A LAW-OF-THE-WALL SHIFT CORRELATION FOR STOCHASTICALLY ROUGH SURFACES

by

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A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of

Master of Applied Science

in Aerospace Engineering

Ottawa-Carleton Institute for Mechanical & Aerospace Engineering

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Abstract

Wind tunnel experiments were performed on several flat plates uniformly covered with stochastic (i.e. random) roughness. Velocity profiles in the boundary layer were measured at five streamwise stations. The momentum integral method was used to determine the local skin friction coefficients. It was shown that the skin friction coefficients thus determined were consistent with the velocity-defect law for turbulent boundary layers in mild pressure gradients. Law-of-the-wall plots of the data were made and the shift due to roughness of the logarithmic portion of the velocity profile, Δu^+ , was determined for each surface.

Three-dimensional digital maps of the rough surfaces were made by a third party using a laser scanning device. Several statistical parameters of the roughness configurations were calculated using the raw surface scan data. A correlation model to relate the shifts, Δu^+ , to the roughness configuration parameters was sought. Several key configuration parameters were identified. A technique for relating stochastic surface roughness to standard sand-grain roughness was developed. However, at the time of writing, a successful correlation had not yet been found.

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List of Symbols

A	Slope of log-law region of defect law, law-of-the-wall, law-of-the-wall shift curve, shift-curve-intercept curve (Eqns. 2.6, 2.12, 2.18, and 4.21)
A_F	Total frontal area of a series of discrete roughness elements
A_S	Plan surface area of wall
A_W	Upstream-facing wetted area of a discrete roughness element
B	Intercept of log-law region of law-of-the-wall plot (Eqn. 2.12)
b_m	Discrete roughness element mean width
C	Intercept of log-law region of law-of-the-wall intercept-shift curve (Eqn. 2.18)
C_f	Skin friction coefficient, $\frac{\tau_w}{\frac{1}{2}\rho U_e^2}$
C_{p_o}	Total pressure coefficient, $\frac{p_o - P_{c2}}{P_{c1} - P_{c2}}$
C_{p_s}	Static pressure coefficient, $\frac{p_s - P_{c2}}{P_{c1} - P_{c2}}$
C_q	Local dynamic pressure coefficient, $\frac{q_e}{q_{ref}}$
D	Intercept of log-law region of defect-law curve (Eqn. 2.6)
D_{as}	3D area-scale fractal dimension (Eqn. 3.58)
D_{ls}	2D length-scale fractal dimension (Eqn. 3.25)
d	Effective origin shift (Eqn. 2.20)
E	Intercept of log-law portion of “universal” shift curve (Eqn. 4.21)
$G(\omega)$	Power spectral density function (Eqn. 3.32)

H	Boundary layer shape factor, $\frac{\delta^*}{\theta}$
k	Discrete element roughness height
k^+	Roughness Reynolds number, $\frac{k u_\tau}{\nu}$
k_2	Secondary roughness parameter (Eqn. 4.21)
k_s	Standard-sand roughness height (Nikuradse)
k_s^+	Standard-sand roughness Reynolds number, $\frac{k_s u_\tau}{\nu}$
k_t	Total roughness parameter (Eqn. 4.23)
k_t^+	Total roughness Reynolds number, $\frac{k_t u_\tau}{\nu}$
L	Effective surface sample length
L_o	Original surface sample length
L_{ox}	Original surface sample length in x -direction
L_{oy}	Original surface sample length in y -direction
L_x	Effective surface sample length in x -direction
L_y	Effective surface sample length in y -direction
m	Power spectral density function moments (Eqn. 3.34)
N_{AREA}	2D normalized area (Eqn. 3.19)
N_{VOL}	3D normalized volume (Eqn. 3.53)
n	Number of surface data points
n_x	Number of surface data points in x -direction
n_y	Number of surface data points in y -direction
P_{c1}	Wind tunnel settling chamber static pressure
P_{c2}	Wind tunnel contraction static pressure
P_{cA}	3D summit count per unit area (Eqn. 3.51)
P_{cL}	2D peak count per unit length (Eqn. 3.17)
p	Pressure
p_o	Total pressure

p_s	Static pressure
q	Dynamic pressure, $\frac{1}{2}\rho u^2$
q_e	Freestream dynamic pressure, $\frac{1}{2}\rho U_e^2$
q_{ref}	Reference dynamic pressure, $P_{c1} - P_{c2}$
R	Gas constant for air, 287 kJ/kg·K
Re	Reynolds number
R_a	2D mean absolute height (Eqn. 3.1)
R_{al}	2D shortest linear autocorrelation length (Eqn. 3.28)
R_K	Modified roughness Reynolds number (Eqn. 2.3.4)
R_k	2D nominal core roughness (Eqn. 3.11)
R_{ku}	2D surface kurtosis (Eqn. 3.4)
R_L	2D developed length ratio (Eqn. 3.20)
R_p	2D maximum height
R_{pk}	2D peak height above core roughness (Eqn. 3.12)
R_{pm}	2D average maximum peak height (Eqn. 3.8)
R_q	2D RMS height (Eqn. 3.2)
R_{sc}	2D mean peak curvature (Eqn. 3.24)
R_{sk}	2D surface skewness (Eqn. 3.3)
R_{sm}	2D average profile element width (Eqn. 3.23)
R_t	2D height range (Eqn. 3.7)
R_v	2D minimum height
R_{vk}	2D valley height below core roughness (Eqn. 3.13)
R_{vm}	2D average minimum valley height (Eqn. 3.9)
R_z	2D Ten-point height (Eqn. 3.10)
$R(\tau)$	Autocovariance function (Eqn. 3.29)
S_a	3D mean absolute height (Eqn. 3.35)

S_{al}	3D shortest areal autocorrelation length (Eqn. 3.61)
S_{dr}	3D developed area ratio (Eqn. 3.54)
S_k	3D nominal core roughness (Eqn. 3.45)
S_{ku}	3D surface kurtosis (Eqn. 3.38)
S_m	2D stochastic element width
S_p	3D maximum height
S_{pk}	3D summit height above core roughness (Eqn. 3.46)
S_{pm}	3D average maximum summit height (Eqn. 3.42)
S_q	3D RMS height (Eqn. 3.36)
S'_q	3D RMS height per unit length
S_{sc}	3D mean summit curvature (Eqn. 3.57)
S_{sk}	3D surface skewness (Eqn. 3.37)
S_{sm}	3D average surface element area
S_t	3D height range (Eqn. 3.41)
S_v	3D minimum height
S_{vk}	3D valley height below core roughness (Eqn. 3.47)
S_{vm}	3D average minimum valley height (Eqn. 3.43)
S_z	3D Ten-point height (Eqn. 3.44)
S'_z	3D Ten-point height per unit length
$S\Delta_q$	3D RMS slope (Eqn. 3.55)
$S\lambda_q$	3D average spatial wavelength (Eqn. 3.56)
T	Temperature
U_e	Freestream velocity
U_{ref}	Reference velocity
u	Flow velocity in x -direction
u^+	Non-dimensional velocity, $\frac{u}{u_\tau}$

u_τ	Friction velocity, $\sqrt{\frac{\tau_w}{\rho}}$
V_{cA}	3D valley count per unit area (Eqn. 3.52)
V_{cL}	2D valley count per unit length (Eqn. 3.18)
x	Longitudinal distance along test surface and along wind tunnel test section
y	Lateral distance along test surface (surface metrology) and vertical distance from surface (fluid flow)
y^+	Non-dimensional distance above surface, $\frac{yu_\tau}{\nu}$
y_o	Original y -datum for deterministic roughness
z	Vertical distance from surface (surface metrology)

Greek Symbols

β	Pressure gradient parameter, $\frac{\delta^*}{\tau_w} \frac{dp}{dx}$
Δu^+	Law-of-the-wall intercept shift (Fig. 2.8)
Δx	Longitudinal surface step size
Δy	Lateral surface step size
Δ_a	Mean absolute slope
Δ_q	2D RMS slope (Eqn. 3.21)
δ	Boundary layer thickness
δ^*	Boundary layer displacement thickness (Eqn. 2.1)
κ	von Karman constant for law-of-the-wall slope
λ	Deterministic roughness spacing parameter (Eqn. 2.21)
λ_c	Surface upper wavelength cutoff
λ_q	2D average spatial wavelength
λ_{eff}	Waigh's effective spacing parameter (Eqn. 2.22)
λ_s	Surface lower wavelength cutoff
μ	Fluid dynamic viscosity

ν	Fluid kinematic viscosity
θ	Boundary layer momentum thickness (Eqn. 2.2)
ρ	Fluid density
$\rho(\tau)$	Autocorrelation function (Eqn. 3.31)
τ	Autocorrelation/autocovariance distance between points
τ_w	Wall shear stress
τ_x	Autocorrelation/autocovariance distance between points in x -direction
τ_y	Autocorrelation/autocovariance distance between points in y -direction
ω	Power spectral density function surface frequency
ω_x	Surface frequency in x -direction
ω_y	Surface frequency in y -direction

Subscripts

e	Freestream
$exp.$	Experimental
$inner$	Inner region of boundary layer
k	Roughness based
m	Mean
max	Maximum
min	Minimum
o	Origin, original
p	Peak, summit
ref	Reference
s	Sand-grain
t	Total
v	Valley

w	Wall
x	In x -direction, based on x -distance
y	In y -direction, based on y -height
τ	Based on shear stress

Superscripts

+	Non-dimensional inner-region variable
---	---------------------------------------

Abbreviations

$2D$	Two-dimensional
$3D$	Three-dimensional
ACF	Autocorrelation function
$ACVF$	Autocovariance function
BF	Blue Formica surface
CFD	Computational Fluid Dynamics
CSH	Coarse Shingle surface
CSP	Coarse Spackle surface
FSH	Fine Shingle surface
FSP	Fine Spackle surface
MIM	Momentum integral method
$PSDF$	Power spectral density function
RMS	Root-mean-square
RST	Rust surface
$r.a.$	Relative area
$s.o.o.$	Scale of observation
TAR	Tar paper surface

<i>VIN</i>	Vinyl surface
<i>WF</i>	White Formica surface

Chapter 1

Introduction

In most engineering applications involving wall-bounded fluid flows, a smooth surface is ideal. This is because a smooth surface will usually impart less resistance to the flow, will tend to delay transition to energy-consuming turbulent flow, and will tend to minimize heat transfer to the surface. In practice though, surfaces are not always smooth. Manufacturing processes, special protective coatings, degradation due to pitting and corrosion, and other effects cause engineering surfaces to be, or to become, rough. In some cases however, turbulent flow and increased heat transfer are desired. In any case, the flow over rough surfaces must be understood, so that its effects can be accounted for in the design process. Not only does the effect of roughness need to be understood on a qualitative level for design guidelines, but with the advent of advanced fluid flow simulation in recent years, the quantitative effects of roughness must be determined and modelled. This modelling takes the form of semi-empirical relations, as solution of the full Navier-Stokes equations for rough surfaces is still prohibitively demanding on computing resources.

Past studies of rough surfaces initially involved close-packed sand-grain roughness, as performed by Nikuradse (1933) and Prandtl and Schlichting (1934). Research then pro-

gressed to two-dimensional and then three-dimensional deterministic (or discrete-element) roughness, as done by Dvorak (1969), Simpson (1973), and Dirling (1973), to name but a few. Research on stochastic (or randomly rough) surfaces has been limited, with Tarada (1990) carrying out one of the few investigations. Stochastic surface research has been held back, in part due to the lack of descriptive statistical parameters to characterize the roughness and the lack of technology needed to accurately capture a digital map of the surface for analysis. In recent years these tools have become available, so fruitful research can now be conducted.

The aim of the current work is to model stochastically rough surfaces in terms of their effect on fluid flow. Several statistical parameters characterizing the roughness geometry are determined, and a correlation between these parameters and the aerodynamic effects of roughness is sought.

Chapter 2 of this thesis reviews the behaviour of turbulent boundary layers and current knowledge concerning effects of roughness on their behaviour. Chapter 3 gives an introduction to the field of surface metrology, in order to facilitate understanding of the process of surface mapping and characterization. Chapter 4 outlines the experimental methods and apparatus used to conduct the research, that is, of the wind tunnel experiments, data reduction, and surface measurements. The experimental surfaces used in the research are described in Chapter 5 and a discussion of results is presented in Chapter 6. Finally, conclusions and recommendations are presented in Chapter 7.

Chapter 2

Boundary Layer Behaviour

2.1 Fundamentals

A boundary layer is a thin region of viscous fluid flow between a surface and the outer, effectively inviscid, freestream flow. It is characterized by a velocity profile, ranging from zero velocity at the surface to the freestream velocity at the boundary layer edge. The flow in the boundary layer can be either laminar (orderly) or turbulent (chaotic). The boundary layer thickness tends to increase with increased distance along the surface.

For laminar incompressible flat-plate (i.e. zero-pressure gradient) boundary layer flow, Blasius (1908) developed the well known self-similar solution, which states that all boundary layer velocity profiles, regardless of Reynolds number or skin friction coefficient, are self-similar; in other words, they all collapse onto one curve if plotted in terms of suitable similarity parameters. These parameters as well as the universal curve are shown in Fig. 2.1.

u is the flow velocity, U_e is the freestream velocity, y is the distance from the wall, and δ is the boundary layer thickness. A laminar boundary layer tends to be unstable, and will

Figure 2.1: Blasius Laminar Boundary Layer Profile for Zero Pressure Gradient. Reproduced from Clauser (1956).

tend to become turbulent through the process of transition. Roughness, excessive freestream turbulence, and adverse pressure gradients can cause earlier transition to turbulent flow. Turbulent boundary layers involve chaotic flow patterns, with vortices and eddies. Turbulent layers tend to have a fuller profile and a higher skin friction coefficient. Boundary layers can be described by certain parameters, as defined in the following section.

2.1.1 Boundary Layer Parameters

Boundary Layer Thickness Definition

The boundary layer thickness, δ , is ideally defined as the height y from the wall where the local velocity reaches the freestream velocity. For experimental data reduction though, the boundary layer height is usually defined as the height at which the normalized velocity, u/U_e , reaches 0.99, or 99% of the freestream velocity. This is due to scatter in the data around the boundary layer edge making the edge difficult to detect precisely.

Displacement Thickness

The displacement thickness is an integral length scale of the boundary layer, related to the mass flux deficit produced by the friction at the wall. For zero pressure gradient flows, its value tends to increase with downstream distance. Its definition is

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_e}\right) dy \quad (2.1)$$

Momentum Thickness

The momentum thickness is an integral length scale of the boundary layer, related to the momentum flux deficit produced by the friction at the wall. For zero pressure gradient flows, its value tends to increase with downstream distance. Its definition is

$$\theta = \int_0^\delta \frac{u}{U_e} \left(1 - \frac{u}{U_e}\right) dy \quad (2.2)$$

Shape Factor

The shape factor is the ratio of displacement thickness to momentum thickness, and is a parameter that describes the shape of the boundary layer velocity profile. Its definition is

$$H = \frac{\delta^*}{\theta} \quad (2.3)$$

Zero-pressure gradient laminar boundary layers have a constant shape factor of ~ 2.6 , while zero-pressure gradient turbulent boundary layers on rough surfaces have variable shape factors, in the approximate range of 1.2 - 2.5 (Clauser (1956), Fig. 20).

2.2 Turbulent Boundary Layers on Smooth Surfaces

The turbulent boundary layer can be divided into two regions, an inner and an outer layer, based on the turbulence structure in each layer. The outer layer has large scale fluid motions and a long time history, while the inner layer has smaller scale motions and tends to be in an energy equilibrium state. The division between the layers is generally given as $y/\delta \simeq 0.1$ (Clauser (1956)).

A plot of some turbulent boundary layers for both smooth and rough surfaces, using the conventional velocity profile parameters u/U_e and y/δ , is given in Fig. 2.2. These profiles do not collapse onto one universal curve. This is because as flow conditions, including roughness, change, the skin friction coefficient, C_f , affects the velocity profile shape.

Figure 2.2: Turbulent boundary layer velocity profiles for smooth and rough flat plates. Reproduced from Schetz (1993).

2.2.1 The Defect Law

In order to collapse the turbulent profiles (for zero pressure gradient) onto one universal curve, C_f must become part of the parameters used to plot such a curve. Suitable parameters can be identified by considering the fluid dynamics of the outer layer. As theorized by von Karman (1930), the outer-region dynamics are dominated by turbulence, are not directly affected by viscosity, and are also dependent on outer region properties. He defined a velocity defect ($U_e - u$) and postulated the following dependency relationship:

$$U_e - u = f\left(\tau_w, \rho, y, \delta, \frac{dp}{dx}\right)$$

where

$$\tau_w = \frac{1}{2} \rho U_e^2 C_f$$

is the shear stress at the wall, ρ is the fluid density, C_f is the local skin friction coefficient, p is the static pressure, and x is the horizontal distance along the surface. Then by dimensional analysis the following results:

$$\frac{u - U_e}{u_\tau} = f\left(\frac{y}{\delta}, \frac{\delta}{\tau_w} \frac{dp}{dx}\right) \quad (2.4)$$

where u_τ is the *friction velocity*, a velocity scale based on the wall shear stress τ_w . It is defined by the following equation:

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}} = U_e \sqrt{\frac{C_f}{2}} \quad (2.5)$$

Equation 2.4 is the *defect law*. For zero-pressure gradient flows, the defect parameter $(u - U_e)/u_\tau$ is a function of y/δ only, and can be used to collapse the curves of Fig. 2.2 onto one universal curve as shown in Fig. 2.3.

Figure 2.3: Defect-law plot of turbulent boundary layer velocity profiles for smooth and rough flat plates. Reproduced from Clauser (1956).

Since the defect law is based on the assumption that the fluid viscosity is unimportant in the outer layer, it only applies to the outer region of the flow and not in the near-wall region. The inner layer will be a function of different properties (see section 2.2.2), but where the two regions overlap, the functions must be equal, and this leads to the *log-law*, where the velocity varies as the logarithm of the height parameter (see Clauser (1956) for derivation, after Millikan, 1937):

$$\frac{(u - U_e)}{u_\tau} = A \log \frac{y}{\delta} + D \quad (2.6)$$

The values of the constants A and D are 5.62 (as per Coles and Hirst, 1968, as stated in White (1991)) and -2.5 (from Clauser (1956)) respectively. Fig. 2.4 illustrates the logarithmic portion of the defect-law plot.

The effect of pressure gradients on the defect law plot is to shift the curve away from

Figure 2.4: Defect-law plot of turbulent boundary layer velocity profiles for smooth and rough flat plates – logarithm scale. Reproduced from Clauser (1956).

the zero-pressure gradient case. In the case of favourable pressure gradients, there is less “defect” and so the curve shifts downward. For adverse pressure gradients there is more defect and so the curve shifts upward (refer to Fig. 2.5). Note that the pressure gradient parameter β , as given by Clauser (1956), is

$$\beta = \frac{\delta^*}{\tau_w} \frac{dp}{dx} \quad (2.7)$$

Clauser gives arguments in favour of using δ^* rather than δ , as in equation 2.4, for this parameter. From Fig. 2.5 it can be seen, by comparing the values of β , that the defect plot is much more sensitive to favourable pressure gradients ($\beta < 0$) than to adverse pressure gradients ($\beta > 0$). In fact, even a mildly favourable pressure gradient can cause a significant shift in the defect-law curve.

Figure 2.5: Defect-law plot of turbulent boundary layer velocity profiles in various pressure gradients. Reproduced from Herring and Norbury (1967).

2.2.2 The Law-of-the-Wall

As already mentioned, the defect law does not extend down to the near-wall region, and so a different “law” for this region will be required. Prandtl (1926) theorized that in the inner layer, the flow would be dependent only on the near wall parameters:

$$u_{inner} = f(\tau_w, \rho, \mu, y)$$

Since the flow is dependent on near-wall parameters, suitable velocity and length scale parameters must be used, different from those used in the outer layer. Keeping the friction velocity as a normalizing parameter, the following non-dimensional velocity and length parameters are used:

$$u^+ = \frac{u}{u_\tau} \tag{2.8}$$

$$y^+ = \frac{yu_\tau}{\nu} \quad (2.9)$$

Dimensional analysis then yields the *law-of-the-wall* relation

$$u^+ = f(y^+) \quad (2.10)$$

This relationship is illustrated in Fig. 2.6. Note that u^+ is Reynolds number independent. This is true for low to high Reynolds numbers, but for very low values of Re , u^+ is also a function of Re . For most practical aeronautical applications however, the Reynolds number is usually high enough for u^+ to be Re -independent. See Bergstrom *et al.* (2001) for more detail. Some recent investigations (Panton (2000), Buschmann (2001), and Afzal (2001)) have shown that there may or may not exist a universal law-of-the-wall, but for the current work it shall be assumed that such a universal law-of-the-wall exists.

There are three main regions on a wall-law plot. The first is the *laminar sublayer*, which is a very thin region of flow near the wall in which the turbulent fluctuations are damped out by the wall. In this region a simple analysis yields

$$u^+ = y^+ \quad (2.11)$$

Note that this is consistent with equation 2.10 and is confirmed by experiment. The next region is the overlap region, as previously discussed in section 2.2.1. This region is described by a logarithmic velocity profile in near-wall coordinates:

$$u^+ = A \log y^+ + B \quad (2.12)$$

From experimental data, the value of the constant A is 5.62 as noted before, and B is taken as 5.0 as per Coles and Hirst, 1968, as stated in White (1991). The log-law region can

Figure 2.6: Law-of-the-wall plot. Reproduced from Schetz (1993).

alternatively be expressed as

$$u^+ = \frac{1}{\kappa} \ln y^+ + B \quad (2.13)$$

where $\kappa = 0.41$. Research has also been successfully conducted on using power laws to describe the velocity profile in the near-wall region (see Bergstrom *et al.* (2001) for more detail) but this thesis will focus on the logarithmic law-of-the-wall relationship.

The third region of the plot (Fig. 2.6), is the wake region, which corresponds to the outer region of the boundary layer. The wake region is described by the defect law, as outlined in section 2.2.1.

The effect of adverse pressure gradients on the law-of-the-wall is to reduce the overlap layer, or logarithmic region, by increasing the wake region, but it is important to note that the logarithmic relation, equation 2.12, remains unaffected (see Fig. 2.7).

Figure 2.7: Pressure gradient effect on the law-of-the-wall. Reproduced from White (1991).

2.3 Turbulent Boundary Layers on Rough Surfaces

2.3.1 Defect Law for Rough Surfaces

Since roughness is a wall property, and it has been shown that wall properties directly affect only the flow in the near-wall region, the primary effect of roughness is to change the wall shear stress, τ_w , and thus the friction velocity, u_τ , and the skin friction coefficient, C_f . The outer region flow is only affected indirectly by the roughness through its effect on the velocity scale u_τ . In the defect law, the velocity defect, $u - U_e$, is normalized by u_τ so the defect law applies equally to smooth and rough walls (Clauser (1956) – see Fig. 2.3).

2.3.2 Law-of-the-Wall for Rough Surfaces

In terms of the law-of-the-wall, the above mentioned change in wall shear stress shows up as a reduction in u^+ and an increase in y^+ , seen as a downward shift on the law-of-the-wall plot (see Fig. 2.8). The law-of-the-wall for rough surfaces can then be written as:

$$u^+ = A \log y^+ + B - \Delta u^+ \quad (2.14)$$

where $\Delta u^+ = \Delta u / u_\tau$ is the downward shift due to roughness. This shift is a function of the roughness geometry and flow conditions. Depending on these conditions, three flow regimes can be identified:

- **aerodynamically smooth**, where the roughness is “small” enough to keep the laminar sublayer intact, and where viscous effects still dominate, and drag on the roughness elements is negligible.
- **transitionally rough**, where the laminar sublayer starts to be disrupted and both the viscosity and form drag of the roughness elements affect the flow.
- **fully rough**, where the laminar sublayer is destroyed and the form drag of the roughness elements is the dominant source of the wall shear stress and the viscosity has no direct importance.

For the fully-rough flow regime, in the inner region (but not too near the roughness) the mean velocity profile should be a function of the distance y from the wall, of the roughness height k , of ρ and of τ_w , but not of viscosity μ . That is,

$$u = f(\tau_w, \rho, y, k) \quad (2.15)$$

Figure 2.8: Law-of-the-wall plot with roughness effect. Reproduced from Kuethe and Chow (1976).

Then, by dimensional analysis,

$$\frac{u}{u_\tau} = f\left(\frac{y}{k}\right) \quad (2.16)$$

As before, in the overlap zone between the inner and outer layers of the boundary layer, the function must be logarithmic and the logarithmic law-of-the-wall should thus be of the form

$$u^+ = A \log \frac{y}{k} + B - C \quad (2.17)$$

Note that this is compatible with equation 2.16. This implies that in the fully rough regime, Δu^+ in equation 2.14 has the form

$$\Delta u^+ = A \log k^+ + C \quad (2.18)$$

where

$$k^+ = \frac{k u_\tau}{\nu} \quad (2.19)$$

and C is a function of the roughness geometry. Again, this applies only in the fully rough region. Fig. 2.9 shows the relationship between Δu^+ and k^+ .

Figure 2.9: Law-of-the-wall intercept-shift due to roughness. Reproduced from Schetz (1993).

On the basis of experiment, the three roughness regimes are delimited by (White (1991)):

$$\begin{aligned} k^+ < 5 & \quad \text{aerodynamically smooth} \\ 5 \leq k^+ \leq 50 & \quad \text{transitionally rough} \\ 50 < k^+ & \quad \text{fully rough} \end{aligned}$$

2.3.3 The Displacement Height

In the preceding analysis, equations 2.6, 2.14, and 2.17 make use of the height y above the nominal surface. However, the origin of this nominal surface is not easily determined for rough walls, due to their inherent unevenness. For deterministic roughness, which has a well-defined wall on top of which the roughness elements sit, a displacement height can be used to determine the effective origin:

$$y = y_o - d \quad (2.20)$$

See Fig. 2.10 for illustration of the variables, and refer to Waigh (1996) for more information. Jackson (1981) has shown by theoretical arguments that the effective origin is at the height about which the drag forces of the roughness elements have zero moment.

Figure 2.10: Displacement height for deterministic roughness. Reproduced from Waigh (1996).

For stochastic roughness, there is no well-defined wall, due to the inherent randomness of the surface (see Fig. 2.12). However, the effective origin can be found by a simple method outlined by Clauser (1956). If the experimental boundary layer profile data is plotted on a defect-law plot, the effective origin can be found in a trial and error fashion by simply

adjusting the effective origin until the logarithmic part of the overlap layer is in fact a straight line (on a log-scale plot). If the effective origin is not chosen correctly, the near wall profile will curve upward or downward from this straight line (see Fig. 2.11). Refer to Reda *et al.* (1974) for another example. Of course, care must be taken when using this method for transitionally rough surfaces, where in the near wall region the plotted curve is supposed to deviate downward from the straight logarithmic part. Unfortunately the process outlined by Clauser is difficult to implement with any accuracy in practice. Jackson's analysis can also apply to stochastically rough surfaces, but the issue of purely stochastic surfaces having no well-defined elements reduces its applicability. It remains as a future research area to correlate the effective origin to the stochastic surface geometry.

Figure 2.11: Use of defect-law plot to determine the effective origin of stochastic roughness. Note: S_a is centre-line-average height.

Figure 2.12: Example of stochastic roughness (2D profile).

2.3.4 Current Intercept-Shift Correlations for Different Roughness Types

As previously stated, the intercept C of the log portion of the law-of-the-wall intercept shift curve (equation 2.18) is a function of the roughness geometry. It is a function of geometry only, the flow conditions U_e and ν being accounted for in u_τ . When the topic of “roughness correlation” is discussed, what is implied is that attempts are made to correlate C to the various parameters describing the roughness geometry. Usually this is done using data in the fully-rough regime only. Dealing with the transitionally rough regime would be somewhat more complex.

There are two types of roughness, deterministic and stochastic. Deterministic roughness consists of well-defined independent discrete roughness elements located in an orderly manner on a well-defined wall, such as shown in Fig. 2.10. Stochastic roughness is a random or chaotic surface which has no well-defined elements, as shown in Fig. 2.12. These are the two extremes of roughness classification, but there lies a large grey area between these extremes in which many rough surfaces fall. An example would be a surface composed of somewhat randomly shaped lumps laid out in a deterministic fashion, such as apple trees lined up in an orchard, or something of that nature. However, the bulk of past research work has been conducted on these two roughness extremes, as it is best to first understand the simplest cases before proceeding to the more complex ones.

Sand-Grain Roughness

Closely-packed sand-grain roughness, from experiments done by Prandtl and Schlichting (1934) and Nikuradse (1933), is a unique form of roughness in a sense that it requires only one parameter to characterize it, the sand-grain height k_s . This type of roughness is often used as a standard, with other roughness types being characterized in terms of an “equivalent standard-sand roughness height”. The two main parameters required to characterize a rough surface are a height parameter (usually given as k) and a spacing parameter (usually given as λ). Since “standard” sand-grain roughness has the individual sand grains packed together as tightly as possible, the spacing parameter becomes common for any size sand grains. Thus only the height parameter, given as k_s , is necessary to characterize this roughness type. Due to this one-parameter characterization, there is no dependance of the sand-grain intercept C on any other roughness parameter, i.e. it has a constant value. According to re-worked data of Nikuradse, $C \simeq 4$ (Grigson (1984)). Fig. 2.9 shows this single curve for sand-grain roughness.

Deterministic Roughness

For discrete element roughness, it is accepted that the height of the individual elements, k , is sufficient as a height parameter. The spacing parameter, however, has been a topic of much research over the years, with contributions from Morris, Betterman, Dvorak, Simpson, Wooding, and Dirling (Waigh (1996)). Simpson used a spacing parameter of

$$\lambda = \frac{A_S}{A_F} \quad (2.21)$$

where A_F is the total frontal area of the roughness elements and A_S is the plan surface area of the wall (see Waigh (1996) for more detail). Using this parameter, the correlation curves

are shown in Fig. 2.13.

Figure 2.13: Simpson's correlation between λ and C . Reproduced from Waigh (1996).

One can see the relatively poor correlation between C and λ . This suggests that A_F and A_S are insufficient in describing the roughness. Waigh (1996) devised an improved “effective spacing parameter”, using additional descriptive properties of the roughness elements:

$$\lambda_{eff} = \lambda \left(\frac{k}{b_m} \right)^a \left(\frac{A_W}{A_F} \right)^b \quad (2.22)$$

where λ is Simpson's spacing parameter, (k/b_m) is the spanwise aspect ratio, (A_W/A_F) is the element bluntness, and a and b are constants depending on the spacing regime.

There are two spacing regimes for deterministic roughness: the dense spacing regime, where the roughness elements are so closely packed together that the flow skims over top, resulting in a drop in resistance; and the sparse spacing regime, where the elements are sufficiently separated so that they individually contribute to the flow resistance.

Using the above effective spacing parameter, Waigh found the following correlations:

$$C = 10.56 \log \left(\lambda \left(\frac{k}{b_m} \right)^{0.87} \left(\frac{A_W}{A_F} \right)^{0.44} \right) - 7.59 \quad (2.23)$$

$$C = -5.75 \log \left(\lambda \left(\frac{k}{b_m} \right)^{0.55} \left(\frac{A_W}{A_F} \right)^{1.38} \right) + 5.78 \quad (2.24)$$

where equation 2.23 is for the dense spacing regime, and equation 2.24 is for the sparse spacing regime. See Fig. 2.14 for illustration.

Figure 2.14: Waigh's correlation between an effective λ_{eff} and C . Top: dense spacing regime. Bottom: sparse spacing regime. Reproduced from Waigh (1996).

Stochastic Roughness

Stochastic roughness has no spacing regimes per se, as there are no well defined elements to space. As will be shown in subsequent sections, there are quite different parameters used to describe random roughness. These are mostly statistical parameters based on treating the surface as a random function. This topic will be expanded upon in Chapter 3.

Chuah *et al.* (1982) performed experiments in an attempt to validate a new roughness function proposed by Musker and Lewkowicz. Musker used statistical parameters to define a modified roughness Reynolds number (in place of k^+):

$$R_K = \frac{S_q u_\tau}{\nu} (1 + a \Delta_a) (1 + b S_{sk} S_{ku})$$

where S_q is the RMS roughness height, Δ_a is the mean absolute slope, S_{sk} is the skewness of the height distribution, S_{ku} is the kurtosis of the height distribution, and $a = 0.5$ and $b = 0.2$ are empirical constants. Musker then proposed a roughness function

$$\frac{u}{u_\tau} = \frac{1}{\kappa} \ln y + X$$

where

$$X = A \log \frac{7.752 R_K}{\exp -0.005 R_K + 0.44 R_K}$$

According to Chuah *et al.*, however, Musker's equations were not very successful, failing to find a universal roughness correlation. The idea is nevertheless sound, Musker recognizing that k alone is insufficient to characterize stochastic surfaces, and that additional statistical parameters must be used.

Acharya *et al.* (1986) noted that previous researchers attempted to find a relationship between R_a (the two-dimensional centre-line average height) and k_s . These extremely simple

equations were merely rough estimates, with error bounds up to 50%, and only applied to the select few surfaces that were used in their experiments. Acharya's experiments found that different surfaces had different multiplicative coefficients for relating R_a to k_s , confirming the poor results of previous work, such as Bammert and Sandstede (1976) who obtained a correlation $k_s = R_a^{0.877}$, and Koch and Smith (1976) who obtained $k_s = 6.2 R_a$.

It is obvious then, that attempts to correlate stochastic roughness to the intercept-shift have been few and far between, and unsuccessful at that. The aim of the current research is to define a proper roughness parameter in order to obtain a successful correlation.

Alternative Methods of Modelling Rough-Wall Flow

The discussion so far has been on modelling rough-wall flow by replicating the law-of-the-wall for rough surfaces. Another approach is the so-called *discrete element method*. This method involves adding roughness modelling terms (for the form drag of individual elements and flow blockage factors) directly to the momentum equation governing the flow, along with turbulence models (see Taylor *et al.* (1985) for more detail). This is relatively easily done for deterministic roughness, but for stochastic roughness the problem of characterizing the surface remains. Taylor *et al* suggested using average dimensions of the dominant stochastic elements. Tarada (1990) went further and developed several models based on finding equivalent deterministic elements for the stochastic surfaces, based on relations to the statistical properties of the surfaces. According to Tarada, these methods provided reasonable predictions for rough-wall flow.

Chapter 3

Surface Metrology

Deterministic roughness characterization and classification is a relatively simple topic in comparison to that for stochastic roughness. Deterministic surfaces are usually manufactured to a specific set of measurements of the individual elements, so measuring the basic surface geometry is not a consideration, and calculating configuration parameters is simple mathematics.

Stochastic roughness on the other hand, involves much more complex methods of measurement and characterization, due to the randomness of the surface. Stochastic surfaces were never fully understood in the past, and little research was done on the subject. This was due to several reasons, a main one being that measurement of the surfaces was very difficult without the proper technology. Only within the last twenty years or so has the measurement technology become available to properly measure stochastic surfaces to a sufficient degree of accuracy. The tribology and wear fields initiated the research into surface metrology, and developed many parameters to describe surfaces.

Surface metrology is not a topic usually discussed in the fluid mechanics field, so this

chapter serves as an introduction to the topic, describing surface classification, methods of measurement, and the various descriptive parameters, in order to give a background for the work done in this thesis.

3.1 Surface Classification

3.1.1 Defining “Roughness”

The definition of roughness is not as simple as it seems. To the lay-person, a rough surface is one that is simply not smooth, but to a metrology expert, roughness falls under a more detailed set of guidelines. The first aspect to consider is the **scale of observation**. How “rough” a surface is depends on the context in which one looks at it. To a fluid dynamicist studying atmospheric boundary layers, pavement is smooth, but when looking at the small scale boundary layers usually seen in the aeronautical field, pavement is quite rough.

The second aspect to consider is how the actual surface differs from the nominal, or intended, surface. The intended surface is that according to manufacturing specifications. Compared to a flat plate, a series of egg-crates for example, is “rough”, but when looking solely at an egg-crate, it is intended to be its shape, and so is not rough. However, looking at the *surface* of the egg-crate, it can be smooth *or* rough. Again, it all depends on what the intended surface is, the deviation from it, and the scale of observation.

The intended surface is usually called the **form** of the surface. Form is not considered roughness. Reducing our scale of observation (i.e. zooming in on the surface), the next geometrical feature is the **form error**, which defines any large-scale deviations from the nominal surface, such as a large dent in the hood of a car. Zooming in further, the next feature is **waviness**. Waviness is considered too large-scale to be roughness.

Another feature is **lay**, which is unintended *directional* striations deviating from the nominal surface. Upon further magnification, we finally come to **roughness**, which is defined as the smallest wavelength deviations from the nominal surface. **Texture** encompasses roughness, waviness, and lay. Figs. 3.1 and 3.2 illustrate the above concepts.

Figure 3.1: **Roughness, waviness, and lay.** Reproduced from **Precision-Devices-Inc (2001).**

The different surface features can thus be classified according to wavelength bands. When studying a surface feature, such as roughness, the roughness wavelength must be captured and the waviness and form wavelengths must be removed, by the process of **filtering**, using select **wavelength cut-offs**. Filtering can be done by electronic, mechanical, or mathematical means, depending on the measurement method. In fluid dynamics we are generally only interested in roughness, as form and waviness error are usually not a factor due to the precise requirements (and thus manufacturing) of fluid dynamic surfaces.

Figure 3.2: Wavelength illustration of roughness, waviness, and form error. Reproduced from Precision-Devices-Inc (2001).

3.1.2 Roughness Feature Definition

Profiles vs. Surface Maps

Now that roughness has been properly defined, the features that comprise roughness must be identified. First the distinction between a two-dimensional profile and a three-dimensional surface map must be made. A 2D profile is a line profile taken across the surface, resulting in a plot of z (height) versus x (distance), as shown in Fig. 3.4. A 3D surface map is a plot of $z = f(x, y)$ and is shown in Fig. 3.3. Profiles can be sufficient data for unidirectional (2D) surfaces, but for multi-directional surfaces, a 3D surface map must be used to capture the surface geometry.

Figure 3.3: An example of a 3D surface map.

Isotropy

An isotropically rough surface is one whose roughness is the same in any direction, showing no signs of lay or directionality. One might be tempted to think, therefore, that a profile trace anywhere on the surface would be sufficient to characterize the surface, but this is not the case. A single profile can easily pass between summits and valleys on the surface, and may even be a smooth line. Taking several profiles and averaging the properties derived from them would improve the process, but then one might as well opt for the full surface

scan and be sure of proper results.

In terms of fluid dynamics or tribology, isotropy makes the surface analysis much more simple, since the flow or surface contact motion is not affected by the orientation of the surface. As an example, consider drag-reducing riblets on the wing of an aircraft. Depending on the lay direction, the flow will be vastly different. Conversely, the flow over sandpaper is independent of the direction.

2D Features vs. 3D Features

The **mean line** of a surface is taken as the height datum, $z = 0$. A **peak** can be defined as any point *on a profile* above the mean line that is greater than its two nearest neighbours, or the same definition, but above the $+S_a$ line (centre-line average height above the mean line). These definitions depend on the industry standard one chooses to observe (see section 3.3). A **summit** is defined in the same manner, but for a surface map. A **2D valley** has the same definition but occurs below the mean line on a profile, and a **3D valley** occurs on a surface map. See Figs. 3.4 and 3.5 for illustration.

Figure 3.4: A 2D profile illustrating peaks and local vs. global definition. Reproduced from Precision-Devices-Inc (2001).

Figure 3.5: A 2D profile illustrating valleys and local vs. global definition. Reproduced from Precision-Devices-Inc (2001).

Local vs. Global Features

Figs. 3.4 and 3.5 best illustrate the concept of local and global features. Global features are the more dominant feature, physically and statistically, with local features usually being neglected in such parameters as peak count, for example.

3.2 Measurement Techniques

3.2.1 Measurement Considerations

Before measuring a surface, many questions must first be answered, in order to choose the proper measurement technique to provide the necessary data. These questions include:

- What is the nature of the surface? Is it 2D or 3D, requiring only a profile or a full surface map?
- What sample size is required to fully capture the surface?
- What resolution and range is necessary in the measuring sensor to accurately capture the surface features?

- What wavelength cut-offs and level of filtering will be used?
- What sampling interval should be used?
- Are there budgetary constraints? (Surface scanning can be expensive).

Once these questions are answered, the proper measurement method can be chosen.

Measuring Profiles vs. Surface Maps

To measure a profile, a simple linear trace across the surface is needed. A surface scan involves either performing a series of profile scans and assembling them into a surface map, if a contact method is used, or scanning points over an area and assembling them, if a non-contact method is used.

Axis System and Datum

The axis system used in industry is a standard x - y - z orthogonal coordinate system, with x as the main longitudinal direction (for profiling, and opposite to the lay direction if any), y as the lateral direction, and z as the vertical direction, or height parameter. See Fig. 3.6 for illustration.

All measurement systems measure the surface relative to some datum. This datum is usually a physical reference, such as the location of the sensor. The captured z -data is usually referenced to this datum, and then can be referenced to the mean line with post-processing.

Figure 3.6: An illustration of the axis system orientation and a profile with sectioning plane. Reproduced from Precision-Devices-Inc (2001).

Sample Sizing

Budgetary and resource constraints aside, the larger the sample size taken, the better. However, the minimum sample size necessary is that which fully represents the surface. This can be determined from the following criterion: the values of the descriptive parameters, such as RMS height, should not change if the sample size is enlarged or moved to another location on the larger surface, but will change if the sample size is reduced. In essence, one can take a running average of several descriptive parameters, and once the average does not change, the minimum sample size is found. An example would be taking the topography of a mountain range. One would not use one or two mountains to represent the whole range. Instead, a few dozen would be necessary to fully represent the landscape surface.

In practice however, scanning and re-scanning surfaces can be resource-draining, so with some judgment and experience, a sample size can be judged to be adequate.

Resolution, Range, and Sampling Rate

The **vertical resolution** of an instrument is the smallest height change that can be measured. The appropriate sensor/measurement method must be chosen for the given surface, in order to adequately capture the surface geometry. If the large-scale roughness is the only interest, then a lower resolution sensor can be used. As an example, the geometry of an array of golf balls needs to be measured, but the dimples on the golf balls may be unimportant. The **lateral resolution** is the smallest measurable distance that the sensor can be moved in the x - y plane.

The **vertical range** is the sensing range of the sensor in the z -direction. Generally, when increased vertical range is needed the resolution tends to be lowered. The **lateral range** is usually a function of the equipment geometry, and not the sensor, and can limit sample sizes. The appropriate sensor with the required vertical range must be chosen to capture the whole surface.

The **sampling rate** (or alternatively the *step size*), is the number of samples taken per unit length (or the horizontal distance between samples points). The higher the sampling rate the better, as more small-scale features will be captured.

In surface metrology it is crucial to use an adequate sampling rate to capture the surface geometry of interest, as the surface descriptive parameters can vary greatly with improper selection.

Fig. 3.7 illustrates the difference the step size can make, in scanning the same surface. For high resolutions, the step size can be limited by the lateral resolution. Sampling rate is also tied in to the wavelength cut-offs: if only waviness is to be measured, a roughness-scale step size will not be used.

Figure 3.7: An illustration of the effect of step size on the captured surface, for the same measured surface (different sample locations). Top: 20 μm step size. Bottom: 500 μm step size.

Filtering

There are two applications for filtering, the first being to capture the desired surface wavelengths, as discussed previously, and also to remove noise in the captured signal and data point outliers. Fig. 3.8 shows the difference between a pre-noise-filtered surface map and the filtered map.

3.2.2 Measurement Methods

Contact Stylus Profiling

This method is the oldest and simplest in use today. It involves dragging a stylus across the surface, the vertical motion of the stylus being digitized through a transducer. The transducer can be an electromagnetic coil for example. Since this is a “contact” method, it can be destructive if the surface is fragile. Also, the resolution may be limited by the size of the stylus tip, usually made of diamond.

Optical Interferometry

This method makes use of phase-shifted interference fringe patterns to detect surface height changes. A combination of light reflected from the rough surface and an internal reference surface produces the fringe pattern. As the roughness height changes, the phase shift changes and a computer algorithm makes use of the shift to produce a map of the surface. See San-Diego-Magnetics (2002) for more information. Interferometry is a non-contact method, offering resolutions down to the nanometer scale.

Figure 3.8: An illustration of the effect of filtering out noise and data outliers on the captured surface. Top: unfiltered. Bottom: filtered.

Laser Triangulation

Laser triangulation involves directing a laser beam vertically downward onto the surface and recording its position on a light sensitive detector. The sensor is then moved laterally, and if the surface height changes, the reflected beam will shift from its previous position. Fig. 3.9 illustrates the above concept. The shift is related to the height change by the geometry of the device's optics. This technique gives resolutions down to the micrometer scale, and is usually suitable for the roughness scales that are of interest in aeronautics.

Figure 3.9: A schematic of a laser triangulation surface measurement device. Reproduced from CyberOptics-Corporation (1992).

Scanning Probe Microscopy

The general field of Scanning Probe Microscopy encompasses several techniques such as Atomic Force Microscopy, Scanning Tunnelling Microscopy, Lateral Force Microscopy, and Magnetic Force Microscopy. These methods provide surface imaging with atomic or near-atomic resolution. Scanning Electron Microscopy is also a related method.

3.3 Roughness Parameters

In recent years attempts to standardize roughness parameters have occurred, however these have not been entirely successful. There exist a hundred or more parameters in use in the industry, many of which have been devised by a worker simply for a certain project, with no application to other areas. There are many variations on what are essentially the same parameters, where certain aspects to the parameter are open to subjective interpretation. The main current standards are ANSI/ASME B46.1-1995, Surface Texture (Surface Roughness, Waviness and Lay) and DIN 4776, 1990, Measurement of Surface Roughness. There are also many ISO standards for surface metrology procedures and quality. The parameters presented here try to follow the above standards, but may deviate in certain areas.

There are many parameters that are interrelated, with known relations, or relations that may make themselves apparent during data processing. There are also some parameters that are redundant, but exist as a matter of choice or clarity. The following sections present only those parameters the author has initially judged to be applicable to the current work. With the experimental surfaces all being isotropic, no direction-specific parameters are presented.

3.3.1 2D Parameters

Fig. 3.10 shows an actual surface and its discretized form for use in digital analysis. The mean line is the zero-height datum. Each data point is represented by a bar whose width is the step size, Δx . The number of sample points is n and the original sample length L_o , and effective sample length, L are

$$L_o = (n - 1)\Delta x$$

$$L = n\Delta x$$

Figure 3.10: The effect of discretizing a surface, showing change in sample length and surface representation.

The definitions of the various statistical parameters are presented next, with their analytical expression, numerical expression and their units. Note that $[-]$ represents a dimensionless parameter.

R_a - mean absolute height

$$R_a = \frac{1}{L_o} \int_0^{L_o} |z(x)| dx = \frac{1}{n} \sum_{i=1}^n |z_i| \quad [\text{length}] \quad (3.1)$$

R_a is an amplitude parameter, defined as the average of the magnitude of the height distribution, also known as the centre-line average height. It is insensitive to the spatial distribution or polarity (i.e. bias toward peaks or valleys) of the roughness.

R_q - root-mean-square height

$$R_q = \sqrt{\frac{1}{L_o} \int_0^{L_o} z(x)^2 dx} = \sqrt{\frac{1}{n} \sum_{i=1}^n z_i^2} \quad [\text{length}] \quad (3.2)$$

R_q is an amplitude parameter, defined as the RMS height or the standard deviation of the height distribution. It is insensitive to the spatial distribution or polarity of the roughness.

R_{sk} - skewness of height distribution

$$R_{sk} = \frac{1}{R_q^3} \frac{1}{L_o} \int_0^{L_o} z(x)^3 dx = \frac{1}{R_q^3 n} \sum_{i=1}^n z_i^3 \quad [-] \quad (3.3)$$

The skewness indicates a bias towards peaks or valleys. A skewness of zero indicates an even height distribution; a positive value indicates bias toward peaks; and a negative value indicates bias toward valleys (see Fig. 3.11). The skewness thus gives a measure of the peak versus valley bias, or “polarity”, of the roughness. Since the height values are cubed, the

maxima will be the dominant factors, and since maxima are sensitive to sample size, R_{sk} can suffer from poor repeatability if the sample size is not chosen carefully.

Figure 3.11: Probability density curves of the height distribution, with various skewness values.

R_{ku} - kurtosis of height distribution

$$R_{ku} = \frac{1}{R_q^4} \frac{1}{L_o} \int_0^{L_o} z(x)^4 dx = \frac{1}{R_q^4 n} \sum_{i=1}^n z_i^4 \quad [-] \quad (3.4)$$

The kurtosis describes the probability density of the height distribution. A Gaussian distribution of heights will have an R_{ku} of 3. If the surface has a larger than normal number of peaks and valleys, $R_{ku} > 3$, and if the surface is slowly varying, i.e. having few peaks and valleys, $R_{ku} < 3$ (see Fig. 3.12). The kurtosis is insensitive to spatial distribution or the polarity of a surface. Repeatability is also an issue, just as for the skewness.

Figure 3.12: Probability density curves of the height distribution, with various kurtosis values.

R_p - maximum height

$$R_p = z_{max} \quad [\text{length}] \quad (3.5)$$

R_p is the height of the highest peak. Since it is a single measurement, it has repeatability problems and as such is of limited significance.

R_v - minimum height

$$R_v = z_{min} \quad [\text{length}] \quad (3.6)$$

R_v is the depth of the lowest valley. Since it is a single measurement, it has repeatability problems and as such is of limited significance.

R_t - height range

$$R_t = z_{max} - z_{min} \quad [\text{length}] \quad (3.7)$$

R_t is the height range that the profile spans. Since it is based on single measurements, it has repeatability problems and as such is of limited significance.

R_{pm} - average maximum peak height

$$R_{pm} = \frac{1}{m} \sum_{i=1}^m z_{max_i} \quad [\text{length}] \quad (3.8)$$

where m is the number of peaks chosen to average over (usually 5 or 10). R_{pm} is the average of the highest m number of peaks. Since it is based on multiple measurements, it is more robust and significant than R_p .

R_{vm} - average minimum valley height

$$R_{vm} = \frac{1}{m} \sum_{i=1}^m z_{min_i} \quad [\text{length}] \quad (3.9)$$

where m is the number of valleys chosen to average over (usually 5 or 10). R_{vm} is the average of the lowest m number of valleys. Since it is based on multiple measurements, it is more robust and significant than R_v .

R_z - ten-point height (height range)

$$R_z = R_{pm} - R_{vm} \quad [\text{length}] \quad (3.10)$$

Since R_z is based on multiple measurements, it is far more significant than R_t . It is useful in distinguishing surfaces that may have a similar R_a or R_q but different maxima. Note that the name “ten-point height” is used when R_{pm} and R_{vm} are based on 5 peaks/valleys. This is in general an industry standard, and is used in the present work.

R_k - nominal core roughness range

$$R_k = 2 R_a \quad [\text{length}] \quad (3.11)$$

This and the following five parameters are derived from the “bearing ratio curve” (see Michigan-Metrology (2001) for more information). R_k is the range in which the core of the roughness exists, outside of which are the high peaks and deep valleys.

R_{pk} - peak height above core roughness

$$\begin{aligned} R_{pk} &= R_p - R_a \quad [\text{length}] \\ &= R_{pm} - R_a \quad [\text{length}] \end{aligned} \quad (3.12)$$

The above choice of using R_p or R_{pm} to calculate R_{pk} is subjective. R_{pk} is the height

range that the peaks occupy over the $+R_a$ line.

R_{vk} - valley height below core roughness

$$\begin{aligned} R_{vk} &= R_v + R_a && [\text{length}] \\ &= R_{vm} + R_a && [\text{length}] \end{aligned} \quad (3.13)$$

The above choice is similarly subjective. R_{vk} is the height range that the valleys occupy below the $-R_a$ line.

R_{pk}/R_k - peak to core range ratio

$$R_{pk}/R_k = \frac{R_{pk}}{R_k} \quad [-] \quad (3.14)$$

This parameter gives the ratio of the peak range to the nominal core range. The larger the ratio the higher the peaks, relative to the roughness core.

R_{vk}/R_k - valley to core range ratio

$$R_{vk}/R_k = \frac{|R_{vk}|}{R_k} \quad [-] \quad (3.15)$$

This parameter gives the ratio of the valley range to the nominal core range. The larger the ratio the deeper the valleys, relative to the roughness core.

R_{pk}/R_{vk} - peak range to valley range ratio

$$R_{pk}/R_{vk} = \frac{R_{pk}}{|R_{vk}|} \quad [-] \quad (3.16)$$

This parameter gives the ratio of the peak range to the valley range. A ratio greater than 1 means the peaks are higher than the valleys, and vice versa.

P_{cL} - peak count per unit length

$$P_{cL} = \frac{\# \text{ peaks}}{L} \quad \left[\frac{1}{\text{length}} \right] \quad (3.17)$$

Peaks are dominant structures and P_{cL} gives a quantitative description of the number of peaks over a profile, but no height-magnitude description. See section 3.1.2 for the definition of peaks.

V_{cL} - valley count per unit length

$$V_{cL} = \frac{\# \text{ valleys}}{L} \quad \left[\frac{1}{\text{length}} \right] \quad (3.18)$$

Valleys are dominant structures and V_{cL} gives a quantitative description of the number of valleys over a profile, but no height-magnitude description. See section 3.1.2 for the definition of valleys.

N_{AREA} - normalized area

$$N_{AREA} = \frac{1}{L_o} \int_0^{L_o} (R_a - z(x)) dx = \frac{1}{n} \sum_{i=1}^n (R_a - z_i) \quad \text{for all } z(x) < R_a \quad [\text{length}] \quad (3.19)$$

The normalized area gives the area on a profile between the $+R_a$ line and the surface, divided by the sample length L (see Fig. 3.13).

Figure 3.13: Hatched area shows the area between the $+R_a$ line and the surface.

R_L - developed length ratio

$$R_L = \frac{1}{L_o} \int_0^{L_o} \sqrt{1 + \left(\frac{dz}{dx}\right)^2} dx = \frac{1}{L} \sum_{i=2}^n \sqrt{(z_i - z_{i-1})^2 + \Delta x^2} \quad [-] \quad (3.20)$$

The developed length ratio is the profile length divided by the sample length. The profile length is the arc length of the surface, i.e. the distance along the surface. R_L gives

a quantitative measure of the “complexity” of a surface profile. The higher the ratio, the larger the surface features and/or the more superimposed small-scale features there are.

Δ_q - RMS slope

$$\Delta_q = \sqrt{\frac{1}{L_o} \int_0^{L_o} \left(\frac{dz}{dx} \right)^2 dx} = \sqrt{\frac{1}{n} \sum_{i=2}^n \left(\frac{z_i - z_{i-1}}{\Delta x} \right)^2} \quad [-] \quad (3.21)$$

The *RMS* slope gives a measure of the average slope of the surface. The polarity of the slope is lost. Note that the mean slope is zero, and as such is not included in equation 3.21.

λ_q - average spatial wavelength

$$\lambda_q = 2\pi \frac{R_q}{\Delta_q} \quad [\text{length}] \quad (3.22)$$

The average spatial wavelength is derived simply from the *RMS* height and *RMS* slope. The smaller the wavelength the more closely packed the average roughness “elements” are.

R_{sm} - average profile element width

$$R_{sm} = \frac{1}{N} \sum_{i=1}^N S_{m_i} \quad [\text{length}] \quad (3.23)$$

where N is the number of elements and S_m is the element width, defined as the distance between successive upward (or downward) zero-crossings (see Fig. 3.14). R_{sm} can be useful

if the surface has well-defined “lumps”, but for purely stochastic surfaces its applicability is limited.

Figure 3.14: Definition of profile element width and mean value. Reproduced from Thomas (1999).

R_{sc} - mean peak curvature

$$R_{sc} = \frac{1}{N} \sum_{j=1}^N \left[\frac{d^2 z}{dx^2} \Big|_{peak} \right]_j = \frac{1}{N} \sum_{j=1}^N \left(\frac{z_{i-1} - 2z_i + z_{i+1}}{\Delta x^2} \right)_j \quad \left[\frac{1}{\text{length}} \right] \quad (3.24)$$

where N is the number of peaks and the subscript i represents the data point at the peak. For numerical calculation, a 3-point first order central difference method is used to calculate the second derivative of the height. This calculation is only performed for peaks, although a “mean surface curvature” could also be calculated by applying the above equation to the whole surface.

D_{ls} - length-scale fractal dimension

$$D_{ls} = 1 - \left[\frac{\Delta \log (\text{rel.length})}{\Delta \log (\text{scale.of.obs.})} \right]_{\text{linear region}} \left[\frac{1}{\text{length}} \right] \quad (3.25)$$

This parameter is found by performing a length-scale fractal analysis on the surface profile. This involves fitting the surface with smaller and smaller line segments (i.e. reducing the scale of observation) and determining the relative length of the profile for each scale of observation (length of the individual line segments).

$$\text{scale of obs.} = \frac{L_o}{2^r} \quad \text{for } r = 0, 1, 2 \dots \log_2(n-1) \quad [\text{length}] \quad (3.26)$$

$$\text{relative length} = \frac{1}{L_o} \sum_{s=0}^{2^r-1} \sqrt{(z_{i_2} - z_{i_1})^2 + \left(\frac{L_o}{2^r}\right)^2} \quad [-] \quad (3.27)$$

where

$$\begin{aligned} i_1 &= 1 + \left(\frac{n-1}{2^r}\right)s \\ i_2 &= 1 + \left(\frac{n-1}{2^r}\right)(s+1) \end{aligned}$$

and “linear region” denotes the linear region of the log-log plot. Once this data set is found, a log-log plot of the relative length vs. scale of observation is made, as shown in Fig. 3.15. Note that the surface may or may not be a fractal surface. The parameter name “fractal dimension” refers to the method used.

The slope of this linear region is denoted by the bracketed term in equation 3.25. The length scale fractal dimension can be written more simply as:

$$D_{ls} = 1 - \text{slope}$$

Figure 3.15: A log-log plot of the fractal analysis parameters.

The highest relative length ratio is the parameter R_L , as shown previously. The higher the slope, the higher the fractal dimension, and the more “complex” a surface is.

R_{al} - shortest linear autocorrelation length

$$R_{al} = \tau|_{\rho(\tau)=a} \quad [\text{length}] \quad (3.28)$$

where a can be 0, 0.1, or 0.2, depending on choice. This parameter is explained in the

following lines. The *autocovariance* function (ACVF) is

$$\begin{aligned} R(\tau) &= \frac{1}{L-\tau} \int_0^{L-\tau} z(x) z(x+\tau) dx \\ R(\tau_i) &= \frac{1}{n-i} \sum_{k=1}^{n-i} z_k z_{k+i} \quad [\text{length}] \end{aligned} \quad (3.29)$$

where

$$\tau_i = i \Delta x \quad \text{for } i = 0, 1, 2, \dots, \frac{n}{2} \quad [\text{length}] \quad (3.30)$$

is the distance between multiplied z -points, as shown in figure 3.16, and n is the number of sample points. Subscript k refers to the data points.

Figure 3.16: Construction of the autocovariance function. Reproduced from Thomas (1999).

For $\tau=0$, $R(\tau_0)=R_q^2$, and as such, the ACVF is normalized by R_q^2 to give the *autocorrelation* function (ACF):

$$\rho(\tau) = \frac{R(\tau)}{R_q^2} = \frac{1}{R_q^2(n-i)} \sum_{k=1}^{n-i} z_k z_{k+i} \quad [-] \quad (3.31)$$

The autocorrelation function gives a plot of correlation function $\rho(\tau)$ vs. distance between points (τ) as shown in Fig. 3.17. The closer the successive points are together,

i.e. the smaller the τ , the more correlated they will be; in other words, they will be related to each other since they occupy the same region. As the distance between points moves farther apart, the points will become less and less correlated. The above is for a given surface. When comparing two separate surfaces, the geometry of the surfaces will affect their correlation functions, as illustrated in Fig. 3.17. The shortest linear autocorrelation length, R_{al} , as given by equation 3.28 is the distance τ where the autocorrelation function drops to either 0 or 0.1 or 0.2, depending on choice. Usually the shorter the average wavelength or the more complex the surface, the smaller the R_{al} .

Figure 3.17: A comparison of autocorrelation functions for two different surfaces. Reproduced from Thomas (1999).

Frequency Analysis

The *power spectral density function* (PSDF) is directly related, by a Fourier-transform relationship, to the autocovariance function:

$$G(\omega/2\pi) = 4 \int_0^{\infty} R(\tau) \cos(\omega)\tau \, d\tau \quad (3.32)$$

or

$$G(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\tau) e^{-i\omega\tau} d\tau \quad (3.33)$$

The PSDF, $G(\omega)$, has moments

$$m_n = \int_{\omega_1}^{\omega_2} \omega^n G(\omega) d\omega \quad (3.34)$$

where

$$\begin{aligned} m_0 &= \text{variance of the height distribution, } R_q^2 \\ m_2 &= \text{variance of the slope distribution, } \Delta_q^2 \\ m_4 &= \text{variance of the curvature distribution} \end{aligned}$$

Since $G(\omega)$ is directly related to $R(\tau)$, it is unnecessary to perform a spectral analysis, however some prefer to compute the PSDF rather than the ACF, giving ease of computation as the main reason.

3.3.2 3D Parameters

Fig. 3.18 shows an actual surface and its discretized form for use in digital analysis. The mean plane is the zero-height datum. Each data point is represented by a rectangular prism whose sides have the dimension of the step sizes in their respective directions: in the x -direction, Δx , and in the y -direction, Δy . The number of sample points in the x -direction is n_x , and in the y -direction, n_y . The total number of sample points is $n = n_x \cdot n_y$. The original sample length in the x -direction is L_{ox} , and in the y -direction, L_{oy} . The effective sample length in the x -direction is L_x , and in the y -direction, L_y . The original sample area

is $L_{ox} \cdot L_{oy}$ and the effective sample area is $L_x \cdot L_y$.

$$L_{ox} = (n_x - 1)\Delta x$$

$$L_{oy} = (n_y - 1)\Delta y$$

$$L_x = n_x \Delta x$$

$$L_y = n_y \Delta y$$

The parameters are given next, with their analytical expression, numerical expression and their units. Note that $[-]$ represents a dimensionless parameter. Since most of the 3D parameters are extensions of, or analogous to, the 2D parameters, only the necessary equations will be given. Note that the prefix “R” designated 2D parameters, while for 3D parameters, “S” is used instead. Also, the analytical expressions will be applicable to any rectangular sample area, while **the numerical expressions have been reduced based on the sample area being square and the step sizes being equal**, i.e. $n_x = n_y$ and $L_x = L_y$.

S_a - mean absolute height

$$S_a = \frac{1}{L_{ox}L_{oy}} \int_0^{L_{oy}} \int_0^{L_{ox}} |z(x, y)| dx dy = \frac{1}{n} \sum_{i=1}^n |z_i| \quad [\text{length}] \quad (3.35)$$

Figure 3.18: The effect of discretizing a surface, showing the change in surface representation. Top: actual surface. Bottom: discretized surface.

S_q - root-mean-square height

$$S_q = \sqrt{\frac{1}{L_{ox}L_{oy}} \int_0^{L_{oy}} \int_0^{L_{ox}} z(x,y)^2 dx dy} = \sqrt{\frac{1}{n} \sum_{i=1}^n z_i^2} \quad [\text{length}] \quad (3.36)$$

S_{sk} - skewness of height distribution

$$S_{sk} = \frac{1}{R_q^3} \frac{1}{L_{ox}L_{oy}} \int_0^{L_{oy}} \int_0^{L_{ox}} z(x,y)^3 dx dy = \frac{1}{R_q^3 n} \sum_{i=1}^n z_i^3 \quad [-] \quad (3.37)$$

S_{ku} - kurtosis of height distribution

$$S_{ku} = \frac{1}{R_q^4} \frac{1}{L_{ox}L_{oy}} \int_0^{L_{oy}} \int_0^{L_{ox}} z(x,y)^4 dx dy = \frac{1}{R_q^4 n} \sum_{i=1}^n z_i^4 \quad [-] \quad (3.38)$$

S_p - maximum height

$$S_p = z_{max} \quad [\text{length}] \quad (3.39)$$

S_v - minimum height

$$S_v = z_{min} \quad [\text{length}] \quad (3.40)$$

S_t - height range

$$S_t = z_{max} - z_{min} \quad [\text{length}] \quad (3.41)$$

S_{pm} - average maximum summit height

$$S_{pm} = \frac{1}{m} \sum_{i=1}^m z_{max_i} \quad [\text{length}] \quad (3.42)$$

where m is the number of summits chosen to average over (usually 5 or 10). S_{pm} is the average height of the highest m number of summits.

S_{vm} - average minimum valley height

$$S_{vm} = \frac{1}{m} \sum_{i=1}^m z_{min_i} \quad [\text{length}] \quad (3.43)$$

where m is the number of 3D valleys chosen to average over (usually 5 or 10). S_{vm} is the average height of the lowest m number of valleys.

S_z - ten-point height (height range)

$$S_z = S_{pm} - S_{vm} \quad [\text{length}] \quad (3.44)$$

Note that the name “ten-point height” is used when S_{pm} and S_{vm} are based on 5

summits/valleys. This is in general an industry standard, and is used in the present work.

S_k - nominal core roughness range

$$S_k = 2 S_a \quad [\text{length}] \quad (3.45)$$

S_k is the range in which the core of the roughness exists, outside of which are the high summits and valleys.

S_{pk} - summit height above core roughness

$$\begin{aligned} S_{pk} &= S_p - S_a & [\text{length}] \\ &= S_{pm} - S_a & [\text{length}] \end{aligned} \quad (3.46)$$

The above choice of using S_p or S_{pm} to calculate S_{pk} is subjective. S_{pk} is the height range that the summits occupy over the $+S_a$ line.

S_{vk} - valley height below core roughness

$$\begin{aligned} S_{vk} &= S_v + S_a & [\text{length}] \\ &= S_{vm} + S_a & [\text{length}] \end{aligned} \quad (3.47)$$

The above choice is similarly subjective. S_{vk} is the height range that the 3D valleys occupy below the $-S_a$ line.

S_{pk}/S_k - summit to core range ratio

$$S_{pk}/S_k = \frac{S_{pk}}{S_k} \quad [-] \quad (3.48)$$

This parameter gives the ratio of the summit range to the nominal core range. The larger the ratio the higher the summits, relative to the roughness core.

S_{vk}/S_k - valley to core range ratio

$$S_{vk}/S_k = \frac{|S_{vk}|}{S_k} \quad [-] \quad (3.49)$$

This parameter gives the ratio of the 3D valley range to the nominal core range. The larger the ratio the deeper the valleys, relative to the roughness core.

S_{pk}/S_{vk} - summit range to valley range ratio

$$S_{pk}/S_{vk} = \frac{S_{pk}}{|S_{vk}|} \quad [-] \quad (3.50)$$

This parameter gives the ratio of the summit range to the 3D valley range. A ratio greater than 1 means the summits are higher than the valleys, and vice versa.

P_{c_A} - summit count per unit area

$$P_{c_A} = \frac{\# \text{ summits}}{L_x L_y} \left[\frac{1}{\text{length}^2} \right] \quad (3.51)$$

Summits are dominant structures and P_{c_A} gives a quantitative description of the number of summits over a surface, but no height-magnitude description. See section 3.1.2 for the definition of summits.

V_{c_A} - 3D valley count per unit area

$$V_{c_A} = \frac{\# \text{ 3D valleys}}{L_x L_y} \left[\frac{1}{\text{length}^2} \right] \quad (3.52)$$

N_{VOL} - normalized volume

$$\begin{aligned} N_{VOL} &= \frac{1}{L_{ox} L_{oy}} \int_0^{L_{oy}} \int_0^{L_{ox}} (S_a - z(x, y)) \, dx \, dy \\ &= \frac{1}{n} \sum_{i=1}^n (S_a - z_i) \quad \text{for all } z(x, y) < S_a \quad [\text{length}] \end{aligned} \quad (3.53)$$

The normalized volume gives the volume on a surface between the $+S_a$ line and the surface, divided by the sample area $L_x L_y$. See Fig. 3.13 for the profile diagram.

S_{sm} - average surface element area

For profiles this parameter is simple to calculate since finding the successive upward (or downward) zero-crossings is relatively easy. For a surface however, one needs to find the perimeter of zero-crossings that surround a summit or valley, and for purely stochastic surfaces this can become a complex programming exercise. Due to the limited applicability of this parameter for purely stochastic surfaces, it has not been used in the present work.

S_{dr} - developed area ratio

$$\begin{aligned}
 S_{dr} &= \frac{1}{L_{ox}L_{oy}} \int_0^{L_{oy}} \int_0^{L_{ox}} \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} dx dy \\
 &= \frac{1}{L_x L_y} \sum_{j=1}^{n_y-1} \sum_{i=1}^{n_x-1} \left[\frac{1}{2} \Delta x \Delta y \cdot \right. \\
 &\quad \cdot \sqrt{1 + \left(\frac{z_{i+1,j} - z_{i,j}}{\Delta x}\right)^2 + \left(\frac{z_{i,j+1} - z_{i,j}}{\Delta y}\right)^2 + \left(\frac{(z_{i+1,j} - z_{i,j})(z_{i,j+1} - z_{i,j})}{\Delta x \Delta y}\right)^2} + \\
 &\quad + \frac{1}{2} \Delta x \Delta y \sqrt{1 + \left(\frac{z_{i+1,j+1} - z_{i,j+1}}{\Delta x}\right)^2 + \left(\frac{z_{i+1,j+1} - z_{i+1,j}}{\Delta y}\right)^2} + \dots \\
 &\quad \left. \dots + \left(\frac{(z_{i+1,j+1} - z_{i,j+1})(z_{i+1,j+1} - z_{i+1,j})}{\Delta x \Delta y}\right)^2 \right] \quad [-] \quad (3.54)
 \end{aligned}$$

The developed area ratio is the surface area divided by the sample area. The surface area is the total wetted area, and is found by mapping the surface with triangles whose side lengths are the step sizes Δx and Δy . Fig. 3.19 illustrates the above method. S_{dr} gives a quantitative measure of the “complexity” of a surface profile. The higher the ratio, the larger the surface features and/or the more superimposed small-scale features there are. See section 4.3.3 for additional calculation info.

Figure 3.19: A surface mapped with an example triangle element. Reproduced from Dong *et al.* (1994).

$S\Delta_q$ - RMS slope

$$\begin{aligned}
 S\Delta_q &= \sqrt{\frac{1}{L_{ox}L_{oy}} \int_0^{L_{oy}} \int_0^{L_{ox}} z'(x,y)^2 dx dy} \\
 &= \sqrt{\frac{1}{(n_x-1)(n_y-1)} \sum_{j=2}^{n_y} \sum_{i=2}^{n_x} \left[\left(\frac{z_{i,j} - z_{i-1,j}}{\Delta x} \right)^2 + \left(\frac{z_{i,j} - z_{i,j-1}}{\Delta y} \right)^2 \right]} \quad [-] \quad (3.55)
 \end{aligned}$$

where

$$z'(x,y) = \sqrt{\left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2}$$

$S\lambda_q$ - average spatial wavelength

$$S\lambda_q = 2\pi \frac{S_q}{S\Delta_q} \quad [\text{length}] \quad (3.56)$$

S_{sc} - mean summit curvature

$$\begin{aligned}
 S_{sc} &= \frac{1}{N} \sum_{k=1}^N \left[z''(x, y) |_{\text{summit}} \right]_k \\
 &= \frac{1}{2N} \sum_{k=1}^N \left(\frac{z_{i-1,j} - 2z_{i,j} + z_{i+1,j}}{\Delta x^2} + \frac{z_{i,j-1} - 2z_{i,j} + z_{i,j+1}}{\Delta y^2} \right)_k \left[\frac{1}{\text{length}} \right] \quad (3.57)
 \end{aligned}$$

where

$$z''(x, y) = -\frac{1}{2} \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right)$$

and N is the number of summits. A 3-point first order central difference method is used for calculating the second derivative of the height for the N summits.

D_{as} - Area-scale fractal dimension

$$\begin{aligned}
 D_{as} &= 2 - 2 \cdot \text{slope} \\
 &= 2 - 2 \left[\frac{\Delta \log(\text{rel.area})}{\Delta \log(\text{scale.of.obs})} \right]_{\text{linear region}} \left[\frac{1}{\text{length}^2} \right] \quad (3.58)
 \end{aligned}$$

This parameter is found by performing an area-scale fractal analysis on the surface. This involves fitting the surface with smaller and smaller triangular elements (i.e. reducing the scale of observation) and determining the relative area of the surface for each scale of observation (area of the individual triangles). See Fig. 3.20 for illustration.

Figure 3.20: An example of the process of area-scale fractal analysis, mapping the surface with triangular elements. Reproduced from Brown (2000).

$$\text{s.o.o.} = \frac{L_{ox}L_{oy}}{2 \cdot 2^{2r}} \quad \text{for } r = 0, 1, 2 \dots \frac{\log_2((n_x - 1)(n_y - 1))}{2} \quad [\text{length}^2] \quad (3.59)$$

$$\begin{aligned} \text{r.a.} = & \frac{1}{L_{ox}L_{oy}} \sum_{t=0}^{2^r-1} \sum_{s=0}^{2^r-1} \left[\frac{1}{2} \Delta X \Delta Y \sqrt{1 + \left(\frac{z_{i_2,j_1} - z_{i_1,j_1}}{\Delta X} \right)^2 + \left(\frac{z_{i_1,j_2} - z_{i_1,j_1}}{\Delta Y} \right)^2} + \dots \right. \\ & \dots \left(\frac{(z_{i_2,j_1} - z_{i_1,j_1})(z_{i_1,j_2} - z_{i_1,j_1})}{\Delta X \Delta Y} \right)^2 + \frac{1}{2} \Delta X \Delta Y \sqrt{1 + \left(\frac{z_{i_2,j_2} - z_{i_1,j_2}}{\Delta X} \right)^2} + \dots \\ & \left. \dots \left(\frac{z_{i_2,j_2} - z_{i_2,j_1}}{\Delta Y} \right)^2 + \left(\frac{(z_{i_2,j_2} - z_{i_1,j_2})(z_{i_2,j_2} - z_{i_2,j_1})}{\Delta X \Delta Y} \right)^2 \right] \quad [-] \quad (3.60) \end{aligned}$$

where

$$\begin{aligned}
 \Delta X &= \frac{(n_x - 1)\Delta x}{2^r} \\
 \Delta Y &= \frac{(n_y - 1)\Delta y}{2^r} \\
 \Delta x &= x\text{-direction step size} \\
 \Delta y &= y\text{-direction step size} \\
 i_1 &= 1 + \frac{n_x - 1}{2^r} s \\
 i_2 &= 1 + \frac{n_x - 1}{2^r} (s + 1) \\
 j_1 &= 1 + \frac{n_y - 1}{2^r} t \\
 j_2 &= 1 + \frac{n_y - 1}{2^r} (t + 1)
 \end{aligned}$$

and “s.o.o.” is scale of observation; “r.a.” is relative area; and “linear region” denotes the linear region of the log-log plot.

Once this data set is found, a log-log plot of the relative area vs. scale of observation is made, as shown in Fig. 3.15 (replace relative length with relative area on the y-axis). The slope of this linear region is denoted by the bracketed term in equation 3.58. The highest relative area ratio is the parameter S_{dr} , as shown previously.

S_{al} - shortest areal autocorrelation length

$$S_{al} = \tau \mid_{\rho(\tau_x \tau_y)=a} \quad [\text{length}] \quad (3.61)$$

where a can be 0, 0.1, or 0.2, depending on choice. An explanation of this parameter

follows. The *autocovariance* function (ACVF) is

$$\begin{aligned}
 R(\tau_x, \tau_y) &= \frac{1}{(L_x - \tau_x)(L_y - \tau_y)} \int_0^{L_y - \tau_y} \int_0^{L_x - \tau_x} z(x, y) z(x + \tau_x, y + \tau_y) dx dy \\
 R(\tau_{x_i}, \tau_{y_j}) &= \frac{1}{(n_x - i)(n_y - j)} \sum_{p=1}^{n_y - j} \sum_{q=1}^{n_x - i} z(x_k, y_p) z(x_{k+i}, y_{p+j}) \quad [\text{length}] \quad (3.62)
 \end{aligned}$$

where

$$\tau_{x_i} = i \Delta x \quad \text{for } i = 0, 1, 2 \dots \frac{n_x}{2} \quad [\text{length}] \quad (3.63)$$

$$\tau_{y_j} = j \Delta y \quad \text{for } j = 0, 1, 2 \dots \frac{n_y}{2} \quad [\text{length}] \quad (3.64)$$

are the distances between multiplied z -points.

For $\tau_x = \tau_y = 0$, $R(\tau_{x_0}, \tau_{y_0}) = S_q^2$, and as such, the ACVF is normalized by S_q^2 to give the *autocorrelation* function (ACF):

$$\begin{aligned}
 \rho(\tau_x, \tau_y) &= \frac{R(\tau_x, \tau_y)}{S_q^2} \\
 &= \frac{1}{S_q^2 (n_x - i)(n_y - j)} \sum_{p=1}^{n_y - j} \sum_{q=1}^{n_x - i} z(x_k, y_p) z(x_{k+i}, y_{p+j}) \quad [-] \quad (3.65)
 \end{aligned}$$

The autocorrelation function gives a plot of correlation function $\rho(\tau_x, \tau_y)$ vs. distance between points (τ_x, τ_y) as shown in Fig. 3.21. This is a single quadrant plot, with the origin based at a corner of the surface sample. Depending on how the analysis is done, however, a four-quadrant (i.e. sample-centred) plot can be made, as shown in Fig. 3.22.

The shortest linear autocorrelation length, S_{al} , as given by equation 3.61 is the distance $\sqrt{\tau_x^2 + \tau_y^2}$ where the autocorrelation function drops to either 0 or 0.1 or 0.2, depending on choice. Usually the shorter the average wavelength or the more complex the surface, the smaller the S_{al} .

Figure 3.21: A single-quadrant areal autocorrelation plot.

Frequency Analysis

Similarly to the one-dimensional PSDF of equation 3.33, the two-dimensional power spectral density function is directly related to the autocovariance function:

$$G(\omega_x, \omega_y) = \frac{1}{2\pi^2} \int_0^\infty \int_0^\infty R(\tau_x, \tau_y) \cos(\omega_x \tau_x + \omega_y \tau_y) d\tau_x d\tau_y$$

Figure 3.22: A sample-centred four-quadrant areal autocorrelation plot.

Since $G(\omega_x, \omega_y)$ is directly related to $R(\tau_x, \tau_y)$, it is unnecessary to perform a spectral analysis, however some prefer the ease of computing the PSDF compared to the ACF.

Chapter 4

Experiment Description and Data Reduction Methods

The objective of the wind tunnel tests was to obtain boundary layer profiles at different stations on the test surfaces. These profiles are used to determine the skin friction coefficient and to plot the inner layer velocity profile (u^+ vs. y^+), in order to obtain the intercept-shift due to roughness (see section 2.3.2). The apparatus and methods used are outlined in the next sections of this chapter.

The objective of the surface scanning was to capture the geometry of the rough surfaces in order to calculate the descriptive parameters to be used in the intercept-shift correlation (see section 3.3). The equipment and analysis methods are outlined in later sections of this chapter, and the chapter concludes with an outline of the method used for correlating the intercept shifts.

4.1 Apparatus for Boundary Layer Measurements

4.1.1 Wind Tunnel and Test Section

The Carleton University closed circuit low speed wind tunnel was used for the experiments. The tunnel has a contraction ratio of 12:1, contracting from a circular cross-section settling chamber to a rectangular cross-section test section. The test section is 20" high by 30" wide by 72" long (0.51 m x 0.76 m x 1.83 m) with no tapering. There is a bleeder gap between the rear of the test section and the tunnel diffuser. See Fig. 4.1 for illustration.

The tunnel is powered by a 45 kW induction motor belt-driving a magnetic clutch which in turn belt-drives the tunnel fan. The magnetic clutch is controlled by a potentiometer dial giving a reasonable speed resolution. The maximum test section airspeed is approximately 75 m/s, allowing a Reynolds number per unit length of approximately 5 million per metre.

The test section has a Plexiglas side wall for viewing, and an access door as the other side. The ceiling has static pressure taps located above the probing stations. There are nine probing stations, five along the longitudinal centreline of the test section and four off centreline, as depicted in Fig. 4.2. Since the test surfaces were mounted or fabricated on 7/16" MDF board, a ramp was necessary at the front of the section to guide the airflow from the tunnel contraction floor to the test surface.

4.1.2 Traverse Gear and Pitot Probe

Velocity profiles were measured by traversing a Pitot probe across the boundary layer at the measurement stations. The traverse gear was mounted to the underside of the test section. It consists of a mounting block to which is attached guide rails and a stepper motor and lead screw to drive a probe mounting block (see Fig. 4.3). The stepper motor is

Figure 4.1: Schematic of Wind Tunnel and Experimental Setup

Figure 4.2: Top: Layout of the surface-plate. Bottom: Details of surface mounting and ramp. Note: all dimensions in millimetres.

Figure 4.3: Traverse Gear. Modified from Baumann (1999).

a five-wire type with a 1.8° step. The lead screw is a brass screw of $1/4''$ diameter and has a pitch of 20 revolutions per inch. A 1.8° step corresponds to a vertical movement of the probe of 0.00635 mm. Since the probe only had to move in one vertical direction (upwards), backlash in the traverse gear was not a concern, and thus was not measured.

The pitot probe was mounted to the probe mounting block, which traverses vertically. The probe consisted of a hollow circular brass tube of $3/32''$ inside diameter to which was soldered a 1.0 mm outside diameter stainless-steel tube with a flattened mouth. The brass tube was placed inside a larger $1/4''$ outside diameter brass tube for mounting in the probe mounting block. The probe was leak tested with success. Fig. 4.4 is a picture of the actual probe, with dimensions, and Fig. 4.5 shows the measurements of the probe mouth.

Figure 4.4: A photograph of the flat-mouth pitot probe.

Figure 4.5: Pitot probe mouth measurements, in millimetres.

4.1.3 Pressure Transducer and Data Acquisition System

A combined pressure transducer and data acquisition system, the Pressure Systems Inc. (PSI) Model 9010 Intelligent Pressure Scanner, was used for the pressure measurements in the boundary layer traverses and the tunnel contraction pressure difference. The pressure ports were hooked up to the static taps and Pitot probe with Tygon tubing, with soldered brass adapters if necessary.

The PSI system features 16 silicon DH200 pressure transducers, 8 absolute (310 kPa range) and 8 differential (± 5 kPa). In the present experiments only 3 of the differential channels were used. Details regarding the measured pressures are presented later, in section 4.2.5. The PSI unit has a 12 bit sequential analog to digital converter built in, and transmits the data to a computer through a serial RS-232 interface port. The unit is powered by an external 15 Volt DC power supply. The unit is also supplied with 100 psi nitrogen gas, to provide pressure for a pneumatic manifold which is used to re-zero the calibration offsets online.

Calibration of the unit was done by doctoral candidate A. Mahallati, as the unit was shared with him during the experiments. A sample calibration graph is shown in Fig. 4.6. The uncertainty of pressure measurements is estimated at ± 2 Pa with 95% confidence when re-zeroing the offsets every 50 sample points (Mahallati (2002)). A sampling rate of 16 samples/sec per channel was used, as fixed by the PSI system when using 3 channels. A total number of samples of 300 was used for each data point, giving a sampling time of 18.75 seconds, as suggested by Mahallati (2002). This suggestion came as a result of Mahallati's boundary layer measurements which showed that approximately 200 samples was the lower limit to properly capture the mean flow, as determined by taking a running average of the flow velocity measurements. The PSI unit was reset and re-zeroed before each traverse and re-zeroed each half hour of traverse time, corresponding to approximately 50 data points.

Figure 4.6: Pressure Transducer Calibration Graph. Courtesy of Mahallati (2002).

4.1.4 Computer and Software Setup

An Intel 486/33 MHz PC was used for data storage and control of the PSI unit and stepper motor, through a program ("Scanner") written by A. Mahallati. The software imports an ASCII data file of boundary layer height points to be taken, in conjunction with settings governing the pitch of the lead screw and the degree-step of the stepper motor. See Appendix A for an example traverse input file.

The software also controls the pressure transducers, with options to calibrate, reset, and re-zero the system. A graphical display of the pressure values is given during the traverse runs. The data is output to three files, the first being a record of the average pressure differentials for each channel used, the second outputs the static, total, and dynamic pressure coefficients, and the third is a record of the traverse, with information on probe movement time, sampling time etc. Appendix B contains examples of these output files.

4.1.5 Miscellaneous Apparatus

The tunnel air temperature was measured with a thermometer inserted into the settling chamber. Its accuracy was verified to be sufficient with the use of a portable thermometer known to be accurate. The atmospheric pressure was measured with a Bourdon-type barometer, previously verified by the department technicians and other researchers. The temperature and atmospheric pressure were used only to determine the air density.

4.2 Boundary-Layer Experiments and Data Reduction

4.2.1 Test Matrix

Ten surfaces were tested, with boundary layer traverses made at each of the five centreline stations shown in Fig. 4.2. In addition, traverses were made at the off-centreline stations for two surfaces, to check for flow two-dimensionality. The test matrix is shown in Table 4.1. The surfaces will be described in Chapter 5.

Table 4.1: Experimental Test Matrix

surface	centreline traverses	off-centreline traverses
Blue Formica	x	x
Coarse Shingle	x	
Coarse Spackle	x	x
Fine Shingle	x	
Fine Spackle	x	
Rust	x	
Tar Paper	x	
Vinyl	x	
White Formica	x	
Smooth	x	

4.2.2 Wind Tunnel Settings and Test Conditions

The tunnel airspeed was set to 50 m/s, giving a Reynolds number per metre of approximately 3×10^6 . This speed was based on pressure measurements made at the first probe station. There was some small variation in Reynolds number (about 1%) due to temperature change in the tunnel, but this has an insignificant effect on the boundary layer data. This was verified by performing a few traverses at different Reynolds numbers (1.5×10^6 and 4.1×10^6) and noting the change in boundary layer parameters.

The air density was calculated using the ideal gas law:

$$\rho = \frac{p}{RT}$$

where $R = 287 \text{ kJ/kg}\cdot\text{K}$ is the gas constant for air, p is the absolute tunnel static pressure, and T is the tunnel temperature. The dynamic viscosity, μ , was determined using Sutherland's Law:

$$\mu = \mu_o \left(\frac{T}{T_o} \right)^{\frac{3}{2}} \cdot \frac{T_o + 110}{T + 110}$$

where $\mu_o = 1.7894 \times 10^{-5} \text{ Pa}\cdot\text{s}$ and $T_o = 288.16 \text{ K}$.

4.2.3 Boundary Layer Measurements

In order to use the proper traverse input file, the approximate boundary layer thickness needed to be determined. This was achieved using a quick traverse with sparse spacing and a low sampling time. A range of input files was made based on different boundary layer thicknesses, and at times experience allowed the author to choose the appropriate input file without a quick traverse.

The vertical spacing between sampling points was varied according to height in the

boundary layer. Near the wall, a higher number of sample points is necessary to capture the greater rate of change in velocity, so steps of 0.09525 mm (corresponding to 15 1.8° steps of 0.00635 mm) were used for the first ~ 2.5 mm of the boundary layer. For the range of ~ 2.5 mm to ~ 10.5 mm, a sample spacing of 0.19050 mm was used (corresponding to 30 1.8° steps of 0.00635 mm). For the range of ~ 10.5 mm to ~ 20.5 mm, a sample spacing of 0.28575 mm was used (corresponding to 45 1.8° steps of 0.00635 mm). For the range of ~ 20.5 mm and upward, a sample spacing of 0.47625 mm was used (corresponding to 75 1.8° steps of 0.00635 mm). After the boundary layer traverse, a measurement was taken at the highest point attainable by the traverse gear outside the boundary layer (123.99 mm), in order to measure local freestream conditions. This measurement was later used to find the local dynamic pressure coefficient C_q (see section 4.2.5). The probe then returned to the surface to once again measure wall conditions. See Appendix A for an example traverse input file.

The boundary layer traverses were performed as follows: The probe was aligned with the flow direction to an accuracy of approximately $\pm 0.5^\circ$, well within the accepted limits for pitot probe yaw accuracy. Using the controlling software, the probe was manually positioned on the surface “wall”, occasionally with some flex, or rebound, of the probe (i.e. bent upward, which is later corrected for). Note that this “wall” is not well-defined for stochastic surfaces, so the probe was placed on the surface summits (see section 4.2.5, “*y*-Datum Determination”). For the small-scale (relative to the probe size) rough surfaces, the summits were sufficiently close together to constitute a “wall” that the probe could come in contact with, but for the Coarse Spackle and Fine Spackle surfaces however, the roughness is of a sufficiently large scale that the probe could pass between summits. To avoid undue uncertainty, the probe for these two spackle surfaces was placed at the summit height, and not placed down between the summits. Therefore there was no flex in the probe for these two surfaces. The traverse was then started on an automatic survey run using the

appropriate input file.

The waiting time between each sampling point was approximately 6 seconds, in order for the disturbances due to the probe movement to dissipate. During each run, the pressure transducers were re-zeroed every 50 points or so, depending on the boundary layer thickness.

4.2.4 Flow Two-Dimensionality Check

As noted in Table 4.1, boundary layer traverses were made for two surfaces at lateral stations for streamwise stations 1 and 3. This was done to check for flow two-dimensionality, as any cross-flow, or lateral velocity would invalidate the use of the two-dimensional momentum integral method for determining C_f (see section 4.2.5). The flow was confirmed as two-dimensional if the velocity profiles at the 3 spanwise stations were identical to a sufficient accuracy.

4.2.5 Boundary Layer Data Analysis

y-Datum Determination

As mentioned in section 4.2.3, at times the probe was bent up from the surface as a result of bringing it down too far towards the surface. This resulted in the tip of the probe being raised off the surface, thus giving a higher velocity reading. In order to correct for this and determine the proper probe position datum, the lowest velocity measure was taken as the point where the probe was at the surface.

In order to find the proper $y = 0$ datum, the probe mouth size had to be accounted for. Referring to Fig. 4.5, the half-height of the probe is 0.23 mm. This value was added to the probe-bend-corrected datum, so that when the probe was resting on the surface, the first

reading was considered to be at the centre of the probe mouth, 0.23 mm above the surface. No displacement correction was applied as suitable information is not available for Pitot tubes with a flattened mouth like the present Pitot tube.

Furthermore, for the rough surfaces the probe was considered to be on the highest peaks of the surface, using the roughness parameter S_{pm} of the surface in question as the height above the mean line (see section 4.3.3 for details on the S_{pm} parameter). Using the method of Clauser (1956), outlined in section 2.3.3, the displacement height, d , was found and used to find the true $y = 0$ datum.

Boundary Layer Thickness Definition

For the experimental data reduction, the boundary layer thickness, δ , was defined as the height at which the normalized velocity, u/U_e , reached 0.999, or 99.9% of the freestream velocity. The use of this relatively high value, versus 0.99 or 0.995 used in much earlier work, was enabled by the relatively low data point scatter in the velocity profile at the edge of the boundary layer, as can be seen in Fig. 4.7.

Velocity Profile Determination

As mentioned in section 4.1.3, three of the differential pressure channels of the PSI system were used in the experiments. Referring to Fig. 4.1, there is a static pressure tap, P_{c1} , located at the settling chamber where the velocity is approximately zero and another, P_{c2} , near the end of the contraction. The Pitot probe sensed the total pressure, p_o . For each profile the static tap located above the probe, on the ceiling of the test section, sensed the static pressure, p_s .

P_{c2} was used as the reference pressure for all three differential channels. That is, channel

Figure 4.7: An example of the velocity profile near the boundary layer edge. Data taken from Blue Formica surface, station 4.

1 measured $p_o - P_{c2}$, channel 2 measured $p_s - P_{c2}$, and channel 3 measured $P_{c1} - P_{c2}$.

The controlling software automatically calculates the total and static pressure coefficients:

$$C_{p_o} = \frac{p_o - P_{c2}}{P_{c1} - P_{c2}}$$

$$C_{p_s} = \frac{p_s - P_{c2}}{P_{c1} - P_{c2}}$$

The dynamic pressure coefficient, C_q , relates the local dynamic pressure at the probing station, q_e , to that given by the contraction pressure difference $P_{c1} - P_{c2}$. Calibration data (Nituch (1972)) shows that at the start of the empty test section the freestream dynamic

pressure is related to $P_{c1} - P_{c2}$ by

$$\frac{1}{2}\rho U_e^2 = \frac{P_{c1} - P_{c2}}{0.91} \quad (4.1)$$

In this thesis a reference velocity, denoted U_{ref} , is defined as

$$U_{ref} = \sqrt{\frac{2(P_{c1} - P_{c2})}{\rho}} \quad (4.2)$$

Due to the buildup of the boundary layer displacement thickness, δ^* , the velocity will not be uniform throughout the test section, and so the local C_q must be measured to find the true local freestream velocity:

$$\begin{aligned} C_q &= \frac{\frac{1}{2}\rho U_e^2}{\frac{1}{2}\rho U_{ref}^2} \\ &= \frac{U_e^2}{U_{ref}^2} \\ &= \frac{P_o - P_s}{P_{c1} - P_{c2}} \quad \text{at freestream} \\ &= C_{p_o} - C_{p_s} \quad \text{at freestream} \end{aligned} \quad (4.3)$$

Combining all the above, the normalized velocity results:

$$\frac{u}{U_e} = \frac{C_{p_o} - C_{p_s}}{C_q} \quad (4.4)$$

Boundary Layer Parameter Calculation

The parameters describing the boundary layer were outlined in section 2.1.1. For computerized data reduction, a numerical method of calculating δ^* and θ must be used. A

simple discretized numerical integration scheme was used:

$$\delta^* = \sum_{i=1}^{\delta/\Delta y} \left[\left(1 - \frac{\bar{u}_{ei} + \bar{u}_{ei-1}}{2} \right) \Delta y \right] \quad (4.5)$$

$$\theta = \sum_{i=1}^{\delta/\Delta y} \left[\frac{\bar{u}_{ei} + \bar{u}_{ei-1}}{2} \left(1 - \frac{\bar{u}_{ei} + \bar{u}_{ei-1}}{2} \right) \Delta y \right] \quad (4.6)$$

Note that the Δy steps are not always equal, as outlined in section 4.2.3.

Pressure Gradient Calculation

Due to the buildup of the boundary layer displacement thickness, δ^* , there was a slight pressure gradient along the test section. Since the differential pressure transducers were referenced to the contraction pressure P_{c2} , the actual total and static pressures were not measured. However, the pressure gradient can be obtained from the variation in the local C_q 's as outlined below. The freestream total pressure can be assumed to remain constant (Bernoulli's equation). Therefore, recalling that $U_e^2 = U_{ref}^2 C_q$ from equation 4.3,

$$\begin{aligned} \frac{dp}{dx} &= -\frac{1}{2} \rho \frac{dU_e^2}{dx} \\ &= -\rho U_e \frac{dU_e}{dx} \\ &= -\rho U_{ref}^2 \sqrt{C_q} \frac{d\sqrt{C_q}}{dx} \\ &= -\frac{1}{2} \rho U_{ref}^2 \frac{dC_q}{dx} \\ &= -(P_{c1} - P_{c2}) \frac{dC_q}{dx} \end{aligned}$$

The defect law pressure gradient parameter, β , can then be expressed as

$$\begin{aligned}\beta &= \frac{\delta^*}{\tau_w} \frac{dp}{dx} \\ &= \frac{\delta^*}{C_f C_q} \frac{dC_q}{dx}\end{aligned}\quad (4.7)$$

where C_f is the skin-friction coefficient.

C_f Calculation

The coefficient of skin friction

$$C_f = \frac{\tau_w}{\frac{1}{2} \rho U_e^2}$$

must be determined in order to calculate the friction velocity, u_τ , which is in turn used to calculate u^+ , y^+ , and k^+ . These latter parameters are needed to plot the law-of-the-wall curve and the intercept-shift curve. There are several ways to obtain C_f , including measuring it directly with the use of force balances, or indirectly with the use of Preston tubes, and also indirectly by equation or graphical means. The following methods are those used in the current work.

▷ Clauser Plot

The Clauser plot is an expanded law-of-the-wall plot with C_f removed from the plot variables. Starting with the law-of-the-wall equation (2.12) and substituting the friction velocity equation (2.5) and rearranging, the equation

$$\frac{u}{U_e} = \left[A \log \frac{y U_e \sqrt{\frac{C_f}{2}}}{\nu} + B \right] \sqrt{\frac{C_f}{2}} \quad (4.8)$$

gives a plot of normalized velocity versus Reynolds number (based on height, y) for different values of C_f , for the log-law region only (Fig. 4.8).

Figure 4.8: A Clauser plot based on experimental smooth-wall data for known values of C_f . Reproduced from Baumann (1999).

To use a Clauser plot, the experimental boundary layer profile is plotted in terms of u/U_e vs. Re_y and where the experimental points match the plots of equation 4.8, they share the same C_f . Since the Clauser plot is based on an expanded law-of-the-wall for **smooth walls**, it can only be used for determining friction coefficients of smooth-wall flow. The Clauser plot can also be used for pressure-gradient flow since the law-of-the-wall is insensitive to pressure-gradient.

▷ Defect-Law Plot

The defect-law plot is an expanded defect-law plot (as in Fig. 2.4) with C_f removed from the plot variables. Starting with the defect-law equation (2.6) and substituting the friction velocity equation (2.5) and rearranging, the equation

$$\frac{u}{U_e} = \left[A \log \frac{y}{\delta} + D \right] \sqrt{\frac{C_f}{2}} + 1 \quad (4.9)$$

gives a plot of normalized velocity versus normalized boundary layer height for different values of C_f , for the log-law region only. Alternatively, a plot of experimental curves for known C_f 's can be used over the whole outer region, that is, up to $y/\delta = 1.0$ (Fig. 4.9).

Figure 4.9: Defect-law plot based on experimental data (curved lines) and defect-law equation (straight lines) for known values of C_f . Note: $\beta = 0$.

To use a defect-law plot, the experimental boundary layer profile is plotted in terms of u/U_e vs. y/δ and where the experimental points match the given profiles, they share the same C_f . The defect-law plot can be used for smooth or rough surfaces, as the defect-law applies to both. When a pressure gradient is involved however, the appropriate defect plot

must be used, corresponding to the actual value of the pressure gradient parameter, β .

▷ Momentum Integral Method

The momentum integral method, developed by von Karman, is based on applying the x-wise momentum equation to a control volume encompassing a vertical slice of length dx of the boundary layer. Assuming two-dimensional flow, the resulting equation is

$$\frac{C_f}{2} = \frac{d\theta}{dx} + (2 + H) \frac{\theta}{U_e} \frac{dU_e}{dx} \quad (4.10)$$

When the pressure gradient is zero, equation 4.10 reduces to

$$\frac{C_f}{2} = \frac{d\theta}{dx} \quad (4.11)$$

The pressure gradient term in equation 4.10 can alternatively be expressed as

$$\frac{dU_e}{dx} = -\frac{1}{\rho U_e} \frac{dp}{dx}$$

giving

$$\frac{C_f}{2} = \frac{d\theta}{dx} - (2 + H) \frac{\theta}{\rho U_e^2} \frac{dp}{dx} \quad (4.12)$$

or using equation 4.3 in equation 4.10:

$$\frac{C_f}{2} = \frac{d\theta}{dx} + (2 + H) \frac{\theta}{\sqrt{C_q}} \frac{d\sqrt{C_q}}{dx} \quad (4.13)$$

▷ Equation Methods

The logarithmic defect-law equation (2.6) can be solved for C_f :

$$C_f = 2 \left[\frac{\frac{u}{U_e} - 1}{A \log \frac{y}{\delta} + D} \right]^2$$

By calculating this C_f for each boundary layer coordinate $(u/U_e, y/\delta)$, a graph of C_f vs. y/δ can be obtained. Since the defect-law plot can be used by matching the experimental data points to the defect-law equation lines (see “Defect-Law Plot” section above), the friction coefficients should be equal along the portion of the defect-law plot where the log-law applies. This gives a horizontal line of C_f ’s on the graph of C_f vs. y/δ and an average can be taken (to account for scatter). In essence, this is just a calculation method versus the graphical method of using the defect-law plot.

Alternatively, if the shift-curve intercept C is known for a specific roughness k , then the law-of-the-wall (equation 2.12) and the intercept-shift equation (2.18) can be combined with the defect law (equation 2.6) to give the following skin friction law

$$\sqrt{\frac{2}{C_f}} = A \log \left(\frac{\delta}{k} \right) - D + B - C \quad (4.14)$$

which applies only for fully rough flow. From this one can see the value of determining a correlation for the shift-curve intercept, C .

Curve Fitting Techniques

In order to calculate the pressure gradient term (in whatever form) for use in the β parameter or in the momentum integral method, a curve fit must be used to describe the variation of C_q . Also, the change in momentum thickness, θ , must be described. This is a more involved procedure than it would first seem.

For incompressible flat-plate zero-pressure gradient turbulent boundary layers, Blasius gives the following variation of boundary layer height and C_f with x -distance (Anderson (1991)):

$$\delta = \frac{0.37x}{Re_x^{1/5}}$$

$$C_f = \frac{0.0148}{Re_x^{1/5}}$$

If δ grows as the 5th root of x -distance, then so too should δ^* and θ (or at least follow the trend). Since the buildup of δ^* causes a slightly favourable pressure gradient along the wind tunnel test section, and thus an increase in U_e with x -distance, then the change in C_q should be related to the change in δ^* and thus δ . The derivative of C_q would follow an inverse trend to that shown by C_q itself, the slope starting out high and tapering off, but not in a linear fashion. Similarly, since C_f is directly related to the change in θ (eqn. 4.11), θ should follow the trend of C_f , which also varies to the 5th root.

The above behaviour must govern the choice of curve fit. In this respect, a linear curve fit is not applicable, and even a quadratic fit is not applicable since the derivative of a quadratic function is a linear function. Due to some scatter in the data, polynomials are not usable since their derivatives easily deviate from the proper trend. Three separate curve types were tried:

$$\begin{array}{ll} \text{Generalized Hyperbola} & y = a - \frac{b}{(1 + cx)^{\frac{1}{d}}} \\ \text{Bent Hyperbola} & y = a + bx - \sqrt{(c + bx)^2 + d^2} \\ \text{general power law} & y = a + bx^{\frac{1}{c}} \end{array} \quad (4.15)$$

where a , b , c , and d are constants varying with each curve fit. The general power law gave the best and most consistent fits, so it was used to fit both the C_q (or $\sqrt{C_q}$) and θ data. Fig. 4.10 shows an example curve fit. The software CurveExpert 1.3 was used for all curve fits.

Analytically differentiating the power law function gives the derivative of θ or C_q vs. x :

$$y' = \frac{b}{c} x^{\left(\frac{1}{c}-1\right)} \quad (4.16)$$

Figure 4.10: An example of a power law (eqn. 4.15) curve fit for θ vs. x .

Intercept-Shift Calculation

Fig. 4.11 gives an experimental example of the intercept-shift of the law-of-the-wall plot due to roughness. The intercept of the smooth-wall line (5.0) minus the intercept of the log-law portion of the experimental plot gives the shift:

$$\Delta u^+ = B_{smooth} - B_{exp}. \quad (4.17)$$

One can also combine the defect law with the law-of-the-wall and solve for B_{exp} :

$$B_{exp.} = -A \log \left(\frac{\rho U_e \delta}{\mu} \right) \sqrt{\frac{C_f}{2}} + D + \sqrt{\frac{2}{C_f}} \quad (4.18)$$

Figure 4.11: An example of a law-of-the-wall plot showing the shift due to roughness.

k^+ , C-Intercept Calculation

The roughness Reynolds number k^+ is calculated using equation 2.19. The roughness height k was taken as either S_q or S_z , for reasons that will become clear later.

Once the shifts and roughness Reynolds numbers are calculated, the shift curve can be plotted, as shown in Fig. 2.9. In order to find the C -intercept, a logarithmic curve fit was applied to the data points:

$$\Delta u^+ = A \log k^+ + C \quad (4.19)$$

An example plot is shown in Fig. 4.12.

Figure 4.12: An example of a shift curve with a logarithmic curve fit to experimental data.

4.2.6 Boundary Layer Code Analysis

An existing Carleton University boundary layer calculation code based on the Head and Patel (1969) entrainment method was used to check/compare the experimental boundary layer behaviour and C_f values determined from the momentum integral method.

The input for the code includes experimental values of H , θ , Re , and U_e for an initial starting point x . The roughness parameters k and λ (after Dvorak (1969)) are input as well. In this case S_z was used for k and λ was iterated manually until the calculated C_f for station 2 matched the experimental C_f at this station. Calculations were started at station 2 since at station 1 the boundary layer was probably not yet at equilibrium with the surface

roughness (see section 6.1.3). Pressure gradient information was input in terms of values of U_e at the measurement stations.

The output of the code included δ , δ^* , θ , H , and C_f vs. x -distance. See Appendix G for sample input and output files.

4.3 Surface Metrology Apparatus and Methods

4.3.1 Surface Scanning Preparation

Before the surfaces can be scanned, the height range, vertical and lateral resolutions, and scan area must be decided on, with any budgetary constraints considered. These parameters were determined for each surface through careful examination of the surfaces using photographic microscopes, magnifying and measuring tools, and proper judgement (as guided by the principles set out in section 3.2.1). The vertical resolution was determined by the height range since the scanning sensors are chosen by height range requirements (see section 4.3.2). Table 4.2 lists the various parameters and their values.

Table 4.2: Surface Scan Parameters

surface	scan area (mm x mm)	lateral res. (μm)	vertical res. (μm)	total points
Blue Formica	10 x 10	40	3.8	63001
Coarse Shingle	12 x 12	40	10.2	90601
Coarse Spackle	45 x 45	150	20.3	90601
Fine Shingle	8 x 8	25	3.8	103041
Fine Spackle	20 x 20	100	10.2	40401
Rust	10 x 10	25	2	160801
Tar Paper	8 x 8	20	2	160801
Vinyl	13 x 13	50	3.8	68121
White Formica	10 x 10	40	3.8	63001

The surfaces also had to be inspected for any form or waviness that would have to be filtered out to capture the roughness. This was not a problem for any surface except Rust. The Rust sample had some curvature (form) to it, which was later removed (see section 5.1.6). The other surfaces were fabricated in such a way as to have a flat base upon which the roughness sat, so form or waviness was not a concern.

4.3.2 Surface Scanning Device

The surface scans were contracted out to ASC International (now Laser Vision Technologies - www.lvtechnologies.com). A laser triangulation method was used with a CyberOptics CyberScan 212 system (www.cyberoptics.com).

The system incorporates a sturdy machined flat base for mounting of the samples. The lateral resolution (in x and y) is $1\mu\text{m}$ and the diameter of the laser spot is $5\mu\text{m}$. The software can acquire a maximum of 4000 steps per scan length. The datum is the plane along which the laser sensor sits. 10 readings per second can be taken. The raw data is exported in ASCII format, in a table of x - y - z values. Table 4.3 gives a list of the available laser sensors and their height ranges and vertical resolutions.

Table 4.3: CyberScan 212 Laser Sensors

sensor	height range (mm)	vertical res. (μm)
PRS-15	0.15	0.38
PRS-30	0.30	0.76
PRS-40	0.41	1.0
PRS-80	0.81	2.0
PRS-150	1.5	3.8
PRS-400	4.1	10.2
PRS-800	8.1	20.3
PRS-1600	16.3	40.6

Since there was no form or waviness in the surfaces, no upper wavelength cut-off (λ_c) was used, although typical values are in the range of 0.8 to 8.0 mm. A low-pass filter with a lower wavelength cut-off $\lambda_s = 0.002667$ mm was used to remove white noise from the scans (see Fig. 3.8). Standard values of λ_s are usually given as $\lambda_c/300$.

4.3.3 Roughness Parameter Calculation

Since the present work concerns surface maps and not 2D profiles, only the 3D roughness parameters were calculated. The raw ASCII data (see Appendix C for sample) was imported into a graphing and analysis program (Mathsoft Axum 5.0) and modified. The original total number of data points was determined from the scan area and lateral resolution, but once noise-filtering was performed on the data, an outer perimeter of points two-wide was eliminated, giving a net scan area. This outer perimeter showed up as garbage values in the raw data, and was removed, the x and y values being re-zeroed. Using the discrete surface representation outlined in section 3.3.2, an effective scan area was made. Table 4.4 lists the modified scan areas.

Table 4.4: Surface Scan Modified Areas

surface	original scan area (mm x mm)	points per side	total points	net scan area (mm x mm)	points per side	total points	effective scan area (mm x mm)
Blue Formica	10 x 10	251	63001	9.84 x 9.84	247	61009	9.88 x 9.88
Coarse Shingle	12 x 12	301	90601	11.84 x 11.84	297	88209	11.88 x 11.88
Coarse Spackle	45 x 45	301	90601	44.40 x 44.40	297	88209	44.55 x 44.55
Fine Shingle	8 x 8	321	103041	7.90 x 7.90	317	100489	7.925 x 7.925
Fine Spackle	20 x 20	201	40401	19.60 x 19.60	197	38809	19.70 x 19.70
Rust	10 x 10	401	160801	9.90 x 9.90	397	157609	9.925 x 9.925
Tar Paper	8 x 8	401	160801	7.92 x 7.92	397	157609	7.94 x 7.94
Vinyl	13 x 13	261	68121	12.80 x 12.80	257	66049	12.85 x 12.85
White Formica	10 x 10	251	63001	9.84 x 9.84	247	61009	9.88 x 9.88

Once the raw data was conditioned as above, it was exported as an ASCII file (sample in Appendix D) for input to computer programs which calculate the various roughness parameters. These codes (written by the author in C) are listed in Appendix E. Appendix F contains sample output files from these programs.

The first code found the mean height and then zeroed the individual heights around this mean plane. This code calculated the parameters S_a , S_q , S_{sk} , S_{ku} , S_p , S_v , S_t , S_k , N_{VOL} , $S\Delta_q$, and $S\lambda_q$, along with a single-quadrant $\rho(\tau_x, \tau_y)$ data set. From this $\rho(\tau_x, \tau_y)$ data set, correlation lengths were calculated for several directions, starting along the τ_x line and sweeping through a 90° arc up to the τ_y direction. These correlation lengths were output to a spreadsheet program (Microsoft Excel 2000), corrected and interpolated if necessary, and the shortest correlation length S_{al} was found. Note that the correlation length is defined in the present work as the length $\sqrt{\tau_x^2 + \tau_y^2}$ where $\rho(\tau_x, \tau_y) = 0.2$ (i.e. $a = 0.2$ for equation 3.61).

The parameters involving summits and valleys were found manually through the use of the graphing and analysis program. The summits and valleys were isolated on an overhead-view topographical plot by eliminating all points below S_a for summits and above $-S_a$ for valleys. Fig. 4.13 illustrates this method.

As outlined in Chapter 3, the choice of what defines an isolated summit or valley is somewhat subjective. In the present work, a bounding box of 10x10 data points was made around a summit/valley, and anything outside this could be counted as another isolated feature. If a long ridge was present, it was counted as more than one feature, the exact number depending on the size and judgment of the author. P_{cA} and V_{cA} were determined in this manner.

The parameters S_{pm} and S_{vm} were evaluated using the 5 highest summits and 5 lowest valleys. These were found manually using the above technique. S_z was then calculated

Figure 4.13: An example of a topographical plot showing isolated valleys. The darker the points the lower the height of those points.

manually based on these two parameters. The parameters involving S_{pk} and S_{vk} were calculated based on S_{pm} and S_{vm} , and were calculated manually.

The second code performed the fractal analysis on the surfaces. As discussed in section 3.3.2, the scale of observation is the area of an individual triangular element fitted to the surface. The total number of triangles must be $2 \cdot 2^{2r}$, a power of two, therefore the number of surface data points must reflect this. Also, a sufficient number of scales of observation is necessary to obtain a fractal log-log plot as shown in Fig. 3.15. This was achieved by mirroring the surface several times to generate a larger virtual surface, and then cut off at two sides to make the number of data points a power of two.

Once the mirrored surface was generated by the code, S_a was re-calculated to verify that the mirrored surface's properties did not deviate from the original's (by the act of cutting off the two ends). The difference was shown to be negligible. The code then performed the fractal analysis, generating the fractal log-log plot. This plot was then imported into a spreadsheet program and the linear region was manually determined, as was the slope and D_{as} . S_{dr} was manually determined as the highest relative area value.

4.4 Method Used to Develop a Law-of-the-Wall Shift Correlation

4.4.1 Analysis of Roughness Parameters For Suitability in Shift-Correlation Model

Once all the roughness parameters were calculated, they were studied to determine their suitability for use in a correlation model. Aspects such as statistical significance, repeatability, and redundancy were considered. In the end, only the most descriptive, independent, and robust parameters made the cut, so to speak. These final parameters were then used to find a correlation, with some further elimination of parameters. The results of this work will be outlined in Chapter 6.

4.4.2 Correlation Model

The correlation model used is based on finding a combination of roughness parameters that adequately define the roughness. As discussed in Chapter 2, the roughness parameter k is, by itself, insufficient for describing the roughness. That is, the law-of-the-wall shift Δu^+ of equation 2.18 depends not only on k or k^+ , but also on some other parameter

or parameters that depend on the roughness configuration. The objective is to identify suitable parameters and establish a correlation between the law-of-the-wall shift, C , and k^+ , and these additional parameters. Once that relation is found, it should be possible to incorporate these additional roughness parameters into a new effective-height, or total-roughness, parameter, denoted k_t^+ .

If the shift relation is currently written as (equation 2.18)

$$\Delta u^+ = A \log k^+ + C$$

where C is a function of the roughness configuration, the goal is to have

$$\Delta u^+ = A \log k_t^+ + E \quad (4.20)$$

where E is a constant. Therefore C must be of the form

$$C = A \log k_2 + E \quad (4.21)$$

so that

$$k_t^+ = k^+ k_2 \quad (4.22)$$

or

$$k_t = k k_2 \quad (4.23)$$

where k_2 is a function of one or more roughness parameters other than k . k_2 must be non-dimensional in order to keep k_t^+ non-dimensional.

The form of k_2 was chosen to be

$$k_2 = (aX^b + cY^d + eZ^f + \dots)^g \quad (4.24)$$

where X , Y , and Z represent roughness parameters or combinations thereof, and a , b , c etc. are constants. To be consistent with past work, k was chosen to be a dimensional height parameter. k_t then has the form

$$k_t = k (aX^b + cY^d + eZ^f + \dots)^g \quad (4.25)$$

Fig. 4.14 illustrates the goal of the correlation.

Figure 4.14: The goal of the correlation model is to collapse the curves on the left onto one “universal” curve on the right, through use of a “total” roughness parameter, k_t^+ .

4.4.3 Model Coefficient Determination

The constants a , b , c etc. were planned to be determined with the use of a computer program designed to vary the coefficients and fit a logarithmic curve through the shift points $(\Delta u^+, k^+)$, the best curve fit determining the optimal coefficients. The coefficients could be iterated step-wise, or more intelligently through use of a genetic algorithm for example. A spreadsheet was used to graph the shift points, and vary the coefficients to get a sense of

the changes that occurred. This approach proved to be quite effective and so the coefficients were determined by simple trial and error, guided by some judgement.

4.4.4 Relation to Standard Sand-Grain Roughness

The Prandtl-Schlichting curve for close-packed sand-grain roughness is often used as a standard in fluid mechanics. Other roughnesses are often related to it, that is, they are assigned a suitable equivalent standard-sand roughness height, k_s , since many CFD programs have k_s as the roughness input. This requires a method for determining suitable values of k_s for any given roughness configuration. Since the proposed correlation model gives a single curve, it can easily be related to the sand-grain curve through a simple multiplicative constant, that is

$$k_t = a k_s \quad (4.26)$$

For the fully-rough region the shift curve equation for sand-grain roughness is (Grigson (1984))

$$\Delta u^+ = A \log k_s^+ - 4.0 \quad (4.27)$$

The constant a in equation 4.26 can be determined by combining the shift curve equation for standard-sand roughness and that for stochastic roughness:

$$\begin{aligned} A \log k_s^+ - 4.0 &= A \log k_t^+ + E \\ \log \frac{k_s}{k_t} &= \frac{E + 4.0}{A} \\ \frac{k_s}{k_t} &= 10^{\frac{E+4.0}{A}} \\ k_s &= k_t \left[10^{\frac{E+4.0}{A}} \right] \end{aligned} \quad (4.28)$$

4.4.5 Relation to Waigh's Spacing Parameter

In the sparse spacing regime, Waigh's shift curve intercept relation is

$$\begin{aligned} C &= -5.75 \log \left(\lambda \left(\frac{k}{b_m} \right)^{0.55} \left(\frac{A_W}{A_F} \right)^{1.38} \right) + 5.78 \\ &= -5.75 \log \lambda_{eff} + 5.78 \end{aligned}$$

as noted in section 2.3.4. The constant 5.75 is close to the constant $A = 5.62$ (the difference due to curve fit leeway), and will be taken as 5.62, as it should be equal, as outlined by the derivation in section 4.4.2 (equation 4.21, where λ_{eff} is analogous to k_2). If equation 4.21 and Waigh's relation are equated, the following results:

$$\begin{aligned} A \log k_2 + E &= -A \log \lambda_{eff} + 5.78 \\ A \log (k_2 \lambda_{eff}) &= 5.78 - E \\ k_2 \lambda_{eff} &= 10^{\frac{5.78-E}{A}} \\ \lambda_{eff} &= \frac{10^{\frac{5.78-E}{A}}}{k_2} \end{aligned} \tag{4.29}$$

The inverse relationship between λ_{eff} and k_2 makes physical sense, for as k_2 increases, the roughness level increases, and λ_{eff} decreases. However a decrease in λ_{eff} for the sparse spacing regime results in an increase in roughness as well. This derivation works only for the sparse spacing regime, but since the dense spacing regime only spans a spacing parameter range of $1 \sim 10$ and the sparse spacing regime spans a range of $10 \sim 1000$, it is deemed applicable in most cases.

Chapter 5

Experimental Surfaces

At the outset of this thesis, it was necessary to find suitable surfaces that would encompass the range of roughness required to give a useful variation in intercept-shifts. It was decided that a few surfaces would be mildly rough, some would have medium roughness, and some would be very rough. It was also planned that the surfaces would try to reflect engineering surfaces if possible. Nine surfaces were chosen as an adequate number, since time and resources did not allow more than that. A smooth surface was also tested to give a baseline and to validate the experiments.

The surfaces were required to be robust, needing to withstand 50 m/s airflow, since they had to be scanned after the wind tunnel tests. If they were damaged during the tests or during shipping to the contracted scanners, then the scanned surfaces would not reflect those that were tested. The surfaces were also chosen to be isotropic, as the analysis would be much simpler (see section 3.1.2).

The following sections present the surfaces and their roughness parameters. Note that the surface maps are not all scaled the same in the height direction. All the surfaces were

mounted or fabricated upon 7/16" MDF Ranger board, a very fine wood particle board with a high density and good rigidity. This board was chosen for its resistance to deformation, and for ease of mounting/bonding the surfaces.

5.1 Surfaces

5.1.1 Blue Formica

Formica is a brand of plastic laminate veneer that counter tops are usually made of. This surface is designated "Blue" since the actual sample is blue, and to distinguish it from the White Formica surface. Qualitatively speaking, this surface is only slightly rough, with low rounded summits with a smooth micro-finish. The model number is "1762-41 Midnight Dust, Sand Finish". It was acquired from a local hardware store. It was bonded to the MDF board with a water-based contact cement.

Digital Representation

Table 5.1: Blue Formica Digital Information

step size (μm)	vertical res. (μm)	effective scan area (mm x mm)	total points	mirrored scan area (mm x mm)	total points
40	3.8	9.88 x 9.88	61009	41.0 x 41.0	1050625

The following figures depict the surface in various ways. Fig. 5.1 is an overhead shaded-relief view, giving a close approximation to an actual photograph of the surface. Fig. 5.2 is an overhead topographical view, giving a good indication of the surface's slopes and dominant features. Note the gentle rolling nature of the surface. Fig. 5.3 contains a meshed surface map and also a colour-contoured map that better distinguishes the maxima.

Figure 5.1: A shaded-relief overhead view of the Blue Formica surface.

Figure 5.2: A topographical overhead view of the Blue Formica surface.

Figure 5.3: Top: A meshed surface map of the Blue Formica surface. Bottom: A colour-contoured map. (x, y, z in mm)

Roughness Parameters

The calculated 3D roughness parameters are listed in Table 5.2.

Table 5.2: Blue Formica 3D Roughness Parameters

Blue Formica		note: any dimensional parameters given in terms of mm			
parameter	value	parameter	value	parameter	value
S_a	0.02512	S_{vm}	-0.06955	P_{cA}	0.461
S_q	0.03131	S_z	0.16436	V_{cA}	0.318
S_{sk}	0.41614	S_k	0.05023	N_{VOL}	0.02963
S_{ku}	2.83323	S_{pk}	0.06969	S_{dr}	1.00629
S_p	0.10605	S_{vk}	-0.04443	$S\Delta_q$	0.11239
S_v	-0.07756	S_{pk}/S_k	1.38742	$S\lambda_q$	1.75042
S_t	0.18361	S_{vk}/S_k	0.88453	D_{as}	2.00236
S_{pm}	0.09481	S_{pk}/S_{vk}	1.56853	S_{al}	0.80717

Figure 5.4: A meshed side view of the single-quadrant areal autocorrelation function for the Blue Formica surface.

Fig. 5.4 is a side view of the areal autocorrelation function, and Fig. 5.5 is an isometric view of the same function. Since $\rho(\tau_x, \tau_y)$ drops to zero well within the sample side lengths for all the surfaces, it can be said, in general, that the surface sample sizes chosen were adequate. Fig. 5.6 is a 4-quadrant areal autocorrelation function (in this case, simply the single-quadrant function mirrored, to provide an example). Fig. 5.7 contains the plots used in the calculation of P_{cA} and V_{cA} respectively (see section 4.3.3).

Figure 5.5: A colour-contoured isometric view of the single-quadrant areal autocorrelation function for the Blue Formica surface. (τ_x, τ_y in mm)

Figure 5.6: A 4-quadrant areal autocorrelation function of the Blue Formica surface.
(τ_x, τ_y in mm)

Figure 5.7: Top: overhead view of the Blue Formica isolated summits. Bottom: the isolated valleys. ($z > S_a$ for summits; $z < -S_a$ for valleys)

5.1.2 Coarse Shingle

This surface is a roofing sheet, with coarse-grit crushed-slate grains embedded in tar, like a roofing shingle, but a continuous sheet instead. Qualitatively speaking, this surface is quite rough, due to the large grains. The maker is EMCO Building Products Group. The model number is “RR 401 CSA Spec A 123.2-M Type M”. It was acquired from a local hardware store. It was bonded to the MDF board with a water-based contact cement.

Digital Representation

The important digital information is listed in Table 5.3.

Table 5.3: Coarse Shingle Digital Information

step size (μm)	vertical res. (μm)	effective scan area (mm x mm)	total points	mirrored scan area (mm x mm)	total points
40	10.2	11.88 x 11.88	88209	41.0 x 41.0	1050625

The following figures depict the surface in various ways. Fig. 5.8 is an overhead shaded-relief view, giving a close approximation to an actual photograph of the surface. Fig. 5.9 is an overhead topographical view, giving a good indication of the surface’s slopes and dominant features. Note the large flat-topped grains, and the steep drop-offs where the grains are embedded in the tar. There are also some deep valleys between the grains. Fig. 5.10 contains a standard meshed surface map and also a colour-contoured map that better distinguishes the maxima and minima.

Shaded Relief - Topographical View - Black and White

Figure 5.8: A shaded-relief overhead view of the Coarse Shingle surface.

Topographical View - Color Shading Added

Figure 5.9: A topographical overhead view of the Coarse Shingle surface.

Figure 5.10: Top: A meshed surface map of the Coarse Shingle surface. Bottom: A colour-contoured map. (x, y, z in mm)

Roughness Parameters

Table 5.4: Coarse Shingle 3D Roughness Parameters

Coarse Shingle		note: any dimensional parameters given in terms of mm			
parameter	value	parameter	value	parameter	value
S_a	0.14637	S_{vm}	-0.71255	P_{cA}	0.319
S_q	0.18523	S_z	1.15228	V_{cA}	0.411
S_{sk}	-0.60377	S_k	0.29274	$NVOL$	0.16305
S_{ku}	3.33615	S_{pk}	0.29336	S_{dr}	1.53366
S_p	0.50851	S_{vk}	-0.56618	$S\Delta_q$	1.16444
S_v	-0.80784	S_{pk}/S_k	1.00212	$S\lambda_q$	0.99948
S_t	1.31635	S_{vk}/S_k	1.93407	D_{as}	2.20940
S_{pm}	0.43973	S_{pk}/S_{vk}	0.51814	S_{al}	0.54077

Figure 5.11: A colour-contoured isometric view of the single-quadrant areal autocorrelation function for the Coarse Shingle surface. (τ_x, τ_y in mm)

Figure 5.12: Top: overhead view of the Coarse Shingle isolated summits. Bottom: the isolated valleys. ($z > S_a$ for summits; $z < -S_a$ for valleys)

5.1.3 Coarse Spackle

This surface comprises large-scale ceiling spackle paint applied to the MDF board. Qualitatively speaking, this surface is very rough, due to the large spackle peaks. The make and model is “BEHR Premium Plus Ceiling Texture - Popcorn Finish”. It is a flat latex based paint and was applied directly to the MDF board with a large-scale spackle roller, “EZ Paint Canada, Deep Texture Cover”. It was acquired from a local hardware store.

Digital Representation

The important digital information is listed in Table 5.5.

Table 5.5: Coarse Spackle Digital Information

step size (μm)	vertical res. (μm)	effective scan area (mm x mm)	total points	mirrored scan area (mm x mm)	total points
150	20.3	44.55 x 44.55	88209	153.75 x 153.75	1050625

The following figures depict the surface in various ways. Fig. 5.13 is an overhead shaded-relief view, giving a close approximation to an actual photograph of the surface. Fig. 5.14 is an overhead topographical view, giving a good indication of the surface’s slopes and dominant features. Note the large, steep peaks on top of a mostly flat surface, with shallow wide valleys. Fig. 5.15 contains a standard meshed surface map and also a colour-contoured map that better distinguishes the maxima and minima.

Figure 5.13: A shaded-relief overhead view of the Coarse Spackle surface.

Figure 5.14: A topographical overhead view of the Coarse Spackle surface.

Figure 5.15: Top: A meshed surface map of the Coarse Spackle surface. Bottom: A colour-contoured map. (x, y, z in mm)

Roughness Parameters

Table 5.6: Coarse Spackle 3D Roughness Parameters

Coarse Spackle		note: any dimensional parameters given in terms of mm			
parameter	value	parameter	value	parameter	value
S_a	0.45111	S_{vm}	-0.94711	P_{cA}	0.028
S_q	0.57161	S_z	3.66006	V_{cA}	0.032
S_{sk}	0.98050	S_k	0.90222	N_{VOL}	0.53753
S_{ku}	4.28356	S_{pk}	2.41334	S_{dr}	1.30624
S_p	3.09771	S_{vk}	-0.49600	$S\Delta_q$	0.84910
S_v	-0.97893	S_{pk}/S_k	2.67489	$S\lambda_q$	4.22979
S_t	4.07664	S_{vk}/S_k	0.54976	D_{as}	2.12871
S_{pm}	2.71295	S_{pk}/S_{vk}	4.86560	S_{at}	2.02914

Figure 5.16: A colour-contoured isometric view of the single-quadrant areal autocorrelation function for the Coarse Spackle surface. (τ_x, τ_y in mm)

Figure 5.17: Top: overhead view of the Coarse Spackle isolated summits. Bottom: the isolated valleys. ($z > S_a$ for summits; $z < -S_a$ for valleys)

5.1.4 Fine Shingle

This surface is a roofing sheet, with fine sand grains embedded in tar, like a roofing shingle, but a continuous sheet instead. Qualitatively speaking, this surface is quite rough, due to the large number of sharp sand grains. The maker is EMCO Building Products Group. The model number is “RR 303 CSA Spec A 123.2-M Type S”. It was acquired from a local hardware store. It was bonded to the MDF board with a water-based contact cement.

Digital Representation

The important digital information is listed in Table 5.7.

Table 5.7: Fine Shingle Digital Information

step size (μm)	vertical res. (μm)	effective scan area (mm x mm)	total points	mirrored scan area (mm x mm)	total points
25	3.8	7.925 x 7.925	100489	25.625 x 25.625	1050625

The following figures depict the surface in various ways. Fig. 5.18 is an overhead shaded-relief view, giving a close approximation to an actual photograph of the surface. Fig. 5.19 is an overhead topographical view, giving a good indication of the surface’s slopes and dominant features. Note the large number of small, yet jagged, peaks. Fig. 5.20 contains a standard meshed surface map and also a colour-contoured map that better distinguishes the maxima and minima.

Figure 5.18: A shaded-relief overhead view of the Fine Shingle surface.

Figure 5.19: A topographical overhead view of the Fine Shingle surface.

Figure 5.20: Top: A meshed surface map of the Fine Shingle surface. Bottom: A colour-contoured map. (x, y, z in mm)

Roughness Parameters

Table 5.8: Fine Shingle 3D Roughness Parameters

Fine Shingle		note: any dimensional parameters given in terms of mm			
parameter	value	parameter	value	parameter	value
S_a	0.07811	S_{vm}	-0.59092	P_{cA}	3.821
S_q	0.10703	S_z	1.14227	V_{cA}	3.582
S_{sk}	-0.05572	S_k	0.15622	N_{VOL}	0.09208
S_{ku}	5.11956	S_{pk}	0.47324	S_{dr}	2.20301
S_p	0.69327	S_{vk}	-0.51281	$S\Delta_q$	1.77825
S_v	-0.65992	S_{pk}/S_k	3.02932	$S\lambda_q$	0.37817
S_t	1.35319	S_{vk}/S_k	3.28261	D_{as}	2.59343
S_{pm}	0.55135	S_{pk}/S_{vk}	0.92284	S_{al}	0.14341

Figure 5.21: A colour-contoured isometric view of the single-quadrant areal autocorrelation function for the Fine Shingle surface. (τ_x, τ_y in mm)

Figure 5.22: Top: overhead view of the Fine Shingle isolated summits. Bottom: the isolated valleys. ($z > S_a$ for summits; $z < -S_a$ for valleys)

5.1.5 Fine Spackle

This surface comprises small-scale wall spackle paint applied to the MDF board. Qualitatively speaking, this surface is quite rough, due to the spackle peaks. The make and model is “BEHR Premium Plus Sand Wall Texture - Sand Finish”. It is a flat latex based paint and was applied directly to the MDF board with a small-scale spackle roller, “Bennett, Heavy Texture Stucco Roller”. It was acquired from a local hardware store.

Digital Representation

The important digital information is listed in Table 5.9.

Table 5.9: Fine Spackle Digital Information

step size (μm)	vertical res. (μm)	effective scan area (mm x mm)	total points	mirrored scan area (mm x mm)	total points
100	10.2	19.70 x 19.70	38809	102.5 x 102.5	1050625

The following figures depict the surface in various ways. Fig. 5.23 is an overhead shaded-relief view, giving a close approximation to an actual photograph of the surface. Fig. 5.24 is an overhead topographical view, giving a good indication of the surface’s slopes and dominant features. Note the medium, slightly rounded peaks, with shallow wide valleys. Fig. 5.25 contains a standard meshed surface map and also a colour-contoured map that better distinguishes the maxima and minima.

Figure 5.23: A shaded-relief overhead view of the Fine Spackle surface.

Figure 5.24: A topographical overhead view of the Fine Spackle surface.

Figure 5.25: Top: A meshed surface map of the Fine Spackle surface. Bottom: A colour-contoured map. (x, y, z in mm)

Roughness Parameters

Table 5.10: Fine Spackle 3D Roughness Parameters

Fine Spackle		note: any dimensional parameters given in terms of mm			
parameter	value	parameter	value	parameter	value
S_a	0.14901	S_{vm}	-0.35023	P_{cA}	0.070
S_q	0.18552	S_z	1.06090	V_{cA}	0.093
S_{sk}	0.55498	S_k	0.29803	N_{VOL}	0.17437
S_{ku}	3.24210	S_{pk}	0.56166	S_{dr}	1.07808
S_p	0.78272	S_{vk}	-0.20122	$S\Delta_q$	0.40713
S_v	-0.38231	S_{pk}/S_k	1.88458	$S\lambda_q$	2.86316
S_t	1.16503	S_{vk}/S_k	0.67517	D_{as}	2.03428
S_{pm}	0.71067	S_{pk}/S_{vk}	2.79127	S_{al}	1.38005

Figure 5.26: A colour-contoured isometric view of the single-quadrant areal autocorrelation function for the Fine Spackle surface. (τ_x, τ_y in mm)

Figure 5.27: Top: overhead view of the Fine Spackle isolated summits. Bottom: the isolated valleys. ($z > S_a$ for summits; $z < -S_a$ for valleys)

5.1.6 Rust

This surface is a rusted steel plate. The rust was adequately uniform over the whole plate. Qualitatively speaking, this surface is slightly rough, due to the numerous small-scale peaks. It was acquired from a local scrap yard, and mounted to the MDF board.

Digital Representation

The important digital information is listed in Table 5.11.

Table 5.11: Rust Digital Information

step size (μm)	vertical res. (μm)	effective scan area (mm x mm)	total points	mirrored scan area (mm x mm)	total points
25	2	9.925 x 9.925	157609	51.255 x 51.255	4198401

The following figures depict the surface in various ways. Fig. 5.28 is an overhead shaded-relief view, giving a close approximation to an actual photograph of the surface. Fig. 5.29 is an overhead topographical view, giving a good indication of the surface's slopes and dominant features. Note the large number of small-scale features, and numerous small peaks. As mentioned previously, the Rust sample had some form to it (due to a slight bend in the steel plate sample) that needed to be removed. This was accomplished by fitting a plane to the surface by the linear least-squares method and using this plane as the height datum. This was an adequate solution since the bend in the plate was small, and the actual sampled area was much smaller than the plate sample, so that the curvature, but not the slope, was negligible. Fig. 5.30 contains the form-error meshed surface map and the corrected map. Fig. 5.31 is a colour-contoured map of the form-error surface, and Fig. 5.32 is the corrected surface.

Figure 5.28: A shaded-relief overhead view of the Rust surface.

Figure 5.29: A topographical overhead view of the Rust surface.

Figure 5.30: Top: A meshed surface map of the form-error Rust surface. Bottom: the corrected surface. (x, y, z in mm)

Figure 5.31: A colour-contoured surface map of the form-error Rust surface. (x, y, z in mm)

Figure 5.32: A colour-contoured surface map of the corrected Rust surface. (x, y, z in mm)

Roughness Parameters

Table 5.12: Rust 3D Roughness Parameters

Rust		note: any dimensional parameters given in terms of mm			
parameter	value	parameter	value	parameter	value
S_a	0.01370	S_{vm}	-0.05770	P_{cA}	2.497
S_q	0.01791	S_z	0.15458	V_{cA}	2.304
S_{sk}	1.07265	S_k	0.02740	N_{VOL}	0.01656
S_{ku}	4.94372	S_{pk}	0.08318	S_{dr}	1.02127
S_p	0.11258	S_{vk}	-0.04400	$S\Delta_q$	0.20902
S_v	-0.06520	S_{pk}/S_k	3.03577	$S\lambda_q$	0.53843
S_t	0.17778	S_{vk}/S_k	1.60584	D_{as}	2.01020
S_{pm}	0.09688	S_{pk}/S_{vk}	1.89045	S_{al}	0.40604

Figure 5.33: A colour-contoured isometric view of the single-quadrant areal autocorrelation function for the Rust surface. (τ_x, τ_y in mm)

Figure 5.34: Top: overhead view of the Rust isolated summits. Bottom: the isolated valleys. ($z > S_a$ for summits; $z < -S_a$ for valleys)

5.1.7 Tar Paper

This surface is a construction tar paper. It is a matrix of fibres and tar, which gives it a moderate roughness, due to some medium scale rounded bumps, but more so from the small scale surface fibres. The maker is EMCO Building Products Group. The model number is “BP RF403 #15 Asphalt Felt Plain”. It was acquired from a local hardware store. It was bonded to the MDF board with a water-based contact cement.

Digital Representation

The important digital information is listed in Table 5.13.

Table 5.13: Tar Paper Digital Information

step size (μm)	vertical res. (μm)	effective scan area (mm x mm)	total points	mirrored scan area (mm x mm)	total points
20	2	7.94 x 7.94	157609	40.98 x 40.98	4198401

The following figures depict the surface in various ways. Fig. 5.35 is an overhead shaded-relief view, giving a close approximation to an actual photograph of the surface. Fig. 5.36 is an overhead topographical view, giving a good indication of the surface’s slopes and dominant features. Note the large number of small scale features, and the many fibres laid across the surface. Fig. 5.37 contains a standard meshed surface map and also a colour-contoured map that better distinguishes the maxima and minima.

Shaded Relief - Topographical View - Black and White

Figure 5.35: A shaded-relief overhead view of the Tar Paper surface.

Topographical View - Color Shading Added

Figure 5.36: A topographical overhead view of the Tar Paper surface.

Figure 5.37: Top: A meshed surface map of the Tar Paper surface. Bottom: A colour-contoured map. (x, y, z in mm)

Roughness Parameters

Table 5.14: Tar Paper 3D Roughness Parameters

Tar Paper		note: any dimensional parameters given in terms of mm			
parameter	value	parameter	value	parameter	value
S_a	0.04298	S_{vm}	-0.13676	P_{cA}	1.079
S_q	0.05293	S_z	0.30589	V_{cA}	5.457
S_{sk}	0.37963	S_k	0.08595	N_{VOL}	0.05030
S_{ku}	2.77425	S_{pk}	0.12615	S_{dr}	1.41576
S_p	0.19946	S_{vk}	-0.09378	$S\Delta_q$	0.99323
S_v	-0.14061	S_{pk}/S_k	1.46774	$S\lambda_q$	0.33484
S_t	0.34007	S_{vk}/S_k	1.09110	D_{as}	2.20890
S_{pm}	0.16913	S_{pk}/S_{vk}	1.34517	S_{al}	0.85659

Figure 5.38: A colour-contoured isometric view of the single-quadrant areal autocorrelation function for the Tar Paper surface. (τ_x, τ_y in mm)

Figure 5.39: Top: overhead view of the Tar Paper isolated summits. Bottom: the isolated valleys. ($z > S_a$ for summits; $z < -S_a$ for valleys)

5.1.8 Vinyl

This surface is a residential grade vinyl floor covering. Qualitatively speaking, this surface is not very rough, with a plateau and shallow crater surface with very small scale dimples, which give traction for the floor, but have little effect on the aerodynamics. The make and model is “Toughguard AGSTARSTEP-000”. It was acquired from a local floor coverings store. It was bonded to the MDF board with a water-based contact cement.

Digital Representation

The important digital information is listed in Table 5.15.

Table 5.15: Vinyl Digital Information

step size (μm)	vertical res. (μm)	effective scan area (mm x mm)	total points	mirrored scan area (mm x mm)	total points
50	3.8	12.85 x 12.85	66049	51.25 x 51.25	1050625

The following figures depict the surface in various ways. Fig. 5.40 is an overhead shaded-relief view, giving a close approximation to an actual photograph of the surface. Fig. 5.41 is an overhead topographical view, giving a good indication of the surface’s slopes and dominant features. Fig. 5.42 contains a standard meshed surface map and also a colour-contoured map that better distinguishes the maxima and minima.

Shaded Relief - Topographical View - Black and White

Figure 5.40: A shaded-relief overhead view of the Vinyl surface.

Topographical View - Color Shading Added

Figure 5.41: A topographical overhead view of the Vinyl surface.

Figure 5.42: Top: A meshed surface map of the Vinyl surface. Bottom: A colour-contoured map. (x, y, z in mm)

Roughness Parameters

Table 5.16: Vinyl 3D Roughness Parameters

Vinyl		note: any dimensional parameters given in terms of mm			
parameter	value	parameter	value	parameter	value
S_a	0.02318	S_{vm}	-0.09737	P_{cA}	0.509
S_q	0.02940	S_z	0.17309	V_{cA}	0.363
S_{sk}	-0.51526	S_k	0.04635	N_{VOL}	0.02603
S_{ku}	3.18426	S_{pk}	0.05254	S_{dr}	1.01865
S_p	0.08004	S_{vk}	-0.07419	$S\Delta_q$	0.19516
S_v	-0.11443	S_{pk}/S_k	1.13355	$S\lambda_q$	0.94637
S_t	0.19447	S_{vk}/S_k	1.60065	D_{as}	2.00885
S_{pm}	0.07572	S_{pk}/S_{vk}	0.70818	S_{al}	1.22606

Figure 5.43: A colour-contoured isometric view of the single-quadrant areal autocorrelation function for the Vinyl surface. (τ_x, τ_y in mm)

Figure 5.44: Top: overhead view of the Vinyl isolated summits. Bottom: the isolated valleys. ($z > S_a$ for summits; $z < -S_a$ for valleys)

5.1.9 White Formica

Formica is a brand of plastic laminate veneer that counter tops are usually made of. This surface is designated “White” since the actual sample is white, and to distinguish it from the Blue Formica surface. Qualitatively speaking, this surface is only slightly rough, with low rounded summits with a smooth micro-finish. The model number is “Wilsonart 4604-50 Pearl Sand, Premium #50 Finish”. It was acquired from a local hardware store. It was bonded to the MDF board with a water-based contact cement.

Digital Representation

The important digital information is listed in Table 5.17.

Table 5.17: White Formica Digital Information

step size (μm)	vertical res. (μm)	effective scan area (mm x mm)	total points	mirrored scan area (mm x mm)	total points
40	3.8	9.88 x 9.88	61009	41.0 x 41.0	1050625

The following figures depict the surface in various ways. Fig. 5.45 is an overhead shaded-relief view, giving a close approximation to an actual photograph of the surface. Fig. 5.46 is an overhead topographical view, giving a good indication of the surface’s slopes and dominant features. Note the gentle rolling nature of the surface. Fig. 5.47 contains a standard meshed surface map and also a colour-contoured map that better distinguishes the maxima and minima.

Shaded Relief - Topographical View - Black and White

Figure 5.45: A shaded-relief overhead view of the White Formica surface.

Topographical View - Color Shading Added

Figure 5.46: A topographical overhead view of the White Formica surface.

Figure 5.47: Top: A meshed surface map of the White Formica surface. Bottom: A colour-contoured map. (x, y, z in mm)

Roughness Parameters

Table 5.18: White Formica 3D Roughness Parameters

White Formica		note: any dimensional parameters given in terms of mm			
parameter	value	parameter	value	parameter	value
S_a	0.02352	S_{vm}	-0.07702	P_{cA}	0.307
S_q	0.02905	S_z	0.15122	V_{cA}	0.492
S_{sk}	-0.12694	S_k	0.04705	N_{VOL}	0.02682
S_{ku}	2.65576	S_{pk}	0.05068	S_{dr}	1.00721
S_p	0.08525	S_{vk}	-0.05350	$S\Delta_q$	0.12036
S_v	-0.08035	S_{pk}/S_k	1.07715	$S\lambda_q$	1.51669
S_t	0.16560	S_{vk}/S_k	1.13709	D_{as}	2.00310
S_{pm}	0.07420	S_{pk}/S_{vk}	0.94729	S_{al}	0.58140

Figure 5.48: A colour-contoured isometric view of the single-quadrant areal autocorrelation function for the White Formica surface. (τ_x, τ_y in mm)

Figure 5.49: Top: overhead view of the White Formica isolated summits. Bottom: the isolated valleys. ($z > S_a$ for summits; $z < -S_a$ for valleys)

5.1.10 Smooth

A smooth surface was tested to give a baseline for the rough surfaces. The surface used was a 1/8" thick aluminum sheet, which was wet-sanded using 400 grade metal sand paper in the direction of airflow. The plate was mounted directly to the test section floor, and therefore did not need a ramp.

5.2 Other Potential Surfaces

It would have been ideal to test a wider range of surfaces, encompassing more of the engineering application field, but time and other resources were limited. This section gives an outline of other potential surfaces that would have proved interesting or useful.

Rust is common on engineering surfaces, although undesirable. While one level of rust was tested in the current work, different ranges of rust could be tested. Dirt accumulation often occurs on vehicles in motion, and as such it was initially desired to test a surface with dirt, oil, and rubber deposits, as is often encountered on road cars. This surface would require some diligence in fabricating though. A high level of mud deposits would also be of interest, as off-road vehicles often encounter this situation, as would be a surface covered with insect remains, as windshields are often covered with them after long drives. Insect remains are also of concern on leading edges of aircraft lifting surfaces. Chipped or peeled paint, as well as pitted surfaces from stones etc, would be of interest in vehicle and aircraft aerodynamics too.

A host of surfaces covered in different fluid droplets, such as water or oil, would be of high interest, but measuring the fluid-surface combination would prove very difficult. Fabrics would also prove tough to model due to their porosity. Of a more simple nature, sandblast or cast surfaces would be easy to manufacture and test. There are many different types

of concrete and stuccos available as well. Many industrial coatings have some roughness to them that might be of interest. Acid-etched surfaces are another possibility. Frost, as accumulated on aircraft wing, would be of vast interest, but would need to be preserved in order to be measured (Kind and Lawrysyn (1991)).

5.3 Comparison of Surfaces

A comparison of the various surfaces is made in this section. For brevity, the surface names are abbreviated as follows:

Blue Formica	BF
Coarse Shingle	CSH
Coarse Spackle	CSP
Fine Shingle	FSH
Fine Spackle	FSP
Rust	RST
Tar Paper	TAR
Vinyl	VIN
White Formica	WF

Examining the surface images and parameter results, some qualitative comparisons of the surfaces can be made.

Height

In terms of overall roughness, the height parameters S_a , S_q , and S_z are the most significant, and as such the CSP, CSH, FSP, and FSH surfaces are the most rough. They are in general an order of magnitude higher than the other five surfaces.

Spacing

The wavelength parameter $S\lambda_q$ can give a good relationship to the average spacing of the roughness elements. The smaller the spacing, the greater the roughness (in general). The BF, WF, and VIN surfaces are gentle rolling surfaces, and as such have large wavelengths, even though their height is low. The TAR and RST surfaces have a small wavelength due to the large number of small-scale surface features, as illustrated in their respective surface images. The spackle surfaces, CSP and FSP have large wavelengths, this due to the relative sparseness of their large peaks. The shingle surfaces have smaller wavelengths, each reflected by their sand grain size, with CSH having a larger wavelength than FSH.

The shortest autocorrelation length, S_{al} , can also give a spacing relation, and tends to mimic the average wavelength. Looking at each surface's plot of autocorrelation function, one can see the very short S_{al} of FSH, RST, and CSH.

Slope

The average slope, $S\Delta_q$, can give a relation to the jaggedness of a surface, and is related to the wavelength by the height parameters. The CSH, FSH, and TAR surfaces have the highest slopes, as can be seen from their topographic plots. This is due to the nature of the surfaces, being comprised of sand grains or fibres embedded in a tar matrix, which can give steep drop-offs where the two meet. The Formica surfaces have very low slopes, confirming the rolling nature of their surfaces.

Complexity

The two fractal parameters S_{dr} and D_{as} give a measure of the “complexity” of a surface, and are somewhat related to the above parameters. Surfaces with small wavelengths, high

slopes, and many features will have a high complexity. As such, the shingle and TAR surfaces have high complexities, the FSH surface standing out in particular, as can be seen by its surface images. The spackle surfaces have low complexities, again due to the sparseness of their large peaks, while the Formica and VIN surfaces are also simple, but due to their rolling nature. Even though the RST surface has many features, they are very small, and as such, it has a low complexity.

Summits and Valleys

The number of features on a surface also affects its roughness level, and the summit and valley counts P_{cA} and V_{cA} reflect this. The FSH, TAR, and RST surfaces have a large number of features which enhance their roughness level. The spackle surfaces are on the opposite end of the spectrum, having few (but large) features.

Another aspect to summits and valleys is the bias towards each. As outlined by Baumann (1999), protrusion roughness elements are approximately ten times more aerodynamically rough than equal size depression roughness elements. From this one can surmise that summits are far more aerodynamically rough than valleys. So a surface with a bias toward summits will tend to have a greater roughness. The skewness parameter S_{sk} gives the best measure of this. S_{pk}/S_{vk} also gives a good relation. The CSP and RST surfaces are very peaky, as indicated by high skewness values, while the CSH and VIN surfaces are biased toward valleys.

5.4 Summary of Roughness Parameters

Table 5.19: Summary of Roughness Parameters [part 1]

parameter	units	Surface				
		Blue Formica	Crse. Shingle	Crse. Spackle	Fine Shingle	Fine Spackle
S_a	mm	0.02512	0.14637	0.45111	0.07811	0.14901
S_q	mm	0.03131	0.18523	0.57161	0.10703	0.18552
S_{sk}	–	0.41614	-0.60377	0.98050	-0.05572	0.55498
S_{ku}	–	2.83323	3.33615	4.28356	5.11956	3.24210
S_p	mm	0.10605	0.50851	3.09771	0.69327	0.78272
S_v	mm	-0.07756	-0.80784	-0.97893	-0.65992	-0.38231
S_t	mm	0.18361	1.31635	4.07664	1.35319	1.16503
S_{pm}	mm	0.09481	0.43973	2.71295	0.55135	0.71067
S_{vm}	mm	-0.06955	-0.71255	-0.94711	-0.59092	-0.35023
S_z	mm	0.16436	1.15228	3.66006	1.14227	1.06090
S_k	mm	0.05023	0.29274	0.90222	0.15622	0.29803
S_{pk}	mm	0.06969	0.29336	2.41334	0.47324	0.56166
S_{vk}	mm	-0.04443	-0.56618	-0.49600	-0.51281	-0.20122
S_{pk}/S_k	–	1.38742	1.00212	2.67489	3.02932	1.88458
S_{vk}/S_k	–	0.88453	1.93407	0.54976	3.28261	0.67517
S_{pk}/S_{vk}	–	1.56853	0.51814	4.86560	0.92284	2.79127
P_{cA}	1/mm ²	0.461	0.319	0.028	3.821	0.070
V_{cA}	1/mm ²	0.318	0.411	0.032	3.582	0.093
N_{VOL}	mm	-0.02963	0.16305	0.53753	0.09208	0.17437
S_{dr}	–	1.00629	1.53366	1.30624	2.20301	1.07808
$S\Delta_q$	–	0.11239	1.16444	0.84910	1.77825	0.40713
$S\lambda_q$	mm	1.75042	0.99948	4.22979	0.37817	2.86316
D_{as}	1/mm ²	2.00236	2.20940	2.12871	2.59343	2.03428
S_{al}	mm	0.80717	0.54077	2.02914	0.14341	1.38005

note: $m=5$ for S_{pm} and S_{vm} ; $a=0.2$ for S_{al}

Table 5.20: Summary of Roughness Parameters [part 2]

parameter	units	Surface			
		Rust	Tar Paper	Vinyl	White Formica
S_a	mm	0.01370	0.04298	0.02318	0.02352
S_q	mm	0.01791	0.05293	0.02940	0.02905
S_{ak}	–	1.07265	0.37963	-0.51526	-0.12694
S_{ku}	–	4.94372	2.77425	3.18426	2.65576
S_p	mm	0.11258	0.19946	0.08004	0.08525
S_v	mm	-0.06520	-0.14061	-0.11443	-0.08035
S_t	mm	0.17778	0.34007	0.19447	0.16560
S_{pm}	mm	0.09688	0.16913	0.07572	0.07420
S_{vm}	mm	-0.05770	-0.13676	-0.09737	-0.07702
S_z	mm	0.15458	0.30589	0.17309	0.15122
S_k	mm	0.02740	0.08595	0.04635	0.04705
S_{pk}	mm	0.08318	0.12615	0.05254	0.05068
S_{vk}	mm	-0.04400	-0.09378	-0.07419	-0.05350
S_{pk}/S_k	–	3.03577	1.46774	1.13355	1.07715
S_{vk}/S_k	–	1.60584	1.09110	1.60065	1.13709
S_{pk}/S_{vk}	–	1.89045	1.34517	0.70818	0.94729
P_{cA}	1/mm ²	2.497	1.079	0.509	0.307
V_{cA}	1/mm ²	2.304	5.457	0.363	0.492
$NVOL$	mm	0.04656	0.05030	0.02603	0.02682
S_{dr}	–	1.02127	1.41576	1.01865	1.00721
$S\Delta_q$	–	0.20902	0.99323	0.19516	0.12036
$S\lambda_q$	mm	0.53843	0.33484	0.94637	1.51669
D_{as}	1/mm ²	2.01020	2.20890	2.00885	2.00310
S_{al}	mm	0.40604	0.85659	1.22606	0.58140

note: $m = 5$ for S_{pm} and S_{vm} ; $a = 0.2$ for S_{al}

Chapter 6

Results and Discussion

The experimental results are presented in the following sections. The results of the boundary layer analysis for each surface are presented in section 6.2. The law-of-the-wall shift plots are presented in section 6.3 and an error analysis follows in section 6.4. Section 6.5 gives the results of the roughness parameter analysis, indicating the final parameters chosen for the correlation model. The results of the correlation exercise are presented in section 6.6, followed by the relationship of a theoretical correlation model to Schlichting's sand-grain roughness etc. in section 6.7.4.

6.1 Preliminaries

Before the boundary layer results are presented, a note on the use of the defect-law plot must be made, along with a note on the applicability of results for the first and last probing stations. The results of the check on flow two-dimensionality will be shown as well.

6.1.1 Defect-Law Plot Use

The two main indirect methods of finding C_f for boundary layers are the defect-law plot and the momentum integral method. It would have been desirable to use both, but as discussed in section 2.2.1, the defect-law plot is sensitive to pressure gradient, especially favourable pressure gradients. As will be seen later, there is a mild pressure gradient in the wind tunnel test section, due to the buildup of δ^* . This favourable pressure gradient has β values in the range of approximately 0.01 to ~ 0.1 (depending on roughness level), confirming its mild nature. Although mild, investigation showed that these β values could be expected to have sufficient effect on the defect-law profiles that using the defect-law plot approach to determine C_f would be open to question. The problem is that, as discussed in section 2.2.1 and shown in Fig. 2.5, there is little data for defect plots with various β values. Secondly, the β values for these experiments are not constant throughout the test section, but display a consistent variation from approximately 0.01 to ~ 0.1 . Fig. 6.1 illustrates the above, showing the variation of β with lengthwise distance along the test section. From the figure it can be seen that the rougher the surface, the higher the β values, i.e. the higher the pressure gradient. This is because a rougher surface will produce a larger boundary layer thickness, which results in a larger displacement thickness, δ^* . As mentioned before, the buildup of δ^* causes the (mild) pressure gradient, so the larger the δ^* , the larger the pressure gradient. Fig. 6.1 confirms this, with increasing roughness (smooth $\rightarrow$ White Formica $\rightarrow$ Coarse Shingle $\rightarrow$ Coarse Spackle) causing an increase in β .

As a result of the issues involved with using the defect-law plot, it was decided to use the momentum integral method as the primary method of determining C_f and to check the C_f values thus determined for consistency with interpolated estimates of the defect-law profiles for the appropriate β values, as shown in Fig. 6.2. This plot shows experimental boundary layers plotted on a defect-law plot with the known plots of various β 's. The

Figure 6.1: A plot showing the variation of β with x -distance along the test section, for various surfaces.

experimental defect-law curves were generated using C_f values deduced by the momentum integral method. The β values for each measurement station can be calculated based on these C_f values, as per equation 4.7. If the experimental plots fall in the interpolated range of β values that match the calculated β values, then the momentum integral method can be said to be consistent with the defect-law, giving confidence in the method's accuracy. Fig. 6.2 shows this method for the Coarse Shingle surface. Comparison with the β values of Fig. 6.1 shows the consistency. The same consistency was observed for all the surfaces.

For the smooth surface, the Clauser plot can also be used in addition to the momentum integral method to determine C_f since it is insensitive to pressure gradient.

Figure 6.2: A defect-law plot showing experimental boundary layer results along with known curves for various β values. (β curves from Herring and Norbury (1967))

6.1.2 Flow Two-Dimensionality

A check on the spanwise flow uniformity is shown for the Blue Formica surface in Fig. 6.3. The data is for station 3, for the centre, left, and right stations (see Fig. 4.2). Since the boundary layer profiles are all very nearly identical, the flow can be considered spanwise-uniform, or two-dimensional.

Figure 6.3: Boundary layer profiles for station 3 centre, left, and right.

6.1.3 Use of Measurement Stations 1 and 5

The first probing station is located 0.3175 m after the start of the roughness, which begins immediately after the ramp (see Fig. 4.2). A typical boundary layer thickness for this station is about 15 mm, which gives a ratio of $x/\delta \simeq 20$. The step change in roughness will cause the boundary layer to be in a non-equilibrium state for some distance and the inner and outer layer similarity arguments are inapplicable (Antonia and Luxton (1971)). Pendergrass and Arya (1984) showed that at the step change in roughness, the local C_f considerably overshoot the equilibrium value and took some distance downstream to return to an equilibrium state. Antonia and Luxton (1971) found that the readjustment of the boundary layer to the equilibrium condition takes approximately twenty boundary layer

thicknesses from the start of the roughness. Therefore the data for station 1, where $x/\delta \simeq 20$, will reflect a boundary layer that may or may not be in equilibrium. Another possible cause of non-equilibrium is the fact that station 1 is also close the ramp at the leading edge of the test surface. The data for station 1 was therefore processed but not used in determining the final correlations.

Probe station 5 is located about 10 cm upstream of the bleeder slot (see Fig. 4.1). As such, the flow may be affected by the influx of air into this bleeder slot, so the results for station 5 should also be interpreted with caution.

6.2 Boundary Layer Profile and Skin Friction Results

The boundary layer profile and C_f results for each surface are presented here. The law-of-the-wall shifts, Δu^+ , and the C -intercepts will be given later, in section 6.3.

A few observations on the results presented in this section can be made. The rougher surfaces, such as Coarse Spackle, have larger boundary layer heights than the smoother surfaces. The increased resistance creates a greater velocity defect, and thus δ^* and θ are larger for the rougher surfaces. The shape factors are also higher for the rougher surfaces. Due to the larger δ^* 's, the rougher surfaces tend to have higher C_q 's. Obviously the increased resistance generated by the larger roughness elements will cause a higher C_f , and thus a higher law-of-the-wall shift, Δu^+ .

6.2.1 Smooth Surface

Table 6.1 lists the boundary layer parameters for the smooth surface. “Stn. #” represents the probing station number, and x is measured from the start of the roughness, after the ramp (see Fig. 4.2).

Table 6.1: Smooth Surface Boundary Layer Results

Smooth Surface			$Re/L \simeq 3.0 \times 10^6$ [1/m]		$U_e = 50$ m/s
Stn. #	x (m)	δ (mm)	δ^* (mm)	θ (mm)	H
1	0.317	12.898	1.637	1.265	1.294
2	0.676	18.041	2.312	1.794	1.289
3	1.031	23.566	2.970	2.311	1.285
4	1.393	28.328	3.548	2.764	1.284
5	1.724	32.615	4.381	3.385	1.294

Stn. #	C_q	β	C_f (MIM*)	C_f (Clauser)	B -intercept**
1	0.9312	-0.018	0.00340	0.00325	4.465
2	0.9436	-0.027	0.00309	0.00300	4.942
3	0.9567	-0.036	0.00295	0.00290	5.001
4	0.9694	-0.044	0.00287	0.00280	4.826
5	0.9797	-0.054	0.00285	0.00275	4.857

* “MIM” $\equiv$ momentum integral method

** B -intercepts found using C_f ’s from MIM; see equation 2.12 for definition of B

The variation in C_q exhibits the slightly favourable pressure gradient in the test section. The values of H confirm the turbulent nature of the boundary layers (see section 2.1.1). For smooth wall boundary layers the value of H is usually close to 1.4; it is not clear why the present values are significantly lower. However, of primary importance is that the C_f values are reasonable. The C_f ’s given by the momentum integral method (MIM) and by the Clauser plot agree well. Using the C_f ’s from the MIM, the B -intercepts of the logarithmic portion of the law-of-the-wall are shown. Ignoring station 1 (as discussed in

section 6.1.3), the average of the remaining four intercepts is 4.91, which is quite similar to the standard value of 5.0 as given by Coles and Hirst (White (1991) – see section 2.2.2). The above consistency supports the validity of the experimental method and data reduction techniques.

Figure 6.4: Boundary layer profiles for station 1 to 5, Smooth surface.

Fig. 6.4 shows the increase in boundary layer thickness with x -distance. Fig. 6.5 depicts the normalized boundary-layer profiles, with y normalized by δ . Though difficult to see due to the small variation in C_f , the similarity to Fig. 2.2 is evident, with higher C_f boundary layers having more defect. The smaller the x -distance, the higher the C_f .

Figure 6.5: Normalized boundary layer profiles for station 1 to 5, Smooth surface.

Figure 6.6: Clauser plot of Smooth surface boundary layer profiles for station 1 to 5.

Figure 6.7: Law-of-the-wall plot for Smooth surface.

Fig. 6.6 is a Clauser plot for stations 1 to 5. The C_f 's can be read off from this graph, using the log-law portion of the curve. Fig. 6.7 is a law-of-the-wall plot. The curves fall approximately on the standard $B=5.0$ logarithmic line. Figs. 6.8 and 6.9 show the variation and curve fit of θ and $\sqrt{C_q}$. Note that due to the sensitivity of the curve fitting procedure, for station 5 θ was not used in the curve fit as it appeared to be an outlier. The goodness of these curve fits is typical of that for the fits to the θ and C_q data for the rough surfaces.

Figure 6.8: θ vs. x variation and curve fit for Smooth surface.

Figure 6.9: $\sqrt{C_q}$ vs. x variation and curve fit for Smooth surface.

6.2.2 Blue Formica

Table 6.2: Blue Formica Boundary Layer Results

Blue Formica y -datum = S_{vm} $Re/L \simeq 3.0 \times 10^6$ [1/m] $U_e = 50$ m/s					
Stn. #	x (m)	δ (mm)	δ^* (mm)	θ (mm)	H
1	0.317	13.443	1.874	1.433	1.307
2	0.676	19.158	2.599	2.011	1.292
3	1.031	24.587	3.224	2.512	1.283
4	1.393	30.302	3.886	3.027	1.284
5	1.724	35.732	4.661	3.602	1.294

Stn. #	x (m)	C_q	β	C_f (MIM)	Δu^+
1	0.317	0.9546	-0.037	0.00371	2.449
2	0.676	0.9712	-0.040	0.00335	2.028
3	1.031	0.9873	-0.043	0.00318	1.935
4	1.393	1.0012	-0.046	0.00307	1.949
5	1.724	1.0034	-0.051	0.00297	1.846

Figure 6.10: Normalized boundary layer profiles for Blue Formica surface.

6.2.3 Coarse Shingle

Table 6.3: Coarse Shingle Boundary Layer Results

Coarse Shingle y -datum = $-2 S_a$ $Re/L \simeq 3.0 \times 10^6$ [1/m] $U_e = 50$ m/s					
Stn. #	x (m)	δ (mm)	δ^* (mm)	θ (mm)	H
1	0.317	16.202	3.471	2.294	1.513
2	0.676	25.822	4.988	3.399	1.455
3	1.031	33.823	6.040	4.273	1.414
4	1.393	41.920	7.239	5.122	1.413
5	1.724	49.159	8.325	5.953	1.398

Stn. #	x (m)	C_q	β	C_f (MIM)	Δu^+
1	0.317	0.9574	-0.031	0.00739	10.563
2	0.676	0.9780	-0.045	0.00612	9.834
3	1.031	0.9970	-0.055	0.00554	9.492
4	1.393	1.0142	-0.065	0.00521	9.305
5	1.724	1.0247	-0.073	0.00502	9.243

Figure 6.11: Normalized boundary layer profiles for Coarse Shingle surface.

6.2.4 Coarse Spackle

Table 6.4: Coarse Spackle Boundary Layer Results

Coarse Spackle y -datum = mean line $Re/L \simeq 3.0 \times 10^6$ [1/m] $U_e = 50$ m/s					
Stn. #	x (m)	δ (mm)	δ^* (mm)	θ (mm)	H
1	0.317	20.508	5.723	3.481	1.644
2	0.676	33.557	8.170	5.156	1.584
3	1.031	44.511	9.454	6.357	1.487
4	1.393	55.941	11.463	7.591	1.510
5	1.724	65.942	13.341	8.890	1.501

Stn. #	x (m)	C_q	β	C_f (MIM)	Δu^+
1	0.317	0.9630	-0.039	0.01139	14.848
2	0.676	0.9853	-0.060	0.00923	14.364
3	1.031	1.0138	-0.072	0.00826	14.078
4	1.393	1.0325	-0.088	0.00776	14.033
5	1.724	1.0537	-0.101	0.00750	14.098

Figure 6.12: Normalized boundary layer profiles for Coarse Spackle surface.

6.2.5 Fine Shingle

Table 6.5: Fine Shingle Boundary Layer Results

Fine Shingle y -datum = $-S_a$ $Re/L \simeq 3.0 \times 10^6$ [1/m] $U_e = 50$ m/s					
Stn. #	x (m)	δ (mm)	δ^* (mm)	θ (mm)	H
1	0.317	15.718	3.112	2.118	1.469
2	0.676	24.672	4.458	3.137	1.421
3	1.031	32.863	5.655	4.032	1.403
4	1.393	40.293	6.732	4.837	1.392
5	1.724	n.a.	n.a.	n.a.	n.a.

Stn. #	x (m)	C_q	β	C_f (MIM)	Δu^+
1	0.317	0.9538	-0.027	0.00680	9.679
2	0.676	0.9728	-0.040	0.00580	9.165
3	1.031	0.9966	-0.050	0.00537	9.050
4	1.393	1.0070	-0.060	0.00511	9.007
5	1.724	n.a.	n.a.	n.a.	n.a.

n.a. – no data available for station 5, as errors were found in the post-processing

Figure 6.13: Normalized boundary layer profiles for Fine Shingle surface.

6.2.6 Fine Spackle

Table 6.6: Fine Spackle Boundary Layer Results

Fine Spackle y -datum = mean line $Re/L \simeq 3.0 \times 10^6$ [1/m] $U_e = 50$ m/s					
Stn. #	x (m)	δ (mm)	δ^* (mm)	θ (mm)	H
1	0.317	15.418	3.212	2.200	1.460
2	0.676	24.848	4.659	3.275	1.423
3	1.031	33.420	6.110	4.279	1.428
4	1.393	40.564	6.982	4.988	1.400
5	1.724	48.184	8.259	5.885	1.403

Stn. #	x (m)	C_q	β	C_f (MIM)	Δu^+
1	0.317	0.9526	-0.033	0.00690	9.784
2	0.676	0.9722	-0.037	0.00595	9.454
3	1.031	0.9868	-0.042	0.00551	9.404
4	1.393	1.0079	-0.044	0.00519	9.193
5	1.724	1.0208	-0.048	0.00502	9.194

Figure 6.14: Normalized boundary layer profiles for Fine Spackle surface.

6.2.7 Rust

Table 6.7: Rust Boundary Layer Results

Rust y -datum = S_{vm} $Re/L \simeq 3.0 \times 10^6$ [1/m] $U_e = 50$ m/s					
Stn. #	x (m)	δ (mm)	δ^* (mm)	θ (mm)	H
1	0.317	14.672	1.900	1.451	1.310
2	0.676	20.291	2.774	2.121	1.308
3	1.031	27.530	3.447	2.669	1.292
4	1.393	33.817	4.195	3.251	1.290
5	1.724	n.a.	n.a.	n.a.	n.a.

Stn. #	x (m)	C_q	β	C_f (MIM)	Δu^+
1	0.317	0.9556	-0.034	0.00428	4.477
2	0.676	0.9738	-0.039	0.00378	3.370
3	1.031	0.9965	-0.041	0.00353	3.639
4	1.393	1.0056	-0.045	0.00338	3.529
5	1.724	n.a.	n.a.	n.a.	n.a.

n.a. – no data available for station 5 due to dimensional limits of the rust plate

Figure 6.15: Normalized boundary layer profiles for Rust surface.

6.2.8 Tar Paper

Table 6.8: Tar Paper Boundary Layer Results

Tar Paper		y -datum = mean line	$Re/L \simeq 3.0 \times 10^6$ [1/m]	$U_e = 50$ m/s	
Stn. #	x (m)	δ (mm)	δ^* (mm)	θ (mm)	H
1	0.317	15.924	2.230	1.711	1.303
2	0.676	22.782	3.074	2.409	1.309
3	1.031	30.498	3.939	3.035	1.298
4	1.393	37.165	4.707	3.617	1.301
5	1.724	43.356	5.407	4.184	1.292

Stn. #	x (m)	C_q	β	C_f (MIM)	Δu^+
1	0.317	0.9490	-0.026	0.00455	5.407
2	0.676	0.9661	-0.034	0.00400	4.707
3	1.031	0.9783	-0.043	0.00376	4.688
4	1.393	0.9967	-0.050	0.00361	4.614
5	1.724	1.0057	-0.056	0.00352	4.615

Figure 6.16: Normalized boundary layer profiles for Tar Paper surface.

6.2.9 Vinyl

Table 6.9: Vinyl Boundary Layer Results

Vinyl y -datum = $\frac{1}{2} S_{vm}$ $Re/L \simeq 3.0 \times 10^6$ [1/m] $U_e = 50$ m/s					
Stn. #	x (m)	δ (mm)	δ^* (mm)	θ (mm)	H
1	0.317	14.356	1.929	1.459	1.322
2	0.676	20.261	2.666	2.058	1.295
3	1.031	26.548	3.457	2.677	1.291
4	1.393	31.786	3.939	3.068	1.284
5	1.724	37.120	4.634	3.602	1.287

Stn. #	x (m)	C_q	β	C_f (MIM)	Δu^+
1	0.317	0.9531	-0.030	0.00400	3.580
2	0.676	0.9700	-0.032	0.00358	3.030
3	1.031	0.9841	-0.035	0.00339	2.975
4	1.393	1.0012	-0.036	0.00324	2.809
5	1.724	1.0041	-0.039	0.00316	2.822

Figure 6.17: Normalized boundary layer profiles for Vinyl surface.

6.2.10 White Formica

Table 6.10: White Formica Boundary Layer Results

White Formica		y -datum = S_{vm}	$Re/L \simeq 3.0 \times 10^6$ [1/m]	$U_c = 50$ m/s	
Stn. #	x (m)	δ (mm)	δ^* (mm)	θ (mm)	H
1	0.317	13.811	1.930	1.473	1.310
2	0.676	19.717	2.683	2.064	1.300
3	1.031	25.241	3.331	2.593	1.284
4	1.393	30.766	3.912	3.047	1.284
5	1.724	35.719	4.586	3.563	1.287

Stn. #	x (m)	C_q	β	C_f (MIM)	Δu^+
1	0.317	0.9498	-0.024	0.00405	3.631
2	0.676	0.9675	-0.035	0.00351	2.692
3	1.031	0.9804	-0.044	0.00327	2.422
4	1.393	0.9971	-0.051	0.00312	2.205
5	1.724	1.0053	-0.059	0.00305	2.206

Figure 6.18: Normalized boundary layer profiles for White Formica surface.

6.2.11 Comparison of Experimentally Determined C_f 's with Predictions of a Boundary Layer Code

Table 6.11 lists the experimentally determined (i.e. using the momentum integral method) skin friction coefficients and those determined by the boundary layer code described in section 4.2.6.

The purpose of determining C_f using the boundary layer code was not to compare the exact values of C_f between the two methods, but rather to compare the *trend* in C_f vs. x -distance. Since the code has λ , the spacing parameter, as an input, and this was iterated manually until the C_f 's for station 1 and 2 somewhat matched the momentum integral method C_f 's, the exact values are irrelevant, as they were forced to be equal. However, by forcing the first and second station C_f 's to be approximately equal (between methods), the trend in C_f vs. x -distance could be compared for stations 3 to 5. The point of this exercise is that since the momentum integral method is sensitive to the curve fitting procedure, as discussed in section 4.2.5, the trend in C_f could vary significantly if the curve fits were poor. As can be seen in Table 6.11, the C_f 's as determined by the momentum integral method and by the code agree very well. This gives confidence that the curve fits were acceptable, and thus that the C_f values are correct.

6.3 Law-Of-The-Wall Shifts

The law-of-the-wall plots for each station are shown in Figs. 6.19 to 6.23. The values of the shifts, Δu^+ , are listed in Table 6.12a. Recall that Δu^+ is the vertical shift between the smooth-wall log-law and the parallel logarithmic portions of the profile plots in Figs. 6.19

Table 6.11: Experimental vs. Code C_f 's

Surface	Stn. #	C_f (MIM)	C_f (code)	Surface	Stn. #	C_f (MIM)	C_f (code)
Smooth	1	0.00340	0.00317	Fine Spackle	1	0.00690	0.00712
	2	0.00309	0.00289		2	0.00595	0.00582
	3	0.00295	0.00278		3	0.00551	0.00544
	4	0.00287	0.00269		4	0.00519	0.00522
	5	0.00285	0.00262		5	0.00502	0.00502
Blue Formica	1	0.00371	0.00374	Rust	1	0.00428	0.00442
	2	0.00335	0.00332		2	0.00378	0.00375
	3	0.00318	0.00317		3	0.00353	0.00353
	4	0.00307	0.00305		4	0.00338	0.00337
	5	0.00297	0.00295		5	n.a.	n.a.
Coarse Shingle	1	0.00739	0.00694	Tar Paper	1	0.00455	0.00476
	2	0.00612	0.00592		2	0.00400	0.00410
	3	0.00554	0.00555		3	0.00376	0.00380
	4	0.00521	0.00529		4	0.00361	0.00366
	5	0.00502	0.00509		5	0.00352	0.00355
Coarse Spackle	1	0.01139	0.01144	Vinyl	1	0.00400	0.00400
	2	0.00923	0.00880		2	0.00358	0.00352
	3	0.00826	0.00826		3	0.00339	0.00333
	4	0.00776	0.00776		4	0.00324	0.00322
	5	0.00750	0.00751		5	0.00316	0.00311
Fine Shingle	1	0.00680	0.00686	White Formica	1	0.00405	0.00394
	2	0.00580	0.00569		2	0.00351	0.00346
	3	0.00537	0.00536		3	0.00327	0.00327
	4	0.00511	0.00507		4	0.00312	0.00316
	5	n.a.	n.a.		5	0.00305	0.00307

to 6.23. Recall also, from Chapter 2, equation 2.18, that

$$\Delta u^+ = A \log k^+ + C$$

for fully rough flow, where k^+ is defined as ku_τ/ν and k is a measure of the roughness height. Table 6.12b lists the values of k^+ when S_z is used for k and Table 6.12c lists the values of k^+ when S_q is used for k .

For deterministic roughness, the individual element height k is normally used for obvious reasons. For stochastic roughness, there are three main height parameters, S_a , S_q , and S_z . Since S_a and S_q are in general related (as will be shown later), they are not independent, and can be considered essentially equivalent. S_q is chosen since it is used in the calculation of S_{sk} and other parameters. S_z is not directly related to either S_a or S_q , so it is an independent height parameter - two surfaces can have the same RMS height, S_q , but different height ranges, S_z . The final choice of using S_q or S_z will be discussed later, in section 6.7.3.

Since S_z gives the height range of the roughness, it is more directly related to k than S_q is, and as such, it is more suitable to delimit the different roughness regimes, as shown in Fig. 2.9. Fig. 6.24 is a plot of the shifts versus an S_z -based k^+ , that is, versus $S_z u_\tau/\nu$.

The fully rough flow regime is usually delimited by $k^+ > 50$ and $\Delta u^+ > 5$, and from Fig. 6.24 it can be seen that only the CSH, CSP, FSH, and FSP surfaces are in this regime. The other five surfaces are in the transitionally rough regime. As such, these 5 surfaces cannot be used in the correlation model for C , as equation 2.18 only applies to the fully rough regime. Note that station 1 was not used for these plots, for reasons mentioned earlier. Fig. 6.25 is a plot of the shifts versus an S_q -based k^+ , that is, versus $S_q u_\tau/\nu$. Since S_q is in general an order of magnitude smaller than S_z , so too will its k^+ be.

For the four fully rough surfaces, the C -intercept can be found as outlined in section 4.2.5. A logarithmic curve of slope $A=5.62$ fit to the points for stations 2-5 yielded the values in Table 6.13. Note that depending on use of S_z or S_q for k^+ , different C values will result.

Figure 6.19: Law-of-the-wall plot for the rough surfaces, station 1.

Figure 6.20: Law-of-the-wall plot for the rough surfaces, station 2.

Figure 6.21: Law-of-the-wall plot for the rough surfaces, station 3.

Figure 6.22: Law-of-the-wall plot for the rough surfaces, station 4.

Figure 6.23: Law-of-the-wall plot for the rough surfaces, station 5.

Table 6.12: Law-Of-The-Wall-Shift Data Summary

	Blue Formica	Coarse Shingle	Coarse Spackle	Fine Shingle	Fine Spackle	Rust	Tar Paper	Vinyl	White Formica
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(a)

Stn. #	Δu^+	Δu^+	Δu^+	Δu^+	Δu^+	Δu^+	Δu^+	Δu^+	Δu^+
1	2.449	10.563	14.848	9.679	9.784	4.477	5.407	3.580	3.631
2	2.028	9.834	14.364	9.165	9.454	3.730	4.707	3.030	2.692
3	1.935	9.492	14.078	9.050	9.404	3.639	4.688	2.975	2.422
4	1.949	9.305	14.033	9.007	9.193	3.529	4.614	2.809	2.205
5	1.846	9.243	14.098	n.a.	9.194	n.a.	4.615	2.822	2.206

(b)

Stn. #	$k^+ (S_z)$	k^+	k^+	k^+	k^+	k^+	k^+	k^+	k^+
1	20.82	206.85	825.28	200.17	187.48	21.47	43.755	23.14	20.29
2	20.17	192.76	755.02	184.11	173.73	20.13	40.848	21.99	18.94
3	19.82	188.78	713.97	178.41	169.87	19.77	40.330	21.52	18.79
4	19.32	181.55	682.65	174.37	162.34	19.03	39.113	20.93	17.89
5	18.80	176.35	665.42	n.a.	157.23	n.a.	37.80	20.39	17.45

(c)

Stn. #	$k^+ (S_q)$	k^+	k^+	k^+	k^+	k^+	k^+	k^+	k^+
1	3.97	33.25	128.89	18.76	32.78	2.49	7.57	3.93	3.90
2	3.84	30.99	117.92	17.25	30.38	2.33	7.07	3.74	3.64
3	3.78	30.35	111.50	16.72	29.71	2.29	6.98	3.66	3.61
4	3.68	29.19	106.61	16.34	28.39	2.21	6.77	3.56	3.44
5	3.58	28.35	103.92	n.a.	27.50	n.a.	6.54	3.46	3.35

Table 6.13: C-Intercepts for Fully Rough Surfaces

surface	Coarse Shingle	Coarse Spackle	Fine Shingle	Fine Spackle
C-intercept (S_z)	-3.36	-1.86	-3.59	-3.16
C-intercept (S_q)	1.10	2.67	2.19	1.10

Figure 6.25: Intercept-shift curve based on $k = S_q$.

6.4 Error Analysis

Time constraints prevented a thorough error analysis, however an error bound on the C -intercepts was obtained using a simple method. The shift curve data points (k^+ , Δu^+) proved to be extremely sensitive to C_f , specifically the variation of C_f with station number. Since C_f is found using the momentum integral method, which uses derivatives of θ and C_q , it is itself sensitive to the curve fit procedure. Therefore the largest error was assumed to come from the curve fit procedure.

Two cases above and below the correct C value were obtained by modifying a single data point of θ to generate a curve fit with a sharper drop off in slope and one with less of a drop off in slope. The θ value at this point was raised and lowered by 2.5%. These changes reflected the error that might be found if a data point was off, but not readily detectable as an outlier. From the data analysis work, it became apparent that any greater percentage change would be detectable as an outlier, and would not be used in the curve fit, and so no error would result. The following tables outline the numerical results. θ at station 2 was varied and k^+ was based on S_z .

Table 6.15 illustrates the changes in C_f and the shifts that occur due to the variation in θ . Table 6.16 gives the C -intercepts for each case. The intercepts were found by fitting the log equation

$$y = A \log(x) + \text{intercept}$$

to the shift points. The larger of the two errors (θ 2.5% high or θ 2.5% low) was taken as the error bound, given as ± 0.18 .

A note on the possible error at station 1 must be made. As discussed in section 6.1.3, there will be a C_f overshoot after the step change in roughness, and this may be felt at station 1. As can be seen in the boundary layer results, there is a high drop off in C_f from

Table 6.14: Error Analysis Results - θ Variation

Stn. #	correct θ (m)	lowered θ (m)	raised θ (m)
1	0.00229398		
2	0.00339933	0.00331435	0.00348431
3	0.00427297		
4	0.00512243		
5	0.00595340		

Table 6.15: Error Analysis Results - Parameter Variation

Stn. #	correct θ_2			lowered θ_2			raised θ_2		
	$k^+ (S_z)$	C_f	Δu^+	$k^+ (S_z)$	C_f	Δu^+	$k^+ (S_z)$	C_f	Δu^+
1	211.42	0.00739	10.563	203.27	0.00683	9.808	219.16	0.00794	11.232
2	191.24	0.00612	9.834	190.72	0.00609	9.779	191.77	0.00615	9.891
3	184.70	0.00554	9.492	187.78	0.00573	9.844	182.03	0.00539	9.179
4	176.20	0.00521	9.305	181.46	0.00553	9.945	171.68	0.00495	8.726
5	170.42	0.00502	9.243	177.01	0.00542	10.079	164.83	0.00470	8.484

Table 6.16: Error Analysis Results - C Variation

correct- θ_2 C	lowered- θ_2 C	raised- θ_2 C	error bound
-3.07	-2.89	-3.24	± 0.18

station 1 to 2. This may be due to the boundary layer readjusting after the step change, but it may also be attributed to error in the curve fit procedure. For any curve fit, the curve at the first and last points will always be compromised since there is only one other point (the second or penultimate) to control the curve. The points inside have two points either side of them to control the curve. Thus at station 1, the curve fit may be steeper than it should be, especially given the use of a power law curve. This would show up as a higher than expected C_f . This is yet another reason why the results for station 1 must be

treated with caution and why the C_f results for this station were not used in establishing the correlations. Similar conditions apply for station 5.

6.5 Roughness Parameter Analysis Results

This section discusses the results of the roughness parameter analysis, outlining the initial choices for use in the correlation model, elimination of parameters due to redundancy, and the final selection for the correlation-model trials.

The configuration parameters outlined in Chapter 3 were those initially found in the literature by the author and judged to perhaps have some applicability. Upon further consideration, once the parameters were calculated and studied, some were eliminated. Among those were the single point parameters S_p , S_v , and S_t , for reasons outlined in Chapter 3. S_{pk} , S_{vk} , and S_k were not used since these height or height region parameters are not as significant or as simple as S_q or S_z . As such, the parameters based on these (S_{pk}/S_k , S_{vk}/S_k , and S_{pk}/S_{vk}) were eliminated as well.

The remaining parameters were studied more thoroughly, and some were found to be redundant. Fig. 6.26 shows the relationship of S_a and S_q for the present test surfaces. They appear to be linearly related, but in actual fact, there is no mathematical relationship between the two parameters. Only for a pure sine wave does a multiplicative constant between the two exist – $S_q = 1.11 S_a$. For general surfaces the relationship varies, but an average multiplicative constant of $S_q = 1.25 S_a$ has been established from various surface finish procedures, such as turning, milling, grinding, and machining Tabenkin (2000). For the present surfaces, a linear curve fit (not shown) to the S_q vs. S_a data yields a constant of $S_q = 1.27 S_a$, which is in good agreement with the average reported by Tabenkin. The 1.25 constant seems to be applicable to most surfaces, and so S_q and S_a can be considered to be

related, in general. As such, S_a can be eliminated from the selection, since S_q is chosen as a result of convenience, it being used for calculating other parameters such as S_{sk} and S_{ku} .

Figure 6.26: A plot of S_q vs. S_a for the present test surfaces, showing the linear relationship between the two.

Fig. 6.27 shows the relationship of N_{VOL} and S_a for the present test surfaces. Since they are almost linearly related, in general, and S_a is linearly related to S_q , then N_{VOL} can be eliminated as well. N_{VOL} is related to S_a because the normalized volume is in effect a height parameter as well (see section 3.3.2).

Fig. 6.28 is a plot of S_{dr} vs. D_{as} , the two fractal parameters, for the present test surfaces. Looking at Fig. 3.15, one can see that S_{dr} is the highest point on the y -axis, i.e. the largest relative area ratio. D_{as} is related to the slope of the log-log region through equation 3.58. It is evident that the higher the slope, the higher the largest relative area ratio will be, so D_{as} should be related to S_{dr} , as shown in Fig. 6.28. Therefore D_{as} will be eliminated, as S_{dr} is more robust and easier to calculate.

Figure 6.27: A plot of N_{VOL} vs. S_a for the present test surfaces, showing the linear relationship between the two.

Figure 6.28: A plot of S_{dr} vs. D_{as} for the present test surfaces, showing the linear relationship between the two.

Fig. 6.29 shows the relationship between $S\Delta_q$ and S_{dr} for the present test surfaces. Based on the analytical expressions for each, one would not immediately see the relationship, but closer inspection yields the answer. S_{dr} is a measure of the “complexity” of the surface, and a more complex surface will have a higher number of features per unit area. In order to have this higher number of features, they must be spaced more closely, and as such, the surface becomes more “jagged”, having higher slopes, which is what $S\Delta_q$ is a measure of. Since these two parameters are related, $S\Delta_q$ will be eliminated.

Figure 6.29: A plot of $S\Delta_q$ vs. S_{dr} for the present test surfaces, showing the relationship between the two.

Further analysis yields more eliminations from the final parameter selection. Since S_z is calculated from S_{pm} and S_{vm} , these two parameters can be eliminated. $S\lambda_q$ is based on S_q and $S\Delta_q$ and so it can be removed as well. P_{cA} and V_{cA} give a measure of how many features exist per unit area, but since S_{dr} is also somewhat of a measure of this, and the calculation of P_{cA} and V_{cA} is open to some subjectivity, these two parameters will not be

used either.

The above eliminations yield the final six parameters, listed in Table 6.17, to be used in the correlation trials. The significance of each is shown.

Table 6.17: **Final Parameter Selection for Correlation Trials**

parameter	significance
S_q	average height parameter
S_z	height range parameter
S_{sk}	summit/valley bias
S_{ku}	height distribution relation
S_{dr}	surface complexity
S_{al}	correlation length / wavelength parameter

To characterize a randomly rough surface, an average or RMS height parameter is needed as is a height range parameter. This is a very important characteristic of stochastic surfaces, versus the deterministic surfaces studied to date. For typical deterministic roughness, as studied by Dvorak (1969), Simpson (1973) etc., all the roughness elements were uniform, and as such, they had the same height, k . Therefore only one height parameter was necessary to characterize the roughness. However, if deterministic surfaces with *non-uniform* roughness elements were studied, i.e. elements with different heights, then what would the term k apply to? The height range or the average height? This issue exists for stochastic surfaces, since they have no well-defined elements. Therefore, one can see that *two* height parameters are required to characterize random surfaces, an average height and a height range, because, as stated earlier, two surfaces can have identical average heights but vastly different height ranges, and thus very different effects on fluid flow. For this reason, the RMS height, S_q , and the height range, S_z , are both required to characterize the surfaces. One of these parameters will be used as the main height parameter k .

A summit/valley indicator (skewness) is necessary since a surface with summits has more

resistance than a corresponding surface with valleys instead of summits (Baumann (1999)). The kurtosis gives a relation of the height distribution to a normal Gaussian distribution, and may prove useful. The surface complexity gives a measure of many surface properties, such as feature count and slope. The autocorrelation length can be used as a wavelength parameter.

6.6 Correlation Results

The correlation attempts for the C -intercept are presented for the $k = S_q$ case. A successful correlation was not found, and suspected reasons for the lack of success are given.

6.6.1 C-Intercept Correlation

$k = S_q$ Model

As outlined in section 4.4.2, the C -intercept will be a function of k_2 , a non-dimensional term containing several roughness configuration parameters other than the main height parameter used for k . The relation for C is proposed to be of the form (equation 4.21)

$$C = A \log k_2 + E$$

and k_2 is proposed to be of the form (equation 4.24)

$$k_2 = (aX^b + cY^d + eZ^f + \dots)^g$$

where X , Y , and Z represent roughness parameters or combinations thereof, and a , b , c etc. are constants. The various correlation models attempted are now presented.

▷ Trial 1

The initial correlation model for k_2 was chosen to be

$$k_2 = \left[a(S_{dr})^b + c \left(\frac{S_z}{S_{al}} \right)^d + e S_{sk} + f (S_{ku} - 3) \right]^g \quad (6.1)$$

Since k_2 has to be non-dimensional, S_z and S_{al} are arranged together, resulting in a slope term (height divided by length). A smaller S_{al} for a given S_z results in a higher slope and a higher roughness. A higher S_z results in a higher roughness. A higher S_{dr} also results in higher roughness. A positive S_{sk} corresponds to a summit biased surface and will increase the roughness. A valley bias surface will have a negative S_{sk} and reduce the roughness modifier. The kurtosis has 3 subtracted from it to give a Gaussian distribution a kurtosis modifier value of zero. If the surface is slowly varying, it will have less roughness than a “spiky” surface, and have a kurtosis less than 3. This results in a negative kurtosis modifier, which reduces the roughness.

A successful correlation could not be obtained with this model however. The results seemed to indicate that the S_{ku} term was insignificant. This makes sense as the S_{dr} term encompasses its aspects. A slowly varying surface (small S_{ku}) will have a low complexity (small S_{dr}) and a surface with a larger than normal number of peaks and valleys will have a high complexity. The S_{ku} term could thus be dropped with confidence.

▷ **Trial 2**

The second correlation model for k_2 was chosen to be

$$k_2 = \left[a(S_{dr})^b + c \left(\frac{S_z}{S_{al}} \right)^d + e S_{sk} \right]^f \quad (6.2)$$

This model did not correlate either. The correlation procedure seemed to indicate that the S_{al} term was not necessary. This makes sense as the S_z/S_{al} term is a slope term, and as already shown in section 6.5 and Fig. 6.29, S_{dr} is related to $S\Delta_q$, the RMS slope. So S_{dr} could also be assumed to be related to the S_z/S_{al} term. Also, a surface with a small average wavelength (low S_{al}) will tend to have a high complexity (high S_{dr}). Thus S_{al} could then be dropped from the model with confidence. The height range parameter S_z is still kept however.

▷ **Trial 3**

The third correlation model for k_2 was chosen to be

$$k_2 = \left[a(S_{dr})^b + c \left(\frac{S_z}{S_q} \right)^d + e S_{sk} \right]^f \quad (6.3)$$

S_z still needed to be normalized so S_q was chosen for this purpose. S_z/S_q gives a height range ratio (to RMS height). This seems theoretically sound at first, as a surface with a higher S_z than another, but with the same S_q , will have a higher roughness and larger S_z/S_q term. However, upon closer inspection, it appears flawed. For two surfaces with the same S_z but different S_q 's, the lower S_q surface will have a lower roughness, but will increase the

S_z/S_q term.

This model did correlate though, but the coefficient of the S_z/S_q term, c , was negative. This is completely contrary to the model format, where each term is supposed to add to the roughness level. The negative coefficient underlines the contrary effects described in the previous paragraph. As such, this model was deemed unsuccessful, and is viewed with a large amount of skepticism. Since only four surfaces are used in the correlation, this correlation could be non-indicative of the theory behind the correlation, i.e. a coincidence. A model should make physical and theoretical sense as well as being successfully correlated.

▷ Trial 4

The fourth correlation model for k_2 was chosen to be

$$k_2 = \left[a(S_{dr})^b + c \left(\frac{S_q}{S_z} \right)^d + e S_{sk} \right]^f \quad (6.4)$$

As a further investigation of the third model, it was decided to invert the S_z/S_q term, and observe the results. The same contrary effects outlined in the third model discussion still apply. This model correlated as well, as it should, seeing as how the same parameters are used. However, the coefficients generated were such that the resulting k_2 (and thus k^+) values were of the order of 10^{12} . This is ridiculously large, as k_2 is supposed to act merely as a modifier to k and thus k_t^+ (equation 4.22) should still be in the normal range of previous k^+ values. These results confirm the inapplicability of the third and fourth correlation models.

Further theoretical consideration reveals the following. The S_z term must be an independent, or stand-alone, term, so that the S_q value that is already factored into k

does not affect the S_z term. In other words, the S_z term must capture the *magnitude* of S_z , i.e. the height of the large summits and valleys, and not describe the *relative* geometry of these dominant features. Therefore S_z cannot be normalized by another height parameter, such as S_q , or even by a length parameter, such as S_{al} or $S\lambda_q$, since these ratios give descriptive parameters of the relative geometry, such as aspect ratio or slope. These parameters are intrinsic to the surface, and as such are unsuitable for normalizing S_z . To use an aerodynamic analogy, normalizing S_z by S_q would be akin to normalizing the lift of a wing by its drag to determine its lift coefficient. This gives the lift-to-drag ratio instead, and does not describe the magnitude of the lift (relative to other wings), as the lift coefficient does. The drag is an intrinsic property of the wing, in the same fashion as the lift, as they are both based on the pressure distribution over the wing. This is similar to a surface, where S_z and S_q , for example, are both based on the surface geometry. If one changes the pressure distribution, or surface geometry, both parameters will change, and so their effects will not be separate, or independent, of each other. Returning to the wing analogy, what is done in practice is to determine the lift coefficient by normalizing the lift by an external, or reference, parameter, namely the freestream dynamic pressure. An additional parameter is also required and an unrelated parameter, the surface area of the wing, is used. So it is apparent that it is necessary to normalize S_z by such an external, or reference, parameter, one that is common to any surface. However, for surface metrology, it seems that there exists no such parameter. The surface geometry is a fixed system; it does not change if surrounding conditions change. Using the sample size length is inappropriate since for sample sizes greater than the minimum required to capture the surface, the configuration parameters do not change, i.e. S_z etc. is independent of any sample length scale. Using the minimum sample size required to characterize a surface would also be inappropriate, as two surfaces with equal S_z values, and thus equal roughness components due to S_z , can have different minimum sample sizes.

What is sought is a non-dimensional parameter that describes the magnitude of S_z relative to other surfaces, and not to a surface's own features. Until this parameter is found, it seems that a successful correlation will not be obtainable.

▷ **Trial 5**

Since including the S_z parameter as part of the k_2 term yielded no results, it was decided to remove the S_z parameter from k_2 and combine it with the main height parameter k . This would create a combined dimensional height parameter involving S_q and S_z , while leaving the k_2 term non-dimensional, and a function solely of S_{dr} and S_{sk} .

The main height parameter model was chosen to be of the form

$$k = S_q \left(\frac{S_z}{S_q} \right)^c \quad (6.5)$$

and k_2 is of the form

$$k_2 = \left[a(S_{dr})^b + e S_{sk} \right]^f \quad (6.6)$$

S_z can be normalized by S_q in the main height parameter since it directly modifies S_q , and not indirectly as it did in the previous k_2 models. Since the coefficient determination method used was a manual iteration method, only two values of c were tried, 0.5 and 0.75. Neither was successful.

A different combination of S_q and S_z was then tried:

$$k = S_q + c S_z \quad (6.7)$$

with k_2 remaining as in equation 6.6. For the surfaces tested, a loose relation of $S_q \simeq 0.15$

was found, and so c was chosen to equal 0.15, to give the S_q and S_z terms approximately the same magnitude. This model did not yield a successful correlation either.

The models in Trials 5 may hold promise for future work, as the concept of a combined, dimensional effective height parameter involving S_q and S_z seems theoretically sound.

6.7 Theoretical Correlation Results

As stated above, a successful correlation was not obtained, however as a demonstration of the methods involved in the correlation procedure, theoretical results and an analysis of these results will be presented, to give an understanding of said methods and to provide a base for future work. The theoretical results include a Δu^+ vs. k^+ relation, using a new k_t^+ term incorporating the additional roughness parameters, along with a discussion on the use of S_z vs. S_q as the main height parameter.

6.7.1 Theoretical Correlation Model

As noted previously, S_z is required to be normalized by some external or reference length. As an exercise, a unit length can be used, giving

$$S'_z = \frac{S_z}{\text{unit length}} \quad (6.8)$$

This is not a true normalization, and the choice of a unit length is arbitrary, as any length can be used, since it is common to all surfaces. Another reason a unit length cannot be used, is that if a surface is scaled up or down, its C -intercept on the shift curve would not change. Its k^+ and Δu^+ values would change in such a way as to keep the intercept constant, the location on the shift curve moving up or down along the log-law line. If the

intercept is constant, then k_2 must remain constant. However, since S'_z would increase or decrease with the scaling of the surface, then k_2 would change.

Using the S'_z term, a correlation model for k_2 was chosen to be

$$k_2 = \left[a(S_{dr})^b + c(S'_z)^d + e S_{sk} \right]^f \quad (6.9)$$

This model was “successfully” correlated, with the following coefficients determined, shown in Table 6.18.

Table 6.18: **S_q-Model Coefficients**

coefficient	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
value	1.85	0.20	214	1.20	0.122	4.33

This gives the final model:

$$k_2 = \left[1.85(S_{dr})^{0.20} + 214(S'_z)^{1.20} + 0.122 S_{sk} \right]^{4.33} \quad (6.10)$$

Table 6.19 lists the values of C and k_2 for the four fully rough surfaces. Fig. 6.30 illustrates the correlation, with error bounds on the C values.

Table 6.19: **S_q-Model C and k₂ Values**

surface	C-intercept	k ₂
CSH	1.10	20.342
CSP	2.67	38.688
FSH	2.19	31.777
FSP	1.10	20.263

The final relation for C is

$$C = A \log k_2 - 6.25 \quad (6.11)$$

Figure 6.30: The S_q -model correlation plot.

The linear regression coefficient of the curve fit, r^2 , was 0.99999996. The software package CurveExpert 1.3 was used for the curve fits. Table 6.20 shows the individual parameter values (within the brackets of equation 6.10) for each surface.

Table 6.20: S_q -Model Individual Parameter Values

surface	S_{dr} term	S'_z term	S_{sk} term
CSH	2.015	0.064	-0.0737
CSP	1.952	0.255	0.1196
FSH	2.167	0.063	-0.0068
FSP	1.878	0.058	0.0677

Examining each term's relative size and range, it is obvious that S_{dr} is the main operator, with the S'_z and S_{sk} terms acting as modifiers to it. This makes physical sense as S_{dr} encompasses so many properties of a surface, defined overall as the “complexity”. The skewness and height range have a greater percentage range in their values, illustrating that the dominant summits and valleys, and a surface's bias towards each, is important in distinguishing surfaces. The terms and coefficients inside the bracket of equation 6.10 act to align the C values with a common intercept, E . The power coefficient f then aligns this linear curve fit to the slope A .

k = S_z Model

For this model, S_z and S_q are simply interchanged. S_z is now used as the main height parameter k , and S_q is used in the k_2 parameter as a modifier. Based on the analysis for the S_q model the same correlation equation is used:

$$k_2 = \left[a(S_{dr})^b + c(S'_q)^d + e S_{sk} \right]^f \quad (6.12)$$

where S'_q is normalized in the same manner as S'_z was. This model was “successfully” correlated as well, with the following coefficients determined, shown in Table 6.21.

Table 6.21: **S_z-Model Coefficients**

coefficient	a	b	c	d	e	f
value	1.35	0.29	26300	1.10	0.530	0.60

This gives the final model:

$$k_2 = \left[1.35(S_{dr})^{0.29} + 26300(S'_q)^{1.10} + 0.530 S_{sk} \right]^{0.60} \quad (6.13)$$

Table 6.22 lists the values of C and k_2 for the four fully rough surfaces. Fig. 6.31 illustrates the correlation, with error bounds on the C values.

Table 6.22: S_z -Model C and k_2 Values

surface	C-intercept	k_2
CSH	-3.36	2.036
CSP	-1.86	3.763
FSH	-3.59	1.853
FSP	-3.16	2.207

The final relation for C is

$$C = A \log k_2 - 5.09 \quad (6.14)$$

The linear regression coefficient of the curve fit, r^2 , was 0.99999984. Table 6.23 shows the individual parameter values (within the brackets of equation 6.13) for each surface.

Table 6.23: S_z -Model Individual Parameter Values

surface	S_{dr} term	S'_q term	S_{sk} term
CSH	1.528	2.063	-0.3200
CSP	1.459	7.125	0.5197
FSH	1.697	1.128	-0.0295
FSP	1.380	2.066	0.2941

Examining each term's relative size and range, it is obvious that S_{dr} again is a main operator, but that the S'_q term is larger, and has a far greater variation in its values. This indicates that S_q captures more of the height property of a surface than S_z does, as in the $k = S_q$ correlation model, the S_z term was small. The S_{sk} term is again a modifier with a large value percentage range.

Figure 6.31: The S_z -model correlation plot.

6.7.2 Final Shift Correlation

$k = S_q$ Model

As outlined in section 4.4.2, a new k^+ can be obtained using the additional roughness parameters used to correlate C . A new “total roughness parameter” can be derived:

$$k_t = k k_2$$

Substituting in the roughness parameters, the following results:

$$k_t = S_q \left[1.85(S_{dr})^{0.20} + 214(S'_z)^{1.20} + 0.122 S_{sk} \right]^{4.33} \quad (6.15)$$

which gives

$$k_t^+ = \frac{k_t u_\tau}{\nu} \quad (6.16)$$

The final shift correlation model can then be given by equation 4.20, with the E -intercept inserted:

$$\Delta u^+ = A \log k_t^+ - 6.25 \quad (6.17)$$

Again, note that this is for the fully rough region only. Using this new correlation, the graph of Δu^+ vs. k_t^+ results, shown in Fig. 6.32. Observe how the individual shift curves have collapsed onto one “universal” curve, similar to the standard-sand-roughness curve.

k = S_z Model

As for the S_q model, the same analysis is applied to the S_z model.

$$k_t = S_z \left[1.35(S_{dr})^{0.29} + 26300(S'_q)^{1.10} + 0.530 S_{sk} \right]^{0.60} \quad (6.18)$$

The final shift correlation model can then be given by equation 4.20, with the E -intercept inserted:

$$\Delta u^+ = A \log k_t^+ - 5.09 \quad (6.19)$$

Using this new correlation, the graph of Δu^+ vs. k_t^+ results, shown in Fig. 6.33.

Figure 6.32: The new S_q -model shift curve, using k_t^+ instead of k^+ .

6.7.3 S_q vs. S_z Main Amplitude Parameter Choice

The correlation analysis has been performed for both cases of using $k = S_q$ and $k = S_z$. Arguments for both will now be given.

Using S_z as the main height parameter has the advantage of being equal to the k height of deterministic roughness. k is the height of the individual roughness elements, which are equal for uniform deterministic roughnesses, and represents the total height, or height range. S_z represents the height range for a stochastic surface. Therefore using S_z is more in keeping with tradition, being similar to k . This also gives a similar range of k^+ values

Figure 6.33: The new S_z -model shift curve, using k_t^+ instead of k^+ .

on the shift curve, which gives consistent k^+ values for the different roughness regimes.

Using S_q as the main height parameter has the advantage of statistical significance. For a purely stochastic surface, there are no well defined elements, and so S_z does not give a general measure of element height, but rather only the height range of the surface. S_q is a more robust and descriptive parameter, not relying solely on the highest summits and lowest valleys. The significance of S_q versus S_z is supported by the coefficients in their respective k_2 equations. When S_q is used as the main height parameter, the contribution of the S'_z term is small, showing that S_z is merely a modifier to the S_q term. However, when using S_z as the main height parameter, the contribution of the S'_q term is large, and

varies significantly, showing that S_q is a significant parameter. In the end, it makes little difference, as the k_t^+ values differ from the S_z and S_q -model k^+ values anyway, eliminating the primary advantage of using S_z as the main height parameter.

6.7.4 Relation to Previous Correlations

Standard Sand-Grain Roughness

As stated in section 4.4.4, to relate the height parameters for standard sand-grain and stochastic roughnesses, a simple multiplicative constant is used:

$$k_t = a k_s$$

Inserting the appropriate value of E , for the S_q -model or for the S_z -model, into equation 4.28, the following results:

$$k_s = 0.398 k_t \quad (S_q\text{-model}) \quad (6.20)$$

$$k_s = 0.640 k_t \quad (S_z\text{-model}) \quad (6.21)$$

Expanding this in terms of k_t yields the following:

$$k_s = 0.398 S_q \left[1.85(S_{dr})^{0.20} + 214 (S'_z)^{1.20} + 0.122 S_{sk} \right]^{4.33} \quad (S_q\text{-model}) \quad (6.22)$$

$$k_s = 0.640 S_z \left[1.35(S_{dr})^{0.29} + 26300 (S'_q)^{1.10} + 0.530 S_{sk} \right]^{0.60} \quad (S_z\text{-model}) \quad (6.23)$$

Waigh's Spacing Parameter

In section 4.4.5, the relation of k_2 to Waigh's spacing parameter was given by equation 4.29:

$$\lambda_{eff} = \frac{10^{\frac{5.78-E}{A}}}{k_2}$$

Substituting the shift curve intercept E , for both correlation models results in:

$$\lambda_{eff} = \frac{138.2}{k_2} \quad (S_q\text{-model}) \quad (6.24)$$

$$\lambda_{eff} = \frac{86.0}{k_2} \quad (S_z\text{-model}) \quad (6.25)$$

Expanding this in terms of λ_{eff} and k_2 yields the following.

$$\lambda \left(\frac{k}{b_m} \right)^{0.55} \left(\frac{A_W}{A_F} \right)^{1.38} = \frac{138.2}{\left[1.85(S_{dr})^{0.20} + 214(S'_z)^{1.20} + 0.122 S_{sk} \right]^{4.33}} \quad (6.26)$$

$$\lambda \left(\frac{k}{b_m} \right)^{0.55} \left(\frac{A_W}{A_F} \right)^{1.38} = \frac{86.0}{\left[1.35(S_{dr})^{0.29} + 26300(S'_q)^{1.10} + 0.530 S_{sk} \right]^{0.60}} \quad (6.27)$$

where equation 6.26 is for the S_q model and equation 6.27 is for the S_z model.

6.7.5 Skin Friction Law

As shown by equation 4.14, a skin friction equation results from combining the law-of-the-wall and intercept-shift-curve equations with the defect law:

$$\sqrt{\frac{2}{C_f}} = A \log \left(\frac{\delta}{k} \right) - D + B - C$$

where C is not a constant, but a function of roughness geometry. This functional relationship for stochastically rough surfaces is still unknown, despite the attempts of the current work.

Substituting in the theoretical newfound relation for C , the following results:

$$\sqrt{\frac{2}{C_f}} = A \log \left(\frac{\delta}{k_t} \right) - D + B - E \quad (6.28)$$

where E is a constant, its value depending on which height parameter model is used. Different relations for k_t will be applied, again, depending on the parameter model used. Substituting in $A = 5.62$, $D = -2.5$, $B = 5.0$, and the appropriate value of E for the model chosen, the following results:

$$\sqrt{\frac{2}{C_f}} = 5.62 \log \left(\frac{\delta}{k_t} \right) + 13.75 \quad (S_q\text{-model}) \quad (6.29)$$

$$\sqrt{\frac{2}{C_f}} = 5.62 \log \left(\frac{\delta}{k_t} \right) + 12.59 \quad (S_z\text{-model}) \quad (6.30)$$

These equations, if based on real correlations, would be of much benefit, since if the roughness parameters are known (to obtain k_t), and δ is known for a given boundary layer and x -location, the local skin friction coefficient for that given x -location can be calculated, as would be done in a CFD routine.

6.7.6 Summary of Theoretical Results

$$C = A \log \left[1.85(S_{dr})^{0.20} + 214(S'_z)^{1.20} + 0.122 S_{sk} \right]^{4.33} - 6.25 \quad (S_q\text{-Model})$$

$$C = A \log \left[1.35(S_{dr})^{0.29} + 26300(S'_q)^{1.10} + 0.530 S_{sk} \right]^{0.60} - 5.09 \quad (S_z\text{-Model})$$

$$k_t = S_q \left[1.85(S_{dr})^{0.20} + 214 (S'_z)^{1.20} + 0.122 S_{sk} \right]^{4.33} \quad (S_q\text{-Model})$$

$$k_t = S_z \left[1.35(S_{dr})^{0.29} + 26300 (S'_q)^{1.10} + 0.530 S_{sk} \right]^{0.60} \quad (S_z\text{-Model})$$

$$\Delta u^+ = A \log k_t^+ - 6.25 \quad (S_q\text{-Model})$$

$$\Delta u^+ = A \log k_t^+ - 5.09 \quad (S_z\text{-Model})$$

$$k_s = 0.398 k_t \quad (S_q\text{-Model})$$

$$k_s = 0.640 k_t \quad (S_z\text{-Model})$$

$$\lambda \left(\frac{k}{b_m} \right)^{0.55} \left(\frac{A_W}{A_F} \right)^{1.38} = \frac{138.2}{\left[1.85(S_{dr})^{0.20} + 214 (S'_z)^{1.20} + 0.122 S_{sk} \right]^{4.33}} \quad (S_q\text{-Model})$$

$$\lambda \left(\frac{k}{b_m} \right)^{0.55} \left(\frac{A_W}{A_F} \right)^{1.38} = \frac{86.0}{\left[1.35(S_{dr})^{0.29} + 26300 (S'_q)^{1.10} + 0.530 S_{sk} \right]^{0.60}} \quad (S_z\text{-Model})$$

$$\sqrt{\frac{2}{C_f}} = A \log \left(\frac{\delta}{k_t} \right) - B + C + 6.25 \quad (S_q\text{-Model})$$

$$\sqrt{\frac{2}{C_f}} = A \log \left(\frac{\delta}{k_t} \right) - B + C + 5.09 \quad (S_z\text{-Model})$$

Chapter 7

Conclusions and Recommendations

From the experimental and analytical results, several conclusions can be drawn.

1. Aerodynamic effects of nine different stochastic roughness configurations were investigated in wind tunnel experiments.
2. Laser scans were done on samples of each of the rough surfaces to produce detailed 3D digital maps. A variety of statistical configuration parameters were evaluated for each surface.
3. Using the momentum integral method, the skin friction coefficients of the nine surfaces were successfully found. These C_f values were in good consistency with the defect law.
4. Only four of the nine surfaces were found to be in the fully-rough regime.
5. Intercept values of the velocity-shift curves were evaluated and, for the fully-rough cases, attempts at correlating the intercepts in terms of some of the statistical roughness-configuration parameters were made.

6. Several roughness parameters are interrelated, with only a few required to characterize the roughness, aerodynamically speaking. Some of these key parameters were identified: the RMS height, the height range, the surface relative area ratio, and the skewness of the surface height distribution.
7. A correlation between several roughness parameters and the C -intercept for stochastically rough surfaces was not successfully determined. However, insight into the methods needed to find a successful correlation was obtained.
8. As a method exercise, using theoretical results, a “total” roughness parameter which completely describes the roughness was found, resulting in a “universal” shift curve with a constant intercept E . Equivalent values of other parameters used to characterize roughness can be easily obtained from the total-roughness parameter.

7.1 Suggestions for Future Research

In order to continue the current work, several suggestions are made:

- The unknown “external length-scale” parameter needed to normalize S_z must be determined. Alternatively, a combined dimensional height parameter involving S_q and S_z must be determined.
- A wider range of roughness samples should be tested, with a greater variation in surface types. This would generate a more valid and robust correlation, if one is found.
- The experiments should be run at varying speeds, particularly higher speeds, to give a greater span of k^+ values, extending from the transitionally rough to fully rough

regimes. A correlation extending to the transitionally rough regime could then be attempted.

- The wind tunnel should be configured to generate a zero-pressure gradient flow-field in the test section, so that the defect-law plot method can also provide reliable skin friction coefficients, which can be compared with those from the momentum integral method.
- If a correlation model is successfully determined, use the stochastic roughness parameters to characterize deterministic surfaces, and find if the stochastic correlation model is applicable to these surfaces.
- Determine any relationship between the effective y -origin for stochastic surfaces and the roughness parameters.

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Appendix A

Sample Traverse Input File

Below is a short sample traverse input file. The first number is the number of traverse points. The left column contains the x position values (zero in this case since only a vertical traverse is being done) and the right column contains the y position values (in mm).

```
22
0.00000 0.00000
0.00000 4.09575
0.00000 8.09625
0.00000 12.00150
0.00000 16.00200
0.00000 20.00250
0.00000 23.90775
0.00000 28.19400
0.00000 32.00400
0.00000 36.29025
0.00000 40.10025
0.00000 43.91025
0.00000 48.19650
0.00000 52.00650
0.00000 56.29275
0.00000 60.10275
0.00000 63.91275
0.00000 68.19900
0.00000 72.00900
0.00000 76.29525
0.00000 123.99645
0.00000 0.00000
```

Appendix B

Sample “Scanner” Program Raw Output Data

Below is a sample of the output file containing the average pressure differentials for each channel used. The columns contain, from left to right, the x position, the y position (in mm), $P_o - P_{c2}$, $P_s - P_{c2}$, and $P_{c1} - P_{c2}$ (all in Pascals).

0.00000	0.00000	501.700256	62.913860	1550.596680
0.00000	0.09525	484.867035	62.713215	1550.753662
0.00000	0.19050	517.978516	62.635025	1550.554077
0.00000	0.28575	556.445618	62.923397	1550.788086
0.00000	0.38100	586.997375	63.225574	1550.069214
0.00000	0.47625	612.709839	63.372677	1551.009521
0.00000	0.57150	631.812683	63.189407	1546.374146
0.00000	0.66675	635.429932	61.223724	1510.475098
0.00000	0.76200	654.015320	60.690754	1513.437256
0.00000	0.85725	671.017029	61.127117	1511.831543
0.00000	0.95250	690.957703	61.534153	1512.724487
0.00000	1.04775	706.195496	61.584953	1512.327759
0.00000	1.14300	719.744385	61.160118	1511.130493
0.00000	1.23825	735.556030	61.834381	1511.528931
0.00000	1.33350	741.050110	61.610222	1511.642334
0.00000	1.42875	757.975952	61.372044	1512.129761
.	.	.	.	.
0.00000	16.57350	1462.825439	61.078770	1466.005127
0.00000	16.85925	1461.799683	60.671165	1464.111938
0.00000	17.14500	1462.628784	60.447388	1464.987915
0.00000	17.43075	1461.968750	61.018002	1464.317383
0.00000	17.71650	1463.062866	60.824291	1465.380981
0.00000	18.00225	1462.686157	60.350716	1464.731567
0.00000	18.28800	1463.371460	60.635468	1466.275635
0.00000	18.57375	1463.368896	60.603645	1465.643311
0.00000	18.85950	1463.073975	60.750874	1465.547974
0.00000	19.14525	1464.886475	61.011295	1467.462769
0.00000	19.43100	1463.089233	60.416901	1464.877319
0.00000	19.71675	1463.398438	61.218269	1465.296143
0.00000	20.00250	1464.312866	60.983215	1466.468994
0.00000	123.99645	1449.843018	59.275177	1456.729492
0.00000	0.00000	473.720215	60.605854	1467.004150

Below is a sample of the output file containing the calculated pressure coefficients. The first number is the number of data points. The columns contain, from left to right, the x position, the y position (in mm), C_{p_o} , C_{p_s} , and $C_{p_{Pc1}}$ (which is always 1).

```

105
0.00000 0.00000 0.323553 0.040571 1.000000
0.00000 0.09525 0.312667 0.040438 1.000000
0.00000 0.19050 0.334062 0.040391 1.000000
0.00000 0.28575 0.358814 0.040572 1.000000
0.00000 0.38100 0.378688 0.040785 1.000000
0.00000 0.47625 0.395041 0.040854 1.000000
0.00000 0.57150 0.408578 0.040861 1.000000
0.00000 0.66675 0.420690 0.040529 1.000000
0.00000 0.76200 0.432136 0.040097 1.000000
0.00000 0.85725 0.443841 0.040428 1.000000
0.00000 0.95250 0.456761 0.040675 1.000000
0.00000 1.04775 0.466964 0.040718 1.000000
0.00000 1.14300 0.476296 0.040469 1.000000
0.00000 1.23825 0.486634 0.040906 1.000000
0.00000 1.33350 0.490228 0.040754 1.000000
.
.
0.00000 16.57350 0.997837 0.041659 1.000000
0.00000 16.85925 0.998427 0.041435 1.000000
0.00000 17.14500 0.998395 0.041256 1.000000
0.00000 17.43075 0.998401 0.041666 1.000000
0.00000 17.71650 0.998426 0.041502 1.000000
0.00000 18.00225 0.998609 0.041198 1.000000
0.00000 18.28800 0.998026 0.041350 1.000000
0.00000 18.57375 0.998454 0.041344 1.000000
0.00000 18.85950 0.998319 0.041448 1.000000
0.00000 19.14525 0.998253 0.041572 1.000000
0.00000 19.43100 0.998783 0.041241 1.000000
0.00000 19.71675 0.998711 0.041775 1.000000
0.00000 20.00250 0.998536 0.041580 1.000000
0.00000 123.99645 0.995279 0.040686 1.000000
0.00000 0.00000 0.322915 0.041308 1.000000

```

Below is a sample of the output file containing a record of the traverse. The first several lines may be entered into the Scanner program (the defaults are shown here). The zero offsets are shown where the PSI system was re-zeroed.

```
Date: 07-26-2001
Time: 15:15:11
Data Acquisition: Automatic
Data Mode: RUN
Test Section: TUNNEL
Ambient Pressure: 750.7 mm Hg
Ambient Temperature: 23 deg C
Active Channels: 1 2 8
Samples per channel: 300
Units: Pa
Zero Offsets:      -1.990284      -2.098153      -1.186485
Probe Point 1 => ( 0.00000, 0.00000), Moving: 6.969 sec.,
Sampling: 19.340 sec., I/O: 0.000 sec., Total: 26.309 sec.
Probe Point 2 => ( 0.00000, 0.09525), Moving: 6.973 sec.,
Sampling: 19.340 sec., I/O: 0.000 sec., Total: 26.313 sec.
Probe Point 3 => ( 0.00000, 0.19050), Moving: 6.969 sec.,
Sampling: 19.340 sec., I/O: 0.000 sec., Total: 26.309 sec.
Probe Point 4 => ( 0.00000, 0.28575), Moving: 6.969 sec.,
Sampling: 19.344 sec., I/O: 0.000 sec., Total: 26.313 sec.
Probe Point 5 => ( 0.00000, 0.38100), Moving: 6.969 sec.,
Sampling: 19.328 sec., I/O: 0.000 sec., Total: 26.297 sec.
Probe Point 6 => ( 0.00000, 0.47625), Moving: 6.980 sec.,
Sampling: 19.332 sec., I/O: 0.000 sec., Total: 26.313 sec.
.
.
Probe Point 55 => ( 0.00000, 7.71525), Moving: 7.027 sec.,
Sampling: 19.332 sec., I/O: 0.000 sec., Total: 26.359 sec.
Zero Offsets:      -1.735576      -1.859893      -0.632071
Probe Point 56 => ( 0.00000, 7.90575), Moving: 6.980 sec.,
Sampling: 19.332 sec., I/O: 0.000 sec., Total: 26.313 sec.
.
.
Probe Point 105 => ( 0.00000, 0.00000), Moving: 38.391 sec.,
Sampling: 19.340 sec., I/O: 0.000 sec., Total: 57.730 sec.
Total time: 2883.37109375 sec.
```

Appendix C

Sample Raw Surface Scan Data

Below is a sample of the raw data output from the surface scans. The “1.70141E38” are the garbage values, as a result of the filtering.

x,y,z(height) data in mm

```

0.00000    0.00000 1.70141E38
0.49987    0.00000 1.70141E38
0.99974    0.00000 1.70141E38
1.49962    0.00000 1.70141E38
.
.
0.00000    0.99974 1.70141E38
0.49987    0.99974 1.70141E38
0.99974    0.99974    0.09320
1.49962    0.99974    0.10227
1.99949    0.99974    0.11141
.
.
28.49270    0.99974    0.16070
28.99258    0.99974    0.14781
29.49245    0.99974 1.70141E38
29.99232    0.99974 1.70141E38
0.00000    1.49962 1.70141E38
0.49987    1.49962 1.70141E38
0.99974    1.49962    0.10131
1.49962    1.49962    0.10613
.
.
27.99283    28.99258    0.26373
28.49270    28.99258    0.24543
28.99258    28.99258    0.22937
29.49245    28.99258 1.70141E38
29.99232    28.99258 1.70141E38
0.00000    29.49245 1.70141E38
0.49987    29.49245 1.70141E38
.
.
28.99258    29.99232 1.70141E38
29.49245    29.99232 1.70141E38
29.99232    29.99232 1.70141E38

```


Appendix D

Sample C Program Surface Data Input File

Below is a sample of the conditioned data for input into the C programs to calculate the roughness parameters. The columns are x, y, and z data (all in mm).

```
0.0 0.0 -0.07959
0.5 0.0 -0.07052
1.0 0.0 -0.06138
1.5 0.0 -0.05836
2.0 0.0 -0.03908
2.5 0.0 -0.02559
3.0 0.0 -0.01912
3.5 0.0 -0.01864
4.0 0.0 -0.01841
.
.
26.5 0.0 0.00505
27.0 0.0 0.00031
27.5 0.0 -0.01209
28.0 0.0 -0.02498
0.0 0.5 -0.07148
0.5 0.5 -0.06666
1.0 0.5 -0.05475
1.5 0.5 -0.03885
2.0 0.5 -0.02492
.
.
23.0 28.0 0.13151
23.5 28.0 0.14220
24.0 28.0 0.14771
24.5 28.0 0.14132
25.0 28.0 0.13341
25.5 28.0 0.12544
26.0 28.0 0.11820
26.5 28.0 0.09750
27.0 28.0 0.09094
27.5 28.0 0.07264
28.0 28.0 0.05658
```

Appendix E

Computer Codes

Note: not all computer code listed was used in the final work.

```
/* C program to calculate surface roughness parameters for Blue Formica */
/* scanned surface from sorted xyz data */
/* This code is copyright 2002 Michael Ferrari */

#include <stdio.h>
#include <math.h>
#include <malloc.h>

#define linepoints 247
#define points 61009
#define dx 0.040
#define dy 0.040
#define PI 3.1415926539

/*****

typedef double array1[linepoints];
typedef double array2[linepoints];

/* main program */

void main()

{
array1 *zdata1;
array2 *zdata2;

double x,y,z, zsum, zbest, zmean, a, b;
int i,j,p,q;

double Sa, Sq, Ssk, Sku, Sp, Sv, St, Sk, NVOL1, NVOL2, Sdelq;
double Sdelqsk, Sdelqku, Slambdaq, Sgamma;
double Sc, Sal[(linepoints+1)/2][2], Sst[(linepoints+1)/2][2];
double Xdelq, Ydelq, Xdelqsk, Ydelqsk, Xdelqku, Ydelqku;
double Rotauxy[(linepoints+1)/2][(linepoints+1)/2], Taux, Tauy;
double Stauxy[(linepoints+1)/2][(linepoints+1)/2];

FILE *filein, *fileout1, *fileout2, *fileout3, *fileout4,
```

```

*fileout5, *fileout6;
filein = fopen("BF c data.txt", "r");
fileout1 = fopen("BF temp.txt", "w");
fileout2 = fopen("BF Rotau.txt", "w");
fileout3 = fopen("BF Sal's.txt", "w");
fileout4 = fopen("BF Stau.txt", "w");
fileout5 = fopen("BF Sst's.txt", "w");
fileout6 = fopen("BF Sgamma.txt", "w");

fprintf(fileout1, "Blue Formica roughness parameters\n\n");
fprintf(fileout2, "Blue Formica Ro_tau values\n\n");
fprintf(fileout2, "Tau_x  Tau_y  Ro_Tau\n\n");
fprintf(fileout3, "Taux  Tauy  Sal  RoTau\n\n");
fprintf(fileout4, "Tau_x  Tau_y  S_Tau\n\n");
fprintf(fileout5, "Tau_x  Tau_y  Sst  S_Tau\n\n");
fprintf(fileout6, "Tau_x  Tau_y  S_Tau\n\n");

zdata1 = malloc(sizeof(array1) * linepoints);
zdata2 = malloc(sizeof(array2) * linepoints);

/* read in the data */

for (j=0; j<=(linepoints-1); ++j) { /* j corresponds to y */
for (i=0; i<=(linepoints-1); ++i) { /* i corresponds to x */
fscanf(filein, "%lf %lf %lf", &x, &y, &z);
zdata1[j][i] = z;
}
}

/* calculate best fitting plane */

zsum = 0;
for (j=0; j<=(linepoints-1); ++j) {
for (i=0; i<=(linepoints-1); ++i) {
zsum = zsum + zdata1[j][i];
}
}
zbest = zsum/points;
fprintf(fileout1, "zbest %8.5f\n", zbest);

```

```
/* calc zero'd z values */

for (j=0; j<=(linepoints-1); ++j) {
for (i=0; i<=(linepoints-1); ++i) {
zdata1[j][i] = zdata1[j][i] - zbest; /* zdata1 now holds zero'd z values */
}
}

/* check to see mean is zero */

zsum = 0;
for (j=0; j<=(linepoints-1); ++j) {
for (i=0; i<=(linepoints-1); ++i) {
zsum = zsum + zdata1[j][i];
}
}
zmean = zsum/points;
fprintf(fileout1, "zmean %8.5f\n", zmean);

/* calc Sa */

zsum = 0;
for (j=0; j<=(linepoints-1); ++j) {
for (i=0; i<=(linepoints-1); ++i) {
zsum = zsum + fabs(zdata1[j][i]);
}
}
Sa = zsum/points;
fprintf(fileout1, "Sa %8.5f\n", Sa);

/* calc Sq */

zsum = 0;
for (j=0; j<=(linepoints-1); ++j) {
for (i=0; i<=(linepoints-1); ++i) {
zsum = zsum + zdata1[j][i]*zdata1[j][i];
}
}
```

```
}
Sq = sqrt(zsum/points);
fprintf(fileout1, "Sq %8.5f\n", Sq);

/* calc Ssk */

zsum = 0;
for (j=0; j<=(linepoints-1); ++j) {
  for (i=0; i<=(linepoints-1); ++i) {
    zsum = zsum + zdata1[j][i]*zdata1[j][i]*zdata1[j][i];
  }
}
Ssk = zsum/points/(Sq*Sq*Sq);
fprintf(fileout1, "Ssk %8.5f\n", Ssk);

/* calc Sku */

zsum = 0;
for (j=0; j<=(linepoints-1); ++j) {
  for (i=0; i<=(linepoints-1); ++i) {
    zsum = zsum + zdata1[j][i]*zdata1[j][i]*zdata1[j][i]*zdata1[j][i];
  }
}
Sku = zsum/points/(Sq*Sq*Sq*Sq);
fprintf(fileout1, "Sku %8.5f\n", Sku);

/* calc Sp */

Sp = 0;
for (j=0; j<=(linepoints-1); ++j) {
  for (i=0; i<=(linepoints-1); ++i) {
    if (zdata1[j][i] > Sp) {
      Sp = zdata1[j][i];
    }
  }
}
fprintf(fileout1, "Sp %8.5f\n", Sp);
```

```
/* calc Sv */

Sv = 0;
for (j=0; j<=(linepoints-1); ++j) {
  for (i=0; i<=(linepoints-1); ++i) {
    if (zdata1[j][i] < Sv) {
      Sv = zdata1[j][i];
    }
  }
}
fprintf(fileout1, "Sv %8.5f\n", Sv);

/* calc St */

St = Sp - Sv;
fprintf(fileout1, "St %8.5f\n", St);

/* calc Sk */

Sk = 2*Sa;
fprintf(fileout1, "Sk %8.5f\n", Sk);

/* calc NVOL1 and 2 */

for (j=0; j<=(linepoints-1); ++j) { /* zdata2 now hold Sa-cut-off z data */
  for (i=0; i<=(linepoints-1); ++i) {
    if (zdata1[j][i] > Sa) {
      zdata2[j][i] = Sa;
    }
    else {
      zdata2[j][i] = zdata1[j][i];
    }
  }
}

zsum = 0;
for (j=0; j<=(linepoints-1); ++j) {
  for (i=0; i<=(linepoints-1); ++i) {
    zsum = zsum + zdata2[j][i];
  }
}
```



```
}
}
zbest = zsum/points;
NVOL1 = (Sa-zbest);
fprintf(fileout1, "NVOL1 %8.5f\n", NVOL1);

zsum = 0;
for (j=0; j<=(linepoints-1); ++j) {
  for (i=0; i<=(linepoints-1); ++i) {
    zsum = zsum + (Sa - zdata2[j][i]);
  }
}
NVOL2 = zsum/points;
fprintf(fileout1, "NVOL2 %8.5f\n", NVOL2);

/* calc Sdelq */

zsum=0;
for (j=1; j<=(linepoints-1); ++j) {
  for (i=1; i<=(linepoints-1); ++i) {
    a = (zdata1[j][i] - zdata1[j][i-1])/dx;
    b = (zdata1[j][i] - zdata1[j-1][i])/dy;
    zsum = zsum + a*a + b*b;
  }
}
Sdelq = sqrt(zsum/(linepoints-1)/(linepoints-1));
fprintf(fileout1, "Sdelq %8.5f\n", Sdelq);

/* calc Sdelqsk */

zsum=0;
for (j=1; j<=(linepoints-1); ++j) {
  for (i=1; i<=(linepoints-1); ++i) {
    a = (zdata1[j][i] - zdata1[j][i-1])/dx;
    b = (zdata1[j][i] - zdata1[j-1][i])/dy;
    zsum = zsum + pow((a*a + b*b),3);
  }
}
Sdelqsk = zsum / ((linepoints-1)*(linepoints-1)*Sdelq*Sdelq*Sdelq);
fprintf(fileout1, "Sdelqsk %8.5f\n", Sdelqsk);
```

```
/* calc Sdelqku */

zsum=0;
for (j=1; j<=(linepoints-1); ++j) {
for (i=1; i<=(linepoints-1); ++i) {
a = (zdata1[j][i] - zdata1[j][i-1])/dx;
b = (zdata1[j][i] - zdata1[j-1][i])/dy;
zsum = zsum + pow((a*a + b*b),4);
}
}
Sdelqku = zsum / ((linepoints-1)*(linepoints-1)*Sdelq*Sdelq*Sdelq*Sdelq);
fprintf(fileout1, "Sdelqku %8.5f\n", Sdelqku);

/* calc Slambdaq */

Slambdaq = 2*PI*Sq/Sdelq;
fprintf(fileout1, "Slambdaq %8.5f\n", Slambdaq);

/* calc Sc */

zsum=0;
for (j=1; j<=(linepoints-2); ++j) {
for (i=1; i<=(linepoints-2); ++i) {
a = (zdata1[j][i+1] + zdata1[j][i-1] - 2*zdata1[j][i])/dx/dx;
b = (zdata1[j-1][i] + zdata1[j+1][i] - 2*zdata1[j][i])/dy/dy;
zsum = zsum + a+b;
}
}
Sc = -zsum/2/(linepoints-2)/(linepoints-2);
fprintf(fileout1, "Sc %8.5f\n", Sc);

/* calc Xdelq */

zsum=0;
b=0;
for (j=0; j<=(linepoints-1); ++j) {
for (i=1; i<=(linepoints-1); ++i) {
```

```
a = (zdata1[j][i] - zdata1[j][i-1])/dx;
zsum = zsum + a*a;
}
b = b + sqrt(zsum/(linepoints-1));
}
Xdelq = b/linepoints;
fprintf(fileout1, "Xdelq %8.5f\n", Xdelq);

/* calc Ydelq */

zsum=0;
b=0;
for (i=0; i<=(linepoints-1); ++i) {
for (j=1; j<=(linepoints-1); ++j) {
a = (zdata1[j][i] - zdata1[j-1][i])/dy;
zsum = zsum + a*a;
}
b = b + sqrt(zsum/(linepoints-1));
}
Ydelq = b/linepoints;
fprintf(fileout1, "Ydelq %8.5f\n", Ydelq);

/* calc Xdelqsk */

zsum=0;
b=0;
for (j=0; j<=(linepoints-1); ++j) {
for (i=1; i<=(linepoints-1); ++i) {
a = (zdata1[j][i] - zdata1[j][i-1])/dx;
zsum = zsum + a*a*a;
}
b = b + zsum/((linepoints-1)*Xdelq*Xdelq*Xdelq);
}
Xdelqsk = b/linepoints;
fprintf(fileout1, "Xdelqsk %8.5f\n", Xdelqsk);

/* calc Ydelqsk */

zsum=0;
```

```
b=0;
for (i=0; i<=(linepoints-1); ++i) {
for (j=1; j<=(linepoints-1); ++j) {
a = (zdata1[j][i] - zdata1[j-1][i])/dy;
zsum = zsum + a*a*a;
}
b = b + zsum/((linepoints-1)*Ydelq*Ydelq*Ydelq);
}
Ydelqsk = b/linepoints;
fprintf(fileout1, "Ydelqsk %8.5f\n", Ydelqsk);

/* calc Xdelqku */

zsum=0;
b=0;
for (j=0; j<=(linepoints-1); ++j) {
for (i=1; i<=(linepoints-1); ++i) {
a = (zdata1[j][i] - zdata1[j][i-1])/dx;
zsum = zsum + a*a*a*a;
}
b = b + zsum/((linepoints-1)*Xdelq*Xdelq*Xdelq*Xdelq);
}
Xdelqku = b/linepoints;
fprintf(fileout1, "Xdelqku %8.5f\n", Xdelqku);

/* calc Ydelqku */

zsum=0;
b=0;
for (i=0; i<=(linepoints-1); ++i) {
for (j=1; j<=(linepoints-1); ++j) {
a = (zdata1[j][i] - zdata1[j-1][i])/dy;
zsum = zsum + a*a*a*a;
}
b = b + zsum/((linepoints-1)*Ydelq*Ydelq*Ydelq*Ydelq);
}
Ydelqku = b/linepoints;
fprintf(fileout1, "Ydelqku %8.5f\n", Ydelqku);
```

```

/* calc Rtauxy */

for (q=0; q<=((linepoints-1)/2); ++q) {
for (p=0; p<=((linepoints-1)/2); ++p) { /* this will vary Taux and Tauy */
zsum=0;
for (j=0; j<=((linepoints-1)-q); ++j) { /* this will vary x & y locations */
for (i=0; i<=((linepoints-1)-p); ++i) {
zsum = zsum + zdata1[j][i]*
zdata1[j+q][i+p];
}
}

Rotauxy[q][p] = zsum/(linepoints-p)/(linepoints-q)/Sq/Sq;
Taux = p*dx;
Tauy = q*dy;
fprintf(fileout2, "%5.2f %5.2f %8.5f\n", Taux, Tauy, Rotauxy[q][p]);
}
}

/* calc Sal's */

for (q=0; q<=((linepoints-1)/2); ++q) {
for (p=0; p<=((linepoints-1)/2); ++p) { /* this will vary Taux and Tauy */
Taux = p*dx;
Tauy = q*dy;
if (Rotauxy[q][p] <= 0.2) {
Sal[q][0] = sqrt((Taux-dx)*(Taux-dx) + Tauy*Tauy);
Sal[q][1] = sqrt(Taux*Taux + Tauy*Tauy);
fprintf(fileout3, "%5.2f %5.2f %8.5f %8.5f\n", Taux-dx, Tauy, Sal[q][0],
Rotauxy[q][p-1]);
fprintf(fileout3, "%5.2f %5.2f %8.5f %8.5f\n", Taux, Tauy, Sal[q][1],
Rotauxy[q][p]);
break;
}
}
if (Taux == 0) break;
}

/* calc Stauxy */

```

```

for (q=0; q<=((linepoints-1)/2); ++q) {
for (p=0; p<=((linepoints-1)/2); ++p) { /* this will vary Taux and Tauy */
zsum=0;
for (j=0; j<=((linepoints-1-q)); ++j) { /* this will vary x & y locations */
for (i=0; i<=((linepoints-1-p)); ++i) {
zsum = zsum + (zdata1[j][i] - zdata1[j+q][i+p]) *
(zdata1[j][i] - zdata1[j+q][i+p]);
}
}
}

```

```

Stauxy[q][p] = zsum/(linepoints-p)/(linepoints-q);
Taux = p*dx;
Tauy = q*dy;
fprintf(fileout4, "%5.2f %5.2f %10.7f\n", Taux, Tauy, Stauxy[q][p]);
}
}

```

```

/* calc Sst's */

```

```

a=0.8*2*Sq*Sq;
for (q=0; q<=((linepoints-1)/2); ++q) {
for (p=0; p<=((linepoints-1)/2); ++p) { /* this will vary Taux and Tauy */
Taux = p*dx;
Tauy = q*dy;
if (Stauxy[q][p] >= a) {
Sst[q][0] = sqrt((Taux-dx)*(Taux-dx) + Tauy*Tauy);
Sst[q][1] = sqrt(Taux*Taux + Tauy*Tauy);
fprintf(fileout5, "%5.2f %5.2f %8.5f %10.7f\n", Taux-dx, Tauy,
Sst[q][0], Stauxy[q][p-1]);
fprintf(fileout5, "%5.2f %5.2f %8.5f %10.7f\n", Taux, Tauy, Sst[q][1],
Stauxy[q][p]);
break;
}
}
if (Taux == 0) break;
}

```

```

/* grab Sst's for Sgamma */

```

```

a=0.8*2*Sq*Sq;

```

```
zsum=0;
for (q=0; q<=((linepoints-1)/2); ++q) {
for (p=0; p<=((linepoints-1)/2); ++p) { /* this will vary Taux and Tauy */
Taux = p*dx;
Tauy = q*dy;
if (Stauxy[q][p] <= a) {
zsum = zsum + Stauxy[q][p];
fprintf(fileout6, "%5.2f %5.2f %10.7f\n", Taux, Tauy, Stauxy[q][p]);
}
else break;
}
if (Taux == 0) break;
}
Sgamma = zsum*dx*dy;
fprintf(fileout1, "Sgamma %13.10f\n", Sgamma);

free(zdata1);
free(zdata2);

fclose(filein);
fclose(fileout1);
fclose(fileout2);
fclose(fileout3);
fclose(fileout4);
fclose(fileout5);
fclose(fileout6);

}
```

```
/* C program to perform fractal analysis on Blue Formica scanned surface */
/* from sorted xyz data */
/* This code is copyright 2002 Michael Ferrari */

#include <stdio.h>
#include <math.h>
#include <malloc.h>

#define linepoints1 247
#define linepoints2 1025
#define points1 61009
#define points2 1050625
#define dx 0.040
#define dy 0.040
#define rmax 10

/*****

typedef double array1[linepoints1];
typedef double array2[linepoints2];

/* main program */

void main()

{
array1 *zdata1;
array2 *zdata4;

double zmean, zsum, zbest, Sa, rel_area, scale_obs;
double x,y,z,a,b,c,d, Area, Area1, Area2, delx, dely;
int i,j,r,s,t,q1,q2,p1,p2, qlines, plines;

FILE *filein, *fileout1, *fileout2;
filein = fopen("BF c data.txt", "r");
fileout1 = fopen("BF mirrored data.txt", "w");
fileout2 = fopen("BF fractal.txt", "w");
```



```
fprintf(fileout1, "Blue Formica mirrored data\n");
fprintf(fileout1, "x      y      z\n\n");
fprintf(fileout2, "Blue Formica fractal analysis\n\n");
fprintf(fileout2, "Expanded surface properties:\n");

zdata1 = malloc(sizeof(array1) * linepoints1);
zdata4 = malloc(sizeof(array2) * linepoints2);

/* read in the data */

for (j=0; j<=(linepoints1-1); ++j) { /* j corresponds to y */
for (i=0; i<=(linepoints1-1); ++i) { /* i corresponds to x */
fscanf(filein, "%lf %lf %lf", &x, &y, &z);
zdata1[j][i] = z;
}
}

/* calculate best fitting plane */

zsum = 0;
for (j=0; j<=(linepoints1-1); ++j) {
for (i=0; i<=(linepoints1-1); ++i) {
zsum = zsum + zdata1[j][i];
}
}
zbest = zsum/points1;

/* calc zero'd z values */

for (j=0; j<=(linepoints1-1); ++j) {
for (i=0; i<=(linepoints1-1); ++i) {
zdata1[j][i] = zdata1[j][i] - zbest; /* zdata1 now holds zero'd z values */
}
}

/* copy zdata1 to zdata4 */
```

```

for (j=0; j<=(linepoints1-1); ++j) {
for (i=0; i<=(linepoints1-1); ++i) {
zdata4[j][i] = zdata1[j][i]; /* zdata4 now holds zdata1 values */
}
}

/* mirror data in x direction */

for (j=0; j<=(linepoints1-1); ++j) {

for (i=0; i<=((linepoints1-1)-1); ++i) {
zdata4[j][i+linepoints1] = zdata4[j][((linepoints1-1)-i)];
/* mirror about 246 */
}
for (i=0; i<=((linepoints1-1)*2-1); ++i) {
zdata4[j][i+((linepoints1-1)*2+1)] = zdata4[j][((linepoints1-1)*2-1)-i];
/* mirror about 492 */
}
for (i=0; i<=((linepoints2-2 - (linepoints1-1)*4); ++i) {
zdata4[j][i+((linepoints1-1)*4+1)] = zdata4[j][((linepoints1-1)*4-1)-i];
/* mirror about 984 */
}
}

/* mirror data in y direction */

for (i=0; i<=(linepoints2-1); ++i) {

for (j=0; j<=((linepoints1-1)-1); ++j) {
zdata4[j+linepoints1][i] = zdata4[((linepoints1-1)-j)][i];
/* mirror about 246 */
}
for (j=0; j<=((linepoints1-1)*2-1); ++j) {
zdata4[j+((linepoints1-1)*2+1)][i] = zdata4[((linepoints1-1)*2-1)-j][i];
/* mirror about 492 */
}
for (j=0; j<=((linepoints2-2 - (linepoints1-1)*4); ++j) {
zdata4[j+((linepoints1-1)*4+1)][i] = zdata4[((linepoints1-1)*4-1)-j][i];
/* mirror about 984 */
}
}

```

```
}

/* write out mirrored data */

for (j=0; j<=(linepoints2-1); ++j) {
for (i=0; i<=(linepoints2-1); ++i) {
fprintf(fileout1, "%5.2f %5.2f %8.5f\n", i*dx, j*dy, zdata4[j][i]);
}
}

/* check to see mean is zero */

zsum = 0;
for (j=0; j<=(linepoints2-1); ++j) {
for (i=0; i<=(linepoints2-1); ++i) {
zsum = zsum + zdata4[j][i];
}
}
zmean = zsum/points2;
fprintf(fileout2, "zmean %8.5f\n", zmean);

/* calc Sa */

zsum = 0;
for (j=0; j<=(linepoints2-1); ++j) {
for (i=0; i<=(linepoints2-1); ++i) {
zsum = zsum + fabs(zdata4[j][i]);
}
}
Sa = zsum/points2;
fprintf(fileout2, "Sa %8.5f\n\n", Sa);

fprintf(fileout2, "scale_obs      rel_area\n\n");

/* fractal analysis */

for (r=0; r<=rmax; ++r) { /* varies scale of observation */
```

```

qlines = pow(2,r);
plines = pow(2,r);
Area = 0;
delx = (linepoints2-1)*dx/plines;
dely = (linepoints2-1)*dy/qlines;
/* calc 1st row of triangles (up and down facing) */
for (s=0; s<=(qlines-1); ++s) { /* varies y line */
for (t=0; t<=(plines-1); ++t) { /* varies x line */
q1 = (linepoints2-1)*s/qlines;
q2 = (linepoints2-1)*(s+1)/qlines;
p1 = (linepoints2-1)*t/plines;
p2 = (linepoints2-1)*(t+1)/plines;

a = (zdata4[q1][p1] - zdata4[q1][p2])/delx;
b = (zdata4[q1][p1] - zdata4[q2][p1])/dely;
c = (zdata4[q2][p1] - zdata4[q2][p2])/delx;
d = (zdata4[q1][p2] - zdata4[q2][p2])/dely;

Area1 = (delx*dely/2) * sqrt(1 + a*a + b*b + a*a*b*b);
Area2 = (delx*dely/2) * sqrt(1 + c*c + d*d + c*c*d*d);
Area = Area1 + Area2 + Area;
}
}

rel_area = Area/((linepoints2-1)*dx*(linepoints2-1)*dy);
scale_obs = ((linepoints2-1)*dx*(linepoints2-1)*dy) / (2*pow(2,2*r));

fprintf(fileout2, "%12.7f %15.10f\n", scale_obs, rel_area);
}

free(zdata1);
free(zdata4);

fclose(filein);
fclose(fileout1);
fclose(fileout2);

}

```

```
/* C program to grab Stau and Tau's for Sst line to plot log-log curve */
/* This code is copyright 2002 Michael Ferrari */

#include <stdio.h>
#include <math.h>
#include <malloc.h>

#define linepoints 247
#define dx 0.040
#define dy 0.040
#define tx 0.04
#define ty 0.40

/*****

typedef double array1[(linepoints+1)/2];

/* main program */

void main()

{
array1 *Sdata1;

int i,j;
double Taux, Tauy, Taumag;

FILE *filein, *fileout1;
filein = fopen("BF Stau.txt", "r");
fileout1 = fopen("BF topothesy.txt", "w");

fprintf(fileout1, "Blue Formica min Stau line for topothesy calc\n");
fprintf(fileout1, "Taux  Tauy  Taumag  Stau\n\n");

Sdata1 = malloc(sizeof(array1) * ((linepoints+1)/2));

/* read in the data */
```

```
for (j=0; j<=(linepoints-1)/2; ++j) {
for (i=0; i<=(linepoints-1)/2; ++i) {
fscanf(filein, "%lf %lf %lf", &Taux, &Tauy, &Sdata1[j][i]);
}
}

/* grab points along Sst line */

j=0;
i=0;
Tauy=0;
Taux=0;
Taumag=0;

while (Tauy <= (linepoints-1)/2*dy) { /* using Tauy since ty>tx */
fprintf(fileout1, "%5.2f %5.2f %7.4f %10.7f\n", Taux, Tauy,
Taumag, Sdata1[j][i]);
++i;
j=j+10; /* can't use i=i+tx/dx because of conversion from int to double */
Taux = dx*i;
Tauy = dy*j;
Taumag = sqrt(Taux*Taux + Tauy*Tauy);
}

free(Sdata1);

fclose(filein);
fclose(fileout1);
}
```

Appendix F

Sample C Program Output Files

This is a sample output file containing data to plot the areal autocorrelation function (dimensions in mm).

Blue Formica Rho_tau values

Tau_x	Tau_y	Ro_Tau
0.00	0.00	1.00000
0.04	0.00	0.99626
0.08	0.00	0.98528
0.12	0.00	0.96827
0.16	0.00	0.94613
.	.	.
4.76	0.00	0.03965
4.80	0.00	0.03526
4.84	0.00	0.03097
4.88	0.00	0.02677
4.92	0.00	0.02275
0.00	0.04	0.99347
0.04	0.04	0.98962
0.08	0.04	0.97870
0.12	0.04	0.96173
0.16	0.04	0.93961
0.20	0.04	0.91292
.	.	.
4.56	4.92	-0.12959
4.60	4.92	-0.13534
4.64	4.92	-0.14084
4.68	4.92	-0.14601
4.72	4.92	-0.15087
4.76	4.92	-0.15533
4.80	4.92	-0.15932
4.84	4.92	-0.16252
4.88	4.92	-0.16475
4.92	4.92	-0.16580

This is a sample output file containing the calculated areal autocorrelation lengths (in mm).

Taux	Tauy	Sal	RhoTau
1.16	0.00	1.16000	0.20362
1.20	0.00	1.20000	0.18946
1.16	0.04	1.16069	0.20033
1.20	0.04	1.20067	0.18612
1.12	0.08	1.12285	0.21075
1.16	0.08	1.16276	0.19548
1.12	0.12	1.12641	0.20431
1.16	0.12	1.16619	0.18914
1.08	0.16	1.09179	0.21240
1.12	0.16	1.13137	0.19638
1.08	0.20	1.09836	0.20274
1.12	0.20	1.13772	0.18703
.			
.			
0.60	0.60	0.84853	0.20861
0.64	0.60	0.87727	0.19300
0.52	0.64	0.82462	0.21224
0.56	0.64	0.85041	0.19729
0.44	0.68	0.80994	0.21253
0.48	0.68	0.83235	0.19903
0.36	0.72	0.80498	0.20881
0.40	0.72	0.82365	0.19730
0.28	0.76	0.80994	0.20065
0.32	0.76	0.82462	0.19157
0.08	0.80	0.80399	0.20244
0.12	0.80	0.80895	0.19863
-0.04	0.84	0.84095	0.05007
0.00	0.84	0.84000	0.17996

This is a sample output file containing the calculated fractal analysis data.

Blue Formica fractal analysis

Expanded surface properties:

zmean -0.00070

Sa 0.02505

scale_obs	rel_area
838.8608000	1.0000004314
209.7152000	1.0000062388
52.4288000	1.0000118050
13.1072000	1.0000499177
3.2768000	1.0002720961
0.8192000	1.0010120337
0.2048000	1.0026273091
0.0512000	1.0042878938
0.0128000	1.0053561221
0.0032000	1.0059944215
0.0008000	1.0062854409

Appendix G

Sample Boundary Layer Code Input and Output Files

This is a sample input file containing info to calculate the boundary layer parameters. The first line contains the initial H and θ , followed by λ and k , an initial guess for C_f , Re/L , U_e/U_{ref} , and the first x -distance. The second line contains U_{ref} and a reference length, c , both taken as 1.0 in this case. The left column is the U_e/U_{ref} values, and the right column is the x -distances.

```
1.513 0.00229398 23 0.00115228 0.00700 3054575 1 0.3175
1.00 1.00
5
1.010709 0.6763
1.020478 1.0319
1.029233 1.3938
1.034550 1.7240
```

This is a sample output file containing the calculated boundary layer parameters. The first series of values are those at intermediate steps, while the last series are the values for the specified x -points.

ROUGH WALL BOUNDARY LAYER CALCULATION

LAMBDA= 23.00 K/C=0.0011523 REYNO= 3054575.

C = 1.000 UREF = 1.000

Results at extra steps:

	U/Uref	H	THETA/C	DELS/C	Cf	UKONU
1	1.001339	1.473557	0.002442	0.013788	0.006980	208.214700
2	1.002677	1.501913	0.002583	0.013993	0.006681	203.973200
3	1.004016	1.535182	0.002717	0.014249	0.006393	199.802900
4	1.005355	1.547972	0.002845	0.014725	0.006225	197.424000
5	1.006693	1.549382	0.002970	0.015334	0.006120	196.009200
6	1.008032	1.546536	0.003092	0.015994	0.006042	195.005000
7	1.009370	1.542411	0.003212	0.016667	0.005975	194.184200
8	1.010709	1.537775	0.003330	0.017338	0.005916	193.481100
1	1.012337	1.532461	0.003485	0.018210	0.005841	192.564800
2	1.013965	1.527164	0.003637	0.019073	0.005774	191.760500
3	1.015594	1.522113	0.003786	0.019926	0.005712	191.038000
4	1.017222	1.517355	0.003932	0.020766	0.005655	190.382600
5	1.018850	1.512881	0.004077	0.021594	0.005602	189.785000
1	1.022229	1.504851	0.004393	0.023387	0.005489	188.495900
2	1.023980	1.500665	0.004563	0.024363	0.005434	187.862800
3	1.025731	1.496535	0.004731	0.025328	0.005383	187.300300
4	1.027482	1.492579	0.004897	0.026281	0.005336	186.792400
5	1.029233	1.488880	0.005059	0.027221	0.005291	186.326900
1	1.030562	1.488173	0.005253	0.028268	0.005227	185.440700
2	1.031891	1.485473	0.005444	0.029354	0.005175	184.757900
3	1.033221	1.482250	0.005631	0.030446	0.005129	184.171200
4	1.034550	1.478963	0.005817	0.031532	0.005087	183.643700

X/C	U/Uref	Hconv	THETA/C	DELTASTAR/C	DELTA/C	Cf	Utau*K/NU
0.3175	1.00	1.513	0.002294	0.00347	0.01705	0.00694	207.4
0.6763	1.01	1.538	0.003330	0.00512	0.02246	0.00592	193.5
1.0319	1.02	1.509	0.004219	0.00636	0.02877	0.00555	189.2
1.3938	1.03	1.489	0.005059	0.00753	0.03475	0.00529	186.3
1.7240	1.03	1.479	0.005817	0.00860	0.04013	0.00509	183.6