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High-speed Interconnect Analysis in the Presence of Incident Electromagnetic Fields

by

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**A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of
Master of Engineering**

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Abstract

Current trends in high end designs are causing interconnects to become an important performance limiting factor, thus increasing the need for efficient and accurate interconnect analysis. With novel techniques presented in this thesis, moment matching algorithms are used for the linear time and frequency domain analysis of both frequency depended interconnects and transmission lines illuminated by incident fields. This provides a significant CPU improvement over current methods, and allows for efficient nonlinear analysis.

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List of symbols

γ	Propagation constant on a transmission line.
Δt	Time step in a conventional transient analysis.
$\Phi(x)$	Function expressing the current in a non-linear element.
ϵ	Permittivity
σ	Conductivity.
Ψ	Magnetic flux.
μ	Permeability.
$\beta_x, \beta_y, \beta_z$	Propagation constant of the incident field in the x, y and z directions. Defined in (3.39)-(3.41).
θ_E	Angle defining the polarization of the incident wave (defined in Figure 3.3).
θ_p, ϕ_p	Define the incidence angle of the interfering EM fields (defined in Figure 3.3).
φ	Matrix of the transmission line parameters (defined in (3.60)).
$\psi(s)$	Source vector including lumped sources and source due to EMI.
Δ_i	Intermediate moments in the calculation of the incident field moments (defined in (3.98)).
$\Gamma(s)$	Fraction of the distributed source that is independent of z . (defined in (3.57)).

a_i	Coefficients of the numerator of the Padé expansion.
A	Transmission line stamp (defined in (3.69)).
$b_k(s)$	Vector representing the voltage and current source associated with incident field coupling to interconnect k .
b_i	Coefficients of the denominator of the Padé expansion.
B	Transmission line stamp (defined in (3.69)).
B_y^i	Magnitude of the incident magnetic field in the y-direction.
$\vec{B}^i$	Incident magnetic field.
$\vec{B}^s$	Scattered magnetic field.
$\vec{B}$	Total magnetic field (scattered and incident).
c	Capacitance associated with a lumped interconnect model.
C	Capacitance per unit length of a transmission line.
D	Matrix of transmission line parameters (defined in (3.55)).
D_k	Current selector matrix for transmission line set k (defined in (3.71)).
e_x, e_y, e_z	Magnitude of the projection of unity vector in the direction of the electric field to the x, y, and z axis (defined in (3.36)-(3.38)).
e_ϕ	Vector representing the lumped sources in the network ϕ .
$\vec{E}$	Total electric field (scattered and incident).
E_o	Magnitude of incident electric field.
$\vec{E}^i$	Incident electric field.
$\vec{E}^s$	Scattered electric field.
E_x	Magnitude of the electric field in the x-direction.
E_x^s	Magnitude of the scattered electric field in the x-direction.
E	Matrix of transmission line parameters (defined in (3.55)).
f	Frequency.
$F(z, s)$	Vector of the distributed current and voltage source due to EMI (defined in (3.56)).

g	Conductance associated with a lumped interconnect model.
G	Conductance per unit length of a transmission line.
h	Distance between the two conductors of a transmission line.
$\mathbf{H}_\phi$	Matrix representing memoryless elements in the network ϕ .
$\mathbf{H}(s)$	Intermediate term in the incident field moment calculation (defined in (3.95)).
I_D	Distributed current source due to EMI
J	Current in a current source.
l	Inductance associated with a lumped interconnect model only in Sec. 2.2.1.
l	Length of the transmission line (everywhere except Sec. 2.2.1).
L	Inductance per unit length of a transmission line.
L_i	Coefficients of the polynomial approximation of the inductance per unit length as a function of frequency f (defined in (4.3)).
L'_i	Coefficients of the polynomial approximation of the inductance per unit length as a function of s (defined in (4.3)).
m_i	Moments of the system at the desired output node.
$\mathbf{M}_n$	Vector moment of all the nodes in the system.
N_t	Number of transmission lines in the network.
N_ϕ	Number of nodal variables due to lumped components.
$\mathbf{Q}(s)$	Intermediate term in the incident field moment calculation (defined in (3.94)).
r	Resistance associated with a lumped interconnect model only in Sec. 2.2.1.
R	Resistance per unit length of a transmission line.
R_i	Coefficients of the polynomial approximation of the resistance per unit length as a function of frequency f (defined in (4.4)).
R'_i	Coefficients of the polynomial approximation of the resistance per unit length as a function s (defined in (4.4)).

s_0	Expansion point during a CFH hop.
s	Laplace variable ($j\omega$).
$T(s)$	State transmission matrix in the transmission line stamp (defined in (3.66)).
U	Unity matrix.
$w_{c1}(t)$	Equivalent delayed time domain voltage source at the near end of the transmission line.
$w_{c2}(t)$	Equivalent delayed time domain voltage source at the far end of the transmission line.
$W_{c1}(s)$	Equivalent frequency domain voltage source at the near end of the transmission line.
$W_{c2}(s)$	Equivalent frequency domain voltage source at the far end of the transmission line.
W_ϕ	Matrix representing memory elements in the network ϕ .
W	Matrix representing memory elements in the network.
$x(t)$	Vector representing the node voltages and currents in the time domain.
$X(s)$	Vector representing the node voltages and currents (output vector) in the frequency domain.
$Y(s)$	Modified nodal admittance matrix.
Y_i	Moments of the MNA matrix.
Y_{TL}	Transmission line admittance $G + sC$.
Z_0	Characteristic impedance of a transmission line.

Abbreviations

AWE	Asymptotic waveform evaluation.
CFH	Complex frequency hopping.
EM	Electromagnetic (field).
FFT	Fast fourrier transform.
IFFT	Inverse fast fourrier transform.
IWR	Iterative waveform relaxation.
LMS	Linear multi-step.
MCM	Multi-chip module.
MMT	Moment matching techniques
MNA	Modified nodal admittance.
PCB	Printed circuit board.
RC	Resistor-capacitor (network).
RLC	Resitor-inductor-capacitor (network).

Chapter 1

Introduction

1.1 Background and motivation

Current industrial trends are moving towards more complex designs, higher operating frequencies, sharper rise times, shrinking device sizes and low power consumption. This, coupled with increasing integration of analog circuits with digital blocks, has made interconnects and packaging very important[1]. As devices are optimized and reduced in size, the proportion of the delay and signal degradation attributed to interconnects is becoming dominant. This is significantly increasing the importance of accurate interconnect analysis and have presented many challenges to circuit simulation. Interconnect effects (Fig. 1.1) such as delay, ringing and cross-talk represent critical factors in high frequency system design, and several authors have studied these effects and proposed simulation techniques [2]-[12]. Recently moment matching techniques such as Asymptotic Waveform Evaluation (AWE)[13]-[15] and Complex Frequency Hopping (CFH) [16]-[24] have proven to be efficient and reliable techniques for interconnect analysis, providing 100 to 1000 times

speed-up over conventional simulation methods. CFH is well suited for VLSI and PCB designs where a large number of interconnects has to be considered. It has been applied to networks containing lumped elements and lossy coupled transmission lines[21] with non-linear terminations[16],[24]. The objective of this thesis is to account for the frequency dependence of the transmission line parameters and analyze immunity problems due to incident field coupling to transmission lines, while still benefiting from the advantages of using moment matching techniques.

Figure 1.1: Interconnect effects

Electrically long interconnects function as spurious antennas to pick up emissions from other nearby electronic systems[27]. This makes susceptibility to emissions a major concern to current system designers of high-frequency products[28]. Hence the availability of interconnect simulation tools that include the effect of incident fields is becoming an important design requirement. Incident field coupling to transmission lines has been a problem of interest for many years. Various modeling approaches are possible, the most common one being an extension to the telegrapher's equations[29]. This approach con-

serves the quasi-TEM assumption, and models the incident field effects as distributed sources along the transmission lines. Various publications have addressed the solution of the transmission line equations in the presence of incident fields[27]-[39]. In general, current simulation techniques rely on solving the system equations at each frequency point to obtain the frequency response. Convolution is then used to obtain the time domain response of systems with nonlinear terminations. For large systems, these techniques could be very expensive in terms of CPU time and memory requirements, and in many cases suffer from numerical instability and ill-conditioning problems.

Another issue facing interconnect simulation is the frequency dependence of the transmission line parameters [40]-[46]. Distributed transmission line models have traditionally been analyzed with the assumption that the per unit length resistance, inductance, capacitance and conductance (R , L , C and G) are constant with respect to frequency. This assumption holds for the low frequency region, but as the operating frequencies increase the change in the resistance and inductance becomes significant and cannot be ignored[42],[43]. While full wave models could be used to analyze interconnects at high frequencies, in most cases it is sufficient to keep the quasi-TEM assumption, but account for the frequency dependence of R and L . Current SPICE-like simulation techniques have great difficulty handling interconnects as will be discussed in detail in Chapter 2. Those same problems would still affect the simulation of frequency dependent interconnects. The approach used in this thesis is to analyze the frequency dependent interconnects using moment matching techniques. This ensures the accuracy of the simulation in addition to reducing the CPU cost.

1.2 Contributions

The subject of this thesis is the simulation of high speed interconnects with moment matching techniques. The main focus is on the incident field coupling to transmission lines and the frequency dependence of the transmission line parameters. Specifically the main contributions are:

1. A frequency domain derivation of the method of characteristics is given that provides an insight into the theoretical reason why lossy interconnects are difficult to analyze (Sec. 2.3.3).
2. A simplified derivation of the telegrapher's equations in the presence of incident fields is provided based on what was presented in [27]. These are then put in a convenient form for analysis with moment matching techniques in the case of incident uniform plane waves (Sec. 3.2 and Sec. 3.3).
3. A technique for determining the moments of the source vector in the presence of incident fields is derived (Sec. 3.5.4).
4. A technique is developed for the determination of the moments of the output of a system based on the moments of the source vector, and those of the modified nodal admittance (MNA) matrix (Sec. 3.5.3).
5. A technique for the determination of the moments of the MNA matrix is given for the case of interconnects with frequency dependent parameters (Sec. 4.4 and Sec. 4.5).

1.3 Organization of the thesis

After the introduction in the present chapter, transmission line modelling and the difficulties they cause in circuit simulation are covered in Chapter 2. Commonly used interconnect simulation techniques are also presented in that chapter, and justifications are given for the choice of moment matching techniques in this work. In Chapter 3, the algorithm for the analysis of incident field coupling to transmission lines using moment matching techniques is developed. The frequency dependence of the transmission line parameters is introduced in Chapter 4. In addition, the derived algorithm for the simulation of frequency dependent interconnects using moment matching techniques is presented. Conclusions and proposed further research are given in Chapter 5.

Chapter 2

Interconnect simulation

2.1 Introduction

As was discussed in Chapter 1, interconnect analysis is one of the most important challenges facing circuit simulators. As in any other simulation problem, two important factors come into effect: the model used to represent the interconnect, and the solution algorithm. Depending on the desired accuracy and the frequency of operation, different models may be necessary, and each model presents different challenges for simulation. In this chapter, various interconnect models are presented, as well as two simulation algorithms. Insight is also given into the inherent difficulties of transmission line simulation in the context of nonlinear circuit simulation. The moment matching algorithm presented here is used as a basis for the research in the remainder of this thesis.

Figure 2.1: Interconnect hierarchy

2.2 Interconnect models

Interconnections can be at various levels of the design hierarchy, such as packaging structures, multichip modules (MCM), printed circuit boards (PCB), backplanes and on-chip dies (Fig. 2.1). The top view and cross section of an example interconnect system is shown in Fig. 2.2. Depending on the operating frequency and nature of the structure these interconnects can be modelled as lumped (RC or RLC), lossy coupled distributed transmission lines, full-wave sections, or measured linear subnetworks

Figure 2.2: Top view and cross-sectional view of an interconnect system

2.2.1 Lumped models

In the past, interconnect models have been generally restricted to RC tree models. RC trees are RC circuits with capacitors from all nodes to ground (no floating capacitors, and no resistors to ground). The signal delay through RC trees were often estimated using a form of the Elmore delay[47], which provided a dominant time constant approximation for monotonic step responses. However, with increasing signal speeds, and in diverse technologies such as Bipolar, BiCMOS or MCMs, RC tree models are no longer adequate. In Bipolar circuits, lumped-element interconnect models may require the use of inductors or grounded resistors, which are not compatible with RC trees. Even for MOS circuits operating at higher-frequencies, the effects of coupling capacitances may need to be included in the delay estimate. RLC circuits with non-equilibrium initial conditions may have responses which are non-monotonic. This typically results in visible signal ringing in the waveform. A single time constant approximation with Elmore delay is not generally sufficient for such circuits. Two time-constant models have been shown to improve the accuracy, but only as compared to RC tree monotone response approximations. In addition, lumped interconnect circuits extracted from layouts contain a large number of nodes

which make the simulation highly CPU intensive. Fig. 2.3 shows a general lumped component model for an interconnect, where l is the inductance, r is the resistance, g is the conductance and c is the capacitance of the interconnect.

Figure 2.3: Lumped transmission line model

2.2.2 Distributed models

At relatively higher signal-speeds, the electrical length of interconnects becomes a significant fraction of the wavelength giving rise to signal distorting effects that do not exist at lower frequencies. Consequently, the conventional lumped impedance interconnect models become inadequate and distributed transmission line models with per unit length parameters are needed (Fig. 2.4). The basic model is the simple delay-line (or lossless line). More complicated models include per-unit-length loss (either in the direction of the travelling wave or due to dielectric substrate loss), or coupling between adjacent transmission lines, where the coupling may be resistive, inductive, capacitive or a combination of these. The models may even require nonuniform per-unit-length parameters to accurately model different geometries [48].

Figure 2.4: Distributed transmission line model.

The main problem in simulating these high-frequency models lies in the fact that distributed elements, while formulated in terms of partial differential equations, are best described in the frequency-domain. Nonlinear terminations on the other hand can only be given in the time-domain. These simultaneous formulations cannot be handled by a traditional ordinary differential equation solver such as SPICE.

2.2.3 Full-wave models

At further sub-nanosecond rise times the cross-sectional dimensions of the interconnects are of the same order as the wave length. In this case, an interconnect block needs to be treated as a dispersive structure and field components in the direction of propagation can no longer be neglected[8]. Consequently, even the distributed models based on quasi-TEM approximations become inaccurate in describing the interconnect performance. In such situations full-wave models which take into account all possible field components, and satisfy all boundary conditions, are required in order to give an accurate estimation of the high-frequency effects.

Figure 2.5: Interconnect cross-section/full-wave analysis.

However, circuit simulation of full-wave models is highly involved[49]. The information that is obtained through a full-wave analysis is in terms of field parameters such as modal propagation constant, modal characteristic impedance, and current-eigenvectors etc. However, a circuit simulator requires the information in terms of currents, voltages and circuit impedances. This demands a generalized method to combine modal results into circuit simulators in terms of a full-wave stencil. Another important issue, is the CPU cost of a full-wave analysis associated with each interconnect subnetwork at each frequency point of analysis. A typical high-frequency interconnect circuit needs thousands of frequency points to get an accurate response while sweeping up to a few GHz. It would become prohibitively CPU expensive to simulate such a circuit, because of the combined cost involved (i.e, the cost of the evaluation of a full-wave model to obtain modal parameters and the cost of the computation of the circuit response at each point).

2.2.4 Measured subnetworks

In practice, it may not be possible to obtain accurate analytical models for interconnects because of the geometric inhomogeneity and associated discontinuities. To handle such situations, modelling techniques based on measured data have been proposed in the litera-

ture [50],[51]. In general, the behaviour of high-speed interconnects can easily be represented by measured frequency-dependent scattering parameters, or time-domain terminal measurements. Time-domain data could be obtained by time-domain reflectometry (TDR) measurements or electromagnetic techniques[52]. The tabulated data, regardless of how they are obtained, are considered as measured data. One important factor to note, is that the subnetwork may be characterized by large sets of time-domain data, making the system over-determined. This kind of measurement data in a large number of sets is essential in a practical environment as the measurements are usually contaminated by noise and the use of only a single set of measurements may lead to inaccurate results. Including all available sets of measurements in the simulation helps in reducing the impact of noise on the network response. However, handling such over-determined situations in circuit simulation is a tedious and CPU expensive process.

2.3 Interconnect simulation

Simulation of interconnects is associated with two major bottlenecks. First, these networks consist of hundreds or thousands of components, such as resistors, capacitors, inductors, transmission lines and other levels of interconnect models. At the terminations there generally exist nonlinear elements such as drivers and receivers. If only lumped RLC models are considered, ordinary differential equation solvers such as SPICE may be used for simulation purposes. However, the CPU cost would be large, owing to the fact that SPICE is mainly a nonlinear simulator and does not handle large RLC networks efficiently (at least one L/U decomposition is required at every time step as discussed in Sec. 2.3.1). The second major difficulty is the mixed frequency time domain problem that arises when distributed elements are to be analysed (see Sec. 2.3.2). These two difficulties lead to intense research in the CAD area during recent years to develop efficient tech-

niques for obtaining transient solutions involving interconnects for accurate signal integrity analysis.

2.3.1 Background on circuit simulation

Before addressing the problems of interconnect simulation, it would be useful to provide a brief summary of circuit simulation with lumped elements. Whether in the frequency or time domain, the first step is to set up the Modified Nodal Admittance matrix (MNA) [53] which is a set of simultaneous equations representing the network. For example consider the small circuit in Fig. 2.6. Its MNA equations are

$$\left(\begin{bmatrix} G_1 & 0 & 1 \\ 0 & G_2 & -1 \\ 1 & -1 & 0 \end{bmatrix} + s \begin{bmatrix} 0 & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & -L \end{bmatrix} \right) \mathbf{X}(s) = \begin{bmatrix} J(s) \\ 0 \\ 0 \end{bmatrix} \quad (2.1)$$

The above equation representing a very simple 2 node circuit, has the same form as any MNA matrix representing a lumped linear circuit which is normally in the form of a very large matrix equation

$$[\mathbf{W} + s\mathbf{H}]\mathbf{X}(s) = \mathbf{e}(s) \quad (2.2)$$

which can also be written in the time domain

$$\mathbf{W}\mathbf{x}(t) + \mathbf{H}\dot{\mathbf{x}}(t) = \mathbf{e}(t) \quad (2.3)$$

Figure 2.6: Simple linear circuit

Frequency domain simulation is conventionally done by solving (2.2) at each frequency point through L/U decomposition and forward backward substitution. For time domain simulation, difference equations based on the Linear Multi-Step (LMS)[54] techniques are used. The most common of these integration formulae is the trapezoidal rule which yields the following difference formula

$$\mathbf{x}(t + \Delta t) = \mathbf{x}(t) + \Delta t \frac{(\dot{\mathbf{x}}(t) + \dot{\mathbf{x}}(t + \Delta t))}{2} \quad (2.4)$$

$$\left[\mathbf{W} + \frac{2}{\Delta t} \mathbf{H} \right] \mathbf{x}(t + \Delta t) = \left[\frac{2}{\Delta t} \mathbf{H} - \mathbf{W} \right] \mathbf{x}(t) + [\mathbf{e}(t) + \mathbf{e}(t + \Delta t)] \quad (2.5)$$

Note that the trapezoidal rule is an LMS method of order 2, which is the highest possible order that could insure absolute stability [55]. Being limited to such a low order, forces the simulators to use small step sizes in order to ensure accuracy. Obtaining the transient response requires the solution of (2.5) at every time step. In this case only one L/U decomposition is needed unless a change in the time-step is required. SPICE, being a nonlinear simulator, does not take advantage of this fact and performs at least one L/U decomposi-

tion at each time-step. This is due to the fact that very few circuits are purely linear. In fact in most cases the circuit consists of a large linear network connected to highly nonlinear drivers and receivers. The MNA equations of such nonlinear networks, assuming only lumped components is of the form

$$\mathbf{W}\mathbf{x}(t) + \mathbf{H}(\dot{\mathbf{x}}(t)) + \Phi(\mathbf{x}) = \mathbf{e}(t) \quad (2.6)$$

where $\Phi(\mathbf{x})$ is a nonlinear function of $\mathbf{x}$. The difference equation based on the trapezoidal rule therefore becomes

$$\begin{aligned} \left[\mathbf{W} + \frac{2}{\Delta t} \mathbf{H} \right] \mathbf{x}(t + \Delta t) + \Phi(\mathbf{x}(t + \Delta t)) \\ = \left[\frac{2}{h} \mathbf{H} - \mathbf{W} \right] \mathbf{x}(t) + (\mathbf{e}(t) + \mathbf{e}(t + \Delta t)) - \Phi(\mathbf{x}(t)) \end{aligned} \quad (2.7)$$

In the case of nonlinear elements Newton iteration is used to solve (2.7) at each time step, which requires at least one L/U decomposition. The $\mathbf{W}$ and $\mathbf{H}$ matrices are usually very large. Also the number of L/U decompositions is very high due to the low order of the integration formula. This causes the CPU cost of a time domain analysis to become prohibitive. When distributed transmission lines are to be analyzed as well, the situation is even worse as will be seen in the following sections.

2.3.2 Mixed frequency/time domain problem

Analysing interconnects is one of the most important challenges facing current commercial simulators. The problems related to interconnect simulation could appear in various forms depending on the approach used for the solution, but they are all based on one fundamental issue. The MNA equations of the lumped section of the nonlinear network can only be expressed in the time domain as in (2.7). The transmission lines however, are characterized by partial differential equations whose solution is best expressed in the fre-

quency domain. All the methods proposed in the literature for transmission line simulation have attempted with varying success to solve this system of simultaneous equations which comprise both frequency and time domain equations. Two such techniques will be discussed in the following sections.

2.3.3 Method of characteristics

The method of characteristics was developed by Branin[56] in 1967. It transforms the partial differential equations of a transmission line into ordinary differential equations. Some authors have proposed extensions to this method to allow it to handle lossy transmission lines, for example the iterative waveform relaxation techniques (IWR) introduced by Chang[5]. The method of characteristics remains however most practical for lossless lines for reasons that will become clear in this section.

Transmission lines are characterized by partial differential equations in space and time (the Telegrapher's equations). These equations are derived for the case of incident field coupling to transmission lines in Chapter 3. The ordinary form of the Telegrapher's equations will be stated here without incident field coupling

$$\begin{aligned}\frac{\partial}{\partial z}V(z) &= -(R + sL)I(z) \\ \frac{\partial}{\partial z}I(z) &= -(G + sC)V(z)\end{aligned}\tag{2.8}$$

where R , L , C and G are the per unit length resistance, inductance, capacitance and conductance respectively, and z is the spatial variable in the direction of propagation. An analytical solution for (2.8) was derived in [57] for two conductor lines which provides the Y parameters of the two port transmission line network

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{Z_0(1 - e^{-2\gamma l})} \begin{bmatrix} 1 + e^{-2\gamma l} & -2e^{-\gamma l} \\ -2e^{-\gamma l} & 1 + e^{-2\gamma l} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (2.9)$$

where γ is the propagation constant on the line, and Z_0 is the characteristic impedance.

V_1 and I_1 are the terminal voltage and current at the near end of the line, and V_2 and I_2 are the terminal voltage and current at the far end of the line as shown in Fig. 2.7.

Figure 2.7: Y parameters of transmission line network

The Y parameters of the transmission line, are complex functions of s , and in most cases cannot be directly transformed into an ordinary differential equation in the time domain. The method of characteristics succeeded in doing such a transformation, but only for lossless transmission lines. Although this method was originally developed in the time domain [56] using what was referred to as characteristic curves (hence the name), a short alternative derivation in the frequency domain will be presented here. The frequency domain approach gives more insight as to the limitations of this technique and possible solutions to those limitations.

By re-arranging the terms in (2.9) the following expressions are obtained

$$\begin{aligned} V_1 &= Z_0 I_1 + e^{-\gamma l} [2V_2 - e^{-\gamma l} (Z_0 I_1 + V_1)] \\ V_2 &= Z_0 I_2 + e^{-\gamma l} [2V_1 - e^{-\gamma l} (Z_0 I_2 + V_2)] \end{aligned} \quad (2.10)$$

W_{c1} and W_{c2} are then defined as

$$\begin{aligned} W_{c1} &= e^{-\gamma l} [2V_2 - e^{-\gamma l} (Z_0 I_1 + V_1)] \\ W_{c2} &= e^{-\gamma l} [2V_1 - e^{-\gamma l} (Z_0 I_2 + V_2)] \end{aligned} \quad (2.11)$$

which reduces (2.10) to

$$\begin{aligned} V_1 &= Z_0 I_1 + W_{c1} \\ V_2 &= Z_0 I_2 + W_{c2} \end{aligned} \quad (2.12)$$

Now by substituting the terms $Z_0 I_1$ and $Z_0 I_2$ in (2.11) with their values from (2.12) we obtain,

$$\begin{aligned} W_{c1} &= e^{-\gamma l} [2V_2 - e^{-\gamma l} (2V_1 - W_{c1})] \\ W_{c2} &= e^{-\gamma l} [2V_1 - e^{-\gamma l} (2V_2 - W_{c2})] \end{aligned} \quad (2.13)$$

Noting the symmetry in (2.13), allows for a convenient recursive expressions for W_{c1} and W_{c2} ,

$$\begin{aligned} W_{c1} &= e^{-\gamma l} [2V_2 - W_{c2}] \\ W_{c2} &= e^{-\gamma l} [2V_1 - W_{c1}] \end{aligned} \quad (2.14)$$

A lumped model of the transmission line can then be deduced from (2.12) and (2.14), as shown in Fig. 2.8. The expressions of W_{c1} and W_{c2} are still however in the frequency domain. A time domain expression is needed in order to mix the time domain nonlinear equations of the lumped elements with the transmission line stamp. This is where the lossless line restriction has to be imposed. For the case of lossless lines, the propagation constant is purely imaginary and the inverse Laplace of (2.14) can be taken analytically by replacing $e^{-j\beta}$ with a time shift (or delay)

$$\begin{aligned} w_{c1}(t + \tau) &= 2v_2(t) - w_{c2}(t) \\ w_{c2}(t + \tau) &= 2v_1(t) - w_{c1}(t) \end{aligned} \quad (2.15)$$

Each transmission line can therefore be modeled by two impedances and two voltage controlled voltage sources with time delay. While this transmission line model is in the time domain and can be easily linked to time domain circuit simulation, the time shift affects the stability of the integration formula and causes the step size to significantly decrease therefore increasing CPU time. For lossy lines, the propagation constant is not purely imaginary and can therefore not be replaced by a pure delay. In that case analytical expressions for w_{c1} and w_{c2} cannot be found in the time domain although some numerical techniques were proposed such as the iterative waveform relaxation techniques (IWR) [5][41]. Transmission line simulation has been therefore generally limited to lossless lines, and suffer from excessive CPU requirements due to the size of the MNA matrix and the large number of time steps required.

Figure 2.8: Method of Characteristics interconnect model

2.3.4 Interconnect simulation through moment matching

Recently, several powerful interconnect simulation algorithms were developed based on moment-matching and Padé approximation. A more detailed discussion of these techniques can be found in Chapter 3, as this concept was used in the simulation of incident field coupling to transmission lines. In this section we will focus on the contribution of these techniques to efficient and accurate lossy coupled transmission line simulation.

Moment matching techniques could be classified under the banner of order reduction techniques. A very large system containing a large number of poles is reduced to a system with a fraction of the number of poles in the original system, but behaving exactly like the original up to a user specified highest frequency of interest. CPU savings of up to a thousand times faster than conventional techniques are possible as the need to handle large matrices is significantly reduced. While being a frequency domain technique, moment matching bridges the gap to time domain nonlinear simulation[24]. In fact, because it is a frequency domain technique, moment matching is a natural way of simulating transmission lines which are best characterized in the frequency domain. The link to nonlinear simulation comes from the fact that the output of moment matching techniques is, instead of a simple frequency response, a set of poles and residues representing the linear system. These poles and residues can then be used to analytically obtain a time domain state space representation which SPICE can easily simulate. This technique is practical because usually the nonlinear elements are drivers and receivers and their number is very small compared to the total number of elements in the system. Consider for example the system in Fig. 2.9. The linear section is considered as a multi-port network and analyzed using MMTs. The poles and residues obtained are used to generate a time-domain state space representation which is easily linked to time domain simulators.

Figure 2.9: Macromodelling with MMTs

Moment matching techniques significantly reduce the CPU time of large simulations, while at the same time providing a convenient solution to the mixed frequency/time domain problem encountered in interconnect analysis. The only disadvantage of MMTs is the complexity of the implementation as will be apparent from later discussions in this thesis. However, this results in a significant reduction in the computational complexity and CPU time.

Chapter 3

Lines Excited by Incident Fields

3.1 Introduction

Electrically long interconnects function as spurious antennas to pick up emissions from other nearby electronic systems[27]. This makes susceptibility to emissions a major concern to current system designers of high-frequency products[28]. Hence the availability of interconnect simulation tools including the effect of incident fields is becoming an important design requirement.

This chapter presents a technique for the simulation of transmission lines excited by incident fields. Based on complex frequency hopping (CFH) [21], the proposed method[61],[62] takes full advantage of the benefits moment matching techniques outlined in the previous chapter. The transmission line model derived from the modified telegrapher's equations is used to account for the incident fields. A generalized stamp that includes EMI effects in a form suitable for moment matching techniques is derived, as well as a new

moment-generation algorithm for transmission lines excited by incident fields. The proposed technique provides a speed-up of one to two orders of magnitudes over conventional methods while maintaining good accuracy.

The transmission line equations are first derived in Sec. 3.2. The result is a set of inhomogeneous partial differential equations similar to the Telegrapher's equations but with additional distributed sources. In Sec. 3.3 incident plane waves are analyzed and the distributed sources due to EMI are expressed in a convenient form for use with moment matching techniques. A solution of the transmission line equations in the presence of incident plane waves is presented in Sec. 3.4, thus providing a stamp for such transmission lines. Sec. 3.5 presents the complex frequency hopping (CFH) technique and develops the necessary algorithms to allow the analysis of the interconnect stamp developed in Sec. 3.4.

3.2 Transmission line equations

The method employed in this thesis for modelling incident field coupling to transmission lines is to express this coupling as distributed sources along the line. This approach was first introduced by Taylor[29] and later expressed in more convenient forms and generalized to multi-conductor lines by various authors[30],[31],[32]. In this section, a derivation of the transmission line equations in the presence of incident fields is shown. The effect of the incident field will appear as distributed sources added to the Telegrapher's equations. In order to improve the clarity of the presentation the derivation in this section is for the case of two conductor lines, but as shown in [32], this approach is general for lossy coupled multi-conductor lines.

3.2.1 Derivation of the first Telegrapher's equation

Consider a transmission line illuminated by an incident EM field (Fig. 3.1). Applying Faraday's law

$$\nabla \times \vec{E} = -j\omega \vec{B} \quad (3.1)$$

where $\vec{E}$ and $\vec{B}$ are the phasors of the total electric and magnetic fields respectively. Note that these include both incident and scattered fields. Stoke's theorem is then applied to (3.1),

$$\oint_C \vec{E} \cdot d\vec{l} = \int_S (\nabla \times \vec{E}) \cdot d\vec{s} = -j\omega \int_S \vec{B} \cdot d\vec{s} \quad (3.2)$$

Figure 3.1: Interconnect excited by an arbitrary incident field

Expanding the integral in (3.2) and noting that the integration is confined to the x-z plane

$$\int_0^h E_x(x, z) dx + \int_z^{(z+\Delta z)} E_z(h, z) dz - \int_0^h E_x(x, z + \Delta z) dx - \int_z^{(z+\Delta z)} E_z(0, z) dz = -j\omega \int_S \vec{B}^i \cdot d\vec{s} - j\omega \int_S \vec{B}^s \cdot d\vec{s} \quad (3.3)$$

Note that the magnetic field $\vec{B}$ has been separated into its incident and scattered components $\vec{B}^i$ and $\vec{B}^s$. Now recalling the definition of voltage between two points,

$$V(z) = -\int_0^h E_x(x, z) dx \quad (3.4)$$

$$V(z + \Delta z) = -\int_0^h E_x(x, z + \Delta z) dx \quad (3.5)$$

and using the definition of per unit length inductance (L)[27] of the line in terms of the line current and the flux (Ψ) of the scattered magnetic field through the inductance loop around the surface S

$$L\Delta z = \frac{\Psi}{I} = \frac{\int_S \vec{B}^s \cdot d\vec{s}}{I} \quad (3.6)$$

equation (3.3) can be re-written in the form

$$[V(z + \Delta z) - V(z)] + \left[\int_z^{(z+\Delta z)} E_z(h, z) dz - \int_z^{(z+\Delta z)} E_z(0, z) dz \right] = -j\omega L\Delta z I - j\omega \int_S \vec{B}^i \cdot d\vec{s} \quad (3.7)$$

Equation (3.7) is then divided by Δz and the limit is taken as Δz approaches zero. By invoking the fundamental theorem of calculus and the mean value theorem we obtain

$$\lim_{\Delta z \rightarrow 0} \frac{-\int_S \vec{B}^i \cdot d\vec{s}}{\Delta z} = \lim_{\Delta z \rightarrow 0} \frac{\int_z^{(z+\Delta z)} \int_0^h B_y^i(x, z) dx dz}{\Delta z} = \int_0^h B_y^i(x, z) dx \quad (3.8)$$

$$\frac{\partial}{\partial z}V(z) + sLI + [E_z(h, z) - E_z(0, z)] = s \int_0^h B_y^i(x, z) dx \quad (3.9)$$

Now we note that the voltage drop along the line due to conductor losses is

$$\Delta z E_z = I \frac{R \Delta z}{2} \quad (3.10)$$

where $R/2$ is the per-unit length resistance of each of the two conductors. The current in the signal conductor is assumed to be equal to, and opposite in direction to the current in the reference conductor. This is equivalent to considering only differential mode currents and neglecting common mode currents. Neglecting common mode current is a valid assumption in this case because we are only interested in the final result at the termination where only differential mode current flows (this can be easily verified by applying the charge continuity equation). Based on this assumption

$$E_z(h, z) - E_z(0, z) = I \frac{R}{2} + I \frac{R}{2} = IR \quad (3.11)$$

Where R is the per unit length resistance of the transmission line. The final form of the first Telegrapher's equation in the presence of EMI is therefore

$$\frac{\partial}{\partial z}V(z) + (sL + R)I = s \int_0^h B_y^i(x, z) dx \quad (3.12)$$

3.2.2 Derivation of the second telegrapher's equation

In order to obtain the second Telegrapher's equation, it is more convenient to use the point form of Maxwell's equations. Ampere's law is applied to the surface in between the conductors, in order to relate the scattered electric and magnetic fields.

$$\nabla \times \vec{H}^s = \sigma \vec{E}^s + j\omega\epsilon \vec{E}^s \quad (3.13)$$

We now equate the components of (3.13) in the x direction, and recall the assumption that the scattered magnetic field is transverse and has no components in the z direction (i.e the current is only flowing in the z direction)

$$\frac{\partial}{\partial z} H_y^s = -\sigma E_x^s - s\epsilon E_x^s \quad (3.14)$$

Integrating (3.14) in the transverse plane from 0 to h

$$\frac{\partial}{\partial z} \int_0^h H_y^s dx = -\sigma \int_0^h E_x^s dx - s\epsilon \int_0^h E_x^s dx \quad (3.15)$$

The left hand side of (3.15) can be related to the current on the line and the per unit length inductance using (3.6) and (3.8). The right hand side is obtained by using the definition of the scattered voltage

$$\frac{L}{\mu} \frac{\partial}{\partial z} I(z, s) = -\sigma V^s(z, s) - \epsilon s V^s(z, s) \quad (3.16)$$

Now recalling the identities relating the per-unit length parameters in homogeneous media[27]

$$LC = \mu\epsilon \quad (3.17)$$

$$GL = \sigma\mu \quad (3.18)$$

The second Telegrapher's equation can be written in terms of the scattered voltage

$$\frac{\partial}{\partial z} I(z, s) = -GV^s(z, s) - sCV^s(z, s) = -Y_{TL}V^s(z, s) \quad (3.19)$$

where Y_{TL} is the transmission line admittance

$$Y_{TL} = G + sC \quad (3.20)$$

The Telegrapher's equations in terms of the total voltage are needed however, in order to account for terminal conditions, and conveniently link to the other nodes of the network. The total voltage is the voltage due to both the scattered electric field, and the incident electric field.

$$V(z) = -\int_0^h E_x^s(z, s) dx - \int_0^h E_x^i(z, s) = V^s(z, s) - \int_0^h E_x^i(z, s) \quad (3.21)$$

By applying the definition of the total voltage in (3.21) to (3.19) The second Telegrapher's equation in terms of the total voltage and current is obtained

$$\frac{\partial}{\partial z} I(z, s) + (G + sC)V(z, s) = -Y_{TL} \int_0^h E_x^i(z, s) dx \quad (3.22)$$

3.2.3 A convenient form of the Telegrapher's equations

The transmission line equations derived in Sec. 3.2.1 and Sec. 3.2.2 in the presence of incident fields are

$$\frac{\partial}{\partial z} V(z, s) + (sL + R)I(z, s) = s \int_0^h B_y^i(x, l) dx \quad (3.23)$$

$$\frac{\partial}{\partial z} I(z, s) + (G + sC)V(z, s) = -Y_{TL} \int_0^h E_x^i(z, s) \quad (3.24)$$

The additional sources shown on the right hand side of the telegrapher's equations are expressed in terms of the incident electric and magnetic fields. A more convenient expression of those sources in terms of the electric field only can be derived. Applying Faraday's law for the incident fields around the contour of the surface S in Fig. 3.1 as was done in (3.3)

$$\begin{aligned}
& \int_0^h E_x^i(x, z) dx + \int_z^{(z+\Delta z)} E_z^i(h, z) dz - \int_0^h E_x^i(x, z + \Delta z) dx - \int_z^{(z+\Delta z)} E_z^i(0, z) dz = \\
& -j\omega \int_S \vec{B}^i \cdot d\vec{s}
\end{aligned} \tag{3.25}$$

By dividing by Δz and allowing it to approach zero, a similar technique to the one used to determine the identities in (3.7) and (3.8) can be used to deduce

$$s \int_0^h B_y^i(x, z, s) dx = -\frac{\partial}{\partial z} \int_0^h E_x^i(x, z, s) dx + [E_z^i(h, z, s) - E_z^i(0, z, s)] \tag{3.26}$$

The transmission line equations in the presence of electromagnetic interference can therefore be expressed in terms of the total voltage and current and the incident electric field

$$\frac{\partial}{\partial z} V(z, s) + (sL + R)I(z, s) = V_D \tag{3.27}$$

$$\frac{\partial}{\partial z} I(z, s) + (G + sC)V(z, s) = I_D \tag{3.28}$$

where

$$V_D = -\frac{\partial}{\partial z} \int_0^h E_x^i(x, z, s) dx + [E_z^i(h, z, s) - E_z^i(0, z, s)] \tag{3.29}$$

$$I_D = -Y_{TL} \int_0^h E_x^i(x, z, s) dx \tag{3.30}$$

3.2.4 Multiconductor transmission lines

While the derivation of the coupling in the previous sections was shown for a two conductor line, the results were proven to be general for multiconductor transmission lines[32] such as the ones in Fig. 3.2. The general form of equations (3.27) to (3.30) for an n conductor transmission line is therefore

$$\frac{\partial}{\partial z} V(z, s) + (sL + R)I(z, s) = V_D \quad (3.31)$$

$$\frac{\partial}{\partial z} I(z, s) + (G + sC)V(z, s) = I_D \quad (3.32)$$

where R , L , C and G are $n \times n$ per unit length resistance, inductance, capacitance and conductance matrices, and

$$V_D = \begin{bmatrix} \dots \\ -\frac{\partial}{\partial z} \int_{\text{reference cond}}^{\text{conductor } i} \vec{E}_t^i d\vec{l} + [E_z^i(i^{th} \text{ conductor}, z, s) - E_z^i(\text{reference cond.}, z, s)] \\ \dots \end{bmatrix} \quad (3.33)$$

$$I_D = \begin{bmatrix} \dots \\ -Y_{TL} \int_{\text{reference cond}}^{\text{conductor } i} \vec{E}_t^i(\vec{z}) d\vec{l} \\ \dots \end{bmatrix} \quad (3.34)$$

where $\vec{E}_t^i$ is the component of the incident field in the transverse plan (x-y plane in Fig. 3.2), and E_z is the value of the incident electric field in the direction of the propagation along the line (z-direction).

Figure 3.2: Multiconductor lines.

3.3 Interference by incident plane waves

For many practical cases, the distance between the interferer and the tested system is large compared to the wave length of the incident signal. In such cases the analysed transmission lines are situated in the far field of a spherical incident field which could locally be approximated by a uniform plane wave. In the following section the transmission line equations will be analysed for the case of uniform plane waves, and their solution put in a form that is compatible with simulators based on moment matching techniques.

A frequency domain representation of an incident uniform plane wave is given by[63]

$$\vec{E}^i = E_o(e_x\hat{a}_x + e_y\hat{a}_y + e_z\hat{a}_z)e^{-j\beta_x x}e^{-j\beta_y y}e^{-j\beta_z z} \quad (3.35)$$

where

$$e_x = \sin\theta_E \sin\theta_p \quad (3.36)$$

$$e_y = -\sin\theta_E \cos\theta_p \cos\phi_p - \cos\theta_E \sin\phi_p \quad (3.37)$$

$$e_z = -\sin\theta_E \cos\theta_p \sin\phi_p + \cos\theta_E \cos\phi_p \quad (3.38)$$

$$\beta_x = -\beta \cos\theta_p \quad (3.39)$$

$$\beta_y = -\beta \sin\theta_p \cos\phi_p \quad (3.40)$$

$$\beta_z = -\beta \sin\theta_p \sin\phi_p \quad (3.41)$$

A closed form formulation of the distributed sources current and voltage sources defined in (3.30) and (3.29) is derived in Sec. 3.3.1 and Sec. 3.3.2 respectively for the case of incident plane wave excitation. The Telegrapher's equations are then expressed in a convenient matrix representation in Sec. 3.2.3.

Figure 3.3: Uniform plane wave

3.3.1 Distributed current sources

The distributed current sources are obtained by applying the definition in (3.30) and integrating the incident electric field between the two conductors

$$I_D = -Y_{TL} \int_0^h E_x^i(z, s) dx \quad (3.42)$$

where E_x^i , the component of the incident field in the x direction becomes, in the case of incident plane wave excitation

$$E_x^i = E_o e_x (e^{-j\beta_x x} e^{-j\beta_y y} e^{-j\beta_z z}) \quad (3.43)$$

Integrating (3.42) in the $y=0$ plane

$$I_D = -Y_{TL} \int_0^h E_o e_x (e^{-j\beta_x x} e^{-j\beta_z z}) dx \quad (3.44)$$

$$\begin{aligned} I_D &= -jY_{TL} E_o e_x e^{-j\beta_z z} \left[\frac{e^{-j\beta_x x}}{\beta_x} \right]_{x=0}^{x=h} \\ &= -jY_{TL} E_o e_x e^{-j\beta_z z} \left[\frac{e^{-j\beta_x h} - 1}{\beta_x} \right] \\ &= -jY_{TL} E_o e_x e^{-j\beta_z z} \left[\frac{\sum_{n=1}^{\infty} \frac{(-j\beta_x h)^n}{n!}}{\beta_x} \right] \\ &= -Y_{TL} E_o e_x e^{-j\beta_z z} \sum_{n=1}^{\infty} \frac{(-j\beta_x)^{n-1} h^n}{n!} \end{aligned} \quad (3.45)$$

The propagation constant β in the above formulation can be replaced by the Laplace variable s by noting that

$$\beta_x = -\beta \cos \theta_p = -\omega \sqrt{\epsilon \mu} \cos \theta_p \quad (3.46)$$

$$-j\beta_x = s \sqrt{\epsilon \mu} \cos \theta_p \quad (3.47)$$

and similarly

$$-j\beta_z = s \sqrt{\epsilon \mu} \sin \theta_p \sin \phi_p \quad (3.48)$$

Therefore the distributed current along the line can be expressed as

$$I_D = -Y_{TL} E_o e_x e^{zs \sqrt{\epsilon \mu} \sin \theta_p \sin \phi_p} \sum_{n=0}^{\infty} \frac{h^{n+1} (\sqrt{\epsilon \mu} \cos \theta_p)^n}{(n+1)!} s^n \quad (3.49)$$

3.3.2 Distributed voltage source

The distributed voltage sources are obtained by applying (3.29)

$$V_D = -\frac{\partial}{\partial z} \int_0^h E_x^i(x, z, s) dx + [E_z^i(h, z, s) - E_z^i(0, z, s)] \quad (3.50)$$

The first term of the above equation is evaluated by following the same procedure as in Sec. 3.3.1 and differentiating with respect to z ,

$$\begin{aligned} V_{D1} &= -\frac{\partial}{\partial z} \int_0^h E_x^i(x, z, s) dx \\ &= -\sqrt{\epsilon \mu} \sin \theta_p \sin \phi_p \mathcal{E}_o e_x e^{zs \sqrt{\epsilon \mu} \sin \theta_p \sin \phi_p} \sum_{n=1}^{\infty} \frac{h^n (\sqrt{\epsilon \mu} \cos \theta_p)^{n-1}}{n!} s^n \end{aligned} \quad (3.51)$$

The second term is simply expressed as

$$\begin{aligned}
V_{D2} &= E_z^i(h, z, s) - E_z^i(0, z, s) \\
&= E_o e_z e^{-j\beta_z z} (e^{-j\beta_x h} - 1) \\
&= E_o e_z e^{-j\beta_z z} \sum_{n=1}^{\infty} \frac{(-j\beta_x h)^n}{n!} \\
&= E_o e_z e^{s\sqrt{\epsilon\mu} \sin\theta_p \sin\phi_p z} \sum_{n=1}^{\infty} \frac{(\sqrt{\epsilon\mu} \cos\theta_p h)^n s^n}{n!}
\end{aligned} \tag{3.52}$$

Finally the distributed voltage due to incident plane wave coupling is

$$\begin{aligned}
V_D &= V_{D1} + V_{D2} \\
&= E_o e^{zs\sqrt{\epsilon\mu} \sin\theta_p \sin\phi_p} \sum_{n=1}^{\infty} \frac{e_z (\sqrt{\epsilon\mu} \cos\theta_p)^n - e_x \sqrt{\epsilon\mu} \sin\theta_p \sin\phi_p (\sqrt{\epsilon\mu} \cos\theta_p)^{n-1}}{n!} h^n s^n
\end{aligned} \tag{3.53}$$

3.3.3 Telegrapher's equations in for incident plane waves

Drawing on the results of the previous two sections, we can now express the telegrapher's equations in the presence of incident plane waves in a convenient matrix form

$$\frac{d}{dz} \begin{bmatrix} V(z, s) \\ I(z, s) \end{bmatrix} = -(\mathbf{D} + s\mathbf{E}) \begin{bmatrix} V(z, s) \\ I(z, s) \end{bmatrix} + \mathbf{F}(z, s) \tag{3.54}$$

where

$$\mathbf{D} = \begin{bmatrix} 0 & R \\ G & 0 \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} 0 & L \\ C & 0 \end{bmatrix} \tag{3.55}$$

and

$$\mathbf{F}(z, s) = E_o \Gamma(s) e^{zs\sqrt{\epsilon\mu} \sin\theta_p \sin\phi_p} \tag{3.56}$$

were

$$\Gamma(s) = \left[\begin{aligned} & \sum_{n=1}^{\infty} \frac{e_z(\sqrt{\epsilon\mu}\cos\theta_p)^n - e_x\sqrt{\epsilon\mu}\sin\theta_p\sin\phi_p(\sqrt{\epsilon\mu}\cos\theta_p)^{n-1}}{n!} h^n s^n \\ & - e_x Y_{TL} \sum_{n=0}^{\infty} \frac{h^{n+1}(\sqrt{\epsilon\mu}\cos\theta_p)^n}{(n+1)!} s^n \end{aligned} \right] \quad (3.57)$$

The expression in (3.56) is convenient because the dependence on z was isolated in the exponential, and the function $\Gamma(s)$ is a pure function of s whose moments are easily evaluated. This formulation was derived for a two conductor line, but as was shown in Sec. 3.2.4 the same technique is applicable to multiconductor lines. Additional complexity is added to the process since the lines are generally not in the same plane. In this case the integration becomes a 2-dimensional problem and the phase difference of the incident wave between the conductors has to be accounted for.

3.4 Transmission line stamp

The multiconductor transmission line (MTL) stamp is derived from the solution of Telegrapher's equations. In this section, the solution of the transmission line equations including distributed sources due to incident fields is presented. It is then put in a form suitable for moment matching techniques. Consider the telegrapher's equations defined in (3.54). They can be written in the form

$$\frac{d}{dz} \mathbf{x}(s, z) = -\boldsymbol{\varphi}(s) \mathbf{x}(s, z) + \mathbf{F}(z, s) \quad (3.58)$$

where

$$\mathbf{x}(s, z) = \begin{bmatrix} \mathbf{V}(z, s) \\ \mathbf{I}(z, s) \end{bmatrix} \quad (3.59)$$

and

$$\varphi(s) = \mathbf{D} + s\mathbf{E} \quad (3.60)$$

Due to the addition of the distributed sources $\mathbf{F}(z, s)$, equation (3.58) is an inhomogeneous first order differential equation in z . Its solution can be found from the sum of the homogeneous solution and the particular solution. The homogeneous equation is

$$\frac{\partial}{\partial z} \mathbf{x}(z, s) + \varphi(s) \mathbf{x}(z, s) = 0 \quad (3.61)$$

for which the solution can be simply as

$$\mathbf{x}(z, s) = e^{-(z-z_0)\varphi(s)} \mathbf{x}(z_0, s) \quad (3.62)$$

The homogeneous solution in (3.62) is often referred to as the state transmission matrix (STM) in linear systems literature. The particular solution of (3.58) can then be found using the method of variation of parameters

$$\mathbf{x}(z, s) = \int_{z_0}^z e^{(z-\rho)\varphi(s)} \mathbf{F}(s, \rho) d\rho \quad (3.63)$$

The final solution for the inhomogeneous Telegrapher's equations is the sum of the homogeneous solution (STM) and the particular solution

$$\mathbf{x}(z, s) = e^{-(z-z_0)\varphi(s)} \mathbf{x}(z_0, s) + \int_{z_0}^z e^{(z-\rho)\varphi(s)} \mathbf{F}(s, \rho) d\rho \quad (3.64)$$

If the near end of the interconnect is taken to be at the origin of the axis ($z_0 = 0$) and a transmission line of length l ($z = l$) is considered, the near and far end voltage and current can be related by

$$\begin{bmatrix} \mathbf{V}(l) \\ \mathbf{I}(l) \end{bmatrix} = \mathbf{T}(s) \begin{bmatrix} \mathbf{V}(0) \\ \mathbf{I}(0) \end{bmatrix} + \mathbf{b}(s) \quad (3.65)$$

where the STM is given by,

$$\mathbf{T}(s) = e^{-\Phi l} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \quad (3.66)$$

$$\mathbf{b}(s) = e^{-\Phi l} \int_0^l e^{\Phi z} \mathbf{F}(z, s) dz \quad (3.67)$$

The transmission line stamp for the case when incident fields are present therefore becomes

$$\mathbf{A} \begin{bmatrix} V(0) \\ V(l) \end{bmatrix} + \mathbf{B} \begin{bmatrix} I(0) \\ I(l) \end{bmatrix} = \mathbf{b}(s) \quad (3.68)$$

where

$$\mathbf{A} = \begin{bmatrix} -T_{11} & 1 \\ T_{21} & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} T_{12} & 0 \\ -T_{22} & 1 \end{bmatrix} \quad (3.69)$$

3.5 CFH in the presence of EMI

Moment matching is a model reduction technique whereby the Taylor series expansion of the network equations is used to generate, a low-order transfer function approximation. Complex frequency hopping (CFH) extends the process to multiple expansion points in the complex plane near or on the imaginary axis using a binary search algorithm. With a minimized number of frequency point expansions, enough information is obtained to enable the generation of an approximate transfer function that matches the original function up to a user defined highest frequency of interest. In this section an algorithm is presented that would allow the use of CFH for the analysis of incident field coupling to transmission lines.

3.5.1 Formulation of the network equations

Consider a linear network ϕ which contain linear lumped components and N_t lossy transmission lines excited by incident fields. Assume the network ϕ has N_ϕ nodal variables. In the case of incident field excitation, the modified nodal admittance matrix (MNA) matrix equations for the network with impulse input excitation can be formulated as,

$$Y(s)X(s) = \Psi(s) \quad (3.70)$$

or

$$\begin{bmatrix} (H_\phi + sW_\phi) & D_1 & \dots & D_{N_t} \\ A_1 D_1^t & B_1 & 0 & 0 \\ \dots & 0 & \dots & 0 \\ A_{N_t} D_{N_t}^t & 0 & 0 & B_{N_t} \end{bmatrix} \begin{bmatrix} V(s) \\ I_1(s) \\ \dots \\ I_{N_t}(s) \end{bmatrix} = \begin{bmatrix} e_\phi(s) \\ b_1(s) \\ \dots \\ b_{N_t}(s) \end{bmatrix} \quad (3.71)$$

Where

$V(s) \in \mathcal{R}^{N_\phi}$ is the vector of the N_ϕ variables describing the subnetwork ϕ which includes node voltage waveforms appended by independent voltage source current, linear inductor current, nonlinear capacitor charge and nonlinear inductor flux waveforms,

H_ϕ, W_ϕ are constant matrices describing the lumped memoryless and memory elements of network ϕ respectively,

$e_\phi(s)$ is a vector with entries determined by the independent voltage and current sources,

with elements $d_{i,j} \in \{0, 1\}$ and with

with a maximum of one non-zero in each row or column, is a selector matrix that maps $\mathbf{I}_k \in \mathbb{R}^2$, the vector of currents entering the transmission line subnetwork k , into the node space $\mathbb{R}^{N_\phi}$ of the network ϕ .

$\mathbf{b}_k(s)$ represents the effect of incident electromagnetic interference on transmission line set k ,

$\mathbf{A}_k$ and $\mathbf{B}_k$ relate the terminal voltages $\mathbf{V}_k$ and currents $\mathbf{I}_k$ of transmission line set k by the following expression.

$$\mathbf{A}_k \mathbf{V}_k + \mathbf{B}_k \mathbf{I}_k = \mathbf{b}_k \quad (3.72)$$

The subscript k will be dropped for simplicity in the remainder of the thesis.

To illustrate the above formulation method, consider the network shown in Fig. 3.4. This network can be described by (3.71), where

$$\mathbf{W} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 \\ 0 & 0 & C_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.73)$$

$$\mathbf{H} = \begin{bmatrix} G_1 & -G_1 & 0 & 1 \\ -G_1 & G_1 & 0 & 0 \\ 0 & 0 & G_2 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (3.74)$$

$$\mathbf{D}_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad (3.75)$$

$$\mathbf{I}_1(s) = \begin{bmatrix} I_a & I_b \end{bmatrix}^t \quad (3.76)$$

$$\mathbf{V}(s) = \begin{bmatrix} V_1 & V_2 & V_3 & I_E \end{bmatrix}^t \quad (3.77)$$

$$\mathbf{e}(s) = \begin{bmatrix} 0 & 0 & 0 & V_i(s) \end{bmatrix} \quad (3.78)$$

The network equations therefore are,

$$\begin{bmatrix} s\mathbf{W} + \mathbf{H} & \mathbf{D} \\ \mathbf{A}(s)\mathbf{D}^t & \mathbf{B}(s) \end{bmatrix} \begin{bmatrix} \mathbf{V}(s) \\ \mathbf{I}_1(s) \end{bmatrix} = \begin{bmatrix} \mathbf{e}(s) \\ \mathbf{b}(s) \end{bmatrix} \quad (3.79)$$

where $\mathbf{A}(s)$, $\mathbf{B}(s)$ and $\mathbf{b}(s)$ relate the terminal voltages and currents of the transmission line network by,

$$\mathbf{A}(s) \begin{bmatrix} V_2 \\ V_3 \end{bmatrix} + \mathbf{B}(s) \begin{bmatrix} I_a \\ I_b \end{bmatrix} = \mathbf{b}(s) \quad (3.80)$$

where,

$$\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \quad (3.81)$$

The circuit equations are therefore,

$$\begin{bmatrix}
 G_1 & -G_1 & 0 & 1 & 0 & 0 \\
 -G_1 & G_1 + sC_1 & 0 & 0 & 1 & 0 \\
 0 & 0 & G_2 + sC_2 & 0 & 0 & 1 \\
 1 & 0 & 0 & 0 & 0 & 0 \\
 0 & A_{11} & A_{12} & 0 & B_{11} & B_{12} \\
 0 & A_{21} & A_{22} & 0 & B_{21} & B_{22}
 \end{bmatrix}
 \begin{bmatrix}
 V_1 \\
 V_2 \\
 V_3 \\
 I_E \\
 I_a \\
 I_b
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 0 \\
 0 \\
 V_i(s) \\
 b_1(s) \\
 b_2(s)
 \end{bmatrix}
 \quad (3.82)$$

Figure 3.4: An example circuit containing distributed components

3.5.2 Background on MMTs

A comprehensive coverage of moment matching techniques will not be attempted in this thesis. The reader is referred to [23] for more details on the algorithm and a comprehensive list of references. Instead, a summary of the basic principles is provided, with more attention given to those aspects of MMTs that were modified and built upon in this work.

A. Conventional simulation techniques

It would be useful to briefly introduce conventional simulation techniques before going too deeply into new simulation methods. The conventional method of solving the MNA equations in (3.71) is through L/U factorization in the frequency domain, or Linear Multi-Step (LMS) integration techniques such as the Euler Method in the time domain. These techniques, while proven very accurate and stable, suffer from extensive CPU requirements due to the large number of costly L/U factorization required. A typical frequency domain analysis would require about 1000 L/U factorizations. The cost of time domain simulation is even higher. Circuits generally include a large number of poles and these often include some very fast poles caused by stray capacitance for example. Even though such poles would normally have little effect on the results, they cause the LMS algorithm to significantly reduce the step size in order to stay within the stability region. The small step size would result in a large number of L/U decompositions and consequently a higher CPU cost. In addition to these practical CPU problems, the conventional techniques are also set back by the lack of an efficient and reliable algorithm that would allow the analysis of lossy distributed elements such as lossy coupled transmission lines. The latter problem is discussed in more detail in Sec. 2.3.

B. Underlying concepts behind moment matching techniques.

The power of MMTs is based on the realization that systems of any practical size will have a large number of poles, with only a few of those poles having any significant practical effect on the transfer function. This effect is most easily visualized in RC networks (with only real poles) as illustrated in Fig. 3.5, where it is clear that the pole p_2 will have little effect on the final result. MMTs capitalize on this fact by using Padé approximation[58],[59] to determine the relatively few dominant poles of the system. Only those

dominant poles are considered when evaluating the response. Instead of obtaining the response at each frequency point, MMTs are numerical techniques that determine accurate analytical expressions for the impulse response of the network.

Figure 3.5: Dominant RC poles

C. Padé approximation

Cauchy provided the foundation of Padé approximants in 1821 and many others have developed and investigated them since. Baker[58] is credited with being the first to apply Padé approximation to the circuit simulation field in 1975. In this section, a brief description of how Padé is applied to MMTs is presented along with some of their relevant properties. Padé approximants employ a rational function approximation to fit the desired

response. This type of approximation has proven to be very suitable for network transfer functions. MMTs use Padé to fit a rational function to the moments of the system transfer function. For example, for the following transfer function with moments m_i ,

$$h(s) = m_0 + m_1 s + m_2 s^2 + \dots \quad (3.83)$$

a set of equations could be derived to obtain a rational approximation of order L/M for $h(s)$

$$h(s) = \frac{a_0 + a_1 s + a_2 s^2 + \dots + a_L s^L}{1 + b_1 s + \dots + b_M s^M} \quad (3.84)$$

where the coefficients a_i and b_i are obtained by

$$\begin{bmatrix} m_{L-M+1} & m_{L-M+2} & \dots & m_L \\ m_{L-M+2} & \dots & \dots & m_{L+1} \\ \dots & \dots & \dots & \dots \\ m_L & m_{L+1} & \dots & m_{L+M-1} \end{bmatrix} \begin{bmatrix} b_M \\ b_{M-2} \\ \dots \\ b_1 \end{bmatrix} = - \begin{bmatrix} m_{L+1} \\ m_{L+2} \\ \dots \\ m_{L+M} \end{bmatrix} \quad (3.85)$$

$$\begin{aligned} a_0 &= m_0 \\ a_1 &= m_1 + b_1 m_0 \\ a_2 &= m_2 + b_1 m_1 + b_2 m_0 \\ &\dots \\ &\dots \\ a_L &= m_L + \sum_{i=1}^{\min(L, M)} b_i m_{L-i} \end{aligned} \quad (3.86)$$

As can be easily seen from the above equations, the algorithm for finding the rational Padé approximation is relatively simple. Determining the optimum order of the approximation is however a less obvious problem to solve, and is discussed in [23]. Although Padé approximants have proven to be very useful for reducing the order of the system and

determining the dominant poles, some of their inherent properties have dictated that a single expansion often gives inaccurate results. This has led to the development of multi-point expansion techniques as will be seen in the following sections. The following is a list of those properties which have the most impact on MMTs.

1. Although the matrix in (3.85) is a Toeplitz matrix, experience shows that it is illconditioned if its size is large. This implies that one can only expect to detect 6 to 8 accurate poles from a single expansion.
2. Padé often produces unstable poles on the right hand side of the complex plane.
3. Like other approximants, Padé accuracy deteriorates as we move away from the expansion point.
4. Padé provides no estimates for error bounds.

D. Summary of a single expansion MMT algorithm

Obtaining a low order approximation of the network transfer function using a single Padé expansion is commonly referred to in the literature as Asymptotic Waveform Evaluation (AWE). This type of order reduction is generally accurate for RC networks, but could fail for some RLC networks and networks with distributed elements for reasons that will be discussed at the end of this section. The AWE expansion is normally done at the origin of the complex plane. Fig. 3.7. outlines the general steps of an AWE simulation. First the moments of the MNA matrix are obtained. This procedure is normally simple, but gets more involved when distributed elements are present (see Sec. 4.4). The CPU cost of this step is negligible compared to L/U decomposition of the large MNA matrix. The next step is to obtain the moments of the output vector. The recursive formulae used at this stage are derived in Sec. 3.5.3 for the case of incident field excitation. The CPU cost of obtaining all these moments is one L/U decomposition. This is the same cost of one frequency point analysis or one time step using conventional techniques. After obtaining the

system moments, the elements corresponding to the desired output are selected from the moment vector. Padé is then applied using (3.85) and (3.86) to obtain a rational transfer function. The poles and residues of the system transfer function can then be obtained from the rational transfer function. Note that the approach used in this thesis was to evaluate the transfer function in order to obtain the frequency response without explicitly calculating the poles and residues of the system.

Figure 3.6: Pole distribution of an example RLC network

It was mentioned at the beginning of the previous paragraph that AWE (or a single point expansion) often fails for non RC networks. This is due to the fact that the poles of an RC network are all on the real axis and therefore an expansion at the origin could clearly determine which ones are dominant and which ones are not, as can be seen Fig. 3.5. In the case of RLC it is possible to have poles with similar damping constants, but different resonant frequencies, as is illustrated in Fig. 3.6. In this example all the dominant poles happen to be within the radius of convergence of the Padé expansion, but one can construct an RLC network where some dominant poles will be missed. In systems containing distributed elements the number of dominant poles will be significantly higher, and it is

normally very difficult to capture them all with a single Padé expansion (note that due to the illconditioning of the Padé matrix, only about six to eight accurate poles can be expected from a single expansion). This has prompted the development of multi-point expansion techniques such as complex frequency hopping.

Figure 3.7: General steps of Moment Matching.

E. Complex Frequency Hopping

For systems with many dominant poles such as transmission line networks, a single Padé expansion will not be able to capture all the dominant poles as illustrated in Fig. 3.8. For such networks, Complex Frequency Hopping (CFH) is used. CFH extends the process of Moment Matching to multiple expansion points in the complex plane near or on the imaginary axis using a binary search algorithm. With a minimized number of frequency point expansions, enough information is obtained to enable the generation of an approximate transfer function that matches the original function up to a predefined highest frequency of interest. By using the information from all the expansion points, CFH ensures that all dominant poles are captured as illustrated in Fig. 3.8. In addition CFH provides clear bounds on the region of accuracy.

Figure 3.8: Pole distribution of network with distributed elements.

Complex Frequency Hopping relies on a binary search algorithm[20][21] to determine the expansion points and to minimize the number of expansions. Different methods have been developed for the implementation of the binary search algorithm. The one used in

this thesis is referred to as the transfer function approach[20], this is a variation of CFH that does not explicitly evaluate the poles and zeros of the network but uses the transfer functions obtained at various hops (expansions) to evaluate the frequency response. IFFT is then used for transient analysis. The output of CFH consists of a set of transfer functions corresponding to the various expansion points as illustrated in Fig. 3.9. Each transfer function is checked for consistency with adjacent transfer functions, which guarantees accuracy within the region of validity.

Figure 3.9: CFH output

F. Binary search algorithm for the transfer function approach

The binary search algorithm employed in CFH is best implemented recursively. It is therefore difficult to intuitively separate it into clear steps. Instead the following set of rules will be used as an explanation,

1. At least two expansions are required: one at the origin (zero frequency) and one at the highest frequency of operation.

2. Each transfer function must match its adjacent transfer functions at the mid frequency point between the two.
3. If two transfer functions do not match at the mid frequency point, an additional expansion is performed at that point.
4. Steps 2 and 3 are repeated until no new expansions are required.

3.5.3 System moments in the presence of incident fields

As mentioned in the previous section, one of the important steps of the CFH algorithm is obtaining the system moments. In this section, recursive formulae will be derived for the calculation of the system moments in the case of incident field excitation. In order to perform a CFH expansion at the frequency s_0 we start with MNA equations given in (3.70). First the Taylor expansion of the MNA matrix is obtained as outlined in Sec. 4.4. Also in the case of incident field coupling the moments of the vector $\psi(s)$ are needed. The details of the latter are derived in Sec. 3.5.4. Therefore starting from the MNA equations

$$Y(s)X(s) = \psi(s) \quad (3.87)$$

one could obtain

$$\sum_n Y_n (s - s_0)^n \sum_n M_n (s - s_0)^n = \sum_n \psi_n (s - s_0)^n \quad (3.88)$$

where the coefficients Y_n and ψ_n are the moments of the MNA matrix and the input vector respectively. The moments M_n of the output vector $X(s)$ are the unknowns to be calculated. Recursive formulae for M_n can be derived by expanding (3.88) and equating equal powers of $(s - s_0)$

$$Y_0 M_0 = \Psi_0 \quad (3.89)$$

$$Y_0 M_n = - \sum_{r=1}^n Y_r M_{n-r} + \Psi_n \quad (3.90)$$

Note that Y_0 does not change on the right hand side, and CPU cost of determining all the moments is only one L/U decomposition. The moments of the transfer function at the desired output correspond to one row of the moment vectors M_n . These moments are used in the Padé approximation.

3.5.4 Incident field moments

In our discussion of the system moment calculation in Sec. 3.5.3 we have assumed the moments of the source vector ψ to be available. The vector ψ is directly dependent of the incident EM fields, and was defined in (3.71) as a function of the sources $b(s)$ induced by those fields. In this section, an algorithm is derived to determine the moments of $b(s)$ and consequently the moments of $\psi(s)$. The lumped sources due to EMI were defined in (3.67) as

$$b(s) = e^{-\Phi l} \int_0^l e^{\Phi z} F(z, s) dz \quad (3.91)$$

By noting the definition of $F(z, s)$ in (3.56), and that of the transmission line stamp $T(s)$ in (3.66), $b(s)$ can be written as

$$\begin{aligned} b(s) &= T(s) \int_0^l e^{\Phi z} E_o \Gamma(s) e^{zs\sqrt{\epsilon\mu}\sin\theta_p \sin\phi_p} dz \\ &= E_o T(s) Q(s) \Gamma(s) \end{aligned} \quad (3.92)$$

The moments of $b(s)$ are needed are therefore defined in terms of T_i , Q_j and Γ_k , the

moments of $T(s)$, $Q(s)$ and $\Gamma(s)$ respectively

$$b_k = E_o \sum_i \sum_j T_i Q_j \Gamma_{k-i-j} \quad (3.93)$$

The moments of $T(s)$ are calculated using the Matrix Exponential method[21], and those of $\Gamma(s)$ are obtained from (3.57). Finally a recursive technique can be derived for the calculation of the moments of $Q(s)$ as follows

$$\begin{aligned} Q(s) &= \int_0^l e^{\varphi z} e^{zs\sqrt{\epsilon_r\mu_r}\sin\theta_p\sin\phi_p} dz \\ &= \int_0^l e^{H(s)z} dz \end{aligned} \quad (3.94)$$

where

$$H(s) = \varphi + Ks = H_0 + H_1s \quad (3.95)$$

and

$$K = \frac{\sqrt{\epsilon_r\mu_r}}{c} U \sin\theta_p \sin\phi_p \quad (3.96)$$

where U is the unity matrix. Note that the operand under the integral in (3.94) is an exponential of a matrix multiplied by an exponential of a scalar. While the summation rule cannot be used to combine two matrix exponentials, it is possible to do so if one of the two exponentials is a scalar, but only after its operand is multiplied by the unity matrix as was done in (3.96).

By analytically performing the integration in (3.94) and noting that

$$H(s)Q(s) = (e^{H(s)l} - U) \quad (3.97)$$

and setting

$$\Delta(s) = e^{H(s)l} - U = \sum_i \Delta_i s^i \quad (3.98)$$

the following equation can be deduced

$$(H_0 + H_1 s) \sum_i Q_i s^i = \sum_i \Delta_i s^i \quad (3.99)$$

where the coefficients Q_i are the moments of $Q(s)$, and Δ_i are the Taylor coefficients of the right hand side of (3.97). By equating equal powers of s in (3.99), the following recursive formulae for the calculation of the moments Q_i can be derived

$$H_0 Q_0 = \Delta_0 \quad (3.100)$$

$$H_0 Q_i = -H_1 Q_{i-1} + \Delta_i \quad (3.101)$$

The coefficients Δ_i are determined as follows

$$\Delta(s) = e^{(H_0 + H_1 s)l} - U = P(s) - U = \sum_i P_i s^i - U \quad (3.102)$$

where U is the unity matrix, and coefficients P_i can be obtained from

$$P_i = \sum_{j=1}^{\infty} P_{i,j} \quad (3.103)$$

where $P_{i,j}$ are given recursively by

$$P_{i,j} = \frac{(H_0 P_{i,j-1} + H_1 P_{i-1,j})l}{i+j} \quad (3.104)$$

where l is the length of the transmission line and

$$P_{0,0} = U \quad (3.105)$$

$$P_{i,j} = 0 \quad (i < 0, j < 0) \quad (3.106)$$

The ∞ in (3.103) is only a formality of description. In general only about 20 to 30 terms

are needed if the eigenvalues of $\mathbf{H}l$ are inside the unit circle in the complex plane at the highest frequency. For electrically long lines, the eigenvalues will be outside the unit circle, and the line can be efficiently divided into subsections by noting that

$$e^{\mathbf{H}l} = e^{\frac{\mathbf{H}l}{2}} e^{\frac{\mathbf{H}l}{2}} \quad (3.107)$$

which implies that the moments of a line can be generated by squaring the half line moments. If $\mathbf{P}_n^{(\frac{1}{2})}$ represents the half line moments, and $\mathbf{P}_n$ the full line moments, then

$$\mathbf{P}_n = \sum_{i=1}^n \mathbf{P}_i^{(\frac{1}{2})} \mathbf{P}_{n-i}^{(\frac{1}{2})} \quad (3.108)$$

Note that the above procedure is not equivalent to discretizing the line, as the number of states in the MNA is still the same and lines are still treated as distributed elements.

Figure 3.10: Basic steps in the calculation of Q_i

The algorithm for the calculation of the moments of the incident field equivalent sources $\mathbf{b}(s)$, can therefore be summarized as follows. First $\mathbf{b}(s)$ is factored in three functions of s ($\Gamma(s)$, $T(s)$, and $Q(s)$) as shown in (3.92). The moments of each of those functions are then separately evaluated and equation (3.93) is used to determine the moments of $\mathbf{b}(s)$. The moments of $\Gamma(s)$ are simply given in (3.57), and those of $T(s)$ are obtained using the matrix exponential technique[21]. The procedure to obtain the moments of $Q(s)$ covered in this section, is outlined in Fig. 3.10.

3.5.5 Summary of the algorithm

After presenting the algorithm for applying moment matching in the simulation of incident field coupling to transmission lines an attempt will now be made to summarize the basic steps involved in such a simulation. Fig. 3.11 gives an outline of the algorithm, with detailed steps given below.

1. Setup the MNA equations
2. Obtain the transmission line moments and expand $Y(s)$ as shown in Chapter 4.
3. Calculate the incident field moments $\mathbf{b}_i$ defined in (3.93).
4. Calculate the system moments $\mathbf{M}_i$ using (3.89) and (3.90).
5. Apply Padé to the moments of the desired output.
6. Repeat steps 2 to 5 at each CFH hop until convergence
7. Evaluate the transfer functions within their respective regions of validity in order to obtain the frequency response.
8. Use IFFT to determine the time domain response

Figure 3.11: Basic steps of incident field simulation with CFH

3.6 Special cases

The results in the previous sections, although general to all plane waves, are somewhat complicated. In this section we will consider some important special cases presented in (Endfire, Sidefire and Broadside excitation), in order to illustrate the procedures described in the previous sections.

3.6.1 Endfire Excitation

The first case is referred to as endfire excitation (Fig. 3.12, a), for which the wave is propagating in the positive z direction and the electric field is polarized in the positive x direction ($\theta_p = 90^\circ$, $\phi_p = -90^\circ$ and $\theta_E = 90^\circ$). For this incidence and polarization we have

$$\begin{aligned} e_x &= 1 & \beta_x &= 0 \\ e_y &= 0 & \beta_y &= 0 \\ e_z &= 0 & \beta_z &= \beta \end{aligned} \quad (3.109)$$

The distributed sources due to EMI defined in (3.56) therefore become

$$F(z, s) = E_o \Gamma(s) e^{-sz\sqrt{\epsilon\mu}} \quad (3.110)$$

where

$$\Gamma(s) = \begin{bmatrix} \sqrt{\epsilon\mu}hs \\ -hY_{TL} \end{bmatrix} \quad (3.111)$$

Also $H(s)$ defined in (3.95) becomes

$$H(s) = \Phi - Ks = D + s(E + U\sqrt{\epsilon\mu}) \quad (3.112)$$

where $\mathbf{D}$ and $\mathbf{E}$ are defined in (3.55) as functions of the transmission line parameters, and $\mathbf{U}$ is the identity matrix. Knowing E_o , $\Gamma(s)$ and $\mathbf{H}(s)$ one can then easily apply the algorithm of Sec. 3.5.4 to obtain the incident field moments.

Figure 3.12: Special cases of incident waves.

Analytical formulae for the analysis of endfire excitation of single transmission lines with lumped terminations were derived in [27]. The results of these were obtained for a 20 cm long buried microstrip line (cross section shown in Fig. 3.13) with $50\ \Omega$ terminations. These were compared in the frequency domain to the results from CFH (see Fig. 3.14) to verify the accuracy of the CFH algorithm. The amplitude of the input field was 1V/m. A buried microstrip was chosen to simulate a homogeneous environment around the transmission line.

Figure 3.13: Buried microstrip line

Figure 3.14: Endfire excitation, comparison with analytical results

3.6.2 Sidefire Excitation

Similarly, for Sidefire excitation (Fig. 3.12, *b*), the wave is propagating in the positive x direction with the electric field polarized in the positive z direction ($\theta_p = 180^\circ$, $\phi_p = 0^\circ$, $\theta_E = 0^\circ$). In this case we have

$$\begin{aligned} e_x &= 0 & \beta_x &= \beta \\ e_y &= 0 & \beta_y &= 0 \\ e_z &= 1 & \beta_z &= 0 \end{aligned} \quad (3.113)$$

The distributed sources due to EMI defined in (3.56) therefore become

$$F(z, s) = E_o \Gamma(s) \quad (3.114)$$

where

$$\Gamma(s) = \begin{bmatrix} \sum_{n=1}^{\infty} \frac{(-\sqrt{\epsilon\mu})^n}{n!} h^n s^n \\ 0 \end{bmatrix} \quad (3.115)$$

Also, with $K = 0$, $H(s)$ defined in (3.95) becomes

$$H(s) = \phi - Ks = D + sE \quad (3.116)$$

The results of the proposed technique for the case of sidefire excitation were compared in Fig. 3.15 with analytical results published in [27]. The test structure was a 20 cm long two-wire line with 2 mm separation and per unit length parameters of: $C=87\text{pf/m}$, $L=382\text{pF/m}$, and $R=0.35\ \Omega/\text{m}$. The amplitude of the incident field was taken to be 1 V/m.

Figure 3.15: Sidefire excitation: Comparison with analytical results

3.6.3 Broadside Excitation

Finally for Broadside excitation (Fig. 3.12, *c*), the incident field is propagating in the negative *y* direction with the incident field polarized in the positive *x* direction ($\theta_p = 90^\circ$, $\phi_p = 0^\circ$, and $\theta_E = 90^\circ$). For this incidence and polarization, $\mathbf{b}(s)$ can be expressed as,

$$\mathbf{b}(s) = \mathbf{E}_o \mathbf{T}(s) \mathbf{Q}(s) \Gamma(s) \quad (3.117)$$

where $\Gamma(s)$ is

$$\Gamma(s) = \begin{bmatrix} 0 \\ -Y_{TL}h \end{bmatrix} \quad (3.118)$$

the moments of $\mathbf{T}(s)$ are found using the matrix exponential technique, and those of $\mathbf{Q}(s)$ are found following the procedure in Fig. 3.10 with

$$\mathbf{H}(s) = \mathbf{H}_0 + \mathbf{H}_1 s = \mathbf{D} + s\mathbf{E} \quad (3.119)$$

Analytical formulae for the analysis of broadside excitation of single transmission lines with lumped terminations were derived in [27]. The results of these were obtained for a 20 cm long buried microstrip line (cross section shown in Fig. 3.13) with 50Ω terminations. As for the previous two cases the accuracy of the algorithm was verified as shown in Fig. 3.16 with an input field of 1V/m.

Figure 3.16: Broadside excitation: Comparison with analytical results

3.7 Numerical results

3.7.1 Comparison with analytical results

A time domain example was also used to verify the accuracy of the proposed algorithm, consider the simple example shown in Fig. 3.18 which consists of a transmission line terminated with resistive loads $R_1 = R_2 = 50\Omega$ in the presence of an incident uniform plane wave. The electric field has a 1V/m amplitude and a 1ns rise/fall time. As in example 1 the response of this circuit can be evaluated using the analytical solution described in [27].

The time domain response of this circuit is shown in Fig. 3.18, which shows that the CFH results are consistent with the analytical results. Note that the CFH algorithm requires only one L/U decomposition of the system MNA for each frequency hop (2 L/U decompositions in the case of this example). Conventional techniques, require one L/U decomposition for each frequency point. For 1000 frequency points, this translates into a speed-up of 500:1.

Figure 3.17: Circuit for example 1

Figure 3.18: Transient response of circuit in example 2

3.7.2 Example

To demonstrate the efficiency of the method, a relatively large interconnect network extracted from the layout of an industrial printed circuit board, consisting of 458 resistors, inductors and capacitors and 12 transmission lines is considered. The incident field is a uniform plane wave with electric field of 1V/m amplitude and 5ns rise/fall time. The number of hops required was 5. In Fig. 3.19 the CFH generated transient response is compared with the FFT response obtained using 1024 frequency point solutions and as seen, the two responses are indistinguishable. It is to be noted that the dominant cost of a frequency hop is the L/U decomposition of the MNA matrix which is approximately the CPU cost of one frequency point analysis using conventional techniques [11],[23]. The CFH speed-up ratio obtainable in this example is approximately 200:1 over conventional methods.

Figure 3.19: Transient response for example 3

Chapter 4

Frequency dependent Interconnects

4.1 Introduction

It was shown from measurements [42],[43] that neglecting the effect of the frequency dependence of the transmission line parameters leads to significant errors in the results. In this chapter, the problem of the frequency dependence of the interconnect per-unit length parameters is introduced. An algorithm for the inclusion of a frequency dependent interconnect model in moment matching techniques (MMTs) is presented. In Sec. 3.5 a description of MMTs was presented with an emphasis on complex frequency hopping (CFH) which is useful for interconnect analysis. An important step of the CFH algorithm was the determination of the moments or the derivatives of the modified nodal admittance matrix (MNA) with respect to frequency, and the transmission line parameters are the most important factor to consider in this step. Our goal in this chapter is to provide a technique to evaluate those moments in the case of frequency dependent interconnects. First Sec. 4.2 provides some background on the nature of the frequency dependence of the

transmission line parameters and the physical reasons behind it. Sec. 4.3 then presents a simple modelling technique for the interconnect parameters this is compatible with MMTs (an algorithm that requires continuous derivatives). In Sec. 4.4 the system moments are derived as a function of the transmission line moments, which are discussed in Sec. 4.5.

4.2 Frequency dependent interconnects

As the operating frequency increases the resistance per unit length, and inductance per unit length of the transmission line vary. This is largely due to varying current distribution in the conductor and ground plane caused by the induced electric field. This effect can be separated into three parts: Skin effect, Edge effect and proximity effects.

4.2.1 Edge and proximity effects

The edge and proximity effects dominate the low to medium frequency region in terms of their effect on the interconnect parameters. The edge effect is when the current starts concentrating near the sharp edges of the conductor thus raising the resistance. It affects both the signal and ground conductors, but is more pronounced on the signal conductors. The proximity effect causes the current to concentrate in the sections of the ground plane that are close to the signal conductor. This modifies the magnetic field between the two conductors therefore reducing the inductance per unit length. It also raises the resistance per unit length as more current is crowded in the ground plane under the conductor. The proximity effect appears at medium frequencies. Current densities in a microstrip showing these two effects are shown in Fig. 4.1, with the dimensions of that microstrip given in Fig. 4.3. While both effects need to be accounted for, the proximity effect seems to have more significance, especially in its effect on the inductance.

4.2.2 Skin effect

The skin effect is due to the current being concentrated in a thin layer at the conductor surface. It is pronounced mostly at high frequencies on both the signal and the ground conductors. The thickness of the layer is related to the skin depth, and decreases with increasing frequency. This results in the resistance being proportional to the square root of the frequency at very high operating frequencies. The magnetic fields inside the conductors are also reduced due to skin effect. This reduces the internal inductance and therefore the total inductance. At even higher frequencies, as the internal inductance approaches zero and the edge and proximity effects are fully pronounced, the inductance becomes essentially constant. Fig. 4.2 shows the skin effect on the signal conductor of the microstrip in Fig. 4.3.

Figure 4.1: Current density and proximity and edge effects

Figure 4.2: Current density showing skin effect

Figure 4.3: Conductor over a ground plane

4.2.3 Typical behaviour of R and L

The frequency plots of R and L of the microstrip in Fig. 4.3 are shown in Fig. 4.4. These plots present a typical behaviour of R and L in general. L starts off as essentially constant until the edge and proximity effects get into effect. The edge effect causes the resistance to increase, and the current crowding under signal conductor (due to the proximity effect) causes the inductance to decrease and resistance to increase. As we go higher in frequency, the edge and proximity effects become fully pronounced and will cause little additional change in R and L , but the skin effect becomes significant. Initially the inductance is reduced due to skin effect because of the reduction of the magnetic fields inside the conductors, but as the contribution of those magnetic fields to the overall inductance becomes insignificant L becomes essentially constant at very high frequency. The resistance on the other hand becomes a direct function of the skin depth and therefore varies with the square root of the frequency.

Figure 4.4: Inductance with respect to frequency

Figure 4.5: Resistance with respect to frequency

4.3 Modelling of R and L

4.3.1 Parameter extraction

In this research polynomial fitting was used in the algorithm for linking moment matching techniques to the frequency dependent transmission line parameters. The data used in the fitting could be obtained from measurements, empirical formulae or electromagnetic simulation[40],[45],[42]. The data used in this thesis was obtained through numerical solutions of R and L with respect to frequency. For low frequencies, the DC approximation of

current distribution (uniform throughout the conductor cross section) gives good accuracy. The situation is similar for very high frequencies where the surface currents screen the interior of the conductors. Both cases can be handled (in two dimensions) by the Laplace equation which is used by static field solvers. The transition between the low (dc) and high frequency range can be described as the quasi-stationary region. This region is characterized by a non-uniform current distribution. In this case the solution of the Helmholtz equations is required. A combination of boundary elements (BEM) method and finite elements method (FEM) is used in this case[45][44].

4.3.2 Approximating polynomials

By generating many data points using empirical equations or field solvers, the least square method is used to fit a polynomial to the resistance and inductance matrices as functions of frequency. Due to the complex dependence of the resistance and inductance on the frequency, it is difficult for a single polynomial to fit the R and L curves over a wide frequency range, therefore more than one polynomial are used, and each is valid in its own frequency range. When expanding around a given point on the complex frequency axis, the polynomial that is valid in that region is used. For example, When expanding around $s = s_o$, the polynomials valid in a the frequency range containing s_o have the form

$$L(f) = L_0 + L_2 f^2 + L_4 f^4 + \dots \quad (4.1)$$

$$R(f) = R_0 + R_2 f^2 + R_4 f^4 + \dots \quad (4.2)$$

Equations (4.1) and (4.2) written in the Laplace domain have the form

$$\begin{aligned} L(s) = L_0 + L_2 \left(\frac{1}{2\pi j} \right)^2 s^2 + L_4 \left(\frac{1}{2\pi j} \right)^4 s^4 + \dots = \\ L'_0 + L'_2 s^2 + L'_4 s^4 + \dots \end{aligned} \quad (4.3)$$

$$R(s) = R_0 + R_2 \left(\frac{1}{2\pi j} \right)^2 s^2 + R_4 \left(\frac{1}{2\pi j} \right)^4 s^4 + \dots =$$

$$R'_0 + R'_2 s^2 + R'_4 s^4 + \dots \quad (4.4)$$

Only even powers of f were used in order to ensure that the result will be real when converted to the s domain. These relatively simple polynomial models naturally supply frequency derivatives which are then used in moment matching techniques.

4.4 Moments of the MNA matrix

An important step in the algorithm of MMTs is the determination of the moments of the MNA matrix. These moments are simple to obtain in the case of networks with lumped elements, but are more complex in the case of distributed components such as transmission lines. The MNA equations were defined in (3.71) as,

$$Y(s) = \begin{bmatrix} (H_\phi + sW_\phi) & D_1 & \dots & D_{N_t} \\ A_1 D_1^t & B_1 & 0 & 0 \\ \dots & 0 & \dots & 0 \\ A_{N_t} D_{N_t}^t & 0 & 0 & B_{N_t} \end{bmatrix} \quad (4.5)$$

where H_ϕ and W_ϕ are constant with respect to s and represent the lumped elements. A_i and B_i are complex functions of s and represent the stamp of transmission lines. If the network has only lumped elements, the moment will simply be H_ϕ and W_ϕ , but in the case of transmission line network it would depend on the frequency derivatives of A and B which represent the transmission line stamp introduced in (3.72),

$$A_k(s)V_k + B_k(s)I_k = b_k(s) \quad (4.6)$$

In [13],[14] a method based on determining the eigenvalues and eigenvectors of the system in (4.6) have been proposed to determine the derivatives of A and B . This method suffers from rounding errors which increase with each new moment generated[23]. To eliminate this problem, a matrix exponential technique was proposed in[21]. This method, while very accurate and suitable for use with moment matching techniques, is limited to interconnects with constant parameters. In practical cases however, the resistance per unit length (R) and the inductance per unit length (L) are complex functions of frequency. Sec. 4.5 introduces a technique for determining the Taylor coefficients (or derivatives) of A and B (also referred to as the transmission line moments) in the case of frequency dependent interconnects. Once the transmission line moments are obtained the moments of the MNA matrix $Y(s)$ are calculated using

$$A_k = \sum_i A_k^{(i)} s^k \quad B_k = \sum_i B_k^{(i)} s^k \quad (4.7)$$

where $A_k^{(i)}$ and $B_k^{(i)}$ are derived from the transmission line moments. The moments Y_i of $Y(s)$ therefore become

$$Y_0 = \begin{bmatrix} H_\phi & D_1 & \dots & D_{N_t} \\ A_1^{(0)} D_1^t & B_1^{(0)} & 0 & 0 \\ \dots & 0 & \dots & 0 \\ A_{N_t}^{(0)} D_{N_t}^t & 0 & 0 & B_{N_t}^{(0)} \end{bmatrix} \quad (4.8)$$

$$Y_1 = \begin{bmatrix} W_\phi & 0 & \dots & 0 \\ A_1^{(1)} D_1^t & B_1^{(1)} & 0 & 0 \\ \dots & 0 & \dots & 0 \\ A_{N_t}^{(1)} D_{N_t}^t & 0 & 0 & B_{N_t}^{(1)} \end{bmatrix} \quad (4.9)$$

$$Y_r = \begin{bmatrix} 0 & 0 & \dots & 0 \\ A_1^{(r)} D_1^t & B_1^{(r)} & 0 & 0 \\ \dots & 0 & \dots & 0 \\ A_{N_t}^{(r)} D_{N_t}^t & 0 & 0 & B_{N_t}^{(r)} \end{bmatrix} \quad ;(r \geq 2) \quad (4.10)$$

Finally the moments Y_i are used in the CFH algorithm to obtain the transfer function of the network as outlined in Sec. 3.5. The procedure for calculating the transmission line moments ($A^{(r)}$ and $B^{(r)}$) is developed in the following section.

4.5 Transmission line moments

The transmission line moments are needed for determining the MNA moments. As mentioned in the previous section, two methods have been previously proposed for extending moment matching techniques to the analysis of distributed circuits. The eigenvalue moment method which suffers from round-off errors as the number of moments increases, and the matrix exponential method which significantly improves accuracy. However, the latter technique was limited to the analysis of distributed elements with frequency independent parameters. A generalization of the moment exponential method is presented here, that uses the polynomial models in (4.3) and (4.4) to determine accurate moments of frequency dependent interconnects. This method is based on the evaluation of the transmission line moments or the Taylor expansion of $T(s)$ in (3.66), which represents a complex function of s .

$$T(s) = e^{-\phi l} = e^{-(D+sE)l} = e^{(D'+sE')l} \quad (4.11)$$

where,

$$D' = -D = \begin{bmatrix} 0 & -R \\ -G & 0 \end{bmatrix} \quad E' = -E = \begin{bmatrix} 0 & -L \\ -C & 0 \end{bmatrix} \quad (4.12)$$

where R , L , C and G are the per unit length transmission line resistance, inductance, capacitance and conductance respectively. An important property of matrix exponentiation is that while it parallels scalar exponentiation in most ways, it does not follow the normal exponentiation rule. That is,

$$(A \cdot B \neq B \cdot A) \leftrightarrow (e^A e^B \neq e^{A+B}) \quad (4.13)$$

The matrices must commute if they are to follow exponent addition. So in the general case of transmission line parameters,

$$e^{(D'+sE')l} \neq e^{D'l} e^{sE'l} \quad (4.14)$$

In addition, for the cases where R and L are frequency dependent, D' and E' are functions of s .

$$\begin{aligned} D'(s) + sE'(s) &= \begin{bmatrix} 0 & -R'_0 \\ -G & 0 \end{bmatrix} + \begin{bmatrix} 0 & -L'_0 \\ -C & 0 \end{bmatrix} s + \\ &\begin{bmatrix} 0 & -R'_2 \\ 0 & 0 \end{bmatrix} s^2 + \begin{bmatrix} 0 & -L'_2 \\ 0 & 0 \end{bmatrix} s^3 + \dots = \sum_{i=0}^{n_o} F_i s^i \end{aligned} \quad (4.15)$$

In order to get the Taylor expansion of $T(s)$ we assume a series in the form,

$$T(s) = e^{(D'+sE')l} = T_0 + sT_1 + s^2T_2 + \dots \quad (4.16)$$

The coefficients T_i can then be calculated as follows,

$$T_i = \sum_{j=1}^{\infty} C_{i,j} \quad (4.17)$$

where the $C_{i,j}$ are given recursively by,

where

$$C_{i,j} = \frac{1}{j} \sum_{m=0}^{n_o} F_m C_{i-m,j-1} \quad (4.18)$$

where F_m and n_o are the coefficients and order of the expansion in (4.15), and,

$$C_{0,0} = U \quad (4.19)$$

$$C_{i,j} = 0 \quad ;(i < 0, j < 0) \quad (4.20)$$

In general only about 20 to 30 terms are needed (4.17) if the eigenvalues of ϕl are inside the unit circle in the complex plane at the highest frequency. If the line is too long, it can be efficiently divided into subsections by noting that,

$$e^{-\phi l} = e^{-\frac{\phi l}{2}} e^{-\frac{\phi l}{2}} \quad (4.21)$$

That is the moments of a line can be generated by squaring the half line moments. If the half-line moments are $C_n^{(\frac{1}{2})}$ represents the half line moments, then,

$$\begin{aligned} & C_0 + sC_1 + s^2C_2 + \dots \\ &= \left(C_1^{(\frac{1}{2})} + sC_2^{(\frac{1}{2})} + s^2 \left(C_3^{(\frac{1}{2})} \right) + \dots \right) \\ & \times \left(C_1^{(\frac{1}{2})} + sC_2^{(\frac{1}{2})} + s^2C_3^{(\frac{1}{2})} + \dots \right) \end{aligned} \quad (4.22)$$

giving,

$$C_n = \sum_{i=1}^n C_i^{(1/2)} C_{n-i}^{(1/2)} \quad (4.23)$$

Using this technique, the line can be subdivided by a power of 2 and the moments of the smallest section calculated. After obtaining the transmission line moments one can simply calculate the derivatives of A and B but observing the transmission line stamp in Sec. 3.4 and noting that,

$$A = \begin{bmatrix} -T_{11} & 1 \\ T_{21} & 0 \end{bmatrix} \quad B = \begin{bmatrix} T_{12} & 0 \\ -T_{22} & 1 \end{bmatrix} \quad (4.24)$$

4.6 Numerical examples

4.6.1 Incident field coupling

An interconnect (with dimensions shown in Fig. 4.3, on page 77) was analysed in the presence of EMI. The incident plane waves were in the z direction. The length of the line was taken to be 5 cm with 50Ω terminations. The analysis was done with and without taking into account the frequency dependence of the transmission line parameters. As can be seen from the graph, a deviation exists at high frequency.

4.6.2 Matched line

A 10 cm long line was analysed with 50Ω terminations (almost matched compared to its characteristic impedance of 46Ω). The input was a 1 Volt 0.6 ns pulse with a rise/fall time of 0.1 ns. In Fig. 4.8 the impulse response is shown with and without accounting for the

frequency dependence of the parameters. The amplitude drop is mostly due to skin effect. The ripple in frequency dependent case is due to the mismatch between the load and the line impedance (due to the varying inductance). In Fig. 4.9 the transient response is shown. Note the variation in both the amplitude and delay.

4.6.3 Non-matched line

The terminations used in Sec. 4.6.2 were replaced with 100Ω resistors. The frequency and transient responses are shown in Fig. 4.10 and Fig. 4.11 respectively. The input was a 1 Volt 0.6 ns pulse with 0.1 ns rise/fall time. As can be seen from the graphs, the difference between the frequency dependent case and the constant parameter assumption becomes larger when the lines are mismatched.

4.6.4 Coupled lines

The circuit in Fig. 4.6 was analysed with a 12 cm long coupled line. The input was taken to be a 1 Volt 0.6 ns pulse with 0.1 ns rise/fall time. The effect of neglecting the frequency dependence of the transmission line parameters can be seen both in the frequency and time domain responses (Fig. 4.12 and Fig. 4.13).

Figure 4.6: Coupled line circuit.

4.6.5 Large example

To demonstrate the efficiency of the method, a relatively large interconnect network extracted from the layout of an industrial printed circuit board, consisting of 458 resistors, inductors and capacitors and 12 transmission lines is considered. The CFH algorithm converged to the same response as using the conventional method, with only 11 L/U decompositions as opposed to 1024. A comparison of the results with and without frequency dependence of the transmission lines is shown in Fig. 4.14 and Fig. 4.15, for the frequency and time domain responses respectively.

Figure 4.7: Incident field coupling to frequency dependent interconnect.

Figure 4.8: Frequency domain response (50Ω termination)

Figure 4.9: Time domain response (50Ω termination)

Figure 4.10: Frequency domain response (100Ω termination)

Figure 4.11: Transient response (100Ω termination)

Figure 4.12: Frequency response (Coupled line example)

Figure 4.13: Transient response: coupled lines

Figure 4.14: Large example: frequency response

Figure 4.15: Large example: transient response

Chapter 5

Conclusions and future research

5.1 Conclusion

A novel technique has been developed for the analysis of frequency dependent transmission lines in the presence of incident electromagnetic fields. The new method is based on the recently developed moment matching techniques. This approach was proven to be as accurate as current techniques published in the literature while providing a significant improvement in CPU time and additional flexibility in nonlinear simulation. The advantages of using MMTs for such simulation are listed below.

1. The most obvious advantage of using MMTs for the simulation of frequency dependent interconnects and the incident field coupling is the CPU speed-up. While MMTs are more complex to develop and implement, their complexity is at the analytical level and results in a reduction in the computational complexity at the simulation level. What would have required thousands of L/U decompositions using

conventional techniques can be achieved with about 2 to 10 L/U decompositions using MMTs with comparable accuracy.

2. The approach presented in this thesis for the analysis of EMI presents no theoretical restrictions on the number of transmission lines affected by EMI and their physical placement in the system. Previously published methods either encounter theoretical limits to their ability to handle complex circuits or simply face prohibitive CPU requirements.
3. Despite being a numerical technique, the approach presented here provides analytical results which are very useful for bridging the gap between frequency domain simulation and time domain simulation with nonlinear elements.

5.2 Further research

Various related topics have been investigated for further research. Some of these are listed below.

5.2.1 Multiple transmission lines

In practical cases many transmission lines in a system are sensitive to incident fields, and the simulation of the combined effect of the incident fields on all the traces is necessary. These interconnects would have different orientation and physical location on the board, and the network would contain many other distributed lines in addition to the lumped elements. The method presented in this thesis can be extended to handle such cases by accounting for the different orientation and phase delay between the conductors. Conventional simulation tools would require a very high CPU cost and are normally limited to very simple circuits in the time domain.

Figure 5.1: Multiple transmission line excitation

5.2.2 Inhomogeneous medium

The various simulation techniques proposed for the analysis of incident field coupling to interconnects have in general treated the medium surrounding the interconnects as homogeneous as far as the incident field is concerned. Imposing such conditions for inhomogeneous structures, such as the multi-layered medium in Figure 5.2, introduces errors in the results. A solution has been proposed in [33] to account for such inhomogeneity. It is based on the assumption that the traces are embedded in a planar layered inhomogeneous medium, and requires the calculation of the fields in each layer separately. An initial investigation of this problem has shown this approach can be used to improve the accuracy of the method presented in this thesis.

Figure 5.2: Multi-layered Inhomogeneous medium

5.2.3 Nonlinear simulation

While conventional techniques can handle the analysis of coupling to linear circuits (albeit with significant CPU requirements), time domain nonlinear simulation is often impossible to perform for most circuits of practical sizes. This has led to various simplifications of the transmission line model (such as assuming lossless lines), which compromises the accuracy of the simulation. MMTs have proven to be very useful for bridging the gap between linear and nonlinear simulation through Macromodelling[24][17]. While finding a practical macromodel for the incident field presents many additional challenges, initial investigation indicates that such an approach is possible and would retain the same accuracy and much of the CPU savings of moment matching. The EMI macromodel could appear as additional ports in the network, or more likely as additional sources on existing ports as shown in Figure 5.3.

Figure 5.3: Incident field macromodel

5.2.4 Sensitivity analysis

Sensitivity analysis has proven to be very useful in design and optimization. Techniques were developed to perform sensitivity analysis on interconnects using AWE[64]. In addition to being limited to a single expansion that approach is based on the eigenvalue moment method and therefore suffers from inaccuracies in higher order moments. Avoiding these inaccuracies is therefore needed. In addition, performing sensitivity analysis based on interconnects using multiple expansions provides a significant improvement in

the accuracy and reliability of the results. These techniques could also be used for sensitivity analysis with respect to incident fields.

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IMAGE EVALUATION TEST TARGET (QA-3)

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