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BEHAVIOUR OF WEBS OF ROLLED STEEL SHAPES SUBJECTED TO CONCENTRATED LOADS

by

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B. Eng. (Hons) - University of Oran, Algeria

M.A.Sc. - University of Ottawa, Canada

**A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of
DOCTOR OF PHILOSOPHY***

**DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING
CARLETON UNIVERSITY
OTTAWA, CANADA**

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Behaviour of Webs of Rolled Steel Shapes Subjected to Concentrated Loads

submitted by

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in partial fulfillment of the requirements for
the degree of Doctor of Philosophy

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Abstract

Concentrated loads from wheel loads or gantry girders, purlins or beams onto the main frame members of buildings, and roller loads during launching of plate or box girders can cause local failure of the web into the form of elastic or inelastic web instability (web crippling) or web yielding. Since the early 1930's, about 653 tests have been conducted on built-up rolled sheet steel shapes and welded plate girders to determine the effects of web thickness, flange rigidity, load eccentricity and material properties on the web's capacity. Many formulae have been developed from this work for determining the ultimate capacity of the web and over the years, the empirical and semi-empirical expressions by Graham et al (1959) and Roberts (1981) have been adapted and adopted as design rules by specifications and standards in North America. The modified Roberts solution for web crippling contains a contribution to web buckling capacity from both the flange and the web where only the contribution of the web should be accounted for as the flanges may be fully utilized in bending. It was developed for interior loads only and is based on section sizes and failure mechanisms that bear no resemblance to the range of W and WWF shapes produced by Canadian and American mills. The lack of a unified approach to describe the web capacity for a range of behaviour (including elastic web buckling which is currently not existent) and the application of the modified Roberts solution that is suspect have generated the need for this research.

Therefore, the objective of this study was to develop rational expressions to predict the ultimate load-carrying capacity of webs of these rolled shapes. A state-of-the-art review is presented along with a summary of an analytical finite element study of the web crippling behaviour of a wide range of rolled W and WWF shapes. Based on the numerical results, the equivalent column concept was developed, and subsequently verified experimentally, for both interior loads and end reactions, for a large range of W shapes. Use of the equivalent column concept and the equivalent slenderness parameter, λ_e , in one of the column curves given in Clause 13.3.1 gives a simple, easy to comprehend, and unified

approach for determining the web yielding, inelastic and elastic web buckling capacity of sections subjected to concentrated loads. The mean test to predicted ratio and the corresponding coefficient of variation for the proposed design rules, with the simplified K value of 0.6 which represents realistic load and boundary conditions, are 1.063 and 0.098, respectively.

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List of Symbols

a	Half shear distance.
a_1, a_2	Constants.
A	Area of equivalent column.
b	Width of the flange.
B_r	Factored bearing resistance as defined by CAN/CSA-S16.1-M89.
c	Length over which the load acts.
d	Overall depth of web plate.
D	Plate rigidity.
$[D]$	Elastic matrix.
$[D]_{ep}$	Elasto-plastic constitutive matrix.
E	Modulus of elasticity.
E_{st}	Strain hardening modulus.
f	Yield function.
F_y	Minimum yield stress.
G	Shear modulus.
h	Depth of the web.
H	Hardening function.
$[H]$	Transformation matrix.
$[H]^T$	Transpose of the transformation matrix.
I	Moment of inertia.
I_1, I_2, I_3	Stress invariants.
J'_1, J'_2, J'_3	Deviatoric stress invariants.
k	Distance from outer face of the flange to web toe of fillet.
k_c	Critical buckling coefficient.
K	Effective length factor.
l, L	Span length.
M_f	Plastic moment in the flange.

M_w	Plastic moment in the web per unit width of web.
M_u	Ultimate global bending moment capacity of web.
M_{u0}	Panel's ultimate capacity under global bending moment only.
n	Number of tests.
N	Length of bearing of an applied load.
P	Direct load on web.
P_{cr}	Elastic buckling load.
P_{ewb}	Elastic buckling capacity as defined by Eq. (3.16), $K = 0.6$.
$P_{exp.}$	Ultimate load determined experimentally.
P_{Lusas}	Ultimate load predicted by LUSAS FEA.
$P_{pred.}$	Ultimate load predicted by the equivalent column concept, Option A (13.3.1).
$P_{Roberts81}$	Ultimate load predicted by Roberts (1981) solution.
$P_{S16.1}$	Ultimate load predicted by S16.1 Clause 15.9(b).
P_u	Ultimate direct load that can be carried by the girder.
P_{u0}	Ultimate direct load in absence of global shear or bending.
P_y	Yield load.
P_{yield}	Ultimate load predicted by S16.1 Clause 15.9(a).
P_{y10t}	Yielding capacity based on $w (c + 10 t) F_y$.
Q	Plastic potential.
Q_u	Ultimate global shear capacity.
Q_{u0}	Panel's ultimate capacity under global shear only.
r	Radius of gyration.
t	Plate or flange thickness.
w	Thickness of the web.
V	Coefficient of variation.
x, y, z	Rectangular coordinate system.
$\bar{X}$	Mean value.
y_0	Initial out-of-plane deflection.
α	Distance between yield lines.

α_{Lusas}	Distance between yield lines determined by LUSAS FEA.
$\alpha_{Roberts}$	Distance between yield lines determined by Roberts (1981) solution.
β	Distance between plastic hinges formed in the flange or reliability index.
β_T	Target reliability index.
β_1	Distance between edge of the load and plastic hinge .
β_2	Distance between plastic hinges.
δ_h	Horizontal deflection.
δ_v	Vertical deflection.
δ_{ij}	Kronecker delta.
η	Length of the web beneath the load which yielded in compression.
$\epsilon_a, \epsilon_b, \epsilon_c$	Strains in directions a, b, and c.
ϵ_f	Failure strain.
ϵ_{ij}	Strain vector.
$d\epsilon_{ij}$	Total strain increment.
$(d\epsilon_{ij})_e$	Elastic strain increment.
$(d\epsilon_{ij})_p$	Plastic strain increment.
ϵ_{st}	Strain hardening strain.
ϵ_u	Ultimate strain.
ϵ_x, ϵ_y	Strains in x and y directions.
ϵ_y	Yield strain.
$\epsilon_1, \epsilon_2, \epsilon_3$	Principal strains.
γ_{xy}	Shear strain.
λ	Slenderness parameter.
λ_e	Equivalent slenderness parameter.
$d\lambda$	Plastic multiplier.
ν	Poisson's ratio.
Φ	Resistance factor.
ρ	Mean of measured value to nominal value.
σ_b	Global bending stress.

σ_f	Flange yield stress.
σ_{ij}	Stress vector.
$d\sigma_{ij}$	Stress increment.
σ'_{ij}	Deviatoric stress vector.
$d\sigma'_{ij}$	Deviatoric stress increment.
σ_p	Stress at the proportional limit.
σ_u	Ultimate stress.
σ_w	Web yield stress.
σ_y	Yield stress.
σ_{ys}	Static yield stress.
σ_x, σ_y	Principal stresses in x and y directions.
$\sigma_1, \sigma_2, \sigma_3$	Principal stresses.
$\bar{\sigma}$	Effective or equivalent stress.
θ	Angle that web makes with the horizontal when crippling.
$\theta_a, \theta_b, \theta_c$	Orientation of the rosette.
θ_1	Orientation of principal strains.

Chapter 1

INTRODUCTION

1.1 General

Concentrated loads from wheel load on gantry girders, purlins or beams onto the main frame members of buildings, and roller loads during launching of plate and box girders can cause local failure of the web in the form of elastic or inelastic web instability (web crippling) or web yielding, which may curtail the members overall capacity. Schematic diagrams of these basic failure modes are shown for both wide flanges and built-up girders in Fig. 1.1 and for the web's capacity versus a slenderness parameter in Fig. 1.2. The latter figure is analogous to that for column or plate buckling where the web's behaviour varies from elastic buckling to yielding as the web becomes stockier. The figure also indicates that there may be some post-buckling strength or increased capacity due to restraint provided by the compression flange to the web for both elastic and inelastic buckling regions. To prevent these modes of premature failure the designer will either stiffen and strengthen the web with transverse and/or longitudinal stiffeners or increase its capacity by increasing the web thickness. From an economical point of view, it is best to keep the fabrication simple and to use as few web stiffeners as possible.

Since 1906, about 653 tests have been conducted on built-up rolled sheet steel shapes and welded plate girders to determine the effects of web thickness, flange rigidity, load eccentricity, and material properties on the web's capacity. In addition, analytical and numerical solutions for elastic behaviour of web plates and the ultimate capacity of stiffened plate girder web panels have been developed. In general, the analytical and numerical solutions put-forth were developed for plate girder web panels with assumed idealized boundary conditions, geometry, material behaviour, and load configurations and neglect the effects of material and geometric nonlinearities and geometric imperfections which play an important role in determining the web capacity. The remaining portion of analytical work deals with the development of plastic mechanisms for the prediction of failure loads. The corresponding equations are semi-empirical in nature.

Many formulae have been developed from this work for determining the ultimate capacity of the web. Over the years, several of these empirical and semi-empirical expressions have been adapted and adopted as design rules by specifications and standards in North America. These, however, are only applicable to steel sections with a slenderness ratio h/w and for flange to web thickness ratio t/w values that are generally outside the range of most of the W shapes and WWF shapes produced by Canadian and American Mills. There are no specific design rules for hollow structural sections (HSS).

Definitions used throughout this thesis for section geometry; loading and support conditions; cross section types; loading and failure conditions are shown in Figs. 1.3, 1.4, 1.5, and 1.6, respectively.

1.2 State-of-the-Art Review

Fig. 1.7 is a flow chart which illustrates the development of our knowledge base and the research conducted on the behaviour of webs under concentrated edge loads.

Also included in the following sections the analytical and experimental research on web

buckling: elastic and inelastic behaviour, the effect of global bending stresses, and the applicability of the current S16.1 and LRFD design provisions.

1.2.1 Web Buckling: Elastic Behaviour

At the beginning of this century, Sommerfield (1906) and Timoshenko (1910) provided the first approximate solutions to the problem of the buckling of a simply supported plate subjected to equal and opposite concentrated forces applied at midpoints of the long edges as shown in Fig. 1.4(a). A more rigorous solution of this problem was presented by Leggett (1937) about 30 years later. Leggett used a stress function in the form of an infinite series to represent the membrane forces in the plate prior to buckling. Leggett's solution has been confirmed using the finite element method. The results indicate that Timoshenko's approximate solution is about 25% in error for plates having l/h equal to unity. The error however, reduces to approximately 12.5% for long plates with l/h approaching infinity.

Girkman (1936) was the first investigator to study the problem of buckling of a rectangular plate subjected to discrete edge loading at the middle of one longitudinal edge and supported at its two vertical edges as shown in Fig. 1.4(b). Zetlin (1955) provided a more detailed study of the problem of buckling of flat rectangular plates subjected to loads distributed over short portions of the edges. Zetlin assumed the plate to be simply supported along all four edges and that the applied load, symmetrically distributed about the mid-span section, was supported by shear stresses, distributed parabolically over the two ends as shown in Fig. 1.4(e). The solutions of the partial differential equations, made by means of Rayleigh-Ritz energy method give a buckling coefficient, k_c , from which the critical buckling load can be calculated as;

$$P_{cr} = k_c \frac{\pi^2 D c}{l^2} \quad (1.1)$$

where, P_{cr} = elastic buckling load, N

k_c = buckling coefficient,

l = length of the web plate panel, mm

c = length of over which the load acts, mm

D = plate rigidity given by $\frac{Et^3}{12(1-\nu^2)}$, $N.mm$

E = modulus of elasticity, MPa

t = plate thickness, mm

ν = Poisson's ratio.

White and Cottingham (1962) studied the buckling of rectangular web plate when loaded and supported as illustrated in Fig. 1.4(d). The authors used the finite difference as a numerical solution to the problem. Seven different load combinations were investigated for clamped and simply supported plates having height-to-width ratios of 1/3 to 3. The results were presented in the form of charts, with the buckling coefficients k_c plotted as a function of the plate aspect ratio h/l for each loading case.

Rockey and Bagchi (1970) solved similar problem as in Fig. 1.4(d). They presented a solution of buckling of the web plate of a conventional plate girder under the action of a partial edge load applied in the plane of the web and perpendicular to the flange. The authors used the finite element method including geometric nonlinearity. Based on these analyses, curves were developed illustrating relationship between the applied buckling edge load and the aspect ratio, flange to web thicknesses ratio, and the ratio of the length of the patch load to the width of the plate.

Khan and Walker (1972) obtained solutions for the problem illustrated in Figs. 1.4(b) and 1.4(c) using a modified form of the potential energy equation for plate buckling as suggested by Alfutov and Balabukh (1967). The applied loads are distributed over a finite length c . The elastic buckling coefficient are within 5% of those calculated using the finite element solutions of Rockey and Bagchi (1970).

Bagchi and Rockey (1975) continued this work on web behaviour, to determine the post-

buckling behaviour of a web plate under partial edge loading. The authors presented the results of a large deflection elastic finite element solution. Comparisons were made with existing experimental results conducted by Rockey et al (1972), Khan and Walker (1972), Skaloud and Novak (1972), Rockey and Bagchi (1970), and Bergfelt (1971) and on three additional test specimens by the authors. The results were also compared with the results obtained from the finite element analysis and the trend of the data was found to be in close agreement.

The presence of either in-plane moment or shear will reduce the applied edge load necessary to buckle the panel. These effects were studied and presented in a series of papers by Khan and Johns (1975) and Khan, Johns and Hayman (1977). The modified potential energy equation proposed by Alfutov and Balabukh (1967) was used to study the buckling of plates subjected to a variety of loading conditions. In their studies, Khan et al (1972, 1975 and 1977) included the effects of localized edge loading combined with or without in-plane bending and shear. The resulting work culminates with an expression for elastic buckling of webs which is similar to that of thin plates, Bleich (1959), and is given as;

$$P_{cr} = k_c \frac{\pi^2 D}{h} \quad (1.2)$$

where, P_{cr} = elastic buckling load, N
 h = depth of the plate, mm .

The results of combined loadings were presented in the form of interaction curves for the buckling coefficient k_c and the loadings.

1.2.2 Web Crippling and Web Yielding: Inelastic Behaviour

Residual stresses, initial imperfections, and localized loading coupled with global response are but a few of the factors which influence the inelastic behaviour of webs under concen-

trated loads and make the problem of developing a rigorous closed form solution complex if not impossible. Some simplified solutions based on assumed collapse mechanisms, as deduced from experimental observations, have been developed, but, these have limited applicability. The experimental and analytical research leading to the development of those empirical or semi-empirical formulae for predicting the ultimate loads of plate girders subjected to loading over a finite length, with or without in-plane bending moment, and shear are presented in this section.

Ketchum and Draffin (1932) tested 66 light I-shape beams, as shown in Fig. 1.5(a), for compression of the web over the end support bearing block. The tests were planned to give information on the inelastic buckling mode of failure seen in standard beams. The authors concluded that the failure of the beams was generally due to the inelastic buckling of the web. In the cases of very narrow bearing blocks and a distance between the stiffener and the bearing block of $h/2$ or less the failure is rather direct yielding of the material at the web-flange junction.

Winter and Pian (1946) reported on the results of 136 tests carried out to determine the ultimate strength of thin-walled webs subject to concentrated loads or reactions. Of the 136 tests performed on 18 different types of simply supported beams, 110 were made on double-web specimens, and the remaining 26 tests were carried out on single-web specimens as shown in Fig. 1.5(b). Four different loading conditions, simulated in the test program are illustrated in Fig. 1.6. For the tested range, the test specimens of equal web thickness had approximately the same bearing resistance, irrespective of the depth of the specimen. The types of failure revealed that crippling of the web consisted principally in lateral distortion of the part of the web directly adjacent to the applied load (highly stressed regions). The failure did not occur under a simple yielding but rather an elastic or plastic instability beneath the applied load. In addition, the bearing resistance for interior loading was roughly twice the bearing resistance for end loading. The authors reasoned that this was the case as the stresses were distributed symmetrically for webs that were loaded on the interior and to one side only for webs that were loaded on the

end resulting in webs having only half the strength. Based on the test results and the conclusions put-forth, Winter and Pian developed the following two empirical formulae to predict the web crippling per web;

$$P_u = \left(15 + 3.25 \left(\frac{c}{w} \right)^{0.5} \right) w^2 \sigma_w \quad (\text{Interior loading}) \quad (1.3)$$

$$P_u = \left(10 + 1.25 \left(\frac{c}{w} \right)^{0.5} \right) w^2 \sigma_w \quad (\text{End loading}) \quad (1.4)$$

where, P_u = ultimate load, *lb.*

w = web thickness, *in.*

σ_w = web yield stress, *lb./sq.in.*

As part of an experimental program designed to investigate the strength, stress distribution, and stiffening requirements of two-way and four-way direct welded beam-to-column connections, Graham et al (1959) tested 11 column stubs to determine a column web buckling criterion. The schematic diagram of the test and load application shown in Fig. 1.6(c) simulates the compression on the column web generated by the compressive force in the beam flanges framing into the connection. Based on experimental evidence, an estimate of the strength of the compression region of a column was obtained by assuming that the resistance of the column web is equal to:

$$P_u = w (t + 5 k) \sigma_y \quad (1.5)$$

where, P_u = resistance supplied by column web, *N*

t = beam flange thickness (load length), *mm*

σ_y = yield stress, *MPa*

k = distance from outer face of the flange to web toe of fillet, *mm.*

Eq. (1.5) was adopted into the working stress standards for web yielding under concentrated loads and further revised for subsequent limit states standards giving the current

expressions in Clause 15.9 (i) (a) and (b) in CAN/CSA-S16.1-M89, CSA (1989).

In the early sixties, Granholm (1961) carried out a series of tests on slender plate girders to investigate the behaviour of webs subjected to bending, shear and localized edge loading. Based on results of these tests Granholm proposed the single formula for predicting the ultimate load for web crippling:

$$P_u = 8.5 w^2 \quad (1.6)$$

where, P_u = ultimate load, kN .

Good agreement is noted between the ultimate experimental loads and the predicted loads by Eq. (1.6) with a test to predicted ratio of 1.17 and a coefficient of variation of 0.12.

Bossert and Ostapenko (1967) tested 10 stiffened plate girders, with a web slenderness ratio of 296, under a uniformly distributed edge load over one panel to determine the failure load. In addition, the finite difference method was used to determine the elastic buckling strength of the plates. Based on this numerical and experimental data the authors developed an empirical relationship between the elastic buckling load and ultimate strength incorporating the influence of bending stresses and aspect ratio.

The work started by Granholm (1961) was continued by Bergfelt et al during the period of 1968 to 1971. Extensive experimental investigations were carried out to know the influence of parameters such as the web and the flange thicknesses, the flange and web yield stresses, the girder depth and length of load area on the web ultimate load-carrying capacity under edge loading. In these tests, it was demonstrated that failure involved a combination of yielding and buckling of the web, and that the ultimate carrying capacity of the web under edge loading can be increased by about 1.7 times with an increase in the flange to web thickness ratio $\frac{t}{w}$ from 1.87 to 4.69. The influence of the length of the load c was examined, and an increase in the ultimate capacity up to a maximum of 30% was found possible with an increase in the c from 0.0 to 180.0 mm. Bergfelt

(1971) proposed an empirical equation for predicting the ultimate load for web crippling to improve Granholm's formula by introducing the effect of the modulus of elasticity E .

Skaloud and Novak (1972) studied the influence of flange stiffness upon the ultimate load behaviour and incremental collapse of thin webs subjected to partial edge load. A total of 21 tests were performed. The analysis of the experimental results shows the considerable influence of flexural stiffness the flange has on the post-buckled performance of the web. Because of the short spans of the test specimens the flange capacity was not fully utilized to carry the bending moments. Hence, even after the apparent inelastic instability of the web, the web was able to sustain further load (post-buckling strength) as a result of the bending resistance of the flange. This increase in apparent capacity represents the flanges contribution to carrying the load. This contribution would diminish of course as the requirements of the flange to carry bending moments increase.

Rockey, Elgaaly and Bagchi (1972) conducted a study to obtain a relationship between the ultimate load capacity of a web panel, as shown in Fig. 1.5(c), and its elastic buckling load. In the study, a linear elastic finite element model was used to determine the elastic buckling load of a rectangular web plate subjected to concentrated load as shown in Fig. 1.4(b). The results of the finite element analysis were presented in the form of an interaction curve relating the concentrated load to the in-plane moment or in-plane shear. The ultimate load was determined experimentally. The test results showed that an increase of the web slenderness h/w from 94 to 325 resulted in an increase of the load ratio P_u/P_{cr} from 0.59 to 2.17. This ratio increases from 1.01 to 1.48 with the increase in the relative load length c/l from 0.1 to 0.5. Failure of all test panels was defined by the formation of localized yield lines under the applied load. Each yield line corresponds closely to a segment of a circle with a horizontal length equal to that of the concentrated load.

Elgaaly et al (1973 and 1975) presented the results from tests on 20 open thin-walled specimens with the cross-section shown in Fig. 1.5(c) in which the objective of the experimental research program was to investigate the overall behaviour of web panels subjected

to concentrated loading together with in-plane bending moments or uniform shear. The results from tests have shown that failure occurred by local crippling just under the concentrated load and that the presence of in-plane bending moment or shear reduces the web's ultimate load-carrying capacity. Empirical interaction relationships, relating the ultimate load ratio P_u/P_{u0} to the ultimate bending moment ratio M_u/M_{u0} and the ultimate shear ratio Q_u/Q_{u0} were developed. The subscript "0" in P_u , M_u , Q_u indicates the web ultimate capacity due to one in the absence of the other two.

Further tests by Bergfelt and Lindgren (1974) to study the effect of parameters such as web and flange thicknesses and material properties, resulted in several semi-empirical formulae to predict the ultimate capacity of the web under a concentrated load. The authors modelled the web crippling failure under a point load (semi spherical rocker) using the three hinge flange model developed by Bergfelt (1971). The beam described by the authors is analogous to a fixed ended beam (top flange) on an elasto-plastic foundation (web). This mechanism forms as follows:

- At small load (elastic range), the flange behaves as a beam on an elastic foundation consisting of the web.
- At increasing load (yielding starts), a plastic hinge forms in the top flange just under the point load.
- The web starts yielding below the formed hinge, then the yielding region in the web extends in depth and width.
- Vertical deformations of the top flange and out-of-plane deformations of the web start increase rapidly.
- The failure occurs when two plastic hinges form in the negative moment region on either side of the point load leading to a three hinge mechanism.

In the case where the load is transferred through a very stiff plate, hinges form in the flange at each end of the load plate creating a four hinge mechanism.

Based on experimental results and some assumptions as to how the flange interacts with the web, Herzog (1974), Dubas and Gehri (1975 and 1978), and Skaloud and Drdacky (1975), proposed empirical expressions to predict the web crippling load of web plate girders under concentrated loading. Skaloud and Drdacky web crippling formula has been incorporated in the Czechoslovakian design rules for plate girders.

Experimental evidence, Elgaaly et al (1972, 1973, and 1975), has shown conclusively that the buckled shape of crippled webs is formed like an arc of a circle (maximum out-of-plane displacement) and lies directly under the load. Plastic hinges form in the loaded flange perpendicular to the length of the beam (two at the edge of the loaded surface and two additional at a distance β on either side of the first two, see Fig. 1.8) and yield lines in the web along the flange web junction and along the arc described previously. Roberts and Rockey (1979) presented a lower bound plastic mechanism model based on the development of four plastic hinges and three yield lines as shown in Fig. 1.8. Using the conservation of energy and minimizing the ultimate load with respect to β , the authors developed two semi-empirical formulae to predict web crippling and web yielding with the lesser of the two governing the webs capacity.

$$P_u = \frac{4M_f}{\beta} + (4\beta + 2c - 2\eta) \frac{M_w}{\alpha \cos \theta} \quad (\text{Web crippling}) \quad (1.7)$$

$$P_u = 2(4M_f \sigma_w w)^{0.5} + \sigma_w w c \quad (\text{Web yielding}) \quad (1.8)$$

where, P_u = ultimate load, N

β = location of plastic hinges defined in Fig. 1.8, mm

θ = angle of yield lines defined in Fig. 1.8, mm

α = location of yield lines defined in Fig. 1.8, mm

η = length of the web which yielded in direct compression, mm

M_w = web plastic moment per unit width of web, $N.mm/mm$

M_f = plastic moment capacity of the flange, $N.mm$.

Roberts (1981) conducted 29 tests on slender short span plate girders subjected to local

edge loading in the plane of the web. The objectives of the experimental research were to continue the previous work; to explore the effect of geometric parameters (w , t , d , σ_w , σ_f) on the behaviour and ultimate load; and to refine the previous expressions, Roberts et al (1979). Adopting the same collapse mechanism, Roberts simplified the web crippling equation by modifying α and neglecting few terms. The web yielding expression was not changed. The web crippling formula can be written as follows:

$$P_u = 0.5 w^2 \left(E \sigma_w \frac{t}{w} \right)^{0.5} \left(1.0 + \frac{3c}{d} \left(\frac{w}{t} \right)^{1.5} \right) \quad (\text{Web crippling}) \quad (1.9)$$

where, P_u = web crippling load, N

t = flange thickness, mm

d = web panel depth, mm .

Modified versions of the mechanism solutions were proposed by Roberts and Chong (1981) for situations where the edge loading is uniformly distributed between vertical web stiffeners. It was assumed that three plastic hinges form in the flange, two adjacent to and one mid-way between the vertical web stiffeners. Again following the same steps described in Roberts and Rockey (1979) mechanism solution and using the assumption of the simplified solution developed by Roberts (1981), Roberts and Chong (1981) derived two formulae for ultimate loads. The collapse load is taken as the lesser of the two values.

Roberts and Markovic (1983) conducted a series 10 tests on plate girders each subjected to local edge loads in the plane of the web, with varying geometric parameters; web thicknesses from 3 mm to 10 mm, web slenderness ratio from 50 to 166, and flange to web thickness ratio from 1.01 to 1.97. The objective of this study was to determine the accuracy of Roberts (1981) formulae in predicting the load carrying capacity of the web. Tests results for rolled I-beams, with web thickness, web slenderness ratio, and flange to web thickness ratio ranging from 4.1 mm to 13.0 mm, 17. to 45.8, and 1.39 to 1.75, respectively, from Dubas and Gehri (1978), were added to the database and compared with Roberts (1981) formulae. In all the cases where the shapes were stocky ($h/w \leq 50$),

the predicted yielding capacity of the web was smaller than the predicted crippling load. In the case of $t/w \leq 3$, the predicted web crippling capacity is grossly underestimated. The authors stated that all of the tested shapes failed by bending (eventhough the yielding load was smaller), and that the predicted crippling load is satisfactory if $t/w \geq 3$ and $c/d \leq 0.2$. In the case of $t/w \leq 3$, the ratio of 3 should be in the crippling formula.

Elgaaly and Nunan (1989) researched the behaviour of rolled section web under eccentric edge compressive load with the eccentricity of the load varying from 0.0 to 12.7 mm. Tests were conducted on 22 test specimens. As would be expected the load carrying capacity was reduced as the eccentricity was increased. A linear elastic finite element model was developed to evaluate the test results. The elastic model limits the comparison to the initial stages of loading prior to any large out-of-plane displacement and inelastic behaviour (local yielding).

Shimizu, Yabana, and Yoshida (1989) conducted a series of 7 tests on web plate girders subjected to concentrated load in the plane of the web. The objective of the experiment was to develop a new collapse model for loaded web plates (concentrated loads) based on the Roberts and Rockey's (1979) mechanism solution. On each test model, two yield lines formed, encircling the bearing block. The upper portion of the web plate remained at a right angle to the upper flanges and the location of the four plastic hinges in the top flange was not the same. The procedure to estimate the ultimate load however, is similar to that adopted by Roberts and Rockey (1979). Following the steps of that solution, Shimizu et al (1989) derived the following web crippling load:

$$P_u = \frac{4M_f}{\beta_2} + (4\beta_1 + 4\beta_2 + 2c - 2\eta) \frac{M_w}{\alpha \cos \theta} \quad (\text{Web crippling}) \quad (1.10)$$

where, P_u = web crippling load, N

β_1 = distance between edge of the load and plastic hinges, mm

β_2 = distance between plastic hinges, mm .

Shimizu, Horii, and Yoshida (1989) presented elasto-plastic analyses to study the effects of the upper flange torsional stiffness and the length over which the load acts as a ratio of the panel length on the behaviour of the loaded web plates. An iterative numerical analysis method known as the Dynamic Relaxation Method (DRM) was used as a numerical tool. It was concluded that for very stiff flanges (almost fixed junction) only two yield lines form, whereas for a thin flange (flexible junction) two to three yield lines would form as the torsional stiffness lessen.

Elgaaly et al (1990, 1991, and 1992) conducted a series of experimental and analytical studies researching the behaviour of rolled shapes (produced by American mills), with stiffened and unstiffened webs subjected to in-plane and eccentric concentrated edge loads. For unstiffened webs with concentrated in-plane edge loads, shapes with stocky webs, $h/w \leq 60$, failed by yielding and those with slender webs, $h/w \geq 100$, failed by crippling. As expected, the ultimate strength of the web decreased with increasing eccentricity (with respect to the plane of the web) of the load. The test results were compared with those determined from a finite element analysis using NONSAP, isoparametric shell elements, geometric and material nonlinearity, and an improved convergence criterion. The mean test to predicted value and the corresponding coefficient of variation were 1.02 and 0.06, respectively.

1.2.3 Effect of Global Bending Stresses

Skaloud and Novak (1972), Elgaaly and Rockey (1973), Herzog (1974), and Roberts and Rockey (1979) have concluded from their research that;

- the in-plane normal stresses due to bending will decrease the webs capacity to carry in-plane concentrated loads, and
- that the flange ability to provide restraint to the web under load diminishes as their capacity is used to carry in-plane normal forces due to bending.

The reduced capacity is given by multiplying the webs capacity by the reduction factor given in its general form in Eq. (1.11):

$$\left[1 - \left(\frac{\sigma_b}{\sigma_w}\right)^{a_1}\right]^{\frac{1}{a_2}} \quad (1.11)$$

where, σ_b = in-plane normal stress due to bending, *MPa*

a_1, a_2 = curve fit constants evaluated for the given researcher.

Note, most beams of practical lengths will reach their moment capacity before their webs capacity is reached. The flange contribution should be neglected for these cases. Only for very short beams or beams on elastic foundations will the webs capacity for concentrated load govern the design.

1.2.4 Applicability of Current Design Provisions

Both the American specification, Load Resistance Factor Design for Structural Steel Buildings (1986), and the Canadian standard CAN/CSA-S16.1-M89 have similar empirical semi-rational expressions to describe the capacity of webs of rolled steel shapes for the local failure modes of web crippling and web yielding. Variations in the constants results from the system of units used. Illustrated by the expressions in Clause 15.9 of CAN/CSA-S16.1-M89, with modified nomenclature, these are given for web yielding and crippling for an interior load point, for example, by Eqs. (1.12) and (1.13) respectively.

$$B_r = 1.10 \Phi w (5k + c) \sigma_w \quad (1.12)$$

$$B_r = 300 \Phi w^2 \left[1 + 3 \left(\frac{c}{d}\right) \left(\frac{w}{t}\right)^{1.5}\right] \sqrt{\sigma_w \frac{t}{w}} \quad (1.13)$$

where, B_r = factored bearing resistance, *N*

Φ = resistance factor, 0.9.

The applicability of these design equations is in question as;

1. the web yielding equations are based on Graham et al tests on welded interior beam-to-column connections, Graham et al (1959). Differences in application of load (load transfer from the beam flange to the column web), its small load length, the beam flange thickness and weld, along with the calibration coefficient 1.10 (calibration to the previous working stress standard) make the applicability of Eq. (1.12) for web yielding suspect; and
 2. the web crippling equations are based on an assumed collapse mechanism derived by Roberts (1981), and as deduced from the experimental work of Bergfelt et al (1968, 70, 71), Skaloud and Novak (1972), Bergfelt and Lindgren (1974), Bagchi and Rockey (1975) Drdacky and Novotny (1977), Bergfelt (1979), and Roberts (1981). Of these, there are a limited number of tests with a similar set of geometric parameters as those produced in Canadian mills, and/or tests with appropriate boundary conditions at the load points, and/or tests with practical load lengths. A summary of the range of geometric data for existing test data, W and WWF shapes is listed in Table 1.1 and shown graphically by histograms for web thickness, web slenderness, flange to web thickness ratio, flange thickness, and flange slenderness in Figs. 1.9 to 1.13, respectively. As developed, the semi-empirical web crippling is restricted to sections and load lengths that meet the following geometric parameters, $t/w \geq 3$ and $c/d \leq 0.2$. As shown in Fig. 1.11, 97% of the shapes produced by Canadian mills have values of $t/w < 3$ leading to a gross underestimation of the crippling capacity of most W and WWF shapes unless the lower limit of 3 is set, Roberts (1981), (not verified by appropriate tests). The other limit, $c/d \leq 0.2$ restricts the load length (non practical values) for shallow sections; i.e. W360, $c \leq 72 \text{ mm}$. Note, these limitations do not appear in either the standard or the commentary.
- More recent tests were conducted by Elgaaly and Nunan (1989) on eccentrically loaded webs and by Elgaaly et al (1990, 1991, 1992) on geometrically similar sections with varying stiffening details and stiffened panel lengths. The latter tests (the only

ones applicable) were compared with the equations for web yielding and crippling in LRFD, AISC (1986), (similar to those found in S16.1-M89, CSA 1989) and were reported to be satisfactory in most cases. Insufficient data was given to make a statistically relevant comparison. The range of sections tested covers a limited range of available shapes.

As described later in this thesis, the three yield line collapse mechanism developed by Roberts (1981) is dependent upon web thickness, web slenderness, and flange to web thickness ratio parameters. The collapse mechanism of rolled shapes, especially with thicker webs in the range of those produced in Canadian mills, is different, where only two yield lines may develop within the web. Residual stresses, initial imperfections, and localized loading coupled with global response (in-plane normal stresses due to bending) are but a few of the factors which influence the inelastic behaviour of webs subjected to concentrated loads and must be incorporated into a series of design expressions for practical rolled section sizes. Currently, design standards and specifications do not consider the behaviour of webs of HSS under concentrated loads.

1.3 Objectives and Scope

The objectives of this analytical and experimental study are:

1. to compare finite element predictions with existing experimental evidence to calibrate the analytical model;
2. to conduct a numerical study to examine the behaviour of rolled shapes (W and WWF) when subject to concentrated loads;
3. to select and test a representative sample of W and WWF shapes that will describe the full range of behaviour from web yielding to inelastic web buckling to elastic web buckling;

4. to compare finite element predictions with the response of this test group; and
5. to develop rational mathematical models to describe the behaviour of webs of W and WWF shapes subjected to concentrated loads. From these, simplified design expressions will be developed for inclusion in the standard.

The testing and analytical comparisons of WWF shapes are part of a follow-up Master theses. The study of the behaviour of HSS subjected to concentrated loads lies outside the scope of this work.

1.4 Outline of the Thesis

The thesis first presents a general introduction describing the failure of webs subjected to concentrated loads, a state-of-the-art review, and a discussion about the applicability of current design rules given in North American standards and specifications.

Chapter 2 details the application of the finite element method in modeling the tested shapes and others using the FEA LUSAS program. Simplified design rules to predict failure loads are developed in Chapter 3. A detailed description of the experimental program is given in Chapter 4. The test results are presented in Chapter 5. Comparisons between the test results, the finite element predictions, and loads predicted using Roberts solution, Roberts (1981), and the equations for web crippling in S16.1-M89 are presented in Chapter 6. Conclusions and recommendations from this study are given in Chapter 7.

Figure 1.1: Failure Modes of W and WWF Shapes.

Figure 1.2: Ultimate Bearing Capacity of Webs.

Figure 1.3: Definitions of Section Geometry and Loads.

Figure 1.4: Loading and Support Conditions for Rectangular Plate Subjected to Edge Load.

Figure 1.5: Typical Cross-Section Shapes Tested.

Figure 1.6: Loading and Failure Conditions.

Figure 1.7: Research Development for Web buckling.

Figure 1.7: Research Development for Web buckling con't.

Figure 1.7: Research Development con't.

Figure 1.8: Roberts Mechanism of Failure.

Figure 1.10: Web Slenderness Histogram.

Figure 1.11: Flange to Web Thickness Ratio Histogram.

Figure 1.12: Flange Thickness Histogram.

Figure 1.13: Flange Slenderness Ratio Histogram.

Table 1.1: Range of Geometric Data for existing tests, W and WWF Shapes.

Reference	n	t mm	w , mm	d , mm	h/w ,	$b/2t$,	c , mm	Type of Shape
Ketchum and Draffin (1932)	66	4.34-5.72	3.05-4.83	152.4-304.8	42.5-68.3	10.2-11.7	12.7-152.4	R.I.B.
Winter and Pian (1946)	136	1.17-3.75	1.17-7.51	190.1-205.7	13.7-172.1	18.6-54.0	20.3-88.9	R.S.S.
Zetlin (1955)	20	0.152	0.152	76.2 - 304.8*	501.3-2005.3	---	4.7-254.0	Hat
Granholm (1961)	11	9.00-10.00	2.20-4.60	308.0-620.0	129.6-193.5	10.0	120.0-150.0	W.P.G.
Bossert and Ostapenko (1967)	10	15.88	2.88-3.09	914.4	285.8-309.2	6.4	711.2-1447.8	W.P.G.
Bergfelt et al (1968,70,71)	23	6.00-15.30	2.00-3.40	300.0-700.0	144.0-335.0	3.3-12.5	0.0-200.0	W.P.G.
Khan and Walker (1972) †	30	---	---	---	90-200	---	---	Plate
Skaloud and Novak (1972)	21	4.97-30.50	2.00-3.00	500.0-1000.0	190.0-395.6	1.0-14.5	50.0-200.0	W.P.G.
Rockey et al (1972)	28	0.94-3.25	0.94-3.25	304.8*	93.8-324.3	48.2 - 56.7†	30.5-152.4	H.I.W.
Elgaaly and Rockey (1973)	14	0.94-2.59	0.94-2.59	304.8*	117.6-324.3	46.7 - 55.5†	61.0	H.I.W.
Bergfelt and Lindgren (1974)	88	2.90-18.30	2.09-5.83	507.8-736.6	120.1-241.4	9.6-19.4	0.0-100.0	W.P.G.
Elgaaly (1975)	18	0.94-1.52	0.94-1.52	304.8*	200.0-324.3	51.9 - 53.7†	61.0	H.I.W.
Bagchi and Rockey (1975)	3	12.7	3.25	635.0	187.6	6.0	50.0-75.0	W.P.G.

Table 1.1: Range of Geometric Data for existing tests, W and WWF Shapes con't.

Reference	n	t mm	w , mm	d , mm	h/w ,	$b/2t$,	c , mm	Type of Shape
Drdacky and Novotny (1977)	16	9.90-16.00	3.57-4.01	300.0-600.0	66.9-162.5	1.5-2.6	30.0-60.0	W.P.G.
Dubas and Gehri (1978)	26	5.70-28.10	4.10-18.00	100.0-362.0	15.7-45.8	4.4-9.9	0.0	W.P.G.
Bergfelt (1979)	22	5.03-15.50	2.00-3.03	300.0-800.0	129.4-394.9	9.7-12.4	40.0	W.P.G.
Roberts (1981)	29	3.05-20.06	0.99-4.94	250.0-750.0	72.9-498.9	3.7-24.4	50.0	W.P.G.
Roberts and Markovic (1983)	10	5.94-10.05	3.01-9.95	500.0	50.3-166.1	14.9-25.3	0.0-100.0	W.P.G.
Elgaaly and Nunan (1989)	22	8.89	6.35	304.8	45.2	5.7	61.0	W
Shimizu et al (1989)	7	9.00	6.00	1018.0	166.7	16.7	180.0-500.0	W.P.G.
Elgaaly et al (1990, 91, 92)	53	5.08-13.37	2.97-8.97	251.0-635.1	54.5-213.7	6.1-12.2	60.3-381.0	W
CISC Handbook	107	20.00-60.00	11.00-60.00	350.0-2000.0	6.3-97.0	3.3-12.5	—	WWF
CISC Handbook	87	6.40-35.60	5.00-22.10	152.0-635.0	11.7-63.6	4.2-11.5	—	W

Notes: * Value of h is given.

† Value of l/t for cold-formed steel as specified in CAN3-S136-M84.

‡ Values of all parameters are not available, but the following ranges are given:
 $l/h = 4.0$ or 5.75 and $0.25 \leq c/h \leq 1.0$.

R.I.B.: Rolled I-Beams.

W.P.G.: Welded Plate Girders.

R.S.S.: Rolled Sheet Steel.

H.I.W.: Hat section with Inclined Webs.

Chapter 2

FINITE ELEMENT ANALYSIS

2.1 General

A finite element analysis which includes initial imperfections (out-of-straightness), material and geometric nonlinearity is being used to investigate the behaviour of webs of W and WWF shapes (produced in Canadian mills) that are subjected to concentrated in-plane loads. The analysis will provide a direct comparison with available and new experimental data and will serve as a vehicle for numerical studies related to the influence of parameters such as the loaded flange boundary conditions, flange-web thickness ratio, and end reactions on the behaviour of the web.

The FEA LUSAS program was selected for the finite element analysis study. The beams were modelled with an assemblage of 8-node semi-loof shell elements which account for both in-plane and out-of-plane bending deformations. The nonlinear behaviour was modelled using von Mises Huber-Hencky associative flow rule isotropic hardening law and an explicit integration scheme for the plastic strains. The Newton-Raphson iteration procedure was used to achieve equilibrium. Listed in Table 2.1 are the limits of the convergence criteria used in the present analysis.

2.2 Verification of the FEA LUSAS Program

The FEA LUSAS program has been tested and compared with classical solutions by Timoshenko (1955) and Massonet and Save (1965); and with experimental results for tests on steel shapes with large deformations and inelastic behaviour by Driver and Kennedy (1987) and Roberts (1981), to verify the applicability of the type of elements, material law and solution strategy selected.

2.2.1 Cantilever Beam Loaded at its Tip

A rectangular cantilever beam with a point load at the free end, shown in Figs. 2.1(a) and (b) was discretized into 20 shell elements. The load deflection curves determined from LUSAS were compared to closed formed solutions developed by Timoshenko (1955). The first solution for a beam constructed of an elastic perfectly plastic material tests the program's ability to model the nonlinear behaviour of the material. As shown in Fig. 2.1(c), the load deflection curves predicted by Timoshenko (1955) and the FEA LUSAS program are virtually the same throughout, with a small variance of 2% at ultimate. The curves for the second solution, for an elastic beam subjected to large nonlinear deflections are shown in Figs. 2.1(d) and (e) in a non dimensional form for the load (PL^2/EI) versus the horizontal deflection (δ_h/L) and the load versus vertical deflection (δ_v/L), respectively. Again the curves compare well, verifying the nonlinear geometric predictive capacity of the program.

2.2.2 Simply Supported Beam Under a Mid-Span Point Load

A simply supported beam loaded at its middle by a point load as shown in Figs. 2.2(a) and (b) was discretized into 20 shell elements. The load deflection curves determined by LUSAS were compared to the closed formed solution developed by Massonet and Save

(1965) to verify the modelling of the boundary conditions and to verify the program's ability to predict the nonlinear behaviour of the material. Again, the curves are virtually synonymous, with first yield, development of the plastic hinge and ultimate coinciding.

2.2.3 Cantilever I-Beam Loaded at its Tip

The third example case involves the testing of a cantilever beam of I-shape cross section loaded at its free end. This case was considered because the test data, material and geometric properties are well known, Driver and Kennedy (1987), and both inelastic behaviour and large deformations occurred. The geometry, material properties and finite element idealization of the beam are shown in Figs. 2.3(a) and (b). The load was applied monotonically up to failure which occurred when a plastic hinge formed at the fixed end. The load deflection curves predicted from beam theory (elastic) and LUSAS FEA are plotted along with the test curve, Driver and Kennedy (1987), in Fig. 2.3(c). The variance in the slope of the elastic portion of the curve is due to an elastic rigid body rotation of the support (fixed end) in the test which is not modelled in the analysis. If the curve predicted by LUSAS was rotated about the origin to match the initial slope, simulating an elastic rigid body rotation, then the remaining portion overlays the test curve, verifying the program's ability to predict large deformation inelastic behaviour.

2.2.4 Inelastic Buckling of a Plate Girder

A fourth comparison was made with test results of Roberts (1981) plate girder A1-3 which was simply supported and uniformly loaded at the mid-span over a finite length c . This comparison was essential in determining whether or not LUSAS, with the proper boundary conditions, could be used to accurately model the inelastic buckling behaviour of webs of W and WWF shapes. Fig. 2.4(a) shows the plate dimensions, material properties and loading, and Fig. 2.4(b) the corresponding finite element idealization of half the

beam (symmetry). The out-of-plane deformation of the web, Roberts (1981), showed a localized deformation just beneath the top flange with a maximum negative out-of-plane deformation at $0.07 d$ from the top flange and a maximum positive out-of-plane deformation at about $0.35 d$ from the top flange. The ultimate load was $9.04 kN$. Different boundary conditions were tested to determine which best matched the behaviour of test girders. The first set of boundary conditions (B.C. 1), restrain out-of-plane deformation on all four edges of the web and restrain the vertical deflection of the web along the support. The second set of boundary conditions (B.C. 2) restrain the out-of-plane deformation of the web, restrain the vertical deflection of the bottom edge of the web, and restrain the horizontal and vertical deflection of the web edge along the support. Fig. 2.4(c) illustrates the out-of-plane deflection of the web for both boundary conditions at ultimate. The ultimate load for each case is shown alongside the corresponding curve and with the test to predicted load ratio in brackets. The best comparison for both the deflected shape and the inelastic web buckling capacity is given by B.C. 1. Variations between the experimental load value and that predicted by LUSAS may result from simplifications made in the analysis (no residual stresses and elastic perfectly plastic material) and/or from imperfect load application (eccentrically placed with respect to the web centreline; note, $w = 0.99 mm$). The ultimate load as predicted by Roberts' mechanism solution is $6.08 kN$ which corresponds to a test to predicted load ratio of 1.48.

2.3 Initial Sections Analyzed by LUSAS Program

2.3.1 Description of Initial Sections

Listed in Table 2.2 are the nominal dimensions; flange thickness, t ; web thickness, w ; flange width, b ; depth, d ; web depth, h ; flange slenderness ratio, $b/2t$; web slenderness ratio, h/w ; and the corresponding class of the section of the initial six shapes selected for this preliminary analysis stage to give elastic web buckling, inelastic web buckling, and

web yielding type failures.

The material was assumed to be perfectly elasto-plastic with a yield stress of 300 *MPa*, a modulus of elasticity of 200000 *MPa* and a poisson's ratio of 0.3. Each was analyzed using LUSAS to determine the ultimate load and failure mode. A typical beam adopted in this study, illustrated in Fig. 2.5, has a span equal to 1000 *mm* and a distributed load length of 300 *mm*. The corresponding finite element idealizations for the W410x85 and the remaining shapes are shown in Figs. 2.6(a) and (b), respectively. A total of 20 elements were used to model each flange and 100 elements for the web of the W410x85. The corresponding number of elements used for the remaining shapes was 16 and 104, respectively.

2.3.2 Modelling of W and WWF Sections

The influence of initial imperfections, flanges, and boundary conditions on the behaviour of webs subjected to concentrated loads for selected shapes was investigated.

2.3.2.1 Effect of Initial Imperfections

Two different sets of initial imperfections were tested for the WWF1400x407 section. The first was the elastic buckling mode deflected shape and the second a two dimensional sine wave with the initial imperfections given by;

$$y_0 = \frac{h}{1000} \sin\left(\frac{\pi x}{l}\right) \sin\left(\frac{\pi z}{h}\right) \quad (2.1)$$

where, h = web depth, *mm*

l = span length of the beam, *mm*

x, z = rectangular coordinates, *mm*

y_0 = initial out-of-plane imperfection *mm*.

Similar boundary conditions were considered for both analyses. The out-of-plane displaced shape for the web and the load versus out-of-plane deformation curves for both initial imperfection conditions are illustrated in Figs. 2.7(a) and 2.7(b), respectively. The shape of the web at ultimate is almost identical with the maximum deflection occurring at $h/4$ from the top flange. The ultimate loads are within 0.5%. These preliminary results indicate that the variations considered in initial imperfections had little or no impact on the behaviour of the web. Initial imperfections in the form of a two dimensional sine wave were considered for the remaining initial analytical investigation.

2.3.2.2 Effect of Flanges

The effect of flange torsional and flexural stiffness on the inelastic web crippling capacity of a W410x85 shape was investigated. Three different configurations were investigated; the whole section, the section without the bottom flange, and the section without any flanges. Fig. 2.8 shows the out-of-plane displaced shape of the web along with the corresponding ultimate loads for all three cases. The maximum deflection is similar for all cases and occurs at approximately $0.4 h$ from the top flange. As expected, the maximum out-of-plane deflection is lowest when the whole section is considered (about $2/3$ of the web section alone), as the flanges provide rotational restraint. The ultimate load obtained is 5% less when the bottom flange is neglected and 24% less when both flanges are neglected. Exclusion of the flanges in a finite element analysis would give results which are not applicable to practical problems. The contribution of both flanges is considered for the remaining initial analytical investigation.

2.3.2.3 Effect of Boundary Conditions

The boundary conditions used for Roberts (1981) A1-3 plate girder were tested on the W410x85 section. The first boundary conditions (B.C. 1) simulate the boundary conditions that will be used for the experimental investigation and the second simulate the

conditions expected in crane girders or in the launching of plate girders. The out-of-plane displaced shape of the web and the corresponding ultimate load is shown in Fig. 2.9 for both cases. The out-of-plane displaced shape varies with the maximum deflection occurring at $0.25 h$ and $0.4 h$ for boundary conditions case 1 and case 2, respectively. Although the maximum deflection for case 2 is approximately three times larger than case 1, the web failed by inelastic buckling and the ultimate loads were essentially the same. As expected, when the web becomes stocky and the failure approaches web yielding, the effect of boundary conditions on the ultimate load diminishes. The greatest difference occurs for webs that fail by elastic web buckling. Only case 1 boundary conditions will be considered for the remaining initial analytical investigation.

2.3.3 Summary of the Initial Analytical Investigation

Fig. 2.10 illustrates the changes in the out-of-plane displaced shape of the web, for the initial six shapes listed in Table 2.2, as the load increases to ultimate. As shown, there is a migration of the point of maximum out-of-plane deformation from the web mid-height towards the top flange as the load approaches ultimate. If the point of maximum out-of-plane deformation occurs close to the top flange, WWF1800x510 with h/w of 87.0, then the behaviour tends towards web yielding. If the point of maximum out-of-plane deformation occurs close to the mid-height, W410x39 with h/w of 59.7, then the behaviour tends towards elastic web buckling. Even though the WWF section is more slender, the web thickness is much greater (20 compared to 6) and dominates the probable failure mode. The other shapes failed by inelastic buckling (crippling) and this is corroborated from the predicted load values.

Table 2.3 gives a summary of the out-of-plane displaced shape data for the buckled webs. Listed are; the shape designation, maximum out-of-plane deformation y , the location of this point with respect to the top flange and given as a function of the height h for both the LUSAS prediction, α_{Lusas} and that predicted by Roberts (1981), where $\alpha_{Roberts}$ is

equal to $25 \times w$. Roberts (1981) values for the distance from the top flange to the second yield line are larger in almost every case studied. Note, for the case of W410x85, the second yield line predicted by Roberts (1981) forms very near the bottom flange, $0.72 h$, which is unlikely since the behaviour tends towards yielding and the yielded zone is just beneath the top flange.

Table 2.4 gives a summary of failure loads predicted by different models. Listed are; the shape designation, the elastic and inelastic buckling load predicted by LUSAS (P_{cr} and P_{Lusas}), the inelastic web buckling load (with limitations) as determined by Roberts' (1981) expressions $P_{Roberts}$, the web crippling load (without limitations) and yield load predicted using the expressions given in S16.1 ($P_{S16.1}$ and P_{yield}), and the ratio of elastic buckling load to the inelastic buckling load predicted by LUSAS.

The combined influence of flange web thickness ratio, t/w and web slenderness on the behaviour and ultimate load capacity is demonstrated by comparing the results from W410x39, W610x101, and WWF900x262 where the web thickness is constant. There is slight increase in ultimate load with increase t/w values. With increasing web slenderness from 35.0 for W410x85 to 76.3 for WWF900x262, the probable failure mode changes from inelastic towards elastic web buckling, as indicated by the ratio of the elastic critical buckling load to the inelastic load predicted by LUSAS. For the WWF900x262 this ratio was 1.015. In all cases, S16.1 predicts the highest ultimate loads and Roberts (1981) the lowest (lower bound solution).

2.4 Further Finite Element Analysis to Determine Test Sections

2.4.1 Test Sections Investigated

Further finite element analysis was required to select the appropriate sections that would represent the range of behaviour from elastic web buckling to web yielding for all classes of sections (defined for bending) and for the full range of geometric parameters (w , h/w). The length of beams were selected to give shear span depth ratios greater than 1.5 to advert deep beam behaviour and to prevent flexural failure. Load lengths between 100 *mm* and 300 *mm* were selected to give practical concentrated load lengths or to force a yield failure. The list of sections, geometric properties and corresponding section class used for this analytical study is given in Table 2.5.

Within this list two series of sections were analyzed with different boundary conditions. The first series includes all the seventeen W shapes and five WWF shapes listed with the same boundary conditions as in the preliminary analysis study. The second series, a subset identified by the footnote dagger (†), was analyzed with modified boundary conditions where the top flange was restrained against rotation. Of this group, the sections marked with an asterisk (*) were also analyzed to determine the end reaction capacity. Other than the boundary conditions identified, the constitutive model, analysis, and solution strategy were the same as those determined from the preliminary analysis. An idealized bilinear stress strain curve for steel with a yield stress of 300 *MPa*, a modulus of elasticity of 200000 *MPa*, a strain hardening modulus of 5000 *MPa* and a Poisson's ratio of 0.3 was used. A number of finite element idealizations were used to accommodate the wide range of sizes and shapes with the aspect ratios of the elements being between 1.0 and 4.0 at the critical sections. A typical mesh is shown in Fig. 2.11.

2.4.2 General Behaviour of W and WWF Shapes

The behaviour is best described by the deformed shape of the web, the normal stress distributions and a comparison of failure loads predicted by different solutions.

2.4.2.1 Out-of-Plane Deformation and Mechanisms

This data is presented by Figs. 2.12 to 2.14 which describe typical out-of-plane deformations of the web for elastic buckling, inelastic buckling and web yielding for W shapes with the flange free to rotate and the flange restrained against rotation; and again for WWF shapes with the flange free to rotate, respectively.

The initial imperfections for each beam are in the form of a half sine wave with the maximum imperfection equal to $1/1000$ of the depth at mid-depth. As expected the out-of-plane displacements increase with increasing load. In general, the maximum out-of-plane displacement for W and WWF shapes failing by elastic buckling is located slightly above mid-depth (given by a fraction of h measured from the top flange) and is located closer to the top flange as failure mode is inelastic buckling and closer yet for web yielding. These values are listed in Tables 2.6 and 2.7 for each series of analysis, where α_{Lusas} is the fraction determined from the results of the finite element analysis (based on the coarseness of the finite element mesh) and $\alpha_{Roberts}$ is $25 w$, Roberts (1981). Clearly, $\alpha_{Roberts}$ used by the Roberts' mechanism model is not applicable to sections that fail by web yielding and values for inelastic web buckling cases are not consistent with α_{Lusas} . The deflected shape becomes more pronounced at ultimate loads. The line denoting the maximum displacement along the length of the web and the flange web junction where significant curvature of the web occurs, indicated in Figs. 2.12 to 2.14 as Y1 and Y2, are probable yield lines in a collapse model or mechanism model. Unlike Roberts mechanism which has three yield lines, all W and WWF shapes form only two.

2.4.2.2 Normal Stress Distribution

Figs. 2.15 to 2.23 show typical normal stress distributions along the depth at various locations along the web (figure (a)), and along the length at various depths on the web (figure (b)).

Figs. 2.15 to 2.17 and Figs. 2.18 to 2.20 show typical distributions for W shapes for the boundary conditions flange free to rotate and flange restrained against rotation, respectively; and Figs. 2.21 to 2.23 show typical distributions for WWF shapes with the flange free to rotate.

The normal stresses are in the plane of the web and are normal to the flange. The curves in figures (a) described the rate that the load is taken out of the web in shear. As expected the normal load ($\int \sigma dA$) is equal to the applied load at the top flange and is equal to zero at the bottom flange. The general form of the distribution at ultimate is sinusoidal at the centre of the load. The curves given in figures (b) indicate the length of web that is effective in carrying the normal load. Based on these distributions, a simplified equivalent column model (concept) of the web with the corresponding load distribution and geometry has been developed. Refer to Chapter 3.

The effective column width must be constant for all failure modes and has been determined from data for sections that fail by web yielding. The effective width is calculated by dividing the integral of the normal stress at the critical section at ultimate by the yield stress and web thickness; i.e. $\int \sigma dA$ is statically equivalent to $\sigma_w w \times \text{effective width}$. This is illustrated graphically in Fig. 2.24 for flange free to rotate and flange restrained against rotation; and the results from four analyses are given in Table 2.8. The effective width based on the expression given in the current standard, $c + 5k$, and that of a better fit $c + 10t$ are also listed. Until these values can be verified by experimentation and by further analyses, $c + 5k$ will be used as the effective width.

2.4.2.3 Predicted Failure Loads

The failure loads for elastic buckling, inelastic buckling and web yielding, as defined in section 2.3.3, are given for each test section in Tables 2.9 and 2.10. The failure mode is determined by the lowest value of the best predictions for all three modes of failure (P_{cr} , P_{Lusas} , P_{yield}) and are listed in the last column.

2.4.2.4 Selection of Test Sections

Based on the probable behaviour and failure modes, the availability of sections, flange fixed against rotation, a representative sample (all classes, seven W shapes and five WWF shapes marked with both a dagger (†) and an asterisk (*) in Table 2.5 were selected for the experimental program. The W310x143 is rarely rolled and is still not available. The WWF shapes are being tested and analyzed as a follow-up masters thesis.

The strain distributions and displaced shapes determined by the LUSAS finite element program were used to determine the best location of strain gauges and LVDTs to give a representative description of these fields and sufficient data to verify the proposed equivalent column concept.

(a) Elevation and Cross section

(b) Finite Element Mesh

All dimensions are in mm.
 $\nu = 0.3$
 $E = 200000 \text{ MPa}$
 $\epsilon_{st} = 0.0$
 $\sigma_y = 300 \text{ MPa}$

(c) Load-Deflection Curve

Figure 2.1: Cantilever Beam.

(d) PL^2/EI vs δ_h/L (geometric nonlinearity)

(e) PL^2/EI vs δ_v/L (geometric nonlinearity)

Figure 2.1: Cantilever Beam Con't.

(a) Elevation and Cross section

(b) Finite Element Mesh

All dimensions are in mm.
 $\nu = 0.3$
 $E = 200000 \text{ MPa}$
 $E_{st} = 0.0$
 $\sigma_y = 300 \text{ MPa}$

(c) Load-Deflection Curve

Figure 2.2: Simply Supported Beam.

(a) Elevation and Cross section

(b) Finite Element Mesh

All dimensions are in mm.
 $E = 203027 \text{ MPa}$
 $E_{st} = 5840 \text{ MPa}$
 $\sigma_y = 340 \text{ MPa}$
 $\sigma_u = 471 \text{ MPa}$
 $\nu = 0.3$

(c) Load-Deflection Curves

Figure 2.3: I-shaped Cantilever Beam, Driver and Kennedy (1987).

(b) Finite Element Mesh

All dimensions are in mm.
 $E = 200000 \text{ MPa}$
 $\epsilon_{st} = 0.0$
 $\sigma_w = 193 \text{ MPa}$
 $\sigma_f = 224 \text{ MPa}$
 $\nu = 0.3$

Figure 2.4: Plate Girder Beam (A1-3), Roberts (1981).

Figure 2.5: Features of the Typical Model.

Figure 2.6: Finite Element Meshes Utilized.

Figure 2.7: (a) Out-Of-Plane Displaced Shape of the Web; (b) Load vs Maximum Out-Of-Plane Deformation.

Figure 2.8: Effect of Flanges on Out-Of-Plane Displaced Shape of the Web and Ultimate Load.

Figure 2.9: Out-Of-Plane Displaced Shape of the Web for Different Boundary Conditions.

Figure 2.10: Out-Of-Plane Displaced Shape of the Web.

W 410x39

WWF 1800x510

WWF 1400x407

WWF 900x262

Figure 2.10: Out-Of-Plane Displaced Shape of the Web Con't.

Figure 2.11: Typical Finite Element Mesh and Boundary Conditions.

(a) W410x39 ($l=1800\text{mm}$, $c=200\text{mm}$), elastic/inelastic web buckling

(b) W610x101 ($l=2200\text{mm}$, $c=200\text{mm}$), inelastic web buckling

(c) W310x143 ($l=1200\text{mm}$, $c=100\text{mm}$), Web yielding

Figure 2.12: Out-of-plane Displaced Shape of the Web of W Shapes (flange free to rotate).

(a) W410x39 ($l=1800$ mm, $c=200$ mm), inelastic web buckling

(b) W530x101 ($l=2200$ mm, $c=200$ mm), inelastic web buckling

(c) W310x143 ($l=1200$ mm, $c=100$ mm), web yielding

Figure 2.13: Out-of-plane Displaced Shape of the Web of W Shapes (flange rotation restrained).

(a) WWF1800x510 ($l=8000\text{mm}$, $c=300\text{mm}$), elastic web buckling

(b) WWF500x197 ($l=2200\text{mm}$, $c=200\text{mm}$), inelastic web buckling

(c) WWF350x212 ($l=1500\text{mm}$, $c=100\text{mm}$), web yielding

Figure 2.14: Out-of-plane Displaced Shape of the Web of WWF Shapes (flange free to rotate).

(a) Distributions along the depth at various locations along the web

(b) Distributions along the length at various depths on the web

Figure 2.15: Normal Stress Distribution for W410x39 at Ultimate (flange free to rotate).

(a) Distributions along the depth at various locations along the web

(b) Distributions along the length at various depths on the web

Figure 2.16: Normal Stress Distribution for W610x101 at Ultimate (flange free to rotate).

(a) Distributions along the depth at various locations along the web

(b) Distributions along the length at various depths on the web

Figure 2.17: Normal Stress Distribution for W310x143 at Ultimate (flange free to rotate).

(a) Distributions along the depth at various locations along the web

(b) Distributions along the length at various depths on the web

Figure 2.18: Normal Stress Distribution for W410x39 at Ultimate (flange rotation restrained).

(a) Distributions along the depth at various locations along the web

(b) Distributions along the length at various depths on the web

Figure 2.19: Normal Stress Distribution for W530x101 at Ultimate (flange rotation restrained).

(a) Distributions along the depth at various locations along the web

(b) Distributions along the length at various depths on the web

Figure 2.20: Normal Stress Distribution for W310x143 at Ultimate (flange rotation restrained).

(a) Distributions along the depth at various locations along the web

(b) Distributions along the length at various depths on the web

Figure 2.21: Normal Stress Distribution for WWF1800x510 at Ultimate (flange free to rotate).

(a) Distributions along the depth at various locations along the web

(b) Distributions along the length at various depths on the web

Figure 2.22: Normal Stress Distribution for WWF500x197 at Ultimate (flange free to rotate).

(a) Distributions along the depth at various locations along the web

(b) Distributions along the length at various depths on the web

Figure 2.23: Normal Stress Distribution for WWF350x212 at Ultimate (flange free to rotate).

Figure 2.24: Normal Stress Distribution at the root of fillet for W310x143.

Table 2.1: Convergence Criteria.

Convergence Criterion Type	Limits Suggested by LUSAS	Limits Used in Analysis
Maximum Absolute Residual	Depending on Units used	$< 10^{-1}$
Root Mean Square of the Residual	Depending on Units used	$< 10^{-2}$
Eucledian Displacement Norm	Between 0.1 and 1.0 (reasonable) Between 1E-3 and 0.1 (tight)	$< 10^{-1}$
Eucledian Residual Norm (most versatile)	Between 1.0 and 5.0 (M.N.L.) ¹ less than 0.1 (G.N.L.) ²	$< 10^{-1}$
Work Norm	Between 1E-3 and 1E-1 (slack) Between 1E-6 and 1E-3 (reasonable) Between 1E-9 and 1E-6 (tight)	$< 10^{-6}$

¹ Material nonlinearity.

² Geometric nonlinearity.

Table 2.2: Nominal Dimensions of the Initially Selected Sections.

<i>Designation</i>	<i>t</i>	<i>w</i>	<i>b</i>	<i>d</i>	<i>h</i>	$\frac{b}{2t}$	$\frac{h}{w}$	<i>Class</i>
W 610x101	15	11	228	603	573	7.65	54.6	1
W 410x85	18	11	181	417	381	4.97	35.0	1
W 410x39	9	6	140	399	381	7.95	59.7	1
WWF 1800x510	30	20	500	1800	1740	8.33	87.0	2
WWF 1400x407	30	14	550	1400	1340	9.17	95.7	2
WWF 900x262	30	11	400	900	840	6.67	76.3	2

Table 2.3: Out-Of-Plane Displaced Shape Data.

<i>Designation</i>	<i>y (mm)</i>	α_{Lusas}	$\alpha_{Roberts}$
W 610x101	1.97	0.31 <i>h</i>	0.47 <i>h</i>
W 410x85	0.83	0.25 <i>h</i>	0.69 <i>h</i>
W 410x39	4.42	0.44 <i>h</i>	0.42 <i>h</i>
WWF 1800x510	0.84	0.09 <i>h</i>	0.28 <i>h</i>
WWF 1400x407	4.16	0.16 <i>h</i>	0.29 <i>h</i>
WWF 900x262	4.89	0.31 <i>h</i>	0.32 <i>h</i>

Table 2.4: Predicted Loads by Different Models.

<i>Designation</i>	P_{cr} (kN)	P_{Lusas} (kN)	$P_{Roberts}$ (kN)	$P_{S16.1}$ (kN)	P_{yield} (kN)	P_{cr}/P_{Lusas}
W 610x101	1508	1086	825	1285	1629	1.389
W 410x85	1687	1160	889	1595	1709	1.454
W 410x39	499	464	306	599	908	1.075
WWF 1800x510	7550	2724	2942	3238	3333	2.772
WWF 1400x407	2237	1819	1467	1796	2287	1.230
WWF 900x262	1256	1239	905	1269	1797	1.015

Table 2.5: Nominal Dimensions of the Test Sections.

<i>Designation</i>	<i>c</i>	<i>l</i>	<i>t</i>	<i>w</i>	<i>b</i>	<i>d</i>	<i>h</i>	$\frac{h}{2t}$	$\frac{h}{w}$	<i>Class</i>
	<i>mm</i>	<i>mm</i>	<i>mm</i>	<i>mm</i>	<i>mm</i>	<i>mm</i>	<i>mm</i>			
W 250x131	100	1000	25	15	261	275	225	7.20	14.6	1
W 310x143 [†]	100	1200	23	14	309	323	277	6.75	19.8	1
W 610x113	200	2200	17	11	228	608	573	6.59	51.2	1
W 610x101	200	2200	15	11	228	603	573	7.65	54.6	1
W 530x101 [†]	200	2200*	17	11	210	537	502	6.03	46.1	1
W 530x82 [†]	200	2200*	13	10	209	528	502	7.86	52.7	1
W 460x97	200	1800	19	11	193	466	428	5.08	37.5	1
W 410x85	200	1500	18	11	181	417	381	4.97	35.0	1
W 410x39 [†]	200	1800	9	6	140	399	381	7.95	59.7	1
		1600*								
W 360x45 [†]	200	1500	10	7	171	352	332	8.72	48.1	2
		1400*								
W 360x33	200	1400	9	6	127	349	332	7.47	57.4	1

[†] additional finite element analyses were conducted on these sections with flange rotation restrained.

* additional finite element analyses were conducted on these sections for end reaction ($c=100\text{mm}$) and with flange rotation restrained.

Table 2.5: Nominal Dimensions of the Test Sections Con't.

<i>Designation</i>	<i>c</i> <i>mm</i>	<i>l</i> <i>mm</i>	<i>t</i> <i>mm</i>	<i>w</i> <i>mm</i>	<i>b</i> <i>mm</i>	<i>d</i> <i>mm</i>	<i>h</i> <i>mm</i>	$\frac{h}{d}$	$\frac{h}{w}$	<i>Class</i>
<i>W 310x86</i>	200	1400	16	9	254	310	277	7.79	30.4	1
<i>W 310x52</i>	200	1200	13	8	167	317	291	6.33	38.3	1
<i>W 310x31</i> [†]	200	1400*	7	5	164	306	291	11.0	58.2	3
<i>W 250x45</i>	200	1200	13	8	148	266	240	5.69	31.5	1
<i>W 250x24</i> [†]	200	1200 950*	6	5	145	253	240	11.3	48.2	3
<i>W 200x21</i>	100	800	6	5	133	203	190	10.3	38.2	3
<i>WWF 350x212</i>	100	1500	30	20	350	350	290	5.83	14.5	1
<i>WWF 1800x510</i>	300	6000	30	20	500	1800	1740	8.33	87.0	2
<i>WWF 1400x405</i>	300	5800	30	16	500	1400	1340	8.33	83.7	2
<i>WWF 900x262</i>	300	3900	30	11	400	900	840	6.67	76.3	2
<i>WWF 500x197</i>	200	2200	20	11	500	500	460	12.5	41.8	4

[†] additional finite element analyses were conducted on these sections with flange rotation restrained.

* additional finite element analyses were conducted on these sections for end reaction ($c=100\text{mm}$) and with flange rotation restrained.

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Table 2.6: **Out-Of-Plane Displaced Shape Data (flange free to rotate).**

<i>Designation</i>	<i>c (mm)</i>	<i>l (mm)</i>	<i>y (mm)</i>	α_{Lusas}	$\alpha_{Roberts}$
<i>W</i> 250x131	100	1000	0.51	0.125 <i>h</i>	*
<i>W</i> 310x143	100	1200	0.68	0.125 <i>h</i>	*
<i>W</i> 610x113	200	2200	7.55	0.250 <i>h</i>	0.466 <i>h</i>
<i>W</i> 610x101	200	2200	7.67	0.250 <i>h</i>	0.466 <i>h</i>
<i>W</i> 530x101	200	2200	5.07	0.250 <i>h</i>	0.530 <i>h</i>
<i>W</i> 530x82	200	2200	3.51	0.375 <i>h</i>	0.486 <i>h</i>
<i>W</i> 460x97	200	1800	4.68	0.250 <i>h</i>	0.614 <i>h</i>
<i>W</i> 410x85	200	1500	4.28	0.250 <i>h</i>	0.690 <i>h</i>
<i>W</i> 410x39	200	1800	4.36	0.375 <i>h</i>	0.385 <i>h</i>
<i>W</i> 360x45	200	1500	1.80	0.375 <i>h</i>	0.511 <i>h</i>
<i>W</i> 360x33	200	1400	3.55	0.375 <i>h</i>	0.441 <i>h</i>
<i>W</i> 310x86	200	1400	3.56	0.250 <i>h</i>	0.765 <i>h</i>
<i>W</i> 310x52	200	1200	1.53	0.313 <i>h</i>	0.658 <i>h</i>
<i>W</i> 310x31	200	1400	3.42	0.375 <i>h</i>	0.419 <i>h</i>
<i>W</i> 250x45	200	1200	1.34	0.313 <i>h</i>	0.791 <i>h</i>
<i>W</i> 250x24	200	1200	1.91	0.438 <i>h</i>	0.521 <i>h</i>
<i>W</i> 200x21	100	800	2.21	0.250 <i>h</i>	0.635 <i>h</i>
<i>WWF</i> 350x212	100	1500	0.13	0.125 <i>h</i>	*
<i>WWF</i> 1800x510	300	6000	39.56	0.200 <i>h</i>	0.282 <i>h</i>
<i>WWF</i> 1400x405	300	5800	29.55	0.200 <i>h</i>	0.292 <i>h</i>
<i>WWF</i> 900x262	300	3900	16.39	0.250 <i>h</i>	0.316 <i>h</i>
<i>WWF</i> 500x197	200	2200	11.75	0.250 <i>h</i>	0.573 <i>h</i>

* not applicable for sections that fail by web yielding.

Table 2.7: Out-Of-Plane Displaced Shape Data (flange rotation restrained).

<i>Designation</i>	<i>c (mm)</i>	<i>l (mm)</i>	<i>y (mm)</i>	α_{Lusas}	$\alpha_{Roberts}$
<i>W 310x143</i>	100	1200	2.58	0.125 <i>h</i>	*
<i>W 530x101</i>	200	2200	4.47	0.188 <i>h</i>	0.530 <i>h</i>
<i>W 530x82</i>	200	2200	3.94	0.188 <i>h</i>	0.486 <i>h</i>
<i>W 410x39</i>	200	1800	1.44	0.375 <i>h</i>	0.385 <i>h</i>
<i>W 410x39</i>	200	1600	1.22	0.375 <i>h</i>	0.385 <i>h</i>
<i>W 360x45</i>	200	1500	0.75	0.313 <i>h</i>	0.511 <i>h</i>
<i>W 360x45</i>	200	1400	0.75	0.313 <i>h</i>	0.511 <i>h</i>
<i>W 310x31</i>	200	1400	1.12	0.375 <i>h</i>	0.419 <i>h</i>
<i>W 250x24</i>	200	1200	0.80	0.375 <i>h</i>	0.521 <i>h</i>
<i>W 250x24</i>	200	950	0.72	0.375 <i>h</i>	0.521 <i>h</i>
<i>W 530x101(end)</i>	100	2200	3.04	0.188 <i>h</i>	0.530 <i>h</i>
<i>W 530x82(end)</i>	100	2200	2.22	0.188 <i>h</i>	0.486 <i>h</i>
<i>W 410x39(end)</i>	100	1600	1.27	0.375 <i>h</i>	0.385 <i>h</i>
<i>W 360x45(end)</i>	100	1400	1.05	0.250 <i>h</i>	0.511 <i>h</i>
<i>W 310x31(end)</i>	100	1400	1.08	0.375 <i>h</i>	0.419 <i>h</i>
<i>W 250x24(end)</i>	100	950	0.68	0.375 <i>h</i>	0.521 <i>h</i>

* not applicable for sections that fail by web yielding.

Table 2.8: Stress Distribution due to Yielding.

<i>Designation</i>	$\frac{\int \sigma dA}{\sigma_y w}$ (mm)	<i>c + 5 k (mm)</i>	<i>c + 10 t (mm)</i>
<i>W 310x143 *</i>	324 (<i>c + 5.6 k</i>)	300	329
<i>W 250x131 *</i>	348 (<i>c + 5.9 k</i>)	310	351
<i>WWF 350x212 *</i>	399 (<i>c + 7.3 k</i>)	305	400
<i>W 310x143 **</i>	326 (<i>c + 5.7 k</i>)	300	329

* flange free to rotate.

** flange rotation restrained.

Table 2.9: Predicted Failure Loads by Different Models (flange free to rotate).

<i>Designation</i>	P_{cr} (kN)	P_{Lusas} (kN)	$P_{Roberts81}$ (kN)	$P_{S16.1}$ (kN)	P_{yield} (kN)	<i>Probable Mode of Failure</i>
W 250x131	11556	1629	1774	2398	1575	<i>Yielding</i>
W 310x143	7051	1395	1467	1881	1386	<i>Yielding</i>
W 610x113	1473	956	938	1227	1404	<i>Inelastic buckling</i>
W 610x101	1151	828	825	1084	1284	<i>Inelastic buckling</i>
W 530x101	1606	922	889	1212	1349	<i>Inelastic buckling</i>
W 530x82	969	702	675	935	1082	<i>Inelastic buckling</i>
W 460x97	2251	1027	973	1393	1430	<i>Inelastic buckling</i>
W 410x85	2288	968	889	1330	1349	<i>Inelastic buckling</i>
W 410x39	387	360	307	483	697	<i>Inelastic buckling</i>
W 360x45	620	459	356	592	763	<i>Inelastic buckling</i>
W 360x33	361	327	252	417	622	<i>Inelastic buckling</i>
W 310x86	2197	764	619	1041	1096	<i>Inelastic buckling</i>
W 310x52	1130	565	432	722	878	<i>Inelastic buckling</i>
W 310x31	289	266	187	330	536	<i>Inelastic buckling</i>
W 250x45	1390	564	432	788	790	<i>Inelastic buckling</i>
W 250x24	332	277	187	388	470	<i>Inelastic buckling</i>
W 200x21	395	199	187	297	297	<i>Inelastic buckling</i>
WWF 350x212	18026	2509	2993	3733	2013	<i>Yielding</i>
WWF 1800x510	2504	2866	2941	3238	3333	<i>Elastic buckling</i>
WWF 1400x405	1995	2099	1916	2278	2614	<i>Elastic buckling</i>
WWF 900x262	1335	1226	905	1269	1797	<i>Inelastic buckling</i>
WWF 500x197	2277	1049	905	1263	1252	<i>Inelastic buckling</i>

Table 2.10: Predicted Failure Loads by Different Models (flange rotation restrained).

<i>Designation</i>	P_{cr} (kN)	P_{Lusas} (kN)	$P_{Roberts81}$ (kN)	$P_{S16.1}$ (kN)	P_{yield} (kN)	<i>Probable Mode of Failure</i>
W 310x143	11506	1347	1467	1881	1386	<i>Yielding</i>
W 530x101	3020	998	889	1212	1349	<i>Inelastic buckling</i>
W 530x82	1973	778	675	935	1082	<i>Inelastic buckling</i>
W 410x39	813	422	307	483	697	<i>Inelastic buckling</i>
W 410x39	825	427	307	483	697	<i>Inelastic buckling</i>
W 360x45	1238	475	356	592	763	<i>Inelastic buckling</i>
W 360x45	1240	477	356	592	763	<i>Inelastic buckling</i>
W 310x31	562	316	187	330	536	<i>Inelastic buckling</i>
W 250x24	718	318	187	388	470	<i>Inelastic buckling</i>
W 250x24	729	325	187	388	470	<i>Inelastic buckling</i>
W 530x101(end)	1361	494	441	498	674	<i>Inelastic buckling</i>
W 530x82(end)	896	389	336	373	541	<i>Inelastic buckling</i>
W 410x39(end)	392	209	153	183	349	<i>Inelastic buckling</i>
W 360x45(end)	592	229	178	222	381	<i>Inelastic buckling</i>
W 310x31(end)	273	156	94	122	268	<i>Inelastic buckling</i>
W 250x24(end)	366	158	94	134	235	<i>Inelastic buckling</i>
WWF 350x212	40053	—	2993	3733	2013	<i>Yielding</i>
WWF 1800x510	5067	—	2941	3238	3333	<i>Inelastic buckling</i>
WWF 1400x405	3432	—	1916	2278	2614	<i>Inelastic buckling</i>
WWF 900x262	2505	—	905	1269	1797	<i>Inelastic buckling</i>
WWF 500x197	3489	—	905	1263	1252	<i>Inelastic buckling</i>

Chapter 3

EQUIVALENT COLUMN CONCEPT

3.1 Equivalent Column Concept and Elastic Buckling Solution

The strength of webs for rolled shapes can be determined from an equivalent column concept that reflects the internal force distribution across the width and depth of the web, accounts for the influence of the rotational restraint of the flange at the flange web junction, bearing area, and the two dimensional restraint provided to the plate. This formulation uses the energy conservation method.

Based on the results of a finite element study of selected W and WWF shapes that are representative of three probable modes of failure (elastic web buckling, inelastic web buckling (crippling), and web yielding), the equivalent column dimensions and normal stress distribution shown in Fig. 3.1 were determined.

The elastic buckling solution has been developed for a pinned-ended column with an equivalent uniform load across the width, $c + 5k$, that varies from a maximum at the

root of the fillet on the web near the loaded flange to zero at the far end and is given by the normal stress distribution in Eq. (3.1) and the corresponding load distribution in Eq. (3.2). The distribution of $\sigma(x)$ as a function of σ_{max} is given as

$$\sigma(x) = \frac{\sigma_{max}}{2} \left[1 - \cos \left(\frac{\pi x}{h} \right) \right] \quad (3.1)$$

where, $\sigma(x)$ is related to $P_{cr}(x)$ by,

$$\begin{aligned} P_{cr}(x) &= w \int_0^x \sigma(x) dx \\ P_{cr}(x) &= w \int_0^x \frac{\sigma_{max}}{2} \left[1 - \cos \left(\frac{\pi x}{h} \right) \right] dx \\ P_{cr}(x) &= \frac{w \sigma_{max}}{2} \left[x - \frac{h}{\pi} \sin \left(\frac{\pi x}{h} \right) \right] \end{aligned} \quad (3.2)$$

and the maximum value of P_{cr} becomes,

$$\begin{aligned} P_{cr} &= w \int_0^h \sigma(x) dx \\ P_{cr} &= \frac{w h \sigma_{max}}{2} \end{aligned} \quad (3.3)$$

The load $P_{cr}(x)$, as a function of P_{cr} , is given as

$$P_{cr}(x) = \frac{P_{cr}}{h} \left[x - \frac{h}{\pi} \sin \left(\frac{\pi x}{h} \right) \right] \quad (3.4)$$

The strain energy and potential energy of the system are defined by Eqs. (3.5) and (3.6), respectively.

$$V = \int_0^h \frac{E I}{2} \left(\frac{d^2 y}{dx^2} \right)^2 dx \quad (3.5)$$

$$U = - \int_0^h \frac{P_{cr}(x)}{2} \left(\frac{dy}{dx} \right)^2 dx \quad (3.6)$$

Assuming that the buckled shape of the column can be expressed as a series given by,

$$y = \sum_{i=1}^n a_n \sin \left(\frac{n\pi x}{h} \right) \quad (3.7)$$

The first and second derivatives become,

$$\begin{aligned} \frac{dy}{dx} &= \sum_{i=1}^n a_n \left(\frac{n\pi}{h} \right) \cos \left(\frac{n\pi x}{h} \right) \\ \frac{d^2 y}{dx^2} &= - \sum_{i=1}^n a_n \left(\frac{n\pi}{h} \right)^2 \sin \left(\frac{n\pi x}{h} \right) \end{aligned}$$

Evaluation of the strain energy yields,

$$\begin{aligned}
 V &= \int_0^h \frac{E I}{2} \left(\frac{d^2 y}{dx^2} \right)^2 dx \\
 V &= \int_0^h \frac{E I}{2} \left[- \sum_{i=1}^n a_n \left(\frac{n\pi}{h} \right)^2 \sin \left(\frac{n\pi x}{h} \right) \right]^2 dx \\
 V &= \sum_{i=1}^n \int_0^h \frac{E I}{2} a_n^2 \left(\frac{n\pi}{h} \right)^4 \sin^2 \left(\frac{n\pi x}{h} \right) dx \\
 V &= \sum_{i=1}^n \frac{E I}{2} a_n^2 \left(\frac{n\pi}{h} \right)^4 \int_0^h \sin^2 \left(\frac{n\pi x}{h} \right) dx \\
 V &= \frac{\pi^4 E I}{2 h^4} \sum_{i=1}^n a_n^2 n^4 \int_0^h \sin^2 \left(\frac{n\pi x}{h} \right) dx \\
 V &= \frac{\pi^4 E I}{4 h^3} \sum_{i=1}^n a_n^2 n^4 \quad (3.8)
 \end{aligned}$$

Evaluation of the potential energy yields,

$$\begin{aligned}
 U &= - \int_0^h \frac{P_{cr}(x)}{2} \left(\frac{dy}{dx} \right)^2 dx \\
 U &= - \int_0^h \frac{P_{cr}}{2h} \left[x - \frac{h}{\pi} \sin \left(\frac{\pi x}{h} \right) \right] \left[\sum_{i=1}^n a_n \left(\frac{n\pi}{h} \right) \cos \left(\frac{n\pi x}{h} \right) \right]^2 dx \\
 U &= - \sum_{i=1}^n \int_0^h \frac{P_{cr}}{2h} \left[x - \frac{h}{\pi} \sin \left(\frac{\pi x}{h} \right) \right] a_n^2 \left(\frac{n\pi}{h} \right)^2 \cos^2 \left(\frac{n\pi x}{h} \right) dx \\
 U &= - \frac{P_{cr} \pi^2}{2h^3} \sum_{i=1}^n a_n^2 n^2 \int_0^h \left[x - \frac{h}{\pi} \sin \left(\frac{\pi x}{h} \right) \right] \cos^2 \left(\frac{n\pi x}{h} \right) dx \quad (3.9)
 \end{aligned}$$

Evaluating the integral portion of Eq. (3.9) gives

$$\begin{aligned}
 I &= \int_0^h \left[x - \frac{h}{\pi} \sin \left(\frac{\pi x}{h} \right) \right] \cos^2 \left(\frac{n\pi x}{h} \right) dx \\
 I &= -\frac{h^2}{2\pi^2} + \frac{h^2}{(-1+4n^2)\pi^2} + \frac{-2h^2+h^2\pi^2}{4\pi^2} \quad (3.10)
 \end{aligned}$$

Substituting Eq. (3.10) into (3.9) and simplifying gives,

$$\begin{aligned}
 U &= -\frac{P_{cr} \pi^2}{2h^3} \sum_{i=1}^n a_n^2 n^2 \left(-\frac{h^2}{2\pi^2} + \frac{h^2}{(-1+4n^2)\pi^2} + \frac{-2h^2+h^2\pi^2}{4\pi^2} \right) \\
 U &= -\frac{P_{cr}}{8h} \sum_{i=1}^n a_n^2 n^2 \left(-2 + \frac{4}{(-1+4n^2)} - 2 + \pi^2 \right) \\
 U &= -\frac{P_{cr}}{8h} \sum_{i=1}^n a_n^2 n^2 \left(-4 + \pi^2 + \frac{4}{(-1+4n^2)} \right) \quad (3.11)
 \end{aligned}$$

Minimizing the total energy with respect to a_n yields the solution for the elastic buckling load.

$$\begin{aligned} \frac{d(U + V)}{da_n} &= 0 \\ -\frac{P_{cr}}{8h} \sum_{i=1}^n 2 a_n n^2 \left(-4 + \pi^2 + \frac{4}{(-1 + 4n^2)} \right) + \frac{\pi^4 E I}{4 h^3} \sum_{i=1}^n 2 a_n n^4 &= 0 \\ \sum_{i=1}^n -\frac{P_{cr}}{8h} 2 a_n n^2 \left(-4 + \pi^2 + \frac{4}{(-1 + 4n^2)} \right) + \frac{\pi^4 E I}{4 h^3} 2 a_n n^4 &= 0 \\ \sum_{i=1}^n 2 a_n n^2 \left[\frac{\pi^4 E I}{4 h^3} n^2 - \frac{P_{cr}}{8h} \left(-4 + \pi^2 + \frac{4}{(-1 + 4n^2)} \right) \right] &= 0 \quad (3.12) \end{aligned}$$

The critical elastic buckling load occurs when n is equal to one. Making the appropriate substitutions gives

$$2 a_1 \left[\frac{\pi^4 E I}{4 h^3} - \frac{P_{cr}}{8h} \left(-4 + \pi^2 + \frac{4}{3} \right) \right] = 0 \quad (3.13)$$

For a non-trivial solution the term between square brackets is set to zero.

$$\frac{P_{cr}}{8h} \left(-4 + \pi^2 + \frac{4}{3} \right) = \frac{\pi^4 E I}{4 h^3} \quad (3.14)$$

The critical elastic buckling load for a pinned-ended column with the load distribution described in Fig. 3.1 is,

$$P_{cr} = 2.74 \frac{\pi^2 E I}{h^2} \quad (3.15)$$

3.2 Effective Length Factor

For boundary conditions other than pinned-ended, simply include the effective length factor.

$$P_{cr} = 2.74 \frac{\pi^2 E I}{(K h)^2} \quad (3.16)$$

The following three probable sets of boundary conditions could apply:

- (i) the loaded flange is restrained against lateral displacement and rotation (interior and end reaction concentrated load – deemed to be the most common)

- (ii) the loaded flange is restrained against lateral displacement but is free to rotate (interior concentrated load – the other end of the spectrum)
- (iii) the unloaded flange is free to displace laterally, to rotate, and to warp (end reaction condition).

The corresponding effective length factor is a function of the restraint provided to the web within the equivalent column width by the surrounding web and by the loaded flange (rotational and warping restraint).

$$K = f\left(\frac{c}{h}; \frac{t}{w}\right) \quad (3.17)$$

The relationship between K and c/h ; and t/w were determined for the first two sets of boundary conditions by comparing the elastic buckling load from the finite element analysis with that from Eq. (3.15) for the shapes listed in Table 2.5. The value K is given as the square root of P_{cr} given by Eq. (3.15) divided by the finite element elastic critical buckling load. The values of K were plotted against different functions of geometric properties which represent the restraint of the flange and the surrounding web to determine the degree of correlation that may exist between the two. The following function showed a significant degree of correlation.

$$K = \sqrt{\frac{\left(\frac{c_1}{h}\right)}{\left(\frac{t}{w}\right)}} \quad (3.18)$$

where, c_1 is the value $c + 5k$ (interior load). The function for K was then determined from a linear regression analysis of the appropriate data set to be,

$$K = 0.873 \sqrt{\frac{\left(\frac{c_1}{h}\right)}{\left(\frac{t}{w}\right)}} + 0.125 \quad (\text{flange free to rotate}) \quad (3.19)$$

$$K = 0.513 \sqrt{\frac{\left(\frac{c_1}{h}\right)}{\left(\frac{t}{w}\right)}} + 0.161 \quad (\text{flange rotation restrained}) \quad (3.20)$$

with correlation coefficients for Eqs. (3.19) and (3.20) of 0.95 and 0.97, respectively. The mean values of the ratio of the finite element predicted load to the load predicted by Eq.

(3.16), with K given by Eqs. (3.19) and (3.20) are 0.98 and 0.99 and with coefficients of variation of 0.11 and 0.10, respectively.

The corresponding family of curves for Eqs. (3.19) and (3.20), and the K values calculated for the six test sections which are representative for the range of W shapes produced in Canadian mills, are shown in Figs. 3.2 (a) and (b), respectively. The expressions are too complex for design and a constant value for each case has been recommended.

3.3 Inelastic Buckling and the Application of Column Curves

Critical buckling load as predicted by an elastic and inelastic finite element analysis, as well as the elastic buckling solution predicted by Eq. (3.16), with the K value given by Eqs. (3.19) and (3.20) are plotted on a non-dimensional load versus slenderness graph shown in Figs. 3.3 and 3.4. The critical loads are divided by the yield load of the equivalent column and is given by,

$$P_y = w \sigma_y (c + 5 k) \quad (3.21)$$

The equivalent column breadth, $c + 5 k$ for interior loads, was determined in section 2.4.2.2 to provide a cross-sectional area on which the yield stress acts, that is statically equivalent to the distribution of normal stresses at the root of the fillet on the web, for sections that fail by web yielding. The normal stress distributions and equivalent column breadths were determined from inelastic finite element analyses of a W250x131, a W310x143, and a WWF350x212.

The slenderness parameter λ is obtained by calculating the ratio of the load given by Eq. (3.16) and the yield load P_y to give $\frac{1}{\lambda^2}$, where λ is given by

$$\lambda = 2.09 \left(\frac{K h}{w} \right) \sqrt{\frac{\sigma_w}{\pi^2 E}} \quad (3.22)$$

or expressed as a function of the radius of gyration of the equivalent column as,

$$\lambda_e = 0.604 \left(\frac{K h}{r} \right) \sqrt{\frac{\sigma_w}{\pi^2 E}} \quad (3.23)$$

where, $r = \frac{w}{\sqrt{12}}$.

The equivalent slenderness parameter λ_e is similar to that for columns. Two options have been considered for incorporating the work of this investigation into the behaviour of webs subjected to concentrated loads into the steel standard S16.1.

Option A: Equivalent Web Buckling Slenderness Parameter and S16.1 Column Curves.

Option B: Equivalent Web Buckling Slenderness Parameter and Best Fit Approximation for Inelastic Web Buckling.

The corresponding proposed Clauses and figures illustrating the fit of the equations with respect to the elastic and inelastic finite element data follows.

3.3.1 Option A: Equivalent Web Buckling Slenderness Parameter and S16.1 Column Curves

Clause 15.9 Web Buckling and Yielding

15.9.1

Bearing stiffeners shall be provided where the factored concentrated loads or reactions exceed the factored compressive resistances of webs of rolled beams and welded plate girders. The factored compressive resistance of the web shall be calculated using Clause 13.3.1 with an equivalent slenderness parameter λ_e given by

$$\lambda_e = 0.6 \left(\frac{K h}{r} \right) \sqrt{\frac{F_y}{\pi^2 E}} \quad (3.24)$$

where, r = radius of gyration of the equivalent column cross-section and is $\frac{w}{\sqrt{12}}$.

15.9.2

The effective length factor for interior loads shall be 0.6 when the loaded flange is restrained against rotation and 0.8 when it is free to rotate. The effective length factor for end reactions shall be 0.6 when both flanges are restrained against lateral displacement and rotation and -- when the unloaded flange is free to displace.

15.9.3

The area of the equivalent column is equal to $w(N + 5k)$ for interior loads and $w(N + 2.5k)$ for end reactions, where the length of bearing N shall not be less than k for end reactions.

Figs. 3.5 and 3.6 show Clause 13.3.1 and 13.3.2 with $\Phi = 1.0$; the elastic buckling curve given by Eq. (3.16) and the equivalent slenderness parameter; both the elastic and inelastic finite element buckling loads; and the capacities of the test sections calculated using the current S16.1 rules and the corresponding slenderness parameter, for the interior loading where the loaded flange is free to rotate and restrained against rotation, respectively.

3.3.2 Option B: Equivalent Web Buckling Slenderness Parameter and Best Fit Approximation for Inelastic Web Buckling

Clause 15.9 Web Buckling and Yielding

15.9.1

Bearing stiffeners shall be provided where the factored concentrated loads or reactions exceed the factored compressive resistances of webs of rolled beams and welded plate girders. The factored compressive resistance of the web shall be calculated as follows:

(a) for interior loads with the loaded flange free to rotate

(i) $0.0 \leq \lambda \leq 0.35$

$$B_r = \Phi A F_y$$

(ii) $0.35 \leq \lambda \leq 1.6$

$$B_r = \Phi A F_y (1.168 - 0.180 \lambda)$$

(iii) $1.6 < \lambda$

$$B_r = \Phi A F_y \lambda^{-2}$$

(b) loaded flange restrained against rotation

(i) $0.0 \leq \lambda \leq 0.13$

$$B_r = \Phi A F_y$$

(ii) $0.13 \leq \lambda \leq 1.5$

$$B_r = \Phi A F_y (1.052 - 0.401 \lambda)$$

(iii) $1.5 < \lambda$

$$B_r = \Phi A F_y \lambda^{-2}$$

where, $\lambda = 0.6 \left(\frac{K h}{r} \right) \sqrt{\frac{F_y}{\pi^2 E}},$

r = radius of gyration of the equivalent column cross-section and is $\frac{w}{\sqrt{12}},$

A = area of the equivalent column.

Clauses 15.9.2 and 15.9.3 would be as previously described. Figs. 3.7 and 3.8 illustrate the same information as in Figs. 3.5 and 3.6 with an alternative best fit line inelastic web buckling curve. This set of composite column curves are too complex for inclusion in the standard and are not considered further.

(a) Shear stress distribution and deflected shape

(b) Average normal stress distribution

Figure 3.1: Equivalent Column Concept.

Figure 3.2: Effective Length Factor, K .

Figure 3.3: FEA Predictions and Theoretical Elastic Buckling Column Curve (flange free to rotate).

Figure 3.4: FEA Predictions and Theoretical Elastic Buckling Column Curve (flange rotation restrained).

Figure 3.5: Equivalent Web Buckling Slenderness Parameter and S16.1 Column Curves (flange free to rotate).

Figure 3.6: Equivalent Web Buckling Slenderness Parameter and S16.1 Column Curves (flange rotation restrained).

Figure 3.7: Equivalent Web Buckling Slenderness Parameter and Best Fit Approximation for Inelastic Web Buckling (flange free to rotate).

Figure 3.8: Equivalent Web Buckling Slenderness Parameter and Best Fit Approximation for Inelastic Web Buckling(flange rotation restrained).

Chapter 4

EXPERIMENTAL PROGRAM

4.1 General

The experimental program was designed to obtain the data necessary to describe the strength and behaviour of webs of rolled shapes produced in Canadian mills that are subjected to concentrated in-plane loads at both interior and end reaction locations. This data will be used to validate the finite element modelling and the equivalent column concept described in Chapter 3.

Two series of tests will be conducted of W and WWF shape beams that are simply supported with span lengths varying from 1200 *mm* to 6000 *mm* and with a concentrated load applied at midspan. The first series will investigate web behaviour under the applied load (interior location) and will have doubler plates clamped at the ends to prevent web instability. Subsequently, the web distortions will be pressed out, the doubler plates at the ends removed and bolted to the web under the load. The beam will then be re-tested to investigate the web behaviour over the reaction, the now weaker part. The process will be repeated for the remaining reaction point.

The shapes listed in Table 4.1, a representative sample of those produced in Canadian mills

(class 1, 2, 3 and 4), have been selected to give the full range of anticipated behaviour under load from elastic web buckling to inelastic web buckling to web yielding. The geometric properties (web thickness, flange thickness, depth, and flange width) of the specimens are ranging from 5 mm to 20 mm, 6.4 mm to 30 mm, 253 mm to 1800 mm, and 140 mm to 500 mm, respectively. The web slenderness ratio and length over which the concentrated load acts varies from 14.5 to 87.0 and 100 mm to 300 mm, respectively.

Four analytical models were used to predict the failure loads and mode of failure so that the selection of a representative group of test specimens could be made. The models give the elastic web buckling (LUSAS - finite element method, P_{cr}), the inelastic web buckling capacity (Roberts (1981) semi-empirical relationship, $P_{Roberts81}$, LUSAS - finite element method, P_{Lusas}) and the web yielding capacity (CAN/CSA-S16.1-M89) as shown in Table 4.2. The lowest value gives the probable mode of failure. The span lengths for the corresponding sections were selected to give a minimum shear span to depth ratio of 1.5 and to ensure that the load that causes web failure will be less than that to cause flexural failure (in-plane strength or lateral torsional buckling capacity).

4.2 Test Set-Up

The test set-up consists of a test frame, a 4450 kN hydraulic actuator, knife edge roller assemblies, load plates, electronic and mechanical instrumentation, a data acquisition system and the test specimen. Fig. 4.1 shows schematically a side elevation and cross-section showing details of the set-up. For clarity, the instrumentation and lateral supports at the reactions has not been included. The photograph in Fig. 4.2 shows the overall set-up of the experiment.

4.2.1 Loading System and Support Details

The vertical load will be applied through a series load plates with different load lengths to the top flange of the beam allowing vertical displacement only. Rotation of the flange is prevented by the actuator and load plate. Each beam will be supported by knife-edge-roller assemblies at either end as shown schematically in Fig. 4.3 and in Fig. 4.4 in the photograph.

4.2.2 Lateral Support

To prevent warping of the top flange and distortion of the web over the reactions, the lateral displacement and rotation of the top flange will be restrained with two sets of vertical roller plates butted on both sides to the web of a section clamped to the top flange as shown schematically in Fig. 4.5 and in Fig. 4.6 in the corresponding photograph.

4.2.3 Instrumentation and Measurement

Applied loads, reactions, strains, rotations, and displacements are measured throughout the duration of each test. The locations of the measurements made for strains and deflections are shown in Figs. 4.7, 4.8, 4.9, 4.10 and 4.12 (a), respectively.

A total of 41 gauges were used for each tested shape. Eight uniaxial on the top and bottom flanges and 33 uniaxial on one side only of the web forming 13 rosettes. Two types of gauges were used. Micro-Measurement high elongation strain gauges for rosettes W3, W4, W6, and W7, with a resistance of $120.0 \pm 0.15\%$ ohms, a gauge length of 8 mm, and a gauge factor of $2.055 \pm 0.5\%$ at $24^\circ C$. The remaining gauges were S' wa strain gauges, with a resistance of $119.9 \pm 0.5\%$ ohms, a gauge length of 5 mm, and a gauge factor of $2.12 \pm 1.0\%$ at $20^\circ C$.

These strain gauges were located to correspond with nodes of the finite element mesh in which the strains are reported and to give a reasonable representation of the strain distribution as indicated by a comparison with typical strain contours in the web of W310x31 shown in Fig. 4.8. The in-plane loads carried by the flanges will be determined from strains measured by the strain gauges located on the inside and outside of the top and bottom flange as shown in Fig. 4.7. The strain measurement locations are identified as;

- TF_n - strain gauge located on the top flange, $n = \text{odd}$ (outside surface),
 $n = \text{even}$ (inside surface).
- BF_n - strain gauge located on the bottom flange, n - as above.
- W_n - rosette located on the web surface; n - location number.

Deflections were measured using Linear Variable Differential Transformers (LVDTs). The LVDTs were located to correspond with nodes of the finite element mesh in which the displacements are reported and to give a reasonable description of the displaced shape as indicated by a comparison of Figs. 4.9 and 4.10. Two LVDTs on the Flanges (one on the top and one on the bottom flange) give the vertical displacement of the beam and the relative vertical displacement of one flange with respect to the other. To hold the LVDTs in position, two angles with supporting threaded rods and LVDT brackets were clamped to the loaded flange. Fig. 4.11 shows the manner in which the LVDTs were supported. The location of the LVDTs measuring the out-of-plane displacements of the web and the corresponding support system for the end reaction tests is given in Fig. 4.12 (a) and (b), respectively.

Loads were monitored using a pressure transducer on the pressure line at the 4450 kN actuator. The same hydraulic/pressure transducer configuration was used to calibrate the response of the pressure transducer. The maximum pressure in the pressure line was regulated and kept at levels consistent with the anticipated failure loads to give as much

control as possible in applying the load to the test specimen (± 2 to 3 kN).

The loads, strains and deflections were measured for each load step and recorded using LABCON, Lab Windows, and a Hewlett Packard data acquisition system. The type of measurements, instrumentation used, location, purpose and frequency of measurements are described in Table 4.3. Additional measurements were made to corroborate deflection data, to determine change in section depth, flange rotation, yield line location, and the residual shape of the beam.

4.3 Alignment and Test Procedure

4.3.1 Alignment and Initial Loading

1. Align the test specimen with respect to the test frame (actuator and load plate); and the reaction devices such that the centre of the web top flange junction is centered under the actuator and that the web is vertical.
2. Connect, calibrate and conduct a functional check on instrumentation.
3. Apply an initial load of $1/50^{th}$ of the estimated load capacity of the specimen to seat the test specimen. Unload and record initial (zero) readings.

4.3.2 Test Procedure

1. Apply load monotonically to the beam in appropriate increments (either by displacement or load control) as determined by the load out-of-plane displacement (web) curve until failure occurs.
2. After each load step record electronic and manual readings and conduct a visual inspection. Record the formation of plastic hinges and yield lines; and any other notable observation.

3. After failure unload the beam and record the residual readings. Make a plaster cast of the failed web. Measure the residual shape of the deformed web.

Photographs were taken periodically of the deflected shape, the buckled web, the instrumentation layout, the lateral support system, and of the load and reaction devices.

4.4 Tension Coupons Tests

Tension coupons were taken from the flanges and webs of each shape, as shown in Fig. 4.13, and tested to determine the corresponding stress-strain characteristics. The coupons are cut and milled following ASTM E8M-88 (1988) recommendations using a gauge length of 200 *mm* and a width of 40 *mm*. A typical tension coupon is shown in Fig. 4.14.

The cross-sectional areas of the reduced area of the coupons are determined from 11 measurements of the reduced width and thickness made along the length prior to testing. Pairs of strain gauges mounted on both faces of each coupon, will allow the longitudinal and transverse strains to be measured for strains up to 20,000 μs . Large strains (over 20,000 μs) were measured by using dividers and scale over a 200 *mm* gauge length. The tension coupons are tested in accordance with ASTM E8M-88 (1988) in a 1780 Tinius Olsen testing machine driven by a MTS controller and servo-valve. Loads were recorded directly from the MTS controller.

4.5 Strain Gauge Data Interpretation Program

4.5.1 General

A strain interpretation program has been written to determine the principal stresses from measured strains (elastic or inelastic) for steel subjected to any state of stress given the

uniaxial stress-strain curve for the material and measured strains. Specific states of stress (if known and well defined) such as plane stress, plane strain, axisymmetric, and three dimensional are evaluated uniquely. A direct comparison can then be made between the principal stresses or stress contours as determined experimentally and those evaluated analytically. Internal force distributions can be determined subsequently by integrating the stresses over the cross section.

The program uses well established plasticity algorithms developed for finite element analysis. These are described in sections 4.5.2 to 4.5.4. Verification of the transformation process is described in section 4.5.5.

4.5.2 Procedure for Determining Stresses

The procedure to determine the principal stresses from a given set of measured strains is simple and can be summarized in the following:

- For a given set of strains ϵ_a , ϵ_b , and ϵ_c (rosette), calculate the strains ϵ_x , ϵ_y , and γ_{xy} using Mohr's circle for strain.
- Calculate the principal strains ϵ_1 and ϵ_2 in the plane of measurement.
- Determine the third principal strain ϵ_3 (3-D case) perpendicular to ϵ_1 and ϵ_2 by measurement or calculation.
- Calculate the strain vector ϵ_{ij} by transformation of ϵ_1 , ϵ_2 , and ϵ_3 .
- Calculate the stress vector σ_{ij} from ϵ_{ij} using the elasto-plastic D matrix.
- Calculate the corresponding principal stresses σ_1 , σ_2 , and σ_3 .

4.5.3 Determination of Principal Strains

A three element rosette is used to measure strains at essentially a point in three known directions. Each strain, designated as ϵ_a , ϵ_b , ϵ_c as shown in Fig. 4.15, is oriented with respect to an arbitrary x axis by angles θ_a , θ_b , and θ_c , respectively. Mohr's circle for strain gives the relationships given in Eq. (4.1) which relates the measured strains ϵ_a , ϵ_b , and ϵ_c to strains within the same plane oriented in the arbitrary direction.

$$\begin{Bmatrix} \epsilon_a \\ \epsilon_b \\ \epsilon_c \end{Bmatrix} = \frac{1}{2} \begin{bmatrix} 1 + \cos(2\theta_a) & 1 - \cos(2\theta_a) & \sin(2\theta_a) \\ 1 + \cos(2\theta_b) & 1 - \cos(2\theta_b) & \sin(2\theta_b) \\ 1 + \cos(2\theta_c) & 1 - \cos(2\theta_c) & \sin(2\theta_c) \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (4.1)$$

The principal strains and orientation can be determined from Eqs. (4.2), (4.3), and (4.4) from the in-plane strains determined from Eq. (4.1)

$$\epsilon_1 = \frac{\epsilon_x + \epsilon_y}{2} + \left[\left(\frac{\epsilon_x - \epsilon_y}{2} \right)^2 + \left(\frac{\gamma_{xy}}{2} \right)^2 \right]^{\frac{1}{2}} \quad (4.2)$$

$$\epsilon_2 = \frac{\epsilon_x + \epsilon_y}{2} - \left[\left(\frac{\epsilon_x - \epsilon_y}{2} \right)^2 + \left(\frac{\gamma_{xy}}{2} \right)^2 \right]^{\frac{1}{2}} \quad (4.3)$$

$$\tan(2\theta_1) = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} \quad (4.4)$$

This process gives the principal strains within the plane of the plate in which the strains are being measured. For most applications, tests on structural steel elements, the third principal strain will be perpendicular to this plane and is difficult if not impossible to measure. It must be deduced given the boundary conditions (calculated as a fraction of the principal strains ϵ_1 and ϵ_2), or calculated using Hooke's law in the elastic region and volume consistency in the plastic region.

4.5.4 Elasto-Plastic Stress-Strain Relation

After calculating the principal strains these are then transformed to the strain vector as:

$$\{\bar{\epsilon}\} = [H]\{\epsilon\}[H]^T \quad (4.5)$$

$$\{\epsilon\} = [H]^T\{\bar{\epsilon}\}[H] \quad (4.6)$$

where, $\{\bar{\epsilon}\}$ = vector of principal strains,
 $\{\epsilon\}$ = vector of strains,
 $[H]$ = transformation matrix,
 $[H]^T$ = transpose of the transformation matrix.

The model adopted here is based on the incremental theory of plasticity, in which after initial yielding, the material behaviour will be partly elastic and partly plastic. During any increment of stress, the change in strain is given by

$$d\epsilon_{ij} = (d\epsilon_{ij})_e + (d\epsilon_{ij})_p \quad (4.7)$$

where, $d\epsilon_{ij}$ = total strain increment,
 $(d\epsilon_{ij})_e$ = elastic strain increment,
 $(d\epsilon_{ij})_p$ = plastic strain increment.

The elastic strain increment is related to the stress increment by the following tensor equation

$$(d\epsilon_{ij})_e = \frac{d\sigma'_{ij}}{2G} + \frac{1-2\nu}{E} \delta_{ij} d\sigma_{kk} \quad (4.8)$$

where, G = shear modulus also referred as Lamé's constant,
 E = modulus of elasticity,
 ν = Poisson's ratio,
 $d\sigma'_{ij}$ = deviatoric stress increment,
 δ_{ij} = kronecker delta,
 $d\sigma_{kk}$ = stress increment of diagonal.

The plastic strain increment derived from the flow rule is related to the stress gradient of the plastic potential, Q , and is given as

$$(d\epsilon_{ij})_p = d\lambda \frac{\partial Q}{\partial \sigma_{ij}} \quad (4.9)$$

where, $d\lambda$ is a constant called a plastic multiplier.

The plastic potential is a function of the second and third invariants of the deviatoric stress tensor, J'_2 and J'_3 (Hill (1950)) but as yet, cannot be determined in its general form.

The stress invariants are given by

$$J'_2 = \frac{1}{2} \sigma'_{ij} \sigma'_{ij} \quad (4.10)$$

$$J'_3 = \frac{1}{3} \sigma'_{ij} \sigma'_{jk} \sigma'_{ki} \quad (4.11)$$

where, the deviatoric stress is given by,

$$\sigma'_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk} \quad (4.12)$$

Since the yield function f is also function of J'_2 and J'_3 , then ∂Q is equal to ∂f , giving Eq. (4.13), the basis of the associated theory of plasticity.

$$(d\epsilon_{ij})_p = d\lambda \frac{\partial f}{\partial \sigma_{ij}} \quad (4.13)$$

Eq. (4.13) is the normality condition since $\partial f / \partial \sigma_{ij}$ is a vector directed normal to the yield surface at the stress point under consideration.

Substituting Eqs. (4.8) and (4.13) into Eq. (4.7) gives the complete incremental relationship between stress and strain for elasto-plastic deformation,

$$d\epsilon_{ij} = \frac{d\sigma'_{ij}}{2G} + \frac{1-2\nu}{E} \delta_{ij} d\sigma_{kk} + d\lambda \frac{\partial f}{\partial \sigma_{ij}} \quad (4.14)$$

After some manipulation, the plastic multiplier $d\lambda$ can be obtained in explicit matrix form, Owen and Hinton (1980), and is given as

$$d\lambda = \frac{\{a\}^T \{d_D\} \{d\epsilon\}}{H + \{d_D\}^T \{a\}} \quad (4.15)$$

where, $\{a\}^T$, $\{d_D\}$ and H are defined by Eqs. (4.16), (4.17) and (4.18), respectively.

$$\{a\}^T = \frac{\partial f}{\partial \sigma_{ij}} \quad (4.16)$$

$$\{d_D\} = [D] \{a\} \quad (4.17)$$

$$H = \frac{E_{st}}{1 - \frac{E_{st}}{E}} \quad (4.18)$$

where, $\{a\}$ = flow vector,
 $[D]$ = matrix of elastic constants,
 H = hardening function,
 E_{st} = strain hardening modulus as determined from uniaxial stress-strain curve.

Substitute Eq. (4.15) into Eq. (4.14), to give the incremental stress in terms of incremental strain,

$$\{d\sigma\} = [D]_{ep}\{d\epsilon\} \quad (4.19)$$

where, $[D]_{ep}$ is the elasto-plastic constitutive matrix and is given by,

$$[D]_{ep} = [D] - \frac{\{d_D\}\{d_D\}^T}{H + \{d_D\}^T\{a\}} \quad (4.20)$$

The vectors $\{a\}$ and $\{d_D\}$ are dependent upon the yield function f . The Huber-Hencky von Mises yield criterion best describes the yielding of metals under combined stress and has been selected for the yield function. Yielding occurs when,

$$\sqrt{(3)(J_2')^{\frac{1}{2}}} \geq \bar{\sigma} \quad (4.21)$$

where, $\bar{\sigma}$ = the effective or equivalent stress. Plastic flow occurs when the effective stress equals or exceeds the uniaxial yield stress. The principal stresses are then found from the stress vector by solving for the roots of the third degree equation of principal stresses.

$$\sigma^3 - I_1\sigma^2 + I_2\sigma + I_3 = 0 \quad (4.22)$$

where, I_1 , I_2 , and I_3 are the stress invariants.

The general algorithm of the program is shown in Fig. 4.16 for the r^{th} iteration.

4.5.5 Verification and Discussion

The result from the program were compared to two sets of experimental data from tests on biaxially loaded cylinders conducted by Kennedy and Cheng (1987). A schematic diagram

of the test specimen is shown in Fig. 4.17. All test specimens were loaded axially along their length (monotically up to failure). In the case of biaxially loaded test specimens, internal hydraulic pressure was applied to maintain a circumferential strain of zero at the mid thickness of the tube wall, producing a plane strain stress state.

The uniaxial stress-strain curve, obtained experimentally, was modelled in the strain interpretation program by the trilinear stress-strain curve shown in Fig. 4.18.

A comparison of the longitudinal stress-strain curves (along the length of the cylinder) for a biaxially loaded cylinder as determined experimentally and analytically using the strain interpretation program are shown in Fig. 4.19. The two curves are essentially identical. Small variances elsewhere are due to the definition of the yield stress, σ_y and the trilinear approximation of uniaxial stress-strain curve. Multi-linear uniaxial stress-strain curve would yield even better results.

The same comparison can be made from an examination of principal stress plots (longitudinal stress, σ_x and hoop stress, σ_y) shown in Fig. 4.20. Also shown are the von Mises proportional limit, yield and ultimate loci as determined from measured properties. Again good agreement is observed between the test results and the theoretical results. The hoop stress varies by less than 7 percent.

In conclusion, the strain interpretation program will provide stresses from the measured strains, that will be comparable, in a meaningful way, with those determined from finite element simulations.

Figure 4.1: Schematic Diagram of the Test Set-Up.

Figure 4.2: Photograph of the Test Set-Up.

(a) Top flange is free to displace vertically

(b) Beam simply supported at the ends

Figure 4.3: Schematic Diagram of the Load and Support Details.

Figure 4.4 : Photographs of the Load and Support Details.

Figure 4.5 : Schematic Diagram of the Lateral Supports.

Figure 4.6: Photograph of the Lateral Supports.

Figure 4.7: Identification and Location of Strain Gauges.

CONTOUR VALUE
 A 0.1183E-02
 B -0.7889E-03
 C 0.3945E-03
 D -0.1084E-18
 E 0.3945E-03
 F 0.7889E-03
 G 0.1183E-02

(a) Longitudinal strain contours

CONTOUR VALUE
 A -0.2602E-02
 B -0.2082E-02
 C -0.1581E-02
 D -0.1041E-02
 E -0.5204E-03
 F -0.2168E-18
 G 0.5204E-03

(b) Vertical strain contours

Figure 4.8: Typical Web Strain Contours as Predicted by the Finite Element Analysis at Ultimate.

Figure 4.9: Identification and Location of LVDTs (interior load).

CONTOUR VALUE

A	0
B	2203
C	4406
D	6609
E	8813
F	1102
G	1322

(a) Out-of-plane deformed shape

(b) 2-D deformed shape

(c) 3-D deformed shape

Figure 4.10: Typical Web Out-Of-Plane Displacement Contours and Deformed Shape as Predicted by the Finite Element Analysis.

(a) Elevation

(b) Top view

Figure 4.11: Attachment of LVDTs (interior load).

Figure 4.12: Location and Attachment of LVDTs (End Reaction).

Figure 4.13: Tension Coupon Definition for W-shapes.

Figure 4.13: Tension Coupon Definition for W-shapes Con't.

Figure 4.13: Tension Coupon Definition for W-shapes Con't.

Notation :

- G - Gauge length, mm
- W - Width, mm
- T - Thickness, mm
- R - Radius of fillet, mm
- L - Overall length, mm
- A - Length of reduced section, mm
- B - Length of grip section, mm
- C - Width of grip section, mm.

Figure 4.14: Typical Dimensions for Tension Coupons.

Figure 4.15: General Orientation of a Three Element Rosette.

Figure 4.16: Algorithm of the Strain Interpretation Program.

Figure 4.16: Algorithm of the Strain Interpretation Program Con't.

Figure 4.17: Biaxially Loaded Cylindrical Test Specimen.

Figure 4.18: Uniaxial Stress-Strain Curves (Cylinder 1).

Figure 4.19: Stress-Strain Curves for a Biaxially Loaded Cylinder (Cylinder 2).

Figure 4.20: Von-Mises Yield and Ultimate Failure Locus (Cylinder 2).

Table 4.1: Nominal Dimensions of Tested Shapes.

<i>Designation</i>	<i>c</i>	<i>l</i>	<i>t</i>	<i>w</i>	<i>b</i>	<i>d</i>	<i>h</i>	$\frac{b}{2t}$	$\frac{h}{w}$	<i>Class</i>
	<i>mm</i>	<i>mm</i>	<i>mm</i>	<i>mm</i>	<i>mm</i>	<i>mm</i>	<i>mm</i>			
<i>W</i> 530x101	200	2200	17.4	10.9	210	537	502	6.03	46.1	1
<i>W</i> 530x82	200	2200	13.3	9.5	209	528	502	7.86	52.7	1
<i>W</i> 410x39	200	1600	8.8	6.4	140	399	381	7.95	59.7	1
<i>W</i> 360x45	200	1400	9.8	6.9	171	352	332	8.72	48.1	2
<i>W</i> 310x31	200	1400	7.4	5.0	164	306	291	11.0	58.2	3
<i>W</i> 250x24	200	950	6.4	5.0	145	253	240	11.3	48.2	3
<i>WWF</i> 350x212	100	1500	30	20	350	350	290	5.83	14.5	1
<i>WWF</i> 1800x510	300	6000	30	20	500	1800	1740	8.33	87.0	2
<i>WWF</i> 1400x405	300	5800	30	16	500	1400	1340	8.33	83.7	2
<i>WWF</i> 900x262	300	4000	30	11	400	900	840	6.67	76.3	2
<i>WWF</i> 500x197	200	2200	20	11	500	500	460	12.5	41.8	4

Table 4.2: Predicted Failure Loads by Different Models.

<i>Designation</i>	P_{cr} (<i>kN</i>)	P_{Lusas} (<i>kN</i>)	$P_{Roberts81}$ (<i>kN</i>)	$P_{S16.1}$ (<i>kN</i>)	P_{yield} (<i>kN</i>)	<i>Probable Mode of Failure</i>
<i>W 530x101</i>	3020	998	889	1212	1349	<i>Inelastic buckling</i>
<i>W 530x82</i>	1973	778	675	935	1082	<i>Inelastic buckling</i>
<i>W 410x39</i>	825	427	307	483	697	<i>Inelastic buckling</i>
<i>W 360x45</i>	1240	477	356	592	763	<i>Inelastic buckling</i>
<i>W 310x31</i>	562	316	187	330	536	<i>Inelastic buckling</i>
<i>W 250x24</i>	729	325	187	388	470	<i>Inelastic buckling</i>
<i>W 530x101(end)</i>	1361	494	441	498	674	<i>Inelastic buckling</i>
<i>W 530x82(end)</i>	896	389	336	373	541	<i>Inelastic buckling</i>
<i>W 410x39(end)</i>	392	209	153	183	349	<i>Inelastic buckling</i>
<i>W 360x45(end)</i>	592	229	178	222	381	<i>Inelastic buckling</i>
<i>W 310x31(end)</i>	273	156	94	122	268	<i>Inelastic buckling</i>
<i>W 250x24(end)</i>	366	158	94	134	235	<i>Inelastic buckling</i>
<i>WWF 350x212</i>	40053	—	2993	3733	2013	<i>Yielding</i>
<i>WWF 1800x510</i>	5067	—	2941	3238	3333	<i>Inelastic buckling</i>
<i>WWF 1400x405</i>	3432	—	1916	2278	2614	<i>Inelastic buckling</i>
<i>WWF 900x262</i>	2505	—	905	1269	1797	<i>Inelastic buckling</i>
<i>WWF 500x197</i>	3489	—	905	1263	1252	<i>Inelastic buckling</i>

Table 4.3 : Type of Measurement and Instrumentation.

Instrumentation	Location/Description	Purpose	Frequency of Measurement
Pressure Transducers	Applied Loads	Gives the applied loads.	Discrete readings taken at the end of each load or deformation increment.
	Steel Strains on Web		
Electrical Resistance Gauges	Located on the input pressure line of the actuator.		
	Series of rosettes distributed as shown in Fig. 4.7.	Gives the measurements of strains at discrete locations on the web. These will be converted to principal stresses via the strain interpretation program, giving the distribution of stresses in the web, which in turn permits the determination of internal forces. These distributions will be compared to those from finite element predictions.	Discrete readings taken at the end of each load or deformation increment.
Electrical Resistance Gauges	Steel Strains on Flanges		
	Series of uniaxial strains distributed as shown in Fig. 4.7.	Gives the longitudinal distribution of strains and stresses in the flanges.	Discrete readings taken at the end of each load or deformation increment.
LVDTs	Deflected Shape of Web		
	A series of LVDTs are located on the web and at the load point as shown in Figs. 4.9 and 4.12.	Used to monitor the load deflection response of the web and the flanges under load.	Discrete readings taken at the end of each load or deformation increment.

Chapter 5

TEST RESULTS

5.1 Geometric Properties

The flange thickness, web thickness, web depth, and flange width were measured for each W shape. The number of measurements, the mean value, the coefficient of variation, and the ratio of the mean to nominal value (shown in brackets) are listed in Table 5.1. The mean values were used subsequently, to calculate the sections capacity using the equivalent column concept and in the finite element modelling. The statistical data was used in the determination of the appropriate resistance factor. The distance from the outer face of the flange to the web toe of the fillet, k , is difficult to accurately define, hence the measured values were relatively unreliable, and nominal values of k were used in the calculations.

The maximum initial out-of-parallel (flanges) and out-of-plane straightness (web flatness) were measured for each test specimen and are listed in Table 5.2 along with the corresponding ratios to the web depth. These values lie within the allowable fabrication tolerances as described in the CISC Handbook of Steel Construction 1991, which are

maximum 5 *mm* for web depth of 300 *mm* and under and 6 *mm* for web depth of over 300 *mm* for out-of-parallel, and a ratio to shape depth of 1/150 for web flatness.

5.2 Material Properties

Sixty-eight coupons were tested from the flange tip, flange web junction, and from the web of the six W-shaped sections tested in this series. A summary of the pertinent material properties (E , ν , σ_p , σ_y , σ_{ys} , σ_u , ϵ_y , ϵ_{st} , ϵ_u , ϵ_f , E_{st}) from these tests is given for each section; W250x24, W360x45, W410x39, W310x31, W530x82, and W530x101 in Tables 5.3 to 5.8, respectively. Also listed for the group of coupons taken from the flange and from the web are the mean values and the corresponding coefficients of variation.

Typical stress-strain curves for coupons taken from the flange tips, the flange web junction, and the web for the W530x82 section are shown in Fig. 5.1. For coupons with no definite yield plateau, the 0.2% offset method was used to define the yield stress. The corresponding strain hardening modulus is simply defined as the slope of the tangent to the curve after this point.

5.3 Web Behaviour

A description of the test histories giving dates, failure loads and modes, and observations of initiation of buckling and the formation of yield bands, which relate to inelastic web buckling behaviour, are summarized in Tables 5.9 and 5.10.

The best description of web buckling behaviour is given by the applied load out-of-plane web displacement curves shown in Figs. 5.2 to 5.7 for each section tested. Figures (a) are for interior loads with the loaded flange restrained against lateral displacement and rotation, and Figures (b) and (c) are for end reactions with the restraint specified. Each

graph illustrates the load displacement curve for the LVDT indicated along with the overall buckled shape at ultimate. With the exception of the W410x39 shown in Fig. 5.4, these characteristic curves indicate that the specimens failed by local inelastic web buckling. These curves show little or no out-of-plane displacement up to the load in which yielding begins and lateral stability is lost. The curve for the W410x39, with a web slenderness, h/w of 59.4, exhibits a gradual increase in out-of-plane displacement with increased load that is typical of elastic web buckling. A few of the curves throughout this series show small jumps in displacement (about 0.1 mm) with an increment in load. The slope of the curve on either side of the jump is similar and it is suspected that the jump is related to the sliding and readjusting of the LVDT tip on the steel surface and not with respect to any real displacements. The gradual decrease in load capacity after ultimate is indicative of the continued spread of yielding within the web, local to the load or reaction point.

The corresponding displaced shapes of the web centred under the applied load or centred over the end reaction for the given case are shown in Figs. 5.8 to 5.13 for ultimate and the last load level prior to unloading. In each case, the web was pushed well beyond ultimate to accentuate the location and the yield band formation. The measured dimensions (thickness and length) and location of the first and second yield bands are given in Table 5.10. Fig. 5.14 is a photograph of the second yield band for the interior load case of the W250x24. Yield bands form on the concave side only (the bands appear on opposite sides). The yield bands are circular arcs in shape with one forming close to the flange web junction and the second between $0.25 h$ and $0.5 h$ from the loaded flange. In all cases, the chord length of the circular arc was approximately equal to $c + 5 k$. The displaced shapes, shown in the figures mentioned above, are based on five discrete measurements of the out-of-plane web displacements and the assumption that the flanges were restrained from lateral displacement as specified for each particular case. The out-of-plane web displacement at the load and end reaction points were assumed to be zero.

According to Clause 15.5.1 of S16.1-M89 (CSA (1989)), bearing stiffeners should be provided at unframed ends of single web girders which have a web depth to thickness ratio greater than $1100/\sqrt{F_y}$. None of the sections tested required stiffeners. However, as indicated by the deformed shape of the web for end reaction tests, for the W410x39, the W360x45 and the W310x31 (end reaction 2), given in Figs. 5.10 (b) and (c), 5.11 (b) and (c) and 5.12 (c), respectively, the maximum displacement occurred at the unloaded flange which was free to displace and rotate. End reaction test for end1, for the W310x31, and for the W250x24 were stable up to ultimate. Subsequently, these failed suddenly with lateral displacement and rotation of the unloaded flange. For these two tests, however, the maximum out-of-plane web displacement occurred at the second yield band. Given the lateral instability exhibited by these tests, the ends for the W530x101 and the W530x82 were laterally supported to prevent premature buckling due to web distortion and flange warping.

The principal strains calculated from in-plane strains measured on one side of the web and the corresponding orientation at ultimate for a series of discrete locations, are shown schematically for each interior load test in Figs. 5.15 to 5.20 (refer to Section 4.5.3 for principal strain determination). The first value is associated with the principal strain aligned close to the vertical axis and the second to the horizontal axis. The value in brackets is the orientation of the principal strains with positive values being anti-clockwise. Negative strains indicate compression.

As anticipated, the vertical strain distribution along mid-span (centred under the load) increases to a maximum at $h/8$ from the top loaded flange diminishes to zero at the bottom unloaded flange. The vertical strains become insignificant outside the load affected zone $c + 5k$. More conclusive statements about strain distribution trends cannot be made as the strains were measured on one side only and local bending influenced the values significantly.

Average longitudinal strains measured for top and bottom flanges, the average yield strain as obtained from tension coupon tests, and the ratio of the average bottom strain to the yield strain at ultimate are given for each test in Table 5.11. The ratio suggests that the majority of the flange capacity was being utilized in overall bending. There was no evidence of plastic hinge formation on the loaded flange given the measured strains (interior load only) and lack of any formation of yield lines.

Figure 5.1: Typical stress-strain curves for W530x82.

Figure 5.2: Load vs Out-Of-Plane Web Displacement, W530x101.

Figure 5.2: Load vs Out-Of-Plane Web Displacement, W530x101 Con't.

Figure 5.3: Load vs Out-Of-Plane Web Displacement, W530x82.

Figure 5.3: Load vs Out-Of-Plane Web Displacement, W530x82 Con't.

Figure 5.4: Load vs Out-Of-Plane Web Displacement, W410x39.

Figure 5.4: Load vs Out-Of-Plane Web Displacement, W410x39 Con't.

Figure 5.5: Load vs Out-Of-Plane Web Displacement, W360x45.

Figure 5.5: Load vs Out-Of-Plane Web Displacement, W360x45 Con't.

Figure 5.6: Load vs Out-Of-Plane Web Displacement, W310x31.

Figure 5.6: Load vs Out-Of-Plane Web Displacement, W310x31 Con't.

Figure 5.7: Load vs Out-Of-Plane Web Displacement, W250x24.

Figure 5.8: Out-Of-Plane Web Deformation; W530x101.

Figure 5.8: Out-Of-Plane Web Deformation; W530x101 Con't.

Figure 5.9: Out-Of-Plane Web Deformation; W530x82.

Figure 5.9: Out-Of-Plane Web Deformation; W530x82 Con't.

Figure 5.10: Out-Of-Plane Web Deformation; W410x39.

Figure 5.10: Out-Of-Plane Web Deformation; W410x39 Con't.

Figure 5.11: Out-Of-Plane Web Deformation; W360x45.

Figure 5.11: Out-Of-Plane Web Deformation; W360x45 Con't.

Figure 5.12: Out-Of-Plane Web Deformation; W310x31.

Figure 5.12: Out-Of-Plane Web Deformation; W310x31 Con't.

Figure 5.13: Out-Of-Plane Web Deformation; W250x24.

Figure 5.14: Second Yield Band, Interior Load, W250x24.

Figure 5.15: Strain Distribution Data at Ultimate, 1345 kN, W530x101.

Figure 5.16: Strain Distribution Data at Ultimate, 1180 kN, W530x82.

Figure 5.17: Strain Distribution Data at Ultimate, 553 kN, W410x39

Figure 5.18: Strain Distribution Data at Ultimate, 665 kN, W360x45.

Figure 5.19: Strain Distribution Data at Ultimate, 497 kN, W310x31.

Figure 5.20: Strain Distribution Data at Ultimate, 434 kN, W250x24.

Table 5.1: Cross Sectional Dimensions.

Section	<i>t</i> (mm)			<i>w</i> (mm)			<i>d</i> (mm)			<i>b</i> (mm)		
Designation	<i>n</i>	$\bar{X}$	<i>V</i> (%)	<i>n</i>	$\bar{X}$	<i>V</i> (%)	<i>n</i>	$\bar{X}$	<i>V</i> (%)	<i>n</i>	$\bar{X}$	<i>V</i> (%)
W 530x101	100	17.0	1.8	50	11.0	1.9	36	535.9	0.2	40	207.3	0.3
		(0.976)			(1.009)			(0.998)			(0.987)	
W 530x82	100	13.4	3.4	50	9.9	1.4	39	532.2	0.3	42	212.3	0.2
		(1.004)			(1.037)			(1.008)			(1.016)	
W 410x39	100	8.9	1.0	50	6.7	2.0	40	402.4	0.1	44	139.8	0.3
		(1.017)			(1.047)			(1.009)			(0.998)	
W 360x45	100	9.7	2.0	50	7.1	1.5	40	356.4	0.1	40	173.0	0.3
		(0.992)			(1.026)			(1.012)			(1.012)	
W 310x31	100	7.8	2.4	50	5.7	3.1	42	310.2	0.4	40	165.0	0.3
		(1.051)			(1.150)			(1.014)			(1.006)	
W 250x24	100	6.6	3.2	50	5.0	3.1	40	253.5	0.2	40	147.6	0.2
		(1.031)			(1.004)			(1.002)			(1.018)	

Notes: 1. The ratio of the measured to the nominal value is the value shown in brackets.

2. Variable definitions: *n* = number of measurements; $\bar{X}$ = mean value; *V* = coefficient of variation.

Table 5.2: Initial Imperfections of the Shapes.

Section	Out-of-Parallel (flanges)	Imperfection-to-Depth	Out-of-Plane (web flatness)	Imperfection-to-Depth
Designation	(mm)		(mm)	
W530x101	1.770	0.00330 (1/303)	0.750	0.00140 (1/515)
W530x82	2.761	0.00520 (1/193)	1.305	0.00245 (1/408)
W410x39	0.135	0.00033 (1/2981)	0.710	0.00176 (1/567)
W360x45	0.523	0.00147 (1/681)	0.595	0.00167 (1/599)
W310x31	1.988	0.00640 (1/156)	1.090	0.00351 (1/285)
W250x24	0.359	0.00142 (1/706)	0.155	0.00061 (1/1635)

Table 5.3. Material Properties - W250x24.

Coupon No.	E	ν	σ_p	σ_y	σ_{ys}	σ_u	ϵ_y $\times 10^{-6}$	ϵ_{st} $\times 10^{-6}$	ϵ_u $\times 10^{-6}$	ϵ_f $\times 10^{-6}$	E_{st}
	MPa		MPa	MPa	MPa	MPa	mm/mm	mm/mm	mm/mm	mm/mm	MPa
C1	203564	-	409.7	421.4	391.6	573.3	2048	14516	150000	367104	2172
C2	207488	-	-	384.9	361.6	548.9	1975	17326	150000	580462	3086
C3	200404	-	314.3	396.1	377.8	582.7	2045	11198	150000	541324	5852
C4	207499	-	-	428.5	412.5	570.2	2058	21080	130000	584135	3030
C5	205350	-	-	381.4	358.0	550.7	1839	13429	160000	566074	2480
C6	201184	-	-	381.1	344.1	536.1	1949	9818	150000	388710	2905
$\bar{X}$	204248	-	-	398.9	374.3	560.3	1986	14561	148333	504635	3254
V(%)	1.40	-	-	4.80	6.10	2.90	4.20	26.0	6.10	18.0	37.0
C7	206746	-	-	418.4	-	572.8	2014	15315	150000	385940	2630
C8	203886	0.28	-	428.4	414.6	571.5	2076	11867	140000	421350	-
C9	207437	-	-	445.1	-	582.8	2111	-	185000	504236	1060
C10	199349	0.28	-	416.9	405.5	569.9	2080	16462	190000	431424	-
$\bar{X}$	204355	0.28	-	427.2	410.1	574.3	2070	14548	166250	435738	1845
V(%)	1.60	0.00	-	2.60	1.10	0.90	1.70	13.4	13.0	9.90	42.5

Table 5.4: Material Properties - W360x45.

Coupon No.	E MPa	ν	σ_p MPa	σ_y MPa	σ_u MPa	ϵ_y $\times 10^{-6}$	ϵ_{st} $\times 10^{-6}$	ϵ_u $\times 10^{-6}$	ϵ_f $\times 10^{-6}$	E_{st}	
						mm/mm	mm/mm	mm/mm	mm/mm	MPa	
C11	199963	-	264.3	338.0	329.8	515.8	1656	7604	175000	706036	3756
C12	199822	-	257.1	325.0	312.5	507.1	1563	10848	180000	1009056	3384
C13*	201602	-	300.0	350.0	339.4	521.8	3688†	-	165000	847928	3111
C14*	201129	-	278.6	328.6	302.0	519.5	3625†	-	150000	711304	4510
C15	192600†	-	-	310.0	305.9	491.1	1718†	11428	170000	990407	3757
C16*	202323	-	319.9	364.3	344.4	524.1	3750†	-	165000	742373	2481
$\bar{X}$	200969	-	284.0	335.9	322.3	513.2	1610	9960	167500	834517	3500
V(%)	0.50	-	8.20	5.20	5.10	2.20	2.90	16.9	5.70	15.1	17.9
C17*	200403	-	217.9	400.0	388.4	543.0	3875†	-	125000	754015	2422
C18	201055	-	-	370.7	-	524.4	1899	16581	170000	752476	2080
C19	204759	-	-	373.6	-	526.4	1799	-	165000	886860	1525
C20	202618	0.28	-	366.3	357.7	517.9	1806	14494	170000	647098	1722
C21*	197044	0.29	307.1	385.7	364.5	536.7	3844†	-	125000	792886	3935
$\bar{X}$	201176	0.285	-	379.2	370.2	529.7	1835	15538	151000	766667	2337
V(%)	1.30	1.80	-	3.20	3.60	1.70	2.50	6.70	14.1	10.1	36.6

Notes: * No definite yield plateau.

† Not included in statistic calculations.

† ϵ_y and σ_y determined using 0.2% offset. ϵ_y not included in statistic calculations.

Table 5.5: Material Properties - W410x39.

Coupon No.	E MPa	ν	σ_p MPa	σ_y MPa	σ_{ys} MPa	σ_u MPa	ϵ_y $\times 10^{-6}$	ϵ_{st} $\times 10^{-6}$	ϵ_u $\times 10^{-6}$	ϵ_f $\times 10^{-6}$	E_{st} MPa
C22	206698	-	310.7	327.7	319.0	505.8	2122	6242	21000	651414	3119
C23	204514	-	-	308.8	297.7	480.6	1890	10141	16000	698700	2340
C24	201767	-	300.0	323.1	307.8	504.1	2026	5070	165000	671434	3188
C25	203260	-	-	325.6	321.3	504.6	1643	6705	175000	718911	2128
C26	192214 [†]	-	232.9	318.6	316.3	495.7	2175 [†]	12911 [†]	195000	825293	2361
C27	203590	-	-	334.4	319.5	502.8	1672	8359	160000	737865	2144
$\bar{X}$	203966	-	-	323.0	313.6	498.9	1871	7303	177500	717270	2547
V(%)	0.80	-	-	2.50	2.70	1.80	10.1	24.2	10.6	7.80	17.2
C28	195472	0.29	307.1	350.2	341.1	517.3	2140	10070	14000	627359	3594
C29	203739	0.28	-	363.0	358.8	521.0	1775	13925	19000	881430	1957
C30	177860 [†]	-	-	370.9	-	514.2	1973 [†]	-	18000	557018	1243
C31	204156	-	-	362.9	-	516.5	1773	-	175000	792574	1148
C32	199985	-	-	348.9	336.5	517.5	2055	13201	185000	757094	3285
$\bar{X}$	200838	0.285	-	359.2	345.5	517.3	1936	12399	174000	723094	2245
V(%)	1.70	1.80	-	2.30	2.80	0.40	8.50	13.5	10.2	16.1	45.4

Notes: * No definite yield plateau.

[†] Not included in statistic calculations.

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Table 5.6: Material Properties - W310x31.

Coupon No.	E MPa	ν	σ_p MPa	σ_y MPa	σ_{ys} MPa	σ_u MPa	ϵ_y $\times 10^{-6}$	ϵ_{st} $\times 10^{-6}$	ϵ_u $\times 10^{-6}$	ϵ_f $\times 10^{-6}$	E_{st} MPa
C33	196940	-	-	359.2	-	550.8	2141	15905	130000	463656	3586
C34	206133	-	-	356.6	-	541.4	1591	13765	175000	578245	2529
C35	199272	-	-	354.8	339.5	542.2	2187	12243	150000	613926	3355
C36*	203032	-	300.0	364.3	337.0	550.4	3750 [†]	-	125000	544751	3582
C37	207204	-	-	344.9	-	527.6	1846	11851	175000	585745	4429
C38	203352	-	254.4	352.9	330.2	543.1	3688 [†]	-	150000	645002	3602
$\bar{X}$	202656	-	-	355.5	335.6	542.6	1941	13441	150833	571888	3514
V(%)	1.80	-	-	1.70	1.20	1.40	12.4	11.8	12.9	10.0	15.8
C39	196495	0.28	214.3	392.1	375.7	550.1	3844 [†]	8626	150000	521128	3840
C40	211933	0.28	-	393.7	-	558.0	1857	-	175000	455783	-
C41	206806	-	-	404.4	-	-	1935	15666	-	-	-
C42	199196	-	225.0	391.7	370.7	571.3	3750 [†]	11395	90000	495079	3597
$\bar{X}$	203608	0.28	-	395.5	373.2	559.8	1896	11896	111333	490663	3719
V(%)	3.00	0.00	-	1.30	0.70	1.60	2.10	24.3	65.6	5.50	3.30

Notes: * No definite yield plateau.

[†] ϵ_y and σ_y determined using 0.2% offset. ϵ_y not included in statistic calculations.

Table 5.7: Material Properties - W530x82.

Coupon No.	E MPa	ν	σ_p MPa	σ_y MPa	σ_{ys} MPa	σ_u MPa	ϵ_y $\times 10^{-6}$ mm/mm	ϵ_{st} $\times 10^{-6}$ mm/mm	ϵ_u $\times 10^{-6}$ mm/mm	ϵ_f $\times 10^{-6}$ mm/mm	E_{st} MPa
C43	202871	-	-	370.4	367.9	545.5	1942	13293	220000	797185	3997
C44	206438	-	-	362.7	357.2	547.7	2210	11081	170000	748268	2716
C45	199237	-	-	366.0	349.9	554.1	1992	11408	180000	725729	3111
C46	206503	-	-	374.0	367.5	550.6	1826	15313	170000	993240	3664
C47	205830	-	-	377.7	366.3	544.7	1833	16186	170000	783958	3283
C48	209139	-	-	357.9	-	544.0	1685	13546	185000	726917	3386
C49	197722	-	-	347.4	-	526.2	2065	11861	175000	858332	3169
C50	204959	-	-	366.9	355.0	542.1	2193	14844	150000	980606	3141
$\bar{X}$	204087	-	-	365.4	360.6	544.4	1968	13442	177500	826779	3308
V(%)	1.80	-	-	2.40	1.90	1.40	8.80	13.2	10.5	12.2	10.9
C51*	196384	0.28	307.1	435.7	432.1	591.0	3844 [†]	-	130000	861151	2591
C52	205273	-	-	382.3	379.8	550.6	1854	16915	180000	861380	2651
C53	207256	-	-	382.5	-	552.9	1841	14951	185000	817349	2886
C54	202460	-	-	381.8	378.9	547.0	1954	16619	165000	593476	2757
C55*	201042	0.28	357.1	437.9	409.8	584.1	4063 [†]	-	185000	587555	2480
$\bar{X}$	202483	0.28	-	404.0	400.2	565.1	1883	16162	169000	744182	2673
V(%)	1.90	0.00	-	6.60	5.60	3.30	2.70	5.40	12.3	17.0	5.20

Notes: * No definite yield plateau.

[†] ϵ_y and σ_y determined using 0.2% offset. ϵ_y not included in statistic calculations.

Table 5.8: Material Properties - W530x101.

Coupon No.	E	ν	σ_p	σ_y	σ_{ys}	σ_u	ϵ_y $\times 10^{-6}$	ϵ_{st} $\times 10^{-6}$	ϵ_u $\times 10^{-6}$	ϵ_f $\times 10^{-6}$	E_{st}
	MPa		MPa	MPa	MPa	MPa	mm/mm	mm/mm	mm/mm	mm/mm	MPa
C56	202336	-	300.4	359.7	351.6	525.9	3625 [†]	7788	170000	791474	2655
C57	201813	-	-	339.4	334.8	531.5	1937	9139	185000	925989	2905
C58	198163	-	-	344.8	334.8	533.2	1891	9012	175000	978602	3572
C59*	200869	-	250.0	353.6	-	524.5	3625 [†]	-	170000	972778	2154
C60	203514	-	-	352.8	342.3	527.0	2104	11151	180000	932242	2857
C61	197622	-	-	336.5	332.6	531.0	1673	9086	180000	1080818	2814
C62	200616	-	-	339.8	337.0	525.7	1694	11304	175000	1048866	2880
C63	202324	-	301.0	356.4	341.3	525.2	1947	7676	170000	1056240	3443
$\bar{X}$	200907	-	-	347.9	339.2	528.0	1874	9308	175625	973376	2910
V(%)	1.00	-	-	2.40	1.80	0.60	8.00	14.4	3.00	9.00	14.2
C64*	199896	-	232.1	405.7	386.2	541.8	3875 [†]	-	175000	669975	2270
C65	201634	0.28	-	371.2	370.0	534.2	1782	17500	180000	798605	2956
C66	207528	0.28	-	377.7	374.9	537.7	1803	14651	180000	598299	2388
C67	201262	-	-	366.9	365.6	535.5	1875	16069	185000	713573	2852
C68*	200782	-	-	400.0	366.6	546.8	3938 [†]	-	13000	631385	2435
$\bar{X}$	202220	0.28	-	384.3	372.7	539.2	1820	16073	171000	682367	2580
V(%)	1.30	0.00	-	4.10	2.00	0.90	2.20	7.20	10.7	10.2	10.5

Notes: * No definite yield plateau.

[†] ϵ_y and σ_y determined using 0.2% offset. ϵ_y not included in statistic calculations.

Table 5.9: Test Histories and Observations.

Test Section, Load Location, and Date	Failure Load, kN, and Mode	Maximum Deflection and Location ¹ , mm	Observations
• W530x101 ³	• 1345 kN	• 3.13 mm	• the observations are based on a retest of the section with the
• interior load	• inelastic web	• 0.30 h	unloaded flanges over the supports being laterally supported. ⁴
• 94-03-18	buckling		• two distinct circular arc yield bands formed at ultimate, one just below the fillet and the other centered at 0.27 h below the loaded flange.
• W530x101 ³	• 639 kN ²	• 3.30 mm	• two yield bands (refer to Table 5.10 for dimensions).
• end reaction 1	• inelastic web	• 0.30 h	• additional yield lines propagated radially out from the support and towards mid span.
• 94-03-24	buckling		• gradual failure.
• W530x101 ³	• 618 kN ²	• 3.30 mm	• yield bands start to form at 550 kN,
• end reaction 2	• inelastic web	• 0.26 h	(refer to Table 5.10 for dimensions).
• 94-03-28	buckling		• additional yield lines as end reaction 1.
			• gradual failure.

- Note:
1. Location of the maximum deflection is given as a function of h as measured from the loaded flange.
 2. Reaction load calculated using statics, the applied load, and measured geometry.
 3. The unloaded flange over the supports was restrained against lateral displacement and rotation.
 4. The section was first tested on 94-01-17. The beam reached an ultimate load of 1293 kN with significant warping of the section (no lateral supports for the ends). All measured strains were less than yield and when the beam was unloaded it rebounded close to the original readings.

Table 5.9: Test Histories and Observations con't.

Test Section, Load Location, and Date	Failure Load, kN, and Mode	Maximum Deflection and Location ¹ , mm	Observations
<ul style="list-style-type: none"> • W530x82³ • interior load • 94-03-22 	<ul style="list-style-type: none"> • 1180 kN • inelastic web buckling 	<ul style="list-style-type: none"> • 2.10 mm • 0.31 h 	<ul style="list-style-type: none"> • virtually little or no out-of-plane web displacement at 1100 kN. • at approximately 1160 kN, Luders lines started to form at the edges of the load plate and propagated towards the centre and towards the flange web junction at $c/2 + 2.5$ k from the centre; these continued to spread out to the supports for a distance of about 50 mm on either side. • two distinct circular arc yield bands formed at 1170 kN, one just below the fillet and the other centered at 0.23 h below the loaded flange. • The unloaded flange over the supports was restrained against lateral displacement and rotation.
<ul style="list-style-type: none"> • W530x82³ • end reaction 1 • 94-03-29 	<ul style="list-style-type: none"> • 586 kN² • inelastic web buckling 	<ul style="list-style-type: none"> • 1.92 mm • 0.31 h 	<ul style="list-style-type: none"> • two yield bands formed at ultimate (refer to Table 5.10 for dimensions); these continued to propagate towards mid span. • additional yield lines propagated radially out from the support. • gradual failure.
<ul style="list-style-type: none"> • W530x82³ • end reaction 2 • 94-03-30 	<ul style="list-style-type: none"> • 552 kN² • inelastic web buckling 	<ul style="list-style-type: none"> • 2.26 mm • 0.31 h 	<ul style="list-style-type: none"> • yield bands formed earlier at 475 kN. • additional radial yield lines were also present. • in conjunction with the end reaction failure, the stiffened interior load point was deforming which may have reduced the ultimate load. • sudden buckling failure.

Note: 1. Location of the maximum deflection is given as a function of h as measured from the loaded flange.
2. Reaction load calculated using statics, the applied load, and measured geometry.
3. The unloaded flange over the supports was restrained against lateral displacement and rotation.

Table 5.9: Test Histories and Observations con't.

Test Section, Load Location, and Date	Failure Load, kN, and Mode	Maximum Deflection and Location ¹ , mm	Observations
• W410x39	• 553 kN	• 1.73 mm	• at approximately 492 kN, Ludders lines started to form at the
• interior load	• inelastic web	• 0.31 h	edges of the load plate and propagated towards the centre and
• 94-01-12	buckling		towards the flange web junction at $c/2 + 2.5$ k from the centre;
			corresponds with an increase of out-of-plane web displacement.
			• two distinct circular arc yield bands formed at ultimate, one just
			below the fillet and the other centered at 0.28 h below the
			loaded flange.
• W410x39	• 226 kN ²	• 1.70 mm	• only one yield band formed just above the fillet at 255 kN
• end reaction 1	• inelastic web	• 0.74 h	(refer to Table 5.10 for dimensions).
• 94-01-13	buckling		• out-of-plane web displacements about 0.25 w at ultimate for the
			unloaded flange (maximum along the web); corresponding flange
			rotations occurred.
• W410x39	• 206 kN ²	• 1.25 mm	• no noticeable yield lines.
• end reaction 2	• possible elastic	• 0.74 h	• out-of-plane web displacements about 0.19 w at ultimate for the
• 94-01-13	web buckling		unloaded flange (maximum along the web); corresponding flange
			rotations occurred.

Note: 1. Location of the maximum deflection is given as a function of h as measured from the loaded flange.

2. Reaction load calculated using statics, the applied load, and measured geometry.

Table 5.9: Test Histories and Observations con't.

Test Section, Load Location, and Date	Failure Load, kN, and Mode	Maximum Deflection and Location ¹ , mm	Observations
• W360x45	• 665 kN	• 1.21 mm	• virtually little or no out-of-plane web displacement at 545 kN.
• interior load	• inelastic web	• 0.36 h	• at approximately 620 kN, Luders lines started to form at the edges of the load plate and propagated towards the centre and towards the flange web junction at $c/2 + 2.5$ k from the centre; corresponds with an increase of out-of-plane web displacement.
• 94-01-14	buckling		• two distinct circular arc yield bands formed at 640 kN, one just below the fillet and the other centered at 0.32 h below the loaded flange.
• W360x45	• 257 kN ²	• 2.78 mm	• one yield band formed just above the fillet at 255 kN, the second was less noticeable (refer to Table 5.10 for dimensions).
• end reaction 1	• inelastic web	• 0.78 h	• out-of-plane web displacements about 0.39 w at ultimate for the unloaded flange (maximum along the web); corresponding flange rotations occurred.
• 94-01-15	buckling		
• W360x45	• 272 kN ²	• 4.91 mm	• only one yield band started to form at 256 kN and was complete by 270 kN. (refer to Table 5.10 for dimensions).
• end reaction 2	• inelastic web	• 0.78 h	• out-of-plane web displacements about 0.69 w at ultimate for the unloaded flange (maximum along the web); corresponding flange rotations occurred.
• 94-01-15	buckling		

Note: 1. Location of the maximum deflection is given as a function of h as measured from the loaded flange.
2. Reaction load calculated using statics, the applied load, and measured geometry.

Table 5.9: Test Histories and Observations con't.

Test Section, Load Location, and Date	Failure Load, kN, and Mode	Maximum Deflection and Location ¹ , mm	Observations
• W310x31	• 497 kN	• 1.07 mm	• virtually little or no out-of-plane web displacement at 380 kN.
• interior load	• inelastic web	• 0.56 h	• at approximately 400 kN, Luders lines started to form at the
• 94-01-06	buckling		edges of the load plate and propagated towards the centre and
			towards the flange web junction at $c/2 + 2.5$ k from the centre;
			corresponds with an increase of out-of-plane web displacement.
			• two distinct circular arc yield bands formed at 495 kN, one just
			below the fillet and the other centered at 0.43 h below the
			loaded flange.
• W310x31	• 218 kN ²	• 2.62 mm	• sudden web buckling failure.
• end reaction 1	• inelastic web	• 0.25 h	• two half circular arc yield bands formed about the reaction
• 94-01-10	buckling		plate; refer to Table 5.10 for dimensions.
			• slight rotation of the unloaded flange just after ultimate.
• W310x31	• 224 kN ²	• 4.16 mm	• two yield bands.
• end reaction 2	• inelastic web	• 0.77 h	• significant out-of-plane web displacements, about 0.73 w at
• 94-01-10	buckling		ultimate for the unloaded flange (possibly due to distortions)
			(maximum along the web); corresponding flange rotations occurred.

Note: 1. Location of the maximum deflection is given as a function of h as measured from the loaded flange.
2. Reaction load calculated using statics, the applied load, and measured geometry.

Table 5.9: Test Histories and Observations con't.

Test Section, Load Location, and Date	Failure Load, kN, and Mode	Maximum Deflection and Location ¹ , mm	Observations
<ul style="list-style-type: none"> • W250x24 • interior load • 94-01-20 	<ul style="list-style-type: none"> • 434 kN • inelastic web buckling 	<ul style="list-style-type: none"> • 1.05 mm • 0.43 h 	<ul style="list-style-type: none"> • at approximately 410 kN, Luders lines started to form at the edges of the load plate on the web and propagated towards the centre and out towards the flange web junction at $c/2 + 2.5$ k from the centre. • two distinct circular arc yield bands are visible, one just below the fillet and the other centered at 0.32 h below the loaded flange, at 430 kN. • rapid increase in out-of-plane web displacement. • Luders lines formed near the loaded flange web junction at 170 mm from the end and propagated towards the end. • sudden web buckling failure at 196 kN with slight rotation of the unloaded flange just after ultimate. • two distinct yield bands; refer to Table 5.10 for dimensions.
<ul style="list-style-type: none"> • W250x24 • end reaction 1 • 94-01-24 	<ul style="list-style-type: none"> • 196 kN² • inelastic web buckling 	<ul style="list-style-type: none"> • 0.91 mm • 0.27 h 	

Note: 1. Location of the maximum deflection is given as a function of h as measured from the loaded flange.
2. Reaction load calculated using statics, the applied load, and measured geometry.

Table 5.10: Yield Band Dimensions.

Section Designation	First Yield Band			Second Yield Band		
	Thickness (mm)	Length (mm)	Distances [†] to nearest line farthest line (mm) (mm)	Thickness (mm)	Length (mm)	Distances [†] to nearest line farthest line (mm) (mm)
W530x101	45	390 (380)	15 60	50	410 (380)	110 160
W530x101(end1)	65	200 (190)	95 160	80	220 (190)	160 240
W530x101(end2)	45	180 (190)	30 75	40	210 (190)	100 146
W530x82	55	340 (365)	45 100	65	390 (365)	85 150
W530x82(end1)	65	160 (183)	45 110	55	205 (183)	90 160
W530x82(end2)	50	155 (183)	30 80	85	220 (183)	65 150
W410x39	29	285 (330)	13 41	33	340 (330)	94 124
W410x39(end1)	35	175 (165)	23 58	-	-	-
W410x39(end2)	-	-	-	-	-	-
W360x45	25	305 (335)	12 37	48	360 (335)	85 132
W360x45(end1)	10	180 (168)	50 60	45	165 (168)	50 95
W360x45(end2)	25	155 (168)	45 70	-	-	-
W310x31	17	290 (325)	32 50	44	330 (325)	104 150
W310x31(end1)	35	157 (163)	15 50	45	160 (163)	80 125
W310x31(end2)	35	185 (163)	30 65	50	195 (163)	130 190
W250x24	15	300 (290)	10 25	55	280 (290)	50 105
W250x24(end1)	30	170 (145)	10 40	35	150 (145)	70 105

Note: [†] The distance is measured from the inside edge of the loaded flange. For interior loads this is the top flange and for end reactions this is the bottom flange.

Table 5.11: Measured Strains for the Top and Bottom Flanges.

Designation	$Av.(TF_{1,2})$	$Av.(TF_{3,4})$	$Av.(TF_{5,6})$	$Av.(TF_{7,8})$	ϵ_y	$Av./\epsilon_y^3$
	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	
W530x101	-1276	-1182	1653	1791	1874	0.92
W530x82	-1762	-1879	1754	5343	1968	<i>partially yielded</i>
W410x39	-1454	-1318	8862	1408	1871	<i>partially yielded</i>
W360x45	-1239	-1219	1642	1457	1610	0.96
W310x31	-1964	-1604	5097	3978	1941	<i>yielded</i>
W250x24	-1085	-1086	1641	1518	1986	0.80

Note: 1. $TF_{1,2,3,4}$ are strains measured on the top flange at $c/2 + 2.5 k$ from mid span.

2. $TF_{5,6,7,8}$ are strains measured on the bottom flange at mid span.

3. average bottom flange strain to yield strain ratio.

Chapter 6

DISCUSSION AND COMPARISON

6.1 General

In this Chapter comparisons are made between measured out-of-plane deformations, normal stresses calculated from measured strains, and ultimate loads with the corresponding values determined from an inelastic nonlinear finite element analysis. The analysis modelled the test specimens as three plates (no fillet) with the specified boundary conditions, measured dimensions, measured out-of-plane web flatness, and mechanical properties listed in Tables 5.1, 5.2, and Tables 5.3 to 5.8, respectively. The failure mechanisms, yield bands exhibited experimentally, the effect of boundary conditions, and flange contribution will be discussed. The design methodology, load predictions, and the corresponding statistical analysis including determination of the appropriate resistance factor, are presented.

6.2 Web Buckling Behaviour

Figs. 6.1 and 6.2 illustrate the load out-of-plane web displacement for all the interior load tests and for a typical end reaction test, the W530x101 end1. The experimental curves corresponds to the point of maximum out-of-plane web displacement and the finite element curves with the closest mesh point. The finite element curves are plotted up to ultimate.

The corresponding out-of-plane web displacements at ultimate for all the LVDTs measurements and the distribution along mid span or centred over the end reaction point are given in Figs. 6.3 to 6.8 for the interior load tests and in Fig. 6.9 for the end reaction test of W530x101 end1. The finite element distribution at the location is also shown.

The characteristic experimental and analytical load out-of-plane web deflection curves for all interior load tests and for end reaction tests where the unloaded flange is restrained against lateral displacement and rotation are similar. Variations include different initial slopes and ultimate load values. The average ratio of the ultimate load predicted by the finite element analysis to the corresponding test value is 0.94. This reduction in predicted capacity and subtle variation in initial slope may be attributed to:

1. not modelling the fillet in the finite element analysis. This would lead to a less rigid flange web junction, hence larger rotation at that junction (for the loaded and unloaded flanges) and a diminished web buckling capacity. The observations of increased rotations at the flange web junction are born out by a direct comparison of the deflected shapes at mid span or centred over the end reaction point for the experimental and analytical data as shown in Figs. 6.3 to 6.9.
2. the actual distribution of initial imperfections for the web may be different than that used in the finite element analysis (a double sine wave with the maximum measured imperfection located at mid height, the worst possible case).

3. the misalignment of the centre of the web with respect to the load or the end reaction point (eccentricity of the load with respect to the web centroid) which could only be aligned to the nearest millimeter.

The general deflected shape of the web at ultimate for both the experimental and analytical data are similar, skewed sine curves, with the only variation being the increased rotation at the flange web junction as noted previously. The general deflected shapes indicate end fixity of the web that approaches fixed boundary conditions at both flange web junctions reinforcing the suggested value of K equal to 0.6 for the equivalent column. Comparisons of the maximum measured out-of-plane web deformation with the corresponding deflection selected from the closest discrete mesh point from the finite element analysis given in Table 6.1 indicate good correspondence.

Based on the measured and the residual displacements taken along the length of the beam away from the centre of the load or end reaction point, refer to Tables 6.2 and 6.3, it was found that the out-of-plane web displacement diminished to zero or relatively small value for the region outside $c + 5k$ for interior loads and $c + 2.5k$ for end reactions. This observation coincides with the location and length of the yield bands described in Table 5.10 and corroborates the use of $c + 5k$ or $c + 2.5k$ as equivalent column widths for interior and end reaction loads, respectively.

Other researchers (Roberts et al (1979, 1981), Bergfelt et al (1974, 1979)) have indicated that the flanges will contribute to the web buckling capacity by acting as a beam on an elastic foundation and by providing rotational restraint to the web through its torsional rigidity. This was not the case for this series of tests as the flanges were fully utilized in carrying in plane bending moments as the average strains measured in the flanges approached yield or were yielded. Since the beam is generally designed to utilize its full bending moment capacity, then the flanges cannot and should not be relied on to contribute the local web buckling capacity.

6.3 Comparison of Normal Stress Distributions

Figs. 6.10 to 6.15 show the computed vertical normal stresses at ultimate, for interior load only, at the rosette locations on the nearside of the web for all the tests except for the W530x82 in which the strain gauges were situated on the farside. The first and second yield bands formed on the farside and nearside, respectively (on the concave side of the local out-of-plane bending).

The corresponding finite element stress distributions for nearside, mid web, and farside located along mid span are also shown. The mid web distribution is comparable with the assumed distribution used in the development of the equivalent column buckling capacity. The calculated vertical normal stresses plotted on the same graph show a good to excellent correspondence with respect to trend and magnitude of the corresponding analytical curves. Variations can be attributed to:

1. not modelling the fillet in the finite element analysis, and
2. that the strains are measured over a small zone, approximately 20 *mm* in diameter, not at a point. For large gradations in strains at or near yield band locations, this practical limitation may be significant.

The maximum normal stresses on the nearside for both the test and for the finite element analysis correspond closely to the location of the maximum out-of-plane web deformation. The closeaess of the test results with respect to the vertical normal stress distributions and the close approximation of the mid web distribution as a sine wave for the equivalent column concept suggests that the distribution used for the proposed model is correct.

6.4 Design Methodology, Load Predictions and Statistical Analysis

The equivalent column concept was developed with its implementation into the design standard in mind. The proposed design equations must:

1. give a complete description of the buckling capacity of webs (web yielding, inelastic web buckling, and elastic web buckling), and
2. be based on experimental evidence and contain the appropriate statistical information for the development of the resistance factor, and
3. be simple to use and not foreign to practicing engineers (easy to comprehend).

Figs. 6.16 and 6.17 illustrate the load predicted using the equivalent column concept, for a number of options (previously described in Section 3.3), the loads currently predicted by Clause 15.9 (based on Roberts (1981) solution), and the corresponding experimental data for interior loads and end reaction points, respectively. Although the cluster of data appears to be limited, value of $0.75 \leq \lambda \leq 1.00$, it represents the full range of web slendernesses for W shapes rolled in Canadian mills (with the exception of very stocky shapes which were unavailable). The calculated loads are given in Table 6.4 for elastic web buckling (P_{cr} -finite element elastic buckling solution, P_{ewb} -elastic web buckling based on Eq. (3.16)), inelastic web buckling (P_{Lusas} -inelastic incremental finite element solution, $P_{Roberts81}$ -Roberts (1981), $P_{S16.1}$ -current design provisions, in part are a modified Roberts (1981) formulation), web yielding (P_{yield} -current design provisions, P_{y10t} -current design provisions with a length of $(c + 10 t)$ as supposed to $(c + 5 k)$), the loads calculated using the equivalent column concept with K given by Eq. (3.20) and K simplified for the boundary conditions to a value of 0.6, and the experimental load. Table 6.5 gives the relevant test to predicted statistical data.

A statistical analysis for the determination of an appropriate resistance factor has been conducted for the Option A - Clause 13.3.1, Option A - Clause 13.3.2, Option B, and the current provisions in S16.1. The analysis pertains to cases where the loaded flange and unloaded flange for end reaction are restrained against lateral displacement and rotation. The mean value of the ratio of the measured resistance to the nominal resistance ρ_R is given as

$$\rho_R = \rho_G \times \rho_m \times \rho_p \quad (6.1)$$

where, $\rho_G = \rho_w = 1.0357$, as determined from this test series

$\rho_m = \rho_{\sigma_y} = 1.3010$, as determined from this test series

ρ_p = mean test to predicted ratio for the cases described, refer to Table 6.6.

The coefficient of variation associated with ρ_R is given as

$$V_R = \sqrt{V_G^2 + V_m^2 + V_p^2} \quad (6.2)$$

where, $V_G = V_w = 0.0466$, as determined from this test series

$V_m = V_{\sigma_y} = 0.0660$, as determined from this test series

V_p = coefficient of variation for the test to predicted data sets for the cases described, refer to Table 6.6.

Note: ρ_w and V_w are the statistical parameters for web thickness.

The target reliability index $\beta_T = 3.0$ was used in the calculation of Φ listed in Table 6.6. Setting $\Phi = 0.9$ yields reliability indices of 6.66, 4.25, 6.42, and 6.14 for the cases specified.

Either Option A Clause 13.3.1 or 13.3.2 seem appropriate as replacement design rules for the current provisions in Clause 15.9. Although, the statistics for this limited data set do not suggest a strong difference between the proposed and the existing equations, the existing equations are based on a modified Roberts (1981) formulation that:

1. is semi empirical and represent a best fit to data with geometric parameters that bear little or no resemblance to shapes produced in Canadian mills, and
2. the failure mechanism and formula described by Roberts (1981) are based on the development of three yield line failure mechanism of the web coupled with the development of four plastic hinges (for interior loads only) in the loaded flange. This bears no resemblance to the failure mechanisms in this test series.
3. The test to predicted ratio for interior and end reaction loads for the current provisions are relatively consistent within each data set but not with each other as indicated by the mean and the coefficient of variation in Table 6.5. Note, the Roberts (1981) formulation was developed for interior load only.
4. The Roberts (1981) formula contains a contribution to capacity from both the flange and the web.
5. The modified Roberts (1981) formula was modified for inclusion into the S16.1-M89 (CSA (1989)) and incorporates some variations due to calibration and round off (to make the numerical coefficient simple, a difference of approximately 40%).
6. The restrictions on the use of the modified Roberts (1981) formula in the S16.1 have not been incorporated.

The predicted loads given by the equivalent column concept, Option A Clause 13.3.1, using effective length factors from both Eq. (3.20) and the simplified value K equal to 0.6 are given in Table 6.5. As indicated by the value of 1.037 with the coefficient of variation

of 5.4%, the prediction of the web buckling capacity using K from Eq. (3.20) is excellent. The simplification of K to 0.6 for inclusion in the design standard leads to a slightly higher mean value and a coefficient of variation.

Given the more favorable and consistent statistics for the equivalent column concept; its ability to model the behaviour and internal vertical stress distribution and to predict the web buckling capacity accurately (web contribution only); its ability to incorporate different boundary conditions depending upon the restraint provided by the loaded and unloaded flanges; and that it meets the criteria of giving a complete description for all behaviour and is simple to comprehend and use, the equivalent column concept should replace the current provisions in Clause 15.9 of the S16.1-89 (CSA (1989)) for web crippling and web yielding.

One possible way of including this without having a different resistance factor, Φ , or a large reliability index, β , is to simply multiply the capacity given in Clause 13.3.1 by the ratio of 1.162 to 0.9 (a value of 1.3). Precedence for this has been made elsewhere in the standard. This higher than normal resistance factor is attributed to the test to nominal value of the yield strength of the web being 1.3.

Note, when the load is applied eccentrically to the web, it may be possible to design the web based on the equivalent column and the appropriate beam column interaction equations for strength and in-plane stability only. This has not been verified as it is outside the scope of this thesis.

Figure 6.1: Load vs Out-Of-Plane Web Displacement.

Figure 6.2: Typical End Reaction Behaviour, W530x101 (end1).

Figure 6.3: Out-Of-Plane Distribution Comparison at Ultimate, 1345 kN, W530x101.

Figure 6.4: Out-Of-Plane Distribution Comparison at Ultimate, 1180 kN, W530x82.

Figure 6.5: Out-Of-Plane Distribution Comparison at Ultimate, 553 kN, W410x39.

Figure 6.6: Out-Of-Plane Distribution Comparison at Ultimate, 665 kN, W360x45.

Figure 6.7: Out-Of-Plane Distribution Comparison at Ultimate, 497 kN, W310x31.

Figure 6.8: Out-Of-Plane Distribution Comparison at Ultimate, 434 kN, W250x24.

Figure 6.9: Out-Of-Plane Distribution Comparison at Ultimate, 1277 kN, W530x101 (end1).

Figure 6.10: Stress Distribution Comparison at Ultimate, 1345 kN, W530x101.

Figure 6.11: Stress Distribution Comparison at Ultimate, 1180 kN, W530x82.

Figure 6.12: Stress Distribution Comparison at Ultimate, 553 kN, W410x39.

Figure 6.13: Stress Distribution Comparison at Ultimate, 665 kN, W360x45.

Figure 6.14: Stress Distribution Comparison at Ultimate, 497 kN, W310x31.

Figure 6.15: Stress Distribution Comparison at Ultimate, 434 kN, W250x24.

Figure 6.16: Comparison of Loads Predicted using the Equivalent Column Concept with Interior Load Data.

Figure 6.17: Comparison of Loads Predicted using the Equivalent Column Concept with End Reaction Data.

Table 6.1: Comparison of Out-Of-Plane Deformation.

Shape Designation	Max. Exp. (mm)	Corresp. FEA (mm)	test/predicted
W530x101	3.126	3.052	1.0242
W530x82	2.103	2.042	1.0299
W410x39	1.726	1.590	1.0855
W360x45	1.209	1.125	1.0747
W310x31	1.067	1.070	0.9972
W250x24	1.049	1.000	1.0490

Table 6.2: Typical Residual Out-Of-Plane Deformations Measured for the Interior Load of the W250x24 ($c + 5 k = 285$ mm).

Distance from loaded flange	-180 mm from mid span	At mid span	+180 mm from mid span
0.104 h	0.07	0.26	-0.04
0.208 h	0.20	4.65	0.06
0.333 h	0.42	7.29	0.26
0.479 h	0.23	6.03	0.07
0.646 h	-0.12	3.91	0.29

Table 6.3: Typical Residual Out-Of-Plane Deformations Measured for the End Reaction Load of the W250x24 ($c/2 + 2.5 k = 142.5$ mm).

Distance from loaded flange	At edge	At 180 mm from edge
0.354 h	16.44	3.05
0.458 h	7.43	0.17
0.729 h	-4.80	-2.64
0.875 h	-3.30	-0.67

Table 6.4: Predicted Loads by Different Models.

Section		Elastic Web Buckling		Inelastic Web Buckling		Web Yielding		Web Yielding		Web Yielding	
Designation	P_{cr} (kN)	P_{web} (kN)	P_{Luas} (kN)	$P_{Roberts1}$ (kN)	$P_{S16.1}$ (kN)	P_{yield} (kN)	P_{y101} (kN)	$P^4_{pred.}$ (kN)	$P^5_{pred.}$ (kN)	$P_{exp.}$ (kN)	
W530x101	3124	2542	1142	1030	1400	1742	1562	1269	1179	1345	
W530x82	2181	1768	1050	849	1174	1481	1297	1054	974	1180	
W410x39	950	844	536	368	580	873	696	522	500	553	
W360x45	1345	1319	621	423	702	989	798	646	645	665	
W310x31	866	907	487	287	512	814	633	480	497	497	
W250x24	754	813	436	227	460	672	570	399	428	434	
W530x101(end1)	1404	1271	575	511	571	871	781	634	589	639 ³	
W530x101(end2)	1404	1271	575	511	571	871	781	634	589	618 ³	
W530x82(end1)	985	884	524	405	464	741	649	527	487	586 ³	
W530x82(end2)	985	884	524	405	464	741	649	527	487	552 ³	
W410x39(end1) ¹	449	422	263	184	219	437	348	261	250	226 ³	
W410x39(end2) ¹	449	422	263	184	219	437	348	261	250	206 ³	
W360x45(end1) ¹	639	660	301	211	261	494	399	323	322	257 ³	
W360x45(end2) ¹	639	660	301	211	261	494	399	323	322	272 ³	
W310x31(end1) ²	420	454	241	143	185	407	316	240	249	218 ³	
W310x31(end2) ¹	420	454	241	143	185	407	316	240	249	224 ³	
W250x24(end1) ²	375	406	215	114	160	336	285	199	214	196 ³	

- Notes: 1. lateral displacement and rotation of the unloaded flange occurred (K to be developed).
2. lateral displacement and rotation of the unloaded flange occurred suddenly after ultimate (the maximum out-of-plane displacement occurred at the second yield band).
3. calculated end reactions.
4. $P_{pred.}$ calculated based on K given by Eq. (3.20), and Option A Clause 13.3.1.
5. $P_{pred.}$ calculated based on $K = 0.6$, and Option A Clause 13.3.1.

Table 6.5: Ratios of Experimental to Predicted Loads.

Section	$P_{exp.}/P_{Lusas}$	$P_{exp.}/P_{Roberts81}$	$P_{exp.}/P_{S16.1}$	$P_{exp.}/P_{pred.}$	
Designation				$K_{Eq.3.20}$	$K_{0.6}$
W530x101	1.18	1.31	0.96	1.06	1.14
W530x82	1.12	1.39	1.01	1.12	1.21
W410x39	1.03	1.50	0.95	1.06	1.10
W360x45	1.07	1.57	0.95	1.03	1.03
W310x31	1.02	1.73	0.97	1.03	1.00
W250x24	0.99	1.91	0.94	1.08	1.01
interior $\bar{X}$	1.068	1.568	0.963	1.063	1.082
load $V(\%)$	6.63	14.15	2.59	3.18	7.70
W530x101(end1)	1.11	1.25	1.12	1.01	1.08
W530x101(end2)	1.07	1.21	1.08	0.97	1.05
W530x82(end1)	1.12	1.44	1.26	1.11	1.20
W530x82(end2)	1.05	1.36	1.19	1.05	1.13
W310x31(end1)	0.91	1.52	1.18	0.91	0.88
W250x24(end1)	0.91	1.72	1.23	0.98	0.92
end $\bar{X}$	1.028	1.417	1.177	1.005	1.043
reaction $V(\%)$	9.25	13.28	5.70	6.89	11.76
total $\bar{X}$	1.048	1.493	1.071	1.037	1.063
data set $V(\%)$	7.89	14.17	11.39	5.40	9.78

Note: $P_{pred.}$ is based on Option A Clause 13.3.1.

Table 6.6: Statistical Analysis (flange rotation restrained, $K = 0.6$).

Case	ρ_p	V_p	ρ_R	V_R	Φ	β
Option A - Clause 13.3.1	1.0632	0.0978	1.4326	0.1269	1.1620	6.662*
Option A - Clause 13.3.2	0.8977	0.0975	1.2097	0.1267	0.9815	4.245*
Option B	1.0438	0.0972	1.4064	0.1264	1.1417	6.422*
S16.1-M89	1.0707	0.1139	1.4427	0.1397	1.1458	6.144*

Notes: 1. The values are based on statistical analysis performed on the
W shapes tested.

2. Φ is based on a target reliability index $\beta_T = 3.0$.

3. * values of the reliability index β for $\Phi = 0.9$.

Chapter 7

SUMMARY, CONCLUSIONS, RECOMMENDATIONS, AND FUTURE WORK

7.1 Summary and Conclusions

In all, about 653 tests have been conducted to determine the failure modes and capacity of webs under concentrated loads. Of those used for the development of equations for inelastic web buckling (web crippling), only a very few have geometric properties that are comparable to existing rolled steel, W and WWF shapes, produced in North America. The existing requirements in current standards and specifications are based on Roberts (1981) mechanism solution, which was developed for plate girders with a web thickness less than 5 mm, a web slenderness ratio greater than 70, a flange to web thickness ratio greater than 3, and a load length to depth ratio less than 0.2. This expression has limited applicability since most of the rolled sections have geometric properties that lie outside this range. Hollow structural sections are not considered. The prime objectives of this work was to determine the behaviour of webs subjected to concentrated loads for rolled

steel sections, W and WWF shapes, produced in American and Canadian mills and to develop rational mathematical design rules for inclusion into the Canadian standard for the design of structural steel buildings.

A finite element program called LUSAS has been used to determine the elastic and inelastic web buckling capacity of webs of beams subjected to concentrated in-plane loads for a variety of realistic boundary conditions. The solution strategy, constitutive model and analysis of LUSAS were tested and verified against appropriate classical solutions and experimental data. The program was then used to determine the best analytical model by analyzing the effect of imperfections, flanges, and boundary conditions.

Results of finite element analyses on a wide range of W and WWF sections generated by LUSAS (vertical normal stress distribution) were used in the development of the equivalent column concept. The web is modelled as an equivalent column with simple boundary conditions, constant width, $(c + 5k)$, and a constant vertical normal stress distribution that varies sinusoidally along its length from a maximum at the top flange web junction (equal to the applied concentrated load) to zero at the bottom flange web junction. Based on this, the elastic buckling load, the effective length factor, and an effective slenderness parameter, λ_e , were determined. The elastic buckling, inelastic buckling and yield loads are calculated by substituting λ_e into either Clause 13.3.1 and 13.3.2 of CAN/CSA-S16.1-M89.

The experimental program was designed to validate the finite element predictions and the equivalent column concept, and to determine the appropriate resistance factor. The test data, geometric and material properties, imperfections, load out-of-plane web deformation curves, and vertical normal stress distributions, is presented in both tabular and graphic form. As part of the work, a data post processing program was written to determine the principal strains from strain gauge readings; and to convert these into stresses using the theory of plasticity. Subsequently, using measured geometry, imperfections, and material properties, the elastic web buckling capacity, inelastic web buckling capacity, and web

yielding capacity for all interior and end reaction test sections were determined. The appropriate boundary conditions, restraining the loaded and unloaded flanges against lateral displacement and rotation, were applicable. Failure loads based on the equivalent column concept have been calculated for both interior loads and end reactions and compared with those from the finite element analysis and the test program. A statistical analysis has been conducted on the test data and the appropriate resistance factor calculated. The following conclusions can be drawn from this work:

1. The current design rules given in Clause 15.9 are based on Graham et al (1959) for web yielding and modified Roberts (1981) solution for web crippling. There is no expression for elastic web buckling, for webs with longitudinal stiffeners, and for web capacity of hollow structural sections. The modified Roberts (1981) solution is semi empirical, contains a contribution to web buckling capacity from both the flange and the web (only the contribution of the web should be accounted for as the flanges may be fully utilized in bending), and based on section sizes and failure mechanisms that bear no resemblance to the range of W shapes tested in this thesis and produced by Canadian mills. The Roberts (1981) formulation was developed for interior loads only, yet S16.1 has modified it and applied to end reactions as well. The geometric restrictions specified by Roberts (1981) have not been incorporated into Clause 15.9 nor into the commentary of S16.1. Therefore, the applicability of this set of design equations is suspect.
2. The equivalent column concept is based on elastic and inelastic finite element solution for the full range of W and WWF shapes produced in Canadian mills. The width, $(c + 5 k)$ for interior loads and $(c + 2.5 k)$ for end reactions, was developed based on realistic finite element analysis on sections that failed by yielding. The width multiplied by the yield strength gives a statically equivalent stress block to the actual stress distribution under the load. The finite element analysis indicates that the vertical normal stress distribution directly under the load is sinusoidal in shape with the maximum value at the fillet of the loaded flange and zero at the fillet of the

unloaded flange. This stress distribution was verified experimentally and was used to develop the elastic critical buckling load. The equivalent slenderness parameter, which includes the effect of different boundary conditions through the effective length factor K , was determined by dividing the elastic critical buckling load by the yield load. The effective length factor for restrained flanges was determined from finite element analyses to be 0.6. Based on a comparison between the deflected shape at ultimate determined by the finite element analysis and that measured experimentally, it was concluded that the fillet has to be modelled analytically (by thickening the elements locally), as the analytical curve showed a greater flexibility at the flange web junctions.

3. Use of the equivalent column concept and the equivalent slenderness parameter, λ_e , in one of the column formulation curves given in Clause 13.3.1 gives a simple, easy to comprehend, and unified approach for determining the web yielding, inelastic and elastic web buckling capacity of sections subjected to concentrated loads. The mean test to predicted ratio and the corresponding coefficient of variation for the proposed design rules, with the simplified K value of 0.6, are 1.063 and 0.098, respectively. This can easily be included in S16.1 with a resistance factor of 0.9 and a reliability index of 3.0 by multiplying the capacity determined in Clause 13.3.1 with the equivalent slenderness parameter given in Eq. (3.23) by the calibration value 1.3. This higher than normal resistance factor is attributed to the test to nominal value of the yield strength of the web being 1.5.

7.2 Recommendations and Future Work

During the course of this study on the behaviour of W and WWF shapes subjected to concentrated loads, it became apparent that further analytical investigations and experimental verification were required to broaden the data base for different equivalent slenderness parameter, λ_e , values, WWF shapes, and webs of box girder and plate girder

bridges with longitudinal stiffeners that are launched. Many of the recommendations listed below are currently in progress and others are scheduled to start shortly.

1. Analytical study and experimental verification of the full range of WWF shapes produced in Canadian mills described in Table 4.1. This is currently being done as a Master's thesis.
2. Further tests and the corresponding finite element analysis of very slender plate and box girder webs, with $\lambda_e \geq 1.8$, and with or without longitudinal stiffeners.
3. Further analytical study of different practical boundary conditions (i.e. for unloaded flange free to displace and rotate for end reactions and for longitudinally stiffened webs).
4. There is some analytical and experimental evidence that suggests a better fit for the equivalent column width of $(c + 10 t)$ instead of $(c + 5 k)$ for both the W and especially WWF shapes. This needs to be verified.
5. Future and ongoing finite element models will include a local thickening of the web in the area of the flange web junctions to more precisely model the fillet and its combined effect with the flange in providing rotational restraint to the web.
6. A literature review, tests, and analytical investigation will be required to develop the corresponding design rules for hollow structural sections.

Chapter 8

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