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APPLICATIONS OF MINIMUM-SHIFT-KEYING
IN LOW-DATA-RATE REMOTE SATELLITE
STATIONS AND MICROWAVE PHASE-LOCKED ^{AMPLIFIERS} ~~AMPLIFIERS~~

UNIVERSITY/UNIVERSITÉ

CARLETON UNIVERSITY

DEGREE FOR WHICH THESIS WAS PRESENTED/

GRADE POUR LEQUEL CETTE THÈSE FUT PRÉSENTÉE

Ph. D

YEAR THIS DEGREE CONFERRED/ANNÉE D'OBTENTION DE CE DEGRÉ

1979

NAME OF SUPERVISOR/NOM DU DIRECTEUR DE THÈSE

Dr W. J. Chudobiak, D. J. S. Wight

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APPLICATIONS OF MINIMUM-SHIFT-KEYING IN LOW-DATA-RATE
REMOTE SATELLITE STATIONS AND MICROWAVE PHASE-LOCKED-AMPLIFICATION

by

Surinder Kumar

A Thesis Submitted to the Faculty of Graduate Studies

in Partial Fulfilment

of the

Requirements for the Degree

DOCTOR OF PHILOSOPHY

Department of Electronics

Carleton University

Ottawa, Canada

1978

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The undersigned recommend to the Faculty of Graduate Studies
the acceptance of the thesis:

Applications of Minimum-Shift-Keying in
Low Data Rate Remote Satellite Stations and
Microwave Phase-Locked-Amplification

submitted by Surinder Kumar in partial fulfillment of the
requirements for the degree of Doctor of Philosophy.

Professor W.J. Ludobiak
Supervisor

Professor J.S. Wight
Supervisor

Professor A.R. Boothroyd
Chairman

December 1978

ABSTRACT

The work reported in this thesis presents solutions for two problem areas, both of which employ certain advantageous characteristics of the Minimum-Shift-Keying (MSK) digital modulation technique.

Part A of the thesis deals with the problem of obtaining a high stability frequency reference for a satellite-based data-collection system which collects data from remote unattended ground stations. A method for obtaining this reference from the recovered clock of an MSK modulated command signal is proposed and analyzed. The reference-recovery circuit and the data station transmitter carrier phase noise are analyzed in terms of the resulting degradation of the MSK system bit-error-rate performance. The theoretical results are compared with those obtained from an experimental system and good agreement is demonstrated.

In Part B of the thesis the use of an Injection-Locked-Oscillator Phase-Locked-Amplifier (ILO-PLA) as a high gain microwave amplifier for MSK modulated signals is proposed and analyzed. Based on the theoretical and experimental results, it is demonstrated that an MSK modulated signal is compatible for amplification with a high gain microwave ILO-PLA because of the phase continuous nature of MSK. The criteria used for this evaluation are the spectrum-spreading of the filtered input MSK signal by the ILO-PLA and the bit-error-rate performance degradation introduced by the ILO-PLA. The results show that an ILO-PLA causes only a small spreading of the amplified signal spectrum and results in a bit-error-rate performance which follows the general form of PSK in an ideal channel. In addition the degradation in performance is maintained to a low value (≈ 1.3 dB) even

when the two-sided locking bandwidth is reduced to a value equal to twice the MSK system bit-rate.

The results demonstrate that the spectrum-spreading and bit-error-rate performance of an ILO amplifier is superior for an MSK signal relative to a QPSK signal and further demonstrate that a single-stage high-gain microwave amplifier for MSK signals may be realized using an ILO-PLA.

ACKNOWLEDGEMENTS

I would like to express my deep appreciation of the guidance which my two program supervisors, Prof. W.J. Chudobiak and Prof. J.S. Wight have given me. Their continuous involvement and encouragement was a big factor in successful completion of this thesis work.

I would also like to acknowledge many valuable discussions with Dr. James Cavers of the Department of Systems Engineering and Computing Science at Carleton University, and Dr. Lawrence Stark of the Department of Electronics, Carleton University, Ottawa.

Many fruitful discussions with my fellow graduate students who helped create an atmosphere of encouragement and emotional support are highly appreciated.

Specially I must thank my wife and son for their understanding, encouragement and patience.

Financial assistance by Canadian Commonwealth Scholarship and Fellowship Committee is gratefully acknowledged.

Finally, I would like to thank Mrs. Ruth Thomas for efficient and accurate typing.

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CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

The ever expanding need for data communication has led to the search for better digital modulation techniques. The selection of the modulation method is a major decision in the design of a communication system. This selection is governed by a large number of system design parameters and the ability of the particular modulation method to meet the system design requirement while conserving the frequency spectrum, the transmitter power and circuit complexity. One modulation technique, minimum shift keying (MSK), possesses a number of favourable properties concerning spectrum efficiency, circuit complexity and power conservation. This modulation method has the unique feature of being equivalent to either a continuous phase, frequency shift keying or offset-keyed, sinusoidally-weighted quadrature phase shift keying, and has been a subject of much recent research [8]- [13] [17], [18].

In this thesis we have proposed and investigated the application of this modulation in two areas, one at a system level and the other at circuit level. In each of these areas the MSK approach is shown to have distinct advantages. The first area is a low-bit-rate data communication system for transmitting data via a satellite from remote unattended data stations. The second area pertains to a study of the feasibility of amplifying MSK modulated signals using a phase-locked oscillator as a microwave amplifier (hereafter referred to as a phase-locked-amplifier). We have established in this work that because of certain favourable features, MSK may be used advantageously in these two areas.

The unifying theme of this thesis is the MSK modulation technique which is treated in Chapter 2. A more detailed outline of each of the two problem areas of the thesis is given in the following sections.

1.2 PROBLEM AREAS

1.2.1 The Remote Unattended Low-Bit-Rate Satellite Data Stations

A variety of the satellite based systems of the form shown in Fig. 1.1 have been proposed [1] [2] for transmitting environmental, meteorological and geological data from remote geographic locations using low cost unattended ground stations. These proposed ground stations use various types of sensors to measure the required data and are commanded from a central command station (CCS). At the CCS the data thus collected are processed on a computer.

These stations work on the single channel per carrier (SCPC) mode of the satellite and are allocated a relatively narrow RF bandwidth (a few K Hz) at the microwave transmission frequency of typically 6 G Hz. The narrow RF bandwidth and the large number of such ground stations impose the requirement of a very high-stability frequency reference for the transmitter carrier at each remote station.

The provision of a highly stable frequency reference at each ground station is prohibited by the cost and the prime power considerations. Also, transmitting a stable frequency reference from a central command station to each ground station is complicated by the frequency instability of the data station receiver local oscillator, the satellite local oscillator frequency instability and by the complexity of the ground station circuitry required to regenerate the reference from the received signal.

Fig. 1.1 The Satellite Based Data-Collection System

A technique for providing the frequency reference is introduced and analyzed in this study. In the proposed technique, an MSK modulated command signal is generated at the Central Command Station using a very high stability clock (preferably an atomic standard). Each ground station recovers the clock from the command signal using an MSK clock recovery method. The recovered clock is used as the frequency reference to generate the transmitter carrier at each ground station. It is also used for coherent demodulation at the receiver. All the ground stations thus effectively use the same frequency reference (i.e. the clock signal of the CCS) and essentially have the long term frequency stability of the CCS clock. It may be noted that the recovered clock frequency stability is independent of any small frequency instability due to the satellite transponder local oscillator, and as will be shown later, independent of any local oscillator frequency instability of the data station receiver.

This brief discussion highlights the first problem area for which MSK modulation has been studied and found to be attractive. The second problem area is introduced in Section 1.2.2 below.

1.2.2 The Microwave Phase-Locked-Oscillator as an Amplifier of Digitally Modulated Signals

For angle modulated signals (viz, phase or frequency modulation, digital or analog) the information is contained in the angle and consequently a phase-locked-oscillator whose phase follows the input phase may be used as a high gain single stage microwave amplifier. The use of a phase-locked-oscillator as an amplifier for analog angle modulated signals is well known and such an amplifier is commonly referred to as a Phase Locked Amplifier (PLA). The PLA may be either an Injection Locked

Oscillator (ILO) [3] or a Voltage Tuned Oscillator (VTO-PLA) [4]. In the ILO version, a low power angle-modulated microwave signal is injected into a high power microwave oscillator which phase locks to the weaker signal over a certain frequency range. In the VTO-PLA version, a voltage tuned oscillator is used as the source of microwave power. A sample of the VTO output is phase compared with the modulated input signal in a phase detector. The phase detector output signal is amplified using a wideband DC coupled amplifier, and is fed to the voltage tuning terminal of the VTO to realise a first order phase-lock-loop.

The use of an ILO as the microwave amplifier of Quaternary Phase Shift Keying (QPSK) has been reported by Kuno [5] and Paik [6]. Kuno [5] concluded that the required locking bandwidth (i.e. the frequency range over which the ILO locks to the input signal) is determined by the phase transients at the interbit switching instants. He further concluded that a locking bandwidth equal to five to six times the bit rate is required in general for an ILO-PLA. Since the achievable gain of the PLA is inversely proportional to the square of the locking bandwidth, a large locking bandwidth implies a reduced gain for the ILO-PLA. Paik [6] however measured the performance of a practical QPSK system with an ILO-PLA and observed that at high E_b/N_0 (Bit energy to noise spectral density ratio), the bit-error-rate did not decrease as expected but became high and erratic. Figure 11 of [6] is reproduced here as Fig. 1.2 to illustrate this result. One explanation of this behaviour may be the fact that for QPSK there may be 180° phase transients at some interbit switching instants. A 180° phase transient implies a change of the

Fig. 1.2 The QPSK System Bit-Error-Rate Performance with and without ILO-PLA.

polarity of signal. Therefore if a QPSK signal is band limited¹, the envelope of the filtered QPSK signal goes through zero at the interbit switching instants with 180° phase transients. This happens because of the smearing of the opposite polarity adjacent signals caused by filtering. As a result, the ILO has a much reduced input signal at these instants and may therefore unlock resulting in a high bit-error-rate. In any case from the results of Kuno [5] and Paik [6] it is clear that the performance of the locked oscillator amplifiers for digitally modulated signals is strongly limited by the phase transients at the interbit switching instants.

As will be explained in Chapter 2, for MSK modulation, the ninety degree phase shift of the carrier in a bit interval is obtained by a linear change of the phase with time and the phase continuity is maintained at the interbit switching instants. As a result, an MSK modulated signal is phase continuous in nature and is essentially free of any phase transients. Based on this, it is proposed in this thesis, that this modulation is compatible for microwave amplification by a phase locked amplifier. An analysis and experimental evaluation of the PLA performance for MSK signals is the subject matter of part B of this thesis.

1.3 THESIS OBJECTIVES

1.3.1 Chronological Note

The starting point of the research reported in this thesis was the problem of obtaining a frequency reference for the remote sensor stations.

1 In practical systems bandlimiting is always required in order to limit the transmitted signal spectrum within the allocated channel bandwidth.

In the early stages of the work the feasibility of using the carrier of the received command signal as the frequency reference was investigated. Some circuits for low-level microwave frequency dividers using phase-locked-oscillator were constructed. In this context the PLO circuit was studied and the difficulty of using it as a phase-locked-amplifier for digital microwave signal (as previously reported in literature) was observed. Later the approach of using the received carrier as the frequency reference for the sensor stations was abandoned in favour of using the recovered clock as reference, employing certain favourable characteristics of the MSK modulation. From the phase continuous nature of the MSK modulation, it was inferred that this modulation would also be suitable for realizing a high gain microwave PLA for digitally modulated signals. Thus two significant areas of study, each associated with MSK modulation were identified. At this stage of the work it was decided to devote roughly equal effort in the two areas discussed in Sections 1.2.1 and 1.2.2 above.

1.3.2 Objectives

In part A of this thesis we will be primarily concerned with the development and analysis of a method to obtain the frequency reference at the sensor stations. The theoretical analysis to investigate the feasibility of the proposed system will be carried out and experimental work will be presented to verify the analysis. In part A of the thesis the main objectives are thus:

1. Discussion of the proposed method for obtaining the frequency reference.
- 2/ The theoretical analysis of the carrier and clock recovery system for MSK.

3. An experimental verification of the above analysis.

A tree diagram for the objectives is shown in Fig. 1.3.

In part B of the thesis we will be mainly concerned with the evaluation of PLA performance as an amplifier of MSK modulated signals. The two principal performance criteria used in the study will be the spreading of the filtered spectrum caused by the PLA, and the bit-error-rate performance degradation introduced by the PLA in an MSK system.

In practical communication systems the modulated signal is filtered at the Intermediate Frequency (IF). There may be some spreading of the filtered spectrum when the filtered IF signal is upconverted and amplified by the nonlinear microwave amplifier. Since the spreading of the spectrum may require undesirable narrowband microwave filtering it is important that this does not occur to any appreciable degree.

Any bit-error-performance degradation due to PLA will result in corresponding increase in the required system E_b/N_0 ratio. The available gain of the PLA will be mainly constrained by the permissible degradation in the bit-error-rate performance. In part B of the thesis the main objectives are:

1. To measure the spectrum-spreading effects of the ILO-PLA for the filtered MSK input.
2. To measure the bit-error-rate performance of the ILO-PLA for MSK signals.
3. To present an analytical model of the ILO-PLA for MSK signals.

A tree diagram for these objectives is shown in Fig. 1.3.

Fig. 1.3 The Thesis Objectives

1.4 A GUIDE TO THE THESIS

In this thesis we seek to study the application of MSK to the two problems discussed in Section 1.2 above. Consequently the thesis has been divided into parts A and B. Chapter 2 presents a general description of MSK modulation. The contents of this chapter are relevant to both parts A and B.

Chapter 3 introduces part A by discussing general system design considerations for the satellite sensor stations. The purpose of this chapter is to present general system calculations for the sensor stations.

Chapter 4 describes the design and analysis of the sensor stations. The chapter is divided into six sections. Following an introduction to the chapter in Section 4.1, the proposed scheme for the sensor stations is introduced in Section 4.2. A complete analysis of the MSK carrier and clock recovery circuit is given in Section 4.3. The effect of carrier phase noise on the system bit-error performance is analysed in Section 4.4. The results of a 2.4 Kbits/sec MSK modem are given in Section 4.5. The experimental performance of the reference recovery circuit is compared in this section with the analysis of Section 4.3. Conclusions based on the theoretical and experimental work of Chapter 4 are given in Section 4.6.

Part B of the thesis commences with Chapter 5. This chapter introduces the concept of phase locked amplification. Chapter 6 presents theoretical and experimental spectrum-spreading results for an ILO-PLA used to amplify a 5 M bits/sec, 6 G Hz MSK signal. An analytical model of the ILO-PLA, adapted for MSK signals, is used to compute the spectrum of MSK signals at the ILO output.

Chapter 7 contains the measured bit-error-rate results of the ILO-PLA for MSK signals. To experimentally characterize the performance

of the ILO-PLA the bit-error-rate results were measured by varying the key design parameters of the ILO-PLA.

The conclusions for both parts A and B are summarized in Chapter 8. This chapter also includes the suggestions for future work.

Appendix A gives the details of a combined analytical and numerical method used to compute the spectra in Chapter 6.

CHAPTER 2

MINIMUM SHIFT KEYING (MSK)2.1 INTRODUCTION

The Minimum Shift Keying modulation was patented by Doelz and Heald of Collins Radio Company in 1961 [7]. This form of modulation gained popularity after de Buda [8] [9] introduced a phase continuous and phase coherent frequency shift keying modulation which he termed Fast Frequency Shift Keying (FFSK). As pointed out by de Buda [9] FFSK and MSK differ only in the baseband encoding*. The two modulations have the same spectral properties and are treated equivalently in the literature [10], [11].

For a communication system, the choice of possible digital modulation techniques is based on the properties listed below:

- (i) E_b/N_0 (Bit Energy to Noise Spectral Density Ratio) required to achieve a given bit-error-rate.
- (ii) Bandwidth required for transmission which in turn depends on the spectral properties of the modulation technique.
- (iii) Degradation in bit-error performance and spreading of the spectrum caused by amplitude limiting of the bandlimited signal.
- (iv) Signal synchronization properties.
- (v) Performance for fading channels.
- (vi) Signal interference and noise immunity considerations.
- (vii) Cost and complexity of the implementation of the transmitter and receiver.

* Transformation from MSK encoding to FFSK encoding consists of differentially decoding the MSK data, and then polarity reversing every second bit. Conversely, if in FFSK every second data bit is reversed and the result differentially encoded, then MSK results.

Some of these properties are briefly discussed for MSK modulation in this chapter. The phase continuous nature of MSK is illustrated in Section 2.2 by comparing it with other phase-shift-keying modulations. Two equivalent techniques of realising MSK as a continuous phase FSK or as sinusoidally weighted OK-QPSK are included in Section 2.3. The power spectrum of MSK is given in Section 2.4. The spectrum of the squared MSK signal is briefly reviewed in Section 2.5.

2.2 PHASE SHIFT KEYING

2.2.1 Binary Phase Shift Keying (BPSK)

BPSK is generated by phase-shift-keying of a constant amplitude carrier, with sharp keying transitions between two phase stages separated by π radians.

For an ideal channel, coherent matched filter detection of the BPSK gives a bit error rate [12]:

$$P_b(e) = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} \exp(-y^2/2) dy$$

$$\approx \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}$$

where E_b/N_0 = bit energy to noise spectral density ratio and erfc is complementary error function.

An ideal channel is considered to be one having linear amplitude and phase response, infinite bandwidth, corrupted only by white Gaussian noise. In addition for the ideal channel, perfect carrier and clock references are assumed to be available at the receiver.

2.2.2 Quarternary Phase Shift Keying (QPSK)

In quarternary phase shift keying, two phase quadrature carriers are biphase modulated by alternate data bits. The resulting effect is equivalent to encoding each pair of data bits into one of the four carrier phases differing by 90° as shown in Fig. 2.1(b).

For QPSK, the data sequences which biphase modulate I and Q channels are coincidentally aligned in time sequence. If both the binary components remain the same, no change of state takes place. If only one binary component changes state, then the carrier has phase transitions of $\pm\pi/2$ radians. The change in state of both the binary components results in a phase change of π radians, thus resulting in the reversal of the polarity of the signal. The filtering of QPSK signal causes the envelope to go to zero at the point where polarity reversal had occurred. Amplification of the bandlimited signal by a nonlinear amplifier virtually restores the filtered spectrum to the unfiltered value [13].

2.2.3 Offset Keyed QPSK (OK-QPSK)

In offset keyed QPSK, I and Q channel sequences are offset in time by T (T = bit interval). The switching of two channels does not occur simultaneously thus avoiding phase changes of π radians and consequent polarity reversals. This also alleviates the problem of spectrum restoration in a nonlinear amplifier [13].

Even though any abrupt phase shift of π radians is avoided in OK-QPSK, abrupt $\pi/2$ phase transients are still present. In MSK, these abrupt phase transients also are removed.

Fig. 2.1 The Phasor Diagrams

- (a) MSK
- (b) QPSK

2.2.4 Minimum Shift Keying

In MSK, phase is continuously varied by $\pi/2$ radians over a bit interval. The phase is advanced for a one and retarded for a zero. In addition, the phase continuity is maintained at the interbit switching instant. Absence of any phase discontinuity in MSK leads to the desirable compact power spectrum.

The phasor diagrams of the QPSK and MSK are shown in Fig. 2.1. It may be seen from this figure that MSK is free of phase transients as compared to QPSK which has both 90° and 180° phase transients. The symbol waveforms for QPSK, OK-QPSK and MSK are shown in Fig. 2.2(a) and as this figure shows MSK may be visualized as the OK-QPSK with the sinusoidal symbol weighting in I and Q channels. The modulated carrier time waveform for MSK for an arbitrary six bit data string shown in Fig. 2.2(b) further illustrates the continuous phase nature of MSK.

In Section 2.3, two different but spectrally equivalent representations of MSK and two different techniques of realizing the MSK waveform are presented.

2.3 TWO EQUIVALENT REPRESENTATIONS OF MSK

MSK can be viewed either as a continuous phase FSK with modulation index = $1/2$ (where modulation index is defined as)

$$\frac{f_{\text{mark}} - f_{\text{space}}}{\text{bit rate}}$$

or as OK-QPSK with sinusoidal quadrature antipodal I and Q channel data symbols. Thus, two different hardware techniques can be used to realize MSK, producing basically the same waveform.

Fig. 2.2(a) QPSK, OK-QPSK, MSK, I and Q Channel Symbol Waveforms.
 Symbol interval = $2T$
 Bit interval = T

Fig. 2.2(b) The MSK Waveform for an Arbitrary Six Bit Data String

2.3.1 Continuous Phase FSK Technique

When viewed as continuous phase FSK, we can write the modulated signal waveform as

$$y_{\text{MSK}}(t) = \cos\left(\omega_c t + \frac{\pi u_k t}{2T} + x_k\right) \quad (2.1)$$

$$(k-1)T \leq t \leq kT$$

ω_c = carrier or the centre frequency

u_k = kth data bit = ± 1

x_k = recursive variable designed to maintain the phase continuity at the interbit switching instant

T = bit interval

As can be seen from Eqn. (2.1), the modulation index

$$\frac{\frac{\pi}{2T}(+1) - \frac{\pi}{2T}(-1)}{2\pi/T} = \frac{1}{2}$$

The recursive relation for x_k is obtained by equating phase at the $(k-1)$ th to kth interbit switching instant ($t=(k-1)T$).

$$\omega_c t + \frac{\pi u_k t}{2T} + x_k = \omega_c t + \frac{\pi u_{k-1} t}{2T} + x_{k-1}$$

Putting $t = (k-1)T$ above

$$x_k = x_{k-1} + \frac{\pi(k-1)}{2} (u_{k-1} - u_k) \quad (2.2)$$

Equations (2.1) and (2.2) together define the MSK waveform.

Fig. 2.3 shows the functional diagram of the circuit used to generate an MSK waveform based on its visualization as a continuous phase FSK [42]. In this scheme, two frequencies differing by the clock frequency and locked to the clock are coherently derived by dividing the frequency of a high frequency oscillator which is locked to the clock. Integers m and n in Fig. 2.3 are selected such that

$$\frac{f_{01}}{m} - \frac{f_{01}}{n} = \frac{1}{T} \quad (2.3)$$

One of these frequencies is selected at a time, by a switch which is operated by the data. The signal at the switch output is then a continuous phase FSK with a modulation index equal to one. The output of the switch is frequency divided by two to generate the required modulation index for MSK.

Output frequency of the divide by m counter is further divided by an integer k to generate the clock frequency signal required for phase comparison in the phase lock loop so that

$$f_{01} = \frac{k \cdot m}{T} \quad (2.4)$$

MSK nominal carrier frequency is given by

$$\frac{1}{2} \left(\frac{k}{T} - \frac{1}{2T} \right) = \frac{k}{2T} - \frac{1}{4T} \quad (2.5)$$

This method for generating MSK is particularly attractive for the low bit rate systems and is used in Chapter 4 for a 2.4 kilobits satellite data station.

Fig. 2.3 Generation of MSK as Continuous Phase FSK

2.3.2 MSK Realization as a Special Case of Offset QPSK

Starting from Eqn. (2.1)

$$\begin{aligned}
 y_{\text{MSK}}(t) &= \cos\left(\omega_c t + \frac{\pi u_k t}{2T} + x_k\right) \\
 &= \cos \omega_c t \cos x_k \cos \frac{\pi u_k t}{2T} - \sin \omega_c t \sin \frac{\pi u_k t}{2T} \cos x_k \\
 &\quad - \sin \omega_c t \cos \frac{\pi u_k t}{2T} \sin x_k - \sin x_k \cos \omega_c t \sin \frac{\pi u_k t}{2T}
 \end{aligned} \tag{2.6}$$

From Eqn. (2.2), since $u_k = \pm 1$ and $x_k = 0$, π modulo 2π , we get

$$\sin x_k = 0$$

$$\cos \frac{\pi u_k t}{2T} = \cos \frac{\pi t}{2T}$$

$$\sin \frac{\pi u_k t}{2T} = u_k \sin \frac{\pi t}{2T}$$

Using the above relations in Eqn. (2.6),

$$\begin{aligned}
 y_{\text{MSK}}(t) &= \cos x_k C(t) \cos \omega_c t - u_k \cos x_k S(t) \sin \omega_c t \\
 (k-1)T &\leq t \leq kT
 \end{aligned} \tag{2.7}$$

$$C(t) = \cos \frac{\pi t}{2T} \tag{2.8}$$

$$S(t) = \sin \frac{\pi t}{2T}$$

From Eqns. (2.7) and (2.8), this representation of MSK waveform can be viewed as being composed of two quadrature data channels. The in-phase channel, or I channel, is identified as $\cos x_k C(t) \cos \omega_c t$, where $\cos \omega_c t$ is the carrier, $C(t)$ is the sinusoidal weighting, and $\cos x_k$ is the data dependent term. Similarly, Q channel is identified as $u_k \cos x_k S(t) \sin \omega_c t$, where $\sin \omega_c t$ is the quadrature carrier term, $S(t)$ is the sinusoidal symbol weighting and $u_k \cos x_k$ is the data dependent term.

A circuit for generating MSK waveforms, consistent with this representation, is shown in Fig. 2.4 [10]. This circuit is used for a 5Mbits MSK system in Chapter 5 and the operation of the circuit is described there.

2.4 MSK POWER SPECTRUM

The class of quadrature modulation techniques employing offset time skewed symbol pulses in the quadrature channels can be represented by a first order Markov process. This process is completely described by the state transition probability matrix, initial state probabilities, and the waveforms associated with each state. As shown in Section 2.3, MSK can be considered as a special case of offset QPSK and thus can be represented as a Markov process.

The Markov representation can be used to compute the autocorrelation and its Fourier transform, the power spectral density. This technique is used in Ref. [11] to calculate power spectral density for MSK and OK-QPSK, which is given by:

$$G_{\text{MSK}}(f) = \frac{8P_c T(1 + \cos 2\pi fT)}{\pi^2 (1 - 16T^2 f^2)^2} \quad (2.9)$$

and for OK-QPSK

$$G(f) = 2P_c T \left(\frac{\sin 2\pi fT}{2\pi fT} \right)^2 \quad (2.10)$$

where f = frequency offset from the carrier

and P_c = power in the modulated waveform.

Equations (2.9) and (2.10) are plotted in Fig. 2.5 as a function of f normalized to the binary data rate $R = 1/T$. The MSK spectrum falls off at a rate proportional to $(f/R)^{-4}$ for large values of f/R . In contrast, the offset QPSK spectrum falls off at a rate proportional to $(f/R)^{-2}$.

2.5 SPECTRUM OF THE SQUARED MSK SIGNAL

It was proposed in Chapter 1 that the microwave carrier at the data station would be derived from the recovered clock of the received signal. Consequently, a simple clock recovery circuit will be a distinct advantage. For MSK, this becomes feasible because of the favourable spectrum of the squared MSK signal.

From Eqn. (2.1),

$$\begin{aligned} y_{\text{MSK}}^2(t) &= \cos^2\left(\omega_c t + \frac{\pi u_k t}{2T} + x_k\right) \\ &= \frac{1}{2} + \frac{1}{2} \cos\left(2\omega_c t + \frac{\pi u_k t}{T} + 2x_k\right) \end{aligned}$$

Now,

$$x_k = 0, \pi \text{ modulo } 2\pi$$

$$2x_k = 0, 2\pi \text{ modulo } 2\pi$$

Fig. 2.4 Realization of MSK as Offset Keyed QPSK

Fig. 2.5 The Power Spectral Density for MSK and Offset-QPSK

so that neglecting the DC term, we get

$$y_{msk}^2(t) = \cos\left(2\omega_c t + \frac{\pi u_k t}{T}\right)$$

This signal corresponds to an FSK modulated signal at the carrier frequency, $2\omega_c$ and with modulation index $= \frac{f_{space} - f_{mark}}{2\pi/T} = 1$. This modulation is known as Sunde's FSK in the literature. Its power spectrum, shown in Fig. 2.6, has been derived by Bennet et al in Ref. [14]. An interesting feature of this spectrum is the presence of two discrete spectral lines shown by two δ functions. These spectral lines differ by $\pm 1/2T$ from the carrier frequency and each contains $1/4$ of the total signal power. A simple clock recovery circuit may be implemented by locking two narrow bandwidth PLO's to these spectral lines [8]. Mixing and low pass filtering of the PLO outputs will give the recovered clock signal. This technique is employed in Chapter 4.

Fig. 2.6 The Power Spectral Density of the Squared MSK Signal.

PART A

CHAPTER 3

DATA STATION SYSTEM DESIGN CONSIDERATIONS3.1 INTRODUCTION

One of the problem areas for which a new approach is proposed is concerned with transmitting the sensor data to a central processing station. Some of the system design problems which arise for such a system can be ascribed to the remote operational environment and essentially low data traffic. In this chapter, we describe some of the important design considerations for such a system. Requirement for the system is established in Section 3.2 by indicating a list of potential users of such a system in Canada. Readout time and link design calculations are presented in Sections 3.3 and 3.4 respectively.

3.2 NEED FOR DATA COLLECTION STATIONS

There is a potential requirement of sending environmental data from various sensors to a central processing station.

Considering the widely scattered locations of such sensors (all over Canada, for example) sending the data through a satellite is an obvious choice. Some of the potential users for such a system in Canada are [1]:

- i) Tides and Water Level Branch: Measurement of tides and water levels in Great Lakes, St. Lawrence River, coastal areas in North
- ii) Water Resources Branch: Measurement of water level and temperature, air temperature, precipitation, etc.
- iii) Water Quality: Measurement of water quality using conductivity, ph value, dissolved oxygen and chloride level and other parameters

iv) Atmospheric and Environmental Service (DOE): Measurement of temperature, wind direction and speed, precipitation, hours of sunlight, etc.

v) Other Users:

Earth Physics

Oceanographic Data

Underground Water Resources Survey

Oil and Gas Wells

Pipeline Control Systems

Remote Airstrips

Forestry

Glaciology

Fisheries

Pollution Control

Lighthouse Data.

Typically, a general requirement of transmitting 100 bits of data per hour may be laid down.

3.3 THE READ-OUT TIME

In this section the time required to read-out a given number of sensor stations is calculated. For the purpose of calculating the read-out time, the data read-out method described below, is assumed.

The individual remote stations use Single Channel Per Carrier (SCPC) mode of operation with Frequency Division Multiple Accessing (FDMA) i.e., these stations access the satellite at different frequencies; the time of transmission being controlled by the command signal. Further, it is assumed that the data is read out in groups. The number of stations

in a group is equal to the number of stations which are assigned different frequencies in a total R.F. bandwidth of 45 K Hz (an R.F. bandwidth of 45 K Hz is currently allocated for a single voice channel in SCPC mode of existing communications satellite systems). All the stations in a group transmit at different frequencies simultaneously. Each group of stations has a unique address and is commanded to read from the CCS. With the read-out method described above, the total read-out time T_r for N stations is given by

$$T_r = \frac{N}{n} [T_A + T_a + T_g + D/R] \quad (3.1)$$

where

n is the number of stations in a group

R is the clock rate

T_A = the CCS access time. Since each station transmits at a different frequency, the remote station is identified by the frequency of transmission and no address is transmitted from the data station. Thus,

T_A = Search and Lock Time (SLT) + Bit Synchronization Time (BST)

This time depends on the received S/N ratio and the acquisition time constant of the PLO in the CCS.

T_a = Access time of the remote station group which is equal to the sum of the address decode time, the search and lock time and bit synchronization time.

Since the command signal is transmitted continuously, only one SLT + BST interval is required in the beginning of the n station read-out sequence.

T_g = The time guardband required because of the time differential in transmission time from remote stations in the group. For Canada, this time differential is of the order of 24 ms and assuming each group contains one farthest and one nearest station, a time gap of 24 ms is provided for each group.

D = Number of bits to be transmitted.

The number of stations n in a group (equal to the number of stations using one voice channel RF bandwidth) is given by:

$$n = \frac{B}{b + b_g}$$

where

B = Voice channel RF bandwidth = 45 KHz

b_g = Frequency guardband provided between the various channels in a group

b = Data station RF bandwidth

For MSK, an RF bandwidth equal to 1.5 times the bit rate produces little degradation in the bit-error-rate performance [10]. For our calculations we will assume that a 2.4 K bits/sec clock is used at each station and each station is therefore allocated an RF bandwidth of 3.6 KHz.

Assuming a frequency guardband b_g of .2 KHz, the number of stations in a group is

$$n = \frac{45}{(2.4) \times 1.5 + .2} = 12$$

Assuming $T_a = T_A = 10 \text{ ms}$, $T_g = 24 \text{ ms}$, the read-out time is calculated using Equation 3.1. It may be seen that a read-out time of only 1.427 seconds is required to read data from 200 stations using the RF bandwidth allocated to a single voice channel in SCPC mode.

3.4 LINK DESIGN CALCULATIONS

In this section, simplified link design calculations are carried out to estimate the data station and CCS transmitter power requirements. Sample calculations are carried out assuming a remote station antenna size of four feet diameter. For the satellite and CCS, the antenna gain and the receiver noise figure values used correspond to those for the existing ANIK system.

3.4.1 Data Station to CCS Link

The data station transmits at a nominal frequency of 6 GHz which is converted to a nominal frequency of 4 GHz and is then transmitted to the CCS. Assuming the data station transmitter power to be 1 watt, the received E_b/N_o at the CCS is calculated below.

Data station to satellite link (uplink)

Data station Tx power = 0dBW = +30dBm

4' antenna gain at 6 GHz = 34.5 dB

Feeder loss = 1 dB

Satellite receiver gain to noise temperature ratio

$$G/T = 7 \text{ dB/}^{\circ}\text{K}$$

Data station effective isotropic radiated power

$$(\text{EIRP}) = \text{Tx Power} + \text{Antenna Gain} - \text{Feeder Loss}$$

$$= 0 + 34.5 - 1$$

$$= 33.5 \text{ dBW}$$

Carrier to noise temperature ratio for the uplink is given by

$$\begin{aligned}
 (C/T)_{\text{uplink}} &= \text{EIRP} - \text{Path Loss} + (G/T) \\
 &= 33.5 - 200.70 + 7 \\
 &= -160.2 \text{ dBW/}^{\circ}\text{K}
 \end{aligned}
 \tag{3.2}$$

where 200.70 = typical uplink path loss for Canada.

Satellite to CCS Link (downlink)

The satellite EIRP corresponding to the data station EIRP of 33.5 dBW, has to be calculated. For the ANIK system, curves for calculating the satellite EIRP for a given earth station EIRP are available. These curves are plotted with the x-axis variable as the earth station EIRP back-off from that required to saturate the output TWT amplifier of the satellite. The y-axis variable is the dBs below the saturated power output of the output TWT amplifier.

Now the earth station saturating EIRP = 84 dBW. Since the EIRP for the data station is 33.5 dBW, the effective back-off for the data station = $84 - 33.5 = 50.5 \text{ dB}$.

Referring to the ANIK curves for this back-off, we get the satellite EIRP = 45.5 dB below the saturated EIRP of 35 dBW. Thus, the satellite EIRP = $35 - 45.5 = -10.5 \text{ dBW}$ and consequently

$$\begin{aligned}
 (C/T)_{\text{downlink}} &= \text{EIRP} - \text{Path Loss} + (G/T) \\
 &= -10.5 - 196.96 + 37.5 \\
 &= -169.98 \text{ dBW/}^{\circ}\text{K}
 \end{aligned}
 \tag{3.3}$$

where typical downlink path loss = 196.96 dB.

G/T for the earth station = ~~37.5~~ dB/K. (This figure is taken for the Allan Park Earth Station of the ANIK system.) Now,

$$(C/T)_{\text{total}} = [(C/T)_{\text{up}}^{-1} + (C/T)_{\text{down}}^{-1}]^{-1}$$

$$= -170.4 \text{ dB}$$

Using this value of data station to CCS C/T ratio, the bit energy to noise spectral density ratio

$$E_b/N_o = C/T - K - 10 \log B$$

where K = Boltzmann constant = -228.6 dB

B = bit rate = 2.4 kilobits/sec

Therefore,

$$\frac{E_b}{N_o} = -170.4 + 228.6 - 10 \log(2.4 \times 10^3)$$

$$= 24.94 \text{ dB.}$$

MSK ideally gives a bit error rate of 1×10^{-6} for $E_b/N_o = 10.3 \text{ dB}$, leaving 3 dB for the degradation in the practical system; a fade margin of 11.64 dB is available.*

3.4.2 CCS to Data Station Link

Because of the small size of the data station antenna and because of the limited satellite EIRP, the satellite to data station is the crucial part of this link and virtually determines the C/T of the total link. Therefore for the purpose of calculations below, the C/T of the complete link is assumed to be the same as the satellite to data station component of the link.

* Any other effects such as cross-polarization due to rain are not considered in the system design as these effects are comparatively small at C band.

At a receiver frequency of 4 GHz, a 4 foot diameter data station antenna has a gain equal to 31 dB. For the data station low noise front end at the data station, we assume a GaAs FET uncooled amplifier with a noise temperature of 170°K. If we assume an antenna noise temperature of 130°K, we obtain an effective receiver noise temperature of $170 + 130 = 300^{\circ}\text{K}$. Thus the G/T for data station is $31 - 25 = 6 \text{ dB}/^{\circ}\text{K}$. Now,

$$C/T = \text{satellite EIRP} - \text{Path Loss} + G/T$$

$$\text{and } E_b/N_o = C/T - K - 10 \log B$$

Therefore,

$$\begin{aligned} \frac{E_b}{N_o} &= \text{satellite EIRP} - 196.96 + 6 + 228.6 - 10 \log (2.4 \times 10^3) \\ &= \text{satellite EIRP} + 3.84 \end{aligned} \quad (3.4)$$

- Leaving 10 dB for the fade margin and the practical degradations, the required

$$\frac{E_b}{N_o} = 10.3 + 10 = 20.3 \text{ dB}$$

(where ideally for MSK 10.3 E_b/N_o gives a bit error rate of 1×10^{-6}).

From Equation (3.4) for this value of E_b/N_o we have

$$\begin{aligned} \text{Satellite EIRP} &= 20.3 - 3.84 \\ &= 16.46 \text{ dBW} \end{aligned}$$

A satellite EIRP of 16.46 dBW corresponds to a back-off of 18.54 dB from the saturated output of 35 dBW. Again from the ANIK curves the CCS power required to give this value of EIRP corresponds to 23 dB drive below the saturating earth station power of 84 dBW. Consequently,

$$\text{CCS EIRP} = 84 - 23 = 61 \text{ dBW}$$

The Allan Park Earth Station of the ANIK system has a gain of 63 dB at 6 GHz. Therefore,

$$\text{CCS } T_x \text{ Power} = -2 \text{ dBW}$$

The simplified calculations carried out above show that a data station with a 4 foot antenna, 1 watt transmitter power and 300°K receiver should be feasible.

A detailed analysis of the data station is given in the next chapter.

CHAPTER 4

DATA STATION DESIGN AND ANALYSIS4.1 INTRODUCTION

The frequency stability of the transmitter carrier becomes a very important design consideration for narrow band systems which are allocated a narrow RF bandwidth of a few KHz at the microwave transmission frequency (typically 6 GHz). For the remote sensor data stations which transmit about 100 bits of data per hour, the problem of achieving the required frequency stability is even more acute because the primary power and cost considerations rule out the possibility of providing a local frequency reference at the data station. This suggests that we should consider transmitting a frequency reference of the required stability from a central control station to all the sensor stations.

One possible method of transmitting the frequency reference is to transmit either a single frequency carrier or to use a modulation for the command signal which gives a component at the carrier frequency. In the initial stages of this work such a method was considered. It was realized that because of the receiver L.O instability, the transmitter carrier has to be derived by coherent frequency synthesis from the received carrier signal. Several circuits for low-level coherent frequency dividers were constructed. This approach was abandoned because of the following reasons.

- a) High gain microwave amplification of the received signal is required since the received signal has very low power.
- b) The carrier frequency stability is affected by the frequency instability of the satellite local oscillator.

c) A simple scheme first proposed by the author [21] which uses the recovered clock of a received MSK modulated command signal as the reference offers an attractive alternative.

An analysis and the description of such a method is the subject of this chapter. A block diagram description of the proposed method is included in Section 4.2. An analysis of the MSK reference recovery method is presented in Section 4.3.

Coherent communication systems require both the carrier and clock references for coherent detection. In most practical systems, these references are derived from the received signal. The references thus derived have a residual phase error because of the input noise and the random signal modulation. This reference noise results in the degradation of bit-error-performance*. While the reference recovery analysis is well known in the literature for BPSK, QPSK and OK-QPSK [15] [16], it is not so well known for MSK. Although two recent papers [17] [18] deal with this subject, their results include the effect of the noisy carrier reference alone and the analysis falls short of relating the bit-error-rate to the received E_b/N_0 ratio [19]. Keeping this in view the analysis in Section 4.3 has been conducted to include the effect of the noisy clock and carrier and the bit-error-rate degradation has been related to the received E_b/N_0 ratio with the noise bandwidth of the phase-locked-oscillator in the recovery circuit as the parameter [20]. Thus, the analysis can be applied to a general MSK coherent communication system, and may be used in

* The degradation in bit-error-performance is defined as the increase in the bit-energy to noise spectral density ratio (E_b/N_0) required to compensate the effect of noisy references.

designing the reference recovery system.

One of the apparent limitations of the proposed method will be a higher phase noise of the derived carrier as compared to more conventional satellite earth stations which use a low-noise frequency synthesis technique to obtain the carrier. The effect of this carrier phase noise on the coherent MSK system is estimated in Section 4.4.

Results of an experimental MSK modem are presented in Section 4.5. Conclusions based on the experimental and analytical work are included in Section 4.6.

4.2 THE PROPOSED SCHEME

In the proposed method [1], the carrier reference at the remote stations is derived from an MSK-modulated command signal transmitted continuously from the Central Command Station (CCS) to all the ground stations. The carrier reference at each station is derived from the recovered clock of the received MSK signal. The frequency of the recovered clock is unaffected by the satellite and the data station receiver local oscillator frequencies. As a result, the frequency stability of the regenerated carrier is independent of the frequency-drifts of the satellite and receiver local oscillators and has the same stability as the frequency standard used at the CCS for the clock signal.

The block diagram of the proposed system, including the CCS, the satellite transponder and one ground station is shown in Fig. 4.1. As shown in this figure, the recovered clock is used both for coherent detection and for deriving a transmitter carrier signal. The remote station MSK-modulates the derived carrier with the collected data and transmits it to the CCS on command.

Fig. 4.1 Overall System Functional Diagram Showing One Data Station

4.2.1 The Data Station Receiver

The power spectrum of the squared MSK signal was shown in Fig. 2.6. The salient feature of this spectrum is the presence of two discrete spectral lines differing by the clock frequency. As pointed out by de Buda [8], this feature may be advantageously used to obtain both the carrier and clock references for coherent detection. The references may be recovered by locking two narrow bandwidth PLO's to these spectral lines and by mixing the output of these PLO's. A receiver employing such a technique is illustrated in Fig. 4.2.

As shown in Fig. 4.2 the received signal is amplified by the low noise amplifier and the amplified signal is down converted in two stages. We have proposed the double-down conversion to overcome the problem of L.O frequency drift. The first down converter uses a fixed frequency local oscillator. The frequency stability of this L.O need not be very high as it is followed by a sweeping L.O type down converter. The signal at the output of the second down converter is filtered and squared to generate the spectrum with the discrete spectral lines. The sweeping signal for the second down converter is controlled by the lock detection signals of both the PLO's and the locking is stopped only when both the PLO's are locked. Since there are only two discrete spectral lines differing by the clock frequency, such a scheme avoids any false locking of the PLO's. To recover the carrier and clock the output signals of the two PLO's are mixed and filtered; the low pass filtering yields the clock while the high pass filtering and frequency division by four gives the carrier. The clock signal, thus recovered is used both as the reference signal for the transmitter described in the next section and as the timing signal in the coherent detector.

4.2.2 The Data Station Transmitter

The block diagram of the proposed transmitter is shown in Fig. 4.3. The MSK generation method is basically that described in Section 2.3 in Chapter 2. A brief description of the transmitter circuit is given here.

The recovered clock signal is filtered by a narrow bandwidth filter. A PLO at frequency f_{01} is locked to the filtered clock signal. The output of this PLO is frequency divided by the two counters which divide the frequency by integers m and n respectively. Integers m and n are selected such that

$$\frac{f_{01}}{m} - \frac{f_{01}}{n} = \frac{1}{T}, \text{ the clock rate}$$

The frequency from the divide by m counter is further divided by k to get the clock frequency which is phase compared to the clock signal in the phase-lock-loop of the PLO. The signals at the $\frac{f_{01}}{m}$ and $\frac{f_{01}}{n}$ frequencies, which are locked to the clock, are selected at the clock rate by a switch which is operated by the collected data. The frequency of the switch output is divided by 2, to generate the MSK signal at an IF frequency:

$$IF_1 = \frac{1}{2} \left(\frac{k}{T} - \frac{1}{2T} \right)$$

The IF signal is filtered by the transmitter filter. This filter determines the spectral width of the transmitted signal. The filtered IF signal is upconverted to the microwave transmitter frequency, typically 6 GHz. The carriers used for the frequency upconversion are all frequency locked to the clock. To ease the filtering requirement on the image rejection filters of the mixers, the frequency upconversion is carried out in three steps. A second PLO at frequency $f_{02} = t.f_{01}$ is locked to the signal

Fig. 4.3 The Data Station Transmitter

at frequency f_{01} from the first PLO. The filtered IF_1 signal is mixed with the second PLO output to give a new IF signal IF_2 at frequency $f_{02} + IF_1$. A third PLO at frequency $f_{03} = l.f_{02}$ is locked to the output of the second PLO, and a new IF frequency IF_3 is generated by mixing the output of the third PLO and IF_2 . The output from the third PLO, which is in the range of 100 MHz, is frequency multiplied by p in a SRD multiplier to generate a microwave frequency signal at $p.f_{03}$. The IF_3 signal is mixed with this microwave carrier to generate the transmitter signal.

The frequency of the transmitter signal is given by:

$$f_T = \frac{p.l.t.k.m}{T} + \frac{km}{T} (1 + t + tl) + \frac{1}{2} \left(\frac{k}{T} - \frac{1}{2T} \right)$$

or

$$f_T = \frac{km}{T} (1 + t + t.l + t.l.p) + \frac{1}{2} \left(\frac{k}{T} - \frac{1}{2T} \right)$$

The noise bandwidths of the PLO's in Fig. 4.3 are selected to reduce the phase noise of the carriers generated by these oscillators. Search waveforms may be used to help the acquisition of phase lock loops in Fig. 4.3. To avoid transmission at any undesired frequency, the power supply to the microwave amplifier used as the last stage of the transmitter is switched "ON" only if all the PLO's in Figs. 4.2 and 4.3 are locked.

It may be noted that the only frequency reference used is the recovered clock which has the frequency stability of the standard used at the CCS to generate the clock signal.

In the next section we present an analysis of the MSK carrier and clock recovery circuit.

4.3 AN ANALYSIS OF THE MSK CARRIER AND CLOCK RECOVERY CIRCUIT

As mentioned before, based on the spectrum of the squared MSK signal, the circuit shown in Fig. 4.4 may be used for both the carrier and clock recovery. This circuit is analyzed in this section in three steps.

(a) Starting from the MSK signal and additive Gaussian noise, the S/N ratios in the phase lock loops used in the recovery circuit are calculated. The S/N ratio is calculated by computing the signal and noise components at the squarer output. From the loop S/N ratios, the phase errors at the phase locked oscillator outputs and therefore the recovered carrier and clock phase errors are determined.

(b) In step two the conditional probability of MSK bit-error-rate (conditioned on the carrier and clock phase error) is calculated.

(c) The joint probability density of the carrier and clock phase errors is calculated and the conditional probability of bit error determined in step two above is integrated over all the possible carrier and clock phase errors to determine the overall bit-error-rate.

In the recovery system design, the recovery loop noise bandwidth is the main design parameter, and the bit-error-rate calculations in steps (a) - (c) above are carried out for different noise bandwidths.

4.3.1 The Recovery Loop S/N Ratio

With the input signal $s(t)$ and the additive Gaussian noise $n(t)$, the output of the squarer is given by:

$$r(t) = \{s(t) + n(t)\}^2$$

Fig. 4.4 The MSK Carrier and Clock Recovery Method

The autocorrelation of $r(t)$ yields

$$\begin{aligned}
 R_r(\tau) &= \text{Expected value of } \{r(t) \cdot r(t + \tau)\} \\
 &= E\{(s(t) + n(t))^2 \cdot (s(t + \tau) + n(t + \tau))^2\} \\
 &= E_s\{s^2(t) \cdot s^2(t + \tau)\} + E_n\{n^2(t) \cdot n^2(t + \tau)\} \\
 &\quad + 2 E_s\{s(t) \cdot s^2(t + \tau)\} \cdot E_n\{n(t)\} \\
 &\quad + E_s\{s^2(t + \tau)\} \cdot E_n\{n^2(t)\} + 2 E_s\{s^2(t) \cdot s(t + \tau)\} \cdot E_n\{n(t + \tau)\} \\
 &\quad + E_s\{s^2(t + \tau)\} \cdot E_n\{n^2(t)\} + 2 E_s\{s(t + \tau)\} \cdot E_n\{n^2(t) \cdot n(t + \tau)\} \\
 &\quad + 2 E_s\{s(t)\} E_n\{n(t) \cdot n^2(t + \tau)\} \\
 &\quad + 4 E_s\{s(t) \cdot s(t + \tau)\} E_n\{n(t) \cdot n(t + \tau)\}
 \end{aligned}$$

where E_s denotes the expectation over signal

and E_n denotes the expectation over noise

Now assuming $s(t)$ and $n(t)$ to be zero mean, independent random processes, $s(t)$ is uncorrelated with all of the $(s \times n)$ and $(n \times n)$ components. It is also uncorrelated with the $(s \times s)$ components (the third moments of $s(t)$ are zero).

Thus the autocorrelation of $r(t)$ is reduced to

$$R_r(\tau) = R_{s \times s}(\tau) + R_{s \times n}(\tau) + R_{n \times n}(\tau)$$

where

$$R_{s \times s}(\tau) = E_s\{s^2(t) s^2(t + \tau)\} \quad (4.1)$$

$$R_{s \times n}(\tau) = 4 R_s(\tau) R_n(\tau) + R_s(0) R_n(0) \quad (4.2)$$

$$R_{n \times n}(\tau) = E_n \{n^2(t) n^2(t + \tau)\} \quad (4.3)$$

From Ref. [8]

$$R_{n \times n}(\tau) = R_n^2(0) + 2 R_n^2(\tau) \quad (4.4)$$

The Power Spectral Density of the squarer output is obtained by taking the Fourier transform of its autocorrelation function $R_r(\tau)$

$$F[R_r(\tau)] = F[R_{s \times s}(\tau)] + F[R_{s \times n}(\tau)] + F[R_{n \times n}(\tau)]$$

The three terms in the output power spectral density are considered separately below.

A. sxs Term

The MSK signal may be written as

$$s(t) = A \cos \left(\omega_0 t + \frac{\pi t}{2T} u_k + x_k \right) \quad (4.5)$$

$$(k-1)T \leq t \leq kT$$

where

$$u_k = k\text{th data bit} = \pm 1$$

$$\omega_0 = \text{IF carrier frequency}$$

$$x_k = \text{Recursive variable used to maintain the phase continuity}$$

$$x_k = x_{k-1} + \frac{\pi(k-1)}{2} (u_{k-1} - u_k) \quad (4.6)$$

$$= 0, \pi \text{ modulo } 2\pi$$

The squared signal is

$$\begin{aligned}
 s^2(t) &= A^2 \cos^2 \left(\omega_o t + \frac{\pi u_k t}{2T} + x_k \right) \\
 &= \frac{A^2}{2} + \frac{A^2}{2} \cos \left(2\omega_o t + \frac{\pi u_k t}{T} \right)
 \end{aligned} \tag{4.7}$$

The squared signal corresponds to a DC term and a term at $2\omega_o$ which is frequency shift keyed with a modulation index given by

$$\frac{f_{\text{space}} - f_{\text{mark}}}{\text{Bit Rate}} = 1.$$

The spectrum of this signal has been derived by Bennet and Rice [14] and is shown in Fig. 4.5. The two spectral lines at $f_1 = 2f_0 - \frac{1}{2T}$ and $f_2 = 2f_0 + \frac{1}{2T}$ represent the two input signals to the phase locked oscillators in Fig. 4.5. The continuous spectrum around these lines contributes to the noise. For the signal of Equation 4.7 the area of delta functions at f_1 and f_2 is [14] $\frac{A'^2}{8}$

where the input signal power at $2f_0$ is $\frac{A'^2}{2}$

$$\text{Therefore } \frac{A^4}{8} = \frac{A'^2}{2} \text{ or } A'^2 = \frac{A^4}{4}$$

Consequently the input signal power for the two phase locked oscillators at f_1 and f_2 is

$$\frac{A'^2}{8} = \frac{A^4}{32} \tag{4.9}$$

In order to simplify the analysis we will assume here that the filtering at the transmitter and receiver does not affect the spectrum of the sx_s term. The measured results and the theoretical reasoning to justify this

Fig. 4.5 The Signal and Noise Power Contribution due to the $s \times s$ Term in the Loop Noise Bandwidth B_N .

assumption are included at the end of Section C below. Also the noise bandwidth B_N of the phase-locked-oscillators in Fig. 4.4 may be assumed to be much smaller than $1/T$ and therefore the $s \times s$ term noise density may be considered to be of constant amplitude in the bandwidth B_N as shown in Fig. 4.5. From this figure, the $s \times s$ term noise may be estimated as:

$$.125 A^2 T B_N = \frac{.125 A^4 T B_N}{4} = \frac{A^4 T B_N}{32} \quad (4.10)$$

B. $s \times n$ Term

The power spectral density due to this term is obtained by the convolution of the bandpass signal and noise power spectral densities. The MSK single sided power spectral density for the unfiltered signal is given by [1]:

$$S(f) = \frac{8 P_c T}{\pi^2} \frac{(1 + \cos 4\pi f T)}{(1 - 16T^2 f^2)^2}$$

where

P_c = Power in the modulated carrier

Thus the power spectral density around $2f_0$ due to the $s \times n$ term is given by

$$S_{s \times n}(2f_0 + f) = 4 \cdot \int_{f-B}^B \frac{N_0}{2} \cdot \frac{8 P_c T}{\pi^2} \frac{(1 + \cos 4\pi u T)}{(1 - 16T^2 u^2)^2} du$$

where

N_0 = single sided input noise power spectral density and a rectangular filter with an IF bandwidth equal to $2B$ is assumed.

Substituting $4uT = x$, we have

$$S_{sxn}(2f_0 + f) = \frac{4 N_o P_c}{\pi^2} \int_{4T(f-B)}^{4BT} \frac{1 + \cos \pi x}{(1-x^2)^2} dx \quad (4.11)$$

where the centre frequencies of the PLO's in Fig. 4.4 are $2f_0 \pm \frac{1}{2T}$.

C. sxn Term

Again considering a single sided noise spectral density N_o for the input noise, the input and output noise spectral densities are shown in Fig. 4.6 [22].

For the phase locked oscillator frequency of $2f_0 \pm \frac{1}{2T}$ and assuming this to have a constant amplitude in the PLL bandwidth B_N , the noise due to this term is equal to

$$\frac{N_o^2 B}{2\pi} \left(1 - \frac{1}{2BT}\right) B_N \quad (4.12)$$

From Equations 4.9 to 4.12 and assuming the noise power spectral densities to be constant in the PLL noise bandwidth B_N , and using

$P_c = A^2/2$ and $E_b/N_o = \frac{A^2 T}{2N_o}$, the PLL S/N is given by:

$$S/N = \left[B_N T \left(1 + \frac{3.24 I}{(E_b/N_o)} + \frac{1.272 (BT - .5)}{(E_b/N_o)^2}\right) \right]^{-1} \quad (4.13)$$

where

$$I = \int_{2-4BT}^{4BT} \frac{1 + \cos \pi x}{(1-x^2)^2} dx$$

and the IF bandwidth = $2B$.

In Section A above, the IF filtering was assumed to have no effect on the signal and noise components due to sxs term. At narrow IF bandwidths, the filtering would cause some distortion in the MSK signal, due to the

Fig. 4.6 The Squarer Input and Output Noise Power Spectral Densities

intersymbol interference. An analysis of the effect of this distortion on the squared signal spectrum, presents a complex mathematical problem. The mathematical complexity arises because the analysis has to take into account the effect of filtering followed by a nonlinearity on a non-Gaussian process. However, since 99% of the signal power is contained in a bandwidth equal to 1.1 times the bit rate, filtering with a bandwidth larger than this will have little effect on the MSK signal or the squared spectrum.

A practical verification of the above assumption was carried out by measuring the effect of the prefiltering on the squared signal spectrum. The measurements were conducted with a fifth order Butterworth filter having a fixed bandwidth. The extent of filtering was varied by changing the input MSK signal clock rate. The measured results are shown in Fig. 4.7.

The squarer output spectrum is shown in Figs. 4.7(a) - 4.7(c) for the unfiltered and filtered MSK case with the normalised filter two sided 3 dB bandwidth 2 BT of 1.4, 1.05, .7 respectively. The parameter which affects the performance of the recovery circuit is the level of the discrete spectral lines differing by the clock frequency above the continuous power spectrum*.

* In Fig. 4.7 the spectral line at the centre which is at twice the input IF nominal carrier frequency, arises because of the departure from ideal of the practical circuits used for generating the MSK signal. Mixer feedthrough from the local oscillator to RF port and slight imbalance in amplitude and phase (not differing exactly by 90°) between I and Q channels were observed to be the main reasons for this. However, this spectral line is of no consequence to the discussion of the effect of filtering on the squared spectrum.

As shown in Figs. 4.7(a) and 4.7(b) there was no measurable change in the spectrum for IF bandwidth $2 BT = 1.4$ and 1.05 . However when the bandwidth was changed to $2 BT = .7$, the spectral line level above the continuous spectral density decreased from 16 dB to 12 dB for one line and to 13 dB for the other line as a result of the IF filtering.

These results justify the assumption that the effect of pre-filtering with a bandwidth $2 BT \geq 1.2$ on the squared signal spectrum is negligible. It may be noted here that in practical systems the channel bandwidth for the MSK system will be selected to be larger than 1.2 times the bit rate, as below this bandwidth, there is an abrupt degradation in the bit-error-rate performance [10].

It should also be noted that the assumption regarding negligible effect of prefiltering on the spectrum after the nonlinearity is generally made when analyzing reference recovery systems for the PSK signals [23] [24] [25].

The PLL S/N ratio as calculated above will have some variation with the change in IF bandwidth due to the $s \times n$ and $n \times n$ terms. In Fig. 4.8 the normalised loop S/N ratio $(S/N) \cdot B_N T$ is plotted as a function of the normalised IF bandwidth $2 BT$, with the E_b/N_0 ratio as the parameter. This figure was plotted by numerically computing the integral for various IF bandwidths. As this figure shows there is very little variation in the loop S/N ratio for the bandwidth $2 BT \geq 1.5$.

For the purpose of calculating the bit-error-rate performance, we will henceforth assume $2 BT = 2$. For IF bandwidths in the range of $1.1 \leq 2 BT \leq 1.5$ there may be some improvement in this S/N ratio

Fig. 4.7(a) The Squarer Output Measured Power Spectrum with an Unfiltered MSK (upper trace) and Filtered MSK (lower trace) as the Input. The Filter Normalised 3 dB Bandwidth, $2 BT = 1.4$
 Spectrum Analyser IF BW = .1 MHz, Video BW = .5 KHz
 Scale x axis: 1 MHz/cm
 y axis: 5 dB/cm

Fig. 4.7(b) The Squarer Output Measured Power Spectrum with an Unfiltered MSK (upper trace) and the Filtered MSK (lower trace) as the Input. The Filter Normalised 3 dB BW, $2BT = 1.05$

Spectrum Analyser IF BW = .1 MHz, Video BW = .5 KHz

Scale x axis: 1 MHz/cm
y axis: 5 dB/cm

Fig. 4.7(c) The Squarer Output Measured Power Spectrum with an Unfiltered MSK (upper trace) and the Filtered MSK (lower trace) as the Input. The Filter Normalised 3 dB BW, $2 B/T = .7$
 Spectrum Analyser IF BW = .1 MHz, Video BW = .5 KHz
 Scale x axis: 1 MHz/cm
 y axis: 5 dB/cm

Fig. 4.8 The Phase-Lock-Loop Normalised S/N Ratio as a Function of the Normalised IF Bandwidth with the E_b/N_o Ratio as the Parameter

whereas for $2 BT \leq 1.1$ the effect of $s \times s$ term cannot be neglected and the S/N ratio will degrade rapidly,

From Equation (4.13) with $2 BT = 2$, we have

$$S/N = \alpha = \left[B_N T \left(1 + \frac{15.9}{(E_b/N_o)} + \frac{.636}{(E_b/N_o)^2} \right) \right]^{-1} \quad (4.14)$$

To demonstrate the order of the S/N that may be expected in a practical system, this ratio is plotted in Fig. 4.9 as a function of the E_b/N_o ratio with $B_N T$ as the parameter.

For a S/N ratio of α , the outputs of the phase-locked-oscillators in the recovery circuit of Fig. 4.4 will have phase errors of ϵ_1 and ϵ_2 with a probability density [26]

$$p(\epsilon) = \frac{\exp(\alpha \cos \epsilon)}{2\pi I_0(\alpha)}, \quad (4.15)$$

$$(\epsilon) \leq \pi$$

with the rms phase error σ_ϵ given by

$$\sigma_\epsilon^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n I_n(\alpha)}{n^2 I_0(\alpha)} \quad (4.16)$$

where I_n is the n th order modified Bessel Function of the first kind.

For $\alpha \gg 1$ using $I_0(\alpha) \approx \frac{e^\alpha}{\sqrt{2\pi\alpha}}$

and $\cos \epsilon \approx 1 - \epsilon^2/2$

$p(\epsilon)$ is approximately Gaussian [26]:

Fig. 4.9 The Phase-Lock-Loop S/N Ratio as a Function of the E_b/N_0 Ratio with the Normalised Loop Noise. Bandwidth $B_N T$ as the Parameter

$$p(\epsilon) = \frac{1}{\sqrt{2\pi} \sigma_\epsilon} \exp(-\epsilon^2/2\sigma_\epsilon^2) \quad (4.17)$$

and

$$\sigma_\epsilon^2 \approx \frac{1}{\alpha} \quad (4.18)$$

The models of $p(\epsilon)$ as given by Eqs. (4.15), (4.16) and (4.17), (4.18) have been studied in analyzing the effect of the carrier phase noise on the PSK signals [24]. It has been shown in this reference that for the moderate values of S/N ratio α , both models essentially give the same result.

Our analysis assumes the Gaussian probability density for ϵ_1 and ϵ_2 as given by (4.17) and (4.18).

The phase errors ϵ_1 , ϵ_2 in turn cause the clock phase error $\epsilon' = \epsilon_1 - \epsilon_2$ at the clock frequency $f_c = \frac{1}{T}$ as well as the carrier phase error $\phi_s = \frac{\epsilon_1 + \epsilon_2}{4}$ at the carrier frequency f_o .

In the next two sections we analyze the effects of the clock and carrier phase errors on the MSK bit-error-rate performance.

4.3.2 The Effect of the Recovered Clock Error on the MSK Bit-Error-Rate

In this section we derive the effect of the clock phase error by evaluating the conditional error probability of the correlation detector conditioned on the clock phase error. The correlation detector in both I and Q channels consists of a multiplier-integrator followed by the sample and dump circuit as shown in Fig. 4.10. The synchronizer, which in effect the clock recovery circuit, provides the timing information (i.e. the start and stop times of each incoming signal pulse) to the sample and dump switches. The clock phase error affects the bit-error-rate

Fig. 4.10 The Correlation Detector

because of the timing jitter caused by it in the sync. timing.

A typical signal is shown in Fig. 4.11(a) over a K symbol interval consisting of $K T_s$ seconds. The signal waveform is shown assuming the random epoch ϵ (i.e. the time of the first zero crossing relative to some arbitrary chosen point labeled $t = 0$) to be constant over $K T_s$ seconds. Fig. 4.11(b) shows the same signal waveform as in (a), but received in the presence of noise.

The input signal to the symbol synchronizer may be characterized as

$$y(t) = \sum_{k=0}^{K-1} s(t; a_k, \epsilon) + n(t) = s(t, \epsilon) + n(t) \quad (4.19)$$

where a_k is the polarity (± 1) of the k th transmitted symbol $s(t; a_k, \epsilon)$.

In terms of the basic symbol waveform $p_s(t)$ which is defined to be non-zero over the interval $(0, T_s)$, the received signal may be written as:

$$S(t; a_k, \epsilon) = a_k p_s(t - (k-1)T_s - \epsilon) \quad (4.20)$$

For the MSK waveform, a sinusoidal symbol is used and therefore

$$p_s(t') = \sqrt{2S} \sin \frac{\pi t'}{T_s} \quad (4.21)$$

where $S T_s$ = Symbol Energy.

If $\hat{\epsilon}$ is the sync estimate of ϵ obtained from the symbol synchronizer, the cross-correlator statistics on which the decision is

Fig. 4.11, The Received I or Q Channel Signal over a $K T_s$ Second Interval Without and With Noise

For a given combination of a_{n-1} , a_n , a_{n+1} , the random component of the decision variable Q is obtained by integrating the Gaussian input noise. The integration being a linear operation, the decision variable Q is conditionally Gaussian over a_{n-1} , a_n , a_{n+1} with a variance = $\frac{N_0 S T_s}{2}$.

Therefore the probability of bit error which depends on the mean and variance of the Gaussian variable is given by [27]

$$P_b(e/\epsilon') = \frac{1}{4} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}} \left[r_s(|\lambda|T_s + r_s(T_s - |\lambda|T_s)) \right] \right) + \frac{1}{4} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}} \left[r_s(|\lambda|T_s - r_s(T_s - |\lambda|T_s)) \right] \right) \quad (4.24)$$

where $T_s = 2T = \text{Symbol interval}$

$$\lambda = \frac{\epsilon_c}{T_s}$$

$$r_s(\tau) \triangleq \frac{R_s(\tau)}{S T_s}$$

$$R_s(\tau) = \int_0^{T_s - \tau} p_s(t') p_s(t' + \tau) dt' \quad (4.25)$$

Now using the clock phase error, ϵ' derived in the last section gives

$$\epsilon_c = \frac{\epsilon' T_s}{2\pi}$$

or

$$|\lambda| = \frac{\epsilon_c}{T_s} = \frac{\epsilon'}{2\pi} \quad (4.26)$$

For the MSK waveform $p_s(t')$ is given by Equation (4.21). Substituting for $p_s(t')$ in (4.25)

$$r_s(\tau) = \frac{1}{\pi} \sin \frac{\pi|\tau|}{T_s} + \cos \frac{\pi|\tau|}{T_s} - \frac{|\tau|}{T_s} \cos \frac{\pi|\tau|}{T_s} \quad (4.27)$$

$$-T_s \leq \tau \leq T_s$$

Substituting (4.26) and (4.27) into (4.24) we obtain the desired conditional probability of bit error conditioned on ϵ' as

$$P_b(e/\epsilon') = \frac{1}{4} \operatorname{erfc} \left\{ \sqrt{\frac{E_b}{N_0}} \left(\left(1 - \frac{|\epsilon'|}{\pi} \right) \cos \left(\frac{\epsilon'}{2} \right) + \frac{2}{\pi} \sin \left(\frac{|\epsilon'|}{2} \right) \right) \right\}$$

$$+ \frac{1}{4} \operatorname{erfc} \left(\sqrt{\frac{E_b}{N_0}} \cos \left(\frac{|\epsilon'|}{2} \right) \right) \quad (4.28)$$

$$|\epsilon'| \leq 2\pi$$

This conditional probability of bit error when integrated over all values of ϵ' determines the bit error probability in the presence of the clock phase error.

4.3.3 The Effect of the Recovered Carrier Phase Error on the MSK Bit-Error-Rate

Proceeding with our analysis of the effect of noisy clock and carrier references on the MSK bit-error-rate, we will evaluate in this section the conditional probability of bit error conditioned on the carrier phase error. For sake of clarity and convenience we will use the offset-QPSK description of MSK.

An offset-QPSK signal, with sinusoidal symbols, may be written as

$$x(t) = \begin{cases} A a_0 \cos \omega_0 t \cos \frac{\pi t}{2T} & ; -T \leq t \leq T \\ + A b_1 \sin \omega_0 t \cos \frac{\pi(t+T)}{2T} & ; 0 \leq t \leq T \\ + A b_0 \sin \omega_0 t \cos \frac{\pi(t-T)}{2T} & ; -T \leq t \leq 0 \end{cases}$$

where

a denotes I channel data bit

b denotes Q channel data bit

$$a_0, b_0, b_1 = \pm 1$$

Simplifying, we have

$$x(t) = \begin{cases} A \cos \omega_0 t \cos \frac{\pi t}{2T} & ; -T \leq t \leq T \\ -A \sin \omega_0 t \sin \frac{\pi t}{2T} & ; 0 \leq t \leq T \\ +A \sin \omega_0 t \sin \frac{\pi t}{2T} & ; -T \leq t \leq 0 \end{cases}$$

Since the same analysis applies to the Q channel it is sufficient if we analyze the bit-error-rate for only the I channel. If the signal is coherently detected with a carrier reference $\cos(\omega_0 t + \phi_s)$, ϕ_s being the phase error, the signal at the output of the I channel correlator is:

$$x_0(t) = K \int_{-T}^T x(t) \cos(\omega_0 t + \phi_s) \cos \frac{\pi t}{2T} dt \quad (4.30)$$

where K = constant of integration.

Substituting from (4.29) in (4.30), and neglecting the terms at $2\omega_0$ and simplifying we get

$$x_0(t) = A K \int_{-T}^T \frac{a_0}{4} \cos \phi_s dt - \frac{b_1}{4} \int_0^T \sin \frac{\pi t}{2T} \sin \phi_s dt + \frac{b_0}{4} \int_{-T}^0 \sin \frac{\pi t}{2T} \sin \phi_s dt$$

Considering ϕ_s to be constant in the time interval $-T \leq t \leq T$, the detected signal is given by

$$x_0(t) = A K \left[\frac{a_0 T}{2} \cos \phi_s - \frac{(b_0 + b_1)}{2\pi} T \sin \phi_s \right] \quad (4.31)$$

So far we have not considered the effect of noise. In the presence of white Gaussian input noise, with a single-sided power spectral density of N_0 , the output noise which is obtained by the linear operations of filtering and integration is also Gaussian. As a result, the decision variable at the output of the correlator is conditionally Gaussian, the conditioning being over b_0, b_1 . The bandpass input Gaussian noise may be written as [23]:

$$n(t) = \sqrt{2} [n_1(t) \cos(\omega_0 t + \phi_s) - n_2(t) \sin(\omega_0 t + \phi_s)]$$

where $n_1(t)$ and $n_2(t)$ are both Gaussian with a variance equal to that of input noise (which is $N_0/2$ for a single-sided input noise spectral density N_0). As a result the noise output of the correlator is

$$\begin{aligned}
 n'(t) &= K \int_{-T}^T n(t) \cos(\omega_0 t + \phi_s) \cos \frac{\pi t}{2T} dt \\
 &= \frac{K}{\sqrt{2}} \int_{-T}^T n_1(t) \cos \frac{\pi t}{2T} dt
 \end{aligned}$$

The output noise variance is given by:

$$\sigma_N^2 = \frac{K^2}{2} \cdot \frac{N_0}{2} \int_{-T}^T \cos^2 \frac{\pi t}{2T} dt = \frac{K^2 N_0 T}{4} \quad (4.32)$$

The mean of the decision variable is conditional over b_0, b_1 , so that taking the expectation over b_0, b_1

$$\begin{aligned}
 P[e/\phi_s] &= [\text{Prob. of Error}/b_0 = b_1 = 1] \times [\text{Prob. } b_0 = b_1 = 1] \\
 &+ [\text{Prob. of Error}/b_0 = b_1 = -1] \times [\text{Prob. } b_0 = b_1 = -1] \\
 &+ [\text{Prob. of Error}/b_0 = 1, b_1 = -1] \times [\text{Prob. } b_0 = 1, b_1 = -1] \\
 &+ [\text{Prob. of Error}/b_0 = -1, b_1 = 1] \times [\text{Prob. } b_0 = -1, b_1 = 1]
 \end{aligned}$$

The mean of the Gaussian decision variable changes with the different values of b_0, b_1 ; with the variance being given by (4.32):

For the Gaussian variable with the mean μ and variance σ_N^2 the probability of error is given by [27]:

$$P(e) = \frac{1}{2} \operatorname{erfc} \frac{\mu}{\sqrt{2} \sigma_N} \quad (4.33)$$

By taking the expectation over b_0 , b_1 and using (4.31), (4.32) and (4.33) we have

$$P(e/\phi_s) = \frac{1}{4} \operatorname{erfc}\left\{\sqrt{\frac{A^2 T}{2N_0}} \cos \phi_s\right\} + \frac{1}{8} \operatorname{erfc}\left\{\sqrt{\frac{A^2 T}{2N_0}} \left(\cos \phi_s + \frac{2}{\pi} \sin \phi_s\right)\right\} \\ + \frac{1}{8} \operatorname{erfc}\left\{\sqrt{\frac{A^2 T}{2N_0}} \left(\cos \phi_s - \frac{2}{\pi} \sin \phi_s\right)\right\}$$

using $\frac{A^2 T}{2N_0} = E_b/N_0$ we have

$$\overline{P(e/\phi_s)} = \frac{1}{4} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}} \cos \phi_s + \frac{1}{8} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}} \left(\cos \phi_s + \frac{2}{\pi} \sin \phi_s\right) \\ + \frac{1}{8} \operatorname{erfc} \left(\cos \phi_s - \frac{2}{\pi} \sin \phi_s\right) \quad (4.34)$$

A simplified approximate form of Equation (4.34) is derived below for use in numerical computations.

In Equation (4.31), the first term $(K A \frac{a_0 T}{2} \cos \phi_s)$ may be identified as the signal term since it is determined by the I channel symbol a_0 . The factor $\cos \phi_s$ is due to the reduction of the signal level because of the carrier phase error ϕ_s . The second term in Equation (4.31) $(-KA \frac{(b_0 + b_1)}{2\pi} T \sin \phi_s)$ may be identified as the Q - I channel cross-talk term. This term is determined by b_0 and b_1 and is equal to zero for $\phi_s = 0$.

The r.m.s value of the cross-talk term is equal to

$$\frac{K^2 A^2 T^2}{4\pi^2} \sin^2 \phi_s \mathbb{E}(b_0 + b_1)^2 p(b_0, b_1) \\ = \frac{K^2 A^2 T^2}{2\pi^2} \sin^2 \phi_s \quad (4.35)$$

Obviously there are two random processes involved; the Gaussian noise process, and the random process which generates the cross-talk term in the presence of the carrier phase error ϕ_s .

We may estimate the effect of the cross-talk term by adding the r.m.s variance due to noise to the variance due to cross-talk.

From (4.32) and (4.35) we have

$$\sigma_{N'}^2 = \frac{K^2 N_o T}{4} + \frac{K^2 A^2 T^2}{2\pi^2} \sin^2 \phi_s$$

The mean μ' of the decision variable is given by

$$\mu' = \frac{A K T}{2} \cos \phi_s$$

Therefore the ratio

$$\begin{aligned} \frac{\mu'}{\sqrt{2} \sigma_{N'}} &= \frac{\frac{A K T}{2} \cos \phi_s}{\sqrt{2} \sqrt{\frac{K^2 N_o T}{4} + \frac{K^2 A^2 T^2}{2\pi^2} \sin^2 \phi_s}} \\ &= \sqrt{\frac{\frac{A^2 T^2}{4} K^2 \cos^2 \phi_s}{\frac{K^2 N_o T}{2} + \frac{K^2 A^2 T^2}{2\pi^2} \sin^2 \phi_s}} = \sqrt{\frac{\frac{A^2 T}{2 N_o} \cos^2 \phi_s}{1 + \frac{A^2 T}{2 N_o} \frac{4}{\pi^2} \sin^2 \phi_s}} \end{aligned}$$

$$\text{Using } \frac{E_b}{N_o} = \frac{A^2 T}{2 N_o}$$

$$\frac{\mu'}{\sqrt{2} \sigma_{N'}} = \sqrt{\frac{\frac{E_b}{N_o} \cos^2 \phi_s}{1 + \frac{E_b}{N_o} \frac{4}{\pi^2} \sin^2 \phi_s}}$$

Therefore the probability of error may be written as

$$P(e/\phi_s) = \frac{1}{2} \operatorname{erfc} \frac{\mu'}{\sqrt{2} \sigma_{N'}} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E'_b}{N'_o}} \quad (4.36)$$

$$\text{where } \frac{E'_b}{N'_o} = \frac{\frac{E_b}{N_o} \cos^2 \phi_s}{1 + \frac{E_b}{N_o} \frac{4}{\pi^2} \sin^2 \phi_s} \quad (4.37)$$

$$\frac{E'_b}{N'_o} = \frac{E_b}{N_o} \text{ when } \phi_s = 0.$$

For a given ϕ_s , the $\frac{E'_b}{N'_o}$ given by (4.37) lies between two values [26]:

$$\frac{E'_b}{N'_o} = \frac{E_b}{N_o} \cos^2 \phi_s \quad \text{for BPSK}$$

$$\text{and } \frac{E'_b}{N'_o} = \frac{E_b}{N_o} \frac{\cos^2 \phi_s}{1 + \frac{E_b}{N_o} \sin^2 \phi_s} \quad \text{for QPSK}$$

Thus MSK shows less sensitivity to carrier phase noise than QPSK but has greater sensitivity than BPSK. The carrier phase noise effects in BPSK are limited to signal reduction, there being no cross-talk. However the cross-talk effect in MSK is less than that of QPSK primarily because of two reasons. The first is the offsetting of the I and Q channels, which results in a reduction of the cross-talk effect before it affects the signal channel. The second reason is the sinusoidal symbol shaping which reduces the cross-talk effect.

In the last two sections we have derived the effect of carrier and clock phase errors on the bit-error-rate. In the next section we derive the joint probability density of carrier and clock phase error and the joint conditional probability density of the bit-error-rate.

4.3.4 The Joint Probability Densities $p_1(\phi_s, \epsilon')$ and $P_b(e/\phi_s, \epsilon')$

To evaluate the combined effect of the carrier and clock phase error, we must know the joint probability density $p_1(\phi_s, \epsilon')$ and the joint conditional probability of error $P_b(e/\phi_s, \epsilon')$.

Assuming the phase errors ϵ_1 and ϵ_2 of the recovery circuit phase locked oscillators to be uncorrelated and having a Gaussian probability density of Equation (4.17), we have the joint probability density of ϵ_1, ϵ_2 as:

$$p(\epsilon_1, \epsilon_2) = p(\epsilon_1) \cdot p(\epsilon_2)$$

$$|\epsilon_1| < \pi$$

$$|\epsilon_2| < \pi$$

The probability density $p_1(\phi_s, \epsilon')$ is obtained from $p(\epsilon_1, \epsilon_2)$ by the linear transformation $\phi_s = \frac{\epsilon_1 + \epsilon_2}{4}$, $\epsilon' = \epsilon_1 - \epsilon_2$.

Therefore

$$p_1(\phi_s, \epsilon') = p\left(\frac{4\phi_s + \epsilon'}{2}, \frac{4\phi_s - \epsilon'}{2}\right) \cdot \begin{vmatrix} \frac{\partial \epsilon_1}{\partial \phi_s} & \frac{\partial \epsilon_1}{\partial \epsilon'} \\ \frac{\partial \epsilon_2}{\partial \phi_s} & \frac{\partial \epsilon_2}{\partial \epsilon'} \end{vmatrix}$$

$$\text{or } p_1(\phi_s, \epsilon') = \frac{1}{\pi \sigma_\epsilon^2} \exp\left(-\frac{16\phi_s^2 + \epsilon'^2}{4\sigma_\epsilon^2}\right) \quad (4.38)$$

where σ_ϵ is given by (4.14) and (4.18) and the range of variables ϕ_s and ϵ' of (4.38) is shown in Fig. 4.12.

Assuming ϕ_s to be slowly varying relative to the symbol period and assuming the carrier frequency $f_0 \gg \frac{1}{T}$, the carrier phase noise may be considered as the degradation in the E_b/N_0 ratio as in (4.37), and the clock phase error may be considered as a timing error effect in coherently detecting this degraded signal according to Equation (4.28) such that:

$$P_b(e/\phi_s, \epsilon') = \frac{1}{4} \operatorname{erfc} \left\{ \sqrt{\frac{E'_b}{N'_0}} \left(1 - \frac{|\epsilon'|}{\pi} \right) \cos \frac{\epsilon'}{2} + \frac{2}{\pi} \sin \left(\frac{|\epsilon'|}{2} \right) \right\} \\ + \frac{1}{4} \operatorname{erfc} \left\{ \sqrt{\frac{E'_b}{N'_0}} \cos \left(\frac{\epsilon'}{2} \right) \right\} \quad (4.39)$$

where E'_b/N'_0 is given by Equation (4.37)

4.3.5 The Bit Error Performance Degradation

To estimate the degradation in the bit-error-rate performance due to the combined carrier and clock phase noise and their individual contribution, we have numerically computed the following;

For carrier phase noise alone:

$$P_{b1}(e) = \int_{-\pi/2}^{\pi/2} P_b(e/\phi_s) p(\phi_s) d\phi_s \quad (4.40)$$

where $P_b(e/\phi_s)$ is given by (4.36) and (4.37)

$$\text{and } p(\phi_s) = \frac{2}{\sqrt{\pi} \sigma_\epsilon} \exp \left(-\frac{4\phi_s^2}{\sigma_\epsilon^2} \right) \quad (4.41)$$

σ_ϵ is given by (4.14) and (4.18)

Fig. 4.12 The Integration Region for the Carrier and Clock Phase Noise

For the clock phase noise alone

$$P_{b2}(e) = \int_{-2\pi}^{2\pi} P_b(e/\epsilon') p(\epsilon') d\epsilon' \quad (4.42)$$

where $P_b(e/\epsilon')$ is given by (4.28), and

$$p(\epsilon') = \frac{1}{\sqrt{4\pi} \sigma_e} \text{Exp} \left(-\frac{\epsilon'^2}{4\sigma_e^2} \right) \quad (4.43)$$

For the combined carrier and clock phase noise

$$P_b(e) = \iint P_b(e/\phi_s, \epsilon') p_1(\phi_s, \epsilon') d\phi_s d\epsilon' \quad (4.44)$$

where the region for double integration is as shown in Fig. 4.12 and $P_b(e/\phi_s, \epsilon')$ and $p_1(\phi_s, \epsilon')$ are as given in (4.39) and (4.38).

The integrations were carried out numerically on a Sigma 9 computer. Simpson trapezoidal rules were used for the integration. For the single integration Simpson five point rule was employed whereas for double integration Simpson's two dimensional 9 point rule was used. For the complimentary erfc, a Scientific Program Library (SPL) subroutine was used. For each value of the noise bandwidth B_N , the r.m.s phase error σ_e was calculated using Equations (4.14) and (4.18). The computed value of σ_e was used to obtain the probability densities in Equations (4.42) to (4.44). The number of steps in the integration was increased until a convergence was achieved (the test for convergence was less than one percent change in the computed bit-error-rate from the value computed in the previous step.) The computed results are shown in Figs. 4.13 to 4.15 as plots of bit-error-rate as a function of E_b/N_0 ratio with the normalised loop noise bandwidth

Fig. 4.13 The MSK Bit-Error-Rate as a Function of the E_b/N_0 Ratio Considering the Effect of Noisy Carrier Alone, with the Recovery Loop Normalised Noise Bandwidth as the Parameter

Fig. 4.14 The MSK Bit-Error-Rate as a Function of the E_b/N_0 Ratio Considering the Effect of Noisy Clock Alone with the Normalised Loop Noise Bandwidth B_T as the Parameter.

2

Fig. 4.15 The MSK Bit-Error-Rate as a Function of the E_b/N_0 Ratio Considering the Effect of Both Noisy Clock and Carrier with the Recovery Loop Normalised Noise Bandwidth as the Parameter

$B_N \cdot T$ as the parameter. These results may be used to estimate the performance of the MSK reference recovery circuit.

It may be seen for example that with a normalized loop noise bandwidth $B_N \cdot T \leq .01$, the degradation due to the carrier and clock phase error may be limited to less than .2 dB. The sinusoidal symbol shaping and offsetting of the I and Q channels, results in a relatively low degradation in the bit error performance, for the simple MSK recovery scheme discussed. The degradation rises rapidly for $B_N \cdot T > .03$.

The parameter $B_N \cdot T$ for the practical systems will be based on the practical considerations such as loop acquisition performance, the frequency stability of the local oscillators and the PLL voltage controlled oscillators, as well as the system operating E_b/N_0 ratio. The implication of the results shown in Fig. 4.15 on the performance of an MSK communication system is demonstrated by means of two following examples.

For a 60 Mbits/sec TDMA-MSK system, a PLO noise bandwidth of 600 KHz may be used to give a $B_N \cdot T$ of .01. This will result in about .2 dB degradation due to noisy references at an operating bit-error-rate of 1×10^{-6} .

Now consider an unattended SCPC 32 Kilobits/sec satellite station. Such a station may be used for instance for a single voice channel with delta modulation. Because of the temperature stability and loop acquisition considerations, a PLL noise bandwidth of 1 KHz may be used. This will yield a $B_N \cdot T \approx .03$ and, therefore, for a nominal bit-error-rate of 10^{-6} , a 0.5 dB degradation in the bit-error performance will occur due to the noisy references in coherent MSK detector.

Considering again the proposed system for the sensor stations described in Section 4.2, the reference recovery analysis of Section 4.3 will be used to determine the PLL noise bandwidths as well as the frequency sweep rate for the search waveforms.

For a second order phase-lock-loop with a damping constant $\zeta = .707$, the noise bandwidth B_N is given by [29]

$$B_N = .53 \omega_n \text{ Hz}$$

where ω_n = loop natural frequency in rad/sec.

For the search waveform, the maximum rate of frequency search for a 90 percent or greater confidence of achieving lock is given by [29]:

$$D = \frac{\omega_n^2}{2} \left(1 - \frac{1}{\sqrt{\alpha}}\right) \text{ rad/sec}^2 \quad (4.45)$$

where α = loop S/N ratio given by (4.14).

Substituting for ω_n from above in (4.45), we have

$$D = .567 B_N^2 \left(1 - \frac{1}{\sqrt{\alpha}}\right) \text{ Hz/sec.} \quad (4.46)$$

The maximum permissible frequency search rate would limit the acquisition time required to achieve lock of the PLO's from an initial unlocked condition.. For example from Fig. 4.15, at an operating bit-error-rate of 1×10^{-6} , if only .2 dB degradation in bit error performance due to noisy references is allowed, a $B_N \cdot T$ of .01 (or $B_N = 24 \text{ Hz}$ for 2.4 kilobits/sec bit rate) is required. Using Fig. 4.9 and Equation (4.46), we can conclude that the frequency search rate will be limited to 277 Hz/sec. Thus in the presence of an initial frequency offset of 277 Hz this may result in as much as one second time delay before locking is achieved. Permitting a .5 dB degradation in the bit-error-rate performance would allow us to increase the frequency search rate to about 2.5 KHz/sec, thereby reducing the locking time by a factor of ten. This increase in frequency search rate would be obtained at the cost of about .5 dB increase

in E_b/N_0 required for a BER of 1×10^{-6} . Experimental results of a 2.4 Kbits/sec MSK designed to verify the analysis given in Section 4.3 will be presented in Section 4.5.

4.4 THE DATA STATION TRANSMITTER CARRIER PHASE NOISE EFFECT ON THE CCS RECEIVER

In the previous section we were mainly concerned with the data station receiver bit-error-performance. In this section we consider the effect of the data station transmitter carrier phase noise on the CCS receiver.

The carrier phase noise in a communication system is contributed by the various oscillators and frequency converters in the system. For multi-channel satellite systems the contribution of carrier phase noise is kept small by using low noise frequency synthesis techniques to generate the carriers at various stages in the system.

For coherent detection at the receiver the carrier is recovered from the received signal. Each carrier tracking loop requires the use of some form of the oscillator tracking loop. The effect of the carrier phase noise can be reduced if the phase lock loop tracks this phase noise and maintains a good correlation between the recovered carrier and the received carrier. Thus the noise bandwidth of the carrier recovery loop should be large enough to track the input carrier phase noise, but it should be low enough to reduce the effect of the receiver thermal noise.

The carrier phase noise effect is an important design consideration for the low-bit-rate systems since the noise bandwidth of the recovery loop has to be kept to a few percent of the bit rate to limit the effect of the receiver thermal noise. This bandwidth is not large enough to track the

large $1/f$ noise of the carrier. The effect of the carrier phase noise for the proposed scheme of Section 4.2 is even more important because of the high phase noise of the transmitter carrier at the data station compared to regular earth stations which use low noise frequency synthesizers. In this section the effect of the carrier phase noise is analyzed for an MSK system. The data station transmitter carrier phase noise will affect the coherent MSK detector at the central command station (CCS). A method for the carrier and clock recovery similar to the one analyzed in Section 4.3, may be used at the CCS. However, the effect of the input carrier phase noise on the two spectral lines in the squared MSK signal spectrum has not been analyzed. We would therefore assume the circuit of Fig. 4.16 for the carrier recovery at the CCS. This circuit makes use of a fourth order non-linearity which would remove the MSK modulation from the input signal resulting in the pure carrier component. The reference recovery circuit of Section 4.3 is assumed for the clock recovery only which will therefore be performed independently of the carrier recovery of Fig. 4.16.

4.4.1 The Carrier Phase Noise Model

The two major components of the carrier phase noise which are important for the low-bit-rate system under consideration are:

- (i) Flicker frequency noise with a phase noise power spectral density given by k_a/f^3 .
- (ii) White frequency noise with a phase noise power spectral density given by k_b/f^2 .

Thus the carrier phase noise power spectral density may be written as

Fig. 4.16 The MSK Carrier Recovery Circuit Assumed at CCS for Calculating the Effect of Data Station Transmitter Carrier Phase Noise

$$S_{\phi}(f) = \frac{k_a}{f^3} + \frac{k_b}{f^2} \quad (4.47)$$

At the output of the fourth order nonlinearity this is increased by a factor of 16 corresponding to a 6 dB/octave increase on squaring. At the output of the carrier recovery phase-locked-oscillator in the CCS receiver the input carrier phase noise causes an output r.m.s phase error σ_{ϵ_1} given by [29]:

$$\sigma_{\epsilon_1}^2 = \int_0^{\infty} \left(\frac{16 k_a}{f^3} + \frac{16 k_b}{f^2} \right) \frac{\left(\frac{\omega}{\omega_n} \right)^4}{1 + \left(\frac{\omega}{\omega_n} \right)^4} df$$

where ω_n = Loop Natural Frequency.

Using the results of Ref. [29], we have

$$\sigma_{\epsilon_1}^2 = \frac{139.36 k_a}{B_{N_1}^2} + \frac{29.64 k_b}{B_{N_1}} \quad (4.48)$$

where B_{N_1} = Recovery Loop Noise Bandwidth and a second order PLL with a damping constant of .707 is assumed.

The carrier phase noise is one source of phase error in the recovered carrier at the CCS, another source being the additive CCS receiver thermal noise.

The r.m.s phase error σ_{ϵ_2} caused by the receiver thermal noise can be determined from the carrier recovery loop S/N ratio due to this noise. The computation of the recovery loop S/N ratio involves calculating the signal and noise powers in the recovery loop noise bandwidth at the output of the fourth order nonlinearity with the signal plus additive Gaussian noise as the input.

The recovery loop S/N ratio for the circuit of Fig. 4.16 has been computed for the QPSK and OK-QPSK. Though these results may be modified somewhat for MSK, to avoid the mathematical complexity we will assume that the results apply to MSK as well.

From [18] we have

$$\sigma_{\epsilon_2}^2 = 1/\alpha$$

$$\alpha = \beta E_b/N_o$$

$$\beta = \frac{1}{B_{N1} \cdot T (14.4 + 55.5(N_o/E_b) + 61.5(N_o/E_b)^2 + 14.02(N_o/E_b)^3)}$$

(4.49)

Since the carrier phase noise and the receiver thermal noise are independent random processes, phase errors ϵ_1 and ϵ_2 caused by the two processes in the recovered carrier at the CSS are uncorrelated. Therefore

$$\sigma_{\epsilon L}^2 = \sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2 \quad (4.50)$$

The r.m.s phase error σ_{ϵ} at the output of the divide by four circuit is given by:

$$\sigma_{\epsilon}^2 = \sigma_{\epsilon L}^2/16 \quad (4.51)$$

For high S/N ratio, the PLL phase error reduces to a Gaussian density function and therefore, as in Section 4.3, a Gaussian probability density function with a zero mean and r.m.s deviation of Equation (4.51) is assumed in this analysis.

4.4.2 The Bit-Error-Rate Performance of the CCS with the Carrier Phase Noise

The conditional probability of MSK bit-error-rate performance conditioned on the carrier phase error ϵ was derived in Section 4.3.3 from Equations (4.36) and (4.37).

$$P(e/\epsilon) = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E'_b}{N'_o}} \quad (4.52)$$

where

$$\frac{E'_b}{N'_o} = \frac{E_b/N_o \cos^2 \epsilon}{1 + \frac{E_b}{N_o} \frac{4}{\pi^2} \sin^2 \epsilon}$$

also

$$p(\epsilon) = \frac{1}{\sqrt{2\pi} \sigma_\epsilon} \operatorname{Exp}(-\epsilon^2/2\sigma_\epsilon^2) \quad (4.53)$$

and σ_ϵ is given by (4.51).

The probability of bit-error-rate is derived by integrating the conditional probability of bit error over all possible carrier phase errors.

Therefore

$$P_e = \int_{-\pi}^{\pi} \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E'_b}{N'_o}} \cdot p(\epsilon) d\epsilon \quad (4.54)$$

In deriving Equation (4.54) the probability of cycle slipping has been neglected. This may be included by considering that the average time between cycle slips for a high S/N ratio loop is given by [29]

$$T_s = \frac{1}{B_{N_1}} \frac{\pi}{4} \operatorname{Exp}\left(\frac{2}{\sigma_{\epsilon L}^2}\right) \quad (4.55)$$

The average duration of the cycle slip may be approximated by [30]

$$T_D = \frac{\pi}{\omega_n} = .53/B_{N1} \quad (4.56)$$

As the bit-error-rate during a cycle slip is equal to .5, the probability of error due to cycle slip is:

$$P_{es} = \frac{1}{2} \frac{T_D}{T_s} = 1.06 \text{ Exp } (-2/\sigma_{\epsilon L}^2) \quad (4.57)$$

Considering that cycle slips are infrequent events, the errors due to noise occur between cycle slips and can be treated as independent so that an overall bit-error-rate is given by

$$P_b(e) = P_e + P_{es} \quad (4.58)$$

where P_e and P_{es} are given by (4.54) and (4.57).

To show the effect of the carrier phase noise on the bit-error-rate, Equation (4.58) was numerically evaluated for an example of carrier phase noise given by Equation (4.59)

$$S_\phi(f) = \frac{1}{f^3} + \frac{.166 \times 10^{-3}}{f^2} \quad (4.59)$$

The bit-error-rate was computed for a clock rate of 2.4 Kbits/sec and the computed results are plotted in Fig. 4.17 as a function of E_b/N_0 ratio, with the carrier recovery loop normalised noise bandwidth $B_{N1} \cdot T$ as the parameter. From this figure a performance degradation of .7 dB is predicted for a bit-error-rate of 1×10^{-6} and $B_{N1} \cdot T = .03$. As mentioned,

Fig. 4.17 The CCS Computed Bit-Error-Rate as a Function of E_b/N_0 Ratio with Carrier Recovery Loop Normalised Noise Bandwidth $B_{N1}T$ as the Parameter.

The Data Station Transmitter Carrier Phase Noise is given by Equation (4.59).

before, the carrier recovery loop noise bandwidth B_{N_1} must be selected to achieve a compromise between the effects of the receiver thermal noise and the carrier phase noise. This can be seen from Fig. 4.17 where the bit error degradation increases rapidly for $E_b/N_0 \geq 8.3$ dB as $B_{N_1} \cdot T$ is changed from .03 to .01. The degradation also increases if $B_{N_1} \cdot T$ is changed from .03 to .04. Consequently for the carrier phase noise example considered, $B_{N_1} \cdot T = .03$ (a PLL noise bandwidth of 72 Hz) gives the least bit-error-degradation of .7 dB at a bit-error-rate of 1×10^{-6} .

4.5 THE EXPERIMENTAL DESIGN

A 2.4 Kbits/sec MSK modem was constructed to experimentally verify the analysis of Section 4.3. The three subsections below describe the experimental modem.

4.5.1 The MSK Modulator

The circuit used for generating a 2.4 Kilobits/sec MSK signal at an IF nominal carrier frequency of 11.4 KHz is shown in Fig. 4.18. The circuit essentially implements the continuous phase FSK technique of generating MSK described in Section 2.3.1.

A 216 KHz PLO is phase locked to the clock signal at 2.4 KHz. Two signals with frequencies of 21.6 KHz and 24 KHz are derived from this PLO by frequency dividing the PLO signal using divide by 10 and divide by 9 counters. These signals differ by the clock frequency and are locked to the clock signal. One of these frequencies is selected at a time by the pseudorandom Non Return to Zero (NRZ) input data. We therefore have at the output of the switch Sundé's FSK at a nominal carrier frequency of 22.8 KHz.

Fig. 4.18 2.4 Kbits/sec, 11.4 KHz MSK Modulator Circuit

This signal is coherently divided by 2 by a J-K flip-flop to generate the MSK signal. The measured power spectral density of the output signal is shown in Fig. 4.19. As may be seen from this figure the measured spectrum closely matches the theoretical spectrum of Fig. 2.5, the mainlobe being 3.6 KHz wide (2.4×1.5) and the first sidelobe being ≈ 22 dB below the mainlobe. Also each sidelobe is 1.2 KHz ($= 2.4 \times .5$) wide and the spectrum falls at a rate of ≈ 12 dB/octave corresponding to the $1/f^4$ factor for MSK.

4.5.2 The Receiver Filter and Reference Recovery System

A sixth order .5 dB ripple Chebycheff active filter was designed for use as the receiver filter. The filter was designed with a centre frequency of 11.4 KHz and a 3 dB bandwidth of 3 KHz. As shown in Fig. 4.20 the filter was implemented in three stages. The centre frequency and Q of each stage was determined from standard tables [31] for .5 dB ripple Chebycheff filters. Because of the requirement for a high gain in the first and third stages these stages consisted of two operational amplifiers in a high gain multiple feedback configuration, whereas one operational amplifier in the same configuration was sufficient to realise the second stage of the filter [32]. The measured frequency response of the filter is shown in Fig. 4.21.

Figure 4.22 shows the circuit used to obtain the reference signals required for the coherent demodulation in the I and Q channels. To recover the references, the filtered received signal is squared using an integrated circuit mixer. Two PLO's centred at 21.6 KHz and 24 KHz are locked to the discrete spectral lines in the squarer output spectrum. The PLO outputs are frequency divided by two. These signals are then filtered using two active low pass filters to obtain the two sinusoidal signals of the form

Fig. 4.19 The 2.4 k Bits/sec Signal Power Spectrum at the Input to the Clock Recovery Circuit.
Scale: X axis: 2 kHz/Div. Y axis: 10 dB/Div.

Fig. 4.20 Sixth Order, 3 KHz BW, .5 dB Ripple Chebycheff Filter with a Centre Frequency = 11.4 KHz

Fig. 4.21 The Measured Frequency Response for the Filter of Fig. 4.20

Scale: X axis = 1 KHz/ Div; Y axis = 1 dB/Div

Fig. 4.22 The Reference Recovery Circuit Used to Recover the References for Coherent Detection of MSK Signal

$$\pm \cos (\omega_c t + \frac{\pi t}{2T}) \text{ and } \pm \cos (\omega_c t - \frac{\pi t}{2T})$$

where ω_c = received IF signal nominal carrier frequency.

Using unity gain amplifiers these signals are summed and subtracted to obtain the reference signals of the form

$$\pm \cos \omega_c t \cos \frac{\pi t}{2T} \text{ and } \pm \sin \omega_c t \sin \frac{\pi t}{2T}$$

The $\pm$ signs arise because of the 180° phase ambiguity introduced as a result of frequency division by 2. In PSK systems this ambiguity must be resolved by differential coding. However, as will become evident in the next section, this ambiguity may be resolved in MSK without differential coding.

The sinusoidal reference signals at frequencies $\omega_c \pm \frac{\pi}{2T}$ are also mixed and low-pass-filtered to obtain a signal at half the clock frequency (hereafter referred to as clock/2). This is a sinusoidal signal. To obtain the square wave signal needed to operate the sample and hold operations, a wide bandwidth PLO is locked to this sinusoidal clock/2 signal. The PLO output is also inverted to obtain the $\overline{\text{clock/2}}$ and clock/2 signals which are required for the I and Q channels.

The power spectrum of the squarer output signal with that of the two PLO's locked to the discrete spectral lines is shown in Fig. 4.23.

The time domain waveforms of the recovered reference signals used for coherent demodulation in the I and Q channels are shown in Fig. 4.24.

4.5.3 The Coherent MSK Detector

Figure 4.25 illustrates the circuit designed to coherently demodulate the MSK signal. As shown in this figure, the received filtered IF signal

Fig. 4.23 The Measured Power Spectrum of the Squared Signal (lower trace) with the Spectra of Two PLLs in the Clock Recovery Circuit

Scale: X axis: 2 KHz/Div. Y axis: 10 dB/Div.

Fig. 4.24 The I and Q Reference Signals Obtained in the Reference Recovery Circuit of Fig. 4.22

Fig. 4.25 The Coherent Detection Circuit for MSK

is multiplied by the I and Q channel reference signals generated by the circuit in the Fig. 4.22. A low frequency balanced mixer (minicircuit labs ZAD-8) was used to implement the multiplication of the received signal with the reference signals. Active second order Butterworth low-pass filters were used at the mixer outputs. These filters were experimentally adjusted to obtain the best bit error performance. Thus at the output of the filters two parallel output data channels were formed, each channel being at 1.2 Kbits/sec. The pulse shape of the demodulated signal is that of the raised-cosine where a mark or "1" is a positive going pulse and a space or "0" is a negative going pulse.

These bipolar raised-cosine data pulse trains were converted to unipolar data using very high gain amplifiers. An advantage of this method is that the decision threshold can be experimentally adjusted to obtain the best bit error performance by simply varying the DC bias at the input of the amplifier. The sample and hold function was accomplished by "D" flip-flops. The signals at the half clock frequencies, obtained in the reference recovery circuit of Fig. 4.22, were used to transfer data from the input to output of the D flip-flops. The data outputs were then interleaved in a serial form by using an exclusive-OR gate.

As mentioned in Section 4.5.3 the phase ambiguity introduced by the divide by two blocks in the reference recovery circuit of Fig. 4.22 may be resolved without the need for differential coding. The signals at the output of divide by two circuits are of the form $\pm \cos(\omega_c t + \frac{\pi t}{2T})$ or $\pm \cos(\omega_c t - \frac{\pi t}{2T})$. Consequently in Fig. 4.22, the signals at the summer output points A and A' are

$$\cos(\omega_c t + \frac{\pi t}{2T}) + \cos(\omega_c t - \frac{\pi t}{2T}) \text{ and } \cos(\omega_c t + \frac{\pi t}{2T}) - \cos(\omega_c t - \frac{\pi t}{2T})$$

$$\text{or } \cos(\omega_c t + \frac{\pi t}{2T}) - \cos(\omega_c t - \frac{\pi t}{2T}) \text{ and } \cos(\omega_c t + \frac{\pi t}{2T}) + \cos(\omega_c t - \frac{\pi t}{2T})$$

$$\text{or } -\cos(\omega_c t + \frac{\pi t}{2T}) + \cos(\omega_c t - \frac{\pi t}{2T}) \text{ and } -\cos(\omega_c t + \frac{\pi t}{2T}) - \cos(\omega_c t - \frac{\pi t}{2T})$$

$$\text{or } -\cos(\omega_c t + \frac{\pi t}{2T}) - \cos(\omega_c t - \frac{\pi t}{2T}) \text{ and } -\cos(\omega_c t + \frac{\pi t}{2T}) + \cos(\omega_c t - \frac{\pi t}{2T})$$

On simplification we get:

$$2 \cos \omega_c t \cos \frac{\pi t}{2T} \text{ and } -2 \sin \omega_c t \sin \frac{\pi t}{2T}$$

$$\text{or } 2 \sin \omega_c t \sin \frac{\pi t}{2T} \text{ and } 2 \cos \omega_c t \cos \frac{\pi t}{2T}$$

$$\text{or } 2 \sin \omega_c t \sin \frac{\pi t}{2T} \text{ and } -2 \cos \omega_c t \cos \frac{\pi t}{2T}$$

$$\text{or } -2 \cos \omega_c t \cos \frac{\pi t}{2T} \text{ and } 2 \sin \omega_c t \sin \frac{\pi t}{2T}$$

If we refer to $\cos \omega_c t \cos \frac{\pi t}{2T}$ as the I channel reference and $\sin \omega_c t \sin \frac{\pi t}{2T}$ as the Q channel reference the signal at point A is either + I reference with the signal at point A' as + Q reference or the signal at A is + Q reference with the signal at A' as + I reference.

Since for +1, -1 or -1, +1 inputs the exclusive -OR gives a +1 output the change in sign of each reference does not affect the result as long as

the other reference is of opposite sign. It may be noted here that since D flip-flops in the sample and hold circuit of Fig. 4.25 change state at the positive going clock edge, the sampling clock/2 and $\overline{\text{clock/2}}$ signals are in effect offset. As the clock/2 signal is obtained by mixing the $\pm \cos(\omega_c t + \frac{\pi t}{2T})$ and $\pm \cos(\omega_c t - \frac{\pi t}{2T})$ signals, the clock/2 signal changes phase by 180° if the reference signal at point A changes from I-reference to Q-reference. This ensures that each channel is sampled by the correct clock/2 signal with an offset sampling being carried out at the other channel.

A practical problem faced in the receiver circuit of Sections 4.5.1 and 4.5.2, was the phase shifts introduced by the low-pass filters at the outputs of the divide by two circuits in the reference recovery circuit of Fig. 4.22. This problem was solved by introducing a corresponding shift in the received signal path before multiplication in the I, Q channel mixer multipliers. The filter phase shifts were then actually trimmed to obtain the maximum eye-openings for the I and Q channels.

4.5.4 The Recovery PLO S/N Ratio Measurement

The experimental set-up of Fig. 4.26 was used to measure and verify the recovery loop S/N ratio calculations of Section 4.3.1. The S/N ratio was measured by filtering the squarer output with a filter of bandwidth B_N centred at one of the discrete spectral lines. Since a major portion of the noise is caused by the $B \times s$ and $s \times n$ terms, the S/N ratio cannot be measured by measuring the signal and noise powers separately. The discrete component signal power was measured using an HP 302 waveanalyser with an IF bandwidth of 6 Hz. To measure the noise power in the filter bandwidth B_N , a phase

Fig. 4.26 The Experimental Set-up Used to Measure the S/N Ratio at the Squarer Output in a Bandwidth B_N

inverted replica of the discrete frequency signal component was added to the filter output so as to cancel the signal. The phase and amplitude of this inverted signal was adjusted to achieve a good cancellation of the signal. The filter output power spectrum before and after the signal cancellation is shown in Fig. 4.27. The theoretical and measured S/N ratio is shown in Fig. 4.28 as a function of the E_b/N_o ratio for a normalised noise bandwidth $B_N T = .04$. The figure shows good agreement between the measured and computed results. The agreement is very good in the E_b/N_o range of 8-12 dB with the theoretical results being a little pessimistic for lower and higher E_b/N_o ratios.

4.5.5 The Bit-Error-Rate and Eye Patterns

The modem described in Sections 4.5.1 - 4.5.3 was used to experimentally determine the bit-error-rate and eye patterns for different reference recovery-loop noise bandwidths. The experimental set-up used for these measurements is shown in Fig. 4.29. The Hewlett-Packard data generator (HP 3760A) and error detector (HP 3761A) were used to generate the pseudorandom non-return to zero (NRZ) data and to measure the bit-error-rate. The General Radio (GR 1390B) white Gaussian noise generator was used with an attenuator to add the required amount of noise. The S/N ratio was measured at the receiver filter output by measuring the signal and noise powers; E_b/N_o was then calculated using the relation

$$E_b/N_o = (S/N) B T \quad (4.60)$$

where B = Receiver filter 3 dB bandwidth

T = Clock time period.

Fig. 4.27 The Filter F₂ Output Power Spectra Obtained in Fig. 4.26 Before and After Signal Cancellation

Fig. 4.28 The Computed and Measured Recovery Loop S/N Ratio as a Function of the E_D/N_0 Ratio for a Normalised Loop Noise Bandwidth $B_N.T = .04$

Fig. 4.29 The Experimental Set-Up Used for the Measurement of Bit-Error-Rate and Eye Diagrams

The recovery loop noise bandwidth B_N was varied by selecting different filters for the phase-lock-loop. For the Signetics 565 PLO used in the receiver there is provision for changing the loop noise bandwidth by adding external filter components. With an internal fixed resistance R_1 the second order PLO filter is formed as shown below

The values of R_2 and C are calculated using the following well-known relations [29]

PLL Natural Frequency

$$\omega_n = \sqrt{\frac{K_o K_d}{\tau_1 + \tau_2}} \quad (4.61)$$

$$\text{The loop damping constant } \zeta = \frac{1}{2} \omega_n (\tau_2 + 1/K_o K_d) \quad (4.62)$$

$$\text{The loop noise bandwidth } B_N = \frac{\omega_n}{8\zeta} \left(1 + 4\zeta^2 + 4\zeta \frac{\omega_n}{K} + \left(\frac{\omega_n}{K} \right)^2 \right) \quad (4.63)$$

where

K_o = The V.C.O gain in rad/sec.volt

K_d = The phase detector constant in Volt/rad

$\tau_1 = R_1 \cdot C$

$\tau_2 = R_2 \cdot C$

$K = K_o K_d$

The filters were designed for different noise bandwidths keeping $\zeta = .707$.

The Eye Diagrams. The eye diagram is the locus of the received baseband waveform over a data bit interval. This locus can be conveniently displayed on the oscilloscope by triggering the oscilloscope externally with the clock and by feeding the correlation detector output before sampling to the scope Y deflection. The pattern displayed on the scope then appears as a human eye. The opening of the eye shows the margin available against noise when sampling the wave. The eye opening represents the overall quality of the transmitter-receiver system and is a popular practical technique used to judge system performance.

Figures 4.30 and 4.31 illustrate the measured eye diagrams and the bit-error-rate with the receiver filter of Fig. 4.20 (Filter Bandwidth $2 BT = 1.25$) as the recovery loop noise bandwidth was varied from $B_N.T = .01$ to $B_N.T = .09$.

As may be seen from Fig. 4.30 the eye opening decreases both horizontally and vertically as the recovery loop noise bandwidth is increased.

Figure 4.31 shows the measured bit-error-rate as the recovery loop noise bandwidth is varied. From the bit-error-rate results of Fig. 4.31, the following points may be observed:

1. For $B_N.T = .09$ and $E_b/N_o < 8.7$ dB the bit-error-rate may lie anywhere in the dashed area. This is an evidence of the strong tendency of the PLO's to unlock. From Fig. 4.8 of Section 4.1.3, the phase lock loop S/N for $E_b/N_o = 8.7$ dB and $B_N.T = .09$ is:

$$B_N.T = .01$$

$$B_N.T = .03$$

$$B_N.T = .09$$

Fig. 4.30 The Measured I and Q Channel Eye Diagrams as the Recovery Loop Noise Bandwidth is Changed from $B_N.T = .01$ to $B_N.T = .09$. The Receiver IF Bandwidth $2BT = 1.2$

$$S/N \approx 4.1 (= 6.1 \text{ dB})$$

This agrees well with the PLO acquisition threshold S/N value of 6 dB [33].

2. The measured bit-error-performance for $B_N T = .01$ is degraded by slightly more than .8 dB for a bit-error-rate of 1×10^{-4} from the ideal channel result calculated theoretically. The bit-error-performance thus shows a good agreement with theory once the .8 dB degradation is accounted for. Part of this degradation can be attributed to the narrow-band receiver filtering with a bandwidth $2B = 1.25/T$. It may be noted that this bandwidth is less than the MSK mainlobe bandwidth of $1.5/T$. As previously determined [11] from a computer simulation of the effect of filtering on MSK, a degradation of about .3 dB is expected for a filter with normalised bandwidth of 1.25. The further degradation of .5 dB results from system hardware limitations. For a bit-error of 1×10^{-6} the degradation increases to 1.2 dB compared to the results of Fig. 4.15.

3. The spread in the bit-error-rate reflects the measurement inaccuracy and the spread in the system performance. This made an accurate determination of the effect of changing $B_N T$ from .01 to .03 difficult. However a reasonable agreement is shown in terms of the rapid increase in BER degradation as the noise bandwidth is increased above $B_N T = .03$.

4.6 CONCLUSIONS

A method which uses the recovered clock of the MSK signal as the frequency reference for the remote sensor stations has been described. The important system design considerations in the implementation of the proposed method have been theoretically and experimentally analyzed.

It may be concluded from the analysis that the MSK reference recovery circuit must be designed as a compromise between the loop acquisition performance and the bit-error-rate performance. The recovery circuit performance curves which may be used to make this compromise have been provided.

The effect of the transmitter carrier phase noise, another important design consideration in low-bit-rate systems, has been analyzed for MSK systems.

The experimental data for the recovery loop S/N ratio and the bit-error-rate as a function of E_b/N_0 ratio show good agreement with the analysis presented in this chapter.

In addition the reference recovery analysis discussed in this chapter shows that MSK has the following important reference recovery advantages as compared to the QPSK.

- A. It requires a simple squaring nonlinearity to recover both the carrier and clock. In comparison QPSK requires a fourth order to recover the carrier. A separate circuit such as the delay and multiply circuit is required to recover QPSK clock.
- B. The carrier and clock are recovered simultaneously. This may be a distinct advantage in burst type of communication systems.
- C. The symbol offsetting and the sinusoidal symbol shaping results in less susceptibility of MSK to the recovered reference phase error.

Part B of the thesis commences with the next chapter. In Chapters 5 to 7 of Part B we describe the performance of an ILQ-PLA for MSK modulated input signals.

PART B

CHAPTER 5

THE INJECTION LOCKED OSCILLATOR -
PHASE LOCKED AMPLIFIER FOR MSK SIGNALS

5.1 INTRODUCTION

Injection locking may be defined as the phenomenon in which a weak signal is injected into a high power oscillator thereby synchronizing its frequency. For microwave oscillators the most common method used to inject the signal is through a circulator which effectively isolates the input and output ports by virtue of its non-reciprocal properties. In general, upon application of the weak input signal, the oscillator frequency is pulled by an amount dependent upon the original frequency difference. If the input frequency is made sufficiently close to the free-running oscillator frequency, the oscillator will become frequency locked to the input signal, and will therefore oscillate at the input signal frequency. The oscillator will remain locked to the input signal frequency over a range of frequencies called the locking range. The locking range depends on the strength of the input signal and the ease or difficulty with which the high power oscillator may be pulled. The latter is the characteristic of a particular oscillator and is associated with the quality factor Q of the oscillator.

The locking bandwidth of an ILO is given by Adler's equation [35].

$$B_L = \frac{2 f_o}{Q_e} \sqrt{\frac{P_i}{P_o}} \quad (5.1)$$

where

B_L = Two sided locking bandwidth

f_o = oscillator frequency

P_i = locking signal power

P_o = The locked oscillator output power

Within the locking range the oscillator phase follows the phase of the input signal with a steady state phase error which depends on the original frequency difference between the input signal and the oscillator. This implies that a low-level phase modulated input signal may be amplified using a higher power injection locked oscillator. The gain of such an amplifier would be limited by the required locking bandwidth. This then offers a mean for realizing a high gain power efficient microwave amplification in a single stage for the phase or frequency modulated signals. In particular for comparatively narrow-band systems, all the transmitter microwave gain may be obtained in a single stage. For example for the sensor station transmitter described in Part A of the thesis, a single one watt ILO-PLA may be used after upconversion.

The application and analysis of an ILO as a microwave amplifier of analog angle modulated signals is well known [3] [4]. The properties of an ILO as a microwave amplifier of digital angle modulated signals (PSK, FSK) has been reported by Kuno [5] and Paik et al [6]. Kuno concluded that the required locking bandwidth of the ILO is determined by the phase discontinuities which occur at the interbit switching instants, and that in practice a locking bandwidth of four to five times the bit rate is required. Paik [6], however, measured the bit-error-rate of an ILO having such a locking bandwidth when amplifying a QPSK modulated signal and observed an anomalous effect. The bit-error-rate at high E_b/N_0 ratios did not decrease rapidly in the expected fashion but remained high. This effect is shown in Fig. 11 of [4], reproduced as Fig. 1.2. Paik suggested that this might be due to the 180° phase discontinuities which cause the

envelope of the filtered signal to go to zero, resulting in the loss of lock at these instants. From the results of Kuno and Paik [5], [6], it may be concluded that the performance of the ILO-PLA for digital signals is limited by the phase transients at the interbit switching instants.

As shown in the MSK phasor diagram of Fig. 2.1, this modulation is essentially a phase continuous process. During the bit interval the phase is changed linearly and the continuity of phase is maintained at the interbit switching instants. Based on this feature of MSK we have proposed [34] in this thesis that this modulation will be more compatible for amplification with a phase locked amplifier as compared to the digital modulation techniques which have phase transients.

In the following two chapters we will seek to verify the performance of an ILO-PLA for MSK signals. To do this we will study two important system performance factors which may be directly related to the characteristics of the ILO-PLA as the microwave power amplifier. While the spectrum spreading effects of an ILO-PLA are treated in the next chapter, the system bit-error performance with an ILO-PLA is the subject of Chapter 7.

CHAPTER 6

THE SPECTRUM-SPREADING EFFECT OF AN INJECTION LOCKED OSCILLATOR -
PHASE LOCKED AMPLIFIER FOR MSK SIGNALS6.1 INTRODUCTION

The spectrum-spreading of the filtered signal by a nonlinear microwave amplifier is an important system design consideration. The spectrum-spreading effect of an ILO-PLA for MSK signals is the subject of this chapter.

In Section 6.2 a brief description of the term spectrum-spreading is included. In Section 6.3 we develop a simplified analytical model of the ILO-PLA which is applied in a computer simulation study of the spectrum-spreading caused by an ILO-PLA for the filtered MSK signals. Section 6.4 includes a description of the combined analytical-numerical technique which was used to compute the power spectrum at the ILO-PLA output. A description of an experimental 5 Mbits/sec MSK system and the theoretical and experimental spectrum-spreading results obtained with this system are presented in Sections 6.5 and 6.6 respectively.

Conclusions based on the experimental and theoretical results are presented in Section 6.7.

6.2 SPECTRUM-SPREADING

Figure 6.1 illustrates the transmitter section of a general digital microwave communication system. The digitally modulated signal is generated from the baseband data at an IF frequency (a common IF frequency is 70 MHz). The spectrum of the modulated signal theoretically extends in frequency from $-\infty$ to ∞ . The modulated signal is then filtered at IF so as to limit the spectrum to meet the channel bandwidth allocated for the system by the

Fig. 6.1 The Power Spectra at Various Points in a Typical Digital Microwave System

frequency regulatory agencies? The filtered signal is upconverted and amplified by the microwave power amplifier. The transmitter amplifier is either a TWT amplifier or an efficient solid state amplifier whose output stages operate as limiting "class C" type devices. In both the cases the amplifier is in effect also a limiter for reasons of efficiency*. Limiting of the filtered signal will result in some increase of the sidelobe power (referred to as spectrum-spreading). In fact it is well known that the hardlimiting of the filtered QPSK signal almost restores the spectrum to the unfiltered value [13] [41]. Any appreciable spreading of the filtered signal spectrum by the limiting amplifier would require the use of microwave filtering after the amplifier which is undesirable and may be impossible for certain narrowband systems. However, since MSK and OK-QPSK are free of 180° phase transients, it is well known [13] that the spectrum-spreading as a result of the limiting of the filtered signal is much less for OK-QPSK and MSK as compared to QPSK.

In succeeding sections the spectrum-spreading of an ILO-PLA for filtered MSK signals will be theoretically and experimentally examined.

6.3 AN ILO MODEL FOR MSK SIGNALS

The phenomenon of injection locking has been well known for many years. A simple theory to describe the behaviour of oscillators under an injected signal, was presented by Adler in a classic paper [35]. The well known result of Adler is in the form of a differential equation for phase given here as Equation 6.1.

* For low-level input signals the ILO output power is more or less independent of the input signal power. Thus an ILO is in effect also a hardlimiter.

$$\frac{d\alpha}{dt} = \Delta\omega_o - \frac{E_i \omega_o}{E_o 2Q_e} \sin\alpha + \phi_i'(t) \quad (6.1)$$

where the input signal is given by

$$e_i = E_i \cos(\omega_i t + \phi_i(t))$$

and the output signal of the locked oscillator is given by

$$e_o = E_o \cos(\omega_i t + \phi_i(t) + \alpha(t))$$

and

$$\Delta\omega_o = \text{Frequency offset} = \omega_i - \omega_o$$

ω_o = Free running frequency of the oscillator

ω_i = Input signal frequency

$\phi_i(t)$ = Phase of the input signal

$\phi_i'(t)$ = Time derivative of $\phi_i(t)$

$\alpha(t)$ = Phase of the output signal relative to the input signal

Q_e = External Q of the oscillator

In our analysis we have used a slightly modified form of Adler's equation [36] to obtain a solution for the output phase. This equation is given by

$$\frac{d\alpha}{dt} = \Delta\omega_o - \frac{E_i \omega_o}{E_o 2Q_e} \frac{\sin\alpha}{1 + \frac{E_i}{E_o} \cos\alpha} + \phi_i'(t) \quad (6.2)$$

This equation takes into account the effect of relatively higher input signal levels and reduces to the Equation (6.1) for $E_i \ll E_o$.

By assuming

$$E_i < E_o$$

$$\sin \alpha \approx \alpha - \alpha^3/6$$

and

$$\cos \alpha \approx 1 - \alpha^2/2$$

Equation (6.2) may be expressed as

$$\frac{d\alpha}{dt} = \Delta\omega_o - \frac{E_i}{E_o} \left(1 - \frac{E_i}{E_o}\right) \frac{\omega_o \alpha}{2Q_e} + \frac{E_i}{E_o} \frac{\omega_o}{2Q_e} \left(1 - \frac{4E_i}{E_o}\right) \frac{\alpha^3}{6} + \phi_i'(t) \quad (6.3)$$

To solve this differential equation for α we have used an iterative technique.

In this technique a first order solution α_1 is obtained by neglecting terms of order α^2 and higher. This solution α_1 is then used to obtain a new solution α_2 which considers an additional effect due to these terms.

Neglecting α^3 term in R.H.S of (6.3) we obtain

$$\frac{d\alpha_1}{dt} = \Delta\omega_o - \frac{E_i}{E_o} \left(1 - \frac{E_i}{E_o}\right) \frac{\omega_o \alpha_1}{2Q_e} + \phi_i'(t) \quad (6.4)$$

(6.4) may be written as

$$\frac{d\alpha_1}{dt} + \omega_m \alpha_1 = \Delta\omega_o + \phi_i'(t) \quad (6.5)$$

where

$$\omega_m = \left(1 - \frac{E_i}{E_o}\right) \frac{E_i}{E_o} \frac{\omega_o}{2Q_e} \quad (6.6)$$

Equation (6.5) is of the form

$$\frac{dy}{dt} + P(t) y = Q(t) \quad (6.7)$$

Using an integrating factor $\mu = e^{\int P(x) dx}$ a solution for such an equation is given by

$$y = e^{-\int P dt} \left[\int Q e^{\int P dt} dt + C \right] \quad (6.8)$$

Comparing (6.5) and (6.7)

$$P(t) = \omega'_m$$

$$Q(t) = \Delta\omega_o + \phi'_i(t)$$

$$\alpha_1(t) = e^{-\int \omega'_m dt} \left[\int (\Delta\omega_o + \phi'_i(t)) e^{\int \omega'_m dt} dt + C \right]$$

$$\alpha_1(t) = \frac{\Delta\omega_o}{\omega'_m} + \frac{\phi'_i(t)}{\omega'_m} + C e^{-\omega'_m t} - \frac{\phi''_i(t)}{\omega'^2_m} \quad (6.9)$$

The constant C is to be determined from the boundary conditions.

For an MSK input signal the phase is changed linearly by 90° in a bit interval, so that in the k th interval phase

$$\phi_i(t) = \frac{\pi U_k t}{2T}$$

where $U_k = k$ th data bit = ± 1 .

For a random data string, the phase for an MSK signal is as shown in Fig. 6.2.

The solution given by Equation (6.9) may be interpreted as the steady state phase error = $\frac{\Delta\omega_o}{\omega'_m} + \frac{\phi'_i(t)}{\omega'_m} - \frac{\phi''_i(t)}{\omega'^2_m}$

accompanied by an exponential phase change from the steady state phase error in the $(k-1)$ th interval to that in the k th interval. Thus the output phase

Fig. 6.2 The ILO Input and Output Phase for a Random String of MSK Modulated Input Signal

will be as shown by the dashed line in Fig. 6.3. provided $\omega'_m T \gg 1$.

Equating the phase error at the $(k-1)$ th to the k th bit switching instant the constant C for the k th bit interval is given by

$$C = -\left(\frac{\pi U_k}{2T \omega'_m} - \frac{\pi U_{k-1}}{2T \omega'_m}\right)$$

Therefore, from Equation (6.9) the phase error $\alpha_1(t)$ in the k th bit interval is given by

$$\alpha_1(t) = \frac{\Delta\omega_0}{\omega'_m} + \frac{\phi'_i(t)}{\omega'_m} - \left(\frac{\pi U_k}{2T \omega'_m} - \frac{\pi U_{k-1}}{2T \omega'_m}\right) e^{-\omega'_m t} - \frac{\phi''_i(t)}{\omega'^2_m} \quad (6.10)$$

$$(k-1)T \leq t \leq K T$$

To determine the effect of higher order terms in Equation (6.3), the value of α_1 obtained from a first order solution is substituted for these terms and a new solution for α is obtained assuming α_1 to be a constant. To keep the analysis simple and meaningful the first order solution used in calculating the effect of higher order terms is taken to be that given by the first two terms in Equation (6.10).

Thus a complete solution for an MSK signal is given by

$$\alpha(t) = \phi_0 + k_1 \phi'_i(t) + k_2 (\phi'_i(t))^2 + k_3 (\phi'_i(t))^3 - \left(\frac{\pi U_k}{2T \omega'_m} - \frac{\pi U_{k-1}}{2T \omega'_m}\right) e^{-\omega'_m t} - \frac{\phi''_i(t)}{\omega'^2_m} \quad (6.11)$$

$$(k-1)T \leq t \leq K T$$

where

$$\phi_o = \frac{1}{\omega'_m} (\Delta\omega_o + \frac{\omega''_m \Delta\omega_o^3}{6\omega'^3_m})$$

$$\omega'_m = (1 - \frac{E_i}{E_o}) \frac{E_i}{E_o} \frac{\omega_o}{2Q_e}$$

$$\omega''_m = (1 - \frac{4 E_i}{E_o}) \frac{E_i}{E_o} \frac{\omega_o}{2Q_e}$$

$$k_1 = \frac{1}{\omega'_m} + \frac{1}{2} \frac{\omega''_m \Delta\omega_o^2}{\omega'^4_m}$$

$$k_2 = \frac{1}{2} \frac{\omega''_m \Delta\omega_o}{\omega'^4_m}$$

This value of $\alpha(t)$ gives the phase error introduced by the ILO. $\alpha(t)$ as calculated above includes the effect of phase nonlinearities as well as the AM to PM effect which arises due to changes in the locking bandwidth as a result of the input signal amplitude variations.

At higher signal levels (lower gains for the ILO-PLA) additional AM-PM effects will be introduced by the variation of the microwave oscillator device impedance with the RF signal level. Taking these effects into account will involve solving two coupled nonlinear differential equations for the output amplitude and phase [37]. This is a difficult task in the case of a phase modulated input signal. For the large gain values, however, (for which we are basically considering the use of ILO-PLA) the assumptions used in the above analysis are quite valid. We may include the additional effect due to any residual AM (i.e. AM-AM) in the output, by calculating the ILO output amplitude from Kurokawa's [38] expression for the

proportional change in the output power ΔP of a negative resistance microwave oscillator

$$\frac{\Delta P}{P_o} = 2 \sqrt{\frac{P_i}{P_o}} \left(\frac{2}{s} - 1 \right) \cos \phi_o \quad (6.12)$$

where

P_i = Input power to the locked oscillator

P_o = Output power of the free-running oscillator

ΔP = Change in output power due to locking

The factor s is a measure of the device negative resistance nonlinearity and is given by

$$s = \frac{1}{R_L} \left. \frac{\partial R}{\partial I} \right|_{I = I_o}$$

where

I = Amplitude of the RF current through the device negative resistance for the locked oscillator

I_o = Value of I for the free-running case

R_L = Load resistance

R = Magnitude of the negative resistance

Using

$$P_{out} = P_o + \Delta P$$

and

$$P_o = \frac{E_o^2}{2R_L}$$

and

$$P_i = \frac{E_i^2}{8R_L}$$

we have from Equation (6.12)

$$E_o = E_o' \left(1 + \frac{1}{2} \frac{E_i}{E_o'} (2/s - 1) \cos \phi_o \right) \quad (6.13)$$

where E_o' represents the output signal amplitude of the free-running oscillator. Equations (6.11) and (6.13) may be used to obtain the ILO output phase and amplitude for the input MSK signal. From Equations (6.11) through (6.13), it is evident that the key circuit parameters which determine the behaviour of the ILO-PLA are the following.

- 1) The gain of the ILO-PLA
- 2) The external Q , Q_e of the ILO
- 3) The frequency offset $\Delta\omega_o$ between the free-running frequency and the input nominal carrier frequency
- 4) The nonlinearity parameter s of the device negative resistance.

6.4 THE COMPUTER SIMULATION

A combined method of analysis and computer simulation, parallel to the one described for PSK spectrum-spreading with a TWT amplifier [40], was used to obtain the spectrum at the ILO output. The computer program requires the measured transmitter filter amplitude and phase response as the input. From this data the real and imaginary parts of the filter impulse response are computed. This impulse response is convolved with the sinusoidal symbol shapes for the MSK symbol waveform to generate two arrays. Eighteen samples per symbol are used in generating these arrays. With these arrays and the ILO model described in the last section and randomly generated data, the ILO-PLA output spectra is obtained using Fast Fourier Transform subroutines.

A complete description of the theoretical basis as well as the program itself is given in Appendix A. The computed power spectra are presented along with the measured results in Section 6.5.

6.5 THE EXPERIMENTAL CIRCUIT

Experimental measurements of the spectrum-spreading results were carried out by designing and implementing a 5 Mbits/sec MSK generator. A 6 GHz Microstrip ILO was designed to test the output spectrum with the 5 Mbits/sec MSK input. The MSK generation circuit, the ILO and the experimental arrangement are described in this section. The results are presented in Section 6.6.

6.5.1 5 M Bits/sec. MSK Generation Circuit

The technique used was consistent with the OK-QPSK view of MSK [39]. The circuit used is shown in Fig. 6.3(a). The MSK nominal carrier frequency was obtained from a 60 MHz signal generator. From this, it was then necessary to generate phase coherent carriers such that the spacing between the two frequencies is exactly one-half the bit rate. This was accomplished by dividing the clock signal by four, low pass filtering the output and multiplying it with the 60 MHz signal using a balanced modulator. The advantage of deriving the carriers in this fashion was that the variations in data-rate do not change the deviation ratio nor do they affect the coherence of the carriers [39]. The carriers at 61.25 MHz and 58.75 MHz were separated by the bandpass filters F_1 and F_2 . Third order Butterworth filters shown in Fig. 6.3(b) were designed to accomplish this. Each filter suppressed the other carrier by more than 25 dB. Further the filter tuning and phase shifter P were adjusted to trim the phase of these carriers so

Fig. 6.3(a) The Circuit Used to Generate 5 M Bits/sec 60 MHz MSK as Offset Keyed - QPSK

Fig. 6.3(b) Circuit Used for Realizing Filters F_1 and F_2 in Fig. 6.3(a)

as to obtain the MSK spectrum which most closely resembled the theoretical form.

Thus, at the output of F_1 and F_2 , we have two signals:

$$\cos \left(\omega_0 t + \frac{\pi t}{2T} \right)$$

and

$$\cos \left(\omega_0 t - \frac{\pi t}{2T} \right)$$

These signals were fed to a hybrid coupler to obtain the following output signals at the sum and difference ports of the hybrid:

$$\cos \left(\omega_0 t + \frac{\pi t}{2T} \right) + \cos \left(\omega_0 t - \frac{\pi t}{2T} \right) = 2 \cos \omega_0 t \cos \frac{\pi t}{2T}$$

$$\cos \left(\omega_0 t + \frac{\pi t}{2T} \right) - \cos \left(\omega_0 t - \frac{\pi t}{2T} \right) = 2 \sin \omega_0 t \sin \frac{\pi t}{2T}$$

These were the I and Q channel components required in Equation (2.7) to generate MSK as OK-QPSK. These quadrature carriers were then amplitude modulated with alternate data bits to produce the modulated signals.

Before modulation, the input data and clock signals were processed as shown in Fig. 6.3. The serial data was divided into two offset I and Q channels. This was done by using two D flip-flops which were clocked by $\text{clock}/2$ and $\overline{\text{clock}/2}$ signals derived by dividing the clock signals. The D inputs for the delay flip-flops were supplied by the input data. Thus, two data channels containing the alternate data bits were formed. The Q outputs of the delay flip-flops were used to modulate as BPSK the I and Q carriers obtained above using the double balanced mixers. Summing of the amplitude modulated signals results in the constant envelope 5 M Bits/sec MSK signal at a nominal carrier frequency of 60 MHz.

The 60 MHz MSK signal was filtered with a fifth order Butterworth filter with a 3 dB bandwidth = 7 MHz. The filtered signal was upconverted to 6 GHz and amplified with an ILO amplifier described in the next section.

6.5.2 The 6 GHz ILO

A 6 GHz, Microstrip, IMPATT oscillator was designed and tested as an ILO-PLA. The microstrip circuit designed is shown in Fig. 6.4. The reactive part of the diode impedance is parallel tuned by two open circuited stubs and the resulting resistive impedance is transformed to 50 ohms by using three series $\lambda/4$ transformers. The input signal is coupled to the microwave oscillator using a circulator to realize an injection locked oscillator.

6.5.3 The Spectrum-Spreading Measurement Method

The technique used to measure the spectrum-spreading effect of the ILO-PLA is shown in Fig. 6.5. The 60 MHz, 5 Mbits/sec MSK signal generated by the circuit of Fig. 6.4 was filtered by a fifth order Butterworth filter with a 3 dB bandwidth equal to 7 MHz. The filtered signal was upconverted by mixing with a 5.960 GHz signal. The upconverted signal was filtered by a Yig filter to remove the image frequency. The output signal from the upconverter was amplified by a linear TWT amplifier. The TWT was operated 20 dB below the saturation level to ensure operation in the linear region. The attenuator at the output of the TWT amplifier was used to control the input signal to the ILO in order to test the ILO at various gains. The ILO output was fed to the spectrum analyser to measure the power spectrum.

Fig. 6.4 6 GHz Microstrip IMPATT ILO

Note: Starting from a value of $\lambda/2$ the two tuning stub lengths were experimentally adjusted to obtain the desired frequency. Compared to a single stub two tuning stubs provide better tuning control. If one stub is overshortened, the other stub can be cut to get the desired frequency.

Fig. 6.5 The ILO-PLA Spectrum-Spreading Test Set-Up

6.6 THE TEST RESULTS

The power spectrum of the MSK signal generated in Fig. 6.3 is shown in Fig. 6.6. As may be seen from this figure the mainlobe is 7.5 MHz wide (1.5×5). The first sidelobe is ≈ 20 dB below the mainlobe compared to the theoretical value of 22 dB. The amplitude and phase of the I-Q carriers used in the MSK transmitter were adjusted to obtain the power spectrum which closely matches the theoretical power spectrum of Fig. 2.5. The filtered spectrum at IF (60 MHz) and the spectrum at the input to the ILO are also shown in Fig. 6.6. Since all amplification prior to the ILO was achieved using linear amplifiers, no appreciable spectrum-spreading occurred prior to the ILO as shown in Fig. 6.6. The power spectrum at the ILO output was measured for various gains and frequency offset values. The measured power spectra are shown in Figs. 6.7 - 6.9 for frequency offset values $\Delta\omega = 0, 2, 4$ MHz respectively. The ILO locking bandwidth, as measured with a single frequency carrier, is also shown in these figures for different gain values. From these figures the following important points may be observed.

1. As expected the hardlimiting by the ILO-PLA does not restore the filtered MSK spectrum to the unfiltered value, as in QPSK.
2. The frequency offset $\Delta\omega$ becomes an important design consideration particularly at high gains. The frequency offset will be determined in practice by the temperature stability of the ILO.

At a gain of 20 dB, as the frequency offset was changed from 0 to 2 and 4 MHz two noticeable effects were an increase in the passband distortion and an increase in the spectral spreading. At $\Delta\omega = 4$ MHz and

Fig. 6.6 The Measured Power Spectra in the Set-Up of Fig. 6.5

Fig. 6.7 Experimental and Theoretical Power Spectral Density at ILO Output.
Freq. Offset = 0 MHz

Fig. 6.8 Experimental and Theoretical Power Spectral Density at ILO Output.
Freq. Offset = 2 MHz

Fig. 6.9 Experimental and Theoretical Power Spectral Density at ILO Output.
Freq. Offset = 4 MHz

gain = 20 dB, a tendency to unlock is indicated by the marked increase in the spectrum-spreading and high passband distortion as seen in Fig. 6.9.

At a gain of 12 dB the effect of changing $\Delta\omega$ from 0 to 2 MHz is only minimal. This is because a change of 2 MHz is only a small percentage of the locking bandwidth of 30 MHz at a gain of 12 dB. From the results of Figs. 6.7 - 6.9, we could conclude that for a locking bandwidth of 9 MHz (corresponding to a gain of 20 dB), the frequency offset should be limited to 2 MHz. An increase in $\Delta\omega$ to 4 MHz would result in a rapid deterioration in spectrum-spreading.

3. From Fig. 6.7 it would seem that if the frequency offset is kept small ($\Delta\omega \approx 0$), increasing the gain actually improves the spectrum-spreading though it increases the passband distortion. While large input signal effects at a lower gain may be one reason for such a behaviour, this result should be interpreted cautiously since for higher $\Delta\omega$ increasing the gain to a value where the ILO becomes unlocked very rapid spreading of the spectrum results.

4. The output spectrum is noticeably nonsymmetrical, i.e., spectrum-spreading is more at the lower frequency side of the spectrum as compared to higher frequency side. With a frequency offset $\Delta\omega \neq 0$ this may be due to the spreading being higher for the frequencies near the edge of the locking region, which would make the spectrum unsymmetrical in the presence of a frequency offset. However, this argument is challenged by the fact that a similar lack of symmetry was observed when $\Delta\omega = 0$ or $\Delta\omega = -2$ MHz. One reason for this lack of symmetry was found to be the nonsymmetrical frequency response of the IF bandpass filter which had about 5 dB difference

in attenuation for frequencies 8 MHz above and below the centre frequency of 60 MHz. However the symmetry differential in the power spectrum in Figs. 6.7 - 6.9 at frequencies 60 ± 8 MHz is about 10 dB. We are unable to provide a satisfactory explanation for this 5 dB difference. The limited time precluded further work required to explain this difference.

5. The envelope of the spectrum at the ILO output was also theoretically computed using the ILO model of Section 6.3 and the computer program described in Section 6.4 and Appendix A. The envelope of the computed spectrum is shown by the dashed lines in Figs. 6.7 - 6.9.

While the agreement with the measured results in the passband is very good, this could not be said for the region removed from the passband.

As explained in Appendix A the filter impulse response was terminated over a time-window to limit the computation time. This amounts to neglecting the effect of filtering outside this time-window, which is equivalent to convolving the filtered signal spectrum with the window Fourier transform. This results in a raising of the power spectrum level, an effect most noticeable in the region where the spectrum level is low.

Another reason for the disagreement between the theoretical and measured results may be the simplified ILO model used in the computation. As mentioned in Section 6.3, this ILO model neglects the effect of the device impedance variation with the RF signal level. As such this model may not be sufficiently accurate to describe the spectrum-spreading effects, particularly since these effects are due to the frequency components with power levels which are more than 30 dB below the passband level. This model may, however, be sufficiently accurate to describe the passband

distortion which would determine the bit-error-rate degradation introduced by the ILO-PLA.

6.6.1 The Spectrum-Spreading for Narrowband MSK Systems

In the experimental arrangement described in Section 6.5.4 above, an IF filter with a 3 dB bandwidth of 7 MHz was used as the transmitter filter for the 5 M Bits/sec MSK signal. The filter bandwidth was thus large enough to pass the mainlobe of the MSK signal spectrum which was 7.5 MHz wide. For some narrowband communication channels, spectrum conservation is a prime consideration and the filtering at IF may be done with a filter which cuts into the mainlobe itself. Although this may result in a degradation in the bit-error-rate performance, it is necessary to meet the frequency regulation requirements. It has been reported previously [41] that such narrowband MSK systems would require narrowband microwave filtering after amplification with a nonlinear microwave amplifier. This conclusion was based on a previously reported result [13], that even though the sidelobes are not regenerated after nonlinear amplification of the filtered MSK signal, the mainlobe is restored to the unfiltered value. Since the mainlobe for MSK is wider than QPSK or OK-QPSK ($1.5 \times$ bit rate for MSK as compared to $1 \times$ bit rate for QPSK), it has been argued [41] that the frequency allocation requirement can only be met for MSK if a microwave filter is used after nonlinear microwave amplification.

To experimentally identify the effect of ILO-PLA hardlimiting on the mainlobe spectrum, we have measured the spectrum at the ILO output with narrowband filtered MSK as the input. As the filter used in our arrangement had a fixed bandwidth of 7 MHz, the narrowband filtering was

obtained by varying the MSK clock rate. The test was carried out for the clock rates of 7 MBits/sec and 10 MBits/sec. The ILO output spectrum was measured for a gain of 25 dB keeping the frequency offset $\Delta\omega = 0$. The measured results are shown in Figs. 6.10 and 6.11. From both these figures it is clear that the mainlobe is not restored as previously reported. This is particularly evident from Fig. 6.11 for the 10 MBits/sec MSK where the filtering effectively cut the mainlobe. The mainlobe was almost completely maintained at its filtered value at the ILO-PLA output. These experimental results do not support the argument given in reference [41]. The deductions in this reference were based on an examination of the simplified computer simulation results of reference [13]. These results were given for $BT = 1.0$. Our experimental results of Fig. 6.11 for $BT = .7$ clearly prove that the filtered MSK signal mainlobe is not regenerated as a result of limiting by an ILO-PLA. The erroneous conclusion of reference [41] may be due to the fact that a filter bandwidth $BT = 1.0$ makes a conclusive judgement about the regeneration of the mainlobe difficult.

6.7 CONCLUSIONS

The spectrum-spreading results obtained in this chapter are in agreement with the previously known result that hardlimiting after filtering does not restore the MSK spectrum to its unfiltered value. An interesting result obtained in this chapter (see Fig. 6.7) shows that for frequency offset equal to zero, the filtered signal-spectrum is maintained (although there is considerable passband distortion) even with a two-sided ILO locking bandwidth of 2 MHz for the 5 MBits MSK signal. For the 5 MBits/sec MSK signal with an ILO-PLA two-sided locking bandwidth of 9 MHz a frequency

Fig. 6.10 The Spectrum-Spreading due to ILO-PLA for a 7 Mbits/sec Filtered MSK signal.
Filter bandwidth = 7 MHz.

Fig. 6.11 The Spectrum-Spreading due to ILO-PLA for a 10 MBits Filtered MSK Signal.
Filter Bandwidth equal to 7 MHz.

offset of 2 MHz causes negligible increase in the spectrum-spreading.

Increasing the frequency offset or decreasing the locking bandwidth (for a higher ILO-PLA gain) causes a rapid increase in the spectrum-spreading.

For the narrowband systems the filtered MSK mainlobe is not regenerated after limiting by the ILO-PLA. This result does not support a previous argument [41] that narrowband MSK systems require microwave filtering after amplification of the filtered signal with a limiting microwave amplifier.

In the next chapter we describe the bit-error-performance results.

CHAPTER 7

THE ILO-PLA BIT-ERROR PERFORMANCE7.1 INTRODUCTION

The bit-error performance of a digital system is determined by the bit energy to noise spectral density ratio required to achieve a certain probability of bit-error. The bit-error performance reflects an overall quality of the digital system. Any degradation in performance must be compensated for by either increasing the transmitter power or by reducing the receiver thermal noise. The degradation in bit-error performance is measured with respect to the performance achievable for an ideal channel where an ideal channel is considered to be one having linear amplitude and phase response, infinite bandwidth, and corrupted only by white Gaussian noise. In addition, for an ideal channel, perfect carrier and clock references are assumed to be available at the receiver. The degradation in bit-error performance is a manifestation of the combined effect of the various factors which cause the communication channel to be non-ideal.

The bit-error performance of a given subassembly of the communication system is evaluated by the additional degradation which may occur as a result of introducing that particular subassembly in the system. In this chapter we seek to experimentally determine the bit-error performance of an ILO-PLA for MSK systems.

As discussed in Chapter 5, the gain of the ILO-PLA determines the locking bandwidth. The degradation in the bit-error performance will increase as the locking bandwidth decreases for the higher gain values. The main result which we seek by the experimental evaluation will be to determine the possible tradeoff between the achievable gain and the bit-

error performance degradation. In determining this tradeoff, we also seek to show that because of the phase continuous nature of MSK, an ILO-PLA does not exhibit a large performance degradation as for QPSK. In addition we will also determine the effect of frequency offset on the bit-error performance. As in Chapter 4, the I and Q channel eye diagrams will also be measured along with the bit-error-rate to present a clear qualitative picture of the ILO-PLA parameter changes. A description of the MSK receiver design to obtain these measurements is presented in Section 7.2. The experimental results are described in Section 7.3, while conclusions are summarized in Section 7.4.

7.2 THE 5 M Bits/sec MSK RECEIVER

A receiver, designed to coherently demodulate the 5 M Bits/sec MSK data from the ILO-PLA output signal of Fig. 6.5 in the previous chapter is shown in Fig. 7.1. Both the carrier and clock references were connected directly from the transmitter. Since the main objective was to test the ILO-PLA performance, obtaining the carrier and clock directly from the transmitter considerably reduces the circuit complexity and avoids introducing an additional variable in the system. The phase of the I and Q channel reference signals were controlled by the phase shifters P_1 and P_2 in Fig. 7.1. The phase shifters were adjusted to obtain the maximum I and Q channel eye openings and the lowest bit-error-rate. The phase shifter P_1 in the received signal path was used to control the phase shift of the received RF signal. Adjusting this phase shifter was equivalent to adjusting both the I and Q channel reference signal phases. The phase shifter P_2 was used to control the phase of the Q reference signal independently.

Fig. 7.1 The 5 M Bits/sec MSK Receiver Designed for the Bit-Error-Rate Measurements

This arrangement was preferred to adjusting both the I and Q channel reference signal phases independently because it offered a practical advantage of adjusting only P_1 to compensate for any phase shift in the signal path which affected I and Q channels equally.

As shown in the receiver circuit of Fig. 7.1, the ILO-PLA output was attenuated to obtain a level suitable for the downconverter. The 6 GHz signal was then downconverted to an IF frequency of 60 MHz. Both the transmitter local oscillator signal (see Fig. 6.6) and the receiver local oscillator signal, required for downconversion were derived by power division from a microwave oscillator (HP 8690). This arrangement avoids any problems due to the frequency instability of the local oscillators. The downconverted signal was filtered at IF with a fifth order, 7 MHz bandwidth IF filter. A signal level of +2 dBm was required at the filter output. For coherent detection, the IF signal was divided in two channels using a power divider. The signals in the two channels were multiplied by the I and Q reference signals (which were hard wired from the transmitter).

Balanced mixers (Minicircuit Labs MCL SRA-1) were used for signal multiplication. The mixer outputs were low-pass-filtered with the second order Butterworth filters. The frequency response of these filters was experimentally adjusted by varying the inductors to achieve the lowest bit-error-rate.

A saturated wideband IC video amplifier (Texas Instruments 733) was used in the open circuit loop to obtain the rectangular pulse signals from the raised cosine bipolar signal at the filter outputs. Finally the sample and hold functions were accomplished by D flip-flops [39]. The D flip-flops

were-clocked by the clock/2 and $\overline{\text{clock}}/2$ signals which were obtained from the Q and $\overline{Q}$ outputs of a divide by two JK flip-flop. The clock input for the divide by two circuit was hardwired from the transmitter. The 2.5 M Bits/sec data signals at the output of sample and hold circuits were then interleaved using an exclusive-OR gate to obtain the recovered 5 M Bits/sec data.

7.3 THE EXPERIMENTAL RESULTS

7.3.1 The Test Arrangement

The arrangement of Fig. 7.2 was used for the bit-error-rate and eye pattern measurements. White noise was added to the signal at the IF filter input. The noise was obtained from the output of a high gain wideband amplifier with a 50 ohms termination at the input. The E_b/N_o ratio was measured from the signal and noise power measurements at point A in the system by using the relation

$$E_b/N_o = (S/N) \cdot B \cdot T \quad (7.1)$$

where

B = IF filter 3 dB bandwidth

$\frac{1}{T}$ = Clock Rate

Different values of E_b/N_o were obtained by varying the noise power with a 1 dB step attenuator. A calibration table in terms of the attenuator reading and the E_b/N_o ratio was thus obtained. This calibration was checked before and after each set of measurements. The noise source was switched off for the eye diagram measurements. In addition the signal level at point A in the test set-up was reduced to -10 dBm for the eye diagram results. The I-Q channel video amplifier outputs were fed to the two Y channels of the oscilloscope. The oscilloscope was triggered externally with the clock signal.

Fig. 7.2 The Test Set-up Used for the BER Measurements

7.3.2 The Locking Bandwidth Results

The locking bandwidth of an ILO-PLA is a key parameter which determines the bit-error-rate degradation. Measurements were conducted using the three microwave oscillators described in Table 7.1.

Table 7.1 Oscillators Used in ILO-PLA

Bit-Error-Rate Performance Testing

*Osc. No.	Configuration	Power Out. (Milliwatts)	Freq. (GHz)	External Q
1	Microstrip IMPATT	10	6	126.5
2	Waveguide IMPATT	410	8.555	60.1
3	Waveguide IMPATT	200	9.187	40.6

The locking bandwidth of an injection locked oscillator is defined as the frequency over which the ILO output frequency is the same as the locking signal frequency. The locking bandwidth can be easily measured by varying the frequency of the single frequency locking signal and by measuring the frequency range over which the ILO output tracks the input signal frequency. The locking bandwidth results for the three oscillators of Table 7.1 are shown in Fig. 7.3. The external Q, Q_e for the oscillators are given in Table 7.1. These values of Q_e were calculated from the locking bandwidth results by using Adler's equation [35] given in Chapter 5 as Equation (5.1).

* Only oscillator No.1 was designed for this research work. This oscillator is the same as the one presented in Fig. 6.5. The other two oscillators were selected from those available.

As may be seen from Fig. 7.3, a higher locking bandwidth can be obtained for the oscillators with lower Q_e for the same locking gain (Ratio of output power to the input locking signal power). Or equivalently for the same locking-bandwidth a higher gain may be obtained with a lower Q oscillator. The locking gain for a two-sided locking bandwidth of 10 MHz is shown in Table 7.2 for the oscillators of Table 7.1.

Table 7.2 The Locking Gain for the Oscillators of Table 7.1 for a Two-Sided Locking Bandwidth of 10 MHz

Osc. No.	Input Power (dBm)	Output Power (dBm)	Locking Gain (dB)
1	-9	10	19
2	-4	25	29
3	-9.66	23	32.66

As the bit-error performance degradation is expected to be a function of the locking bandwidth, a higher gain should be feasible for the lower Q_e oscillator for the same degradation in bit-error performance. This was verified by measuring the bit-error performance for different gains with the three oscillators and the results are presented in the following section.

7.3.3 The Bit-Error Performance and Eye Diagram Results

The measured bit-error-rate performance without the ILO in the system is shown in Fig. 7.4. As a reference, the ideal channel performance is also shown. From this figure it may be seen that at a BER of 1×10^{-6} , the bit-error performance is degraded by 1.5 dB with respect to the reference channel. This degradation arises due to bandlimiting and to the combined

effect of other equipment limitations which contribute to the non-ideal nature of the channel. Any further degradation introduced by the ILO-PLA will be measured with respect to the bit-error-rate curve without the ILO.

The measured bit-error results for the oscillator 1 ILO-PLA are shown in Fig. 7.5 for different gain values. As may be seen from this figure very little degradation ($\approx .15$ dB at BER of 1×10^{-6}) was introduced by the ILO-PLA for a gain of 14 dB. The degradation increased to .3 dB at a BER of 1×10^{-6} as the gain was increased to 20 dB. A rapid degradation in performance (2 dB at 1×10^{-6}) can be noticed as the gain was further increased to 26 dB. It may be seen that the bit-error-rate curves did not exhibit the anomalous behaviour of Fig. 1.1 as reported previously [6] for amplification of QPSK with ILO-PLA. Further a two-sided locking bandwidth of only 9 MHz, (corresponding to a gain of 19 dB from Fig. 7.1) results in less than .3 dB degradation in the performance. This bandwidth is only 1.8 times the bit rate of 5 M Bits/sec.

A further insight into the ILO behaviour may be gained by the eye diagrams shown in Fig. 7.6 as the ILO-PLA gain was increased from 16 dB to 31 dB. Increasing the gain from 16 to 21 dB caused only a marginal closure of the eye diagrams. A rapid closure of the eye diagrams can be noticed as the gain was further increased from 21 to 26 dB. It is interesting to note that even at a gain of 31 dB, where the two-sided locking bandwidth was only 2.5 MHz, the eye was still not completely closed indicating that the system may be still usable though with a very high degradation of the bit-error performance.

Both the bit-error-rate and eye diagram results support the argument that because of the phase continuous nature of MSK, an ILO-PLA presents an

Gain = 16 dB

Gain = 21 dB

Gain = 26 dB

Gain = 31 dB

Fig. 7.6 The I and Q Channel Eye Diagrams with Different Values of the ILO-PLA Gains for the Oscillator No. 1 of Table 7.1

attractive method for amplifying MSK modulated microwave signals.

It may be concluded from the results given so far that a two-sided locking bandwidth of twice the bit-rate caused only approximately .3 dB degradation in performance at a BER of 1×10^{-6} . To further verify this conclusion the bit-error-rate was measured with oscillators 2 and 3 of Table 7.1. As may be seen from the last column of Table 7.1, oscillators 2 and 3 had external Q's of 60.1 and 40.64 respectively compared to an external Q of 126.5 for the oscillator 1. The measured bit-error-rates for the ILO-PLAs with oscillators 2 and 3 are shown in Figs. 7.7(a) and 7.8 respectively. A higher gain was obtained for approximately the same degradation as the external Q was lowered.

The I-channel eye diagrams for oscillator 2 are shown in Fig. 7.7(b) for gains in the range of 30-40 dB. A closure of the eye at higher gains occurs as expected. As in Fig. 7.6 for oscillator 1, the closing of the eye was rapid at higher gain values. For example the vertical eye opening decreased from 4.7 to 4.2 as the gain was increased from 30 to 33 dB. However this opening decreased to 2.4 as the gain was further increased to 36 dB. At a gain of 40 dB the eye was almost completely closed. The eye diagrams of the Fig. 7.7(b) provide a qualitative picture which agrees very well with the bit-error-results of Fig. 7.7(a).

A further confirmation of the conclusion that a locking bandwidth of twice the bit-rate (for example 10 MHz locking bandwidth at 5 M Bits/sec) causes only about .3 dB degradation in the bit-error-rate, is provided by Fig. 7.9 where the measured bit-error-rates for the three oscillators are plotted for a locking bandwidth of 10 MHz. From Table 7.2 a two-sided

Gain = 30 dB

Gain = 33 dB

Gain = 36 dB

Gain = 40 dB

Fig. 7.7(b) The I-Channel Eye Diagrams for Oscillator No.2 as the Gain was Varied from 30 to 40 dB

locking bandwidth of 10 MHz corresponded to gains of 19 dB, 29 dB and 32.66 dB respectively. As may be seen from Fig. 7.9, the performance degradation due to the three ILO-PLA's lies between .3 to .5 dB at a bit-error-rate of 1×10^{-6} for these values of gain. The spread in degradation reflects both the measurement inaccuracies and the system performance variations.

The above results show that a higher gain may be obtained from an ILO-PLA by reducing the external Q of the oscillator. For some wideband analog systems, an ILO external Q of 10 is typically used [43]. For the 5 M-Bits/sec MSK signal, a very high gain (≈ 50 dB) would be feasible in a single stage for such an ILO. In practice a lower limit on the external Q of the oscillator will be imposed by the frequency instability of the oscillator with temperature and by the noise of the oscillator itself. Both the frequency instability and the noise increase as the Q_e is lowered. The Q_e will thus be designed to make a compromise between the frequency stability and noise on one hand and the gain on the other hand. The frequency instability of the oscillator would in effect result in the frequency offset between the ILO free-running frequency and the input signal nominal carrier frequency. The effect of this frequency offset on the bit-error-rate was measured for oscillator No. 1 of Table 7.1. The measured bit-error results are shown in Fig. 7.10 for an ILO-PLA gain of 20 dB with the frequency offset as the parameter. As may be seen from this figure, a frequency offset of 1 MHz resulted in only a minimal degradation of the performance ($< .1$ dB). However increasing the frequency offset to 2 MHz caused a rapid degradation in the performance (1 - 1.5 dB). It may be noted that a frequency offset of 2 MHz at a gain of 20 dB for the

oscillator 1 represents 22.2 percent of the two-sided locking bandwidth of 9 MHz at this gain.

The effect of the frequency offset on the eye diagrams was measured in two series of tests. In the first, at a fixed gain of 20 dB, the eye diagrams were measured by varying the frequency offset from 0 to 2 MHz. In the second series of experiments, the gain was varied from 16 dB to 25 dB while maintaining a fixed frequency offset. These diagrams are presented as Figs. 7.11 and 7.12. The fact that the eye closes at higher frequency offsets is clearly demonstrated by these figures. Figure 7.12 shows that the frequency offset can be tolerated more at the lower gain values as compared to high gains. At higher gain values, and as the frequency offset becomes a large percentage of the locking bandwidth, a rapid degradation in the bit-error performance is indicated by the rapid closure of the eye.

7.4 CONCLUSIONS

Based on the experimental bit-error-rate results given in this chapter, it may be concluded that an ILO-PLA for MSK modulated signals does not display the anomalous behaviour previously reported [5] for QPSK signals, namely large and erratic bit-error-rate at high E_b/N_0 ratios. Further it requires a two-sided locking bandwidth of only twice the MSK modulated signal bit-rate to maintain the degradation in bit-error performance due to ILO-PLA to a small value of the order of .3 dB. A high-gain single-stage microwave amplifier for MSK signals may therefore be realized using an ILO-PLA. A positive or negative frequency offset of about ten percent of the two-sided locking bandwidth causes little further degradation in the performance, while a frequency offset of about twenty-five percent results in a rapid degradation of the performance.

$\Delta\omega = 0$ $\Delta\omega = 1$ MHz $\Delta\omega = 2$ MHz

Fig. 7.11 The Effect of Frequency Offset $\Delta\omega$ on the I and Q Channel Eye Diagrams for an ILO-PLA Gain = 20 dB, Oscillator No.1

Gain = 16 dB

Gain = 20 dB

Gain = 25 dB

Fig. 7.12 The Effect of Changing the Gain on the I and Q Channel Eye Diagrams. The Frequency Offset $\Delta\omega = 2$ MHz.

CHAPTER 8

CONCLUSIONS AND SUMMARY

In this thesis we have proposed and studied the application of MSK modulation to the two following specific problems.

- a) A high stability frequency reference for satellite based sensor stations.
- b) Microwave phase-locked-amplification of digitally modulated signals.

The first problem has been studied in Part A of the thesis. For this problem a new approach of using the recovered clock of the MSK modulated signal command signal as the frequency reference has been proposed and analyzed. The proposed approach provides a frequency reference which is independent of the local oscillator frequency instabilities of the satellite and data station receiver local oscillators. A single frequency reference namely the clock at the command station, is used to generate the transmitter carriers at all the remote data stations. The experimental verification of the analysis has been carried out by constructing a 2.4 K bits/sec. MSK Modem. The main thrust of the work in this area has been to analyze the performance of the MSK carrier and clock recovery circuit and to estimate the effect of transmitted carrier phase noise on the bit-error performance of an MSK receiver. The theoretical and experimental results derived in this section should be ultimately directly applicable to the implementation of sensor data collection system.

In addition the reference recovery results derived in this thesis are of a sufficiently general nature that the results may be used in designing the recovery circuits for MSK communication systems in general.

In Part B of this thesis we have proposed that because of its phase continuous nature, an MSK signal should be compatible for amplification with a high gain microwave injection-locked oscillator phase-locked-amplifier. A verification of this concept has been provided experimentally using an ILO-PLA with a 5 M Bits/sec. MSK signal. The two performance criteria used in this verification were the spectrum-spreading of the filtered signal by an ILO-PLA and the degradation in bit-error performance due to an ILO-PLA.

The spectrum-spreading results are in agreement with the theoretically predicted result [13] that the sidelobe spectrum at the output of a hard-limiter (an ILO-PLA in our case) is not restored to its unfiltered level by hardlimiting of the filtered signal as is the case for QPSK. In addition, experiment conducted using narrow band filtered signals has revealed that the MSK spectrum mainlobe is not restored to its unfiltered value as a result of the hardlimiting by the ILO-PLA. This result does not support the previously held view that the mainlobe would be restored to the unfiltered value by the hardlimiter [41].

A mathematical model of the ILO-PLA which relates the ILO-PLA output to the input MSK signal has been derived for use in computer simulation of MSK systems which use an ILO-PLA as the microwave amplifier. A computer program for calculating the ILO-PLA output power spectrum has been developed. Good agreement was obtained for spectral components within the filter passband. The agreement deteriorated for the low-level sidelobe region. This disagreement may be explained to be either due to the simplified ILO-PLA model used in the computation or due to the termination of the filter impulse response in the computation program.

The bit-error-rate performance of the ILO-PLA has been experimentally evaluated for the MSK modulated input signal. The measured bit-error-rate results were found to follow the general form of an ideal channel theoretical PSK bit-error-rate performance over a measurement range which extended to the low bit-error-rates ($< 10^{-5}$). This result demonstrates that the ILO-PLA is more compatible for MSK modulated signals than for QPSK signals since it has been shown [6] that for the latter case, the bit-error-rate performance is highly degraded and erratic for low bit-error-rates corresponding to high S/N ratios.

From the experimental results we can conclude that an ILO having a two-sided locking bandwidth of only twice the bit-rate will exhibit only a small degradation ($\approx .25$ dB) in the bit-error performance relative to a wide locking bandwidth one. Due to this narrow locking bandwidth requirement, a high-gain, single-stage ILO-PLA may be realized for amplifying MSK modulated signals. In particular for comparatively narrowband MSK systems (≈ 5 M Bits/sec) obtaining all the transmitter microwave gain in a single stage offers an attractive alternative to the multistage stable reflection or three terminal amplifiers.

FUTURE WORK

Future work for the first problem area of MSK-based low-data-rate satellite data stations could be a more accurate determination of the effect of the carrier phase noise on the bit-error-rate performance of a low-bit-rate MSK system. The theoretical analysis for the calculation of the data station transmitter carrier phase noise effect on the CCS

bit-error-rate (see Section 4.4) has indicated that this effect is significant. When conducting this analysis a receiver carrier recovery circuit employing a fourth order nonlinearity was assumed. Further, in order to determine the effect of the additive receiver thermal noise in such a circuit, the results for OK-QPSK were assumed to apply to MSK. A more complete analysis of the carrier phase noise effect in an MSK system would involve determining this effect using a reference recovery circuit employing a second order nonlinearity similar to that of Fig. 4.4.

In the second problem of phase-locked amplification of MSK modulated signals future work could concern the theoretical optimization of the ILO-PLA parameters for MSK signals. This would involve the development of a theoretical model for ILO-PLA which adequately describes the effect of the frequency offset, the locking bandwidth (which is related to the gain and oscillator external Q) and the device impedance variations. This model can then be used in computer optimization of the bit-error performance.

Experimental and theoretical evaluation of the voltage-tuned-oscillator phase-locked-amplifier for the MSK signals and an optimization of the VTO-PLA parameters would be another area in which future effort may yield practically attractive results[44]

In the area of general MSK communication systems the direct MSK modulation of a high power microwave oscillator with the baseband data pulses could be attempted.

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APPENDIX A

COMPUTATION OF THE MSK POWER SPECTRUMAT THE ILO-PLA OUTPUT

A computer program for computing the MSK power spectrum at the ILO-PLA output is developed below. This program is based on a combined analytical and simulation technique which is essentially a modification of that used for calculating the TWT output power spectrum for PSK signals [40].

The MSK signal may be written as:

$$e_1(t) = R_e \left\{ e^{j\omega_c t} \sum_{k=-\infty}^{\infty} a_k \cdot \text{rect}((t - KT)/T) \cdot \cos \frac{\pi(t-KT)}{T} - j b_k \cdot \text{rect}((t-KT-T/2)/T) \cdot \cos \frac{\pi(t-KT-T/2)}{T} \right\} \quad (A.1)$$

where

$R_e(\alpha)$ = Real part of the complex number α

a_k = I channel kth data bit = ± 1

b_k = Q channel kth data bit = ± 1

a_k and b_k are independent random variables

T = symbol interval = 2. Bit interval

$$\text{rect}(x) = \begin{cases} 1 & |x| \leq \frac{1}{2} \\ 0 & \text{elsewhere} \end{cases}$$

The MSK signal of Equation (A.1) is filtered at IF. To obtain the signal at the filter output we use the equivalent low-pass-filter unit impulse response [45].

The output of the BPF with the filter equivalent low-pass impulse $h(t)$ is given by:

$$e_1(t) = R_e \left\{ e^{j\omega_c t} u(t) * h(t) \right\}$$

where

$u(t)$ envelope of the input signal

* denotes convolution

Therefore for the input signal of Equation (A.1), the filter output signal is given by:

$$e_1(t) = R_e \left\{ e^{j\omega_c t} \left[\sum_{k=-\infty}^{\infty} a_k \cdot \text{rect}((t-KT)/T) \cdot \cos \frac{\pi(t-KT)}{T} - j b_k \cdot \text{rect}((t-KT-T/2)/T) \cdot \cos \frac{\pi(t-KT-T/2)}{T} \right] * h(t) \right\} \quad (A.2)$$

If we write

$$R(t) = R_e \left\{ \text{rect} \left(\frac{t}{T} \right) \cdot \cos \left(\frac{\pi t}{T} \right) * h(t) \right\} \quad (A.3)$$

$$I(t) = I_m \left\{ \text{rect} \left(\frac{t}{T} \right) \cdot \cos \left(\frac{\pi t}{T} \right) * h(t) \right\} \quad (A.4)$$

where $I_m(\alpha)$ = Imaginary part of the complex number α
We have from Equation (A.2)

$$e_1(t) = R_e \left\{ e^{j\omega_c t} \left[\sum_{k=-\infty}^{\infty} a_k \cdot [R(t-KT) + j I(t-KT)] - j b_k [R(t-KT-T/2) + j I(t-KT-T/2)] \right] \right\} \quad (A.5)$$

or

$$e_1(t) = x(t) \cos \omega_c t - y(t) \sin \omega_c t$$

where

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \cdot R(t-KT) + b_k \cdot I(t-KT-T/2) \quad (A.6)$$

$$y(t) = \sum_{k=-\infty}^{\infty} a_k \cdot I(t-KT) - b_k \cdot R(t-KT-T/2) \quad (A.7)$$

We may write $e_1(t)$ as:

$$e_1(t) = \sqrt{x^2(t) + y^2(t)} \cos(\omega_c t + \tan^{-1}(\frac{y(t)}{x(t)})) \quad (A.8)$$

For the ILO-PLA input of Equation (A.8) we define

$$x_1 = x(t), \quad y_1 = y(t) \quad (A.9)$$

and

$$x_2 = x(t + \tau), \quad y_2 = y(t + \tau) \quad (A.10)$$

For the input signal $e_1(t)$ of Equation (A.8) the ILO-PLA output is obtained from the Equations (6.11) and (6.13) of Chapter 6.

The output of ILO-PLA may therefore be written as:

$$e_o(t) = R_e \left\{ g(\sqrt{x_1^2 + y_1^2}) \cdot e^{j(\omega_c t + \tan^{-1}(\frac{y_1}{x_1}) + f(x_1, y_1))} \right\} \quad (A.11)$$

where

$$g(A) = E'_o \left[1 + \frac{1}{2} \frac{A}{E'_o} \left(\frac{2}{s} - 1 \right) \sqrt{1 - \left(\frac{\Delta\omega}{\omega_m} \right)^2} \right]$$

$$A = \sqrt{x_1^2 + y_1^2}$$

E'_O = ILO-PLA Free-running oscillator output voltage

s = Oscillator negative resistance device non-linearity parameter

$\Delta\omega$ = Frequency offset

$$f(x_1, y_1) = \phi_O + k_1 \phi'_1(t) + k_2 \phi'^2_1(t) + k_3 \phi'^3_1(t)$$

$$\left(\frac{\pi u_k}{2T \omega'_m} - \frac{\pi u_{k-1}}{2T \omega'_m} \right) e^{-\omega'_m t} - \frac{\phi''_1(t)}{\omega'^2_m}$$

"'" denotes the derivative w.r.t time

$$\phi_O = \frac{1}{\omega'_m} \left(\Delta\omega + \frac{\omega''_m \Delta\omega_O^3}{6 \omega'^3_m} \right)$$

$$\omega'_m = \left(1 - \frac{A}{E'_O} \right) \frac{A}{E'_O} \frac{\omega_O}{2 Q_e}$$

$$\omega''_m = \left(1 - \frac{4A}{E'_O} \right) \frac{A}{E'_O} \frac{\omega_O}{2 Q_e}$$

u_k = kth data bit

$$k_1 = \frac{1}{\omega'_m} + \frac{1}{2} \frac{\omega''_m \Delta\omega_O}{\omega'^4_m}$$

$$k_2 = \frac{1}{2} \frac{\omega''_m \Delta\omega_O}{\omega'^4_m}$$

$$k_3 = \frac{1}{6} \frac{\omega''_m}{\omega'^4_m}$$

Similarly

$$e_o(t + \tau) = R_e \left\{ g(\sqrt{x_2^2 + y_2^2}) \cdot e^{j(\omega_c t + \tan^{-1}(\frac{y_2}{x_2}) + f(x_2, y_2))} \right\}$$

The function $g(\sqrt{x_1^2 + y_1^2})$ represents the AM/AM nonlinearity of the ILO-PLA whereas the effect of AM/PM and phase nonlinearity is included in $f(x_1, y_1)$. In order to obtain the ILO-PLA output power spectrum (i.e., the power spectrum of $e_o(t)$) we first take the autocorrelation of $e_o(t)$ by forming the product $e_o(t) \cdot e_o(t + \tau)$. However, since a_k, b_k are random variables we have to take the ensemble average of $e_o(t) \cdot e_o(t + \tau)$ over a_k, b_k .

To get the product $e_o(t) \cdot e_o(t + \tau)$ we use the identity for complex numbers α and β

$$R_e \alpha \cdot R_e \beta = \frac{1}{2} R_e (\alpha\beta + \alpha\bar{\beta})$$

where $\bar{\beta}$ denotes the complex conjugate of β ; using this identity and neglecting terms at $2\omega_c$

$$e_o(t) \cdot e_o(t + \tau) = \frac{1}{2} R_e \left\{ \frac{g(\sqrt{x_1^2 + y_1^2}) \cdot e^{j f(x_1, y_1)} \cdot (x_1 + jy_1)}{\sqrt{x_1^2 + y_1^2}} \times \frac{g(\sqrt{x_2^2 + y_2^2}) \cdot e^{-j f(x_2, y_2)} \cdot (x_2 - jy_2) \cdot e^{-j\omega_c \tau}}{\sqrt{x_2^2 + y_2^2}} \right\}$$

or/

$$e_o(t) \cdot e_o(t + \tau) = \frac{1}{2} R_e \left\{ H(x_1, y_1) \cdot \bar{H}(x_2, y_2) \cdot e^{-j\omega_c \tau} \right\} \quad (A.12)$$

where

$$H(x(t), y(t)) = \frac{g(\sqrt{x^2(t) + y^2(t)}) \cdot e^{j f(x(t), y(t))} \cdot (x(t) + jy(t))}{\sqrt{x^2(t) + y^2(t)}}$$

(A.13)

" - " denotes complex conjugate

For MSK, the random variable pairs (a_k, b_k) are independently and identically distributed. Since the I and Q channels are offset, the correlation extends over two symbol intervals. The ensemble average over (a_k, b_k) is therefore a periodic function with respect to time with a period equal to $2T$. Taking the Fourier transform with respect to τ , the output power spectral density $w_o(f)$ is given by

$$w_o(f) = \int_{-\infty}^{\infty} e^{j\omega\tau} \frac{1}{2T} \int_{-T}^T \frac{1}{2} R_e \left\{ H(x_1, y_1) \bar{H}(x_2, y_2) e^{-j\omega_c \tau} \right\} dF(x_1, y_1, x_2, y_2) d\tau \quad (A.14)$$

where $F(x_1, y_1, x_2, y_2)$ represents the probability distribution function of x_1, y_1, x_2, y_2 .

From Reference [46], Equation (3.12), the Fourier transform of a bandpass signal $w(t) = R_e \{ u(t) e^{j\omega_c t} \}$ can be written in terms of the equivalent low pass envelope transform $u(f) = \int_{-\infty}^{\infty} u(t) e^{j2\pi ft} dt$ as

$$w(f) = \frac{1}{2} u(f + f_c) + \frac{1}{2} \bar{u}(f_c - f)$$

Using this result the ILQ output spectrum is given by

$$w_o(\omega) = \frac{1}{4} [P(w_c + \omega) + \bar{P}(w_c - \omega)] \quad (A.15)$$

where

$$P(\omega) = \frac{1}{2T} \int_{-\infty}^{\infty} e^{-j\omega\tau} d\tau \int_{-T}^T \int R_e \left\{ H(x_1, y_1) \cdot \bar{H}(x_2, y_2) \right\} dF(x_1, y_1, x_2, y_2) dt$$

or

$$P(\omega) = \frac{1}{2T} \int_{-\infty}^{\infty} e^{-j\omega\tau} d\tau \int_{-\infty}^{\infty} V_0(t) \left\{ \int H(x_1, y_1) \bar{H}(x_2, y_2) dF(x_1, y_1, x_2, y_2) \right\} dt \quad (A.16)$$

where

$V_0(t)$ is a time window

$$V_0(t) = \begin{cases} 1 & |t| < T \\ 0 & |t| \geq T \end{cases}$$

To further simplify we use the transformation of variables t and $t + \tau$ as $t = \eta$, and $t + \tau = \zeta$, which yields

$$P(\omega) = \frac{1}{2T} \int_{-\infty}^{\infty} e^{-j\omega\zeta} d\zeta \int_{-\infty}^{\infty} V_0(\eta) e^{j\eta\omega} d\eta \left\{ \int H(x_1, y_1) \cdot \bar{H}(x_2, y_2) dF[x_1(\eta), y_1(\eta), x_2(\zeta), y_2(\zeta)] \right\} \quad (A.17)$$

which may be written as

$$P(\omega) = E_s \left[\frac{1}{2T} \left\{ \int_{-\infty}^{\infty} e^{-j\omega\zeta} \bar{H}(x(\zeta), y(\zeta)) d\zeta \right\} \times \left\{ \int_{-\infty}^{\infty} V_0(\eta) e^{j\eta\omega} H(x(\eta), y(\eta)) d\eta \right\} \right] \quad (A.18)$$

where E_s denotes the expectation taken on the set s of random variables (a_k, b_k) . To make the computation possible the filter impulse

response has to be terminated over a finite time interval. Therefore it is assumed that the real and imaginary impulse responses $R(t)$ and $I(t)$ are limited within a finite interval and are zero outside this interval. If this interval is assumed to be $2L$ symbols long, $x(t)$ and $y(t)$ may be represented as

$$x(t) = \sum_{k=-L}^L a_k R(t - KT) + b_k I(t - KT - T/2) \quad (A.19)$$

$$y(t) = \sum_{k=-L}^L a_k I(t - KT) - b_k R(t - KT - T/2) \quad (A.20)$$

where L is a finite integer.

For $|\zeta| > (2L + \frac{1}{2})T$, $V_0(n) \cdot H[x(n), y(n)]$ and $\bar{H}[x(\zeta), y(\zeta)]$ become disjointed with respect to random variables (a_k, b_k) since these two functions have no common values (a_k, b_k) .

Therefore for $|\zeta| > (2L + \frac{1}{2})T$, we may write from Equation (A.18)

$$P(\omega) = \frac{1}{T} \int_{-\infty}^{\infty} e^{-j\omega\zeta} E_S \{ \bar{H}[x(\zeta), y(\zeta)] \} d\zeta \int_{-\infty}^{\infty} V_0(n) e^{j\omega n} \cdot E_S \{ H[x(n), y(n)] \} dn \quad (A.21)$$

Since $H[x(n), y(n)]$ is equally likely to take the positive or negative values, its expectation is equal to zero and therefore

$$P(\omega) = 0 \quad \text{for } |\zeta| > (2L + \frac{1}{2})T$$

Thus by introducing a new function $V_{2L}(\zeta)$ such that

$$V_{2L}(\zeta) = 1 \quad |\zeta| < (2L + \frac{1}{2}) T$$

$$= 0 \quad \text{elsewhere}$$

we may write $P(\omega)$ as

$$P(\omega) = E_s \cdot \left[\frac{1}{T} \left\{ \int_{-\infty}^{\infty} V_{2L}(\zeta) e^{-j\omega\zeta} \overline{H} [x(\zeta), y(\zeta)] d\zeta \right. \right. \\ \left. \left. \cdot \left\{ \int_{-\infty}^{\infty} V_0(\eta) e^{j\omega\eta} H x(\eta), y(\eta) d\eta \right\} \right] \right] \quad (A.22)$$

s' = set of random variables (a_k, b_k) , $k = 0, +1, +2 \dots +22$

E'_s = expectation operator representing ensemble averaging over these $(4L + 1)$ random variables.

The Fourier transform of $H[x(t), y(t)]$ is taken in Equation (A.22) over two time windows $V_0(\eta)$ and $V_{2L}(\zeta)$. $V_0(\eta)$ has a length $2T$ for the MSK signal (T = symbol duration) and $V_0(\eta)$ has a length $(4L + 1)T$ where integer L indicates the number of symbol intervals over which the filter impulse response is assumed to extend.

Based on the above analysis the ILO-PLA output spectrum is computed using the following computation steps:

- (1) From the IF filter amplitude and phase response data, the filter impulse response is computed. Note that the filter impulse response will in general have real and imaginary parts.
- (2) Convolve the filter impulse response with the sinusoidal window to obtain the functions $R(t)$ and $I(t)$ of Equations (A.3) and (A.4).

- (3) Choose a sequence (a_ℓ, b_ℓ) from the set of random variables (a_k, b_k) $k = 0, +1, +2 \dots +2L$ where $+L$ is the number of symbol intervals over which the filter impulse response extends. Using $R(t)$ and $I(t)$ computed in step 2 above compute $H(x(t), y(t))$ of Equation (A.13).
- (4) Compute the Fourier transform of $V_0(t) H[x(t), y(t)]$ and $V_{2L}(t) \bar{H}(x(t), y(t))$ in Equation (A.22) and multiply them together.
- (5) Choose another sequence (a_ℓ, b_ℓ) from the set of random variables (a_k, b_k) , $k = 0, +1, +2 \dots +2L$, and perform the same operation performed in step 3 above. Repeat the procedure for another sequence of (a_ℓ, b_ℓ) (The results of reference [40] show that 50 iterations are sufficient to obtain the average).
- (6) Take the average of these products to obtain $P(\omega)$.
- (7) Compute the power spectrum $W_0(\omega)$ at the ILO-PLA output by using Equation (A.15).

It may be noted that the combined analytical and numerical technique given above results in much less computation time [40] compared to a completely numerical simulation technique.

A computer program used to implement the method described above is given below. The results of the computation are discussed in Chapter 6.

```

1.000 C   THE PROGRAMME FILTER
2.000 C   THIS PROG CALCULATES THE FILTER IMPULSE RESPONSE
3.000 C   FROM THE MEASURED FILTER AMPLITUDE AND PHASE
4.000 C   RESPONSE. THE INPUT DATA CONSISTS OF 81 POINTS
5.000 C   OF MEASURED FILTER AMPLITUDE
6.000 C   RESPONSE (ATTENUATION IN DBS) AND 81
7.000 C   POINTS OF MEASURED FILTER PHASE RESPONSE IN
8.000 C   DEGREES, AT FREQ. INTERVALS OF .2 MHZ.
9.000 C   AROUND THE CENTRE FREQ.
10.000 C  THE REAL PART OF THE IMPULSE RESPONSE IS
11.000 C  IN ARRAY HR, AND THE IMAGINARY PART IN ARRAY HI
12.000 C  THE SIMULATION USES 18 SAMPLE POINTS
13.000 C  PER SYMBOL. FOR 5M BITS/SEC. SYSTEM THE
14.000 C  SYMBOL FREQ.=2.5 M HZ., THUS THE SAMPLE FREQ.
15.000 C  IS EQUAL TO 45 MHZ.
16.000 C  THE INPUT DATA FORMAT :9 G.0
17.000 C  THE OUTPUT DATA FORMAT:10 F8.2
18.000    DIMENSION A(161),P(161),HR(126),HI(126)
19.000    FI=22./7.
20.000      P1=176*PI/180
21.000    READ 30,(A(I),I=41,121)
22.000    READ 30,(P(I),I=41,121)
23.000    DO 40 I=1,40
24.000      A(I)=31.+41-I
25.000      A(121+I)=38+I
26.000      P(I)=216+4*(41-I)
27.000      P(121+I)=-641-4*I
28.000 40  CONTINUE
29.000      DO 10 J=1,126
30.000        C=0
31.000        D=0
32.000        DO 15 K=1,161
33.000          A1=10.**(-A(K)/10.)
34.000          B=P(K)*PI/180.
35.000          B1=.8*(J-55)*PI*((K-1)*.2-16.)/18.+B+P1
36.000          C=C+A1*COS(B1)
37.000          D=D+A1*SIN(B1)
38.000 15  CONTINUE
39.000        HR(J)=C
40.000        HI(J)=D
41.000 10  CONTINUE
42.000      WRITE 20,(HR(J),J=1,126)
43.000      WRITE 20,(HI(J),J=1,126)
44.000 20  FORMAT(10 F8.2)
45.000 30  FORMAT(9 G.0)
46.000    STOP
47.000    END

```

```

1.000 C THE PROGRAMME CONVOLVE
2.000 C THIS PROGRAMME CONVOLVES THE COMPUTED
3.000 C FILTER IMPULSE RESPONSE WITH THE SINUSOIDAL
4.000 C WINDOW. THE INPUT IS READ FROM THE OUT PUT
5.000 C FILE OF PROGRAMME FILTER.
6.000 C THE REAL AND IMAGINARY PARTS OF THE RESULT
7.000 C OF CONVOLUTION ARE CONTAINED IN ARRAYS R
8.000 C AND S. THE INPUT AND OUTPUT FORMAT: 10F8.2
9.000 DIMENSION HR(136), HI(136), Q(128), S(128), R(128)
10.000 PI=22./7.
11.000 DO 10 I=1,9
12.000 HR(127+I)=0
13.000 HI(127+I)=0
14.000 10 CONTINUE
15.000 READ 130, (HR(J), J=2, 127)
16.000 READ 130, (HI(J), J=2, 127)
17.000 DO 15 K=1, 128
18.000 Q(K)=0
19.000 S(K)=0
20.000 R(K)=0
21.000 15 CONTINUE
22.000 C GENRATE SINUSOIDAL WINDOW
23.000 DO 20 I=1, 19
24.000 Q(54+I)=SIN(PI*(I-1)/18)
25.000 20 CONTINUE
26.000 C CONVOLVE WINDOW WITH IMPULSE RESPONSE
27.000 A1=0
28.000 A2=0
29.000 DO 30 J=1, 64
30.000 DO 25 K=1, 19
31.000 A1=A1+HR(54+J-1+K)*Q(54+K)
32.000 A2=A2+HI(54+J-1+K)*Q(54+K)
33.000 25 CONTINUE
34.000 R(64+J-1)=A1
35.000 S(64+J-1)=A2
36.000 A1=0
37.000 A2=0
38.000 30 CONTINUE
39.000 B1=0
40.000 B2=0
41.000 DO 40 J=1, 54
42.000 DO 35 K=1, 19
43.000 B1=B1+HR(54-J+K)*Q(54+K)
44.000 B2=B2+HI(54-J+K)*Q(54+K)
45.000 35 CONTINUE
46.000 R(64-J)=B1
47.000 S(64-J)=B2
48.000 B1=0
49.000 B2=0
50.000 40 CONTINUE

```

```
51.000 DO 45 J=1,9
52.000 K1=19-J
53.000 DO 50 K=1,K1
54.000 C1=0
55.000 C2=0
56.000 C1=C1+HR(K)*Q(54+K+J)
57.000 C2=C2+HI(K)*Q(54+K+J)
58.000 50 CONTINUE
59.000 R(10-J)=C1
60.000 S(10-J)=C2
61.000 C1=0
62.000 C2=0
63.000 45 CONTINUE
64.000 WRITE 130,(R(J),J=1,128)
65.000 WRITE 130,(S(J),J=1,128)
66.000 130 FORMAT( 10 F8.2)
67.000 STOP
68.000 END
```

```

1.000 C      THE PROGRAMME ILO
2.000 C      THIS PROGRAMME CALCULATES THE ILO-PLA
3.000 C      OUTPUT POWER SPECTRAL DENSITY FOR THE
4.000 C      INPUT FILTERED MSK SIGNAL.
5.000 C      THE PROG. ILO USES THE OUTPUT OF PROGRAMME
6.000 C      CONVOLVE AS THE INPUT.
7.000 C      THE PROG. USES TWO SCIENTIFIC PROG.LIBRARY
8.000 C      (S.F.L) SUBROUTINES RANDOM AND FOURT
9.000 C      FOR GENERATING THE RANDOM DATA AND FOR
10.000 C     TAKING THE FAST FOURIER TRANSFORMS.
11.000 C     DW= ILO FREQ. OFFSET VALUE
12.000 C     Q1=ILO EXTERNAL Q
13.000 C     G11=ILO -PLA-GAIN IN DBS
14.000 C     S11=ILO DEVICE NEG. RESISTANCE NONLINEARITY
15.000 C     PARAMETER
16.000 C     THE RESULT OFTHE COMPUTATION IS GIVEN AS
17.000 C     THE POWER SPECTRAL DENSITY IN DBS.THE FIRST
18.000 C     1289 POINTS GIVE THE OUTPUT POWER SPECTRAL
19.000 C     DENSITY FOR POSITIVE FREQUENCIES STARTING
20.000 C     FROM FREQ. ZERO AT A FREQ. INTERVAL OF1/45 MHZ.
21.000 C     (=1/SAMPLING FREQ.).THE POWER SPECTRAL DENSITY FOR
22.000 C     THE NEGATIVE FREQUENCIES IS SIMILARLY GIVEN
23.000 C     BY THE LAST 127 POINTS OF THE OUTPUT STARTING
24.000 C     FROM THE POINT 256 OF THE OUTPUT.
25.000 C     THE FILTER IMPULSE RESPONSE IS TERMINATED OVER
26.000 C     FIVE SYMBOL INTERVALS.
27.000 C     THE PROG. IS BASED ON THE THEORY GIVEN ABOVE
28.000 C     COMPLEX P(256),P1(256),P2(256)
29.000 C     DIMENSION A(128),B(128),Q(256),S(512),T(512),S1(512),T1(5
12)
30.000 C     DIMENSION PH(256),PH1(256),D(715),D1(715)
31.000 C     READ(105,130)(A(I),I=1,128)
32.000 C     READ(105,130)(B(I),I=1,128)
33.000 C     N=147
34.000 C     PI=22./7.
35.000 C     GENRATE RANDOM ARRAYS OF +1,-1:D,D1
36.000 C     DO 5 J=1,715
37.000 C     D(J)=RANDOM(N)
38.000 C     D1(J)=RANDOM(N)
39.000 C     IF(D(J).LE.0.5)GO TO 10
40.000 C     D(J)=1
41.000 C     GO TO 120
42.000 10   D(J)=-1
43.000 120  IF(D1(J).LE.0.5)GO TO 15
44.000 C     D1(J)=1
45.000 C     GO TO 5
46.000 15   D1(J)=-1
47.000 5     CONTINUE
48.000 C     I1=0
49.000 C     DO 220 I=1,256
50.000 C     P2(I)=(0.,0.)
51.000 220  CONTINUE

```

```

52.000 200 DO,205 I=1,256
53.000 P1(I)=(0.,0.)
54.000 P(I)=(0.,0.)
55.000 PH1(I)=0
56.000 PH(I)=0
57.000 Q(I)=0
58.000 205 CONTINUE
59.000 DO 215 I=1,512
60.000 S(I)=0
61.000 T(I)=0
62.000 S1(I)=0
63.000 T1(I)=0
64.000 215 CONTINUE
65.000 DO 20 J=1,14
66.000 K=84+18*(J-1)
67.000 DO 25 I=1,128
68.000 S(K+I)=S(K+I)+D(J+11)*A(I)
69.000 T(K+I)=T(K+I)+D(J+11)*B(I)
70.000 S1(K+I)=S1(K+I)-D1(J+11)*B(I)
71.000 T1(K+I)=T1(K+I)+D1(J+11)*A(I)
72.000 25 CONTINUE
73.000 20 CONTINUE
74.000 DO 35 I=1,256
75.000 S(128+I)=S(128+I)+S1(137+I)
76.000 T(128+I)=T(128+I)+T1(137+I)
77.000 35 CONTINUE
78.000 DO 40 I=1,256
79.000 PC1=S(128+I)
80.000 PC2=T(128+I)
81.000 P(I)=CMPLX(PC1,PC2)
82.000 40 CONTINUE
83.000 DO 50 I=1,256
84.000 Q(I)=CABS(P(I))
85.000 50 CONTINUE
86.000 DO 110 I=1,256
87.000 PH(I)=ATAN2(T(128+I),S(128+I))
88.000 110 CONTINUE
89.000 DO 140 I=1,256
90.000 PT=PH(I+1)-PH(I)
91.000 PT1=ABS(PT)
92.000 IF(PT1.LE.PI)GO TO 140
93.000 IF(PT.LE.0)GO TO 145
94.000 PH(I+1)=PH(I+1)-2*PI
95.000 GO TO 140
96.000 145 PH(I+1)=PH(I+1)+2*PI
97.000 140 CONTINUE
98.000 X1=7.5E6
99.000 J=1
100.000 55 PH1(J)=X1*(-11.*PH(J)+18.*PH(J+1)-9.*PH(J+2)+2.*PH(J+3))
101.000 PH1(J+1)=X1*(-2.*PH(J)-3*PH(J+1)+6*PH(J+2)-PH(J+3))
102.000 PH1(J+2)=X1*(PH(J)-6*PH(J+1)+3*PH(J+2)+2*PH(J+3))
103.000 PH1(J+3)=X1*(-2*PH(J)+9*PH(J+1)-18*PH(J+2)+11*PH(J+3))
104.000 J=J+4
105.000 IF(J.GE.256)GO TO 60
106.000 GO TO 55
107.000 C ILO Q=Q1
108.000 60 Q1=130.

```

109.000 C ILO FREQ. OFFSET =DW
 110.000 DW=1E3
 111.000 230 X=Q(1)
 112.000 DO 100 I=2,256
 113.000 IF(Q(I).LE.X)GO TO 100
 114.000 X=Q(I)
 115.000 100 CONTINUE
 116.000 DO 235 I=1,256
 117.000 Q(I)=Q(I)/X
 118.000 P(I)=P(I)/X
 119.000 P1(I)=CONJG(P(I))
 120.000 235 CONTINUE
 121.000 Y1=0
 122.000 DO 225 I=1,256
 123.000 Y1=Y1+Q(I)**2
 124.000 225 CONTINUE
 125.000 Y1=Y1/256.
 126.000 C ILO GAIN=G11 DB
 127.000 G11=30
 128.000 G1=G11+10.*ALOG10(Y1)
 129.000 G=10.**((G1-6.)/20.)
 130.000 DO 65 I=1,256
 131.000 WM1=(1.-Q(I)/G)*Q(I)*6.*10**9/(G*2.*Q1)
 132.000 WM2=(1.-4.*Q(I)/G)*Q(I)*6.*10**9/(G*2.*Q1)
 133.000 PH0=(DW+WM2*(DW**3)/(6.*WM1**3))/WM1
 134.000 TK1=1./WM1+WM2*DW**2/(2.*WM1**4)
 135.000 TK1=TK1/(2.*PI)
 136.000 TK2=WM2*DW/(2.*WM1**4)+(1.+WM2*DW**2/(2.*WM1**3))/WM1**2
 137.000 TK2=TK2/(4.*PI*PI)
 138.000 TK3=WM2/(6.*WM1**4)
 139.000 TK3=TK3/(8.*PI*PI*PI)
 140.000 S11=.81
 141.000 AN= PH0+TK1*PH1(I)+TK2*PH1(I)**2+TK3*PH1(I)**3
 142.000 AM=(1.+Q(I)*(2./S11-1)/(2.*G))*COS(PH0)
 143.000 R=AM*(COS(AN)*S(128+I)-SIN(AN)*T(128+I))/(Q(I)*X)
 144.000 R1=AM*(COS(AN)*T(128+I)+SIN(AN)*S(128+I))/(Q(I)*X)
 145.000 P(I)=CMPLX(R,R1)
 146.000 P1(I)=CONJG(P(I))
 147.000 65 CONTINUE
 148.000 240 DO 70 K=1,110
 149.000 P(K)=(0.,0.)
 150.000 P(K+146)=(0.,0.)
 151.000 70 CONTINUE
 152.000 DO 300 I=1,11
 153.000 P1(I)=(0.,0.)
 154.000 P1(245+I)=(0.,0.)
 155.000 300 CONTINUE
 156.000 NM=256
 157.000 M=1
 158.000 CALL FOURT(P,NM,M,+1,1)
 159.000 CALL FOURT(P1,NM,M,-1,1)
 160.000 DO 75 I=1,256
 161.000 P2(I)=P(I)*P1(I)/1E8+P2(I)
 162.000 75 CONTINUE

```
163.000      I1=I1+14
164.000      IF(I1.LE. 695)GO TO 200
165.000      DO 80 I=1,256
166.000      Q(I)=CABS(P2(I))
167.000      Q(I)=10.*ALOG10(Q(I))
168.000 80    CONTINUE
169.000      A0=Q(1)
170.000      DO 250 I=1,256
171.000      Q(I)=Q(I)-A0
172.000 250   CONTINUE
173.000      OUTPUT G11,DW,Q1
174.000      WRITE(108,130)(Q(I),I=1,256)
175.000 130   FORMAT( 10 F8.2)
176.000 115   STOP
177.000      END
```


END

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FIN