for air-borne sound transmission measurement was also

discussed. Finally, the test plan as well as the analysis procedure was given. The mass-spring model can predict the air-borne TL below the coincidence frequency and the cavity cross modes. The mass-spring resonance mode for the double-leaf walls and the additional mode for the triple-leaf reduce the TL. The TMM [7, 8, 9] is a one-dimensional angle-dependent method for the multi-layered structures that can model the flexibility of the infinite plates and the travelling waves inside the cavities. It is a computationally easy method that is appropriate for low frequencies, where the cavities are not resonant. The finiteness of the structure can be considered by applying the spatial windowing technique to the infinite model [24]. Moreover, the diffuse transmission coefficient can be modelled by calculating the Gaussian weighted integral of angle-dependent transmission coefficients over all angles of incidence [22]. For high frequencies, where all the subsystems including the cavities are resonant in two or three dimensions, the SEA can predict the TL more accurately [25, 28]. This method requires high modal overlaps [26], so it is not suitable for low frequencies. The standard measurement method used in this thesis, ASTM E90-09 [53], as well is based on a three-subsystem SEA model. The measurements are performed on small-scale specimens consisting of homogeneous and isotropic plates. In Chapter 4 the prediction models are validated by comparison against the test results, step-by-step by adding complexity to the structures.

Chapter 4

Results and Discussions

In this chapter the results from sound transmission tests are compared to the ones from prediction models. All the measurements are carried out following the standard test procedure ASTM E-90 explained in Section 3.2. They are presented in graphs with TL values in third-octave bands for the building acoustics frequency range 50 – 6300 Hz. The material properties and the dimensions of the plates, multilayered specimens, the filler wall and the reverberant chambers are found in Appendix A. The sound transmission loss of the specimens are predicted using the theoretical models described in Section 3.1. The results in this chapter are presented according to the sequence presented in Section 3.3. In this thesis a prediction error not greater than 3 dB is considered to be very good.

4.1 Single-leaf wall (“TLA-13-112”)

The specimen in test “TLA-13-112” consists of one layer of Aluminum “Al-1” mounted half way inside the filler wall opening, as shown in Figure 4.1.

Figure 4.1: Test "TLA-13-112": The single-leaf specimen installed in the filler wall

In Figure 4.2 the test results are compared with the predictions by 1- the normal incidence infinite flexible plate model (Equation 3.8, $\theta = 0$), 2- simple diffuse TL estimate (Equation 3.45), 3- angular averaged infinite plate model (Equations 3.8 and 3.47) and 4- Angular averaged finite plate model (Equations 3.8, 3.47 and 3.43).

Figure 4.2: Sound transmission loss of a single plate: measured compared to various prediction models. Measured (“TLA-13-112”): $\blacklozenge$ — (blue solid line), Normal incidence infinite plate model: $\triangle$ (no line), Simple diffuse estimate: $\times$ (no line), Random averaged infinite plate model: $\circ \cdots$ (purple dotted line), Random averaged finite plate model: $\square$ — (red dashed line).

For normal incidence, the impedance of the plate is unaffected by the flexibility (second term in Equation 3.8 is zero). The critical frequency of the plate (calculated using Equation 3.3) is $f_c = 7592$ Hz, which is above the frequency range of interest (similar for all other plates tested in this thesis). The decrease in TL at high frequency region (appearing in the measurements as well as the random averaged models) is due to the onset of the coincidence effect where radiation efficiency of the structure increases significantly.

The Schroeder frequency in each third octave band is calculated using Equation 3.69 and compared to the centre frequency value in Figure 4.3.

Figure 4.3: Schroeder frequency for the large chamber (red $-\square-$) and small chamber (green $-\triangle-$) compared with the frequency value (blue $\cdots$).

In this measurement the Schroeder frequency is at 315 Hz, which means that at this frequency band (and beyond) both rooms have large modal overlap ($M > 1$) and are considered as reverberant.

In the low frequency region the uncertainty in TL measurements, represented by the 95% confidence interval (shown by light blue bands in Figure 4.2), is higher than the uncertainty at higher frequencies. As discussed earlier this happens because of low modal overlap in chambers, which causes spatial variation in sound pressure levels.

The normal incidence models overestimate the TL, because the impedance of the plate decreases with the angle of incidence. A great portion of incident sound is transmitted at higher angles of incidence ($\theta > 0$), therefore the actual TL is lower.

As shown in the figure, among all the models, the angular averaged random incidence finite plate model provides the closest TL prediction to the measurement and an acceptable level of accuracy.

Since the vibro-acoustic model of the plate is validated, next the double-leaf wall test can be performed.

4.2 Double-leaf wall (“TLA-13-131”)

After verification of the plate model, the test “TLA-13-131” is performed on a double-leaf specimen (Figure 4.4) consisting of a 186 mm deep cavity and two Acrylic plates “AC-1” mounted on the ends of the opening.

Figure 4.4: The double-leaf specimen installed in the filler wall

In Figure 4.5 the measured TL is compared to the predictions by various angular averaged finite TMM models. Also shown are the theoretical values of the

resonance frequencies for the normal incidence.

Figure 4.5: The measured and predicted TL of the double-leaf specimen. Measured: $\blacklozenge$ — (blue solid line), random incidence finite TMM: $\square$ — (red dashed line), Gaussian weighted average $\beta = 2$: $\triangle \cdots$ (green dotted line).

Equation 3.15 estimates the resonance frequency of the double-leaf system (Mass-spring-mass) at normal incidence ($\theta = 0$) to be at $f_o = 85.8$ Hz, which is very close to the measured value. However the predicted TL shown in Figure 4.5 does not have a pronounced resonance dip at f_o due to the angular averaging. In the frequency range for which the modal overlap of the chambers are not high enough to ensure a diffuse field, the sound transmission is estimated by individual modes. Therefore the angular weighting technique is less reliable (wave decomposition methods that can model the coupling between the chambers and the structure may be used). The cavity modes (air velocity) are schematically shown in Figure

4.6. The first cavity resonance for the normal incidence (cross mode) occurs at $f_{cav,1} = 922 \text{ Hz}$ and exhibits itself as a dip in all the TL curves in Figure 4.5. Nevertheless the measured dip is not significant because of the superposition of the waves arriving at the structure from all angles, and the damping mechanism in the cavity that is discussed shortly. The tangential modes occur at the frequencies where half a wavelength and its harmonics fit the width or the height of the cavity. At $\sim 140 \text{ Hz}$ and its harmonics, due to the cavity resonance in the tangential directions the velocity at the perimeter becomes high. Therefore a large portion of the energy incident on the source side is lost through the perimeter, causing a reduction in the energy transmission to the receiving side.

Figure 4.6: The cavity modes (air velocity) in the double-leaf specimen. Top: tangential modes; Bottom: cross modes.

4.2.1 Note on the high frequency behaviour

Although the Gaussian weighting improves the TL prediction in most frequency regions, the difference between the measured and predicted results are still large around the cavity resonance. One of the mechanisms that can increase the TL in resonant regions is damping. By taking a closer look at the test wall (the specimen and the filler wall) in Figure 4.7, specifically the cavity between the plates, and with the help of SEA, sources of the TL error can be investigated.

Figure 4.7: The loss mechanism in the cavity: sound transmission (of direct and reflected waves) to the cavity of the filler wall and damping by the insulation.

This perimeter is made of gypsum board “G-1” and has two narrow gaps in the form of slits all around the four sides of the perimeter. These were intended to minimize the structure-borne sound transmission across the filler wall. The incident waves transmitted through the plate arrive at the perimeter of the cavity. Although a

large fraction of the surface area of the perimeter consists of the gypsum board, the sound transmission through the gaps can contribute largely to the overall transmission from the cavity to the filler wall. A substantial portion of the wave that reaches inside the filler wall is dissipated into heat by the cavity insulation.

The TMM model (used here for the prediction of TL of multilayered walls) is formulated for infinite structures. Since in infinite structures there is no interaction between the plate and its boundaries, the method is inherently incapable of dealing with edge losses associated with a finite cavity. Consequently this introduces errors in the TL prediction of the double-leaf wall around the resonance frequencies (such as mass-spring resonance, cavity resonance, etc.). Typically the TL will be under-predicted because of losses at the perimeter that are not accounted for in the TMM model.

4.2.2 Calculating the energy lost through the perimeter of the cavity

This research makes an original contribution by providing a method by which the one-dimensional TMM method can be adapted to account for losses at the perimeter of a finite cavity through the use of SEA. Estimates of the damping loss factor of the cavity are obtained by solving the SEA model of the double-leaf wall for the damping loss factor η_{cav}^d . Estimates are shown to be physically realistic by comparison to a theoretical model. The η^d in SEA is defined for a subsystem as the portion of energy lost in each radian cycle, hence it is a function of time.

Nevertheless, the spaced averaged damping loss factor is directly related to the time averaged damping loss factor, as discussed below, and can be used in the TMM method.

Considering a plane wave in x direction with the damping loss factor η_t , similar to the damping calculated in SEA. It is described in the complete form as in [46]

$$P(x, t) = pe^{-jk_o x} e^{j\omega(1+j\eta_t/2)t} \quad (4.1)$$

The equation is expanded and rearranged as

$$P(x, t) = pe^{j\omega t} e^{-jk_o x} e^{-j\omega \frac{\eta_t}{2} t} \quad (4.2)$$

In the equation above t in $e^{-j\omega \frac{\eta_t}{2} t}$ is replaced by $\frac{x}{c}$ and is rewritten as $e^{-jk_o \frac{\eta_t}{2} x}$.

Hence,

$$P(x, t) = pe^{j\omega t} e^{-jk_o(1-j\frac{\eta_t}{2})x} \quad (4.3)$$

As the spatial damping loss factor η_x is defined as the portion of energy lost in each wavelength, it is proven from the above equation that $\eta_x = \eta_t$. This means that the lost energy in one radian cycle is equal to the lost energy in one wavelength. Therefore the loss factor calculated by SEA can be directly incorporated in the TMM matrix in Equation 3.26 by adding an imaginary part to the real wavenumber

of the air in the cavity [46][35], as shown below

$$k' = k_o \left(1 - j \frac{\eta^t}{2} \right) \quad (4.4)$$

The SEA model

The measurement system was divided into five SEA subsystems: 1- source chamber, 2- Acrylic plate “AC-1”, 3- cavity between the plates, 4- Acrylic plate “AC-1” and 5- receiving chamber, as shown in Figure 4.9. The acoustic power transmission paths consist of non-resonant sound transmission from subsystem i to j , designated by W_{ij} and damped power W_i^d from subsystem i out of the system, i.e. to elements not included in the model [30]. The input sound power generated by loudspeakers inside the source room is also denoted by W_1 . Because of low radiation efficiency of plate modes in frequency range below the critical frequency the resonant sound transmission between rooms and plates is neglected, therefore only forced (direct) transmission between acoustic spaces (rooms and cavity) is included in the model [26]. The filler wall was excluded from the SEA model due to its low sound transmission coefficient.

Figure 4.8: The measured TL of the double-leaf specimen: $\blacklozenge$ — (blue solid line) and the filler wall: $\square \cdots$ (green dotted line).

Figure 4.8 compares the TL of the filler wall to the TL of the double-leaf wall. It shows that the TL difference is at least ~ 16 dB, which means that the specimen has ~ 40 times higher sound transmission coefficient than the filler wall.

Figure 4.9: The SEA subsystems and power transmission paths in the double-leaf wall

The power balance for subsystem 1 is

$$W_1 + W_{31} = W_1^d + W_{13} \quad (4.5)$$

for the cavity

$$W_{13} + W_{53} = W_{31} + W_{35} + W_3^d \quad (4.6)$$

and for the receiving room it is

$$W_{35} = W_{53} + W_5^d \quad (4.7)$$

In SEA, Equations 4.5, 4.6 and 4.7 can be rewritten as

$$\begin{aligned} W_1 + \omega\eta_{31}E_3 &= \omega\eta_1^t E_1 \\ \omega\eta_{13}E_1 + \omega\eta_{53}E_5 &= \omega\eta_3^t E_3 \\ \omega\eta_{35}E_3 &= \omega\eta_5^t E_5 \end{aligned} \quad (4.8)$$

The non-resonant room-to-cavity coupling loss factor, η_{ij} , is given by [29]

$$\eta_{ij} = \frac{13.7S\tau_{ij}}{fV_i} \quad (4.9)$$

where S is the area of the transmission surface, which is a single layer plate, and τ is the transmission coefficient of that plate. For each plate τ is replaced by the measurements on single-leaf plates.¹

For a cavity that is resonance in tangential directions (two-dimensional cavity) the non-resonant cavity-to-room coupling loss factor was calculated in [26] as

$$\eta_{ji} = \frac{\tau_{ij}}{4\pi} \quad (4.10)$$

¹Alternatively, the transmission coefficient of single-leaf specimens can be predicted using mass law or more sophisticated models, but here the measured values are preferred because of higher accuracy.

Craik [32] concluded that the cavity-room coupling loss factors (and opposite) should be higher than what was calculated above. He showed that these loss factors should be multiplied to the term below

$$\frac{\eta_{new}}{\eta_{old}} = \frac{c_o \sigma'}{\omega d} = \sqrt{\frac{c_o U}{f}} \frac{0.45}{2\pi L} \quad (4.11)$$

where U is the perimeter length of the plate.

For the frequencies above the first cavity mode, $\lambda = 2L_z$, the cavity is treated as a chamber, i.e. it can support modes in all three dimensions and the coupling loss factor is calculated using the modal density of a chamber.

Looking back at Equation 4.8 there are two unknowns left in the second and third line, E_3 and η_{3d} , which are calculated by re-arranging those lines and using the total loss factor symbol $\eta_5 = (\eta_{53} + \eta_{5d})$. The power balance equation for the receiving room is re-written as

$$E_3 = \frac{\eta_5 E_5}{\eta_{35}} \quad (4.12)$$

Inserting the E_3 value from Equation 4.12 into the power balance equation for the cavity

$$\eta_{3d} = \frac{\eta_{13} E_1 + \eta_{53} E_5}{E_3} - (\eta_{35} + \eta_{31}) \quad (4.13)$$

Above the first cavity cross mode the cavity is resonant in three dimensions, therefore the cavity is treated as an acoustic chamber and the cavity to room coupling loss factors is calculated from Equation 4.9.

SEA result

The damping loss factor of the cavity, η_3^d , calculated using SEA is plotted in Figure 4.10 along with $\eta_{cav-room}$.

Figure 4.10: The damping loss factor of the cavity, estimated using SEA

One of the main requirements of the SEA model is that both reverberation chambers and the cavity should be resonant. In this analysis, the lowest frequency at which the SEA model is valid is determined by the Schroeder frequency for the cavity.

Confirmation of Approach

The calculated η_{cav}^d in Figure 4.10 is converted into transmission coefficient $\tau_{perimeter}$ using Equation 4.9 and compared to the predicted value to confirm the plausibility of the approach. The prediction introduced here is based on the specifications discussed in Section 4.2.1. The perimeter of the opening in the filler wall consists of two layers of 15.9 mm ($5/8$ in.) gypsum board (the same as the topping material (cladding) of the filler wall) and two lines of gaps with the approximate total width of $10 \sim 15$ mm.

The theoretical transmission coefficient of the perimeter as a function of the gap size is calculated as

$$\tau_{perimeter} = \tau_{gap} r_{gap} + \tau_{gypsum} (1 - r_{gap}) \quad (4.14)$$

where $r_{gap} = \frac{S_{gap}}{S}$. The transmission coefficient through a very narrow, empty slit in the wall is given by Mechel [56] as

$$\tau_{gap} = \frac{2ac_o}{\pi f t^2} \quad (4.15)$$

where a is half the total gap width and t is the thickness of the gypsum board on the perimeter of the cavity.

Since the exact gap width is uncertain, the theoretical $\tau_{perimeter}$ for a range of

plausible gap widths is plotted in Figure 4.11 and the physicality of the SEA predictions are examined.

Figure 4.11: Estimated transmission coefficient of the perimeter of the cavity with various gap sizes, compared to the SEA calculated value. — (grey solid line): calculated SEA, $\times \cdots$ (blue dotted line): 2 mm gap, $\circ - -$ (red dashed line): 15.81 mm gap, $\square \cdots$ (green dotted line): 19 mm gap.

For high frequency range where the SEA is valid, the transmission coefficient estimated using SEA matches with the theoretical results for the 8.5% gap ratio (15.81 mm gap width). Therefore the SEA results are physically plausible.

Damping loss factor η^d in TMM

By inserting the damping loss factor calculated in the complex wavenumber Equation 4.4 the TL is re-calculated as shown in Figure 4.12.

Figure 4.12: TL prediction with damped cavity. measured: $\blacklozenge$ — (blue solid line), prediction with η_d : $\circ$ — (red dashed line), Gaussian weighted average $\eta_d = 0$: $\triangle$... (green dotted line).

The damping mechanism increases the TL at high frequency and makes the TMM prediction match better with the measured data.

The estimated damping loss factor can be used for the prediction of the TL of the triple-leaf specimen.

4.3 Double-leaf wall (“TLA-13-139”)

In test “TLA-13-139” the TL of another double-leaf specimen (Figure 4.13) is measured. The specimen consists of one layer of Acrylic plate “AC-1” mounted on

the boundary of the opening, facing the small chamber, 93 mm deep cavity and the Aluminum plate “AL-1” placed half way inside the aperture. Similar to Section 4.2 the cavity damping is theoretically calculated and validated by theoretical estimation of sound transmission through the perimeter. The results are shown in Figure 4.14.

Figure 4.13: The double-leaf specimen installed in the filler wall

Figure 4.14: TL prediction with damped cavity. measured: $\blacklozenge$ — (blue solid line), Gaussian weighted prediction with damping: $\circ$ — (red dashed line), Gaussian weighted no damping: $\triangle$... (green dotted line).

It seems that the proposed theoretical method for cavity damping is applicable to this structure as well, since incorporating the damping in the TMM model improved the prediction in high frequency range.

4.4 Triple-Leaf Wall (“TLA-13-124”)

The TL measurement “TLA-13-124” is carried out on the triple-leaf specimen (Figure 4.15) consisting of the combination of the previous single-leaf and double-leaf measurements (Sections 4.1 and 4.2), i.e. the “AC-1” Acrylic plate, 93 mm deep

cavity, Aluminum plate “AL-1”, 93 mm deep cavity and the “AC-1” Acrylic plate.

Figure 4.15: The triple-leaf specimen installed in the filler wall

The measured TLs of the triple-leaf structure and the double-leaf structure (where the mid leaf is removed) are compared in Figure 4.16.

Figure 4.16: Comparison of the measured TL for different specimens. Non-perforated triple-leaf: $\circ$ — (blue dashed line), non-perforated double-leaf: $\square$ — (green dashed line).

Adding a third layer in the middle of a double-leaf wall improves the TL in high frequencies but decreases the low frequency TL . According to the calculations by Fahy et al. [9] the TL of multilayered structures in frequencies well above the system resonance is controlled by the total mass of the plates. Since the triple-leaf structure is heavier than the double-leaf, the high frequency TL is higher. On the contrary, in low frequencies the second system resonance introduced by the third leaf decreases the TL around that frequency.

4.4.1 TMM model

The damping loss factor of the perimeter of the cavity is replaced by the value obtained for the narrow double-leaf system “TLA-13-139”. In Figure 4.17 the test results are compared to the no-loss TMM prediction and the damped cavity model. Also the mass-spring and cavity resonances are displayed for the normal incidence transmission.

Figure 4.17: Triple-leaf wall TL prediction with damped cavity. measured: $\blacklozenge$ — (blue solid line), prediction by damped Gaussian weighted average: $\circ$ — (red dashed line), Gaussian weighted average $\eta_d = 0$: $\triangle \cdots$ (green dotted line).

Equations 3.16 and 3.17 estimate the mass-spring resonance frequencies of the triple-leaf system at normal incidence ($\theta = 0$). The first resonance is exactly equal the system resonance of the double-leaf wall in Section 4.2 ($f_{o,1} = 85.8 \text{ Hz}$) and

the higher one is at $f_{o,2} = 158.4$ Hz (less than twice the $f_{o,1}$). The lowest cavity resonance frequency is located at 1844 Hz, twice the value for the double-leaf wall, which is twice as deep as the triple-leaf wall.

In Figure 4.17 similar to the double-leaf case, the TMM model without accounting for losses of the cavity perimeter is less accurate in predicting the TL in the frequency region where the cavity resonance dips occur. The reason is that in the resonant frequency regions the system is damping controlled, meaning that incorrect estimates of damping will cause large errors in the predicted TL.

As it can be seen in the figure, in the high frequency range the TMM model could not correctly estimate the TL and its slope. This is because in the frequency region where the first cross mode in the cavity occurs (and above), the cavity becomes resonant in tangential directions of the cavity (see Figure 4.6) and it starts to behave like an acoustic chamber with resonant modes in all three directions. However, the TMM model used here is an extended version of a one-dimensional model that is no longer reliable for predicting the TL across the cavity.

The finite, Gaussian angular weighted TMM model used in this thesis for the prediction of sound transmission across multilayered walls is built on a number of assumptions that may not be valid in numerous situations. For example, the TMM presumes that:

1. the angular distribution of energy, remains unchanged throughout the whole frequency range and the Gaussian weighting function is limited to $1 < \beta < 2$

[22],

2. the spatial windowing technique, used for incorporating the finiteness of the specimen in TMM, can be applied to multilayered walls that include cavities (adopted from [24]).

These assumptions have not been proven to be always correct, neither is it certain that these assumptions are valid for the problems in this thesis.

4.4.2 TL prediction using SEA

SEA can be an alternative method for TL prediction that involves less assumptions compared to TMM. It is relatively reliable for the high frequency region, where the modal overlap is high enough for all the subsystems, including the cavities in multilayered walls. The TL calculation procedure for the triple-leaf wall in Figure 4.18 is a variation of the procedure in Section 4.2.2. Instead of solving the energy equations for the unknown cavity damping, the TL and reverberation time values found in previous measurements are used to find a rough estimate of the TL for the triple-leaf structure.

Figure 4.18: The SEA subsystems and power transmission paths for the triple-leaf wall

Here, η_{13} , η_{35} , η_{57} and the opposites are calculated from the single-leaf measurements and the total loss factor of the receiving room, η_7 is replaced by the value for the double-leaf in test “TLA-13-139”². The SEA predictions are compared to the measurements and the TMM prediction in Figure 4.19.

²The double-leaf results works only as an approximation for the triple-leaf.

Figure 4.19: Triple-leaf wall TL prediction with damped cavity. measured: $\blacklozenge$ — (blue solid line), TMM prediction: $\circ$ — (red dashed line), SEA prediction: $\square\cdots$ (green dotted line).

The SEA prediction is limited to the high frequency range, where the modal overlap of the cavities are sufficiently high. Two different cavity models are employed for the frequencies below and above the cavity resonance. Below the cavity resonance frequency, the cavity has resonant modes in two dimensions, along the width and height of the plate (but not the depth). Above this frequency, the cavity has resonant modes in all three dimensions.

4.4.3 Combining TMM and SEA

As both TMM and SEA methods have their own advantages and limitations, a proper combination of these methods can lead to a more accurate prediction. The TL curve can be divided into three regions, separated by the vertical lines pertaining to the modal overlap of the cavities.

The TMM model works best for low frequencies where the sound transmission can be described as one-dimensional. For the region where the modal overlap of the cavity becomes high enough to support modes in two dimensions (along the width and the height of the plates) the two-dimensional SEA model is more appropriate (cavity modelled as two-dimensional, cavity-room coupling: Equation 4.10) [26]. Beyond the first cavity cross-mode and after the frequency at which the cavity becomes resonant in three dimensions, the three-dimensional SEA model is the most suitable prediction model (cavity modelled as a chamber, room-room coupling: Equation 4.9) [29]. The combination of the TLs prediction by these models are presented in Figure 4.20.

Figure 4.20: Triple-leaf wall TL prediction by TMM: $\circ$ — (red dashed line), 2D SEA: $\times \cdots$ (orange dotted line), 3D SEA: $\square \cdots$ (green dotted line), compared to measurement: $\blacklozenge$ — (blue solid line)

4.5 Perforated Walls

Utilizing a perforated leaf in multilayered walls may have positive effect on sound insulation. In order to examine this possibility, the sound transmission loss through a perforated single-leaf plate is measured and compared to TMM prediction. After the validation of the model, it is used for the TL prediction of a triple-leaf specimen that features the perforated plate in the middle.

4.5.1 Perforated Single-Leaf Wall (“TLA-13-117”)

The TL test “TLA-13-117” is performed on the single-leaf perforated Aluminum plate “AL-P-1”, positioned in the middle of the test aperture, as sketched in Figure 4.21.

Figure 4.21: The perforated single-leaf specimen installed in the filler wall

In Figure 4.22 the measurement results are compared to the TL predicted by the Gaussian weighted angular averaged TMM model with $\beta = 2$. This model provides an easy impedance-based approach to the TL prediction of perforated (multi-layered) walls. It treats the perforated plate as a usual non-perforated flexible plate that has a complex mass, which was discussed in Section 3.1.2.3.

Figure 4.22: The measured and predicted TL of the perforated single-leaf specimen. Measured: $\blacklozenge$ — (blue solid line), Gaussian weighted average TMM: $\square$ — — (red dashed line).

The theoretical model provides an acceptable prediction of the measurement. This proposed impedance-based model can now be inserted into the transfer matrix formula for the perforated triple-leaf walls.

As the TL of the single-leaf perforated plate is very low, it would be interesting to investigate the actual effectiveness of the perforated plate by comparing the test result of the perforated plate to that of the open aperture (no specimen installed in the opening in the filler wall).

Figure 4.23: Measured TL of the perforated single-leaf plate: $\blacklozenge$ — (blue solid line), and the open aperture: $\square \cdots$ (red dotted line).

As was seen in all the measured data and in the model for finite specimen the TL does not approach zero when $f \rightarrow 0$. As shown in [24] the radiation efficiency of a finite structure is always lower than its corresponding infinite one, hence has a higher TL.

As shown in Figure 4.23 the perforation is effective only in the high frequency region, where the imaginary part of the impedance (Equation 3.40) becomes noticeable.

4.5.2 Perforated Triple-Leaf Wall (“TLA-13-125”)

After validating the prediction model of the perforated single-leaf plate against the test results, the multilayered specimen having a perforated plate in the middle is tested. Then the result is compared to the non-perforated triple-leaf wall, in order to investigate the effect of perforation on TL.

In order to model the specimen, in the TMM formula for the non-perforated triple-leaf specimen the impedance of the middle leaf is replaced by the complex impedance of the single perforated plate Z_{pp} .

The tested specimen resembles the non-perforated triple-leaf specimen in “TLA-13-124”, only here the middle leaf is replaced by the perforated Aluminum plate “AL-P-1”, as drawn in Figure 4.24. In Figure 4.25 the test result is compared to the TMM prediction.

Figure 4.24: The perforated triple-leaf specimen installed in the filler wall

Figure 4.25: Perforated triple-leaf wall TL . measured: $\blacklozenge$ — (blue solid line), prediction with cavity damping: $\circ$ — (red dashed line), Gaussian weighted average no damping: $\triangle$... (green dotted line).

According to Figure 4.25 the best TL prediction is given by the finite, Gaussian-weighted angular-averaged TMM model including the cavity damping value calculated for the narrow double-leaf specimen in “TLA-13-139”.

Figure 4.26 compares the TL of the perforated triple-leaf wall in “TLA-13-125” to the its corresponding non-perforated double- and triple-leaf structures. The non-perforated double-leaf structure is constructed by removing the Aluminum plate in the middle from the perforated triple-leaf test, and the non-perforated triple-leaf structure is made by replacing the perforated mid leaf with the non-perforated Aluminum plate.

Figure 4.26: Comparison of the measured TL for different specimens. Non-perforated triple-leaf: $\circ - -$ (blue dashed line), non-perforated double-leaf: $\square - -$ (green dashed line), perforated triple-leaf: $\times \cdots$ (red dotted line).

Due to the low TL value of the perforated single-leaf specimen (in “TLA-13-117”), the TL of a triple-leaf specimen with a perforated mid layer is very close to that of its corresponding double-leaf specimen in “TLA-13-131” (no mid-leaf). At the first cavity resonance of the deep double-leaf specimen (at 922 Hz) the TL value of the double-leaf is slightly higher than that of the perforated triple-leaf. At this frequency, high pressure difference across the middle leaf results in a high plate velocity. The acoustic intensity absorbed by the perforated plate is proportional to the velocity squared.

$$I_{absorb.pp} = \frac{1}{2} |v_{pp}|^2 Re(Z_{pp}) \quad (4.16)$$

where $I_{absorb, pp}$ is the intensity dissipated by the perforated plate and the v_{pp} is the average velocity of perforated plate. Hence, TL improvement at this cavity resonance frequency is expected.

4.6 Parametric study

In this section the effect of change in geometrical parameters of multilayered specimens on the sound transmission is investigated. The sound transmission data is provided by the theoretical models validated in this thesis. This parametric study assists with the design of optimal tests to be performed on real walls.

4.6.1 Non-perforated triple-leaf structures

4.6.1.1 Thickness of mid leaf

In order to examine the influence of changing the thickness of the mid leaf in triple-leaf structures on the TL, three thicknesses $0.5t$, t and $2t$ have been selected. The triple-leaf structure with the thickness of the mid leaf t is identical to the structure in “TLA-13-124”. The TL predicted by finite Gaussian averaged TMM is presented in Figure 4.27 and the ΔTL (TL difference) between the structures with different thicknesses is shown in Figure 4.28.

Figure 4.27: Changing the thickness of the mid leaf in a triple-leaf wall: TL predicted by TMM. $TL(0.5t)$: $\times \cdots$ (blue dotted line), thickness $TL(t)$: $-\bigcirc-$ (grey solid line), thickness $TL(2t)$: $\square--$ (red dashed line).

Below the first system resonance (at f_{01}), the structure behaves as a rigid mass. Increasing the thickness of the mid leaf will increase its impedance (the equivalent mass). Therefore the multilayered structure with the thickest mid leaf (“ t ”) will have a higher TL. Also it pushes the second system resonance to a lower frequency (Equation 3.17). In high frequency range, after the critical frequency, the TL curve rises with a sharper slope. The thickest plate (“ $2t$ ”) has the lowest critical frequency (Equation 3.3), hence, it has the largest TL.

Figure 4.28: Changing the thickness of the mid leaf in a triple-leaf wall: ΔTL (TL difference) predicted by TMM. $TL(0.5t) - TL(t)$: $\times \cdots$ (blue dotted line), $TL(2t) - TL(t)$: $\square - -$ (red dashed line).

4.6.1.2 Depth of the cavities

The impact of changing the depth of the cavities in triple-leaf structures on the TL is investigated. Three values $0.5L$, L and $2L$ have been chosen as the depth of each cavity. These structures are symmetric, meaning they have identical cavities and topping layers at both ends. The triple-leaf structure with the cavity depth L is identical to the structure in “TLA-13-124”. The TL predicted by finite Gaussian averaged TMM is presented in Figure 4.29 and the ΔTL (TL difference) between the structures with different cavity depths is shown in Figure 4.30.

Figure 4.29: TL predicted by TMM. $TL_{0.5L}$: $\times \cdots$ (blue dotted line), TL_L : $-\bigcirc-$ (grey solid line), thickness TL_{2L} : $\square - -$ (red dashed line).

Figure 4.30: ΔTL (TL difference) predicted by TMM. $TL_{0.5L} - TL_L$: $\times \cdots$ (blue dotted line), $TL_{2L} - TL_L$: $\square - -$ (red dashed line).

Increasing the cavity depth shifts the TL curve to a lower frequency, because the cavity stiffness (Equation 3.14) is inversely proportional to the cavity depth. The depth of the cavity can be designed for TL improvement, depending on the targeted frequency region.

4.7 Summary

The prediction models for the air-borne sound transmission through non-perforated single- double- and triple-leaf as well as the perforated single- and the triple-leaf with perforated mid-leaf structures were validated against the measurements. The parametric studies were performed on triple-leaf structures to find optimum compositions. For the multi-layered structures, the TMM model was accurate in low frequencies, except at the frequencies where the tangential cavity modes occur. The thesis contributed to the calculation of the energy loss through the perimeter of the cavities using SEA and the verification of the physicality of the results by the theoretical method [56]. The damping loss factor was substituted in the prediction models and improved the high frequency prediction. However, the TMM prediction were not as accurate as the SEA in high frequencies, because of the cavities that are resonant in two and three dimensions. The existence of the middle leaf in the non-perforated triple-leaf structure improves the high frequency TL due to the added mass, but reduces the low frequency TL because of the additional

mass-spring resonance. Perforating the middle leaf in the triple-leaf structure will reduce the impact of the second mass-spring resonance on TL, but reduces the high frequency TL. The parametric studies show that a triple-leaf structure with thinner middle leaf will have a higher TL in the low frequencies. Also deeper cavities shift the TL curve to lower frequencies.

Chapter 5

Conclusions

5.1 Summary

Air-borne sound transmission measurements were performed on different small-scale test specimens in order to develop and validate models to predict the air-borne sound transmission across a triple-leaf shear wall. This model can be developed further to model more complex structures including wood framed shear walls. Adding a third layer in the middle of a double-leaf wall improved the air-borne TL in high frequencies (because of higher total mass), but decreases the low frequency TL (due to the second system resonance). The disadvantage of the reduction in low frequency TL outweighs the high frequency TL improvement, since the high frequency TL of the double-leaf structure is already great and can be improved even further by using poro-elastic material in the cavity [5, 57].

The air-borne sound transmission of single-, double- and triple-leaf specimens was predicted using various models for different frequency regions. The mass-spring model was able to provide a first approximation for low frequency TL, but was not applicable to higher frequency regions, where the travelling waves in the cavity and the flexibility of the plates become influential. The solution to this problem was found in the modified Transfer Matrix Method (TMM) model [7, 8, 22, 24]. A summary of the prediction methods discussed in this thesis is given in Table 5.1.

The conventional TMM model predicted the angular-dependent sound transmission through infinitely extended (one-dimensional) systems, but its prediction results could not reasonably match the measurements in the mid and high frequencies. Since the test structures (unless in very high frequencies) are not accurately represented by infinite models, the spatial windowing technique [24] was used to incorporate the finiteness. Moreover, it was learnt that in the reverberation chambers the sound waves are incident on the test structures from all different angles, with the Gaussian weighted angular distribution [22]. The finite, Gaussian-weighted TMM model could accurately predict the measured TL of single-leaf specimens in all frequencies. However, regarding the multilayered structures (double- and triple-leaf specimens), the model could only accurately predict the measured TL below the frequency at which the cavities start to become two-dimensional (resonant in tangential directions of the cavities). Above this frequency the cavities are no longer one-dimensional, hence the cavity damping (dissipation through the perimeter) plays an important role in the sound transmission. Since the damping loss factor of the cavities could not be directly measured, the proposed approach

Method	Assumptions / Restrictions	Cavity			Perforated Mid Leaf	Suitability for Triple-Leaf Walls
		1D	2D	3D		
Mass-Spring Model	Rigid mass motion Spring-like cavity (1D system)	✓			×	Low frequencies (<i>Not accurate</i>)
Transfer Matrix Method	Infinite plates + infinite cavities (1D system)	✓			✓	Low and Mid frequencies (<i>Low frequency not accurate</i>)
Finite TMM	Finite plates + infinite cavities	✓			✓	Low and mid frequencies
Finite TMM + Cavity Perimeter Loss	Finite plates + finite cavities			✓	✓	Low, mid and high frequencies (<i>High frequency not as accurate as SEA + Cavity Perimeter Loss</i>)
Statistical Energy Analysis	High frequencies $M > 1$		✓	✓	✓	High frequencies (<i>Not accurate</i>)
SEA + Cavity Perimeter Loss	High frequencies $M > 1$		✓	✓	✓	High frequencies

Table 5.1: Summary of the prediction methods used in the thesis

to resolve this issue was to estimate the damping loss factor of the cavity in the double-leaf structure using the statistical energy analysis and the TL data obtained from the measurements performed on the constituent single plates. The range of estimated damping loss factor was verified by comparing the SEA values to the ones obtained by the theoretical sound transmission analysis on the gaps and plates of the boundary [56]. There are two ways to incorporate the damping in the predictions. The first one is to extend the finite TMM model by introducing the cavity damping in form of the real part of the wavenumber in the cavities, which is an original contribution. Although this improves the predictions in high frequencies, the SEA model with the same cavity damping predicts the TL much closer to the measured TL. This was expected, because the SEA model can inherently deal with two- and three-dimensional systems, hence no modification to the method was needed. The SEA method is not applicable for lower frequency ranges, where the modal overlap is low. In fact a combination of the modified TMM and the SEA models for different frequency ranges gave a good estimate of the air-borne TL in triple-leaf walls. For the multilayered walls, beyond the frequency where the cavities become resonant in tangential directions of the cavity (along the width and the height), the dissipated energy through the perimeter of the cavities cannot be neglected, and must be explicitly considered in models.

The effect of perforating the mid leaf of the triple-leaf structure on TL was investigated. Measurements were performed on single-leaf perforated plate as well as on a triple-leaf structure with a perforated plate in the middle. Perforating the middle plate improves the low frequency sound insulation by making the triple-leaf

behave like a double-leaf. The impedance of the perforated plate was theoretically derived [38, 39, 40, 41, 42, 43], validated and substituted into the TMM equation for the triple-leaf structure. It was noticed that the perforation was not effective on the first resonance of the system, because the mid leaf has theoretically zero velocity amplitude, hence causes no dissipation. However, perforating the mid leaf shifts the second resonance of the system to a higher frequency range, improving the mid frequency TL. Since the mode shape associated with this resonance involves a moving mid leaf, the perforation will introduce additional damping to the multilayered structure.

5.2 Future Work

Five areas are suggested for future work:

1. Further studying of the mechanism behind the cavity damping and validation of the modified TMM method developed in this thesis for a wide range of multilayered structures. The effect of cavity dimensions and the gap size on the damping has been shown to be important in this thesis, but should be tested on a range of different types of walls. The theoretical study in the thesis is based on a simplified formula [56] and the energy distribution on the perimeter is not accurately known, which may have an impact on the accuracy of TL predictions.
2. To validate the modified TMM method in predicting the air-borne sound

transmission through wood shear walls containing a middle leaf that is perforated, and to determine if an acceptable degree of perforation for acoustical considerations is also acceptable from a structural point of view.

3. Because the thesis showed the benefit of using a perforated mid-leaf, it would be valuable to find the optimum geometry for perforated plates that gives the highest TL. Perforated plates with practical thicknesses, hole diameters and porosity ratios should be tested to investigate the effectiveness of perforation in real life structures.
4. In this thesis a number of multilayered systems with empty cavities (no cavity absorption) was studied. It would be beneficial to look at more general and practical situations, including cases with cavity absorption.
5. To further develop the model to provide an estimate that includes the structure-borne sound transmission path in the models: the test specimens were designed to exclude the structure-borne sound transmission path. However, in real life structures because of the connections between the leaves the structure-borne transmission can be significant.
6. To Perform measurements and to investigate the sound transmission through triple-leaf wood structures.

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Appendices

Appendix A

Specifications of the structures

A.1 Specifications of Plates

Specifications of the constituent plates of the specimens used in TL tests, Table A.1.

Plate (code)	Material	Density ρ [kg/m ³]	Modulus of Elasticity E [Pa]	Thickness t [mm]	Surface dimensions S [m $\times$ m]
AL-1	Aluminum	2712	68.9×10^9	1.6	1.22×1.22
AC-1	Acrylic (Poly methyl methacrylate)	1088	3.3×10^9	4.8	1.22×1.22

(a) Non-perforated plates

Plate (code)	Material	Density ρ [kg/m ³]	Modulus of Elasticity E [Pa]	Thickness t [mm]	Surface dimensions S [m $\times$ m]	Hole diameter d [mm]	Porosity ratio $s\%$
AL-P-1	Aluminum	2712	68.9×10^9	1.6	1.22×1.22	3.2	35

(b) Perforated plate

Table A.1: Specifications of the constituent plates of the specimens.

A.2 Description of the Filler Wall in TL Tests

Figure A.1: The filler wall installed in the test frame

Figure A.2: Inside the filler wall

Cladding 1 (Small Chamber Side)

- Two layers of 16 mm ($\frac{5}{8}$ in.) gypsum board Firecode Type X directly attached.
- Base layer attached using Type S screws 25 mm long spaced 406 mm o.c. at the perimeter and in the field.
- Face layer attached using Type S screws 41 mm long spaced 406 mm o.c. at the perimeter and in the field.

- Screws from base layer to face layer offset by 203 mm.
- Joints of base layer staggered by at least one stud spacing from joints of Cladding 2 gypsum boards.
- Joints of face layer staggered by at least one stud spacing from joints of base layer Cladding 1 gypsum board.
- Gaps along the joints and the perimeter of gypsum boards caulked and taped with aluminum tape.

Framing and Cavity Insulation

- Double stud framing with 20 mm air gap in between.
- 65 mm (2½ in.) base track attached to test frame using screws 13 mm (½ in.) long spaced 610 mm o.c.
- Closed-cell foam placed between the base track and the test frame.
- 65 mm (2½ in.) steel studs spaced 610 mm o.c. attached to base track using screws 13 mm (½ in.) long.
- End studs attached to facility end walls using screws 13 mm (½ in.) long spaced 610 mm o.c. Closed-cell foam placed between the end stud and the facility wall.
- Top track attached to test frame using screws 13 mm (½ in.) long spaced 610 mm o.c.

- Infill element with support structure built to accommodate 1220 mm×1220 mm (4 ft. × 4 ft.) test specimen.
- Cavities between studs filled with 610 mm wide × 610 mm thick steel stud insulation.
- Caulking around the perimeter of the steel frame.

Cladding 2 (Large Chamber Side)

- Base layer attached using Type S screws 25 mm long spaced 406 mm o.c. at the perimeter and in the field.
- Face layer attached using Type S screws 41 mm long spaced 406 mm o.c. at the perimeter and in the field.
- Screws from base layer to face layer offset by 203 mm.
- Joints of base layer staggered by at least one stud spacing from joints of Cladding 1 gypsum boards.
- Joints of face layer staggered by at least one stud spacing from joints of base layer Cladding 2 gypsum board.
- Gaps along the joints and the perimeter of gypsum boards caulked and taped with aluminum tape.

A.3 Test chambers

Figure A.3: Filler wall installed in the test frame inserted between the reverberation chambers.

Figure A.4: The test wall installed between the chambers, an aperture in the filler wall was cut for placing the specimen

Appendix B

Confidence interval for TL

The procedure for calculating the confidence interval for TL is given in Reference [53].

The space averaged level for each frequency band is

$$\bar{L} = 10 \log \left[\frac{1}{n} \sum_{i=1}^n 10^{L_i/10} \right]$$

where L_i is the time averaged level at location i and n the number of microphone positions in each room. In this thesis there are 9 microphone positions in each room. The standard deviation (either for sound pressure level or sound absorption) is calculated as below

$$s(f) = \sqrt{\frac{1}{n-1} \sum_{i=1}^n [X_i(f) - \overline{X(f)}]^2}$$

where $X_i(f)$ is the measured quantity in position i and $\overline{X(f)}$ is the average of the measured quantities at all positions. The 95% confidence interval for $n = 9$ is calculated as

$$\Delta X = 0.769s$$

The confidence interval for TL is measured as

$$(\Delta TL)^2 = (\Delta L_S)^2 + (\Delta L_R)^2 + 18.9 (\Delta A_R/A_R)^2$$

where subscripts S and R denote the source and receiving rooms. Since the TL tests are performed in both directions the confidence interval for the complete measurement is given by

$$\Delta TL_t = \sqrt{(\Delta L_1)^2 + (\Delta L_2)^2}$$