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ANALYSIS OF SHORT RECTANGULAR REINFORCED CONCRETE
COLUMNS SUBJECTED TO BIAXIAL MOMENTS

by

© Alex Y. Sallah, B.Eng.

A Thesis Submitted to the Faculty of
Graduate Studies in Partial Fulfillment
of the Requirements for the Degree of
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ABSTRACT

A parametric method for the design of rectangular columns subjected to moments about two cross sectional axes and axial loads is presented. The method and the associated design aids and equations are based on a comprehensive computer study of square and rectangular columns. The columns were studied for different axial loads, column aspect ratios; f_y , f'_c reinforcement percentages and bar arrangements.

Analysis and design problems solved by the parametric method are presented and compared with the computer solutions. Close agreement was found between the parametric and computer solutions. A comparison between the parametric method and the PCA method is also presented in this study.

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NOTATION

A	$1-\sqrt{2}\beta$, factor used in parametric formulation $R = 1-A\sin^2\theta$
A_s	Area of one reinforcing bar
A_{ST}	Total area of reinforcement in cross section of column
B	Breadth of column cross section
e_x, e_y	Eccentricities of applied load
f'_c	Concrete cylinder stress
f_y	Yield (maximum) stress in steel
F	Constant relating failure surfaces of two identical columns except for depth of cover ratio
H	Over all depth of column cross section
m	$\log 0.5/\log \beta$, exponent used in Load-contour equation: $(M_x/M_{ox})^m + (M_y/M_{oy})^m = 1.0$
M_d	Moment capacity of square column bending about a diagonal
M_{ox}, M_{oy}	Uniaxial moment capacity of column for load acting in Y and X planes, respectively
M_x, M_y	Biaxial moments acting on a column
n	Exponent in the parametric formulation: $R = 1-A\sin^n\theta$
N	Factor used in Pannell's formulation: $M_y = \frac{F \cdot M_{oy} (1-N\sin^2\theta)}{\sec\theta}$
P_u	Factored load acting on cross section of a column
P_{uo}	Factored ultimate axial load acting at plastic centroid of cross section
P_u/P_{uo}	Axial load ratio
P_x, P_y	Ultimate loads acting in X and Y planes, respectively

q $\rho f_y / f'_c$ reinforcement index
 R_θ Non-dimensional radial moment at angle θ to horizontal axis of load-contour
 X, Y Cross sectional axes
 β Originate of non-dimensional load-contour at $\theta = 45^\circ$
 γ Ratio of center to center distance of corner bars of reinforcement on a side to the length of that side
 ρ A_{ST} / BH . reinforcement percentage in cross section of column
 ϕ Capacity reduction factor (= 0.7 for columns with rectangular ties)

CHAPTER 1

INTRODUCTION

1.1 General

The exact mathematical prediction of the ultimate strength of a short rectangular reinforced concrete column loaded eccentrically about two cross sectional axes by hand computation methods is a lengthy and laborious process. It is possible to assume a neutral axis location and fairly quickly compute a load value placed at a theoretically correct position of bi-axial eccentricity. However, the reverse process, that of assuming a load position and then calculating the load, which is more meaningful in design, is only possible by an extremely lengthy trial and error procedure. In practice close approximations are sought.

For a rectangular column cross-section loaded eccentrically about both cross-sectional axes the load-moment interaction surface for one quadrant takes the form shown in Fig. 1.1. A typical horizontal section of this surface, known as load-contour, is shown in Fig. 1.2 in which the intercept at either axis represents the uniaxial moment capacity. Most of the approximations, based on modified forms of Figs. 1.1 or 1.2, seek to relate the biaxial response of a column to its uniaxial capacity.

Whilst the existing hand computation methods for the design of biaxially loaded columns have usually been shown to give satisfactory results they have tended to be, in general, either too approximate, though simple, or relatively complex and burdened

by protracted design aids. The present approach, also based on the load-contour of Fig. 1.2, is an attempt to produce a simpler method for the design of biaxially loaded columns, while attaining a far greater accuracy than other simple approaches.

1.2 Objective and Scope of the Study

The non-dimensional load-contour typically takes the form shown in Fig. 1.3 where the values of M_y and M_x have been divided by axes intercept values M_{oy} and M_{ox} at the appropriate level of P_u . On this diagram can be drawn radial lines, R_θ , at angle θ to the horizontal axis. The radial lines represent the non-dimensional resultants of the ultimate biaxial moments acting on a column section. Two such radial lines are of particular interest to this study. These are the R_{45} and $R_{22.5}$ (or $R_{67.5}$) lines.

The main purpose of this study is to seek a parametric relationship for the non-dimensional load-contour (Fig. 1.3) based on the R_{45} and $R_{22.5}$ values. Pilot studies on this parametric approach have been carried out by Duschene [Ref. 4]. A secondary objective of this study is to establish the Metric version of β vs. P_u/P_o curves [Ref. 10]. The parameter, β [Refs. 9,10], has been defined as the ordinate of the non-dimensional load-contour at $\theta = 45^\circ$. On the load-contour of Fig. 1.2, it has been pointed out in Ref. 9 that β is related to the point P such that at this point the biaxial moment capacities M_{xp} and

M_{yp} are in the same ratio as the uniaxial moment capacities M_{ox} and M_{oy} ; thus at point P

$$\frac{M_{yp}}{M_{xp}} = \frac{M_{oy}}{M_{ox}} \quad 1.1$$

$$\text{or } M_{xp} = \beta M_{ox}, \quad M_{yp} = \beta M_{oy} \quad 1.2$$

The angle, θ_p , made by the radius to point P with the horizontal axis is given by

$$\theta_p = \tan^{-1} \left(\frac{M_{oy}}{M_{ox}} \right) \quad 1.3$$

This alternative definition of β is very useful in that the angle θ_p given by Eqn. 1.3 can be input into a computer program to obtain the coordinates of point P directly for any column.

The computer program used in this study to generate the load contour points accepts the angle θ in degrees at which the load P_u acts with the horizontal cross sectional axis and calculates the ultimate moment (s) that can act on the column at the angle θ . Thus, for $\theta = 0^\circ$ and 90° the uniaxial moment capacities M_{oy} and M_{ox} respectively are calculated, and for $0^\circ < \theta^\circ < 90^\circ$ the ultimate biaxial moments M_x and M_y are calculated. The procedure for calculating β for any column is then as follows:

1. Input $\theta = 0^\circ$ and 90° to calculate M_{oy} and M_{ox} respectively
2. Calculate $\theta = \tan^{-1} (M_{oy}/M_{ox})$
3. Input $\theta = \tan^{-1} (M_{oy}/M_{ox})$ to calculate M_x and M_y

4. The parameter β is computed as

$$\beta = \frac{M_x}{M_{ox}} = \frac{M_y}{M_{oy}}$$

To achieve these twofold objectives, a comprehensive computer analysis of square and rectangular column sections was carried out covering all of the relevant parameters associated with column design. The parameters studied are

1. B/H ratio (or aspect ratio) of the column cross section
2. Number of bars in the face of the column cross section in addition to corner bars - loosely referred to as bar arrangement.
3. Reinforcement percentage, ρ .
4. Concrete strength, f'_c
5. Reinforcement yield strength, f_y .
6. Cover ratio
7. Axial load ratio, P_u/P_{uo}

Two secondary parameters were also investigated. They are

8. q , defined as $\rho f_y / f'_c$
9. Q , defined as $\rho P_u / P_{uo}$

The significance of these two secondary parameters will be investigated further in Chapter 4.

Figure 1.1 One quadrant of Load-Moment Interaction Surface
For a Short Column

Figure 1.2 Load-Contour at Plane of Constant P_u

Figure 1.3 Non-dimensional load-contour at plane of Constant P_u

CHAPTER 2

LITERATURE REVIEW

2.1 General

Several researchers have studied the problem of biaxial bending of reinforced concrete columns and have presented either equations or design curves to be used in conjunction with uniaxial moment capacity curves. Whilst each approach may have been based on rigorous equilibrium and compatibility analysis and therefore could be relied upon to give satisfactory results, the only problem appears to be with the ease a particular method can be applied and the number of associated design aids that must be used. Some researchers have investigated the variables stated in Section 1.2 and those that were found to be unimportant were eliminated. This enabled them to produce condensed design charts of general application. Some of the methods available in the literature will be discussed in the remaining sections of this chapter.

2.2 Review of Previous Work

Fleming and Werner [Ref. 5] in 1965 presented a set of design curves for a square column 8-bar arrangement (4 corner bars and 1 additional bar in each face). Each design curve provided a partial representation of the 3-dimensional interaction surface for the specific bar arrangement. Their method has been noted to be accurate but is severely limited

for practical applications by the large number of design curves that must be produced for all possible column sections and bar arrangements. A full set of design curves was not provided by these authors.

Furlong [Ref. 6] studied the response of two specific column geometries under biaxial bending for two reinforcement percentages (1 percent and 4 percent). The two column geometries were:

1. square column with 4 corner bars and 2 additional bars in each face;
2. square column with 4 corner bars and 2 additional bars in only two opposite faces.

From the results obtained Furlong produced load-contours for different axial load ratios which enabled him to obtain information about the deviation of the moment capacities from the circular curve, that is, the dip in the non-dimensional load-contour (Fig. 1.3). He observed that the maximum deviation from the circular curve occurred at 45° to the major axis of the cross-section. Furlong suggested that the capacity of square columns at load P_u be checked with the following circular form of equation/inequality,

$$\left(\frac{M_x}{M_{ox}}\right)^2 + \left(\frac{M_y}{M_{oy}}\right)^2 \leq 1.0 \quad 2:1$$

where:

M_x = moment component about the x-axis

M_y = moment component about the y-axis

M_{ox} = moment capacity at load P_u when M_{oy} is zero

M_{oy} = moment capacity at load P_u when M_{ox} is zero.

Furlong's study did not cover a wide range of variables and therefore could not be relied upon for design. It, however, provided an insight into the ultimate strength behaviour of columns as depicted on the non-dimensional load-contour (Fig. 1.3).

Weber in 1966 [Ref. 13] presented a method intended to simplify the analysis and design of biaxially loaded square columns. He proposed that the failure surface of square columns with equal amounts of reinforcement in all faces could be approximated by linear interpolation between the capacity for bending on the diagonal and the capacity for bending on a principal axis. Weber's method is heavily dependent on the availability of design charts for bending on the diagonal for all possible parameters encountered in design.

Two papers of particular interest to the present study are those of Bresler [Ref. 2], and Pannell [Ref. 8]. These two papers will be discussed in detail in the following paragraphs.

Bresler [Ref. 2] in 1960 proposed two methods by which the capacity of a short rectangular column subjected to moment about two cross-sectional axes could be computed from the uniaxial failure conditions of the same section. The

first method, known as the Reciprocal Load Method, has the governing equation:

$$\frac{1}{P_u} = \frac{1}{P_x} + \frac{1}{P_y} - \frac{1}{P_{uo}} \quad 2.2$$

where:

P_u is the failure load at eccentricities e_x and e_y

P_x is the failure load at e_y with $e_x = 0$

P_y is the failure load at e_x with $e_y = 0$

P_{uo} is the pure axial failure load. It has been noted that Eqn. 2.2 gives reasonable results for loads above the balanced load (or ultimate loads at low eccentricities) but is inaccurate for loads less than the balanced load. The balanced load is an ultimate load at an eccentricity (called the balanced eccentricity) for which the attainment of yield stress in a tension reinforcement bar takes place simultaneously with the attainment of 0.3% compression strain at a remote concrete fibre. The significance of the compression strain value is that it arbitrarily defines failure in reinforced concrete code requirements. The significance of the balanced points are that they are failure conditions which can be calculated, that is, the strain state is defined, and they provide further points on the failure surface in the toe region of the surface.

The second method, known as the Load Contour Method or the PCA Method [Ref. 10], is based on the non-dimensional load-contour of Fig. 1.3. The general equation for the load-

contour at constant load P_u is

$$\left(\frac{M_x}{M_{ox}}\right)^{m_1} + \left(\frac{M_y}{M_{oy}}\right)^{m_2} = 1.0 \quad 2.3$$

where m_1 and m_2 are experimentally determined exponents based on the geometry and material properties of the column cross-section. M_x , M_y , M_{ox} and M_{oy} have been defined previously. Bresler [Ref. 2] suggested that the exponents be made equal to each other, that is, $m_1 = m_2 = m$. Eqn. 2.3 then reduces to the general ellipse form of,

$$\left(\frac{M_x}{M_{ox}}\right)^m + \left(\frac{M_y}{M_{oy}}\right)^m = 1.0 \quad 2.4$$

Bresler [Ref. 2] reported the calculated values of m to vary from 1.15 to 1.55. Parme et al. [Ref. 9] extended Bresler's work to give a relationship between m and β . This relationship is

$$m = \frac{\log 0.5}{\log \beta} \quad 2.5$$

where β is the ordinate of the non-dimensional load-contour at $\theta = 45^\circ$ or in the physical sense β is that constant portion of the uniaxial moment capacities that can act simultaneously on the column section, that is,

$$M_x = \beta M_{ox} ; M_y = \beta M_{oy} \quad 2.6$$

Substituting Eqn. 2.5 into Eqn. 2.4 gives

$$\left(\frac{M_x}{M_{ox}}\right) \frac{\log 0.5}{\log \beta} + \left(\frac{M_y}{M_{oy}}\right) \frac{\log 0.5}{\log \beta} = 1.0 \quad 2.7$$

Plots of Eqn. 2.7 for different values of β are shown in Fig. 2.1. A design procedure using Eqn. 2.7 is only feasible if the complete range of β values is available.

Of all the interaction equations to date, Eqn. 2.7 is the only one which is capable of giving results close to the actual interaction curve in all cases. Parme et al. [Ref. 9] have reported that the error in overestimation of column strength by Eqn. 2.7 does not exceed 3%. In the course of the present work by comparison with theoretically correct equilibrium solutions the author has consistently found that the accuracy is somewhat better than 3%. Realising that Eqn. 2.7 was too cumbersome to apply, Parme et al. [Ref. 9] suggested two straight line approximations. They are

$$\frac{M_y}{M_{oy}} + \frac{M_x}{M_{ox}} \left(\frac{1-\beta}{\beta}\right) = 1, \quad \text{when } \frac{M_y}{M_{oy}} > \frac{M_x}{M_{ox}} \quad 2.8$$

$$\frac{M_y}{M_{oy}} \left(\frac{1-\beta}{\beta}\right) + \frac{M_x}{M_{ox}} = 1, \quad \text{when } \frac{M_y}{M_{oy}} < \frac{M_x}{M_{ox}} \quad 2.9$$

Eqns. 2.8 and 2.9 have been plotted in Fig. 2.2. As noted in Fig. 2.2, the straight line approximations fall within the theoretical failure surface and are thus conservative. For rectangular column cross sections with equal distribution of reinforcement in all faces a further approximation to be used in Eqns. 2.8 and 2.9 was suggested [Ref. 9], that is,

$$\frac{M_{oy}}{M_{ox}} \approx \frac{B}{H} \quad 2.10$$

where B is the width of the column cross section

H is the depth of the column cross section

The problem of determining the capacity of a column cross section therefore reduces to that of finding the appropriate β value to be used in Eqns. 2.7, 2.8 and 2.9. The β values have been obtained using a computer program prepared in imperial units. Data were accumulated for the parameters stated in Section 1.2. The results from the computer analysis have been reported in a bulletin published by the PCA [Ref. 10]. The report noted that the following parameters had minor effects on β :

1. cover ratio, γ
2. concrete strength, f'_c
3. B/H ratio of the column cross section.

The third parameter may be something of a surprise to the reader. It has been verified here in this study that in consideration of the non-dimensional load-contours (plots of

$\frac{M_y}{M_{oy}}$ vs. $\frac{M_x}{M_{ox}}$) the B/H ratio is of negligible importance.

Values of β were found to be primarily dependent on axial load P_u/P_{uo} ; the reinforcement strength, f_y ; the reinforcement index, q ; and bar arrangement. The report also noted that the bulk of the β values in the most frequently used range of P_u/P_{uo} did not differ by more than 3%. In view of

the small differences noted by the report, only envelopes of lowest β values were plotted for two f_y values and different bar arrangements.

It must be pointed out that the PCA method [Ref. 10] does not in general yield the ultimate biaxial moment capacities of a column along the angle at which the given load, P_u , acts. To obtain the ultimate biaxial moment capacities along the angle at which the given load acts then the following two equations must be solved simultaneously:

$$(M_x/M_{ox}) \frac{\log 0.5}{\log \beta} + (M_y/M_{oy}) \frac{\log 0.5}{\log \beta} = 1.0 \quad \text{and}$$

$$M_y/M_{oy} = M_x/M_{ox} \cdot \tan \theta$$

where

$$\theta = \tan^{-1}(\bar{M}_y/M_{oy})/(\bar{M}_x/M_{ox})$$

M_{ox} and M_{oy} have been defined previously. M_x and M_y are the calculated ultimate biaxial moments that can be sustained by the column at load, P_u . $\bar{M}_x$ and $\bar{M}_y$ are the given biaxial moments acting on the column produced by the biaxially eccentric load, P_u .

Pannell [Ref. 8] in 1960 presented a method by which the design of a short rectangular column of any dimension, cover ratio, and reinforcement distribution can be carried out. Pannell assumed that the load-moment interaction surface [Fig. 1.1] could be represented by a uniaxial load-moment interaction diagram plus a biaxial moment interaction curve.

[Fig. 2.3]. Pannell further assumed that the biaxial moment interaction curve [Fig. 2.3] could be related, by the deviation factor N and a simple trigonometric function, to the uniaxial behavior given as:

$$M_{oy} = \frac{M_y \sec \theta}{1 - N \sin^2 2\theta} \quad 2.10$$

$$\text{where } N = 1 - \frac{M_d}{M_{oy}} \quad 2.11$$

M_d is the radial moment capacity of the column at $\theta = 45^\circ$ to the horizontal axis. M_{oy} has been defined previously.

To take into account the different cover ratios, the factor F was introduced. F is given by

$$F = \frac{M_{oy} \text{ for } \gamma = .8}{M_{oy} \text{ for actual value of } \gamma} \quad 2.12$$

Using this factor, Eqn. 2.10 can be rewritten as

$$M_y = \frac{F \cdot M_{oy} \cdot (1 - N \sin^2 2\theta)}{\sec \theta} \quad 2.13$$

For the special case of a square column with equal distribution of reinforcement on all faces

$$M_{ox} = M_{oy}$$

and therefore

$$M_x = \frac{F \cdot M_{ox} \cdot (1 - N \sin^2 2\theta)}{\csc \theta} \quad 2.14$$

Solving Eqns. 2.13 and 2.14 for the non-dimensional resultant

of the biaxial moments gives

$$R_{\theta} = F.(1 - N\sin^2 2\theta) \quad 2.15$$

In choosing the function $1 - N\sin^2 2\theta$ and fixing the exponent of $\sin 2\theta$ as 2, Pannell argued that this was the simplest expression which satisfied the conditions of smooth transition and tangency. A counter argument is that the exponent of $\sin 2\theta$ could be any number and the conditions will still be satisfied. Clearly, the exponent for $\sin 2\theta$ is not a unique number. The arbitrary fixing of the exponent for $\sin 2\theta$ is the basic flaw in Pannell's formulation and this has resulted in overestimation of column capacity in most cases. Fig. 2.4 shows the plots of Eqn. 2.15 (for $F = 1.0$) and Eqn. 2.7 (PCA Method) for two β values. The difference between the load-contours obtained from these two equations is clearly apparent.

Figure 2.1 Biaxial Moment Relationship (from Ref. 9)

Figure 2.2 Straight line approximation of load-contour at plane of constant P_u

Figure 2.3 Load-contour for Pannell's formulation (Ref. 8)

— PCA Method or Breslers's Method : $(M_x/M_{ox})^m + (M_y/M_{oy})^m = 1$

----- Pannell's Method : $R = 1 - N \sin^2(2\theta)$

Figure 2.4 Comparison of load-contours from Bresler's and Pannell's approaches

CHAPTER 3

PARAMETRIC ANALYSIS

3.1 General

The most common methods of biaxial column analysis used in North America are the Reciprocal Load Method and the PCA Method, both described in the previous chapter. The Reciprocal Load Method is given tacit approval in Canada by having been included in the PCA Metric Design Manual [Ref. 12]. Reference texts describe both methods and often provide detailed charts for obtaining β in the PCA procedure.

The parametric approach developed in the course of the present study is an attempt to provide an accuracy close to that given by the PCA Method while eliminating the need for extensive design aids. It is based on the fact that the load contours, as illustrated in Fig. 1.3, are of simple form whose exact shapes are governed by only a few parameters. The quantity β was introduced in the previous chapter as the ordinate of the non-dimensional load-contour at $\theta = 45^\circ$. It is one alternative for indicating the decrease in strength of a column as the moments are applied with increasing biaxiality of the eccentricities. That is, $\beta = 1$ for a circular column indicating no decrease in strength if a load is applied with equal eccentricities, e_x and e_y . The effect of some of the parameters listed in Section 1.2 are thus of major significance in column behavior under biaxial effects.

3.2 Parametric Representation of the Load-Contour

To be acceptable as a suitable parametric representation of the non-dimensional load-contour the parametric equation must of necessity yield the R_θ values of unity at $\theta = 0^\circ$ and 90° and $\sqrt{2} \times \beta$ at $\theta = 45^\circ$. A further criterion is that the equation must yield the intermediate R_θ values close to the actual interaction curve. Several equations were examined and most of them did not satisfy all the above criteria. The equation which was finally chosen to describe the non-dimensional load-contour is

$$R = 1 - A \sin^n 2\theta \quad 3.1$$

where A and n are unknowns to be determined and are expected to depend on the parameters listed in Section 1.2. θ is the angle in degrees made by any radial line on the load-contour with the horizontal axis. This function has the following attributes:

- (i) it has the value of 1 at $\theta = 0^\circ$ and 90° ;
- (ii) it has symmetry about the 45° radial line;
- (iii) the A parameter is an amplitude of the sine term and controls the extent of the 45° dip;
- (iv) the n parameter controls the shape of the curve between the points at $\theta = 0^\circ$, 45° and 90° .

As a continuous function, Eqn. 3.1 satisfied all of the attributes needed to solve the problem at hand.

For a particular column cross-section, under biaxial moments, A and N are completely determined when R_{45} and $R_{22.5}$

values of the non-dimensional load-contour are known. For square columns with equal amounts of reinforcement in all faces and the same γ ratio on all sides R_{45} and $R_{22.5}$ values can be calculated from the computer output as follows:

1. Input $\theta = 0^\circ$ and 90° to obtain M_{ox} and M_{oy} respectively. M_{ox} and M_{oy} are identical for square columns satisfying the conditions stated previously.

2. Input $\theta = 45^\circ$ to obtain biaxial moments M_x and M_y . For $\theta = 45^\circ$ $M_x = M_y$. Calculate $R_{45} = (\sqrt{2}) \cdot (M_x / M_{ox})$

3. Input $\theta = 22.5^\circ$ to obtain biaxial moments M_x and M_y . Calculate $R_{22.5}$ as $R_{22.5} = \sqrt{[(M_x / M_{ox})^2 + (M_y / M_{oy})^2]}$

Substituting R_{45} in Eqn. 3.1 gives

$$R_{45} = 1 - A \sin^n(2 \times 45^\circ) = 1 - A \text{ or } A = 1 - R_{45}$$

$$\text{but } R_{45} = 1 - \beta \sqrt{2}$$

$$\therefore A = 1 - \beta \sqrt{2} \quad 3.2$$

Eqn. 3.2 shows that the parameter A is related to β by a simple expression which can readily be evaluated. The exponent n is determined as follows.

$$n = \frac{\ln[(1 - R_{22.5})/A]}{\ln(\sin 45^\circ)} \quad 3.3$$

To obtain values of β (hence A) and n , a comprehensive computer analysis of a number of column cross sections was carried out. A description of the column cases considered in the analysis is given in Section 3.3 and a full description of the computer program used is given in Appendix A. The program listing is given in Ref. 7.

3.3 Computer Study

A comprehensive study was carried out on square and rectangular columns using a computer program written by Hartley [Ref. 7]. This program was prepared in accordance with CSA Standard CAN3-A23.3-M77 [Ref. 3] utilizing a parabolic stress-strain relationship for concrete. This program provides a "correct" equilibrium solution to the biaxially loaded column, that is, the algorithm searches for the correct orientation of a neutral axis and assumes that the strain in the cross section is proportional to the distance from the neutral axis. Such programs have been developed by others and are used in the production of column design aids. It is not reasonable to use physical models rather than numerical ones to develop failure criteria since our codes, quite arbitrarily, do define what we term a 'state of failure' in a reinforced concrete element; a state which is difficult to define precisely. The column sections were studied for principal axial load ratios of $P_u/P_{uo} = 0.1, 0.2, \dots, 0.9$, reinforcement percentages of $\rho = 1\%, 2\%, \dots, 8\%$ and for various f'_c and f_y values, bar arrangements, γ ratios, and B/H ratios of the cross-section." All of the computer runs were carried out on a PDP 11/23 resident in the Carleton University Civil Engineering Department.

3.3.1 Square Columns

Square columns with equal amounts of reinforcement in

all faces were considered. The study on square columns was divided into two stages.

1. Interaction points were generated for $\theta = 0^\circ, 22.5^\circ, 45^\circ, 67.5^\circ$ and 90° . These points were used to calculate β , R_{45} and $R_{22.5}$ values. A summary of the column sizes and other parameters considered at this stage are in Table 3.1.
2. Interaction points were generated for $\theta = 5^\circ, 10^\circ, 15^\circ, \dots, 40^\circ$ for a selected number of cases. These points were used to check the accuracy of the parametric formulation (Eqn. 3.1).

3.3.2 Rectangular Columns

Two cases of reinforcement distribution were considered for rectangular columns. They are rectangular columns with equal amounts of reinforcement in all faces (that is, 4 corner bars and an equal number of additional bars in all faces) and rectangular columns with equal amounts of reinforcement in only two opposite faces (that is, 4 corner bars and an equal number of additional bars in only two opposite faces). For rectangular columns with equal amounts of reinforcement in all faces two different arrangements of reinforcing bars were studied. They are

1. the ratio of the center to center distance between the corner bars on a side of the column cross section to the overall length of that side (that is, the cover ratio, γ) was constant for all the 4 sides

of the cross section. This means that the cover on the long and short sides of the column cross section are not identical;

2. constant cover on all sides of the column cross section was provided. This is the usual way of providing cover in rectangular column cross sections and it causes the cover ratios on the long and short sides of the cross section to be different.

Interaction points were generated for $\theta = 0^\circ$ and 90° and for three other intermediate angles. These points were used to calculate β and other R_θ values. The symmetry of the non-dimensional load-contours about the 45 degrees radial line was investigated for the reinforcement distributions and arrangements mentioned previously. Tables 3.2 and 3.3 summarize the cases considered.

Table 3.1 Square Columns with Equal Amounts of Reinforcement
in all faces - Summary of cases considered for
Interaction Points

Column Size (mmxmm)	f_y (MPa)	f'_c (MPa)	γ	Reinf. Pattern	No. of Computer Runs
300x300	300	20	.6	B	72
600x600	300	20	.6, .7, .8	A, B, C, D	864
		30	"	"	"
		40	"	"	"
600x600	350	20	.6, .7, .8	A, B, C, D	864
		30	"	"	"
		40	"	"	"
600x600	400	20	.6, .7, .8	A, B, C, D,	864
		30	"	"	"
		40	"	"	"
Total					7920

Reinforcement Patterns

Table 3.2 Rectangular columns with equal amounts of reinforcement in all faces - summary of cases considered for 5 interaction points

Column size (mmxmm)	f_y (MPa)	f'_c (MPa)	γ	γ'	Reinf. Pattern	No. of Computer Runs
600x300	300	20	.6	.6	A	72
900x300	300	20	.6	.6	A	72
600x300	300	20	.6	.8	A	72
900x300	300	20	.6	.87	A	72
Total						288

Reinforcement Pattern

Table 3.3 Rectangular Columns with equal amounts of reinforcement in only two opposite faces - summary of cases considered for 5 interaction points

Column size (mmxmm)	f_y (MPa)	f'_c (MPa)	γ	γ'	Reinf. Pattern	No. of Computer Runs
600x300	300	20	.6	.8	A,B,C	216
		30	"	"	"	"
		40	"	"	"	"
600x300	400	20	.6	.8	A,B,C	216
		30	"	"	"	"
		40	"	"	"	"
900x300	300	20	.6	.87	A,B,C	216
		30	"	"	"	"
		40	"	"	"	"
Total					1944

Reinforcement Patterns

CHAPTER 4

ANALYSIS OF RESULTS

4.1 Introduction

Each of the variables which could affect the failure surface was examined to find which were the most significant. The non-dimensionalized failure surface is primarily governed by the parameter β [Refs. 9, 10] and so the investigation of the variables will be mainly centered on β . In this chapter some of the results of this investigation of the variables are presented.

4.2 Analysis of Results for Square and Rectangular Columns With Equal Amounts of Reinforcement in All Faces

4.2.1 Effect of B/H Ratio of the Column on β

Three columns of dimensions 300mm x 300mm, 600mm x 300mm and 900mm x 300mm representing B/H ratios of 1, 2 and 3 respectively were studied for the effect of B/H ratio or the aspect ratio of the column on β . The cover ratio for the square column in all cases was 0.6. For the rectangular columns (columns for which $B/H > 1$) two cases of reinforcement arrangement were studied, that is,

1. same cover ratio ($\gamma = 0.6$) on all four sides of the rectangular column
2. same cover on all sides of the rectangular columns as that for the 300mm x 300mm column. This condition

makes the cover ratios on the short and long sides of the rectangular column different.

It was determined that the B/H ratio of the cross-section has no effect on β . In consideration of the non-dimensional load-contours it was discovered that when the same cover ratio was maintained on all sides of the square and rectangular columns the contours for the two columns were identical with symmetry about the 45° radial line. On the other hand when the cover ratios on the short and long sides of the rectangular column are different there is a slight loss of symmetry about the 45° radial line of the load-contour.

The reader might find it somewhat surprising that neither the column aspect ratio (B/H ratio of the column cross-section), nor the alignment of reinforcement mainly in one direction will be an important parameter affecting β . It has been verified not only by the author (see Appendix B), but also in pilot studies leading to this research [Ref. 4] and is also given as a statement of fact in the PCA publication [Ref. 10] that when the load-contours are non-dimensionalized they are symmetrical about the 45° radial line (to all intent and purposes) and have the same β intercept at $\theta = 45^\circ$.

4.2.2 Effects of Number of Bars on the Face of the Column on β and the Uniaxial Moment Capacity of the Column

Square columns with 4, 8, 12 and 16 bar totals made up of 4 corner bars and 0, 1, 2 and 3 additional bars in the face

of the column respectively were studied. The β values for the 8, 12 and 16 bar columns were markedly similar whilst the β values for the 4 bar column were distinctly different from the rest. This difference has also been noted by Parme, Nieves and Gouwens [Ref. 9] and Pannell [Ref. 8]. The curves in Figs. 4.1 and 4.2 illustrate the general form of the variation of β with the number of additional bars in the face of the column and the axial load ratio, P_u/P_{uo} . The difference in β values for the 8, 12 and 16 bar columns was of the order of 1% and did not exceed 3%. In view of the small differences noted for β values for columns with 8 or more bar totals one set of results can be presented for such columns. The results presented in Figs. 4.1 and 4.2 are not surprising. The number of bars in the face of a column is a measure of the continuity of bar distribution on the perimeter of the column. The 4-bar column is geometrically the greatest degree of bar discontinuity. Columns with more than 4 bars may be considered to have a continuous distribution of reinforcement along the perimeter.

The uniaxial moment capacity, M_{ox} or M_{oy} of the column is also dependent on the number of bars in the face of the column. The distinct difference of the 4-bar column uniaxial moment capacities from the capacities of columns with more than 4 bars was again observed. Figs. 4.3 and 4.4 show the typical variation of uniaxial moment capacity with the number of bars in the face of the column and P_u/P_{uo} for reinforcement percentages of $\rho = 3\%$ and $\rho = 6\%$. From the figures it

is noted that the uniaxial moment capacity of the column decreases with increase in the number of bars in the face of the column for a given reinforcement percentage. The decrease in strength for the 16 bar column with respect to the 4 bar column for $0.1 \leq P_u/P_{uo} \leq 0.6$ range from 5% to 25% while the decrease in strength of the 16 bar column with respect to the 8 bar column for $0.1 \leq P_u/P_{uo} \leq 0.6$ range from 2% to 7%. The results in Figs. 4.3 and 4.4 are again not surprising since the uniaxial moment capacity of the column depends on the concentration of the reinforcement and the length of the resisting arm. Geometrically, the 4 bar column for a given reinforcement percentage represents the greatest concentration of reinforcement with a large resisting arm.

Uniaxial Design Charts such as given in the Canadian PCA Design Manual [Ref. 12] are produced for one specific total number of bars in the column cross-section. As mentioned previously, the uniaxial moment capacity varies with the number of bars in the cross-section of the column and so these charts will be in error for other total number of bars different from the one used to produce the Charts. In the design examples given in Chapter 5, the uniaxial moment capacities were obtained from the computer program [Ref. 7] for each bar arrangement.

4.2.3 Effect of Cover Ratio, γ , on β

Three γ ratios of 0.6, 0.7 and 0.8 were studied for square columns. In general, β increased with increase in γ ratio for all P_u/P_{uo} and ρ values. However, the effect of γ on β is not uniform for all P_u/P_{uo} and ρ values as evident in Figs. 4.5 and 4.6. Increases as much as 8% were noted for certain P_u/P_{uo} and ρ values while for some P_u/P_{uo} and ρ values only increases of about 2% were noted. For square columns with equal amounts of reinforcement in all faces the effect of γ ratio on β is given approximately by the following expression:

$$\beta_{\gamma=a} = \beta_{\gamma=.6} + \frac{0.7[(M_{ox})_{\gamma=a} - (M_{ox})_{\gamma=.6}]}{(M_{ox})_{\gamma=a}} \quad 4.1$$

where M_{ox} is the uniaxial moment capacity for the square column at axial load P_u when M_{oy} is zero and $0.6 < a < 1.0$. As mentioned previously, the effect of γ on β is significant in some cases and not significant in some other cases and so it may be considered appropriate to ignore the effect of γ ratio on the design altogether. In a similar study published by the PCA [Ref. 10] the γ ratio was found to be of minor effect for all cases and so was ignored in the design.

4.2.4 Effects of f_y and f'_c on β

Three commonly used f_y values of 300MPa, 350MPa and 400MPa were studied to find the effect of f_y on β . Each f_y value was studied for three f'_c values of 20MPa, 30MPa and

40MPa. In general, for the same f'_c value β decreased with increase in f_y for all P_u/P_{uo} and ρ values. It was noted that β values for the intermediate f_y value of 350MPa could be obtained by interpolating between $f_y = 300\text{MPa}$ and $f_y = 400\text{MPa}$. There was excellent agreement between the interpolated and actual β values. The difference between the interpolated and the actual β values did not exceed 1.5%.

Another investigation was carried out to find the effect on β when f_y was held constant and f'_c changed. It was found that, for the same f_y value, β decreased with increase in f'_c for all P_u/P_{uo} and ρ values. Interpolation between $f'_c = 20\text{MPa}$ and 40MPa for the same f_y value was also found to be valid. The effect of the combination of f_y , f'_c and ρ on β is discussed in Section 4.2.6.

4.2.5 Effect of Reinforcement Percentage, ρ , on β

Eight reinforcement percentages of $\rho = 1\%$, 2% , 3% , ..., 8% were studied for the effect of ρ on β . In general β decreased with increase in ρ for P_u/P_{uo} less than about 0.5 while for P_u/P_{uo} greater than 0.5 β decreased slightly with increase in ρ .

4.2.6 Effect of $q (= \rho f_y / f'_c)$ on β

The variable q usually referred to as the reinforcement index has been widely used in the literature as a design parameter for columns, [Refs. 1,2,8,9 and 10]. The parameter β is said to be dependent on q to the extent that it was found

unnecessary to present β values for different f'_c values [Ref. 10]. This means that β values computed for the same q value but for different f'_c values should be identical. To check the validity of the dependence of β on q in this manner, β values were computed in terms of q for the same f_y value and the three f'_c values of 20MPa, 30 MPa and 40 MPa and the results compared. The q values investigated for $f_y = 300\text{MPa}$ were 0.15, 0.30, 0.45 and 0.60. Thus for $f_y = 300\text{MPa}$ and $f'_c = 20\text{MPa}$, the q values correspond to $\rho = 1\%$, 2%, 3% and 4% respectively; for $f_y = 300\text{MPa}$ and $f'_c = 30\text{MPa}$, the q values correspond to $\rho = 2\%$, 4%, 6% and 8% respectively. It should be noted that the maximum reinforcement percentage permitted in a column by the CSA Code [Ref. 3] is 8%. The β values computed for $f_y = 300\text{MPa}$ have been presented in Figs. 4.7 to 4.10. Values of β were also computed in terms of q for $f_y = 400\text{MPa}$ and in this case the q values studied for $f'_c = 20, 30$ and 40 MPa were 0.2, 0.4, 0.6 and 0.8. These results are also presented in Figs. 4.11 to 4.14.

Figs. 4.11 to 4.14 clearly show that β values computed for the same q value but for different f'_c values are not identical. In each figure, the lowest curve corresponds to β values derived from $f'_c = 40\text{MPa}$, the middle curve from $f'_c = 30\text{MPa}$ and the topmost curve from $f'_c = 20\text{MPa}$. All the β values were computed for the same cover ratio, that is for $\gamma = .6$. The differences between the lowest and topmost β curves range from 3% to 8%. When γ effects are taken into

account then the differences between the lowest β curve for $f'_c = 40\text{MPa}$ and $\gamma = .6$ and the β curve for $f'_c = 20\text{MPa}$ and $\gamma = .8$ range from 6% to 12%. The plots in Figs. 4.7 to 4.14 do not justify the presentation of β values in terms of q whilst ignoring f'_c . In the PCA study [Ref. 10] envelopes of lowest β values were presented in terms of q . The adoption of this approach in this study will make the design for the most commonly used f'_c value of 20MPa unnecessarily very conservative.

4.2.7 Significant Parameters Affecting β and Analytical Expressions for β

The preceding discussions have revealed that β is dependent on f_y , f'_c , ρ , P_u/P_{uo} and to some extent on the number of bars on the face of the column cross section. In particular it was found that

1. β values for the 8, 12 and 16 bar columns did not differ by more than 3%;
2. there was no justification for presenting β values in terms of q whilst neglecting f'_c ; and
3. interpolations for β values for the intermediate f_y and f'_c values are permissible.

Based on the above, it is suggested that two sets of β values be provided. One set for the 4 bar column and the other set for the 8 or more bar column. Each set of β values (for $\gamma = 0.6$) is to be presented as follows:

1. β vs. P_u/P_{uo} in terms of ρ for $f_y=300\text{MPa}$ and for $f'_c = 20\text{MPa}$
 $f'_c = 40\text{MPa}$
2. β vs. P_u/P_{uo} in terms of ρ for $f_y=400\text{MPa}$ and for $f'_c = 20\text{MPa}$
 $f'_c = 40\text{MPa}$

The results in the suggested forms have been presented as plots shown in Figs. 4.15 to 4.18. Values of β for other γ ratios can be computed approximately by expression 4.1. Analytical expressions have been developed for calculating β values for the 8 or more bar column. Values of the parameter A have been computed and presented in Figs. 4.19 to 4.22.

The parameter Q defined in Section 1.2 as $Q = \rho P_u/P_{uo}$ was used in the development of the analytical expressions for β . All the β values were multiplied by the parameter Q . This has the effect of changing the β vs. P_u/P_{uo} curves such as shown in Fig. 4.17 to that shown in Fig. 4.23. It was noted that the transformed curves in Fig. 4.23 had a transition point at or near the axial load ratio, P_u/P_{uo} , of 0.4 for all columns. The analytical expressions were developed for the curves shown in Fig. 4.19 and were then transformed by dividing by the parameter Q to yield the actual β values. The expressions presented here (all for $\gamma = 0.6$) have already been divided by Q .

The expression for $P_u/P_{uo} \geq 0.4$ is given by

$$\beta = \lambda + \frac{(P_u/P_{uo} - 0.4)}{100\rho P_u/P_{uo}} \times b[.1(P_u/P_{uo} + 9.75) + 100\rho - 1] \quad 4.2$$

The expression for $P_u/P_{uo} - 0.4$ is given by

$$\beta = \lambda - \frac{(.4 - P_u/P_{uo})}{\rho P_u/P_{uo}} \times C \left[\left(\frac{P_u}{P_{uo}} \right)^2 (2.6\rho - .035) + \rho \right] \quad 4.3$$

where λ , b and c are as given in Table 4.1

TABLE 4.1 λ , b and c values for Eqns. 4.2 and 4.3

(Eqns. 4.2 and 4.3 are valid for 8 or more bar columns)

f_y (MPa)	f'_c (MPa)	λ	b	c
300	20	$\frac{0.245}{P_u/P_{uo}}$	0.70	0.60
	40	$\frac{0.245}{1.04 P_u/P_{uo}}$	$\frac{0.70}{1.04}$	$\frac{0.60}{1.04}$
400	20	$\frac{(.24-.25\rho)}{P_u/P_{uo}}$	0.65	0.56
	40	$\frac{(.24-.25\rho)}{P_u/P_{uo}}$	0.65	0.56

4.2.8 Determination of the Exponent n in the Parameter Equation

The exponent n in the parametric representation of the non-dimension load contour (Fig. 1.3) is given by

$$n = \frac{\ln[(1 - R_{22.5})/A]}{\ln \sin 45^\circ} \quad 4.4$$

where $A = 1 - \sqrt{2}\beta$ and $R_{22.5}$ is the length of the radial line at 22.5° to the non-dimensional load-contour. The first stage in the determination of the exponent n thus involves accumulation of $R_{22.5}$ values for each A. The $R_{22.5}$ values were computed for square and rectangular columns having equal amounts of reinforcement in each face and the same γ ratio on all sides. The non-dimensional load-contours for these column configurations possess complete symmetry about the 45° radial line. The accumulated $R_{22.5}$ values have been presented on the scatter diagram shown in Fig. 4.24. It is noted from this figure that several slightly different $R_{22.5}$ values occur for the same A value. This does not yield unique values of n for these A values. A polynomial regression analysis was carried out in the second stage to obtain an expression which gives unique n values for each A. The solid line curve plotted in Fig. 4.24 represents the curve through the $R_{22.5}$ values which satisfy Bresler's (or the PCA's) non-dimensional load-contour equation [Ref. 2], that is

$$\left(\frac{M_x}{M_{ox}}\right) \frac{\log 0.5}{\log \beta} + \left(\frac{M_y}{M_{oy}}\right) \frac{\log 0.5}{\log \beta} = 1 \quad 4.5$$

It was further noted that the bulk of the $R_{22.5}$ values occurred on this curve or above it. Almost all of the $R_{22.5}$ values that

occurred below the solid curve occurred within 2.5% of it. This means that the overestimation of column strength by Eqn. 4.5 does not exceed 2.5% at $\theta = 22.5^\circ$ to the horizontal axis of the non-dimensional load-contour. The over-estimation of column strength in some other directions namely, $\theta = 5^\circ, 10^\circ, \dots, 40^\circ$ were also investigated and it was consistently found that the overestimation of column strength in these directions by Eqn. 4.5 also did not exceed 2.5%. The maximum overestimation of column strength by Eqn. 4.5 for most cases is therefore about 2.5%.

In the second stage of the analysis for the exponent n , Eqn. 4.4 was used to calculate n values for all the $R_{22.5}$ values given in Fig. 4.24 and a third degree polynomial regression analysis carried out between A and n to find the line of best fit. The resulting regression equation is

$$n = 1.455 - 4.06A + 6.05A^2 - 1.655A^3 \quad 4.6$$

where $A = 1 - \sqrt{2}\beta$. The correlation index, r^2 , for the regression equation 4.6 is $r^2 = 0.7$ indicating that Eqn. 4.6 accounts for 70% of the variability of the data about the mean $\bar{n}$. A plot of Eqn. 4.6 is presented in Fig. 4.25.

A comparison was made between the parametric method developed in this study and the PCA method [Ref. 10]. There was excellent agreement between the two methods. Appendix C summarizes the results obtained.

4.3 Analysis of Results for Rectangular Columns with Equal Amounts of Reinforcement in Only Two Opposite Faces

Rectangular columns with B/H ratios of 2 and 3 with 6, 8 and 10 bars in the cross section (that is, 4 corner bars and 1, 2 and 3 additional bars respectively in the longer faces only) were studied.

Values of β for these columns were obtained by finding the ultimate biaxial moments which act at an angle of $\tan^{-1}(M_{Oy}/M_{Ox})$, where M_{Ox} and M_{Oy} are the uniaxial moment capacities of the column. This follows directly from the definition of β given in Ref. 9. The β values for these columns and bar arrangements were found to be similar to the β values for square columns with 8 bars equally distributed along the perimeter of the cross section. The differences between these β values were of the order of 1% to 3.5%. As mentioned previously, neither the bar orientation nor the aspect ratio (B/H ratio) of the column cross section has a major effect on β . In view of the small differences noted, separate values of β for rectangular columns with reinforcement equally distributed in only two opposite faces are not given. Instead, these columns are to be analysed and designed for the same β values as that for square columns with 8 or more bars equally distributed along the perimeter of the cross section.

The non-dimensional load-contours for rectangular columns with equal distribution of reinforcement in only two opposite faces of the cross section are not symmetrical

about the 45 degree radial line. This is due to the dissymmetry in reinforcement distribution in the cross section of the column. In most of the cases investigated the dissymmetry of the load-contour about the 45 degree radial line gave rise to an overestimation of column strength of about 3%.

4.4 Accuracy of the Analytical expressions for β and the Parametric Method

The accuracy of Eqns. 4.2 and 4.3 for the calculation of β is given in terms of the ratio (R) of the exact β value from the computer analysis to calculated β value using Eqn. 4.2 or 4.3, that is,

$$R = \beta_{\text{actual}} / \beta_{\text{calc.}}$$

A summary of the results of the calculations for a number of column sections is given in Tables 4.2 to 4.35. The equations give reasonable results. In particular, over the P_u/P_{uo} range from 0.2 to 0.7, the ratio R varied from .97 to 1.04.

The accuracy of the Parametric Method was also evaluated in terms of the ratio (R) of the exact R_θ value from the computer analysis to the calculated R_θ value using the Parametric Method developed in this study, that is,

$$R = \frac{(R_\theta)_{\text{exact}}}{(R_\theta)_{\text{calc.}}}$$

R_θ is given by

$$R_\theta = \sqrt{\left(\frac{M_x}{M_{ox}}\right)^2 + \left(\frac{M_y}{M_{oy}}\right)^2}$$

where M_x , M_y are the biaxial moment capacities of the column at the angle θ and M_{ox} and M_{oy} are the uniaxial moment capacities of the column. Tables 4.36 to 4.51 summarize the results of the calculations for a number of column sections. Again, reasonable results were obtained. Over the P_u/P_{uo} range from 0.1 to 0.8 the ratio R varied from 0.98 to 1.04. Detailed procedures for using the Parametric Method are given in Chapter 5.

Figure 4.1 Variation of β with P_u/P_{uo} and the number of bars on the face of the column for $\rho = 2\%$

Figure 4.2 Variation of β with P_u/P_{u0} and the number of bars on the face of the column for $\rho = 4\%$

Figure 4.3 Variation of uniaxial moment capacity with P_u/P_{uo} and the number of bars on the face of the column for $\rho = 3\%$

Figure 4.4: Variation of uniaxial moment capacity with $\frac{P_u}{P_{uo}}$ and the number of bars on the face of the column for $\rho = 6\%$

Figure 4.5 Variation of β with P_u/P_{u0} and the cover ratio, γ , for $\rho = 6\%$

Figure 4.6 Variation of β with P_u/P_{u0} and the cover ratio, γ , for $\rho = 4\%$

Figure 4.7 Comparison of β values for $q = .15$ and $f_y = 300$ MPa

Figure 4.8 Comparison of β values for $q = 0.3$ and $f_y = 300$ MPa

Figure 4.9 Comparison of β values for $q = .45$ and $f_y = 300 \text{ MPa}$

Figure 4.10 Comparison of β values for $q = 0.6$ and $f_y = 300 \text{ MPa}$

Figure 4.11 Comparison of β values for $q = .2$ and $f_y = 400 \text{ MPa}$

Figure 4.12 Comparison of β values for $q = .4$ and $f_y = 400 \text{ MPa}$

Figure 4.13 Comparison of β values for $q = .6$ and $f_y = 400 \text{ MPa}$

Figure 4.14 Comparison of β values for $q = .8$ and $f_y = 400 \text{ MPa}$

Figure 4.15 B vs. P_u/P_{uo} curves for $f_y = 300$ MPa

4 bar arrangement

Figure 4.16 δ vs. P_u/P_{uo} curves for $f_y = 400$ MPa

4 bar arrangement

Figure 4.17 δ vs. P_u/P_{uo} curves for $f_y = 300$ MPa

8 or more bar arrangement

Figure 4.18 β vs. P_u/P_{uo} curves for $f_y = 400$ MPa

8 or more bar arrangement

Figure 4.19 A vs. P_u/P_{uo} curves for $f_y = 300$ MPa

4 bar arrangement

Figure 4.20 A vs. P_u/P_{uo} curves for $f_y = 400$ MPa

4 bar arrangement

Figure 4.21 A vs. P_u/P_{uo} curves for $f_y = 300$ MPa

8 or more bar arrangement

Figure 4.22 A vs. P_u/P_{uo} curves for $f_y = 400$ MPa

8 or more bar arrangement

Figure 4.23 Simplified β curves used to derive
Eqns. 4.2 and 4.3

Figure 4.24 Scatter of computed $R_{22.5}$ values

Figure 4.25 n vs. A curve

TABLES 4.2-4.35 Accuracy of Eqns. 4.2 and 4.3 for calculating β

TABLE 4.2 Square Column, 8 bars

$f_y = 300 \text{ MPa}$, $\gamma = .6$

$f'_c = 20 \text{ MPa}$, $\rho = 1\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.74	.69	1.07
.8	.685	.675	1.01
.7	.660	.660	1.00
.6	.645	.650	.99
.5	.630	.630	1.00
.4	.610	.610	1.00
.3	.620	.630	.98
.2	.645	.645	1.00
.1	.695	.665	1.05

TABLE 4.3 Square Column, 8 bars

$f_y = 300 \text{ MPa}$, $\gamma = .6$

$f'_c = 20 \text{ MPa}$, $\rho = 3\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.743	.67	1.11
.8	.674	.663	1.02
.7	.660	.655	1.01
.6	.648	.644	1.01
.5	.628	.631	1.00
.4	.605	.613	.99
.3	.582	.591	.98
.2	.596	.591	1.01
.1	.620	.624	.99

TABLE 4.4 Square Column, 8 bars

 $f_y = 300 \text{ MPa}$, $\gamma = .6$ $f'_c = 20 \text{ MPa}$, $\rho = 6\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.745	.667	1.12
.8	.670	.661	1.01
.7	.658	.653	1.01
.6	.650	.644	1.01
.5	.630	.631	1.00
.4	.605	.613	.99
.3	.58	.585	.99
.2	.584	.584	1.00
.1	.602	.619	.97

TABLE 4.5 Square Column, 8 bars

 $f_y = 300 \text{ MPa}$, $\gamma = .6$ $f'_c = 20 \text{ MPa}$, $\rho = 6\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.735	.665	1.11
.8	.666	.659	1.01
.7	.658	.652	1.01
.6	.650	.643	1.01
.5	.633	.631	1.00
.4	.605	.613	.99
.3	.582	.58	1.00
.2	.568	.577	.98
.1	.584	.614	.95

TABLE 4.6 Square Column, 8 bars

 $f_y = 300 \text{ MPa}$, $\gamma = .7$ $f'_c = 40 \text{ MPa}$, $\rho = 1\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.645	.66	.98
.8	.63	.65	.97
.7	.615	.638	.96
.6	.606	.625	.97
.5	.605	.609	.99
.4	.60	.59	1.02
.3	.62	.609	1.02
.2	.65	.622	1.04
.1	.715	.64	1.11

TABLE 4.7 Square Column, 8 bars

 $f_y = 300 \text{ MPa}$, $\gamma = .7$ $f'_c = 40 \text{ MPa}$, $\rho = 3\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.645	.644	1.00
.8	.635	.637	1.00
.7	.62	.629	.99
.6	.615	.62	.99
.5	.607	.607	1.00
.4	.595	.59	1.01
.3	.59	.568	1.04
.2	.615	.568	1.08
.1	.64	.602	1.06

TABLE 4.8 Square Column, 8 bars

 $f_y = 300\text{MPa}$, $\gamma = .7$ $f'_c = 40\text{MPa}$, $\rho = 4\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.658	.642	1.02
.8	.642	.636	1.01
.7	.62	.628	.99
.6	.615	.619	.99
.5	.61	.607	1.01
.4	.595	.59	1.01
.3	.585	.563	1.04
.2	.605	.561	1.08
.1	.62	.595	1.04

TABLE 4.9 Square Column, 8 bars

 $f_y = 300\text{MPa}$, $\gamma = .7$ $f'_c = 40\text{MPa}$, $\rho = 6\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.66	.64	1.03
.8	.645	.634	1.02
.7	.625	.627	1.00
.6	.62	.618	1.00
.5	.615	.606	1.01
.4	.60	.59	1.02
.3	.575	.558	1.03
.2	.59	.554	1.06
.1	.61	.59	1.03

TABLE 4.10 Square Column, 8 bars

 $f_y = 400 \text{ MPa}$, $\gamma = .8$ $f'_c = 20 \text{ MPa}$, $\rho = 1\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.71	.65	1.09
.8	.694	.64	1.08
.7	.658	.63	1.04
.6	.635	.62	1.02
.5	.62	.608	1.02
.4	.60	.595	1.01
.3	.605	.62	.98
.2	.63	.648	.97
.1	.66	.71	.93

TABLE 4.11 Square Column, 8 bars

 $f_y = 400 \text{ MPa}$, $\gamma = .8$ $f'_c = 20 \text{ MPa}$, $\rho = 3\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.69	.63	1.10
.8	.685	.622	1.10
.7	.645	.615	1.05
.6	.62	.607	1.02
.5	.605	.596	1.02
.4	.585	.581	1.01
.3	.56	.564	.99
.2	.565	.57	.99
.1	.59	.62	.95

TABLE 4.12 Square Column, 8 bars

 $f_y = 400 \text{ MPa}$, $\gamma = .6$ $f'_c = 20 \text{ MPa}$, $\rho = 4\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.68	.623	1.09
.8	.68	.617	1.10
.7	.665	.61	1.09
.6	.635	.602	1.05
.5	.615	.59	1.04
.4	.595	.575	1.03
.3	.575	.551	1.04
.2	.565	.551	1.03
.1	.585	.591	.99

TABLE 4.13 Square Column, 8 bars

 $f_y = 400 \text{ MPa}$, $\gamma = .6$ $f'_c = 20 \text{ MPa}$, $\rho = 6\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.675	.615	1.10
.8	.675	.61	1.11
.7	.67	.602	1.11
.6	.64	.593	1.06
.5	.615	.581	1.06
.4	.595	.563	1.06
.3	.575	.53	1.08
.2	.55	.52	1.06
.1	.565	.536	1.05

TABLE 4.14 Square Column, 8 bars

 $f_y = 400 \text{ MPa}$, $\gamma = .7$ $f'_c = 40 \text{ MPa}$, $\rho = 1\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.64	.65	.98
.8	.63	.64	.98
.7	.615	.63	.98
.6	.60	.620	.97
.5	.60	.608	.99
.4	.595	.595	1.00
.3	.60	.62	.97
.2	.63	.648	.97
.1	.69	.71	

TABLE 4.15 Square Column, 8 bars

 $f_y = 400 \text{ MPa}$, $\gamma = .7$ $f'_c = 40 \text{ MPa}$, $\rho = 3\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.645	.627	1.03
.8	.635	.622	1.02
.7	.62	.615	1.01
.6	.602	.607	.99
.5	.59	.596	.99
.4	.575	.58	.99
.3	.555	.564	.98
.2	.58	.57	1.02
.1	.615	.621	.99

TABLE 4.16 Square Column, 8 bars

 $f_y = 400 \text{ MPa}$, $\gamma = .7$ $f'_c = 40 \text{ MPa}$, $\rho = 4\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.645	.623	1.04
.8	.64	.617	1.04
.7	.62	.61	1.02
.6	.602	.602	1.00
.5	.59	.59	1.00
.4	.572	.575	.99
.3	.550	.551	1.00
.2	.565	.551	1.02
.1	.597	.591	1.01

TABLE 4.17 Square Column, 8 bars

 $f_y = 400 \text{ MPa}$, $\gamma = .7$ $f'_c = 40 \text{ MPa}$, $\rho = 6\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.65	.615	1.05
.8	.645	.61	1.06
.7	.625	.602	1.04
.6	.60	.593	1.01
.5	.585	.581	1.01
.4	.568	.563	1.01
.3	.545	.53	1.03
.2	.548	.52	1.05
.1	.57	.536	1.06

TABLE 4.18 Square Column, 12 bars

 $f_y = 300 \text{ MPa}$, $\gamma = .6$ $f'_c = 20 \text{ MPa}$, $\rho = 1\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.74	.69	1.07
.8	.69	.675	1.02
.7	.66	.66	1.00
.6	.64	.65	.98
.5	.63	.63	1.00
.4	.615	.61	1.01
.3	.625	.63	.99
.2	.65	.645	1.01
.1	.69	.665	1.04

TABLE 4.19 Square Column, 12 bars

 $f_y = 300 \text{ MPa}$, $\gamma = .6$ $f'_c = 20 \text{ MPa}$, $\rho = 3\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.735	.67	1.10
.8	.685	.663	1.03
.7	.66	.655	1.01
.6	.64	.644	.99
.5	.625	.631	.99
.4	.61	.613	1.00
.3	.59	.59	1.00
.2	.60	.59	1.02
.1	.625	.624	1.00

TABLE 4.20 Square Column, 12 bars

 $f_y = 300\text{MPa}$, $\gamma = .6$ $f'_c = 20\text{MPa}$, $\rho = 4\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.735	.667	1.10
.8	.685	.661	1.04
.7	.66	.653	1.01
.6	.635	.644	.99
.5	.625	.631	.99
.4	.61	.613	1.00
.3	.585	.586	1.00
.2	.59	.584	1.01
.1	.605	.619	.98

TABLE 4.21 Square Column, 12 bars

 $f_y = 300\text{MPa}$, $\gamma = .6$ $f'_c = 20\text{MPa}$, $\rho = 6\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.735	.665	1.11
.8	.69	.66	1.05
.7	.665	.652	1.02
.6	.63	.643	.98
.5	.62	.631	.98
.4	.61	.613	1.00
.3	.59	.58	1.02
.2	.58	.577	1.00
.1	.595	.614	.97

TABLE 4.22 Square Column, 12 bars

 $f_y = 300 \text{ MPa}$, $\gamma = .6$ $f'_c = 40 \text{ MPa}$, $\rho = 1\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.645	.66	.98
.8	.63	.65	.97
.7	.61	.635	.96
.6	.605	.625	.97
.5	.605	.609	.99
.4	.60	.59	1.02
.3	.615	.609	1.01
.2	.65	.622	1.05
.1	.705	.641	1.10

TABLE 4.23 Square Column, 12 bars

 $f_y = 300 \text{ MPa}$, $\gamma = .6$ $f'_c = 40 \text{ MPa}$, $\rho = 3\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.655	.644	1.02
.8	.64	.637	1.00
.7	.62	.629	.99
.6	.61	.62	.98
.5	.60	.607	.99
.4	.59	.59	1.00
.3	.59	.568	1.04
.2	.605	.568	1.07
.1	.65	.602	1.08

TABLE 4.24 Square Column, 12 bars

 $f_y = 300 \text{ MPa}$, $\gamma = .6$ $f'_c = 40 \text{ MPa}$, $\rho = 4\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.66	.642	1.03
.8	.645	.636	1.01
.7	.625	.628	1.00
.6	.61	.619	.99
.5	.60	.607	.99
.4	.59	.59	1.00
.3	.57	.563	1.01
.2	.59	.561	1.05
.1	.615	.595	1.03

TABLE 4.25 Square Column, 12 bars

 $f_y = 300 \text{ MPa}$, $\gamma = .6$ $f'_c = 40 \text{ MPa}$, $\rho = 6\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.66	.64	1.03
.8	.65	.634	1.03
.7	.63	.627	1.00
.6	.61	.618	.99
.5	.60	.606	.99
.4	.59	.59	1.00
.3	.57	.558	1.02
.2	.58	.554	1.05
.1	.595	.59	1.01

TABLE 4.26 Square Column, 16 bars

 $f_y = 40 \text{ MPa}$, $\gamma = .6$ $f'_c = 20 \text{ MPa}$, $\rho = 1\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.705	.65	1.08
.8	.668	.64	1.04
.7	.64	.63	1.02
.6	.63	.62	1.02
.5	.615	.61	1.01
.4	.60	.595	1.01
.3	.595	.62	.99
.2	.62	.648	.96
.1	.665	.71	.94

TABLE 4.27 Square Column, 16 bars

 $f_y = 400 \text{ MPa}$, $\gamma = .6$ $f'_c = 20 \text{ MPa}$, $\rho = 3\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.68	.63	1.08
.8	.65	.622	1.05
.7	.625	.615	1.02
.6	.61	.607	1.00
.5	.595	.596	1.00
.4	.58	.581	1.00
.3	.56	.564	.99
.2	.56	.57	.98
.1	.585	.621	.94

TABLE 4.28 Square Column, 16 bars

 $f_y = 400 \text{ MPa}$, $\gamma = .6$ $f'_c = 20 \text{ MPa}$, $\rho = 4\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.675	.623	1.08
.8	.64	.617	1.04
.7	.62	.61	1.02
.6	.605	.602	1.00
.5	.59	.59	1.00
.4	.575	.575	1.00
.3	.55	.551	1.00
.2	.54	.551	.98
.1	.57	.591	.96

TABLE 4.29 Square Column, 16 bars

 $f_y = 400 \text{ MPa}$, $\gamma = .6$ $f'_c = 20 \text{ MPa}$, $\rho = 6\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.68	.615	1.11
.8	.64	.61	1.05
.7	.61	.602	1.01
.6	.60	.593	1.01
.5	.585	.581	1.01
.4	.565	.563	1.03
.3	.545	.53	1.03
.2	.535	.52	1.03
.1	.545	.536	1.02

TABLE 4.30 Rectangular Column, 8 bars

 $f_y = 300 \text{ MPa}$ $f'_c = 40 \text{ MPa}, \rho = 4\%$

P_u/P_{uo}	Actual ϕ Value	Calculated ϕ Value	$\frac{\phi_{\text{actual}}}{\phi_{\text{calc}}}$
.9	.65	.642	1.01
.8	.64	.636	1.01
.7	.62	.628	.99
.6	.604	.619	.98
.5	.595	.607	.98
.4	.582	.59	.99
.3	.567	.563	1.01
.2	.59	.561	1.05
.1	.64	.595	1.07

TABLE 4.31 Rectangular Column, 8 bars

 $f_y = 300 \text{ MPa}$ $f'_c = 40 \text{ MPa}, \rho = 6\%$

P_u/P_{uo}	Actual ϕ Value	Calculated ϕ Value	$\frac{\phi_{\text{actual}}}{\phi_{\text{calc}}}$
.9	.66	.645	1.02
.8	.65	.634	1.03
.7	.625	.627	1.00
.6	.605	.618	.99
.5	.595	.606	.98
.4	.58	.59	.98
.3	.56	.558	1.00
.2	.575	.554	1.04
.1	.61	.59	1.03

TABLE 4.32 Rectangular Column, 10 bars

 $f_y = 300 \text{ MPa}$ $f'_c = 40 \text{ MPa}, \rho = 1\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.635	.66	.96
.8	.627	.65	.96
.7	.61	.638	.96
.6	.60	.625	.96
.5	.60	.609	.99
.4	.595	.59	1.01
.3	.61	.61	1.00
.2	.645	.622	1.04
.1	.70	.64	1.09

TABLE 4.33 Rectangular Column, 10 bars

 $f_y = 300 \text{ MPa}$ $f'_c = 40 \text{ MPa}, \rho = 3\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.645	.644	1.00
.8	.635	.637	1.00
.7	.615	.629	.98
.6	.605	.62	.98
.5	.595	.607	.98
.4	.583	.59	.99
.3	.58	.568	1.02
.2	.605	.568	1.06
.1	.65	.602	1.08

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TABLE 4.34 Rectangular Column, 10 bars

 $f_y = 300 \text{ MPa}$ $f'_c = 40 \text{ MPa}, \rho = 4\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.66	.642	1.03
.8	.64	.636	1.01
.7	.62	.628	.99
.6	.605	.619	.98
.5	.595	.607	.98
.4	.58	.59	.98
.3	.57	.563	1.01
.2	.592	.561	1.05
.1	.635	.595	1.06

TABLE 4.35 Rectangular Column, 10 bars

 $f_y = 300 \text{ MPa}$ $f'_c = 40 \text{ MPa}, \rho = 6\%$

P_u/P_{uo}	Actual β Value	Calculated β Value	$\frac{\beta_{\text{actual}}}{\beta_{\text{calc}}}$
.9	.66	.64	1.03
.8	.645	.634	1.02
.7	.625	.627	1.00
.6	.61	.618	.99
.5	.595	.606	.98
.4	.58	.59	.98
.3	.56	.558	1.00
.2	.58	.554	1.04
.1	.61	.59	1.03

TABLES 4.36-4.51 Accuracy of the Parametric Method

(moments are in KNm)

600mmx600mm

TABLE 4.36

Square Column, 8 bars

 $f_y = 300 \text{ MPa}$, $\gamma = .7$ $\theta = 15$ $f'_c = 20 \text{ MPa}$, $\rho = 1\%$

P_u/P_{uo}	.9	.7	.5	.2	.1
Actual					
M_{ox}	120.4	305.3	405.8	374.1	290.7
M_{oy}	120.4	305.3	405.8	374.1	290.7
Actual					
M_x	116.3	285.8	374.0	345.6	279.2
M_y	31.2	76.6	100.2	92.6	74.8
R_θ	1.00	.97	.95	.96	.99
β	.743	.66	.63	.64	.69
n	1.68	1.21	1.09	1.14	1.38
Calculated					
M_x	118.1	286.2	372.7	346.4	278.8
M_y	31.6	76.7	99.9	92.8	74.7
R_θ	1.01	.97	.95	.96	.99
$\frac{(R_\theta)_{\text{actual}}}{(R_\theta)_{\text{calc}}}$	.99	1.00	1.00	1.00	1.00

TABLE 4.37 600mmx600mm
 Square Column, 8bars $f_y = 300 \text{ MPa}$, $\gamma = .7$
 $\theta = 30$ $f'_c = 20 \text{ MPa}$, $\rho = 1\%$

P_u/P_{uo}	.9	.7	.5	.2	.1
Actual					
M_{ox}	120.4	305.3	405.8	374.1	290.7
M_{oy}	120.4	305.3	405.8	374.1	290.7
Actual					
M_x	106.3	250.1	319.3	299.3	247.2
M_y	61.3	144.4	184.3	172.8	142.7
R_θ	1.02	.95	.91	.92	.98
β	.74	.66	.63	.64	.69
n	1.68	1.21	1.09	1.14	1.38
Calculated					
M_x	108.4	249.3	320.0	299.0	247.9
M_y	62.6	143.9	184.7	172.7	143.1
R_θ	1.04	.94	.91	.92	.99
$\frac{(R_\theta)_{actual}}{(R_\theta)_{calc}}$	.98	1.01	1.00	1.00	.99

600mmx600mm
 TABLE 4.38 Square Column, 8 bars $f_y = 300 \text{ MPa}$, $\gamma = .7$
 $\theta = 15$ $f'_c = 40 \text{ MPa}$, $\rho = 3\%$

P_u/P_{uo}	.9	.7	.5	.2	.1
Actual					
M_{ox}	248.1	615.7	844.8	868.9	744.7
M_{oy}	248.1	615.7	844.8	868.9	744.7
Actual					
M_x	230.4	554.8	758.8	792.1	685.4
M_y	61.7	148.7	203.4	212.2	183.7
R_θ	.96	.93	.93	.94	.95
β	.66	.62	.61	.615	.64
n	1.19	1.03	1.00	1.03	1.13
Calculated					
M_x	232.0	557.7	758.1	785.6	689.0
M_y	62.2	149.4	203.1	210.5	184.6
R_θ	.97	.94	.93	.94	.96
$\frac{(R_\theta)_{\text{actual}}}{(R_\theta)_{\text{calc}}}$	.99	.99	1.00	1.00	.99

TABLE 4.39 600mmx600mm
 Square Column, 8 bars $f_y = 300\text{MPa}$, $\gamma = .7$
 $\theta = 30$ $f'_c = 40\text{MPa}$, $\rho = 3\%$

P_u/P_{uo}	.9	.7	.5	.2	.1
Actual					
M_{ox}	248.1	615.7	844.8	868.9	744.7
M_{oy}	248.1	615.7	844.8	868.9	744.7
Actual					
M_x	202.9	475.0	641.5	668.2	601.2
M_y	117.2	274.3	370.4	385.8	347.1
R_θ	.94	.89	.88	.89	.93
β	.66	.62	.61	.62	.64
n	1.19	1.03	1.00	1.03	1.13
Calculated					
M_x	201.8	474.7	641.8	667.9	594.4
M_y	116.5	274.0	370.6	385.6	343.2
R_θ	.94	.89	.88	.89	.92
$\frac{(R_\theta)_{\text{actual}}}{(R_\theta)_{\text{calc}}}$	1.00	1.00	1.00	1.00	1.01

TABLE 4.40 600mmx600mm
 Square Column, 8 bars $f_y = 400\text{MPa}$, $\gamma = .7$
 $\theta = 15$ $f'_c = 20\text{MPa}$, $\rho = 4\%$

P_u/P_{uo}	.9	.7	.5	.2	.1
Actual					
M_{ox}	199.5	487.6	707.4	937.0	900.9
M_{oy}	199.5	487.6	707.4	937.0	900.9
Actual					
M_x	189.8	456.6	637.3	794.6	787.3
M_y	50.8	122.4	170.7	212.9	211.0
R_θ	.99	.97	.93	.88	.91
β	.685	.645	.60	.55	.57
n	1.33	1.14	.97	.84	.89
Calculated					
M_x	190.3	452.2	630.6	794.0	781.2
M_y	51.0	121.2	169.0	212.8	209.3
R_θ	.99	.96	.92	.88	.89
$\frac{(R_\theta)_{\text{actual}}}{(R_\theta)_{\text{calc}}}$	1.00	1.01	1.01	1.00	1.02

TABLE 4.41 600mmx600mm
 Square Column, 8 bars $f_y = 400\text{MPa}$, $\gamma = .7$
 $\theta = 30$ $f'_c = 20\text{MPa}$, $\rho = 4\%$

P_u/P_{uo}	.9	.7	.5	.2	.1
Actual					
M_{ox}	199.5	487.6	707.4	937.0	900.9
M_{oy}	199.5	487.6	707.4	937.0	900.9
Actual					
M_x	171.7	391.1	532.4	651.9	653.1
M_y	99.1	225.8	307.4	376.4	377.1
R_θ	.99	.93	.87	.80	.84
β	.685	.645	.60	.55	.57
n	1.33	1.14	.97	.84	.89
Calculated					
M_x	168.3	390.8	532.0	653.7	650.0
M_y	97.2	225.6	307.1	377.4	375.3
R_θ	.97	.93	.87	.81	.83
$\frac{(R_\theta)_{actual}}{(R_\theta)_{calc}}$	1.02	1.00	1.00	1.00	1.00

TABLE 4.42 .600mmx600mm
 Square Column, 8bars
 $\theta = 15$
 $f_y = 400 \text{ MPa}$, $\gamma = .7$
 $f'_c = 40 \text{ MPa}$, $\rho = 6\%$

P_u/P_{uo}	.9	.7	.5	.2	.1
Actual					
M_{ox}	372.5	828.5	1180.1	1512.7	1431.3
M_{oy}	372.5	828.5	1180.1	1512.7	1431.3
Actual					
M_x	344.0	758.5	1045.9	1279.4	1257.3
M_y	92.2	203.2	280.2	342.8	336.9
R_θ	.96	.95	.92	.88	.91
β	.65	.625	.585	.55	.57
n	1.16	1.06	.93	.83	.89
Calculated					
M_x	346.6	755.7	1037.3	1277.5	1239.8
M_y	92.9	202.5	277.9	342.3	332.2
R_θ	.96	.94	.91	.87	.90
$\frac{(R_\theta)_{\text{actual}}}{(R_\theta)_{\text{calc}}}$	.99	1.00	1.01	1.00	1.01

600mmx600mm
 TABLE 4.43 Square Column, 8 bars
 $\theta=30$

 $f_y = 400 \text{ MPa}, \gamma = .7$
 $f'_c = 40 \text{ MPa}, \rho = 6\%$

P_u/P_{uo}	.9	.7	.5	.2	.1
Actual					
M_{ox}	372.5	828.5	1180.1	1512.7	1431.3
M_{oy}	372.5	828.5	1180.1	1512.7	1431.3
Actual					
M_x	303.0	642.3	869.3	1045.5	1037.7
M_y	174.9	370.8	501.9	603.7	599.1
R_θ	.94	.90	.85	.80	.84
β	.65	.625	.585	.55	.57
n	1.16	1.06	.93	.83	.89
Calculated					
M_x	300.1	646.0	868.8	1050.1	1031.1
M_y	173.3	372.9	501.6	606.3	595.3
R_θ	.93	.90	.85	.80	.83
$\frac{(R_\theta)_{\text{actual}}}{(R_\theta)_{\text{calc}}}$	1.01	.99	1.00	1.00	1.01

TABLE 4.44
600mmx600mm
Square Column, 16 bars
 $\theta = 15$

$f_y = 300 \text{ MPa}$, $\gamma = .7$

$f'_c = 20 \text{ MPa}$, $\rho = 2\%$

P_u/P_{uo}	.9	.7	.5	.2	.1
Actual					
M_{ox}	141.3	350.1	481.5	501.8	441.0
M_{oy}	141.3	350.1	481.5	501.8	441.0
Actual					
M_x	136.5	328.2	440.2	463.8	411.5
M_y	36.6	87.9	118.0	124.3	110.3
R_θ	1.00	.97	.95	.96	.97
β	.74	.66	.63	.635	.66
n	1.65	1.23	1.07	1.10	1.20
Calculated					
M_x	138.5	329.1	439.6	461.3	413.1
M_y	37.1	88.2	117.9	123.6	110.7
R_θ	1.01	.97	.95	.95	.97
$\frac{(R_\theta)_{\text{actual}}}{(R_\theta)_{\text{calc}}}$	.99	1.00	1.00	1.01	1.00

600mmx600mm
 TABLE 4.45 Square Column, 16 bars $f_y = 300 \text{ MPa}$, $\gamma = .7$
 $\theta = 30$ $f'_c = 20 \text{ MPa}$, $\rho = 2\%$

P_u/P_{uo}	.9	.7	.5	.2	.1
Actual					
M_{ox}	141.3	350.1	481.5	501.8	441.0
M_{oy}	141.3	350.1	481.5	501.8	441.0
Actual					
M_x	125.2	287.1	375.8	396.0	359.3
M_y	72.3	165.8	217.0	228.6	207.5
R_θ	1.02	.95	.90	.91	.94
β	.74	.66	.63	.635	.66
n	1.65	1.23	1.07	1.10	1.20
Calculated					
M_x	126.7	287.3	375.9	396.2	359.6
M_y	73.2	165.9	217.0	228.8	207.6
R_θ	1.04	.95	.90	.91	.94
$\frac{(R_\theta)_{actual}}{(R_\theta)_{calc}}$	.98	1.00	1.00	1.00	1.00

TABLE 4.46 600mmx600mm
 Square Column, 16 bars $f_y = 400 \text{ MPa}$, $\gamma = .7$
 $\theta = 15$ $f'_c = 20 \text{ MPa}$, $\rho = 6\%$

P_u/P_{uo}	.9	.7	.5	.2	.1
Actual					
M_{ox}	243.0	597.4	858.3	1179.0	1185.7
M_{oy}	243.0	597.4	858.3	1179.0	1185.7
Actual					
M_x	232.3	549.0	771.1	991.3	1021.5
M_y	62.2	147.1	206.6	265.7	273.7
R_θ	.99	.95	.93	.87	.89
β	.69	.63	.60	.545	.56
n	1.38	1.07	.98	.82	.86
Calculated					
M_x	233.0	545.4	766.5	992.3	1012.7
M_y	62.4	146.1	205.4	265.9	271.3
R_θ	.99	.945	.925	.87	.885
$\frac{(R_\theta)_{\text{actual}}}{(R_\theta)_{\text{calc}}}$	1.00	1.01	1.01	1.00	1.01

TABLE 4.47
600mmx600mm
Square Column, 16 bars
 $\theta = 30$

$f_y = 400\text{MPa}$, $\gamma = .7$
 $f'_c = 20\text{MPa}$, $\rho = 6\%$

P_u/P_{uo}	.9	.7	.5	.2	.1
Actual					
M_{ox}	243.0	597.4	858.3	1179.0	1185.7
M_{oy}	243.0	597.4	858.3	1179.0	1185.7
Actual					
M_x	206.6	466.6	646.9	811.9	837.3
M_y	119.3	269.4	373.5	468.7	483.4
R_θ	.98	.90	.87	.795	.815
β	.69	.63	.60	.545	.56
n	1.38	1.07	.98	.82	.86
Calculated					
M_x	207.2	466.4	647.3	814.4	836.7
M_y	119.6	269.3	373.7	470.2	483.1
R_θ	.985	.90	.87	.80	.815
$\frac{(R_\theta)_{\text{actual}}}{(R_\theta)_{\text{calc}}}$	.99	1.00	1.00	.99	1.00

600mmx300mm
TABLE 4.48 Rectangular Column, 8 bars

 $f_y = 300 \text{ MPa}$
 $f'_c = 40 \text{ MPa}, \rho = 4\%$

P_u/P_{uo}	.8	.6	.5	.2	.1
θ	27.5	53.2	13.5	31.0	12.8
Actual M_{ox}	116.5	202.4	233.2	257.1	210.6
M_{oy}	224.0	366.2	403.4	427.3	383.5
Actual M_x	94.7	104.9	212.0	191.2	204.7
M_y	94.7	253.3	87.9	191.2	84.8
R_θ	.92	.86	.94	.87	1.00
β	.64	.605	.595	.59	.64
n	1.12	.99	.96	.94	1.11
Calculated M_x	95.4	104.4	209.9	188.1	197.3
M_y	95.4	252.0	87.0	188.1	81.7
R_θ	.92	.86	.92	.85	.96
$\frac{(R_\theta)_{\text{actual}}}{(R_\theta)_{\text{calc}}}$	1.00	1.01	1.01	1.02	1.04

600mmx300mm
TABLE 4.49 Rectangular Column, 8 bars

$f_y = 300 \text{ MPa}$
 $f'_c = 40 \text{ MPa}, \rho = 6\%$

P_u/P_{uo}	.8	.7	.3	.2	.1
θ	27.8	12.8	33.3	58.0	13.5
Actual					
M_{ox}	135.9	189.4	342.5	333.9	284.1
M_{oy}	258.1	345.8	520.9	511.8	491.3
Actual					
M_x	113.0	176.2	231.6	147.8	237.2
M_y	113.0	73.0	231.6	357.0	113.2
R_θ	.94	.95	.81	.83	.99
β	.65	.625	.56	.57	.61
n	1.16	1.06	.86	.89	1.00
Calculated					
M_x	112.3	175.9	231.2	147.9	258.7
M_y	112.3	72.9	231.2	357.1	107.2
R_θ	.93	.95	.81	.825	.94
$\frac{(R_\theta)_{\text{actual}}}{(R_\theta)_{\text{calc}}}$	1.01	1.00	1.00	1.00	1.06

600mmx300mm
TABLE 4.50 Rectangular Column, 10 bars

 $f_y = 300 \text{ MPa}$
 $f'_c = 20 \text{ MPa}, \rho = 6\%$

P_u/P_{uo}	.7	.5	.3	.2	.1
θ	55.0	15.1	16.7	17.6	59.1
Actual					
M_{ox}	134.3	200.1	259.6	280.1	253.4
M_{oy}	227.0	306.5	357.4	366.3	366.6
Actual					
M_x	72.6	185.2	225.3	235.2	110.7
M_y	175.3	76.7	93.3	97.4	267.2
R_θ	.94	.96	.905	.88	.85
β	.66	.61	.58	.57	.59
n	1.20	1.01	.91	.88	.935
Calculated					
M_x	72.0	180.2	222.3	234.6	110.8
M_y	174.0	74.7	92.1	97.2	267.3
R_θ	.935	.93	.895	.88	.85
$\frac{(R_\theta)_{\text{actual}}}{(R_\theta)_{\text{calc}}}$	1.01	1.03	1.01	1.00	1.00

TABLE 4.51 600mmx300mm
Rectangular Column, 10 bars

$f_y = 300 \text{ MPa}$

$f'_c = 20 \text{ MPa}, \rho = 4\%$

P_u/P_{uo}	.8	.6	.3	.2	.1
θ	28.1	55.5	16.2	59.4	32.2
Actual					
M_{ox}	78.8	138.3	206.9	206.5	179.6
M_{oy}	147.6	229.7	295.3	294.4	284.8
Actual					
M_x	70.1	71.2	179.4	89.0	137.3
M_y	70.1	171.9	74.3	214.8	137.3
R_θ	1.01	.91	.90	.85	.90
β	.685	.63	.58	.585	.615
n	1.33	1.09	.91	.92	1.02
Calculated					
M_x	67.8	70.7	178.1	88.8	133.9
M_y	67.8	170.6	73.8	214.4	133.9
R_θ	.98	.90	.895	.845	.88
$\frac{(R_\theta)_{\text{actual}}}{(R_\theta)_{\text{calc}}}$	1.03	1.01	1.01	1.00	1.03

CHAPTER 5

ANALYSIS AND DESIGN PROCEDURES FOR COLUMNS SUBJECTED TO BI- AXIAL MOMENTS

5.1 Introduction

This chapter describes the step-by-step procedure for analysis and design of columns subjected to biaxial moments using the parametric method developed in this study. Several solved examples are given to illustrate this new approach. The effect of ignoring the cover ratio, γ , will be highlighted in this chapter.

5.2 Procedures for Analysis

The step-by-step analysis procedure is given as follows:

Given P_u , M_x , M_y , f'_c , f_y , ρ , B , H and γ

1. Obtain M_{ox} , M_{oy} and P_{uo} from uniaxial design charts and calculate M_x/M_{ox} , M_y/M_{oy} and P_u/P_{uo} . The pure axial load P_{uo} can also be obtained from the expression

$$P_{uo} = \phi (.85f'_c B H + (f_y - .85f'_c) A_{ST})$$

where A_{ST} is the total area of reinforcement in the column cross section.

2. Calculate $\theta = \tan^{-1}(M_y/M_{oy})/(M_x/M_{ox})$ and

$$R_\theta = \sqrt{[(M_x/M_{ox})^2 + (M_y/M_{oy})^2]}$$

3. Obtain A from Figs. 4.21 or 4.22 or calculate β from Eqns. 4.2 or 4.3 and calculate A as $A = 1 - \sqrt{2}\beta$
4. Obtain n from Fig. 4.25 and calculate R as $R = 1 - A \sin^2 2\theta$ and compare with R_θ of step 2.

If $R \geq R_\theta$, column is adequate

If $R < R_\theta$, column is inadequate.

5.2.2 Procedure for Design

The step-by-step design procedure is given as follows:

Given P_u , M_x , M_y , f_y and f'_c .

1. Choose B and H of column and bar arrangement. Calculate $\rho = A_s/BH$
2. From Uniaxial Design Charts obtain P_{uo} , M_{ox} and M_{oy} . Calculate M_x/M_{ox} , M_y/M_{oy} , P_u/P_{uo} and

$$\theta = \tan^{-1} \left(\frac{M_y/M_{oy}}{M_x/M_{ox}} \right)$$

3. Obtain A from Figs. 4.21 or 4.22 or calculate β from Eqns. 4.2 or 4.3 and calculate A as

$$A = 1 - \sqrt{2\beta}$$

4. Obtain n from Fig. 4.25 and calculate R as

$$R = 1 - A \sin^n 2\theta. \text{ Calculate}$$

$$M_x = M_{ox} \cdot R \cdot \cos \theta$$

and

$$M_y = M_{oy} \cdot R \cdot \sin \theta$$

and compare with given moments.

5.3 Examples

Eg. 1

Ascertain whether the 700mm x 700mm column containing 16 #30m bars evenly distributed on all the faces with the given data is adequate.

$$P_u = 5232.3 \text{ KN}$$

$$M_x = 485 \text{ KNm}$$

$$M_y = 280 \text{ KNm}$$

$$f_y = 300 \text{ MPa}$$

$$f'_c = 20 \text{ MPa}$$

$$\gamma = 0.6$$

$$\rho = 16 \times 700/700^2 = 2.3\%$$

16 # 30M bars

A_s of one bar = 700mm^2

1. From the computer program [Ref. 7]

$$M_{ox} = M_{oy} = 604.8 \text{ KNm}, P_{uo} = 8049.7 \text{ KN}$$

$$M_x/M_{ox} = 485/604.8 = 0.802$$

$$M_y/M_{oy} = 280/604.8 = 0.463$$

$$P_u/P_{uo} = 5232.3/8049.7 = 0.65$$

2. $\theta = \tan^{-1}(.463/.802) = 30^\circ$

$$R_\theta = [(.463)^2 + (.802)^2] = .926$$

3. From Fig. 4.21 with $P_u/P_{uo} = .65$ and $\rho = 2.3\%$

$$A = .077$$

or from Eqn. 4.2

$$\beta = \frac{.245}{.65} + \frac{.01(.65-.4)}{.023 \times .65} \times .7[(.065 + .975) + 2.3 - 1] = .651$$

$$A = 1 - \sqrt{2\beta} = 1 - \sqrt{2} \times .651 = .079$$

4. From Fig. 4.25 with $A = .079$

$$n = 1.16$$

$$R = 1 - .079 \times \sin^{1.16}(60) = .933 > .926$$

$\therefore$ column is adequate.

(The computer solution of this problem gives $R_{30^\circ} = .933$)

Eg. 2

Design a square column for the load, moments and strengths given, assuming reinforcement equally distributed on all faces.

Given

$$P_u = 808.92 \text{ KN}$$

$$M_x = 129.5 \text{ KNm}$$

$$M_y = 53.64 \text{ KNm}$$

$$f_y = 300 \text{ MPa}$$

$$f'_c = 20 \text{ MPa}$$

$$A_s \text{ of one bar} = 500\text{mm}^2$$

1. Try 400 mm x 400 mm column with 8 #25m bars

$$\rho = 8 \times 500 / 400^2 = 2.5\%$$

2. From the computer program [Ref. 7]

$$M_{ox} = M_{oy} = 166.6 \text{ KNm}, P_{uo} = 2696.4 \text{ KN}$$

$$M_x / M_{ox} = 129.5 / 166.6 = .777$$

$$M_y / M_{oy} = 53.64 / 166.6 = .322$$

$$P_u / P_{uo} = 808.92 / 2696.4 = .3$$

$$\theta = \tan^{-1}(.322 / .777) = 22.5^\circ$$

3. From Fig. 4.21 with $P_u / P_{uo} = .3$ and $\rho = 2.5\%$

$$A = .17 \text{ or from Eqn. 4.3}$$

$$\beta = \frac{.245}{.3} - \frac{(.4 - .3)}{.025 \times .3} \times .6[.3^2(2.6 \times .025 - .035) + .025] = .59$$

$$A = 1 - \sqrt{2} \times .59 = .17$$

4. From Fig. 4.25 with $A = .17$

$$n = 0.92$$

$$R = 1 - .17 \times \sin^{.90}(45) = .876$$

$$M_x = 166.6 \times .876 \times \cos 22.5^\circ = 134.8 \text{ KNm} > 129.5 \text{ KNm} \quad \text{O.K.}$$

$$M_y = 166.6 \times .876 \times \sin 22.5^\circ = 55.8 \text{ KNm} > 53.64 \text{ KNm} \quad \text{O.K.}$$

∴ Design is satisfactory.

(The computer solution of this problem gives the ultimate moment as:

$$M_x = 135.1 \text{ KNm and } M_y = 56.0 \text{ KNm}).$$

Eg. 3

Design a square column for the load, moments and strengths given, assuming reinforcement equally distributed on all faces for $\gamma = 0.8$.

$$P_u = 3583.9 \text{ KN}$$

$$M_x = 240 \text{ KNm}$$

$$M_y = 99.4 \text{ KNm}$$

$$f_y = 400 \text{ MPa}$$

$$f'_c = 20 \text{ MPa}$$

$$A_s \text{ of one bar} = 1000 \text{ mm}^2$$

1. Try 500mm x 500mm column with 8 #35 bars

$$\rho = 8 \times 1000 / 500^2 = 3.2\%$$

2. From the computer program [Ref. 7] with $\gamma = .8$

$$M_{ox} = M_{oy} = 277.69 \text{ KNm}, P_{uo} = 5119.8 \text{ KN}$$

$$M_x/M_{ox} = 240/277.69 = .864$$

$$M_y/M_{oy} = 99.4/277.69 = .358$$

$$P_u/P_{uo} = 3583.9/5119.8 = .7$$

$$\theta = \tan^{-1}(.358/.864) = 22.5^\circ$$

3. From Fig. 4.22 with $P_u/P_{uo} = .7$ and $\rho = 3.2\%$

$$A = .12 \text{ or from Eqn. 4.2}$$

$$\beta = \frac{(.24 - .25 \times .032)}{0.7} + \frac{.01(.7 - .4)}{.032 \times .7} \times .65[(.07 + .975) + (3.2 - 1)] = .614$$

$$A = 1 - \sqrt{2} \times .614 = .13$$

4. From Fig. 4.25 with $A = .13$

$$n = 1.02$$

$$R = 1 - .130 \times \sin 45^\circ = .907$$

$$M_x = 277.96 \times .907 \cos 22.5^\circ = 232.7 \text{ KNm} < 240.0 \text{ KNm}$$

$$M_y = 277.96 \times .907 \sin 22.5^\circ = 96.4 \text{ KNm} < 99.4 \text{ KNm}$$

∴ Design is inadequate

This example was chosen to illustrate the effect of neglecting the cover ratio, γ , in the solution. The computer solution of the same problem gives

$$\beta = .666 \text{ for } \gamma = 0.8$$

$$\therefore \% \text{ underestimation of } \beta = \frac{.666 - .625}{.666}$$

$$= 6.2\%$$

The actual ultimate moments from the computer solution are

$$M_x = 245.5 \text{ KNm and } M_y = 101.7 \text{ KNm}$$

Eg. 4

Ascertain whether the 500mm x 300mm column containing 8 #30M bars on the longer faces with the given data is adequate.

$$P_u = 1013.04 \text{ KN}$$

$$M_x = 76.76 \text{ KNm}$$

$$M_y = 133.0 \text{ KNm}$$

$$f_y = 300 \text{ MPa}$$

$$f'_c = 20 \text{ MPa}$$

$$\gamma = 0.6$$

$$\rho = 8 \times 700 / 300 \times 500 = 3.73\%$$

8 #30M bars

$$A_s \text{ of one bar} = 700 \text{ mm}^2$$

1. From the computer program [Ref. 7] with $\gamma = 0.6$

$$M_{ox} = 157.71 \text{ KNm}, M_{oy} = 200.62 \text{ KNm}, P_{uo} = 2894.36 \text{ KN}$$

$$M_x / M_{ox} = 76.76 / 157.71 = 0.487$$

$$M_y / M_{oy} = 133 / 200.62 = 0.663$$

$$P_u / P_{uo} = 1013.04 / 2894.36 = 0.35$$

2. $\theta = \tan^{-1} (.663 / .487) = 53.7^\circ$

$$R_\theta = \sqrt{(.663)^2 + (.487)^2} = .823$$

3. From Fig. 4.21' with $P_u / P_{uo} = .35$ and $\rho = 3.73\%$

$$A = .16$$

or from Eqn. 4.3

$$\beta = \frac{.245}{.35} - \frac{(.4 - .35)}{.0373 \times .35} \times .6 [(.35)^2 (2.6 \times .0373 - .035) + .0373] = .595$$

$$A = 1 - \sqrt{2} \times .595 = .16$$

4. From Fig. 4.25 with $A = .16$

$$n = .95$$

$$R = 1 - .16 \times \sin^{.91}(2 \times 53.7^\circ) = .847 > .823 \text{ O.K.}$$

∴ Column is adequate.

(Computer solution of this problem gives

$$M_x = 78.82 \text{ KNm}, M_y = 136.57 \text{ KNm and}$$

$$R = \sqrt{[(78.82/157.71)^2 + (136.57/200.62)^2]} = .845)$$

Eg. 5

Design a rectangular column with depth equal to one half the width and reinforcement on only the longer faces.

Given:

$$P_u = 1500.4 \text{ KN}$$

$$M_x = 67.53 \text{ KNm}$$

$$M_y = 252.0 \text{ KNm}$$

$$f_y = 400 \text{ MPa}$$

$$f'_c = 20 \text{ MPa}$$

$$\gamma = .6$$

8 #35M bars

$$A_s \text{ of one bar} = 1000\text{mm}^2$$

1. Try 600mm x 300mm column with 8 #35 bars on the longer faces.

$$\rho = 8 \times 1000 / 600 \times 300 = 4.44\%$$

2. From the computer program [Ref. 7] with $\gamma = .6$

$$M_{ox} = 237.58 \text{ KNm}, M_{oy} = 333.0 \text{ KNm}, P_u/P_{uo} = 4286.8 \text{ KN}$$

$$M_x/M_{ox} = 67.53/237.58 = .284$$

$$M_y/M_{oy} = 252/333 = .757$$

$$P_u/P_{uo} = 1500.4/4286.8 = .35$$

$$\theta = \tan^{-1}(.757/.284) = 69.4^\circ$$

3. From Fig. 4.22 with $P_u/P_{uo} = .35$ and $\rho = 4.44\%$

$$A = .23$$

or from Eqn. 4.3

$$\beta = \frac{(.24 - .25 \times .0444)}{.35} - \frac{(.4 - .35)}{.0444 \times .35} \times .56 [.35^2 (2.6 \times .0444 - .035) + .0444] = .55$$

$$A = 1 - \sqrt{2} \times .55 = .22$$

4. From Fig. 4.25 with $A = .22$

$$n = .83$$

$$R = 1 - .22 \times \sin^{.78}(2 \times 69.4^\circ) = .843$$

$$M_x = 237.58 \times .843 \cos 69.4^\circ = 70.5 \text{ KNm} > 67.53 \text{ KNm} \quad \text{O.K.}$$

$$M_y = 333 \times .843 \sin 69.4^\circ = 262.7 \text{ KNm} > 252.0 \text{ KNm} \quad \text{O.K.}$$

(Computer solution of this problem gives

$$M_x = 71.3 \text{ KNm and } M_y = 265.93 \text{ KNm}).$$

Eg. 6

Design a rectangular column with depth equal to one half the width and reinforcement on only the longer faces.

Given:

$$P_u = 3073.07 \text{ KN}$$

$$e_x = 42.1 \text{ mm}$$

$$e_y = 52.1 \text{ mm}$$

$$f_y = 300 \text{ MPa}$$

8 # 35M bars

$$A_s \text{ of one bar} = 1000 \text{ mm}^2$$

$$f'_c = 40 \text{ MPa.}$$

$$\gamma = .6$$

$$1. \quad M_x = P_u e_y = 3073.07 \times .0521 = 160.0 \text{ KNm}$$

$$M_y = P_u e_x = 3073.07 \times .0521 = 160.0 \text{ KNm}$$

Try 600mm x 300mm column with 10 #30M bars on the longer faces.

$$\rho = 10 \times 700 / 600 \times 300 = 3.9\%$$

2. From the computer program [Ref. 7] with $\gamma = .6$

$$M_{ox} = 216.44 \text{ KNm}, M_{oy} = 377.02 \text{ KNm}, P_{uo} = 5587.4 \text{ KNm}$$

$$M_x / M_{ox} = 160 / 216.44 = .739$$

$$M_y / M_{oy} = 160 / 377.02 = .424$$

$$P_u / P_{uo} = 3073.07 / 5587.4 = .55$$

$$\theta = \tan^{-1}(.424 / .739) = 29.8^\circ$$

3. From Fig. 4.21 with $P_u / P_{uo} = .55$ and $\rho = 3.9\%$

$$A = .14$$

or from Eqn. 4.2

$$\beta = \frac{.245}{1.04 \times .55} + \frac{.01(.55 - .4)}{1.04 \times .039 \times .55} \times .7[(.055 + 9.75) + (3.9 - 1)] = .61$$

$$A = 1 - \sqrt{2} \times .61 = .14$$

4. From Fig. 4.25 with $A = .14$

$$n = 1.0$$

$$R = 1 - .14 \times \sin^{1.0}(2 \times 29.8^\circ) = .879$$

$$M_x = 216.44 \times .879 \cos 29.8^\circ = 165.1 \text{ KNm} > 160.0 \text{ KNm} \quad \text{O.K.}$$

$$M_y = 377.02 \times .879 \sin 29.8^\circ = 165.0 \text{ KNm} > 160.0 \text{ KNm} \quad \text{O.K.}$$

(Computer solution of this problem gives

$$M_x = 164.4 \text{ KNm and } M_y = 164.4 \text{ KNm})$$

CHAPTER 6

SUMMARY AND CONCLUSIONS

The author's contribution is well summarized in Figs. 4.7 to 4.14 and Figs. 4.15 to 4.18 and expressed mathematically in Eqns. 4.2 and 4.4. The design/analysis procedures utilizing these aids are given in the previous chapter. These figures and equations are the result of over 8,000 computer runs and the extrapolation of the resulting data.

The author recommends the use of these aids in the design of columns. He recommends their use over the reciprocal load method for accuracy. He recommends their use over the PCA method (Ref. 2) mainly for simplicity; however, compared with the PCA suggested straight line approximation method, it is possible that the proposed parametric method may in general be more accurate, though less conservative.

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APPENDIX AGeneral Analysis of Short Rectangular Column ComputerProgramA1 General

The Fortran program described in this appendix was written by Hartley [Ref. 7]. It computes the ultimate strength of reinforced concrete column cross sections subjected to biaxial moments and axial loads. The program is capable of analyzing rectangular, circular and any other cross-sectional shape defined by about ten quadrilaterals. Column length effects are not considered in the program. The program has the following calculations options:

1. The computation of a theoretical maximum load from given eccentricities, e_x and e_y .
2. A load contour (at constant axial load) in the e_x and e_y plane in 1, 2, 3 and 4 quadrants in this plane.
3. Overcapacity (or undercapacity) of the column for a given set of load and biaxial moments.

The program is interactive and uses free format for numerical data input.

A2 Units

The program uses the S.I. system of units. The units for the numerical data input are:

Length	mm
Area	mm^2
Load	KN
Stress	N/mm^2 or MPa.

A3 Notation and Program Listing

The notation used in the computer program and program listing are given in Ref. 7.

APPENDIX B

Effect of Aspect Ratio (or B/H Ratio) of Column Cross

Section on β

This appendix shows that neither the column aspect ratio (B/H ratio) nor the alignment of the reinforcement, mainly in one direction has a major effect on β . The discussions that follow apply to columns with the same reinforcement percentage, ρ , same total number of bars in the column cross section and same P_u/P_{uo} . The subscripts R and S used in the discussions refer to rectangular and square columns respectively.

CASE 1. Square and Rectangular Columns with Equal Amounts of Reinforcement in All Faces

Consider two columns of dimensions $H \times H$ and $B \times H$ ($B > H$) having same ρ , P_u/P_{uo} , γ ratio on all sides and same number of bars in the cross section. The ultimate biaxial moments in the two columns are related by

$$(M_x)_R = \frac{(M_{ox})_R}{(M_{ox})_S} \cdot (M_x)_S \quad B.1$$

and

$$(M_y)_R = \frac{(M_{oy})_R}{(M_{oy})_S} \cdot (M_y)_S \quad B.2$$

where $M_{oy} > M_{ox}$. M_x , M_y , M_{ox} and M_{oy} have been defined previously. The uniaxial moment capacities in Eqns. B.1 and B.2 are related by

$$\frac{(M_{ox})_R}{(M_{ox})_S} = \frac{B}{H} \text{ and } \frac{(M_{oy})_R}{(M_{oy})_S} = \left(\frac{B}{H}\right)^2 \quad B.3$$

Rewriting Eqns. B.1 and B.2 gives

$$\frac{(M_x)_R}{(M_{ox})_R} = \frac{(M_x)_S}{(M_{ox})_S} \quad \text{B.4}$$

and

$$\frac{(M_y)_R}{(M_{oy})_R} = \frac{(M_y)_S}{(M_{oy})_S} \quad \text{B.5}$$

Eqns. B.4 and B.5 explain why the non-dimensional load-contours for the two columns are exactly identical. At $\theta = 45$ degrees the ultimate biaxial moments in the square column are equal, that is (M_{45}, M_{45}) . The corresponding ultimate biaxial moments in the rectangular column $(\bar{M}_x, \bar{M}_y)$ are given by

$$(\bar{M}_x)_R = \frac{(M_{ox})_R}{(M_{ox})_S} \cdot M_{45} \quad \text{B.6}$$

and

$$(\bar{M}_y)_R = \frac{(M_{oy})_R}{(M_{oy})_S} \cdot M_{45} \quad \text{B.7}$$

Dividing Eqn. B.7 by Eqn. B.6 gives

$$\frac{(\bar{M}_y)_R}{(\bar{M}_x)_R} = \frac{(M_{oy})_R}{(M_{ox})_R} \cdot \frac{(M_{ox})_S}{(M_{oy})_S} \quad \text{B.8}$$

But for a square column with equal mounts of reinforcement in all faces and same γ ratio on all sides $(M_{ox})_S = (M_{oy})_S$. Eqn. B.8 reduces to

$$\frac{(\bar{M}_y)_R}{(\bar{M}_x)_R} = \frac{(M_{oy})_R}{(M_{ox})_R} \quad \text{B.9}$$

or

$$\frac{(\bar{M}_Y)_R}{(M_{OY})_R} = \frac{(\bar{M}_X)_R}{(M_{OX})_R} = \beta_R \text{ (by definition in Ref. 9)}$$

Rearranging Eqns. B.6 and B.7 gives

$$\frac{(\bar{M}_X)_R}{(M_{OX})_R} = \frac{(M_{45})_S}{(M_{OX})_S} \text{ and } \frac{(\bar{M}_Y)_R}{(M_{OY})_R} = \frac{(M_{45})_S}{(M_{OY})_S}$$

$$\text{But } \frac{(M_{45})_S}{(M_{OX})_S} = \frac{(M_{45})_S}{(M_{OY})_S} = \beta_S$$

$$\therefore \beta_R = \beta_S$$

Case 2: Square column with equal amounts of reinforcement in all faces and rectangular columns with any arrangement of reinforcement in the cross section

Consider two columns of dimensions $H \times H$ and $B \times H$ ($B > H$) having same ρ and P_u/P_{uo} with the square column having the same geometrical characteristics as in Case 1. Expressions B.1 and B.2 are again applicable, however, in this case the strict equalities no longer hold, that is,

$$(M_X)_R \approx \frac{(M_{OX})_R}{(M_{OX})_S} \cdot (M_X)_S \quad \text{B.10}$$

$$(M_Y)_R \approx \frac{(M_{OY})_R}{(M_{OY})_S} \cdot (M_Y)_S \quad \text{B.11}$$

In particular if the ultimate biaxial moments in the square column at $\theta = 45$ degrees are M_{45} , M_{45} then the corresponding ultimate biaxial moments in the rectangular column are given by

$$(\bar{M}_X)_R = b \cdot \frac{(M_{OX})_R}{(M_{OX})_S} \cdot M_{45} \quad \text{B.12}$$

$$(\bar{M}_Y)_R = b \cdot \frac{(M_{OY})_R}{(M_{OY})_S} \cdot M_{45} \quad B.13$$

where b is a variable factor to take account of the dissimilarities in the alignment of the bars in the two columns. The factor b varies from about 0.96 to 1.05.

Rearranging Eqns. B.12 and B.13 gives

$$\frac{(\bar{M}_X)_R}{(M_{OX})_R} = b \cdot \frac{(M_{45})_S}{(M_{OX})_S} \quad \text{and} \quad \frac{(\bar{M}_Y)_R}{(M_{OY})_R} = b \cdot \frac{(M_{45})_S}{(M_{OY})_S}$$

$$\text{or } \beta_R = b \cdot \beta_S$$

$$\text{where } .96 \leq b \leq 1.05$$

APPENDIX C

Comparison of Parametric and PCA Methods

This appendix gives the procedure for obtaining R_θ values from the PCA Method [Ref. 2] and compares these R_θ values with those obtained using the parametric approach developed in this study.

R_θ values from the PCA Method [Ref. 2]

Bresler's load-contour equation [Ref. 2] can be stated in a more convenient form as

$$X^m + Y^m = 1 \quad A.1$$

where

$$m = \log 0.5 / \log \beta$$

$$X = M_x / M_{ox}$$

$$Y = M_y / M_{oy}$$

M_x , M_y , M_{ox} , M_{oy} , β have been defined previously. The equation of the radial line, R_θ , at θ degrees to the X-zxis is given by

$$Y = X \tan \theta$$

The coordinates of the intersection point of the two curves are thus given by

$$X = \frac{1}{(1 + \tan^m \theta)^{1/m}}$$

and

$$Y = \frac{\tan \theta}{(1 + \tan^m \theta)^{1/m}}$$

R_θ is computed as

$$R_\theta = \sqrt{X^2 + Y^2}$$

Table A-1 gives the computed R_0 values using the PCA and the Parametric Methods. As noted from the Table there is excellent agreement between the two methods.

TABLE A.1 R_0 Values from the PCA and the Parametric Methods

β	θ°	R_0 values from	
		PCA Method	Parametric Method
.52	5	.944	.932
	15	.852	.845
	25	.793	.785
	30	.773	.763
	40	.752	.740
.55	5	.955	.948
	15	.874	.875
	25	.819	.822
	30	.801	.803
	40	.780	.780
.58	5	.970	.964
	15	.905	.904
	25	.857	.859
	30	.841	.842
	40	.823	.823
.60	5	.978	.972
	15	.933	.923
	25	.894	.883
	30	.867	.868
	40	.851	.851
.62	5	.984	.980
	15	.941	.940
	25	.906	.907
	30	.893	.894
	40	.879	.879
.65	5	.992	.990
	15	.965	.964
	25	.941	.941
	30	.931	.932
	40	.921	.921
.70	5	.999	.999
	15	.996	.996
	25	.993	.993
	30	.992	.992
	40	.990	.990
.72	5	1.001	1.001
	15	1.006	1.006
	25	1.012	1.012
	30	1.015	1.015
	40	1.018	1.018

		R _g values from	
β	θ°	PCA Method	Parametric Method
.74	5	1.002	1.003
	15	1.014	1.015
	25	1.030	1.030
	30	1.036	1.037
	40	1.045	1.045
.80	5	1.004	1.003
	15	1.030	1.031
	25	1.072	1.075
	30	1.094	1.097
	40	1.127	1.127

END

0	6	H	0	9	H	8	4
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FIN