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**ALIGNMENT DETERMINATION AND ERROR
CHARACTERISATION OF INERTIAL COMPONENTS
FOR USE IN AN AIRBORNE GRAVIMETER**

By

DEREK A. HARTL, B. Eng.

**A Thesis submitted to the
School of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of
Master of Engineering**

**Ottawa Carleton Institute for Mechanical and Aerospace Engineering
Department of Mechanical and Aerospace Engineering
Carleton University, Canada**

April 28, 1995

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Characterisation Of Inertial Components
For Use In An Airborne Gravimeter"**

submitted by Derek A. Hartl in partial fulfilment of the requirements for the degree of
Master of Engineering.

Prof. John S. Sinkiewicz
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Prof. Robert Bell
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April 1995

Department of Mechanical and Aerospace Engineering

ABSTRACT

An airborne gravimeter utilising a gyro stabilised platform is being developed. The accuracy requirements of the system require inertial grade accelerometers and gyroscopes to be used to allow changes of less than a micro g to be detected. Three Q-Flex quartz pendulous type accelerometers are maintained in a local level orientation using two CSG-2 two degree of freedom tuned rotor gyros.

Phase I of the testing involved demonstrating the suitability of the inertial instruments for use in the gravimeter. The error terms were defined and determined by using a precision inertial test table to apply rates or accurately position the instruments. Resolution and temperature dependence were also determined.

Phase II involved mounting the instruments in a inertial sensor block. Model equations were developed to define the gyro and accelerometer outputs. Static testing and autocollimation determined the misalignment angles and the major error terms of the instruments. Lab techniques used in inertial testing are discussed, as are the steps taken to reduce errors.

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List Of Symbols

α	Tuned Rotor Gyro misalignment angle w.r.t inertial space
α_m	Misalignment of accelerometer n, w.r.t X system axis; $m=x,y,z$
α_n	Misalignment of gyro input axis n, w.r.t X system axis; $n=1,2,3,4$
β	Torquer misalignment angle
β_m	Misalignment of accelerometer n, w.r.t Y system axis; $m=x,y,z$
β_n	Misalignment of gyro input axis n, w.r.t Y system axis; $n=1,2,3,4$
γ_m	Misalignment of accelerometer n, w.r.t Z system axis; $m=x,y,z$
γ_n	Misalignment of gyro input axis n, w.r.t Z system axis; $n=1,2,3,4$
δ_l	Misalignment of gravity vector, w.r.t geographic frame; $l=x,y,z$
δ_m	Misalignment of gravity vector, w.r.t system axis; $m=x,y,z$
δ_n	Misalignment angle of accelerometer analysis frame, $n=OA, PA$
Δ	Lumped misalignment term
λ	Latitude
φ	Gimbal angle relative to rotor. Azimuth rotation angle
θ	Angle of angular momentum vector, angle of anisoelastic torque, CMT rotation angle
ψ	Trunion rotation angle
τ	Time Constant (also TC)
ω	Precession Rate
a	Acceleration
a_m	Acceleration component along axis m; $m=OA, PA, IA$
a_n	Acceleration component along axis n; $n=X,Y,Z$
B	Magnetic Flux
B	Bias
B_m	Accelerometer bias of axis m; $m=X,Y,Z$
B_n	Absolute bias of TRG axis n; $n=X,Y$
B_{ii}	Second order bias term of pendulous accelerometer
C	Transformation matrix
CMT	Contraves Motion Table
DTG	Dynamically Tuned Gyroscope
$D(n)_F$	Absolute bias of axis n; $n=X,Y$
$D(n)_n$	Mass unbalance of axis n; $n=X,Y$
$D(n)_m$	Quadrature of axis n; $n=X,Y ; m=Y,X$
$D(n)_s$	Spin axis g sensitive drift term of axis n; $n=X,Y$
$D(n)_{lm}$	G^2 sensitive error terms of tuned rotor gyro of axis n; $n=X,Y ; l,m=X,Y,S$
f	Specific force
F	Force
FOG	Fibre Optic Gyroscope
FOM	Figure of Merit
g	Gravity vector
G	Gravitational acceleration
H	Angular momentum

H_n	Angular momentum about axis n; n=X,Y,Z
i	Current
IA	Input Axis
IGA	Inner Gimbal Assembly
IGS	Inner Gimbal Simulator
I_{gn}	Gimbal inertia of axis n
I_r	Rotor Inertia
I_n	Torquer current of axis n; n=X,Y
k_n	Gyro Torquer scale factor for axis n; n=1,2,3,4
K	Accelerometer Scale Factor
K_L	Rebalance Loop Gain
K_x, K_y	Flexure Stiffness. Gyro scale factor for axis X or Y
l	Length
LSF	Lower Sensor Fixture
m	Mass
M	Applied moment
M_n	Applied moment about axis n; n=X,Y
MU_m	Mass Unbalance of axis m; m=X,Y
MU_n	Mass Unbalance of axis n; n=1,2,3,4
n	Number of coil windings
N	Tuned Speed
N	Torque
N_{OA}	Torque component about accelerometer output axis
OA	Output axis
P	Pendulosity
PA	Pendulous axis
Q_m	Quadrature of axis m; m=X,Y
Q_n	Quadrature of axis n; n=1,2,3,4
r	Radial distance
R	Resistance
RLG	Ring Laser Gyroscope
t	Time
T	Torque
TC	Time Constant
TRG	Tuned Rotor Gyroscope
USF	Upper Sensor Fixture
v	Velocity
V_o	Accelerometer output voltage
X	Axis of gyroscope
X_n	X system axis of IGS
Y	Axis of gyroscope
Y_n	Y system axis of IGS
Z	Axis of gyroscope
Z_n	Z system axis of IGS

1. Introduction

1.1 Project Overview

Gravimetry is the science of measuring and characterising the gravity field. As a first approximation, the Earth's gravity field is quoted as having a constant value of 9.81 ms^{-2} . However, this value represents a global average, and gravity is different from one place on the Earth to the next. One major factor is the latitude dependence. Due to the Earth's rotation, gravity is stronger at the poles than at the equator. Another important factor is altitude, since the magnitude varies inversely as the square of the distance. However, these effects are deterministic, and can be modelled without undue difficulty. Gravimetry is more interested in gravity anomalies, which are local variations in the

Earth's gravity field due to differences in the local density of the Earth's structure. Torge [1] classifies and discusses gravity anomalies in detail.

Gravimetry has been studied as a science for hundreds of years. Indeed, the development of mechanics marked the beginning of the theoretical development of gravimetry in the 17th and 18th centuries. Soon afterwards, pendulum devices were used to measure gravity in discrete locations. By the twentieth century, gradiometers and static gravimeters had been developed, which allowed the gravity profile of small areas to be mapped. The second half of the twentieth century has seen the development of more refined instruments, capable of measuring gravity on the order of 10^{-8} or 10^{-9} ms^{-2} . Many competing designs of gravimeters have been developed, with one of the most widely used being the LaCoste and Romberg design [2]. Traditional designs have required that static measurements be taken. This involves transporting the gravimeter to the desired location, setting up the instrumentation and allowing it to settle, recording the gravity measurements over an extended time to get an averaged reading, and then packing the equipment up and moving on to the next location. This meant that one station could be surveyed in approximately half a day to a full day [1]. Obviously, this labour intensive task made it very difficult to take a large number of readings or to cover a wide area.

Dynamic gravimetry became the obvious solution to this problem. By mounting the gravimeter aboard a moving vehicle, a large amount of data could be collected, over a wide area, in a short period of time. However, difficulty arises in separating the vehicle dynamics from the sensed gravity. Land vehicles suffer from a relatively high shock environment as bumps are traversed. These corrupt the gravimetry data. Combine this

with the fact that land vehicles are limited in the locations to which they can travel, and it is clear that this is not the optimal method of conducting gravity surveys.

Gravimetry measurements can also be conducted at sea. Ships provide a relatively benign environment, which is suitable for measurements on the order of a few μms^{-2} . Here, disturbing vertical accelerations can be filtered out using long averaging times for each data point. This is valid since the vertical motion of the ship due to pitching and swaying in the waves is a long periodic function. Despite the relative success of ship-borne gravimeters, the obvious drawback is that they can only be used at sea. Since much of the interest in gravimetry occurs on land, a third option is needed.

Airborne gravimetry is a concept which has been around for several decades. The idea of placing a gravimeter on board a moving air vehicle allows gravity measurements to be taken anywhere around the world. However, many problems have stood in the way of placing gravimeters on airborne platforms. The most critical of these is vertical height determination. On a ship, the vertical displacements average out to the profile of the geoid; however in the air, the motion is not so easily defined. Since a change in altitude of one meter corresponds to a $3 \mu\text{ms}^{-2}$ change in acceleration due to gravity, height determination is critical. Traditional methods of altimetry fall far short of the ability to accurately resolve height changes on the order of centimetres. However, with the satellite based Global Positioning System (GPS), the type of performance needed is now achievable. Using differential mode, in which a ground station is used to relay error corrections to the airborne platform, measurement of changes in altitude of centimetre precision can be obtained. Both Gleason [3] and Wells et. al. [4] have demonstrated that airborne gravimetry is possible with the aid of differential GPS (DGPS).

An airborne gravimeter is being designed by Sander Geophysics Limited of Kanata, Ontario, with the assistance of Carleton University and numerous subcontractors. The system has been designed to be mounted in a helicopter or fixed wing aircraft. The gravimeter itself will consist of a three axis gimbaled platform which is stabilised by two two-degree-of-freedom gyroscopes to maintain orientation of the platform in inertial space to better than ten seconds of arc. Accelerations will be sensed using three off-the-shelf accelerometers, and the vertical accelerometer must detect changes in input of fractions of micro-g's. These instruments, coupled with the capabilities of DGPS, should allow for the design of a high precision airborne gravimeter. Carleton University has taken on the task of testing the gyroscopes and accelerometers, and defining the misalignment and error terms of the sensors so that calibration of the gravimeter is achieved.

The question still remains, why is knowledge of the gravity field so important? There are several motivating factors which make development of an accurate gravimeter attractive, from scientific, technological and economic standpoints. The gravity field is of interest to geologists, since knowledge of gravity anomalies can be used in oil and mineral exploration. Modelling of the Earth's mantle also aids in the prediction of tidal models, as well as earthquake and volcano activity [3]. Inertial navigation also stands to benefit, since uncertainties in the Earth's gravity profile are regarded as significant navigation and guidance system error sources [5].

1.2 Motivation for Testing

Carleton University's participation in the gravimeter project involves the characterisation of the inertial instruments and determination of their error parameters. This work is crucial to the successful development of a working system. First, it is necessary to assess if the chosen instruments are suitable for use in this system. Laboratory testing can determine the resolution of the accelerometers or gyroscopes. Furthermore, imperfections in the machining and manufacturing of the instruments will cause non-ideal behaviour. Precision testing can determine these error parameters which are then used in error compensation equations to correct the output of the instruments. Other factors influence the performance of inertial instruments, such as temperature or power cycling. These effects can only be determined within a controlled environment, that has been tested and proven in the test laboratory.

Once the instruments have been tested individually, they are integrated within the gravimeter sensor block. Determination of the misalignment angles of the sensor axes to some known orientation becomes necessary. Mirrors mounted on the sensor block are used to establish a frame of reference. Through a combination of optical measurements and laboratory testing, it becomes possible to relate the sensor axes' orientation to the mirror reference frame. This provides the capability of removing the sensor block from the lab environment, installing it onto the gimballed platform, and still being able to determine the positions of the sensor axes, providing initialisation and calibration of the system platform.

1.3 Thesis Outline

Since the scope of the project covers three separate phases of testing, many of the following chapters are divided into subsections which deal exclusively with accelerometer, gyroscope, or system level testing.

Chapter 2 is an introduction to inertial technology, with general discussions about the theory behind the operation and applications of gyroscopes and accelerometers. Specific attention is given to the Q-Flex design of pendulous accelerometers and the CSG-2 dry, tuned, two-degree-of-freedom type gyroscope, since both of these instruments are used in this gravimetry system.

Chapter 3 discusses the equations which are needed to model the behaviour of the individual accelerometers and gyroscopes, as well as the integrated sensor package assembly. The approach is practical in nature, with mathematical formulations developed with laboratory testing procedures. An understanding of the error terms of inertial instruments is necessary in order to define the tests needed to characterise the gyroscopes and accelerometers.

Chapter 4 outlines the tests conducted on the individual sensors, as well as the tests on the completed inner gimbal simulator. The motivation for each test is discussed, with references to the theory developed in Chapter 3. The equipment used in conducting these tests is highly specialised, and its use is discussed in detail.

Chapter 5 presents the results of the tests. The suitability of the instruments for the gravimeter is demonstrated. The error parameters and misalignments are presented and discussed.

Chapter 6 concludes with final remarks and recommendations concerning future testing.

1.4 Objectives

While the overall goal of the Sander Geophysics project is to develop an airborne gravimeter, Carleton University's involvement in the project is much more specific. From an engineering point of view, the scope of work covers areas of technology normally associated with inertial navigation. The remainder of this thesis will be confined to discussions about the hardware involved and its relation to a stabilised platform. An in depth survey of gravimetry is beyond the scope of this work, and will only be mentioned when it relates directly to the subject at hand.

Two separate yet related areas were explored. When the project was first conceptualised, it was necessary to find inertial instruments which would be suitable for the task and to characterise their performance and behaviour. Therefore, individual accelerometer and gyroscope testing was conducted, to provide characterisation of the error terms of the instruments.

The second objective was to integrate the instruments into a sensor block for use on the stabilised platform, and then to test the instruments as a complete system. The major objective of these tests was to determine the misalignment angles of the sensors with respect to a known reference frame. This is critical for defining the performance of the gravimeter.

2. Inertial Instruments

2.1 Overview of Developments in Navigation

The development of inertial components is closely linked with the history of navigation. Since the first time that people started travelling long distances, it was necessary to develop methods for determining where one was and where one was going. This was particularly important for early mariners, since there were no landmarks or roads to guide one's course on the open sea. Buoys and lighthouses were used in the ancient Mediterranean as aids to shoreline navigation. By the eleventh century, the magnetic compass had been invented as a method of roughly determining the direction of North. While stars had been catalogued as far back as the ancient Greeks, it was not until the

fourteenth century that star fixes for navigation purposes came into widespread use. A wide array of instruments were developed to aid in this process, the pinnacle being reached with the precisely machined sextants and octants of the seventeenth and eighteenth centuries.

Long distance navigation didn't start to become reliable until an accurate method of time keeping was developed by John Harrison using a temperature compensated balance-wheel clock [6]. This allowed for longitude to be determined to an accuracy of 30 miles during a trip across the Atlantic from London to the West Indies.

Navigational accuracy continued to improve by small increments over the next century with the introduction of better instruments, time keeping, and navigation charts. However, it wasn't until the twentieth century that quantum leaps occurred in navigational accuracy. By 1906, the gyrocompass was invented as a method of azimuth determination on board ships [7]. With the advent of aviation, it was soon realised that a magnetic compass and an airspeed indicator (based on pitot tubes) was not sufficient to guarantee accurate navigation over long distances. Radio beacons were developed as an aid to pilots, and continue to be used in modern navigation. However, a number of factors, many of which were military in nature, led to the need for the development of inertial navigation systems.

Inertial navigation systems (INS) have a number of advantages over other forms of position and attitude determination methods, which motivated their development. These include [8]:

- An INS is a self-contained system. Unlike radio beacons or star fixes, they do not require any form of external input. This implies that navigation can be done in any

weather or environment, which is of obvious importance for vehicles such as submarines or aircraft. Furthermore, an INS is impervious to radio jamming.

- An INS continuously keeps track of position, heading, and attitude. With other methods, especially celestial observations and certain types of radio beacons, a reading is made and then followed by a series of calculations in order to determine position and heading.
- For civilian applications, an INS presents a relatively low cost, reliable method of navigation and guidance. Without an INS, it is necessary to have a highly skilled human navigator on board the vehicle.

The first place in which an INS was used was Germany during World War II in the V1 and V2 rockets. Following the war, INS development was rapid, led primarily by Charles Draper at the MIT Instrumentation Lab and also by Werner Von Braun's group at the Manned Space Flight Center of NASA in Huntsville, Alabama.. By 1949, inertial navigation packages were being installed on aircraft and by 1954, they were in use on board ships. A landmark in inertial guidance came in 1960 when a Polaris submarine successfully navigated its way under the polar ice cap. Inertial technology continued to advance throughout the 1960s with applications in spaceflight and rocketry.

While most of the fundamentals of inertial technology have not changed significantly over the past couple of decades, the state of the art has continued to be pushed forward through better understanding of material behaviour and improvements in machining techniques. The microcomputer revolution has also aided in these advancements. At the centre of inertial technology are two fundamental instruments: the accelerometer and the gyroscope.

2.2 Accelerometers

2.2.1 BACKGROUND

Instruments which sense and measure linear accelerations are known as accelerometers. They are governed by Newton's second law of motion, which states:

$$\mathbf{F} = m\mathbf{a} \quad 2-1$$

Since gravitational accelerations are indistinguishable from accelerations caused by the motion of a vehicle, the output of an accelerometer actually indicates the combined effects of gravity and vehicular accelerations. Furthermore, mass is an invariant property of a substance. That is, regardless of the location in space or the frame of reference chosen, mass always remains constant in Newtonian mechanics [9]. Therefore, equation 2-1 can be rewritten as a specific force expression:

$$\mathbf{f} = -\mathbf{a} + \mathbf{G} \quad 2-2$$

Note that the sign convention here defines gravity as being positive. This specific force relationship defines the fundamental quantity which an accelerometer measures.

Accelerometers have been mechanised in a number of different ways over the years. However, regardless of their size or shape, all accelerometers have three fundamental components:

1. A proof mass which responds to accelerations.
2. A case or suspension system which serves as a reference frame for the proof mass.
3. A calibrated restraint (pickoff) which relates proof mass displacements to the case.

These components are sufficient to make an accelerometer which is passively controlled.

However, for higher precision instruments, an active control system is often used. These

require a restraint force, usually implemented by a rebalance torquer and a servo loop, in addition to the other basic elements. Though many different types of accelerometers have been devised, there are only a few basic designs which have been widely used or investigated. Most of these designs are pendulous and involve masses which are confined to move only in one well-defined direction, otherwise known as the sensitive or input axis. This results in the need for three accelerometers to be used in any type of three dimensional navigation system.

In principle, an accelerometer could be developed which has three degrees-of-freedom and, therefore, could sense accelerations in any spatial direction. Electrostatic accelerometers have been developed to accomplish just this. By suspending a cube between plates, using electrostatic charges, the voltage necessary to maintain constant air gaps in all directions is monitored and related to applied accelerations. These types of accelerometers have found applications in microgravity experiments in space, but have not proved useful for navigation purposes, due to the higher acceleration inputs.

A Pendulous Integrating Gyro Accelerometer (PIGA) has been developed primarily for inertial high accuracy navigation and guidance applications. Its operation will not be discussed further here, but Broxmeyer [10] includes a discussion on the operational aspects of a PIGA.

It has long been known that if a mass is suspended in such a way as to allow the suspension itself to vibrate (by means of an oscillator), then the acceleration experienced by the mass is related to the natural frequency of the suspension. Early attempts to produce an accelerometer based on this principle were centred around using a string to suspend the mass and causing the string to vibrate. This approach proved to be virtually

unusable in navigation grade accelerometers, primarily due to problems in maintaining tension within the strings. However, due to advances in crystal manufacturing techniques, a number of designs based on vibrating beams have recently been developed. Lawrence [11] devotes an excellent chapter to the discussion of the vibrating beam accelerometer (VBA).

One of the most prevalent designs today is that of the pendulous type accelerometer. Simple versions of these devices operate in an open loop mode, in which the angular deflection of the proof mass produces an output signal. However, this deflection tends to produce cross-axis coupling errors, since the proof mass is not constrained to move in one axis only. Rectification of vibrations can also arise due to this out of axis movement.

The solution to this problem is to operate the accelerometer as part of a closed loop system. In this arrangement, the proof mass is held at null by a torquer which produces a rebalance and/or opposing force. Since during operation the mass always has the same displacement, non-linearity of the pickoff and hinge becomes much less of a concern. It is the torquer which must be made to behave in a linear fashion, which is a substantially easier task. One of the most successful closed loop accelerometer designs is made by Sundstrand Data Control of Redmond Washington.

2.2.2 THE Q-FLEX ACCELEROMETER DESIGN

The Sundstrand Q-Flex (quartz flexure) design of closed loop pendulous accelerometers has been one of the most successful designs in the past two decades. A number of different models have been made, including the QA-2000 and QA-3000, which

Figure 2-1: Cross Section View of Q-Flex Accelerometer (Schematic)

Figure 2-2: Q-Flex Accelerometer Internal Assembly [12]

are being used in this airborne gravimetry system. Each model has subtle improvements over its predecessors, but all function in the same fundamental manner.

A schematic view of the QA-2000 cross section is shown in Figure 2-1. The proof mass is suspended by a hinge to hang between two plates, which act as capacitive pickoffs. If an acceleration is applied to the sensitive axis of the instrument, the size of the air gaps between the capacitor pickoffs will change, causing a larger voltage potential on one pickoff compared to the other. This differential signal is fed back into the servo amp, producing a current through the torquer coils. This current causes a magnetic field which pushes and pulls the proof mass back to the null position. By measuring the torquer current across a resistor and applying a scale factor conversion, the current in milliamps can be related to the applied accelerations.

The unique aspect of the Q-Flex design is the way in which the hinge is mechanised. The pendulum proof mass is made from a disk of amorphous quartz. Around the outer edge of the disk, an annular cut is made which almost encompasses the entire disk. Two small pieces of quartz are left intact, which are thinned down by etching, to serve as hinges between the pendulum and the outer rim of the disk (see Figure 2-2). The advantage to using quartz is that it is almost perfectly elastic in the plane of the section, while non-elastic in the other directions [13].

The torquer coils are mounted on either side of the quartz disk and constrained to move around the permanent magnets. This arrangement is similar in functionality to a loudspeaker driver. Since the coils are attached to the quartz disk, they become part of the pendulum proof mass itself. The electric current which flows through the coils is supplied through a thin conducting layer of metal in the hinges. Deposition of this film is

highly controlled, since differences in thickness from one side of the hinge to the other can result in torques arising from temperature changes.

Ideally, acceleration would only be sensed along one axis and would exhibit perfect linearity and no offset or bias. However, for an actual accelerometer, this is not the case and is part of the justification for Carleton University's role in testing these inertial sensors. The model equation for the pendulous accelerometer is developed in Section 3.3.

2.3 Gyroscopes

2.3.1 BACKGROUND

The gyroscope, which is a mechanism based on the fundamental law of precession, is the most common method of measuring angular rotations. The word itself is derived from the Greek *gyros* meaning "to view" and *skopein* meaning "rotation" [14]. When speaking of gyroscopes, one must be aware that there are two different uses of the word, related, yet subtly distinct. Sometimes, a gyroscope refers simply to a spinning wheel, similar to a child's top or a high school physics demonstration. In engineering, a gyroscope is a device with a spinning rotor that has been mechanised so that it responds to angular rates.

The first type of rotation investigated by scientists was the motion of the Earth. During the 19th century, pendulums had been observed to rotate during the course of the day. In fact, it was the Earth that was rotating about the pendulum, which remained inertially fixed in space. However, pendulums proved to be unsuitable for use in studying angular rotation, as they were susceptible to air currents. This prompted Leon Foucault to devise a gimballed wheel which was constructed to remain fixed in space as the Earth

rotated beneath it [11]. However, it wasn't until this century that the practical application of gyroscopes was realised.

In order to understand how a gyroscope senses rates, it is first necessary to have a grasp of the physics of gyroscopic motion. Starting once again with Newton's second law of motion, it is known that the angular momentum of a body will remain constant, unless a torque is applied to the body. The rate of change of angular momentum is the same as the magnitude of the applied torque:

$$\mathbf{T} = \frac{d\mathbf{H}}{dt} \quad 2-3$$

Referring to Figure 2-3, a spinning wheel is shown as having angular momentum about its spin axis. If a torque is applied to the wheel in a direction orthogonal to the spin direction, the torque causes a change in the angular momentum vector direction. This change, represented by a small change $d\mathbf{H}$, is in the same direction as the torque vector and has a magnitude of

$$d\mathbf{H} = H d\theta \quad 2-4$$

Combining this with equation 2-3, the fundamental law of precession is derived:

$$\mathbf{T} = \frac{d\mathbf{H}}{dt} = \mathbf{H} \times \boldsymbol{\omega} \quad 2-5$$

where $\boldsymbol{\omega}$ is known as the precession rate. This rate acts in the direction perpendicular to the plane of the angular momentum and the applied torque. If a torque is applied to a spinning object, the angular momentum vector precesses itself until it lines up with the applied torque vector. This effect is witnessed when a bicycle rider turns a corner simply by leaning to one side or the other.

Figure 2-3: The Fundamental Law of Precession

Figure 2-4: Precession and Nutation of Rotating Bodies

The motion of the spin vector has been described as precession. However, this vector can experience another form of motion, known as nutation. These two terms are often confused, and some of the literature uses them rather interchangeably. However, they are two distinct effects. An easy way to visualise the motion is to consider a spinning top. Referring to Figure 2-4, it is evident that if the axis of a spinning wheel is not perfectly aligned with its support point on the ground, a torque is produced. This causes the angular momentum vector, and hence the spin axis, of the top to move toward the torque vector. However, as soon as it moves, so does the torque vector. This causes the angular momentum vector to trace out a circular path, forever precessing towards the gravitational torque. However, a spinning body is also capable of rapid cyclic motion, appearing as if the spin axis is rotating about itself. This nutation is observed when a top slows down and appears to start wobbling about its spin axis.

However, some designers have opted to limit the motion of the spinning wheel and constrain it to move only about one axis. This is known as a single degree-of-freedom gyroscope as opposed to a two degree-of-freedom gyroscope which allows freedom of motion of the wheel about two orthogonal axes. While there are many different designs of mechanical gyroscopes, almost all of them fall into one of these two broad categories.

The single degree-of-freedom gyroscope (SDF) blossomed in the late 1940s, spearheaded by work done by Charles Draper at the Instrumentation Labs of MIT. Initial attempts at using both axes of the gyroscope rotor was complex and had cross-coupling effects that yielded mediocre results. This resulted in the concentration on development of high accuracy gyroscopes with only one sensitive axis. As a result, SDF gyroscopes have become the most common type of gyroscope. Broadly speaking, there are two designs in

common use. The rate gyroscope provides an output of the angular rate sensed about the input axis. A gimbal attached to the gyroscope case with a torsion bar is used to restrain motion about the defined axis. The motion of the gimbal and rotor is damped by filling the interior of the casing with a fluid, often a fluorocarbon. A pickoff provides an electrical signal whose magnitude is proportional to the gimbal angle. This type of sensor is open loop in nature.

For high degrees of accuracy, the Rate Integrating gyroscope was developed. In many respects, this gyroscope is constructed similarly to the rate gyroscope, except that the torsion bar is replaced with a ball bearing, and a torquer is added as a closed loop control mechanism. The gyroscope output is derived from the current in the torquer, which is generated by a servo amplifier. Broxmeyer [15], Lawrence [11] and Sinkiewicz et. al. [16] all provide excellent discussions on the SDF gyroscope.

Two degree-of-freedom (TDF) gyroscopes, also known as free gyros, represent a more elegant use of the gyroscopic properties of a spinning wheel, since they make use of all information available rather than constraining movement about one axis. Since a full-fledged inertial measurement unit (IMU) requires rotations to be sensed about three independent axes, an IMU requires only two TDF gyroscopes as opposed to three SDF gyroscopes. This normally results in lower cost, and also provides one extra (redundant) axis.

The most obvious method of designing a TDF gyroscope is by means of a gimbaling system, in which a spinning wheel is allowed to rotate within two gimbals. Since the wheel must be housed within the confines of the gimbals, this necessarily makes

the wheel small, giving the gyroscope a small angular momentum. This generally limits the maximum sensitivity of the instrument.

Similar to the gimbal type TDF gyroscope is the two axis floated gyroscope. Gimbal rings and bearings are used to allow two degree-of-freedom movement, with a flotation fluid now used between the float assembly and the case. This type of sensor operates in a closed loop, and, therefore, has a pickoff providing an error signal that drives the torquers. The two axis floated gyroscope is limited to platform use, since the torquers have low restoring torques in this design. Wrigley et. al. [17] provide a good discussion of these gyroscopes.

The most common type of two degree-of-freedom gyroscope is the dynamically tuned gyroscope. This is the type of gyroscope that is used in this gravimetry system. Section 2.3.2 outlines its design and operation.

Another important class of gyroscopes are those that are based on the constancy of the speed of light. The Sagnac effect is the occurrence of a fringe pattern due to the interference of two counter rotating light beams. A light beam is split and forced in opposite directions around a circular path of travel. If the physical structure on which these paths are mounted is stationary, the light beams arrive at the starting point simultaneously. However, if the base of the paths is rotating, the beams are forced to travel paths that have different effective lengths, causing a phase shift in the interfering beams and an accompanying fringe pattern related to the applied angular rate. Ring laser gyroscopes (RLG) [18,19] and more recently fibre optic gyroscopes (FOG) [20,21] use this principle. The RLG suffers from the physical constraint that the path lengths are necessarily short in order to keep the overall dimensions of the gyroscope small, limiting

its resolution. However, the FOG uses fibre optic cables as the optical wave guide, allowing for coiling and hence very long paths in a small package. While FOG technology is still at least an order of magnitude worse in performance (0.1 deg/hr typical) than high quality mechanical gyroscopes, their relative low cost makes them an attractive option. Coupled with GPS/INS integration, FOGs will most likely be the preferred method of achieving low cost navigation in the future.

2.3.2 THE CSG-2 GYROSCOPE

The Canadian Strapdown Gyroscope, mod 2, (CSG-2) is a dynamically tuned, two degree-of-freedom gyroscope. It was designed and developed by Litton of Canada, with the consultation and support of the Defence Research Establishment Ottawa [22]. A number of design innovations were incorporated into the CSG-2 in an attempt to make a high performance, very low random drift gyroscope. However, its behaviour and design are very much like any other high performance tuned rotor gyroscope. Therefore, the following discussion deals with these types of gyroscopes in general, with specific reference made to the CSG-2 where appropriate.

The Tuned Rotor Gyroscope (TRG), also known as the Dynamically Tuned Gyroscope (DTG) or Dry Tuned Gyroscope, grew out of experiments with spinning wheels supported by universal joints made of two gimbals supported on ball bearings. These designs proved to be rather unstable, with the dynamic inertia effects in the gimbal causing the instabilities. Instead of using gimbals, a flexible joint, or flexure, was introduced in later designs. The problem with a flexure was that it resulted in spring torques which coupled the rotor to the spin shaft, thus making this arrangement an

ineffective inertial reference. If these spring torques were eliminated, the rotor would act as a free body. Early attempts at accomplishing this were done by using magnetic torquers to counteract the spring action of the flexures. However, hysteresis effects in the magnetic field made this solution unreliable.

The solution proved to be to use the dynamics of the gyroscope itself to negate the effects of the spring torques. Consider first a rotor suspended by a gimballed ring, as shown in Figure 2-5. The gimbal ring and rotor are connected to the shaft through means of flexures. Initially, it is assumed that the rotor plane is perpendicular to the spin axis, and the rotor is spinning at some speed N . If the case, and hence the spin shaft, are displaced by an amount ϕ relative to inertial space, the rotor itself remains in the orientation it had before the case rotation. In order to accomplish this, the gimbal ring must travel through an angle of ϕ . This implies that the gimbal is moving relative to the shaft and to the rotor. This obviously leads to energy dissipation, causing torques to arise which tend to reduce the gimbal oscillations, eventually returning the gimbal and rotor to their initial relative orientation perpendicular to the spin axis.

This gimbal oscillation is the key to a dynamically tuned instrument. The physical flexures themselves have positive spring constants, which produce torques on the system. However, at the same time, a negative spring rate arises from the dynamics of the gimbal ring, generating torques caused by the inertia of the gimbals counter to those due to the spring action of the flexures. This negative spring rate is a function of the gimbal inertia and the rotational speed of the system. While the TRG dynamic equations

Figure 2-5: Flutter of Gimbal Ring during Rotation of Shaft

Figure 2-6: Schematic of a Tuned Rotor Gyro [22]

will be dealt with further in Chapter 3, a complete derivation of the equations of motion are developed by Craig [23] and Mansour et. al. [24]. However, the tuning condition can be stated as,

$$N = \sqrt{\frac{K_x + K_y}{I_{gx} + I_{gy} - I_{gz}}} \quad 2-6$$

where K_x and K_y are the x and y flexure stiffnesses, and the I_{gx} , I_{gy} and I_{gz} terms are the gimbal inertias. A flexure suspended rotor running at a speed N results in the rotor behaving as a free body. This is the heart of operation of a dynamically tuned gyroscope.

A schematic representation of a DTG is shown in Figure 2-6. Here, all of the major components common to any tuned rotor gyroscope are seen. The components are housed within a casing that serves both to protect the inner workings of the gyroscope and to provide a reference surface so that the instrument can be accurately mounted. The case is also vital in the heat transfer characteristics of the gyroscope. Most inertial instruments are designed to operate above ambient temperature for several reasons. First, elevating the temperature permits heaters to be used to maintain a precise thermal environment around the instrument. Second, certain gyroscopes do not function properly at low or ambient temperatures. While this is especially true for floated instruments, since the viscosity of the float fluid increases at low temperatures, it also applies to other types of instruments as well, including the DTG. Generally, an operating temperature and associated thermal range is chosen for a gyroscope at the design stage, and the components are then designed accordingly.

The motor, usually a hysteresis, three phase type, is mounted rigidly to the case. The spin shaft is supported by a pair of ball bearings that are also integral to the case

assembly. A mechanical stop, used to limit the angular travel of the rotor, is usually mounted at the end of the shaft itself. The flexures, or hinges, connect the spin shaft to the rotor.

The flexures are one of the key innovations of the CSG-2, setting it apart from other tuned rotor gyroscopes. The flexures are actually machined into the gimbal ring itself, using electro-discharge machining (EDM) techniques [22]. Other designs of flexures have traditionally relied on hinges to attach the gimbal to the rotor and the shaft. In manufacturing integral flexures such as those of the CSG-2, it is critical to ensure that the flexures are all the same thickness. Uneven behaviour of the flexures can lead to large error terms arising in the final gyroscope.

Thermal behaviour of the flexures is also important. If the material chosen is not dimensionally stable over a wide range of temperatures, internal stresses can arise within the flexures, possibly altering their behaviour. Another important consideration is that the X, Y and longitudinal axes of the flexures should intersect at one point, preferably the centre of mass of the gyroscope, and must remain mutually orthogonal at all times. Therefore, if the flexures change size with temperature, these conditions may also change, having a drastic effect on the performance of the gyroscope.

Acceleration and shock forces must also be considered when designing the flexures, especially if the sensor is intended for strapdown in lieu of platform use. On a stabilised platform, the environment is much more benign than if the gyroscope is strapped directly to a moving vehicle. Therefore, in general, an instrument designed for strapdown use could be utilised in either type of navigation system, while the reverse is not necessarily true.

From the tuning condition, Equation 2-6, it is evident that flexure design impacts heavily on the tuned speed of the gyroscope. Since the angular momentum of the rotor increases with the speed of rotation, it is desirable to have as high a tuned speed as is possible. This implies that stiff flexures are desirable. However, a high tuned speed will result in increased bearing wear which reduces the expected life span of the gyroscope. This drives the design to having a low tuned speed with a high rotor inertia (since $H=I\omega$) to obtain good performance. However, the size of the inertia term is constrained by the overall case dimensions. These are trade-offs which must be defined and weighed at the time of gyroscope design. Strategies for optimal hinge design are given by Mansour and Laccini [25].

During operation of a DTG, the case of the instrument is rotated in inertial space, and the rotor itself maintains its orientation. Therefore, there exists relative motion between the case and the rotor, and there must be some method of detecting this motion. This is the purpose of the pickoffs. Numerous types of sensors have been developed to detect motion of the rotor, but the most common type for a DTG, and the one used in the CSG-2, is an electromagnetic variable transformer (inductive pickoffs). A magnetic circuit is established using a core mounted on the case and a magnetically permeable plate mounted on the rotor (see Figure 2-7). The core carries windings for both the primary and secondary coils. A current is driven through the primary coil, establishing a magnetic flux within the circuit whose magnitude is proportional to the gap between the core and the rotor mounted plate. This flux induces an EMF in the secondary windings.

Figure 2-7: Pickoff of a Tuned Rotor Gyro

Figure 2-8: Tuned Rotor Gyro Torquer

The physical pickoffs in a gyroscope come in pairs, with one pair defining the X input axis and the other being orthogonal and defining the Y axis. Consider a rotation about the X input axis. The two air gaps, g_1 and g_2 , between the pickoffs and the rotor change, one of them decreasing and the other increasing. This causes the EMF force in the secondary winding next to the small gap to be larger than that in the opposite pickoff. These two signals are then passed through a differential amplifier circuit which produces a varying voltage output corresponding to the relative rotor motion. Note that if the gaps are equal in magnitude, no electrical output is generated, corresponding to the situation in which the rotor is nulled relative to the case. During fabrication of the gyroscope, this electrical null is adjusted so that it corresponds to the mechanical null, since mismatch of the two can manifest itself as an error in the gyroscope output signal.

Ideally, when the gaps are equal, the two secondary EMFs of a pickoff pair will exactly cancel out. However, residual voltages caused by eddy currents and hysteresis effects in the cores can cause slight variations between the two windings of the secondary pair, both in phase and amplitude. The larger these variations, the noisier the output signal on the gyroscope will be, resulting in a large deadband in the pickoff signal that appears as null. This has an obvious impact on gyroscope performance.

The final consideration necessary in the design of a tuned rotor gyroscope is a method of controlling the orientation of the rotor. Sometimes, this is accomplished external to the gyroscope by servoing the platform on which the instrument is mounted. However, in closed loop operation using rebalance electronics, it is necessary to have an internal method of changing the angular position of the rotor. This is accomplished through the use of torquers, one for each axis. By the fundamental law of precession, it is

obvious that the torquer which responds to rotations about the X axis needs to be oriented in the Y direction, and vice versa.

Figure 2-8 shows a typical design for a DTG torquer. On the case, four coils are mounted, one in each quadrant, through which torquer current, i , is driven. A toroidal permanent magnet is mounted on the rotor that produces a radial magnetic flux, B . The current driven through the coils interacts with the radial magnetic flux, causing a force, F , to be generated such that:

$$F = nli \times B \quad 2-7$$

This force results in a torque being applied to the rotor, causing the rotor to precess. Like the pickoffs, the torquers come in pairs and are wired so that the torques generated by the coils opposite each other add together to produce a net torque. This net torque returns the gyroscope rotor to the case null position of the instrument, through use of an external electronic rebalance loop.

The torquers are designed so that the maximum torque they produce is capable of counteracting the maximum design rate measurable by the gyro. This requirement also defines the maximum power which the torquer coils can dissipate, which in turn establishes the amount of heat which the torquer generates during operation. In static testing of a gyroscope, or during platform operation, torquer heating normally isn't a driving concern. However, in strapdown systems where the gyro torquers are working hard at all times, the effects of torquer heating are sometimes significant. This heating can cause localised hot spots, leading to thermal gradients within the gyro that affect performance. In order to improve heat transfer throughout the gyro, the inner cavity of the gyroscope is filled with a thermally conducting gas, often helium, at slightly above

standard atmospheric pressure. Not only does this gas aid in heat distribution, it also protects the interior of the gyroscope from oxidation.

Since only a fixed amount of room is available for the placement of the torquers, it is necessary to perform a trade-off study when deciding on the size of the coils and magnets. For a given size and torque requirement, an optimum torquer can be designed. These considerations are detailed by Mansour et al [25].

The above features are common to all DTGs, their implementation naturally differing slightly from manufacturer to manufacturer. High quality DTGs are accurately machined instruments. However, even the most precise machining still results in a gyroscope which has error terms, causing less than ideal behaviour. The most significant of these is bias, a constant sensed rate when no input rate is applied. It is important to note that bias is independent of the orientation of the gyroscope and, therefore, always affects performance. Other important error terms are known collectively as g-sensitive and g^2 -sensitive terms. Depending on the construction of the gyro, some or all of these error terms can be significant. Two of the g-sensitive terms, the mass unbalance and the quadrature, will be discussed here due to their overriding importance in the CSG-2.

Consider Figure 2-9. A tuned rotor gyroscope is oriented so that the spin axis is perpendicular to the gravity vector, and the X input axis is aligned with the gravity vector. Ideally, the spinning mass is perfectly balanced in the plane of the wheel. However, if there exists a slight imperfection in mass distribution, it can be modelled as a point mass a distance, d , along the spin axis direction. This tends to cause a torque proportional to the mass and lever arm. Since this torque acts in the Y direction of the gyroscope, a

Figure 2-9: Visualisation of the Mass Unbalance Error Term in a TRG

Figure 2-10: Explanation of the Quadrature Error Term in a TRG

precessional rate appears about the X input axis due to the fundamental law of precession. This is known as mass unbalance drift, often given the symbol MU_x . In order to counteract its effects, a torque equal in magnitude but opposite in direction can be applied about the Y axis direction. Similar arguments can be made to establish the existence of MU_y , the mass unbalance when the Y input axis is aligned with gravity. Since these two terms both arise from the same root cause, namely poorly distributed mass in the spin direction, the terms are almost always similar in magnitude. This assumes that the scale factors for both axes are comparable, which is normally the case in a DTG. It is almost inconceivable to devise a method of affecting one mass unbalance term without changing the other one.

The second major g-sensitive effect is known as quadrature mass unbalance and is given the symbols Q_x and Q_y , respectively. Figure 2-10 helps to illustrate the mechanics which lead to its generation. The DTG has four flexures, two of which connect the rotor to the gimbal, and two others which connect the gimbal to the shaft. For the purposes of this discussion, assume that three of these flexures are 'perfect' and that only one of them has defects in its construction. In reality, all four have some imperfections, but this does not change the basic reasoning behind this discussion.

A flexure can be thought of as a beam that has pin connections at either end. As such, if it is loaded in tension or compression, bending moments are established [26] which need to be compensated for. This means that reaction torques are generated by the dynamic rotor-gimbal system. Since torques are being applied to the wheel, precession occurs.

Returning to Figure 2-10, consider the DTG wheel rotating through 360° . When the defective flexure is in the starting position and lies horizontally, there is no longitudinal force acting on the member. As the wheel rotates a quarter turn, this flexure goes into tension due to the weight of the gimbal-rotor assembly. This tension causes the flexure to twist, thus generating a torque. After half of a revolution, the member is once again horizontal, so that the generated torque falls back to zero. At the three quarter turn mark, the flexure is once again stressed, however this time in compression. The compressive stresses cause a torque to be generated in the opposite sense to those due to the tensile stresses a half turn ago. However, since the rotor position has changed by 180° , the net result is that the torque acts in the same absolute direction as before. Therefore, over the course of a full revolution, the torque continually varies between zero and $T_{\max}$, so that there exists a non-zero mean torque. It is evident that an acceleration along the X axis causes a torque to be experienced about the X axis, leading to a drift appearing along the Y axis. The converse is also true.

These error terms are known as g-sensitive drifts, since they arise due to the action of gravity or acceleration along one axis. There are other types of drift errors that can also arise in a dynamically tuned gyroscope. One such class of error terms is known as the g-squared sensitive drifts. It is possible to define up to nine separate g^2 terms for a DTG, but all of them arise due to the same basic phenomenon: accelerations being sensed by two axes simultaneously. The exact causes vary depending on the axes concerned and the design of the gyro. However, one of the more important causes is anisoelectricity [27]. Anisoelectricity, as the name implies, arises when the stiffness (elasticity) in the axial direction does not match the radial stiffness. Consider Figure 2-11. A point mass is

suspended by two springs which are at right angles to each other in the neutral position. If an acceleration is applied to the mass, and one of the springs, say K_y , is stiffer than the other, then the mass tends to move off the line of action of the force. Since the mass is displaced a perpendicular distance, d , from the force, a torque arises according to:

$$T = \frac{1}{2} m^2 a^2 \left[\frac{1}{K_y} - \frac{1}{K_x} \right] \sin 2\theta \quad 2-8$$

Inspection of this equation reveals that T is maximum when θ is 45° . This is important to remember for the purposes of DTG test definitions, which will be discussed in Section 3.4. For now, it should be noted that the spring mass system of Figure 2-11 is analogous to the rotor gimbal flexure system of a tuned rotor gyro. Therefore, when an acceleration acts along two separate axes, torques can arise inducing drifts which require compensation. In high quality DTGs, these second order acceleration drift terms are often negligible and are, therefore, ignored. However, this cannot be generalised, and each DTG must be evaluated on its own in a controlled laboratory environment before such an assessment is made.

Two other parameters are of significant importance to a DTG, even though they are not error or drift terms. These are known as the Figure of Merit (FOM) and the Time Constant (TC) of the gyroscope. Both of these numbers are good indicators of a gyroscope's performance and quality. The Figure of Merit was first defined by Craig [23] as a parameter which relates the rotor inertia to the gimbal inertias:

$$FOM = \frac{I_r}{I_{gx} + I_{gy} - I_{gz}} \quad 2-9$$

Figure 2-11: Explanation of Anisoclastic Torque in a Tuned Rotor Gyro

Ideal tuning dictates that the denominator equal zero. It turns out that this is not feasible in practice, since an infinite tuned speed would be required. However, by the definition of the FOM, it is evident that the larger the value of the FOM, the closer the DTG comes to being an ideal free body. To achieve a large value, it is necessary that the gimbal inertias be as small as possible when compared to the rotor inertia. If one is comparing two different designs of DTG, the one with the larger FOM usually performs better.

Time Constant (TC or τ) is another important factor in determining gyroscope quality. If a DTG is run open loop, then ideally, the rotor remains fixed in inertial space indefinitely. If the case of the gyro is rotated, the rotor is positioned relative to the case at an angle equal to that in which the case was moved. However, due to torques exerted on the rotor by drag induced by the gas in the case of the gyro, the rotor slowly reorients itself so that the spin axis of the rotor is once again parallel to the case. This behaviour is exponential in time, so it is natural to define a time constant for the DTG as the time it takes for the rotor to return to $1/e$ of the initial angle displaced. For good quality gyroscopes, τ should be large (on the order of a hundred seconds), indicating that there is little internal drag on the rotor, which implies that it is closer to ideal behaviour. Not only can the TC be used to compare two different designs of gyroscope, but it can also be used to monitor the degradation of an individual instrument. If the time constant decreases dramatically, it is likely that the hermetic seal of the case is compromised and that the outside atmosphere has contaminated the gyroscope.

2.4 Summary

An overview of the purpose of accelerometers and gyroscopes and their context within the field of inertial navigation was presented. The specifics of the Q-Flex design of accelerometers and the design of the CSG-2 tuned rotor gyro have been explored. A complete understanding of these instruments is necessary in order to design and implement tests to assess performance.

3. Inertial Component Model Equations

3.1 Introduction

Inertial instruments are governed by the physical laws of motion. In theory, the behaviour and performance of a gyroscope or accelerometer could be perfectly defined in time and space, so that at any given instant, the position and motion of the instrument is known. In practice, such absolute knowledge is unattainable, and so engineers and mathematicians have developed model equations which closely describe the behaviour of the instrument. In other areas of engineering, such models are often only a rough approximation of the state of a system. However, in the field of inertial technology, the performance of high quality instruments can be predicted to better than a part per million

of the applied input (i.e. fractions of micro g predictability of accelerometer outputs and better than 0.01 deg/hr predictability of gyroscope outputs).

This chapter discusses the equations used to model inertial components and systems and the physical implementation of the mathematical model. Using these equations, test procedures are developed which allow for complete characterisation of the instruments.

3.2 Philosophy of Testing

The fundamental goal of testing an inertial instrument is to determine its performance within a controlled environment and simulate the instrument's behaviour in a field environment. A controlled environment implies that as many variables as possible which may affect the instruments have either been eliminated, controlled or measured. In this way, error parameters can be determined based on the observed behaviour of the instrument.

Testing of inertial instruments falls into two broad categories, static and dynamic. Static testing is partially a misnomer, since it only implies that the device under test is not moving relative to the Earth. However, with respect to inertial space, the instrument is still moving and, therefore, experiences Earth rate. Furthermore, the effect of gravity is always present and needs to be taken into account. Dynamic testing implies that some method exists to apply a precisely known rate or acceleration to the instrument, in addition to any effect which may exist due to the Earth.

Many of the error terms used to model the behaviour of inertial instruments are based on sensitivities to gravity or to applied rates. Therefore, it seems to make sense to

place the instruments in orientations where these effects are either maximal or minimal. The obvious way to do this is to statically place an instrument in an orientation that is well known and defined relative to a precisely known orientation. Precise positioning can lead to the error term having either a full effect or no effect on the output, depending upon the orientation. Since many error terms will change sign algebraically simply by inverting the position of the instrument by 180° in the proper sense, the effect of that error term can be removed by algebraically adding the outputs obtained in the two opposite positions.

When devising a method to test inertial instruments, several steps need to be taken. First, a model equation for the instrument output which accounts for all the desired error parameters is developed. Then, using this model, a number of different orientations are defined in which these parameters have known effects. Complementary positions, in which a parameter has either a maximum positive or negative effect on the output, are often crucial to this step, as are positions in which the error term is zero. Mathematical expressions are then used to solve for each of the error parameters, which are easily isolated by adding or subtracting equations from different positions.

There are several methods of positioning inertial instruments, although the use of a motion simulator or a dividing head are most common. Certain characteristics of inertial instruments cannot be determined through use of static positioning. However, for most important error terms, static re-positioning is the preferred method of laboratory determination.

3.3 Testing of Accelerometers

Testing of individual accelerometers is conducted to determine the primary factors affecting accelerometer performance. Bias and scale factor need to be determined. The accelerometer's resolution and sensitivity to temperature have to be measured. Misalignment of the sensor axes is also important; however misalignments are determined in a later phase of testing once the accelerometers are mounted in the actual system assembly. Therefore, misalignment is primarily treated as a lumped parameter for these tests.

In a Q-Flex type accelerometer, accelerations are sensed by measuring the current generated by a torque rebalance loop which is used to maintain a proof mass in an equilibrium position. Therefore, the best place to start development of a model equation is with the Newtonian equation for torque, N , on a mass [28]:

$$N = r \times F = r \times \frac{d}{dt}(mv) \quad 3-1$$

$$N = mr \times a \quad 3-2$$

where m is the proof mass, r is the lever arm from the pivot point to the centre of mass, and a is the absolute acceleration. This equation only applies if the object in question experiences no angular rates or accelerations. Another advantage of static testing is that it makes analysis of the behaviour of the accelerometer simpler.

It should be pointed out that there are a number of different frames of reference which can be used when discussing accelerometer behaviour (see Figure 3-1). The analysis frame is attached to the proof mass pendulum. Therefore, this frame can rotate relative to the case. When the pendulum is in the null position, the analysis

Figure 3-1: Accelerometer Frames of Reference and Misalignment Angles

frame becomes coincident with the sensor frame. Since the origin points are arbitrary, the hinge point is chosen as the origin for both systems to simplify discussion. The reference frame is defined as being attached to the case. The Q-flex accelerometer has a mounting surface which is precision ground to be flat, and this defines the mounting plane; the line perpendicular to this plane defines the third dimension of the reference frame. Finally, there is the Earth frame, defined so that the Z axis is coincident with the local vertical, and the other two axes pointing North and East to form a right handed co-ordinate system.

The general torque equation is a vector expression which accounts for an arbitrary torque in space. Since the torquer coils of the Q-Flex accelerometer are only capable of producing torques about one axis, the output axis component can be stated:

$$N_{OA} = P(-a_{IA} - (\delta_{OA} + \theta)a_{PA} + \delta_{PA}a_{OA}) \quad 3-3$$

where P is the pendulosity of the proof mass, a is the acceleration resolved into accelerometer components, and the other terms are misalignment angles as shown on Figure 3-1. Using the fact that the Q-Flex rebalance loop produces a torque such that $N_{OA} = -K_L\theta$, where K_L is the total loop gain, equation 3-3 can be rewritten as,

$$N_{OA} = -P(B + a_{IA} + \delta_{OA}a_{PA} - \delta_{PA}a_{OA} + \frac{P}{K_L}a_{IA}a_{PA} + B_{II}a_{IA}^2) \quad 3-4$$

where B is a bias term added to account for fixed value torques which arise due to imperfections in the machining process, and B_{II} is a second order non-linearity term.

Equation 3-4 fully describes the torque effects along the accelerometer input axis. However its form is not useful for interpreting the output of an accelerometer. The Q-Flex series of accelerometers outputs a current proportional to the torque generated by the

coils, which in turn is proportional to the sensed acceleration. Therefore, there exists a torquer scale factor, K_T , such that,

$$N_{OA} = -K_T \frac{V_o}{R} \quad 3-5$$

where R is simply a scaling resistor that is used to convert the accelerometer current output into a measured voltage, V_o . This leads to the standard equation for an accelerometer output under static 1 g testing:

$$\frac{V_o}{R} = K(B + a_{IA} + \delta_{OA} a_{PA} - \delta_{PA} a_{OA} + \frac{P}{K_L} a_{IA} a_{PA} + B_{II} a_{IA}^2) \quad 3-6$$

where K is the accelerometer scale factor in mA/g and is equivalent to the pendulosity of the proof mass divided by the torquer scale factor.

The actual mechanisation of the testing will be discussed in Chapter 4, but a few points can be raised to even further simplify Equation 3-6. The way in which the Q-Flex is mounted in the test fixture is such that the output axis is always nominally parallel to the axis of rotation of the dividing head (a device used for accurately positioning the accelerometer; see Section 4.1.1). Therefore, the output axis acceleration component, a_{OA} , is always very close to zero. Since δ_{PA} is also very small, the value of the fourth term in Equation 3-6 is exceedingly small and can be ignored. Furthermore, the coefficients of the g^2 terms are very small, so that the last two terms of Equation 3-6 may also be ignored. Remaining is the simple equation,

$$\frac{V_o}{R} = K(B + a_{IA} + \Delta a_{PA}) \quad 3-7$$

where Δ is a lumped misalignment comprised of δ_{OA} and any misalignments inherent in the mounting fixture itself. This is the basic equation that is used to determine the accelerometer scale factor and its bias.

3.4 Testing of Tuned Rotor Gyroscopes

The need for the testing of individual gyroscopes is similar to that of testing accelerometers. The goal is to fully characterise the output of the instruments so that any non-ideal behaviour can be modelled and compensated. Once this is done, it can be concluded that any remaining output from the gyroscopes must be a measure of the real input rate. Therefore, a model equation needs to be developed which can be used within a laboratory environment to determine the gyroscope error parameters.

As discussed in Chapter 2, a tuned rotor gyroscope obeys the fundamental law of precession. By definition, the angular momentum of an arbitrary body is given by

$$\mathbf{H} = \sum \mathbf{p} \times m\mathbf{v} \quad 3-8$$

By differentiating the angular momentum vector with respect to inertial space, an expression for the applied moment (torque) on a body can be obtained as a function of angular rates and momentum.

$$\dot{\mathbf{H}}_I = \mathbf{M} = \dot{\mathbf{H}}_B + \boldsymbol{\omega} \times \mathbf{H} \quad 3-9$$

where the subscripts, I and B, refer to an inertial frame or a body centred frame, respectively. In a two degree of freedom gyro, the torques of interest are those acting along the X and Y axis directions. Therefore, Equation 3-9 reduces to the familiar equations for a free gyro,

$$\begin{aligned}
 M_x &= \dot{H}_x + H_z \omega_y = I \dot{\omega}_x + H_z \omega_y \\
 M_y &= \dot{H}_y - H_z \omega_x = I \dot{\omega}_y - H_z \omega_x
 \end{aligned}
 \tag{3-10}$$

where H_z is the angular momentum of the gyro in the spin direction.

Development of the complete mathematical description of a tuned rotor gyro is a lengthy and arduous task that is beyond the scope of this discussion. However, several authors have explored the problem in depth. Notably, the landmark papers by Savet [29] and Craig [23] have formed the basis for most of the analysis of tuned rotor gyros in recent years. Another excellent development can be found in [30] which uses much of Savet's and Craig's work as a starting point, but also includes many system level block diagrams which help to clarify some of the mathematics.

During closed loop testing of a DTG, a rebalance torque is applied to the gyroscope by means of the torquer coils. Therefore, by measuring the current applied to keep the rotor at null, the torque and hence the drift experienced by the gyro can be determined. Similar to accelerometer testing, the model equations for a tuned rotor gyro relate a torquer current to the algebraic sum of the applied input and the error parameters. The general rebalance equations can be written as follows:

$$\begin{aligned}
 K_y I_y &= D(X)_F + D(X)_x a_x + D(X)_y a_y + D(X)_z a_z + \omega_x \\
 &\quad + D(X)_{xx} a_x^2 + [D(X)_{yy} a_y^2] + D(X)_{zz} a_z^2 + \\
 &\quad + D(X)_{xy} a_x a_y + D(X)_{xz} a_x a_z + D(X)_{yz} a_y a_z
 \end{aligned}
 \tag{3-11}$$

$$\begin{aligned}
 K_x I_x &= D(Y)_F + D(Y)_y a_y + D(Y)_x a_x + D(Y)_z a_z + \omega_y \\
 &\quad + [D(Y)_{xx} a_x^2] + D(Y)_{yy} a_y^2 + D(Y)_{zz} a_z^2 + \\
 &\quad + D(Y)_{yx} a_x a_y + D(Y)_{xz} a_x a_z + D(Y)_{yz} a_y a_z
 \end{aligned}
 \tag{3-12}$$

Equation 3-11 fully describes the X input axis characteristics, while Equation 3-12 applies to the Y input axis. Refer to Figure 3-2 for a description of the CSG-2 axes. Note that the left sides of the equations have subscripts which are opposite from the axes with which they are dealing. This is because the X input axis responds to torques about the Y axis, and vice versa. The right side of the equations account for the applied rates, ω , as well as the error parameters affecting that axis. The two second order drift terms contained within square brackets cannot be determined with static 1 g testing and, therefore, must be assumed to be negligible during such tests. Wilkinson [31] has shown that this assumption is valid for good quality tuned rotor gyros. These terms are included for the sake of completeness only.

The notation used in these equations is a common means of expressing all of the error parameters in a similar fashion. However, by convention or for historical reasons, certain terms are given special names. $D(X)_F$ is the absolute bias of the X axis and is often given the name, B_x . $D(X)_x$ is the mass unbalance term discussed in Chapter 2, known more commonly as MU_x , while $D(X)_y$ is the quadrature term, often referred to as Q_x . Similar conventions are followed for the Y axis drift terms. These notations are used interchangeably in the literature.

An important point about Equations 3-11 and 3-12 is that most of the error parameters involve acceleration terms along one of the gyro axes. Therefore, by precisely orienting the gyro with respect to the geographic frame of reference (and hence the gravity vector), it would be possible to eliminate the effect of many of these terms, depending upon the orientation chosen. This is done in multiposition testing. For a given position,

Figure 3-2: Definition of the CSG-2 Gyroscope Axes

Figure 3-3: CSG-2 Torquer Misalignment Angles

the orientation of the gravity vector is known, as is the fraction of earth rate which will be sensed by the gyroscope input axis.

With this in mind, a series of positions is defined as shown in Table 3-1. This is a complete test, comprised of 24 positions, which can be used to determine all of the error parameters given in the general rebalance equations for a dynamically tuned gyro. Often, it is not required to determine all of the parameters in the general equation, and so only a subset of these positions is required. For instance, with a high quality gyro such as the CSG-2, the second order terms (g^2) are often insignificant, and to determine just the bias and g sensitive terms, only eight positions need to be used.

One of the important caveats to static gyro testing is that the initial orientation of the gyro axes should be known. By designing special fixturing, it is possible to conduct a series of preliminary tests which allow for initialisation of the gyro. The procedure for this is discussed in more detail in Chapter 4. However, as an overview, the procedure involves levelling the gyro so that the spin axis is parallel to the gravity vector, and then rotating the gyro case to align the axes in a known position. Consider Figure 3-3, which depicts a two-degree-of-freedom gyroscope whose spin axis is vertical and whose X and Y axes are arbitrarily oriented in the geographic plane as shown. If the gyroscope is stationary in this position, then the only rates sensed by the gyro are due to Earth rate and certain error terms:

$$K_x I_x \sin \alpha + K_y I_y \cos(\alpha + \beta) = B_x + D(X)_s + D(X)_m + \omega_E \cos \lambda \sin \theta \quad 3-13$$

$$K_x I_x \cos \alpha - K_y I_y \sin(\alpha + \beta) = B_y + D(Y)_s + D(Y)_m - \omega_E \cos \lambda \sin \theta \quad 3-14$$

Position			Direction of Axes			Acceleration Along Axis (g)			Earth Rate Input	
#	Az.	Tilt	Spin	X	Y	Spin	X	Y	X	Y
1	90	180	Up	N	W	1	0	0	$\omega_e \cos \lambda$	0
2	0	180	Up	W	S	1	0	0	0	$-\omega_e \cos \lambda$
3	270	180	Up	S	E	1	0	0	$-\omega_e \cos \lambda$	0
4	180	180	Up	E	N	1	0	0	0	$\omega_e \cos \lambda$
5	180	270	N	E	Dn	0	0	-1	0	$-\omega_e \sin \lambda$
6	270	270	N	Up	E	0	1	0	$\omega_e \sin \lambda$	0
7	0	270	N	W	Up	0	0	1	0	$\omega_e \sin \lambda$
8	90	270	N	Dn	W	0	-1	0	$-\omega_e \sin \lambda$	0
9	135	270	N	45° Below E	45° Below W	0	$-\cos \frac{\pi}{4}$	$-\cos \frac{\pi}{4}$	$-\omega_e \sin \lambda$ $\cos \frac{\pi}{4}$	$-\omega_e \sin \lambda$ $\cos \frac{\pi}{4}$
10	225	270	N	45° Above E	45° Below E	0	$\cos \frac{\pi}{4}$	$-\cos \frac{\pi}{4}$	$\omega_e \sin \lambda$ $\cos \frac{\pi}{4}$	$-\omega_e \sin \lambda$ $\cos \frac{\pi}{4}$
11	315	270	N	45° Above W	45° Above E	0	$\cos \frac{\pi}{4}$	$\cos \frac{\pi}{4}$	$\omega_e \sin \lambda$ $\cos \frac{\pi}{4}$	$\omega_e \sin \lambda$ $\cos \frac{\pi}{4}$
12	45	270	N	45° Below W	45° Above W	0	$-\cos \frac{\pi}{4}$	$\cos \frac{\pi}{4}$	$-\omega_e \sin \lambda$ $\cos \frac{\pi}{4}$	$\omega_e \sin \lambda$ $\cos \frac{\pi}{4}$
13	90	0	Down	S	W	-1	0	0	$-\omega_e \sin \lambda$	0
14	0	0	Down	W	N	-1	0	0	0	$\omega_e \sin \lambda$
15	270	0	Down	N	E	-1	0	0	$\omega_e \sin \lambda$	0
16	180	0	Down	E	S	-1	0	0	0	$-\omega_e \sin \lambda$
17	90	225.349	Polar N	(90°-λ) Below N	W	$\sin \lambda$	$-\cos \lambda$	0	0	0
18	0	225.349	Polar N	W	(90°-λ) Above S	$\sin \lambda$	0	$\cos \lambda$	0	0
19	270	225.349	Polar N	(90°-λ) Above S	E	$\sin \lambda$	$\cos \lambda$	0	0	0
20	180	225.349	Polar N	E	(90°-λ) Below N	$\sin \lambda$	0	$-\cos \lambda$	0	0
21	90	45.349	Polar S	(90°-λ) Above S	W	$-\sin \lambda$	$\cos \lambda$	0	0	0
22	0	45.349	Polar S	W	(90°-λ) Below N	$-\sin \lambda$	0	$-\cos \lambda$	0	0
23	270	45.349	Polar S	(90°-λ) Below N	E	$-\sin \lambda$	$-\cos \lambda$	0	0	0
24	180	45.349	Polar S	E	(90°-λ) Above S	$-\sin \lambda$	0	$\cos \lambda$	0	0

Table 3-1: Definition of Gyroscope Axes Positions for a 24 Position Test

Notes:

λ is the local latitude angle, equal to 45.349° for the Navlab

Polar N and S refer to the direction in space of the Earth's spin axis.

Positions 1-8, 10, 12 and 13 are used in the 11 position test.

where I_x and I_y are the measured currents in the X and Y torquers, respectively, and λ is the local geographic latitude. By rotating the gyro fixture in the azimuth plane through an angle of α , it is possible to rewrite the above equations in the following form:

$$K_y I_y \cos \beta = B_x + D(X)_s + D(X)_m + \omega_E \cos \lambda \sin(\theta + \alpha) \quad 3-15$$

$$K_x I_x - K_y I_y \sin \beta = B_y + D(Y)_s + D(Y)_m - \omega_E \cos \lambda \sin(\theta + \alpha) \quad 3-16$$

Using this form, it is possible to solve for β , which is the torquer non-orthogonality angle. Generally, these angles are relatively small, and so the α and β angles are normally ignored during the calculation of the other gyro error parameters, especially for high quality gyros where the error terms themselves are small. When developing a closed form solution for the error parameters, it is acceptable to use Equations 3-11 and 3-12 as a starting point.

Mathematical expressions for multiposition testing have been re-developed by McMath [32], McMath and Yeung [33], Carter [34] and Vinnins et. al. [22] for similar sets of gyro tests. However, certain differences exist between the gyroscopes tested, the equipment available for testing, and the definitions and conventions used during testing. Therefore, equations need to be developed specific to the test set up used to evaluate the CSG-2s for this gravimetry project. These are detailed in Appendix A.

3.5 Testing of the Gravimeter Sensor Blocks

The testing program was not limited to testing and evaluating the inertial instruments on an individual basis; integrated system testing was also necessary. The gravimeter inertial cluster is actually comprised of two separate sensor blocks. The lower sensor fixture (LSF) is an accurately machined, thermally controlled block which houses

both of the CSG-2s, as well as the two QA-2000 accelerometers used to measure horizontal accelerations. The upper sensor fixture (USF) is mounted above the LSF and is thermally isolated from the LSF so that the temperature can be controlled separately and more precisely. Its sole purpose is to hold the QA-3000, which is the accelerometer used to sense the vertical disturbances as well as the other navigation parameters. Mounted on the sensor fixtures are three mirrors, one along each of the principle axes, so that an optical sighting can be used to establish a reference frame.

Although the sensor fixtures are designed to tight machining tolerances, it is impossible to guarantee that the sensor axes themselves are perfectly oriented once the instruments are mounted within the blocks. Therefore, tests must be conducted to solve for the misalignment angles of the sensor axes with respect to a reference frame. In this case, an autocollimator is used to establish the orientation of the mirrors with respect to the table axes (see Section 4.3 for a discussion on autocollimation). Therefore, if model equations are developed which relate the sensor outputs to the table position, there exists a common frame of reference with which to relate sensor axis misalignments to the mirror misalignments. Development of these equations was the goal of Sections 3.5.1 and 3.5.2.

Both the accelerometer and gyroscope misalignment angles are measured relative to the system axes, denoted by X_s , Y_s , and Z_s , as shown in Figure 3-4 and Figure 3-5. The system axes frame is separate from the geographic frame defined by the North, East and Down directions. However, by levelling and aligning the motion table (see Chapter 4), these two frames are coincident in the "home" position of the table. Initially therefore, the X_s axis is aligned with North, the Y_s axis is aligned with East, and the Z_s axis is coincident

Figure 3-4: Gravimeter Gyroscope Misalignment Angles in CMT Initial Position

Figure 3-5: Gravimeter Accelerometer Misalignment Angles in CMT Initial Position

with Down. This is important, since it results in the elimination of an intermediate coordinate frame when developing the instrument model equations.

3.5.1 ACCELEROMETERS

Even though the gravimetry project is based on a stabilised platform design, the sensor blocks can be tested in the laboratory on their own in much the same way that a strapdown system is tested. The blocks are rigidly mounted to the motion table, and the reference mirrors are rigidly mounted on the sensor blocks. Therefore, all misalignments of the accelerometers or the mirrors can be referenced to the system axes which are attached to the fixtures themselves.

The development of model equations defining the accelerometer misalignments is based on the orthogonal axis method, which has been used for strapdown system calibration by Staley [35] and Bolduc [36]. This method is only valid if the test table itself can be accurately aligned to the North geographic reference frame.

The aim of these tests is to express the misalignment angles of the accelerometer input axes with respect to the system axes. Since the only input acting upon the instruments is acceleration due to gravity, it is clear that the following relationship holds:

$$\mathbf{a} = \mathbf{C}\mathbf{g} = \begin{bmatrix} \cos\alpha_x & \cos\alpha_y & \cos\alpha_z \\ \cos\beta_x & \cos\beta_y & \cos\beta_z \\ \cos\gamma_x & \cos\gamma_y & \cos\gamma_z \end{bmatrix} \mathbf{g} \quad 3-17$$

where $\mathbf{C}$ is the direction cosine matrix which defines the misalignments of the input axes. The notation is straight forward. The misalignments of the X input axis with respect to the X, Y and Z system axes are denoted by α_x , α_y , and α_z . Similarly, the Y input axis uses

β_x , β_y , and β_z to define its orientation with respect to the system axes. Finally, the Z accelerometer input axis is specified by γ_x , γ_y , and γ_z . The general orientation of the gravity vector with respect to the system axes can also be specified as follows:

$$\mathbf{g} = (\psi)_z (\phi)_x \begin{bmatrix} \cos\delta_x \\ \cos\delta_y \\ \cos\delta_z \end{bmatrix} \quad 3-18$$

where ψ is a rotation about the azimuth axis of the test table, and ϕ is a rotation about the trunion or tilt axis of the motion table. The δ terms are the direction cosines of the gravity vector with respect to the system axes. In order for the rotations to have a reference point, an initial or starting position is defined for the sensor blocks, where both rotation angles are zero. The orientation of the sensor axes are shown in Figure 3-5. The initial position of the motion table is located where the system axes X_s , Y_s , and Z_s correspond in direction to the geographic directions North, East, and Down, respectively. It is also noted that the Z accelerometer nominally experiences a positive one g excitation in this position.

If the test table is levelled precisely, then the gravity vector is aligned with the Down direction (and hence the Z direction of the system co-ordinate frame in the home position). This makes formulation of the accelerometer model equations straight forward. Each accelerometer input axis has three unknowns associated with its misalignment angles, a scale factor term, and a bias term, all needed to completely describe the instrument output. This requires a minimum of five separate positions in which each accelerometer needs to be positioned. Furthermore, two of these positions should be in nominal positive and negative g orientations to achieve a full range of input excitation.

Given the way in which the accelerometers are installed in the sensor fixtures, a six position test can be devised to fully solve for all fifteen unknowns (three accelerometers with five unknowns each):

$$\begin{matrix} \text{Position 1} \end{matrix} \quad \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} -g \cos \alpha_z + B_x \\ -g \cos \beta_z + B_y \\ -g \cos \gamma_z + B_z \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} \quad 3-19$$

$$\begin{matrix} \text{Position 2} \end{matrix} \quad \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} g \cos \alpha_z + B_x \\ g \cos \beta_z + B_y \\ g \cos \gamma_z + B_z \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} \quad 3-20$$

$$\begin{matrix} \text{Position 3} \end{matrix} \quad \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} -g \cos \alpha_x + B_x \\ -g \cos \beta_x + B_y \\ -g \cos \gamma_x + B_z \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} \quad 3-21$$

$$\begin{matrix} \text{Position 4} \end{matrix} \quad \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} -g \cos \alpha_y + B_x \\ -g \cos \beta_y + B_y \\ -g \cos \gamma_y + B_z \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} \quad 3-22$$

$$\begin{matrix} \text{Position 5} \end{matrix} \quad \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} g \cos \alpha_x + B_x \\ g \cos \beta_x + B_y \\ g \cos \gamma_x + B_z \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} \quad 3-23$$

$$\begin{matrix} \text{Position 6} \end{matrix} \quad \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} g \cos \alpha_y + B_x \\ g \cos \beta_y + B_y \\ g \cos \gamma_y + B_z \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} \quad 3-24$$

Close attention must be paid to the negative signs to ensure correspondence to the definitions of the misalignment angles. The above equations are used to determine the accelerometer misalignment angles with respect to the sensor fixture system axes. Appendix B shows the method used to solve for each of the unknowns.

3.5.2 GYROSCOPES

Testing of the gyroscopes, which are housed in the Inner Gimbal Simulator (IGS), to determine misalignment angles is similar in concept to determination of the accelerometer misalignment angles. Two gyros are mounted in the lower sensor fixture. They form a three axis system to measure rates, with a fourth redundant sensitive axis oriented in the Z system axis direction. Ideally, the three axes sense rates about the X_s , Y_s , and Z_s system axes, respectively. However, small misalignments result in the sensitive axes pointing slightly away from the ideal system axes.

Testing is done in a closed loop manner. Each gyroscope has a rebalance loop which supplies current to the torquers; this current is measured and recorded. The philosophy of testing is almost identical to that of multiposition testing of a single gyro, with one important exception. In single gyro testing, the mounting fixture was designed in such a way as to allow for physical reorientation of the gyro input axis. With the IGS, the positions of the gyros are fixed once they have been installed. Therefore, a slightly different approach in developing a model equation is followed, one which closely resembles the orthogonal axis method used in accelerometer testing in Section 3.5.1.

An initial position is defined for the series of tests, which corresponds to the same initial position used in accelerometer system testing. The system axes X_s , Y_s , and Z_s again initially correspond with the North, East, and Down directions. All rotations of the tilt axis (ϕ) and the azimuth axis (ψ) are referenced from here. The orientations of the gyro input axes relative to the sensor axes are shown in Figure 3-4.

Since many of the major error terms of a tuned rotor gyroscope are dependent upon the effect of gravity (see Section 2.3.2), the model equation has to account for the orientation of the gravity vector with respect to the gyro input axes. Once again, the gravity vector can be defined in geographic co-ordinates as,

$$\mathbf{g} = g \begin{bmatrix} \cos\delta_N \\ \cos\delta_E \\ \cos\delta_D \end{bmatrix} \quad 3-25$$

where δ_N is the angle between the gravity vector and North, δ_E is the angle between the gravity vector and East, and δ_D is the angle between the gravity vector and Down. However, since the system axes is used as the reference axis frame, it is more useful to relate the gravity misalignments to it. Since the system reference frame can rotate with the motion table, it is also useful to include rotation angles in the definition. Therefore, the gravity vector becomes,

$$\mathbf{g} = g(\psi)_z(\phi)_x \begin{bmatrix} \cos\delta_x \\ \cos\delta_y \\ \cos\delta_z \end{bmatrix} \quad 3-26$$

Here, δ_x , δ_y , and δ_z are the angles between the gravity vector and the X , Y , and Z axes, respectively, in the initial position. Since the system axes are coincident with the geographic frame in the initial position, it is evident that the δ angles in Equations 3-25 and 3-26 are equal. Furthermore, if it is assumed that the gravity vector acts principally along the Z or Down direction with negligible horizontal components, then Equation 3-26 can be rewritten as,

$$\mathbf{g} = g(\psi)_x(\phi)_x \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad 3-27$$

Another factor to consider is the component of Earth rate which an input axis senses. It is customary to resolve the Earth rate vector into North and Up components as a function of latitude. If one was to observe the Earth from an inertially stationary point above the North pole, the Earth would appear to rotate in a counterclockwise fashion. Therefore, following the right hand rule, the Earth rate vector would be aligned with the North spin pole. Components of this rate are then resolved as follows:

$$\omega = \omega \begin{bmatrix} \cos\lambda \\ 0 \\ -\sin\lambda \end{bmatrix} \quad 3-28$$

where λ is the local latitude angle. Note that the third component has a negative sign since the co-ordinate frames are positive in the down direction.

Now that the external factors affecting the behaviour of the gyroscope are defined in space, it is necessary to define how the error terms manifest themselves. Consider the orientation of the gyro axes in the IGS when in the initial position (see Figure 3-5). This figure is summarised as follows:

	Gyro X (S/N 020)	Gyro Y (S/N 001)
Spin Axis	South	East
X Input Axis	West	South
Y Input Axis	Down	Down

Table 3-2: Gyroscope axes orientations in the IGS initial position

Gyro X derives its name from the fact that its spin axis is mounted parallel to the X system axis, and Gyro Y has its spin axis parallel to the Y system axis.

From the previous discussion, it is evident that there are many different axes that are referred to by the letters X and Y. Each gyro is either X or Y, each gyro has an X and Y input axis, and the system itself has an X, Y and Z co-ordinate frame. To help eliminate some of this confusion, a new labelling convention is used to refer to the input axes of the gyroscopes when contained within the system itself. This is summarised as follows:

- 1: Gyro X, X input axis
- 2: Gyro X, Y input axis
- 3: Gyro Y, X input axis
- 4: Gyro Y, Y input axis

These numbers are used as subscripts for any variable which refers to an input axis. Using this notation, it is possible to define all of the necessary misalignment angles of the gyro input axes with respect to the system axes. The following conventions are observed.

α : Misalignment between X_s and a given input axis.

β Misalignment between Y_s and a given input axis.

γ Misalignment between Z_s and a given input axis.

Therefore, α_1 refers to the misalignment angle between the X input axis of Gyro X and the X system axis. Similarly, γ_4 refers to the angle between the Y input axis of Gyro Y and the Z system axis. The meanings of the other misalignment angles follow directly from the above definitions.

These misalignment angles are the critical parameters which need to be determined during this set of multiposition tests. However, other error parameters must also be considered when developing these model equations. Since the misalignment angles affect

all of the error terms, it is desirable to eliminate as many of these terms as possible to facilitate solving the equations. Doing this must not significantly decrease the validity of the model. Based on testing of the individual CSG-2 gyros (see test results in Section 5.3.5), it has been judged acceptable to ignore the second order error terms and only to include bias and the g sensitive terms for this stage of testing.

The next issue is to deal with how the misalignment angles affect each of the terms in the equation. The applied rate, in this case Earth rate, is sensed along the input axis. Therefore, the rates applied in the system axis frame as shown in Equation 3-28 need to be transformed into the input axis direction. By definition, the bias is unaffected by the orientation of the input axis, so that misalignment angles do not affect its magnitude. This leaves only the g sensitive terms to be handled.

Consider axis 1 of the system, the X input axis of gyro X . Both the mass unbalance, MU_1 , and the quadrature term, Q_1 , arise due to extraneous torques being generated due to the orientation of the gyroscope (but for different reasons, as explained in Section 2.3.2). MU_1 arises when axis 1 is aligned with the gravity vector, while Q_1 arises when axis 1 is perpendicular to the gravity vector. During multiposition testing, the gyro is normally positioned so that only one of MU , Q , or the spin axis drift term affects a given axis. This simplifies analysis greatly. However, once input axis misalignment angles are considered, the situation becomes more complicated. Once the effect of the g sensitive terms are transformed into the input axis direction, the equations become non-linear. This is because terms arise in the model equation which are the product of the g sensitive terms

and the misalignment angles. For example, in developing the general form for the MU_1 term to be used in the model equation, the following formulation would apply:

$$MU(\phi)_x(\psi)_z(C_M)(C_T)(g) \quad 3-29$$

ϕ and ψ are rotations of the motion table, which are known parameters. C_T is a transformation matrix used to determine the direction of the torque vector which causes the drift term, based on the orientation of the gravity vector, g . C_T can be predetermined; however, g in its general form (see Equation 3-18) may be comprised of three direction cosines which need to be solved. C_M is the input axis misalignments, which need to be solved, and MU is the scalar term for the mass unbalance quantity, which is also unknown. As is evident from Equation 3-29, the product of C_M and MU results in a non-linear equation. C_M also appears with similar terms containing Q or the spin axis drift term, and a fourth term which defines the sensed Earth rate. Furthermore, if g is left in the general form of Equation 3-18, then the equations for each of the gyro input axes are coupled together. Therefore, it is not possible to solve for the error terms of axis 1 without also simultaneously solving for axes 2, 3 and 4. Since each axis has 8 unknowns (three misalignment angles, scale factor, bias, and three g sensitive terms), along with three unknowns for the direction of the gravity vector, a minimum of 35 non-linear equations in 35 unknowns are required. This is a very difficult task even with modern numerical methods, and with many of the parameters to be determined having near zero values, numerical solutions become sensitive to initial conditions.

With this in mind, it is advantageous to assume that the gravity vector is perfectly aligned with the Z axis direction. As previously mentioned, this assumption is valid since

the table has been accurately positioned and levelled prior to testing. Not only does this lead to three fewer unknowns to solve for, but it also instantly decouples the model equations for all four axes. Instead of 35 equations in 35 unknowns, the situation becomes 4 sets of 8 equations in 8 unknowns. Unfortunately, each set still includes 8 non-linear equations which can be difficult to deal with.

In order to linearize the equations, a further simplification is needed. It is possible to assume that the misalignment angles have little effect on the value of the error term in question. Making this assumption, Equation 3-29 simplifies to,

$$MU(\phi)_x(\psi)_z(C_T) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad 3-30$$

The only unknown is the scalar MU. A similar argument can be made for the other g sensitive terms. While this greatly simplifies the situation, it is reasonable to question the validity of this assumption. Justification for this step can only be demonstrated by examining the physical meanings and magnitudes of each of the parameters. First of all, the misalignment angles of the input axes with respect to the system axes are assumed to be small. This is a realistic assessment, given the quality of the CSG-2 gyros and the accuracy to which the mechanical fixtures are machined. One would not expect misalignment angles any larger than 10 arc minutes at the worst. Second, the magnitudes of the g sensitive terms themselves are only one or two deg/hr/g (see Section 5.3.5). This is an order of magnitude smaller than the applied rate (i.e. Earth rate). Combining these two facts, it can be concluded that the error introduced by the omission of the misalignment angles is

$$\Delta = MU \cdot \theta = 2 \frac{\text{deg}}{\text{hr/g}} \times \frac{10}{60} \text{deg} \frac{\pi \text{ rad}}{180 \text{ deg}} < 0.006 \frac{\text{deg}}{\text{hr/g}} \quad 3-31$$

where Δ is the error introduced, and θ represents an arbitrary misalignment of the input axis. Worst case scenarios are taken for this calculation. Since this rate is on the same order of magnitude as the random drift of the gyro (see Section 5.3.7), it would be very difficult to differentiate between noise and the effect of the misalignment angles on the g sensitive terms. Therefore, it is reasonable to neglect the effect of the misalignment angles on the g sensitive drift terms.

It is now possible to write equations that fully define the outputs for each of the positions chosen for the multiposition test of the IGS gyros. Table 3-3 lists the orientations of the sensor axes in each of the positions. While there are only eight unknowns for each input axis, implying that only eight positions need to be defined, a slightly different approach is taken. First, an additional constraint is placed upon the solution. Since the misalignment angles are direction cosines defining the orientation of the input axes with respect to the system axes, the following relationship must hold:

$$\sqrt{(\cos \alpha_n)^2 + (\cos \beta_n)^2 + (\cos \gamma_n)^2} = 1 \quad 3-32$$

where the subscript n refers to a general gyro input axis. This condition must be imposed to ensure that the solution is valid and falls within the real plane. This brings the minimum number of equations necessary up to nine. However, the actual number of positions used during testing was 16. This is done for a couple of reasons. First, by using positions that are negative complements, solving for the unknowns becomes straight-forward. Furthermore, by using more positions, more information is incorporated into the solution.

Position			Direction of Axes CSG-2, Serial # 020			Direction of Axes CSG-2, Serial # 001		
#	Az.	Tilt	X	Y	Spin	X	Y	Spin
1	0	180	West	Down	South	South	Down	East
2	270	180	South	Down	East	East	Down	North
3	180	180	East	Down	North	North	Down	West
4	90	180	North	Down	West	West	Down	South
5	0	0	West	Up	North	North	Up	East
6	270	0	North	Up	East	East	Up	South
7	180	0	East	Up	South	South	Up	West
8	90	0	South	Up	West	West	Up	North
9	0	270	West	South	Up	Up	South	East
10	270	270	Up	South	East	East	South	Down
11	180	270	East	South	Down	Down	South	West
12	90	270	Down	South	West	West	South	Up
13	0	90	West	North	Down	Down	North	East
14	270	90	Down	North	East	East	North	Up
15	180	90	East	North	Up	Up	North	West
16	90	90	Up	North	West	West	North	Down

Table 3-3: Definition of Gyroscope Axes Positions Within the Inner Gimbal Simulator for a 16 Position Test

Notes: Position numbers not the same as test defined in Table 3-1
 Positions sufficient to determine misalignment angles, scale factor, bias and
 g sensitive drift terms.

This helps to eliminate any errors which might occur due to small misalignments of the table; incorporating multiple positions 180° apart tends to average out any errors introduced due to table misalignment.

The following are the equations used to solve for the unknowns from multiposition testing. The method for obtaining explicit solutions for each unknown is presented in Appendix C.

$$\text{Position 1} \quad k_1 [\omega_e \cos \lambda \cos \alpha_1 - \omega_e \sin \lambda \cos \gamma_1 + B_1 + Q_1] = I_1 \quad 3-33$$

$$\text{Position 2} \quad k_1 [\omega_e \cos \lambda \cos \beta_1 - \omega_e \sin \lambda \cos \gamma_1 + B_1 + Q_1] = I_2 \quad 3-34$$

$$\text{Position 3} \quad k_1 [-\omega_e \cos \lambda \cos \alpha_1 - \omega_e \sin \lambda \cos \gamma_1 + B_1 + Q_1] = I_3 \quad 3-35$$

$$\text{Position 4} \quad k_1 [-\omega_e \cos \lambda \cos \beta_1 - \omega_e \sin \lambda \cos \gamma_1 + B_1 + Q_1] = I_4 \quad 3-36$$

$$\text{Position 5} \quad k_1 [-\omega_e \cos \lambda \cos \alpha_1 + \omega_e \sin \lambda \cos \gamma_1 + B_1 - Q_1] = I_5 \quad 3-37$$

$$\text{Position 6} \quad k_1 [-\omega_e \cos \lambda \cos \beta_1 + \omega_e \sin \lambda \cos \gamma_1 + B_1 - Q_1] = I_6 \quad 3-38$$

$$\text{Position 7} \quad k_1 [\omega_e \cos \lambda \cos \alpha_1 + \omega_e \sin \lambda \cos \gamma_1 + B_1 - Q_1] = I_7 \quad 3-39$$

$$\text{Position 8} \quad k_1 [\omega_e \cos \lambda \cos \beta_1 + \omega_e \sin \lambda \cos \gamma_1 + B_1 - Q_1] = I_8 \quad 3-40$$

$$\text{Position 9} \quad k_1 [-\omega_e \sin \lambda \cos \alpha_1 - \omega_e \cos \lambda \cos \gamma_1 + B_1 + D(X)_{s1}] = I_9 \quad 3-41$$

$$\text{Position 10} \quad k_1 [-\omega_e \sin \lambda \cos \beta_1 - \omega_e \cos \lambda \cos \gamma_1 + B_1 - MU_1] = I_{10} \quad 3-42$$

$$\text{Position 11} \quad k_1 [\omega_e \sin \lambda \cos \alpha_1 - \omega_e \cos \lambda \cos \gamma_1 + B_1 - D(X)_{s1}] = I_{11} \quad 3-43$$

$$\text{Position 12} \quad k_1 [\omega_e \sin \lambda \cos \beta_1 - \omega_e \cos \lambda \cos \gamma_1 + B_1 + MU_1] = I_{12} \quad 3-44$$

$$\text{Position 13} \quad k_1 [\omega_e \sin \lambda \cos \alpha_1 + \omega_e \cos \lambda \cos \gamma_1 + B_1 - D(X)_{s1}] = I_{13} \quad 3-45$$

$$\text{Position 14} \quad k_1 [\omega_e \sin \lambda \cos \beta_1 + \omega_e \cos \lambda \cos \gamma_1 + B_1 + MU_1] = I_{14} \quad 3-46$$

$$\text{Position 15} \quad k_1[-\omega_e \sin \lambda \cos \alpha_1 + \omega_e \cos \lambda \cos \gamma_1 + B_1 + D(X)_{11}] = I_{15} \quad 3-47$$

$$\text{Position 16} \quad k_1[-\omega_e \sin \lambda \cos \beta_1 + \omega_e \cos \lambda \cos \gamma_1 + B_1 - MU_1] = I_{16} \quad 3-48$$

I_n is the torquer current measured in position n . The scale factor, k_1 , is associated with the X input axis of gyro X; however, it is actually a property of the Y axis torquer. The above numbering scheme has been adopted to keep all subscripts for a set of Equations uniform.

One final word about the meaning of the misalignment angles defined in equations 3-33 to 3-48. Recall that by the fundamental law of precession, torques cause a body with angular momentum to precess such that the rate vector is at right angles to the torque vector. Since the equations for axis 1 (X input axis, gyro X) only include references to the associated Y torquer, the input rates must be being sensed at exactly 90° to the torquer. Therefore, the misalignment angles α_1 , β_1 , and γ_1 are actually effective input axes responding to rates applied perpendicular to the torque. By defining an orthogonal coordinate frame, these misalignment angles can actually express not only the effective input axes, but also the associated torquer misalignment direction cosines. This in fact is what needs to be calculated in a gimbaled system. By providing the appropriate error compensation, the gyroscope behaves as an ideal body with angular momentum and, therefore, remains fixed in inertial space, exactly the effect which is being sought after in a platform system.

3.6 Summary

Model equations are used to define the behaviour of inertial instruments. By accounting for any error terms and compensating for them, the actual physical quantity being sensed can be extracted. These error parameters can only be tested for within a

highly controlled environment, such as an inertial test laboratory, where precise positioning and rates can be applied to the instrument.

Two separate stages of testing are conducted for both gyroscopes and accelerometers. The instruments are tested on an individual basis and then as part of a system which eventually is installed in the airborne gravimeter. Equations have been developed which model all of these situations. The equations can then be used to extract the error parameters from the gyroscopes and accelerometers.

4. Inertial Instrument Testing

4.1 Introduction

This chapter describes the method of testing accelerometers and gyroscopes in detail, as well as the rational behind the design of the test procedures and set-up. Specific discussions about the equipment used in the Defence Research Establishment Ottawa Navigation Laboratory (Navlab) are made. Issues which affect the accuracy of the testing are addressed.

4.2 Accelerometer Testing

4.2.1 TEST EQUIPMENT

The Navlab accelerometer test station is centred around a massive granite slab, providing an exceedingly flat surface on which to define a reference plane. The accelerometer itself is mounted in a test fixture, which is in turn mounted on a dividing head. Support electronics are required to drive and monitor the accelerometer signals. Optical equipment such as autocollimators and mirrors are used to establish a reference frame. Refer to Figure 4-1 and for an overview of the test set-up.

Granite Table

A Starrett Crystal Pink Granite slab was used as the test surface. This stone block, measuring 36" x 48" x 8", is specified to have surfaces which are parallel to better than 3 arcseconds [37]. The slab is supported by rubber pads and rests on a metal frame. Normally, three points are used to define a plane; however in this case, four rubber pads needed to be used in order to properly balance the slab. These rubber pads also serve to isolate the table from vibrations in the floor. The metal frame rests on the floor on four 1-1/16" bolts which are used to level the table.

Dividing Head

The optical dividing head serves as a mount to rotate the accelerometer test fixture in and out of the gravity vector. This instrument was originally designed for precision machining work on lathes. By using the coarse adjustment, the dividing head face can be rotated through a full 360°. The fine adjustment permits precision positioning to better

than 3 arcseconds. The base of the dividing head is machined flat so that it rests flush on the granite slab.

Accelerometer Test Fixture

An aluminum mounting fixture was used to house the accelerometer and to mount it to the dividing head. The fixture is designed to hold the accelerometer such that the input axis is parallel to the dividing head plane. Four cartridge type heaters were inserted into the mounting holes in the block, as well as two RTD temperature monitors. These components are used in conjunction with a TC-300 temperature controller to regulate the temperature of the aluminum block. A surface mount mirror is attached to one side of the fixture as a reference plane for autocollimation.

TC-300 Temperature Controller

The Dynamics Research TC-300 temperature controller is a three channel device designed to proportionally control the temperature of inertial components to better than $\pm 0.1^{\circ}\text{F}$ according to factory specifications [38]. Originally designed for use with a thermistor, it has been modified to accept an RTD for its sensor input. Two RTDs are used in normal operation, one for control and one for monitoring. The control sensor forms one arm of a DC bridge. The opposite arm is a potentiometer that is set to a predetermined resistance, corresponding to the operating temperature. The temperature control circuit delivers power to the heaters such that the RTD and control potentiometer resistances balance. The second RTD is used to monitor the relative temperature of the aluminum block.

Figure 4-1: Accelerometer Test Station

Figure 4-2: Accelerometer Autocollimation Set-up

DC Power Supplies

A dual Anatek 6030 power supply was used to supply the accelerometer with ± 15 volts DC. This supply was chosen for its long term stability (drift of 0.02% over 8 hours), and its low noise and ripple characteristics (1 mV RMS) [39]. Low quality power supplies are often a source of noise in accelerometer outputs, resulting in degraded performance.

DATRON Multimeters

Two Datron 1071A multimeters were used for data collection. One was used to monitor the accelerometer output, while the other monitored the internal temperature of the accelerometer. Since both of these outputs were currents, a Julie Research Labs 1 k Ω resistor was used for each channel to scale the currents into a voltages. These resistors are noted for their high stability and relative insensitivity to atmospheric temperature variations. Each Datron meter is a 7.5 digit multimeter, enabling measurement of changes in accelerometer output of less than a micro-g. The Datron meters are equipped with an IEEE 488.1 compatible bus port, which allows for automated data collection using custom data logging routines on a PDP/11-73 microcomputer. The ability to automatically record data greatly facilitated data collection. Furthermore, the IEEE 488.1 architecture allowed for multiple instruments to communicate on the bus at one time, which meant that accelerometer outputs and temperatures could be recorded simultaneously.

Autocollimator

An autocollimator is an optical device which can be used for aligning a reference mirror to a known orientation. The PA-50 autocollimator used in the Navlab generates a precisely collimated beam of light that is split into two parallel but opposite paths. One

beam stays internal to the autocollimator and serves as a reference. It appears in the centre of the field of view of the ocular as a point of light. The second beam is aimed at the target mirror. If the mirror is nearly perpendicular to the light ray, it is reflected back into the housing of the autocollimator and is visible in the ocular as a second point of light. Adjustment of the mirror until the two light points are coincident ensures that the mirror is perpendicular to the beam of light.

Vertical Levelling Mirror

The vertical levelling mirror (VLM) is a precision instrument which is used to determine the local vertical. It consists of a frame and a base that can be levelled using adjustment screws. A circular bubble level indicates if the base is approximately level. A locking mechanism is then released, allowing the mirror to hang freely within the frame from two fine, gold threads. This allows the mirror to align itself to the local gravity vector, provided it is protected from disturbing air currents.

Bubble Levels

Bubble levels, also known as spirit levels, are used to determine the flatness and levelness of a surface. The levels used vary from coarse measurements with resolutions of only 200 arcseconds per division, to very accurate levels capable of measuring angular deviations of 3 arcseconds.

4.2.2 TEST PROCEDURES

4.2.2.1 Preliminary Considerations

The purpose of accelerometer testing was to fully characterise the instrument's behaviour by investigating a number of different parameters. First, scale factor and bias needed to be determined for use in the model equation. Second, the effect of thermal variations on the accelerometer needed to be investigated. Finally, in order to prove that the Q-Flex accelerometers were capable of measuring changes down to one micro-g, it was necessary to conduct resolution test.

Prior to testing, it was necessary to install the QA-2000 into the test fixture. The mating surfaces of the accelerometer and the aluminum block were cleaned with a degreasing agent to remove contaminants. Silicone heat transfer compound was applied to the mounting screws that attach the QA-2000 to the aluminum block to ensure that heat could travel easily from the aluminum block into the QA-2000. The three screws were then torqued down evenly to hold the accelerometer in place without causing unequal stressing of the mounting flange

With the accelerometer installed, the aluminum block was mounted onto the dividing head face. Similar care was taken to ensure that the mounting surfaces were clean. Quartz washers were used to isolate the fixture from the dividing head. This is important since the whole aluminum block is heated and the dividing head face would act as a heat sink and bleed the thermal energy away from the accelerometer mounting fixture. This could cause thermal stress on the dividing head, but more importantly would cause a

dramatic increase in the power requirements of the heaters. Quartz washers were chosen since they can be precision ground to be flat to within arcseconds.

The test set-up was wired together making extensive use of breakout panels. This enables individual components of the set-up to be removed with greater ease, and allows for easy access to signal and power lines. The wire leads from the dividing head breakout to the table breakout were given extra slack to allow for full 360° rotation of the dividing head during testing.

The dividing head was positioned on the granite surface in the orientation in which it would be used during testing. Using the bubble levels, the granite table was levelled with the shimming bolts at the base of the table frame. Levelling can be achieved to the resolution of the most precise level (3 arcseconds). Once the table had been adjusted, it was important not to add any significant weight to the test surface, as this would compress the rubber supports and shift the level of the slab.

4.2.2.2 Alignment Determination

The reference mirror on the aluminum block is used to establish the orientation of the local gravity vector. The test equipment is set-up on the granite slab as shown in Figure 4-2. The autocollimator is positioned at the same height as the reference mirror and the vertical levelling mirror (VLM) is then positioned between the two instruments. By releasing the locking mechanism on the VLM, the mirror is suspended from two fine gold threads. This allows the mirror to align itself with the local gravity vector. The orientation of the autocollimator is adjusted using a tip tilt fixture until autocollimation off the VLM is achieved. This establishes a light beam which is perpendicular to the gravity

vector. After removing the VLM, the position of the dividing head is adjusted until autocollimation is achieved and the test fixture mirror is aligned to local vertical. The angle of the dividing head is recorded and used as the zero mark during testing. This alignment procedure was repeated several times to gauge the accuracy and repeatability of the method.

4.2.2.3 Accelerometer Testing

Testing investigated several different aspects of accelerometer behaviour. First, the output was characterised by measuring scale factor, bias, and misalignment. Next, effects of temperature on accelerometer output was determined. Finally, the ability to resolve small changes in input was investigated.

Accelerometer characterisation is accomplished through a series of static multiposition tests. Rotating the dividing head through a full 180° will excite the accelerometer with a range of inputs from positive one g to negative one g. By recording the accelerometer outputs in different angular orientations, it is possible to determine scale factor in mA/g, and misalignment.

The number of positions chosen for testing was varied. A quick multiposition test could be conducted using only 4 positions (full positive and negative g input, and both zero input positions). More extensive tests involved using 20, 30 or 40 different positions, equally spaced over a 360° interval. For each data point, the dividing head was manually rotated to its new position and then the accelerometer output and temperature were recorded. The length of time that data was collected at a given position was varied. Using this static data, the accelerometer error terms could be determined.

Temperature can have a significant affect on inertial components. As a result, it is necessary to either control accelerometer temperatures to very tight tolerances, or use a thermal compensation model to correct the accelerometer outputs. In either case, both temperature control stability and temperature sensitivity are important parameters to determine.

Temperature stability is a direct indication of the quality of the temperature controller. By monitoring the temperature over a long period of time, the ability of the controller to hold a specific temperature is measured. In this case, the operating temperature was 68.4°C. In order to lessen the effects of ambient temperature fluctuations, temperature stability tests were also done with the test fixture in a thermal control chamber. This chamber was used to hold the ambient air at a constant 63°C, which means that the TC-300 has to handle fewer changes in the surrounding environment.

Temperature sensitivity was also measured. The QA-2000 was held in a constant orientation, while the block temperature was varied over a 5°C range centred at the operating temperature. Accelerometer temperatures were allowed to stabilise at each new set point before data collection began.

Finally, an important aspect of accelerometer performance is to determine how small a change in acceleration it can resolve. This is especially true for the gravimetry project, since changes of one micro g must be detected. With the accelerometer nominally sensing one g, data collection began. The dividing head was then rotated by a small amount which corresponded to a known change in gravity input (see Appendix D).

Changes in accelerometer output indicate that the instrument is capable of resolving these changes in applied input.

4.3 Gyroscope Testing

4.3.1 INTRODUCTION

The CSG-2 is a two axis, dynamically tuned gyro which is being used to determine attitude and stabilise the gravimetry platform. During testing, two separate methods of controlling the gyro were used. To simulate gyroscopic control of the platform, servo mode was used in which a CSG-2 was used to command one axis of the motion simulator. Alternatively, the gyroscope can be operated using an electronic rebalance loop to control the gyroscope. Conceptually, these are two very different forms of testing a gyroscope. Two separate testing phases were conducted for each gyro. During the first phase, the gyroscopes were tested to determine their suitability for use in the gravimetry project and provide baseline performance data. This required use of existing gyroscope support electronics, which are known to perform to high standards. The second phase saw characterisation of gyroscope performance using the electronics which will be used in the final system. This accomplished two goals. First, gyroscope performance can be affected depending on the support electronics used, so that parameters such as drift and noise can change if the electronics are changed. Second, since the electronics to be used in the gravimetry system were new designs, it was necessary to verify their operation and reliability. Therefore, almost all of the same tests were repeated in both phases, with the only difference being which support electronics were used to excite the gyroscope.

4.3.2 TEST EQUIPMENT

The DREO Navlab is specially equipped to do motion testing of inertial instruments. In fact, it is the only facility of its kind in Canada [40]. The overall layout and test components are illustrated in Figure 4-3. The laboratory has been developed so that data collection has been automated where ever possible.

The Contraves Motion Table

At the heart of the Navlab is the motion simulator manufactured by Contraves Goerz. This motion table is a model 57CD, two axis table, which is controlled by a 30H MPACS level III command system (Modular Precision Angular Control System).

The Navlab currently has two model 57CD tables, which are similar in all respects except for one. Station number 1 is capable of full positioning and rating about two orthogonal axes. Station number 2 has full capability about the azimuth axis, however the trunion (or tilt) axis is limited to positioning only. The trunion axis positioning is constrained to $\pm 180^\circ$, which indicates that mechanical stops prevent full 360° motion of this axis. In all other respects, the tables perform and behave the same. For the majority of gyroscope testing, Station number 2 was used and all future references to the Contraves Motion Table will be with respect to Station number 2.

The tables have been surveyed in place, so that the principle axes of the motion table align with the geographic cardinal directions. (See Section 4.3.3.1 on system testing for more discussion on optical surveying.) Alignment of the table is important in determining what fraction of earth rate is being sensed in a given table orientation. The table itself is capable of positioning accuracy of ± 1 arcsecond, with a resolution capability

2

PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

PRECISIONSM RESOLUTION TARGETS

Figure 4-3: Navlab Layout

of 0.36 arcseconds (0.0001°). The table's rotational motion is supported by mechanical ball bearings, which result in the table axes wobbling by less than ± 3 arcseconds through the course of 360 degrees of rotation. In rate mode, the table is capable of maintaining rates from 0.001 deg/sec all the way to 999.9 deg/sec [41]. This high rate capability is of little interest for the purpose of the gravimeter tests. In addition to rate and position modes, the motion table is also capable of operating in Gyro Servo mode, which allows a gyroscope output signal to be used to drive the table and keep it essentially in an inertial reference frame.

A rate trip switch is included on both axes, which will disconnect the table servos should a pre-set rate be exceeded. This serves as protection for the gyroscope or other instruments on the test table. Note that even though the turnion axis is incapable of rating, it can slew from position to position, and so also includes a rate trip mechanism.

One important aspect of testing inertial components on a motion simulator involves bringing electrical signals off of the table top to rack mounted equipment. This is accomplished through the use of slip rings to pass signals from the rotating table top through cables to the instrumentation racks. The majority of these signal lines are 1 amp, but there are some which are 5 amp rated, specifically for power transmission.

Gyroscope Power Supplies

There are three different types of power supplies required for use in operating a CSG-2 gyroscope. The performance and stability of these supplies is critical. First, the wheel supply (also known as the motor driver or spin supply), is used to supply power to the gyroscope motor. Since each TRG has it's own tuned frequency (See section 2-3), it

is necessary to provide a synchronisation frequency so that the three phase power drives the gyroscope at the right speed. For the first phase of testing, the rack mounted NH Research, Model SF1250W-2 Sine/Square Wave Wheel supply was used. This is a multipurpose gyroscope motor driver, capable of delivering two or three phase power, in either a sine or square wave, with variable voltage levels. An external frequency source, in this case an Adret frequency generator, is used to synchronise the wheel supply output voltage to the proper frequency. The supply is capable of operating in either start or run mode, in which start mode provides variable voltage control to 100% to 150% of the normal operating voltage. This is subject to operator control.

For the second phase of the testing, the Sander Geophysics Limited (SGL) Motor Driver Board was used. This board was specifically designed for use with the gravimetry project, but it still provides some degree of flexibility. The excitation frequency can be set using a keypad interface to the SGL control software. It is capable of delivering two different voltages, for start and run conditions, respectively. Currently, these are chosen by adjusting external DC power supplies, which are used as sources for the motor driver board to produce the three phase. The board is only set up to deliver three phase, sinusoidal power.

Another important supply is known as the microsyn, signal generator, or pickoff driver supply. This supply is used to excite the gyroscope pickoffs by supplying a reference voltage to the primary coils which produces an output in the secondary coils, proportional to the rate being experienced by the gyroscope. The first phase of the testing used the rack mounted NH Research Model SF2599 power source. A single phase AC

output is delivered, which can vary from 1 to 80 volts RMS and range in frequency from 225 Hz to 60 kHz allowing it to be used on a wide range of gyroscope signal generators. The second phase saw use of the Sander Geophysics pickoff driver. This board is hard wired to deliver a sine wave with an RMS voltage of 5V, at a frequency of 53.4 kHz. No direct means are available for changing these values, although the board itself could be altered to operate at different levels. Both the motor and the pickoff driver have been designed to be highly stable, with low noise and drift rates. A third power supply is required to deliver ± 15 VDC to the gyro pre-amp board, but its performance is not critical.

Test fixturing

A specially designed fixture is used to attach and position the CSG-2 gyroscope on top of the Contraves Motion Table. The table top mounting plate is a circular, 14" diameter, aluminum disc, that is mounted on the table's azimuth axis using seven bolts. A three sided steel fixture is mounted to this plate, in which the three sides are machined to accuracies of a tenth of a thousandth of an inch, so that the three sides of the fixture form a corner section which is almost perfectly orthogonal. This three sided fixture provides a mount for a three degree of freedom fixture designed to hold the CSG-2 firmly in place (see Figure 4-4). The orientation of the tip-tilt fixture can be changed using a vernier adjustment. In this way, the gyroscope itself can be levelled. Since the fixture can also rotate about its Z axis, the gyroscope can be positioned within the azimuth plane.

The tip-tilt fixture is fitted with four cartridge heaters embedded within the aluminum housing around the gyroscope. Two RTD type temperature monitors are fitted into the aluminum block as well. The temperature is controlled using the same type of

Figure 4-4: Three Sided Fixture with Tip/Tilt/Rotary Fixture for mounting CSG-2

Figure 4-5: Schematic Representation of CSG-2 Breakout Panel Set-up

controller (TC-300) as was used in the accelerometer testing. In addition, a rack mounted Fluke 2180A digital thermometer is used for temperature data collection over the IEEE bus.

Breakout Panels

In the test and development phase of a project, it is important to be able to easily identify and troubleshoot signals that go to and from the gyroscope. Also, if one component of the test set-up needs to be removed or replaced, it should be possible to do this without dismantling the entire set-up. To this end, it has been found that the use of breakout panels addresses all of these basic concerns. A breakout panel is a box or collection of terminal strips, where signals are brought together to be re-routed. In the gyroscope test set-up, a number of different breakout panels exist. Starting from the gyroscope itself, there are a collection of terminal strips on the three sided fixture. These lead to cables which plug into a table top breakout panel. This panel has test points, so that every signal can be monitored or verified before the signals are passed through connectors to the slip rings. On the instrument rack, a rack mounted breakout panel serves as a common connection point between cables from the table and the rack mounted instruments (see Figure 4-5 for a schematic representation of this layout). This modular design allows for easy removal of components, and also allows breakout panels to be reused for other tests with minimal or no modifications.

Data Monitoring and Collection

Conducting tests is pointless without a means of monitoring what is happening and collecting that information in some manner. A number of different instruments were used for these purposes.

One of the most useful methods for observing behaviour of a tuned rotor gyroscope is to use an oscilloscope to monitor the pickoff outputs using a Lissajous pattern. The pickoff primary voltage is used as the X axis reference, while the pickoff secondary output is used as the Y axis plot. If the gyroscope is operating in the nulled condition, then the Lissajous pattern will appear as a flat horizontal line. If the rotor is not at null, the oscilloscope will display an open oval. Using this method of visualisation, it is easy to imagine the Lissajous pattern as being analogous to rotor motion. Therefore, an oscilloscope is always used to monitor the behaviour of the gyroscope pickoff outputs.

Temperature is constantly monitored during testing. The Fluke thermometer displays gyroscope temperature, whether the instrument is running or not. This temperature can be logged to a computer file if desired (see below). The TC-300 provides the experimenter with an additional visual readout, showing the deviation from the temperature set point.

The PDP-11/73 microcomputer, manufactured by Digital, has a number of custom built tasks for data collection within the Navlab. All data collection and automation is achieved through use of the IEEE 488.1 instrument bus (also known as the GPIB or HPIB bus). Central to this data recording capability is the Motion Table Language (MTL), a simple, structured method of controlling the motion table while at the same time activating

data collection routines. Commands can be initiated either through an interactive keyboard dialogue, or by writing a batch file which will execute a series of MTL commands. In this way, the table can be commanded to enter different modes of operation, such as rating or positioning, while other tasks can be used to collect data from the gyroscopes or other instruments on the table top.

Three data collection tasks are used in gyroscope testing. The LOGTBL task is used to record table position and rate, on either axis. It is also capable of recording temperature from one Fluke thermometer. The LOGTMP task records temperature from up to four thermometers at a time. It is used if the temperature logging ability of LOGTBL is not sufficient. The LOGDAT routine is used to collect data from up to four Datron 1071A multimeters at a time. Voltage, current and resistance can be measured and recorded using this task. The Datrons are the standard method of recording gyroscope data, since by recording torquer voltage in closed loop mode, gyroscope behaviour can be determined with post processing.

All three of these tasks are designed to sample as fast as data points are available from the instruments. This rate can vary due to bus loading, which is caused by multiple instruments trying to use the communications bus at the same time, as well as CPU loading, in which multiple tasks each demand CPU time and therefore slow down operation of an individual data collection task. Generally, data collection is between 1 Hz and 3 Hz, depending on the above factors, and the combination of individual task being used.

The LOGA2D task was required for gyroscope resolution tests to determine if the gyroscope could detect small angular changes. This task collects data from the PDP-11 based analogue to digital 14 bit converter. In burst mode, the LOGA2D task can collect data points at a sample rate of between 6.5 kHz and 9 kHz. Due to CPU constraints, it is not possible to run other tasks while using the A/D board.

4.3.3 TEST PROCEDURES

4.3.3.1 Preliminary Considerations

Before the gyroscope is mounted on the table top, it is necessary to ensure that the motion table is levelled. Similar to the granite slab, the motion table rests on the floor on three levelling screws. Using a wrench, the table can be adjusted in any pitch or roll direction. Once again, spirit levels are used, starting from a coarse approximation with the 200 arcsecond levels, and ending up with the 10 and 3 arcsecond levels for fine adjustment. Levelling of the table is somewhat more involved than for the granite slab. Since the azimuth axis can rotate, there exists the possibility of coning of the azimuth axis. To adjust for this, shimming of the mounting plate and adjustment of the tension in the mounting screws can be done. For this stage of testing (as opposed to system testing), coning is not an over-riding concern, since the tip tilt fixture itself can compensate for coning effects.

Another consideration is balancing of the table. The Contraves Motion Table is inherently bottom heavy about the tilt axis and neutrally stable about the azimuth axis. By mounting fixtures and breakout panels on the table top, this will alter the mass properties

of the table. Even though the table is actively controlled in position mode through the use of servos, it is still desirable to reduce the work load of the electronic control system in the MPACS by balancing both axes so that they are neutrally stable with all of the fixturing in place. The motion table is equipped with 5/8" studs, on which counter weights can be mounted to balance the trunion axis. The breakout panel is equipped with mounting holes so that lead weights can be mounted to balance the azimuth axis. There is no set procedure for achieving a balanced table. It relies upon a common sense iterative approach, in which weights are progressively added to the azimuth and the trunion axes until both are capable of maintaining any arbitrary orientation, simply by the stiction within the bearings.

4.3.3.2 Levelling and Alignment of the CSG-2 Gyroscope

The first step in gyroscope testing is to establish the orientation of the instrument. This is a two stage process. First, the gyroscope is levelled, so that the X and Y axes lie in the horizontal plane, and then the gyroscope is aligned, so that these axes point in known directions.

Levelling is achieved by mounting the gyroscope with the spin axis parallel to the table's azimuth axis, through use of the three sided fixture. The gyroscope is turned on and run at the operating temperature of 68.4°C in open loop mode. That is, no method of controlling the gyroscope, either by use of a rebalance loop or a servo mode is used. The table is then put into rate mode, at a rate that is substantially larger than earth rate, around 30 to 50 degrees per second. The 54 kHz pickoff outputs will be observed to oscillate sinusoidally according to the angular position of the table top. The pickoff outputs can be

monitored using an oscilloscope, but for this procedure, a strip chart recorder has been found to be more useful, since the behaviour over several table rotations can easily be observed. As the table rotates, the sine wave output of the gyroscope pickoffs will oscillate about some average voltage. Figure 4-6 helps illustrate the physical reason for this behaviour. The table rotation and Earth rate sum to a resultant angular velocity, ω_r , being experienced by the case of the gyro. By making ω_r sufficiently large, the effect of Earth rate becomes negligible so that ω_r is effectively the table rate. As the case rotates through 180° , the X pickoff pair changes positions in inertial space, while the rotor maintains its orientation. This results in the pickoff outputs varying sinusoidally. If the case spin axis is not vertical, the sine wave will have a DC offset. By adjusting the verniers on the tip tilt fixture, this offset can be adjusted and reduced to zero. Ideally, adjustment of one vernier will only affect one input axis, however due to azimuth misalignments there will still be some cross coupling. This causes the procedure to be iterative. Adjustment of the verniers occurs while the table is rotating. This makes it difficult to make fine adjustments, since one is required to run around the table at 50 degrees per second while attempting to carefully turn the verniers.

The next step is to align the input axes with the cardinal geographic directions. The tip tilt fixture has a third vernier which is capable of rotating the entire fixture by approximately 3 degrees. Since the two input axes will not be precisely orthogonal, it will only be possible to optimise one axis, and let the second axis fall where it may. For this test, the X axis was chosen to be aligned. Two different procedures were used, the first method being useful for coarse alignment, the second being used for fine tuning.

Figure 4-6: Alignment of the CSG-2 Spin Axis

Both methods operate the gyroscope in closed loop mode and monitor the torquer voltages.

The azimuth axis was positioned so that the gyroscope's X input axis was nominally pointing East. Ideally, the associated torquer voltage will be zero, since no Earth rate is sensed. Bias will cause a small offset from zero, as will misalignments from true East. If the table is now rotated to point west, the bias offset will be the same, but the misalignment effect will be opposite in sign. The voltages in both positions were recorded and the difference, δV , calculated. Rotating the fixture so that the torquer voltage changes by $\delta V/2$, will eliminate the misalignment.

The second method is very similar to the first, except that it relies on longer periods of data recording. By using the first four positions of a multiposition test (See Section 3.4), the misalignment angle α can be determined. By repeating several multiposition tests, and adjusting the rotary fixture to minimise α each time, the misalignment angle between the x input axis of the gyroscope and the geographic coordinate system can be reduced.

Once these levelling and alignment procedures have been conducted, the gyroscope is ready for testing. It is important not to change the settings on the verniers from this point on, as it will invalidate all of the preceding tests. Due to the design of the three sided fixture, it is now possible to reorient the gyroscope so that any axis is parallel to the azimuth axis simply by flipping the fixture onto one of its other sides.

4.3.3.3 Gyroscope Testing

Several different types of gyroscope tests were undertaken. Each test was conducted using both the existing Navlab rack mounted support equipment, and the SGL developed electronics, unless otherwise stated.

Gyroscope Power requirements

The wheel supplies have been designed to provide an initial start up voltage which is higher than the desired run voltage. This is done through manual switching by the operator, or through software timing control in the case of the final SGL design. To measure the power consumed by the gyroscope, it was intended that a differential wattmeter would be used to monitor and record power demand in one of the phases of the gyroscope motor windings. Unfortunately, this instrument did not perform as expected, and so an alternate method of power monitoring needed to be devised.

Using an oscilloscope, the power factor of the motor could be determined by observing the phase shift of the voltage with respect to the current. Once this was determined, it was possible to use the Datron multimeters to record voltage and current for a number of different start/run voltage configurations to determine the combination which would result in the lowest long term power consumption.

Temperature

Temperature is monitored during all gyroscope testing. Therefore, a large quantity of temperature data can be collected without ever doing explicit temperature tests. One of the most useful tests conducted to examine temperature data is the long term random drift test. The gyroscope is placed in position mode and allowed to run closed loop for an

extended period of time, either over night or for a complete weekend. Ideally, the gyroscope output should not change if it remains in the same orientation. By recording temperature during this test as well, the long term temperature stability can be determined, and correlated to the gyroscope output. Temperature measurement is accomplished to a resolution of 0.01°C with the Fluke thermometer.

Gyroscope Resolution

To test the resolution of the gyroscope, the CSG-2 was mounted so that one of the input axes was parallel to the azimuth axis of the table. In this way, if the azimuth axis was moved, that input axis would respond. Using a precision current source, a constant current was injected into the torquers to maintain null at a given position. The MPACS table was then commanded to move by a very small angular increment, ranging from a 10 arcsecond change all the way down to a 1 arcsecond change in table position. The output from the pickoffs was monitored using the PDP A/D board. Data was recorded as the table was stepped back and forth between two given positions. Care must be taken not to command the table to take large angular steps, since the gyroscope is not under active control, but is just being nulled by a constant current.

Servo Testing

Servo testing, in which the motion of the table is controlled by the gyroscope itself acting as part of the feedback loop, is one of the two methods in which gyroscope drift parameters were determined. Servo testing has been a traditional method of testing single degree-of-freedom gyroscopes for gimballed platforms, since it simulates the inertial servo loop mode of the platform. It can just as easily be applied to two degree of freedom

gyroscopes, however it requires that the second axis of the gyroscope be controlled in some auxiliary way, if only a single axis servo loop test facility is available. For these tests, a rebalance loop was used to cage the axis not under servo test.

The CSG-2 was mounted with the X input axis parallel to the azimuth axis (the Y input axis was tested later on in the same manner). The RL-3 (a DREO design) analogue rebalance loop was used to control the Y axis of the gyroscope. The signal from the 54 kHz primary reference was fed into the reference jack on the Gyro Servo module of the MPACS. This module provides the interface between gyroscope signals and the table servo control mechanism.

A second BNC connector, marked FBK on the gyroscope servo module is used to input the gyroscope pickoff outputs. The servo module is equipped with control knobs to adjust the lead/lag characteristics of the signal, as well as to increase the gain of the signal. The module was originally designed to work with signals up to 10 kHz in frequency, however it has been modified to accommodate higher frequencies. Experiments revealed that the gain of the signal still had to be attenuated by a substantial amount. A voltage divider was used to reduce the input voltage to the gyroscope servo module. The ground leg of the divider was equipped with a decade resistance box, so that the gain could be adjusted. Through experimentation, a -60 dB adjustment was found to be needed to the pickoff output signal before inputting it into the MPACS (see Figure 4-7).

The most difficult phase of servo testing lies in closing the servo loop for the first time. Even with proper connections, some adjustments to the gyroscope servo module still were necessary. Most of these adjustments were done in a 'seat of the pants' manner,

Figure 4-7: Gyro Servo Mode Block Diagram

since they depended greatly on the actual gyroscope used. While making these adjustments, it was critical to have the rate trip on the MPACS unit set to a relatively low value (< 20 deg/sec), since incorrect adjustments to the Gyro Servo module will cause the servo loop control to become unstable and the motion table azimuth axis can start to rate in the wrong direction. Once servo control was achieved, fine tuning to remove table vibrations was necessary. While some of this could be done electronically, it was also necessary to make mechanical adjustments to change the natural frequency of the fixture.

Servo testing involves recording the table azimuth rate as the table responds to the servo control of the gyroscope. It is essentially a single axis space integrator. The gyroscope senses earth rate and coupled with all the drift terms, causes the azimuth axis to rate such that the gyroscope pickoff signal is nulled. Since the rates are so small, on the order of earth rate (15 degrees per hour), it was necessary to adjust the rate readout. In normal operation, the MPACS updates at a 1 Hz rate, but only gives rate information to a resolution of 0.0001 deg/s (or 0.36 deg/hr). In order to increase the resolution, the sample rate of the MPACS was slowed down to 0.01 Hz, which results in a rate resolution of 0.0036 deg/hr. Therefore, sample rate was sacrificed to increase resolution.

By positioning the tilt axis such that the gyroscope input axis is in different inertial orientations, all of the first order gyroscope drift parameters can be determined, as outlined in Section 5.3.6.

Gyroscope Multiposition Testing

The second method of determining gyroscope drift parameters is to conduct a multiposition test. By accurately positioning the table in a series of different orientations,

the gyroscope will be excited by different fractions of earth rate and gravity. For each position, an equation can be written which defines the drift terms sensed in that orientation, as well as earth rate. In the general case, with n -multiple positions, a set of n equations in n unknowns can be solved for, thus defining the gyroscope performance parameters.

In order to determine all first and second order drift terms, a 24 position test is used. See Table 3-1 for a breakdown of these positions. If only the first order terms are desired, then only eight of these positions are required. To determine all the first order terms and some of the key second order terms, an 11 position test is often used.

Data collection is straight forward during these tests. The Datron 1071 multimeter is used to record torquer currents (measured as voltages across scaling resistors) on both the X and Y axes, while under control of the RL-3 analogue rebalance loop. The tests are conducted automatically through use of the Motion Table Language program. For a given position, the table is commanded to move to a specific azimuth and tilt angle. A five minute pause then ensues before data collection begins, to allow any transients to settle out which might affect the static measurements. A fifteen minute data collection period is then conducted, in which 10 data points are recorded, each point being the block average of 300 individual data points. This is done to reduce the data files to a manageable size. Since only the average values are used during the calculation of the drift and error parameters, this averaging is considered acceptable. Appendix E shows a typical output from the LOGDAT routine during a multiposition test.

4.4 Inertial System Testing

4.4.1 INTRODUCTION

The completed gravimeter will use a stabilised platform. However, to determine misalignments of the instrument axes with respect to the reference mirrors, the system is tested in much the same way as a strapdown system would be tested. That is, the sensor blocks themselves are fastened to the test table and then tested by positioning or rating the entire sensor fixture. The gyroscopes need to be operated in a closed loop manner for these tests.

Conceptually, many of the same practices and equipment are used to test the system as a whole as were used to test individual instruments. Accelerometer testing differs substantially in that it uses the motion table for precise positioning rather than the dividing head. The following discussion will concentrate mainly on new equipment or procedures which are specific to system testing, with only passing reference made to previously used facilities for the sake of completeness.

4.4.2 TEST EQUIPMENT

Inner Gimbal Simulator

The system under test is known as the Inner Gimbal Simulator, or IGS. It is comprised of an aluminum housing, which is the exact shape and dimensions of the actual inner gimbal used in the gravimeter, with the exception that it has no means of attaching to a middle gimbal ring. The IGS also has mounting holes which allow it to be attached to the motion table. Mounted within the IGS are the upper and lower sensor fixtures (USF

Figure 4-8: Inner Gimbal Simulator, Bottom Cover Removed, View of LSF

Two CSG-2 gyroscopes visible (with large, blue preamp boards, top left and right)
Two QA-2000 accelerometers on bottom left and right faces of LSF

Figure 4-9: Inner Gimbal Simulator, Top Cover Removed, View of USF

Three reference mirrors visible

One QA-3000 accelerometer housed in USF, under Z Reference Mirror

and LSF, respectively). The LSF houses both gyroscopes and the two horizontal accelerometers, while the USF is designed to hold the vertical accelerometer (i.e. the gravimeter). Reference mirrors are mounted along all three system axes so that autocollimation can be performed. See Figures 4-8 and 4-9.

Mounting Plate

A mounting plate (Figure 4-10) was designed as an interface between the motion table top and the bottom of the IGS. The plate was machined to tolerances of 0.1 thousandth of an inch in all critical directions. This included accurate positioning of the bolt hole pattern to accept the mounting of the IGS. The hole pattern for mounting the plate to the table top was intentionally left with oversized holes, so that accurate positioning with the alignment bar could be accomplished (See Section 4.4.3.1 below). Furthermore, the plate was nickel plated. This made the surface resistant to corrosion, to maintain its flatness. It also provided a sufficiently reflective surface to allow for direct autocollimation, without the need for an intermediate mirror.

Autocollimating Theodolyte

A theodolyte was used at two stages of testing of the IGS. Before testing even began, it was used to verify alignment of the motion table. Once the IGS was mounted in place on the table, the autocollimator was then used to determine the misalignment angles of the reference mirrors, with respect to the system axes. A Wild T2 theodolyte was used with the autocollimator attachment. The stated accuracy of repeatedly sighting on an object is 2 arcseconds using the T2 [42]. Use of a theodolyte requires a known reference point, which was established through use of a geodetic site survey.

Figure 4-10: Inner Gimbal Simulator Mounting Plate

Breakout Panel

A table top breakout panel was designed and built for use with the IGS and its associated support electronics. The panel serves as a distribution point to connect signals between the support boards and the IGS, as well as a jumping off point to bring signals from the table top to the rack through the motion table slip rings. The panel was also equipped with fuses and switches, to protect the gyroscope pickoffs, motor circuits, and pre-amp boards. The panel was mounted on the side of a support cage, which also served as a platform to mount the support electronics. (See Figure 4-11).

Temperature Recording

The IGS is equipped with five different temperature sensors. Each accelerometer is capable of outputting a current which is proportional to the internal sensor temperature. Thermistors are also mounted on the upper and lower sensor fixtures to monitor the block temperature. Normally, data recording within the Navlab is conducted on the PDP-11 micro computer. However, temperature collection from the IGS was conducted by using a program called *wiretap*, which ran on a PC platform. Wiretap connects the computer card on board the support electronics box to an external PC, via an RS-232c serial line. Data is collected as a continuous stream, which gives the 5 temperature sensor values along with a time stamp.

Common Equipment

Most of the rest of the test equipment has been detailed in the previous sections. Central to testing is the Motion Table, used to position the IGS. Most data collection is accomplished through use of the PDP-11 data logging routines, specifically LOGDAT.

Figure 4-11: Inner Gimbal Simulator, with Breakout Panel and SGL Electronics

The support electronics are used to drive the gyroscope pickoffs and motors, as well as the LSF and USF heater circuits. Power for these supplies was provided by rack mounted DC power supplies. Oscilloscopes were used to monitor the gyroscope pickoff signals. Since testing was done with the gyroscopes operating in closed loop, the rack mounted rebalance loops were used.

4.4.3 TEST PROCEDURES

4.4.3.1 Preliminary Considerations

The Contraves Motion Table is installed in the Navlab so that its primary axes are aligned with the North-South, East-West cardinal directions. This is done at the time of installation into the facility. However, because of the accuracy requirements of the gravimetry project, a precise table alignment to verify its orientation and quantify its misalignment was desired. Alignment of the motion table is not a trivial task, and requires precise survey techniques and expensive equipment. The most crucial piece of equipment used is a North seeking gyro, known as a GAX. This instrument is mounted upon a theodolite similar to the Wild T2 used in the Navlab. By following a detailed series of procedures [43], it is possible to determine a north line bearing to better than 10 arcseconds of certainty. Appendix F shows an error budget for the procedure. It is important to note that north seeking gyros are seldom used today for surveying, since GPS survey techniques are less expensive, easier to perform, require less operator training and deliver the same level of accuracy. However, GPS is only possible in an outdoor

environment, and so for an accurate alignment procedure within a building, a gyro compass is the only viable option.

The North seeking gyro tells the surveyor the bearing line to geographic North, as opposed to magnetic North. (Actually, gyro compass devices seek East, and merely give an output 90° from this). However, some method to preserve this bearing line once the GAX has been removed is necessary. This is the purpose of the integrated theodolyte. Once a north line has been established, the theodolyte is used to determine a bearing line between the GAX and a reference object (R.O.). The purpose of the R.O. is to allow a theodolyte to be set up in the same spot as the GAX at a later date, and then sight on the reference object to determine the bearing to North. Brass floor plugs are used within the Navlab to mark points over which theodolytes and the GAX are to be set up. The reference object itself is chosen to be an object as far away as possible, since the error introduced on sighting the R.O. decreases as the length of the baseline increases (see Appendix F).

Once the site line to North had been determined, the procedure of aligning and levelling the motion table before the IGS was mounted on it could begin. The first step was to ensure that the table itself was level by using bubble levels. Next, the IGS mounting plate was attached to the CMT table top. The procedure for affixing this plate is illustrated in Figure 4-10. The plate is loosely attached to the CMT top using 4 Allen head bolts. The table trunion axis is then rotated to be horizontal, with the azimuth axis reading precisely 0° so that the mounting plate rests on the two set screws of the alignment bar. A spirit level is then placed on the horizontal plane of the plate, and the set

screws are adjusted until the level shows zero slope. The alignment bar is then left alone. The purpose of this exercise is two-fold. First, it establishes a reference for azimuth rotations, since the axes of the mounting plate are now aligned with the table axes when the azimuth axis is at 0° . Second, it allows the mounting plate to be removed and replaced from the table top in a repeatable manner, simply by butting the face of the plate against the set screws which define a plane.

Once this was done, the trunion axis was returned to the upright position and spirit levels used to assess the levelness of the mounting plate. Metal shims were used to adjust the plate. Note that torquing the plate into place could lead to compression of the shims, so the true level of the plate could not be judged until the bolts were tightened down.

The theodolite was now set up over the reference plug and was initialised using the reference object so that it was facing due South (T2 azimuth reading of $180^\circ 00' 00''$, see Figure 4-12). An optical quality mirror was then attached to the mounting plate, to allow for autocollimation. Experimentation revealed that the mirror was actually unnecessary, since the finish on the plate provided enough reflection to obtain a return beam. This allowed for removal of the mirror to make the test set up simpler. The trunion axis was once again placed in the horizontal position, so that the mounting plate was perpendicular to the line of sight of the theodolite. Using the autocollimator, the CMT base itself was rotated until the return beam coincided with the reference beam. This indicates that the table surface was precisely orthogonal to the line of sight, implying that the motion table was aligned with the geographic co-ordinate frame. To eliminate the effect that an azimuth axis cone angle would have, the azimuth axis was given a constant

Figure 4-12: Use of T2 theodolite (foreground) to align motion table (background)

rate during autocollimation. The table base was then adjusted so that the return beam traced out a circle that was centred around the reference beam. The table was now accurately aligned with North.

It is important to point out that the above procedure is iterative. That is, by changing one parameter, such as the orientation of the base of the table, another factor can be affected, such as the levelness of the table. Therefore, the procedure needed to be repeated a couple of times until no further adjustments needed to be made.

With the table aligned and levelled, the IGS was mounted onto the plate. Quartz washers were used to stand-off the IGS from the plate by a quarter inch. This is done to prevent direct contact of the IGS with the plate, since the plate would act like a heat sink and would tremendously increase the power requirements to the IGS heaters. Quartz was chosen since it is exceedingly flat and a thermal insulator and therefore does not introduce new misalignment angles to the mounting. However, since these washers are brittle, care had to be taken not to over-torque the bolts.

The IGS has a small hole in one side of its outer wall which allows cables to be passed from the gyroscopes, accelerometers, and heaters to the table top breakout panel. All of the wires had been carefully labelled during assembly of the IGS, and all of the appropriate connections were made to the breakout panels.

4.4.3.2 Inner Gimbal Simulator Testing

Three major goals were to be achieved during testing of the IGS. First, the misalignment angles of the gyroscopes within the IGS needed to be determined, as did the accelerometer misalignment angles. Also, so that the instrument misalignments would

have some meaning once the IGS was removed from the motion table, it was necessary to measure the misalignments of the IGS reference mirrors.

Testing of the instruments within the IGS has already been outlined in Chapter 3. Using the developed model equations, a multiposition test sequence was defined for both the gyroscopes and accelerometers. Output from the gyros was recorded by measuring the torquer currents as voltages at the rebalance loops. Since two rebalance loops are required (one for each CSG-2 gyroscope), it was necessary to use both RL-3, which was used in individual gyro testing, and RL-2, which had been previously unused. Functionally, the only difference between the two loops is that RL-3 includes a cross-coupling term to handle accelerations. Past experience in the Navlab has shown that there is very little difference in the performance of the two loops. This needed to be verified during testing. The torquer voltages were recorded using LOGDAT. The scaling resistors were accurately measured in four wire resistance mode to enable the conversion of voltage readings to currents. The data collected was post processed to determine the misalignment angles and other error terms using the equations developed in Chapter 3. Data reduction was done using an Excel spreadsheet to ensure repeatability.

Accelerometer testing of the IGS was very similar to gyro testing. The multiposition tests defined by the model equations from Chapter 3 were used. The accelerometer outputs were connected to resistors and the voltages were measured on the Datron multimeters using the LOGDAT task. Data files were then ported to the PC platform, where Excel spreadsheets were used to analyse the data.

For both types of multiposition tests, the PC based *Wiretap* program was used to record the temperature of the LSF and USF, as well as the accelerometers. While a test was in progress, the temperature could be monitored by viewing the LCD display which was mounted on the support electronics box.

Determination of the mirror misalignment angles required use of the T2 theodolyte. After setting up the instrument over a brass reference plug and sighting on the reference object, the theodolyte was initialised to be level and pointing at $0^{\circ}00'00''$ when facing due North. The motion table was then set to be exactly vertical (trunion angle set to $180^{\circ}00'00''$) and the azimuth axis positioned to $0^{\circ}00'00''$. This placed the IGS so that the X axis mirror was facing North (see Figure 4-13). The theodolyte was then pointed toward the mirror and the azimuth and elevation angles adjusted until the return beam of the autocollimator coincided with the reference beam. These two angles were recorded as the misalignment angles of the mirror plane. An equivalent process would be to align the theodolyte perfectly with the geographic frame and rotate the azimuth and trunion axes of the motion table until autocollimation is achieved. In this case, the mirror misalignment angles will be displayed on the MPACS position displays, rather than the theodolyte angular indicators. The moving of the theodolyte method was chosen simply for operator convenience.

The azimuth and elevation angles of the theodolyte can be used to calculate a direction vector for the normal of the mirror. Once this normal is expressed in system axes co-ordinates, which are aligned with the table axes, then any misalignment angles determined for the gyros or accelerometers can be related to the mirrors. Since the

Figure 4-13: Alignment of the IGS using the T2 theodolite

theodolite can be set up in two different ways for a given position (e.g. a horizontal reading could be 90° or 270° , depending on how the T2 was initialised), then this must be accounted for when calculating the misalignment angles.

Similar procedures were conducted for the Y and Z mirrors as well. The table was positioned so that these mirrors were facing the theodolite, and after autocollimation was achieved, the misalignment angles were recorded. Appendix G shows the values obtained for the mirror misalignments, and how the co-ordinate frame was defined for these calculations.

4.5 Summary

Testing involved three related but separate fields. First, individual accelerometers were tested using an accelerometer test station. Next, individual gyroscopes were tested on the Contraves Motion Table. Finally, all of these instruments were integrated into one system and tested in the IGS. The goal of all these tests was to characterise performance of the instruments, determine their error parameters, and measure their misalignments. The equipment used in these tests has been discussed, as well as the procedures involved in obtaining meaningful results. The test procedures implemented the model equations developed earlier.

5. Results and Discussions

5.1 Introduction

This chapter is divided into four major sections, dealing with evaluation of individual accelerometer and gyroscope testing, followed by testing the instruments while into the IGA. Results are not necessarily presented in a chronological manner, but rather in an order that is a logical progression for the test program. Where possible, results of testing are presented in the form of graphs, tables or charts. Since many tests were conducted multiple times, only representative results are shown for certain tests.

5.2 Accelerometer Testing

Three accelerometers were used in the prototype gravimetry system: two QA-2000s and one QA-3000. Only one of the QA-2000 accelerometers was actually tested by Carleton University. The other two instruments were tested and characterised by Sander Geophysics. Accelerometer testing within the Navlab investigated four major areas; characterisation of the instrument using the model equations developed in Chapter 3, determination of the accelerometer's resolution, long term stability of accelerometer output and sensitivity to temperature.

The majority of accelerometer testing took place on the microflat granite slab, which was levelled to better than 3 arcseconds using the levelling screws and spirit levels. Autocollimation was then used to establish the orientation of the reference mirror on the accelerometer test block as outlined in Chapter 4. Multiple determinations of the local level of the mirror were conducted, and from these an average optical null of $358^{\circ}18'56''$ was determined. This corresponds to the dividing head angle at which the reference mirror is parallel to the gravity vector. This offset from zero must be accounted for when conducting further tests, such as multiposition determinations.

5.2.1 Temperature Sensitivity

Temperature testing was conducted by two separate methods. First, the accelerometer test block itself was temperature controlled using cartridge heaters and the TC-300 temperature controller. The block itself is enclosed in a Plexiglas cover, which holds an insulating packing to isolate the fixture from ambient temperature fluctuations. In an attempt to further isolate the accelerometer, the entire dividing head was placed in a

Thermotron thermal control chamber to tightly control the ambient temperature fluctuations. This was the only phase of accelerometer testing conducted off the granite slab. Therefore, the absolute level of the test set-up could not be guaranteed in the Thermotron, but relative changes in output could be measured.

Figure 5-1 shows a long term stability test of the accelerometer output and its temperature. This temperature is measured directly from the QA-2000 temperature output itself. Early models of Q-Flex accelerometers had no temperature output capability, and therefore external RTDs or thermistors needed to be used. This stability test was conducted by simply positioning the input axis in a nominal 1g field and recording the output. The graph has been scaled so that it clearly shows deviations rather than absolute values of output. A number of observations can be made from this graph. First, it is apparent that the accelerometer output is highly correlated with the variations in temperature of the instrument. This implies that if changes in acceleration of less than a micro-g need to be sensed, then either very tight temperature control or an accurate temperature model is needed. Second, the long term temperature control of the TC-300 is seen to be better than 0.20°C over the course of 60 hours, with short term stability (i.e. for several hours) being significantly less than a tenth of a degree Celsius.

Figure 5-2 shows a similar test, this time conducted with the dividing head in the Thermotron oven. The procedure here was the same as the test in Figure 5-1. However, the thermal chamber was set to 63°C and the TC-300 only had to bring the block temperature up another 5.4°C to operating temperature. This created a thermally benign operating environment. The graph shows that over a 60 hour period, temperature stability

Figure 5-1: Output and Temperature Stability of QA-2000 Accelerometer
(Long Term Static Test, Using Only TC-300 for Control)

Figure 5-2: Output and Temperature Stability of QA-2000 Accelerometer
(Long Term Static Test, Using Thermotron Oven and TC-300 for Control)

Figure 5-3: Variation of QA-2000 Accelerometer Output with Temperature
(Data points represent averaged values)

Figure 5-4: Variation of QA-2000 Accelerometer Output with Temperature
(Data points represent continuous, instantaneous values)

has been improved by over a factor of two, with a peak to peak variation of only 0.07°C being observed. However, the correlation of temperature output to accelerometer output is not as pronounced as in Figure 5-1. A large part of this is due to physical environment of the thermal chamber. The interior is a stainless steel container, which is not isolated from external shocks. Therefore, vibrations from the oven machinery itself, external air conditioners, or even people walking by, can all contribute to changes in the accelerometer output. Some of these vibrations were removed through the use of data averaging, but after several different tests conducted within the Thermotron, it was concluded that further testing within it would not be useful.

The QA-2000 data sheets supplied by Sundstrand provided a temperature model for the accelerometer. However, this model is based on only a few data points, and covers a very large temperature spectrum, typically from below freezing to near 100°C . This is standard practice, since these accelerometers have found use in a wide range of applications. However, for the gravimetry platform, where it is known that the temperature will be tightly controlled, this model is unacceptable. Therefore, a new temperature model was created, covering less than a 5°C range, centred around the operating temperature. Figure 5-3 shows the temperature model developed. Each point represents the average of 45 minutes of data collection. Using a linear curve fit to this set of data, it was determined that a change of 159.35 nA occurs for every degree Celsius increase. This corresponds to a $120.42\text{ }\mu\text{g}/^{\circ}\text{C}$ change. Therefore, temperature control to better than 0.008°C would be required to measure μg changes without temperature modelling. Figure 5-4 shows a similar temperature test to Figure 5-3. The significant

difference here is that the temperature was continuously increased over a short period of time (the entire test spans five minutes), rather than only collecting data at discrete points for long periods of time. The temperature model generated from this test was $159.34 \text{ nA/}^{\circ}\text{C}$, which is in very good agreement with the previous test. The R values for the linear regression for both graphs are almost 1, indicating an extremely good curve fit. This illustrates that the model is valid for instantaneous thermal changes and not just long term trending.

5.2.2 Output Stability

Using the temperature model generated in section 5.2.2, a clearer picture of the QA-2000's long term output stability was generated. The accelerometer was positioned in a one g field and data was collected for a two day period. The thermal model was then used to compensate for changes in accelerometer output that were caused due to fluctuations in temperature in Figure 5-5. Figure 5-6 shows the results, compensated for temperature. A peak-to-peak variation in output of only 5 nA, or $3.7 \mu\text{g}$ was observed. No signs of trending over the course of the 48 hours are evident. Furthermore, short term stability (over the course of several hours) is on the order of one to two nanoamps, or about a μg . This shows that the temperature model allows the accelerometer output to be compensated so that long term stability on the order of micro-gs is attainable.

5.2.3 Characterisation

Multiposition tests were conducted to extract the scale factor, bias, and misalignment terms from the QA-2000. A variation on equation 3-7 was used for data

Figure 5-5: QA-2000 Accelerometer Long Term Output Stability
(No temperature compensation)

Figure 5-6: QA-2000 Accelerometer Long Term Output Stability
(With temperature compensation)

Figure 5-7: Twenty Point Multiposition Test of OA-2000 Accelerometer

Figure 5-8: Effect on OA-2000 Temperature by Rotating the Dividing Head 180°

reduction. By rotating the dividing head through a full 360° rotation, and recording the accelerometer output at discrete points, a graph such as Figure 5-7 was obtained. Each data point is the average of five minutes of accelerometer output data. In addition, five minutes of settling time was allowed at each position before data collection began. This eliminated any thermal transients which might have occurred and allowed for manual adjustment of the temperature of the block to better than $\pm 0.008^{\circ}\text{C}$ of the set point of 68.40°C . If the temperature was not manually compensated, then the block temperature could vary by up to 0.4°C during the course of a full rotation of the dividing head, as illustrated in Figure 5-8. This would skew the results, as discussed in section 5.2.1.

A curve fit to the data points in Figure 5-7 allows the necessary accelerometer characteristics to be determined. Since the accelerometer is placed in positions which follow a circular progression, the curve fit results in a sinusoid which behaves as follows.

$$k \cdot \cos(\theta + (\Delta + \Delta_{opt})) + B \quad 5-1$$

where k is the accelerometer scale factor, in mA/g , B is the bias, in g , Δ is the lumped misalignment term and Δ_{opt} is the misalignment of the mirror from the gravity vector, previously determined to be $358^{\circ}18'56''$. Figure 5-7 shows the results of this curve fit, and Table 5-1 shows the average values of 5 different multiposition tests conducted. The largest residual between the curve fit of Figure 5-7 and the actual recorded values is 14 ppm, with many of the other residuals being as small as 5 ppm.

This multiposition test was based upon 20 equally spaced positions. However, tests were conducted with both a greater and lesser number of positions. A 40 position test was found to generate virtually the same curve fit, with the same size residuals as a 20

Scale Factor Magnitude (mA/g)	1.323260
Bias Magnitude	2.055985×10^{-3}
Misalignment (arc minutes)	3.119
Output Stability (Bias and SF)	6 ppm (peak to peak, over 48 hr period)
Temperature Sensitivity	159.35 nA/°C 120.42 µg/°C
Resolution	< 0.75 µg

Table 5-1: Test Results for QA-2000 accelerometer

position test. Tests with fewer than 20 positions were found to have slightly higher residuals, but even a quick four position test (using $\pm 1g$ and the two 0g positions) yielded results which were only an order of magnitude different from the 20 position test. This indicates that for a quick assessment of a QA-2000, a simple 4 position test would still yield very good results.

5.2.4 Resolution

Resolution tests of the QA-2000 involved turning the dividing head by a known amount, causing a change in the sensed gravitational input to the accelerometer. The accelerometer was placed in a one g orientation (i.e. $358^{\circ}18'56''$) for these tests. This puts the accelerometer output at the top of a cosine curve, which means that a relatively large dividing head rotation is required to produce a small change in accelerometer output. If the same test was conducted with the accelerometer input axis in the horizontal position, only a small change in dividing head angle (a couple of arcseconds) would be required for the same change in accelerometer output. This situation is undesirable, since

Figure 5-9: Resolution Test of QA-2000 Accelerometer
(Nominal 7 Arcsecond Step Inputs)

Figure 5-10: Resolution Test of QA-2000 Accelerometer
(Nominal 0.75 Arcsecond Step Inputs)

positioning the dividing head now becomes the dominating error source (the dividing head is capable of accurately positioning to 2-3 arcseconds). Since a 5 arcminute angle change from 0° is required for a 1 μg change (see Appendix D), the errors due to the dividing head are insignificant.

Figure 5-9 shows a resolution test to detect 7.5 μg changes, while Figure 5-10 shows a test which detects 0.75 μg changes. Each test consisted of recording data for 10 minutes, and rotating the dividing head every minute by an angle which corresponds to the desired change in input. The spikes in output observed in Figure 5-9 are due to trying to accurately position the dividing head. Figure 5-10 clearly shows the 0.75 μg step changes, which proves that the QA-2000 is capable of detecting sub-micro g changes in input. Since the noise level of the Datrons was between 0.3 and 0.4 μV on the scale used, it was difficult to detect smaller changes in gravity, due to the noise of the multimeters. However, these tests imply that the QA-2000 is capable of sensing much smaller changes than 0.75 μg .

Another observation from Figure 5-9 is that there is no significant hysteresis as the accelerometer is moved from one position to another and back again. This further proves that the Q-Flex accelerometers are suitable for this gravimeter, since large hysteresis would create difficulties in accurately resolving small changes in gravity.

5.3 Gyroscope Testing

Two separate CSG-2 gyroscopes were tested individually: serial number 001 and 020. When the CSG-2s were manufactured, each instrument was matched with a similar instrument to facilitate use in one system. That is, S/N 001 and 020 were chosen for the

gravimetry project since they were designed with similar characteristics. In contrast to this, the CSG-2s used in the Heading Reference Unit (HRU), another system under development at the Navlab, have torquer scale factors which are 50% higher than the ones used here [44]. While there is no conceptual problem with using 'mismatched' gyroscopes, it is generally desirable to have similar instruments in a given system.

Each gyroscope was tested individually using both the rack mounted electronics and the SGL designed Gespaq Bus based electronics. Since the latter tests are of greater relevance to the gravimetry project, it is these results which are presented in full. Where applicable, the tests conducted using the rack mounted electronics are presented as a reference baseline for performance, but the individual tests are not discussed to the same level of detail.

5.3.1 Power Consumption

The CSG-2s run on three phase, 400 Hz (nominal) power. However, the actual amount of power required for operation is unspecified. The CSG-2 specification sheets merely state that the power supplies should deliver less than 25V for short duration or 12V for long duration operation. These values are maximums which are meant to protect the gyro motor from damage. In practical operation, it is normally desirable to run the gyros at a much lower power level. This is done for several reasons. First, since the gravimeter will ultimately operate on a helicopter, power available from the aircraft bus is limited. Second, running the gyros at high power levels tends to wear out the motor quicker and thus reduces the lifetime of the gyroscopes. For these reasons, it is desirable to determine the minimum power requirements for these CSG-2s.

Several different configurations were attempted. It was found that the gyro motors would not reach synchronisation with less than 4 volts supplied. Starting the gyros with a voltage above 12 volts resulted in current draws of over 1 amp. This was undesirable, since some of the wiring in the gravimeter will only be rated at 1 amp.

Figure 5-11 shows the results of starting the gyro at 5 volts RMS. Just over 1.3 watts are required to run the gyro at this voltage. Figure 5-12 shows a similar test, with the difference being that the gyro was started using a 8 Volt supply, and then dropped down to 5 Volts once the gyro had reached synchronisation and stabilised. The initial power required is much larger, about 3.3 watts, but once the supply was switched to 5 volts, only 1 watt was required. This represents a significant savings in power over the long run. This concept of using an over-voltage to start a gyroscope has long been employed as a way of conserving power. The initial high voltage magnetises the motor's hysteresis ring differently than a lower voltage would, which results in a lower power consumption. Using this method, it is actually possible to run the gyroscope with less than 4 volts. Tests showed that by starting the gyro with a high voltage, the CSG-2 could subsequently be run with only 2.25 volts. While this results in further power savings, the risk here is that a momentary power fluctuation could cause the gyro to loose synchronisation, and the 2.25 volts would not be sufficient to bring it back up to speed. For this reason, 5 volts was used as the minimum voltage. Eight volts was chosen as a maximum start voltage to limit the current requirements to under an amp. Only three quarters of an amp are required to start the gyro at 8 volts RMS, which still leaves a safety margin for over-current.

Figure 5-11: CSG-2 Power Consumption During Motor Start
(Start and Run Voltage = 5 Volts RMS)

Figure 5-12: CSG-2 Power Consumption During Motor Start
(Start Voltage = 8 Volts RMS, Run Voltage = 5 Volts RMS)

Figure 5-13: Long Term Stability of Pickoff Driver Supplies
(Comparison of SGL and North Hills Designs)

Figure 5-14: Long Term Stability of Wheel Supplies
(Comparison of SGL and North Hills Designs)

5.3.2 Power Supply Stability

Two power supplies are crucial to the operation of a tuned rotor gyroscope: the motor (or wheel) supply and the pickoff driver. Large amounts of noise from either supply can affect performance of the instrument. The rack mounted supplies, manufactured by North Hills electronics, were specifically designed for long term stability with low noise levels. The SGL designed supplies were also designed with these goals in mind, but were unproven prior to testing.

Figure 5-13 and 5-14 show the stability of both types of pickoff and wheel supplies, respectively. Both wheel supplies show comparable noise and stability levels. The North Hills supplies have slightly less noise, with only 0.6 mV peak-to-peak noise levels, compared with 1 mV peak-to-peak for the SGL designed supply. However, Figure 5-13 shows a much noisier signal for the SGL pickoff supply, with just over 10 mV peak-to-peak deviation compared to only 1 mV for the NH supply. These values ignore the spike that occurred at the 2.5 hour mark of the test, which appears to be due to a spike in the lab power itself. Furthermore, the initial half hour trending of the SGL supply is ignored, as it is due to warm up stability. This noise on the pickoff signal could contribute to increased noise in the gyro output signal, which may degrade performance of the gyroscope.

5.3.3 Temperature Sensitivity

As was demonstrated in section 5.2.1, temperature changes can be an important factor in determining the performance of an inertial instrument. Accelerometers show a

significant correlation with temperature changes. Normally, gyroscopes exhibit less of a temperature dependence, but this effect must still be investigated.

Figure 5-15 shows the relationship between the torquer scale factor for CSG-2, serial number 001 and temperature. These scale factors were calculated using results from multiposition tests [Section 5.3.5] conducted at several different temperatures. The X torquer shows a temperature dependence of $-0.764 \text{ deg/hr/mA per degree Celsius}$, while the Y torquer was found to vary by $-0.588 \text{ deg/hr/mA per degree Celsius}$.

In the gravimeter, the temperature control requirements have been dictated by the sensitivity of the accelerometers to temperature. As a design goal, temperature control to better than 0.01°C was aimed for in the temperature controllers. With this in mind, it is seen that the sensitivity of the torquer scale factors to temperature will be less than $-0.01 \text{ deg/hr/mA per hundredth of a degree Celsius}$ for both torquers. Since the standard deviation of the calculated torquer scale factors is approximately 1 deg/hr/mA [5.3.5], there is little point in developing an accurate temperature model for the gyroscopes, since other errors result in torquer scale factor uncertainty being two orders of magnitude larger than thermal corrections. Furthermore, the torquers are not needed in the gravimetry platform except to cage the redundant axis and to re-align the platform. Therefore, further temperature modelling was not considered necessary, and thermal corrections for the gyroscope output should not be necessary given the tight temperature control that will be used in the system.

Figure 5-15: Torquer Scale Factor Variation with Temperature
(CSG-2 Gyroscope, Serial Number 001)

Figure 5-16: Resolution Test of CSG-2, Serial Number 001, X Axis
(10 Arcsecond Step Changes in Azimuth Position)

Figure 5-17: Resolution Test of CSG-2, Serial Number 001, X Axis
(1 Arcsecond Step Changes in Azimuth Position)

Figure 5-18: Resolution Test of CSG-2, Serial Number 001, Y Axis
(10 Arcsecond Step Changes in Azimuth Position)

Figure 5-19: Resolution Test of CSG-2, Serial Number 001, Y Axis
(1 Arcsecond Step Changes in Azimuth Position)

Figure 5-20: Resolution Test of CSG-2, Serial Number 020, X Axis
(10 Arcsecond Step Changes in Azimuth Position)

Figure 5-21: Resolution Test of CSG-2, Serial Number 020, X Axis
(1 Arcsecond Step Changes in Azimuth Position)

Figure 5-22: Resolution Test of CSG-2, Serial Number 020, Y Axis
(10 Arcsecond Step Changes in Azimuth Position)

Figure 5-23: Resolution Test of CSG-2, Serial Number 020, Y Axis
(1 Arcsecond Step Changes in Azimuth Position)

5.3.4 Resolution Tests

Resolution tests were conducted on the CSG-2s to determine the minimum angular increments which could be resolved using the instruments. The gyro was oriented on the motion table so that the axis under test was parallel to the azimuth axis. It was maintained at null using a precision current source. The second axis was controlled using one channel of the RL-3 rebalance loop.

Figures 5-16 to 5-23 show results for 10 arcsecond and 1 arcsecond step increments for both axes of both gyros. Each test consisted of recording the demodulated pickoff output using a 14 bit A/D board. The test began with the gyro in a given position, and then after 1 minute, a step change in position of 10, 5 or 1 arcsecond was commanded to the table. After one minute in this new position, the motion table was commanded to return to its original position. This procedure was repeated for 10 minutes. The graphs show that 10 arcsecond steps are easily resolved by the CSG-2s, and 1 arcsecond steps are still discernible, although the signal-to-noise ratio is becoming small by this point. However, the results indicate that angular changes of at least arcsecond resolution are possible.

Tables 5-2 and 5-3 show the voltage change induced by the 10, 5 and 1 arcsecond step changes for gyro 020. These tables show the voltage deltas at the instance the motion table was commanded to change positions. As evidenced by the graphs, once the motion has stopped and the table is resting in its new position, the gyro output will decay back to null. The time constant of this decay is the same as the time constant of the gyro itself. This behaviour arises due to the fact that the gyro axis is only under partial control.

	10 Arcsecond Step Voltage Change,mV	5 Arcsecond Step Voltage Change, mV	1 Arcsecond Step Voltage Change, mV
1	8.3	4.1	0.9
2	8.2	4.3	0.8
3	8.2	4.4	0.9
4	8.2	4.0	1.0
5	8.3	4.2	1.1
6	8.3	3.9	0.8
7	8.2	4.3	0.9
8	8.2	3.9	0.7
9	8.1	4.0	0.8
10	8.4	4.1	0.9
Mean Volts/Arcsec	0.0008174	0.0008174	0.0008148

Table 5-2: Resolution Test Results for CSG-2, Serial Number 020, X Axis

	10 Arcsecond Step Voltage Change,mV	5 Arcsecond Step Voltage Change, mV	1 Arcsecond Step Voltage Change, mV
1	8.6	4.3	1.1
2	8.2	4.3	1.0
3	8.2	4.0	0.9
4	8.1	4.0	0.9
5	8.8	4.5	0.9
6	8.2	3.9	1.2
7	8.3	4.4	0.8
8	8.1	4.0	0.8
9	8.0	4.1	0.8
10	8.2	4.2	0.7
Mean Volts/Arcsec	0.0008204	0.0008273	0.0008425

Table 5-3: Resolution Test Results for CSG-2, Serial Number 020, Y Axis

The constant current supplied to the torquer does not change during the course of the test, as would normally be the case in an active rebalance loop. Therefore, while the amount of current supplied is sufficient to maintain the gyro at null, once the gyro has moved, it will only return to null at a rate defined by the gyro time constant.

From Tables 5-2 and 5-3, it is seen that the number of volts per arcsecond change is constant for the 10, 5 and 1 arcsecond tests for a given axis. These tests also allow for the calculation of the pickoff scale factor. The demodulator used in these tests was part of the RL-3 circuit. Using a sine wave generator, the gain of the demodulator, in V_{DC} per V_{RMS} was determined to be 0.1316. Using this gain, and the results from the resolution tests, the pickoff scale factor can be determined by working backwards. For example, the pickoff scale factor of gyro 020, X axis, is calculated as follows.

$$k_{POX} = 0.0008174 \frac{V_{DC}}{sec} \times 206.265 \frac{sec}{mRad} \times \frac{V_{RMS}}{0.1316 V_{DC}} = 1.287 \frac{V_{RMS}}{mRad} \quad 5-2$$

Results for the pickoff scale factors are presented in Tables 5-6 and 5-7 of section 5.3.6. Tests were also conducted using the SGL designed demodulator boards and yielded similar results.

5.3.5 Multiposition Tests

Multiposition testing was conducted with both the rack mounted and the SGL designed electronics. The positions defined by the model equations in Chapter 3 were used. For most of the tests conducted, 11 positions tests were used. This allowed for calculation of all of the g sensitive terms, absolute bias, and scale factor, as well as certain key g^2 sensitive terms which are known to affect tuned rotor gyro performance [45].

	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Mean	St. Dev
K_x deg/hr/mA	1654.57	1654.71	1653.78	1654.89	1654.38	1654.89	1654.54	0.4174
K_y deg/hr/mA	1649.34	1649.65	1646.56	1649.09	1648.37	1649.86	1648.81	1.2202
α arcseconds	30.93	182.51	302.27	214.32	255.14	214.73	199.98	92.53
β arcseconds	43.35	145.39	-395.32	73.80	-122.24	105.68	-24.89	203.54
BD_x deg/hr	-1.213	-0.423	-0.467	-0.516	-0.510	-0.485	-0.602	0.3010
BD_y deg/hr	-7.734	-7.551	-7.570	-7.566	-7.553	-7.545	-7.586	0.0731
MU_x deg/hr/g	1.772	1.847	1.804	1.834	1.860	1.881	1.833	0.0394
MU_y deg/hr/g	1.706	1.785	1.731	1.776	1.786	1.804	1.764	0.0377
Q_x deg/hr/g	-0.914	-0.813	-0.804	-0.813	-0.875	-0.895	-0.852	0.0481
Q_y deg/hr/g	-0.889	-0.785	-0.781	-0.777	-0.821	-0.843	-0.816	0.0443
$D(X)_x$ deg/hr/g	-0.051	-0.002	-0.035	-0.0435	-0.0254	-0.0273	-0.0307	0.0170
$D(Y)_x$ deg/hr/g	0.037	0.044	0.021	0.045	0.031	0.048	0.0376	0.0100
$D(X)_{xx}$ deg/hr/g ²	-0.012	0.0051	0.0052	0.0052	0.0029	-0.0093	-0.0006	0.0081
$D(Y)_{yy}$ deg/hr/g ²	-0.037	-0.025	-0.031	-0.036	-0.044	-0.036	-0.0339	0.0067
$D(X)_{xy}$ deg/hr/g ²	0.019	0.034	0.019	0.0174	0.037	0.028	0.0258	0.0085
$D(Y)_{xy}$ deg/hr/g ²	-0.045	-0.051	-0.050	-0.042	-0.049	-0.041	-0.0464	0.0044

Table 5-4: Results from 6 Separate 11 Position Tests Conducted on CSG-2, S/N 020

	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Mean	St. Dev
K_x deg/hr/mA	1483.06	1480.73	1481.14	1482.48	1482.96	1482.30	1482.11	0.9629
K_y deg/hr/mA	1467.34	1467.25	1464.41	1467.31	1467.34	1465.93	1466.60	1.2047
α arcseconds	-105.63	-47.53	160.91	-5.56	-28.63	41.93	2.582	91.518
β arcseconds	2532.03	2616.84	1984.84	2317.83	2506.30	2420.02	2396.31	225.912
BD_x deg/hr	2.308	2.167	1.748	1.786	1.787	1.569	1.895	0.2812
BD_y deg/hr	-2.537	-2.728	-2.147	-2.111	-2.149	-1.908	-2.263	0.3006
MU_x deg/hr/g	0.539	0.522	0.567	0.544	0.519	0.537	0.538	0.0174
MU_y deg/hr/g	0.512	0.519	0.526	0.524	0.496	0.504	0.514	0.0117
Q_x deg/hr/g	-0.987	-0.982	-0.965	-0.982	-0.978	-0.973	-0.978	0.0082
Q_y deg/hr/g	-0.829	-0.832	-0.811	-0.808	-0.817	-0.816	-0.819	0.0095
$D(X)_x$ deg/hr/g	0.060	0.109	-0.051	-0.017	0.0057	-0.032	0.0125	0.0609
$D(Y)_y$ deg/hr/g	0.199	0.114	0.131	0.157	0.141	0.133	0.146	0.0295
$D(X)_{xx}$ deg/hr/g ²	-0.0091	-0.019	0.0011	-0.0067	-0.019	-0.0023	-0.0092	0.0085
$D(Y)_{yy}$ deg/hr/g ²	0.036	0.035	0.027	0.042	0.036	0.026	0.0337	0.0061
$D(X)_{xy}$ deg/hr/g ²	0.023	0.063	0.020	-0.0076	-0.018	0.020	0.0169	0.0283
$D(Y)_{yx}$ deg/hr/g ²	-0.0040	0.082	-0.051	-0.033	0.0005	-0.051	-0.0095	0.0499

Table 5-5: Results from 6 Separate 11 Position Tests Conducted on CSG-2, S/N 001

Tables 5-4 and 5-5 summarise the results of six separate multiposition tests conducted on both CSG-2s. Test-to-test repeatability is indicated by the standard deviations of the six tests. Most of the error parameters show good repeatability and stability, with small standard deviations relative to the size of the parameter. However, two error parameters do not exhibit the same level of repeatability: bias and torquer misalignment angles. Absolute bias was determined to have a large standard deviation, approximately 0.3 deg/hr for all axes except the Y axis of CSG-2, serial number 020. There are two main reasons for this. First, the gyroscopes themselves were turned off between each test. This indicates that each time the gyro is turned back on, slight differences in the absolute bias occurs. Second, between certain tests (e.g. between the tests labelled Test 1 and Test 2 from Table 5-4), the block heaters which maintain the gyro at the operating temperature of 68.4°C were shut off. This thermal cycling appears to have a much more pronounced effect on bias shifts. The effect of thermal cycling on CSG-2 bias terms is discussed further in section 5.4.2., when the results of multiposition testing of the gyroscopes within the system ISA are discussed. The torquer misalignment angles appear to be susceptible to the same effects which cause bias shifts. Since these shifts in the error parameters occur capriciously, there would appear to be no method of predicting what these values will be for a given turn on of the heaters, and to a lesser degree, turn-on of the gyro. Therefore, it may be necessary to re-calibrate the gyros to determine the bias each time the heaters are turned off and on once they are installed in the airborne gravimeter.

5.3.6 Servo Tests

The primary goal of servo testing was to demonstrate that the CSG-2 can be used as a platform stabiliser. The act of slaving the gyro to the servo of the motion table is conceptually identical to using the CSG-2 to maintain the inertial position of the stabilised platform. Therefore, demonstrating this capability within a controlled laboratory environment shows that the CSG-2s are usable in the airborne gravimeter. Servo tests will also indicate the noise level which can be expected when the CSG-2 operates within a servo loop. If the noise level is too high, this might indicate that the output signal from the gyro is not capable of the resolutions required in the airborne gravimeter. Finally, servo tests are another method of determining the first order error terms of a gyro, namely the absolute bias and g sensitive bias terms. These are used as a comparison to the multiposition tests.

One of the challenges of servo testing was to initially close the loop around the gyro. The Contraves motion table had never been used to servo a two degree of freedom gyro of any kind. Furthermore, the bandwidth of the servo module had originally only been designed for 400 Hz to 10 kHz input signals. Since the CSG-2's excitation frequency is 54 kHz, it was necessary to make adjustments to the servo module. A voltage divider to attenuate the signal gain was used prior to injecting the signal into the MPACS module. Adjustments in the lead/lag settings and phasing of the signal were also necessary. After much experimentation and tuning, a stable, stiff servo control of the input axis under test was obtained. Note that testing was conducted with the servoed axis

Figure 5-24: Servo Test of CSG-2, Serial Number 001, X Input Axis
(Input Axis Horizontal, SGL Electronics Used)

Figure 5-25: Servo Test of CSG-2, Serial Number 001, X Input Axis
(Input Axis Vertical, SGL Electronics Used)

Figure 5-26: Servo Test of CSG-2, Serial Number 001, Y Input Axis
(Input Axis Horizontal, SGL Electronics Used)

Figure 5-27: Servo Test of CSG-2, Serial Number 001, Y Input Axis
(Input Axis Vertical, SGL Electronics Used)

Figure 5-28: Servo Test of CSG-2, Serial Number 020, X Input Axis
(Input Axis Horizontal, SGL Electronics Used)

Figure 5-29: Servo Test of CSG-2, Serial Number 020, X Input Axis
(Input Axis Vertical, SGL Electronics Used)

Figure 5-30: Servo Test of CSG-2, Serial Number 020, Y Input Axis
(Input Axis Horizontal, SGL Electronics Used)

Figure 5-31: Servo Test of CSG-2, Serial Number 020, Y Input Axis
(Input Axis Vertical, SGL Electronics Used)

aligned with the motion table azimuth axis, while the second input axis was controlled by one channel of the RL-3 rebalance loop.

Figures 5-24 and 5-25 show a typical servo test of the X axis of CSG-2, serial number 001. These two tests were conducted with the input axis horizontal and vertical, respectively. The procedure used to collect data was slightly different depending upon the orientation of the input axis. When it was vertical, the gyro was allowed to servo for a long period of time (over 60 hours). This provided an indication of the long term stability of the servo control. When the test was conducted with the axis horizontal, the primary goal was to gather data to be used to determine gyro error terms. To do this, it was only necessary to collect servo data when the azimuth axis was near the 0° , 90° , 180° and 270° positions. Using the model equations for the CSG-2 developed in Chapter 3, along with the discussions on error terms from Chapter 2, it is seen that when the X input axis is horizontal and the Y input axis is vertical, the quadrature term Q_x will arise. This corresponds to the situation when the azimuth axis is at 90° or 270° during an X axis horizontal servo test. Therefore, the method of data collection was to place the azimuth axis a few degrees away from 90° (6° for this gyro, at 84° azimuth) and allow the gyro to servo for about an hour, until it reached 96° azimuth. These values were chosen based on previous knowledge that the gyro would servo at between 11 and 13 deg/hr for this axis in these orientations. Other gyros, or other axes would have slightly different conditions. Figure 5-24 shows the one hour of data collected at each of these positions. Similarly, at 0° or 180° table azimuth position, the gyro would only be experiencing the effects of Earth rate bias, and the spin axis g sensitive term, since the Q_x term is zero in these orientations.

Data was collected in a similar manner and is shown on Figure 5-24 as well. Using this data, certain error terms can be calculated. First, Q_x is simply half the difference between the maximum and minimum values of Figure 5-24. Using the average values, this is calculated as,

$$Q_x = \frac{11.6916 - 13.5087}{2} = -0.9585 \text{ deg / hr / g} \quad 5-3$$

The negative sign is consistent with the definition of Q_x from the multiposition tests. The bias term and the spin axis g sensitive term, $D(X)_x$, can also be determined using the data from Figure 5-24. Since the fraction of Earth rate that the X axis sees when horizontal is $\omega_e \sin(\lambda) = 10.7007 \text{ deg/hr}$, the bias is given by,

$$B_x = \frac{13.0789 + 12.6785}{2} - 10.7007 = 2.178 \text{ deg / hr} \quad 5-4$$

The spin axis g sensitive drift is determined by taking half the difference between the servo rates obtained when the azimuth axis is at 0° and 180° . Therefore,

$$D(X)_x = \frac{13.0789 - 12.6785}{2} = 0.2002 \text{ deg / hr / g} \quad 5-5$$

Finally, using the data from Figure 5-25, the value of MU_x can be determined. When the X axis is servoed vertically, the mass unbalance term affects the gyro output in all positions. Using the average from Figure 5-25, and the knowledge that when vertical, the input axis senses $\omega_e \cos(\lambda) = 10.5700 \text{ deg/hr}$ due to Earth rate, the mass unbalance term can be solved for as follows.

$$MU_x = 10.5700 + 2.1780 - 12.0482 = 0.6998 \text{ deg / hr / g} \quad 5-6$$

	Litton 1987 [46]	11 Pos. Test Rack Supplies	Servo Test Rack Supplies	11 Pos Test SGL Supplies	Servo Test SGL Supplies
K_x , (deg/hr/mA)	1663.245	1655.242	-	1654.53	-
K_y , (deg/hr/mA)	1656.858	1649.237	-	1648.81	-
α , (arcsec)	1617.7	20.00	-	199.98	-
β , (arcsec)	1651.0	-241.07	-	-24.89	-
BD_x , (deg/hr)	-7.9872	-0.1834	-0.0667	-0.6024	-0.9190
BD_y , (deg/hr)	0.87537	-7.8949	-8.4134	-7.5865	-7.8274
MU_x , (deg/hr/g)	1.8035	2.0760	2.1357	1.8330	1.9521
MU_y , (deg/hr/g)	1.8535	2.0261	1.9264	1.7646	1.6947
Q_x , (deg/hr/g)	-0.8487	-0.7469	-0.8448	-0.8524	-1.0701
Q_y , (deg/hr/g)	0.8836	-0.7302	-0.7093	-0.8161	-0.8244
$D(X)_x$, (deg/hr/g)	-	-0.0361	-0.0411	-0.0307	-0.4480
$D(Y)_x$, (deg/hr/g)	-	0.0885	0.0484	0.0376	0.0428
$D(X)_{xx}$, (deg/hr/g ²)	-	0.0436	-	-0.0006	-
$D(Y)_{yy}$, (deg/hr/g ²)	-	-0.0166	-	-0.0339	-
$D(X)_{xy}$, (deg/hr/g ²)	-	-	-	0.0258	-
$D(Y)_{xy}$, (deg/hr/g ²)	-	-	-	-0.0464	-
k_{POx} , (V_{RMS} /arcsec)	1.278	1.287	-	-	-
k_{POy} , (V_{RMS} /arcsec)	1.283	1.292	-	-	-
Random Drift σ_x , Horz. deg/hr	0.0033	0.0025	-	0.008	-
Random Drift σ_y , Horz. deg/hr	0.0041	0.0045	-	0.0096	-
Time Constant, τ seconds	96	95.7	-	-	-

Table 5-6: Summary of Test Results for CSG-2, Serial Number 020

	Litton 1987 [47]	11 Pos. Test Rack Supplies	Servo Test Rack Supplies	11 Pos Test SGL Supplies	Servo Test SGL Supplies
K_x , (deg/hr/mA)	1487.935	1483.141	-	1482.11	-
K_y , (deg/hr/mA)	1471.015	1467.253	-	1466.60	-
α , (arcsec)	584.4	35.58	-	2.58	-
β , (arcsec)	-1771.5	-2782.7	-	2396.31	-
BD_x , (deg/hr)	-3.1117	3.0042	3.1133	1.8947	2.1780
BD_y , (deg/hr)	-2.6627	-2.3320	-2.2921	-2.2635	-2.7186
MU_x , (deg/hr/g)	0.4546	0.5921	0.5990	0.5382	0.6998
MU_y , (deg/hr/g)	0.4925	0.5745	1.1235	0.5136	0.2566
Q_x , (deg/hr/g)	-0.9410	-0.9730	-0.9639	-0.9779	-0.9585
Q_y , (deg/hr/g)	-1.1004	-0.8197	-0.7574	-0.8188	-0.8276
$D(X)_x$, (deg/hr/g)	-	-0.0129	-0.0516	0.0125	0.2002
$D(Y)_y$, (deg/hr/g)	-	0.2223	0.1983	0.1460	0.0614
$D(X)_{xx}$, (deg/hr/g ²)	-	0.0109	-	-0.0092	-
$D(Y)_{yy}$, (deg/hr/g ²)	-	0.0008	-	0.0337	-
$D(X)_{xy}$, (deg/hr/g ²)	-	-	-	0.0168	-
$D(Y)_{yx}$, (deg/hr/g ²)	-	-	-	-0.0095	-
k_{POx} , (V_{RMS} /arcsec)	1.4537	1.4562	-	-	-
k_{POy} , (V_{RMS} /arcsec)	1.4018	1.4136	-	-	-
Random Drift σ_x , Horz. deg/hr	0.0031	0.0041	-	-	-
Random Drift σ_y , Horz. deg/hr	0.0048	0.0053	-	-	-
Time Constant, τ seconds	85	84.2	-	-	-

Table 5-7: Summary of Test Results for CSG-2, Serial Number 001

Similar calculations can be made for the other input axes, using data found in Figures 5-26 to 5-31. A summary of the results is found in Tables 5-6 and 5-7, both for tests done using rack mounted supplies, and those done with the SGL electronics.

Results for the error terms calculated from the servo testing are reasonably consistent with those calculated from the multiposition tests, but are not in exact agreement. As previously explained, absolute bias can shift after thermal or power cycling. The g sensitive terms determined are closer in magnitude to those calculated with the multiposition tests. The exception to this is the g sensitive spin axis drift term, which tends to be less predictable during servo tests. Since the values used to calculate the term are very close together in magnitude, any small amount of noise or misalignment of the input axis can have a relatively large effect on the calculation of the drift term.

During long term servo tests, such as in Figure 5-24, a periodic variation is seen. For this particular test, the period is about 30 hours long. This corresponds to the length of time it takes for a complete 360° rotation of the table. Therefore, this sinusoidal variation appears to be a function of the azimuth table position, which seems to indicate a misalignment of the input axis. The effect is much more pronounced for serial number 001 as compared to serial number 020, which shows almost no variation in long term servo rate when the input axis is vertical, which is the expected result. This variation in table servo rate could be due to a couple of different factors. Misalignments of the input axis would result in behaviour consistent with that observed. During the course of testing, it was noted that the tip tilt fixture was not as rigid as would be desired. Since the verniers which position the fixture are spring loaded, as the fixture is reoriented, the self weight can

cause the fixture to shift in position slightly. While the tip tilt fixture is the only method available for aligning the gyro, it would seem that it might also be responsible for introducing misalignment errors. It is hard to assess the magnitude of the errors this would introduce. Note that this would effect not only servo tests, but multiposition tests as well. A redesign of the tip tilt fixture might be worthwhile if future tests are to be conducted.

Using a fast Fourier transform (FFT), it is possible to filter out the periodic variation in the servo rate of Figure 5-25. This result is shown in Figure 5-32. The noise amplitude on the signal is less than 0.04 deg/hr peak-to-peak, and the rate itself gives no indication of trending. Part of this noise on the servo rate is due to the MPACS control modules themselves. Since the MPACS is a general purpose test table, the modules were designed for a wide range of possible gyros. Although tuneable, the modules are not optimised in any way, and do not contain any significant filtering. Another method of assessing noise levels, and therefore quality, of the CSG-2s is to determine their long term random drift.

5.3.7 Random Drift Determination

Random drift is any drift observed in a gyro's output signal which cannot be accounted for by the gyro model equations. Since it is not modelled, it cannot be compensated, and therefore represents a limiting factor in gyro performance. Large random drifts result in degraded performance. As a rule of thumb, gyros with random drifts of less than 0.01 deg/hr are classified as inertial grade instruments, indicating that they are of high quality and are suitable for use in precision inertial systems.

FIGURE 5-32: Servo Test of CSG-2, Serial Number 901, X Axis Vertical
(Filtered Data from Fig 5-25, to remove variation with azimuth angle)

Figure 5-33: Long Term Random Drift Test of CSG-2, Serial Number 020
(X Torquer Voltage measured from RL-3)

Figure 5-34: Long Term Random Drift Test of CSG-2, Serial Number 020
(Y Torquer Voltage measured from RL-3)

Figures 5-33 and 5-34 show a long term random drift test of CSG-2, serial number 020. Both input axes lie in the horizontal plane for this test. The test involved running the gyro in closed loop manner, using RL-3, and monitoring the torquer current. Since the gyro is held stationary for the entire test, the output signal ideally should remain constant. The actual magnitude of the current is not important in these tests, just the change in current necessary to rebalance the gyro. Also, the long term trend is of more interest in these tests, as opposed to the spikes obtained from reading to reading. With this in mind, the average random drift is seen to be 0.008 deg/hr for the X torquer and 0.0096 deg/hr for the Y torquer. These are very low values, which indicate good gyro quality. Furthermore, this drift rate was determined over a long period of time, which indicates good stability of the gyro output.

5.4 Inertial Sensor Assembly Testing

5.4.1 Accelerometer Misalignments within the ISA

Multiposition testing of the gravimeter inertial sensor assembly was conducted to determine misalignment angles of the accelerometers with respect to the system axes. Equations from section 3.5.1 were used to solve for the parameters. The measured misalignments of the reference mirrors are used to relate the instrument misalignments to a fixed reference frame. During data collection, the gyros were run closed loop, even though no data was collected from these instruments. This was done to ensure that the heat flow patterns in the sensor blocks would be the same as in the final system.

Figure 5-35: LSF Temperature Variation During Multiposition Test

Figure 5-36: X Accelerometer Temperature Variation During Multiposition Test

Figure 5-37: Y Accelerometer Temperature Variation During Multiposition Test

Figure 5-38: USF Temperature Variation During Multiposition Test

Figure 5-39: Z Accelerometer Temperature Variation During Multiposition Test

Figure 5-40: Temperature Profile of Navlab Room Temperature
(Measured above the Contraves Motion Table)

Figure 5-41: X Accelerometer Output when IGS is Horizontal

Figure 5-42: Y Accelerometer Output when IGS is Horizontal

Figure 5-43: Z Accelerometer Output when IGS is Horizontal

Before testing began, the effect of rotating the motion table on the ISA temperatures was investigated. Figures 5-35 to 5-39 show the temperature changes as the ISA is rotated from being upright, to being inverted and finally to being horizontal. The temperatures from all three accelerometers, as well as the LSF and USF thermistors are presented. An important observation from these graphs is that for the ten to fifteen minutes following a position change, the temperature behaviour goes through a transient. After this transient settles out, the temperature is stable to better than one one hundredth of a degree Celsius. This suggests that during data collection, a fifteen minute settling period should be used before data collection begins. A second important temperature effect is observed in Figure 5-40, which shows the variation in room temperature over the course of a couple of hours. A sinusoidal variation of 1.5°C for a period of 10 to 15 minutes is observed. Comparing this to Figures 5-35 to 5-39, it is seen that very little of this behaviour affects the ISA temperature. However, the accelerometer output, especially the Z accelerometer in the horizontal orientation, is seen to vary with the same period, as shown in Figures 5-41 to 5-43. This variation does not appear to be due to the sensor fixture blocks themselves changing temperature, since this would show up on one of the temperature sensors. However, room temperature variations can be causing movement of the ISA mounting fixture itself, due to thermal contractions and expansions. This movement would only have to be a couple arcseconds to produce this type of behaviour. The theodolite was used to try to verify this hypothesis, but as the accuracy of the theodolite is only 2 to 3 arcseconds, results were inconclusive. To compensate for

	Mean Values	Standard Deviations
Scale Factor, K_x , mA/g	1.303538797	2.3478×10^{-5}
Scale Factor, K_y , mA/g	1.322520902	4.8039×10^{-6}
Scale Factor, K_z , mA/g	1.269503647	1.7537×10^{-5}
Bias, B_x , g	0.000765621	1.7982×10^{-6}
Bias, B_y , g	0.001533689	3.8837×10^{-6}
Bias, B_z , g	-0.000143458	9.2156×10^{-6}
α_x	0° 4' 51.23"	1.144"
β_x	90° 1' 40.72"	1.3771"
γ_x	90° 1' 0.51"	2.986"
α_y	90° 2' 9.78"	1.448"
β_y	179° 56' 23.67"	0.660"
γ_y	90° 4' 36.69"	1.415"
α_z	89° 55' 33.27"	1.656"
β_z	89° 56' 39.78"	1.347"
γ_z	179° 55' 5.12"	1.492"

Table 5-8: Mean Results from 10 Multiposition Tests Conducted on Q-Flex Accelerometers Mounted in IGS

	Mean Values	Standard Deviations
Scale Factor, K_x , mA/g	1.303520018	1.6282×10^{-5}
Scale Factor, K_y , mA/g	1.322515723	4.5743×10^{-7}
Scale Factor, K_z , mA/g	1.269490476	1.1410×10^{-7}
Bias, B_x , g	0.000765236	7.2583×10^{-7}
Bias, B_y , g	0.001532537	1.0116×10^{-6}
Bias, B_z , g	-0.000146774	3.8237×10^{-6}
α_x	0° 4' 52.45"	1.420"
β_x	90° 1' 40.83"	0.184"
γ_x	90° 0' 59.33"	1.034"
α_y	90° 2' 9.70"	0.177"
β_y	179° 56' 23.07"	0.539"
γ_y	90° 4' 36.12"	0.778"
α_z	89° 55' 31.83"	1.717"
β_z	89° 56' 39.12"	0.576"
γ_z	179° 55' 5.95"	0.967"

Table 5-9: Mean Results from 3 Sets of Multiposition Tests Conducted on Q-Flex Accelerometers Mounted in IGS

this, data was collected over a 10 minute period at each position of the multiposition test to average out the effect of this.

The first set of multiposition tests was conducted over a two week period in which the ISA heaters were kept continuously running. Only the instruments were turned off between test runs. Table 5-8 shows the average values for the ten multiposition tests conducted during this period. The average calculated values of accelerometer misalignments, bias, and scale factor are presented with the standard deviations. The largest standard deviation of any of the misalignment angles is less than three arcseconds, and most are even smaller in value. Since the quoted accuracy of the motion table's positioning capability is only 3 arcseconds, it is unreasonable to expect better results with the test equipment used.

To investigate the effect of turning the heaters off and on again (thermal cycling), two more sets of multiposition tests were conducted. Between each set of tests, the block heaters were turned off, the sensor blocks were allowed to cool to room temperature over night before turning the heaters back on again. Tables 5-9 shows the averages and standard deviations for these three sets of tests. The standard deviations are once again less than three arcseconds for all of the misalignment angles. This shows that thermal cycling has no effect on the calculated values of the misalignment angles or the bias or scale factors.

5.4.2 Gyroscope Misalignments within the ISA

Multiposition testing of the gravimeter inertial sensor assembly was conducted to determine misalignment angles of the gyroscope sensor axes with respect to the system

axes. Equations from section 3.5.2 were used to solve for the parameters. The measured misalignments of the reference mirrors are used to relate the instrument misalignments to a fixed reference frame.

Originally, it was planned to simultaneously test both gyroscopes. To do this, it was necessary to use a second rebalance loop, RL-2, to control one of the gyroscopes. Functionally, RL-2 and RL-3 are identical, except that RL-3 includes some cross axis coupling of the rebalance loops. Previous experience with these two loops has revealed little performance difference between the two loops in static testing. However, once multiposition testing of the gravimetry ISA began, it was realised that the data collected for gyro 001, which was controlled with RL-2, was 2 to 3 times noisier than data collected for gyro 020. Since previous testing had revealed no such difference between individual gyros, it was concluded that RL-2 was responsible for the increased noise level. Therefore, two separate test runs had to be conducted, one on gyro 020 using RL-3, and a second, identical series on gyro 001 using RL-3.

Table 5-10 shows the average values and standard deviations of a series of multiposition tests, as well as the bias, scale factor, first order drift terms and misalignment angles for both gyroscopes within the gravimeter. These series of tests were conducted without shutting down the heaters between tests, although the gyro motors themselves were turned off. The standard deviations on the bias, g sensitive terms, and scale factor terms are relatively small, being the same order of magnitude as the results gathered from individual gyro testing (see section 5.3.5). The misalignment angle standard deviations appear to be very large, especially when compared to the high precision with which the

CSG-2, Serial Number 020					
Axis 1 (X axis)			Axis 2 (Y axis)		
Parameter	Mean	St. Dev	Parameter	Mean	St. Dev
K_1	1650.09	1.387	K_2	1654.51	2.446
B_1	1.0509	0.0252	B_2	-7.7809	0.0234
MU_1	2.0948	0.0621	MU_2	1.8402	0.0389
Q_1	-0.9962	0.0167	Q_2	-0.8898	0.0324
$D(X)_{s1}$	-0.0312	0.0309	$D(Y)_{s2}$	-0.0266	0.0557
α_1	89°52'33.58"	164.88"	α_2	90°6'7.94"	248.51"
β_1	179°45'42.41"	261.32"	β_2	89°46'1.36"	230.36"
γ_1	89°48'25.10"	317.94"	γ_2	0°16'3.79"	115.58"
CSG-2, Serial Number 001					
Axis 3 (X axis)			Axis 4 (Y axis)		
Parameter	Mean	St. Dev	Parameter	Mean	St. Dev
K_3	1465.85	1.174	K_4	1487.27	3.233
B_3	-1.7931	0.0096	B_4	1.1709	0.0135
MU_3	0.4045	0.0144	MU_4	0.2253	0.0329
Q_3	-0.9203	0.0186	Q_4	-0.9864	0.0142
$D(X)_{s3}$	-0.0341	0.0105	$D(Y)_{s4}$	0.0169	0.0107
α_3	179°30'5.89"	146.39"	α_4	90°17'5.06"	141.34"
β_3	90°0'50.43"	165.73"	β_4	89°54'16.71"	77.12"
γ_3	89°30'12.32"	139.84"	γ_4	0°18'3.41"	139.54"

Table 5-10: Mean results from 10 multiposition tests conducted on CSG-2 gyroscopes mounted in IGS

CSG-2, Serial Number 020					
Axis 1 (X axis)			Axis 2 (Y axis)		
Parameter	Mean	St. Dev	Parameter	Mean	St. Dev
K ₃	1465.85	1.174	K ₄	1487.27	3.233
B ₃	-1.7931	0.0096	B ₄	1.1709	0.0135
MU ₃	0.4045	0.0144	MU ₄	0.2253	0.0329
Q ₃	-0.9203	0.0186	Q ₄	-0.9864	0.0142
D(X) _{s3}	-0.0341	0.0105	D(Y) _{s4}	0.0169	0.0107
α_3	179°30'5.89"	146.39"	α_4	90°17'5.06"	141.34"
β_3	90°0'50.43"	165.73"	β_4	89°54'16.71"	77.12"
γ_3	89°30'12.32"	139.84"	γ_4	0°18'3.41"	139.54"
CSG-2, Serial Number 001					
Axis 3 (X axis)			Axis 4 (Y axis)		
Parameter	Mean	St. Dev	Parameter	Mean	St. Dev
K ₃	1467.46	1.0318	K ₄	1490.51	3.0457
B ₃	-1.7173	0.0093	B ₄	1.8146	0.0257
MU ₃	0.3847	0.0031	MU ₄	0.1866	0.0153
Q ₃	-0.9055	0.0115	Q ₄	-1.0149	0.0126
D(X) _{s3}	-0.0530	0.0245	D(Y) _{s4}	0.0142	0.0076
α_3	179°27'32.09"	203.72"	α_4	90°18'25.71"	123.64"
β_3	89°59'3.01"	18.71"	β_4	89°56'10.06"	125.80"
γ_3	89°27'32.97"	203.38"	γ_4	0°18'55.90"	106.73"

Table 5-11: Mean results from 2 sets of multiposition tests conducted on CSG-2 serial number 001 gyroscope mounted in IGS, after a thermal cycle

accelerometer misalignment angles are known. However, these values are consistent with those determined in individual gyro testing. It would appear that the misalignment angles are susceptible to change from turn on to turn on, and that absolute knowledge of the angles is only determinable to a few arcminutes between turn-ons. This is consistent with other tests which have been conducted on the CSG-2 [48].

A second set of tests was conducted on the gyros, after a complete thermal cycle. These results are presented in Table 5-11. This shows that the effect of thermal cycling can have a profound effect on the value of certain error parameters, namely the absolute bias terms. However, the standard deviations for this set of tests is the same order of magnitude as those from the original set of tests. This indicates that once the sensor block is at operating temperature, the error parameters will stay constant as long as the temperature is maintained. Turning the gyros on and off have little effect on the values of the bias, g sensitive terms, and scale factor.

5.5 Summary

A QA-2000 accelerometer and two CSG-2 tuned rotor gyroscopes were tested to determine their error characteristics. Tests revealed that the accelerometer is capable of resolving input changes of less than a micro-g, and the gyroscopes can resolve angular changes of less than an arcsecond. The instruments were then mounted within the gravimeter sensor blocks and multiposition tests were conducted on the entire assembly to determine the error parameters as they will appear within the gravimeter.

6. Conclusions and Recommendations

6.1 Conclusions

This thesis has focused on characterisation of an inertial sensor assembly (ISA) for use in an airborne gravimetry system. While the idea of placing a gravimeter on an aircraft is not a new one, it was an unattainable goal given the state of technology previously. However, recent developments have allowed this prototype gravimeter to be considered. First, accurate altitude determination is achieved through the use of Differential GPS, which allows height determination down to the centimetre level. Second, precise measurement of changes in gravity of less than a micro g is required, through the use of state of the art inertial components. Information from these two separate systems is

integrated using today's high speed computers to process the data in real time. All of these components are vital to the successful implementation of an airborne gravimeter.

Two separate phases of testing were conducted. Individual accelerometers and gyroscopes were tested to determine the performance of the instruments and their suitability for use in the gravimetry project. Following this, the instruments were mounted into the gravimeter's inertial sensor blocks and the instrument cluster was tested to determine error terms and misalignment angles.

Accelerometer testing revealed that:

1. The QA-2000 is capable of reliably and repeatedly detecting changes in acceleration of less than $0.75\mu g$. Noise levels in the data collection instruments prevented detection of smaller changes in input.
2. The output from the QA-2000 is very sensitive to changes in temperature. An output change of $159.35 \text{ nA}/^{\circ}\text{C}$ was measured. This indicates that the temperature should be controlled to better than 0.008°C to keep the accelerometer output stable to better than $1\mu g$.
3. Using the temperature model for the QA-2000, it was demonstrated that the accelerometer output was stable to 4 nA peak to peak over a 48 hour period.
4. The bias and scale factor magnitudes were determined.

Gyroscope testing was conducted on both CSG-2, serial numbers 001 and 020, using a Contraves motion table for accurate positioning. Tests revealed that:

1. The CSG-2 is capable of repeatedly resolving angular changes of 1 arcsecond.

2. The long term random drift of the sensitive axes of the two CSG-2s tested was less than 0.008 deg/hr in the worst case.
3. Starting the gyroscope spin motor at 8 V_{RMS} and running it at 5 V_{RMS} helps to reduce power consumption while still ensuring reliable operation of the gyroscope.
4. The CSG-2 was slaved to the Contraves Motion Table, demonstrating its ability to be used as a method of stabilising the gravimetry platform.
5. The error terms were determined through both closed loop and a servo loop methods. Measuring the torquer currents generated by a rebalance loop and using multiposition tests to determine the error terms was the preferred method of characterising the gyro.

The instruments were mounted in the sensor blocks, which are housed in the inner gimbal simulator and mounted on the motion table. Mounting fixtures and breakout panels were designed. A table alignment procedure was developed to align the motion table to better than 9 arcseconds with respect to the geographic frame of reference. The table was levelled to better than three arcseconds.

Model equations were developed to solve for the misalignment angles and error terms of the accelerometers and gyroscopes mounted in the sensor blocks. Using the results from multiposition testing and optical determination of the orientation of the reference mirrors, it was possible to relate the positions of the sensor axes to those of the mirrors.

Testing of the accelerometers within the sensor block revealed:

1. Misalignment angles with respect to the system axes were determined, with a standard deviation of less than 3 arcseconds.

2. Inversion of the sensor fixture results in a 10 to 15 minute temperature transient before thermal control becomes stable to better than 0.01°C once again.
3. Thermal cycling had no effect on the calculated values of the misalignment angles, bias or scale factor.

Testing of the gyroscopes within the sensor block revealed:

1. Misalignment angles of the gyroscope torquer axes and the effective input axes were determined, with a standard deviation of less than 3 arcminutes. This is consistent with the size of the deviations obtained in individual gyroscope testing.
2. Error terms were determined, with standard deviations of less than 0.05 deg/hr . With the exception of absolute bias, all the error terms had magnitudes in close agreement with those determined in individual gyro testing.
3. Bias magnitudes were stable provided that the sensor block heaters remained on at all times. If the block was allowed to cool to room temperature and then brought back up to operating temperature, the bias would shift unpredictably. Turning the gyro motor on and off had no such effect.

6.2 Recommendations for Future Work.

The inertial sensor assembly has been tested and characterised, and is ready for installation into the gimballed platform. Further work is still necessary before the gravimeter is ready for a test flight.

1. Installation of the ISA into the inner gimbal member.
2. Verification of servo control of the inner stable member. This may involve a few simple tests to demonstrate that the servo control is stable, or it could require

extensive work to tune the servo controller. Adequate safety features should be included to prevent damage to the gyros. A rebalance loop will need to be designed to control the redundant axis in the gravimeter system.

3. Determination of the misalignment angles of the sensor axes with respect to the gimbal resolver angles. An autocollimating theodolite and the reference mirrors will be used to relate the sensor block orientation to the resolver angles.
4. Performance tests should be conducted to determine the stability of the servo control loop and determine if angular drift will be an issue.
5. Initialisation procedures for the gimbaled system need to be developed. As demonstrated in testing, gyro bias will vary significantly if the heaters in sensor block have been turned off and on again. Therefore, a multiposition test will be required, using the gimbaled platform itself to orient the instruments.
6. A method of determining the position of the gravimeter in the aircraft needs to be determined. Autocollimation will be used to determine its orientation.
7. Flight testing over previously surveyed areas will demonstrate system performance.

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APPENDIX A: CSG-2 Multiposition Equations

From Section 3.4, the solutions for the torquer misalignment angles, α and β , can be developed, along with the torquer scale factors, K_x and K_y . By using small angle approximations, equations 3-15 and 3-16 can be rewritten for the first four positions of a multiposition test.

$$\begin{aligned} K_y I_{y1} &= B_x + D(X)_s + D(X)_m + \omega_e \cos \lambda \\ K_x I_{x1} - \beta K_y I_{y1} &= B_y + D(Y)_s + D(Y)_m + \alpha \omega_e \cos \lambda \end{aligned} \quad \text{A-1}$$

$$\begin{aligned} K_y I_{y2} &= B_x + D(X)_s + D(X)_m + \alpha \omega_e \cos \lambda \\ K_x I_{x2} - \beta K_y I_{y2} &= B_y + D(Y)_s + D(Y)_m - \omega_e \cos \lambda \end{aligned} \quad \text{A-2}$$

$$\begin{aligned} K_y I_{y3} &= B_x + D(X)_s + D(X)_m - \omega_e \cos \lambda \\ K_x I_{x3} - \beta K_y I_{y3} &= B_y + D(Y)_s + D(Y)_m - \alpha \omega_e \cos \lambda \end{aligned} \quad \text{A-3}$$

$$\begin{aligned} K_y I_{y4} &= B_x + D(X)_s + D(X)_m - \alpha \omega_e \cos \lambda \\ K_x I_{x4} - \beta K_y I_{y4} &= B_y + D(Y)_s + D(Y)_m + \omega_e \cos \lambda \end{aligned} \quad \text{A-4}$$

where the first equation in each pair refers to the X input axis and the second equation of the pair refers to the Y input axis. Refer to Table 3-1 to determine the orientation of the gyro input axes in a given table position.

By subtracting the X axis equations of A-3 from A-1, the Y torquer scale factor is easily obtained as:

$$K_y = \frac{2\omega_e \cos \lambda}{(I_{y1} - I_{y3})} \quad \text{A-5}$$

Subtracting the X axis equations of A-2 from A-4 and substituting in the expression for the Y torquer scale factor, A-5, yields:

$$\alpha = \frac{(I_{y2} - I_{y4})}{(I_{y1} - I_{y3})} \quad \text{A-6}$$

Turning to the Y axis equations, the X axis torquer scale factor can be determined by subtracting A-4 from A-2.

$$K_x(I_{x4} - I_{x2}) - \beta K_y(I_{y4} - I_{y2}) = 2\omega_e \cos \lambda \quad \text{A-7}$$

Substituting equations A-5 and A-6 into A-7 yields:

$$K_x = \frac{2\omega_e \cos \lambda}{(I_{x4} - I_{x2})} (1 - \alpha\beta) \quad \text{A-8}$$

Since the $\alpha\beta$ term is very small, it is usually ignored, leaving an explicit solution for the X torquer scale factor. Finally, to solve for β , subtract the Y axis equation of A-3 from A-1 to obtain,

$$K_x(I_{x1} - I_{x3}) - \beta K_y(I_{y1} - I_{y3}) = 2\alpha\omega_e \cos \lambda \quad \text{A-9}$$

and then substituting into A-9 the expressions for both torquer scale factors yields the solution for β .

$$\beta = \frac{(I_{x1} - I_{x3})}{(I_{x4} - I_{x2})} (1 - \alpha\beta) - \alpha \quad \text{A-10}$$

As mentioned in Section 3-4, further derivations of the error terms assume that the effects of α and β are negligible. To solve for the bias, g sensitive and g^2 sensitive terms, the equations for the multiposition test are defined, using the axes positions from Table 3-1 and equations 3-11 and 3-12, the general rebalance equations for a tuned rotor gyro. The following equations solve for the X input axis error terms. Development of the Y axis error terms is conceptually identical.

$$K_y I_{y1} = B_x + D(X)_x + D(X)_m + \omega_e \cos \lambda \quad A-11$$

$$K_y I_{y2} = B_x + D(X)_x + D(X)_m \quad A-12$$

$$K_y I_{y3} = B_x + D(X)_x + D(X)_m - \omega_e \cos \lambda \quad A-13$$

$$K_y I_{y4} = B_x + D(X)_x + D(X)_m \quad A-14$$

$$K_y I_{y5} = B_x - Q_x + D(X)_{yy} \quad A-15$$

$$K_y I_{y6} = B_x + MU_x + D(X)_{xx} + \omega_e \sin \lambda \quad A-16$$

$$K_y I_{y7} = B_x + Q_x + D(X)_{yy} \quad A-17$$

$$K_y I_{y8} = B_x - MU_x + D(X)_{xx} - \omega_e \sin \lambda \quad A-18$$

$$K_y I_{y9} = B_x - MU_x \cos \frac{\pi}{4} + Q_x \cos \frac{\pi}{4} - \omega_e \sin \lambda \cos \frac{\pi}{4} \\ + D(X)_{xx} \cos^2 \frac{\pi}{4} + D(X)_{yy} \cos^2 \frac{\pi}{4} - D(X)_{xy} \cos^2 \frac{\pi}{4} \quad A-19$$

$$K_y I_{y10} = B_x + MU_x \cos \frac{\pi}{4} + Q_x \cos \frac{\pi}{4} + \omega_e \sin \lambda \cos \frac{\pi}{4} \\ + D(X)_{xx} \cos^2 \frac{\pi}{4} + D(X)_{yy} \cos^2 \frac{\pi}{4} + D(X)_{xy} \cos^2 \frac{\pi}{4} \quad A-20$$

$$\begin{aligned}
 K_y I_{y11} = & B_x + MU_x \cos \frac{\pi}{4} - Q_x \cos \frac{\pi}{4} + \omega_e \sin \lambda \cos \frac{\pi}{4} \\
 & + D(X)_{xx} \cos^2 \frac{\pi}{4} + D(X)_{yy} \cos^2 \frac{\pi}{4} - D(X)_{xy} \cos^2 \frac{\pi}{4}
 \end{aligned}
 \tag{A-21}$$

$$\begin{aligned}
 K_y I_{y12} = & B_x - MU_x \cos \frac{\pi}{4} - Q_x \cos \frac{\pi}{4} - \omega_e \sin \lambda \cos \frac{\pi}{4} \\
 & + D(X)_{xx} \cos^2 \frac{\pi}{4} + D(X)_{yy} \cos^2 \frac{\pi}{4} + D(X)_{xy} \cos^2 \frac{\pi}{4}
 \end{aligned}
 \tag{A-22}$$

$$K_y I_{y13} = B_x - D(X)_s + D(X)_m - \omega_e \cos \lambda \tag{A-23}$$

$$K_y I_{y14} = B_x - D(X)_s + D(X)_m \tag{A-24}$$

$$K_y I_{y15} = B_x - D(X)_s + D(X)_m + \omega_e \cos \lambda \tag{A-25}$$

$$K_y I_{y16} = B_x - D(X)_s + D(X)_m \tag{A-26}$$

$$\begin{aligned}
 K_y I_{y17} = & B_x - MU_x \cos \lambda + D(X)_s \sin \lambda \\
 & + D(X)_{xx} \cos^2 \lambda + D(X)_{yy} \sin^2 \lambda - D(X)_{xy} \cos \lambda \sin \lambda
 \end{aligned}
 \tag{A-27}$$

$$\begin{aligned}
 K_y I_{y18} = & B_x + Q_x \cos \lambda + D(X)_s \sin \lambda \\
 & + D(X)_{yy} \cos^2 \lambda + D(X)_{xx} \sin^2 \lambda + D(X)_{xy} \cos \lambda \sin \lambda
 \end{aligned}
 \tag{A-28}$$

$$\begin{aligned}
 K_y I_{y19} = & B_x + MU_x \cos \lambda + D(X)_s \sin \lambda \\
 & + D(X)_{xx} \cos^2 \lambda + D(X)_{yy} \sin^2 \lambda + D(X)_{xy} \cos \lambda \sin \lambda
 \end{aligned}
 \tag{A-29}$$

$$\begin{aligned}
 K_y I_{y20} = & B_x - Q_x \cos \lambda + D(X)_s \sin \lambda \\
 & + D(X)_{yy} \cos^2 \lambda + D(X)_{xx} \sin^2 \lambda - D(X)_{xy} \cos \lambda \sin \lambda
 \end{aligned}
 \tag{A-30}$$

$$\begin{aligned}
 K_y I_{y21} = & B_x + MU_x \cos \lambda - D(X)_s \sin \lambda \\
 & + D(X)_{xx} \cos^2 \lambda + D(X)_{yy} \sin^2 \lambda - D(X)_{xy} \cos \lambda \sin \lambda
 \end{aligned}
 \tag{A-31}$$

$$\begin{aligned}
 K_y I_{y22} = & B_x - Q_x \cos \lambda - D(X)_s \sin \lambda \\
 & + D(X)_{yy} \cos^2 \lambda + D(X)_{xx} \sin^2 \lambda + D(X)_{xy} \cos \lambda \sin \lambda
 \end{aligned}
 \tag{A-32}$$

$$K_y I_{y23} = B_x - MU_x \cos \lambda - D(X)_s \sin \lambda + D(X)_{xx} \cos^2 \lambda + D(X)_{yy} \sin^2 \lambda + D(X)_{xy} \cos \lambda \sin \lambda \quad A-33$$

$$K_y I_{y24} = B_x + Q_x \cos \lambda - D(X)_s \sin \lambda + D(X)_{yy} \cos^2 \lambda + D(X)_{xx} \sin^2 \lambda - D(X)_{xy} \cos \lambda \sin \lambda \quad A-34$$

As stated in Section 3-4, the $D(X)_{yy}$ term cannot be solved for using static testing. The reason for this is that no matter what combination of equations are chosen, $D(X)_{xx}$ and $D(X)_{yy}$ will always appear together and cannot be isolated. Therefore, one of the terms must be assumed negligible. As discussed in Chapter 3, it is reasonable to assume that $D(X)_{yy}$ is very small. Therefore, this term is dropped from equation A-11 to A-34, and all of the other error terms can then be isolated.

The g sensitive terms are isolated first. Subtracting A-15 from A-17 gives

$$Q_x = \frac{K_y}{2} (I_{y7} - I_{y5}) \quad A-35$$

Similarly, subtracting A-18 from A-16 results in

$$MU_x = \frac{K_y}{2} (I_{y6} - I_{y8}) - \omega_c \sin \lambda \quad A-36$$

The final g sensitive term is solved for by adding equations A-11 to A-14 and then subtracting A-23 to A-26.

$$D(X)_s = \frac{K_y}{8} (I_{y1} + I_{y2} + I_{y3} + I_{y4} - I_{y13} - I_{y14} - I_{y15} - I_{y16}) \quad A-37$$

Alternatively, $D(X)_s$ can be isolated by subtracting equation A-13 from A-3. This is the solution which must be used during an 11 position test.

$$D(X)_s = \frac{K_y}{2} (I_{y3} - I_{y13}) \quad A-38$$

Next, the absolute bias term is easily isolated by adding equations A-17 and A-15.

$$B_x = \frac{K_y}{2}(I_{y7} + I_{y5}) \quad A-39$$

Finally, the g^2 sensitive terms can be isolated. Adding equations A-18 and A-16 and then subtracting A-17 and A-15 gives,

$$D(X)_{xx} = \frac{K_y}{2}(I_{y8} + I_{y6} - I_{y5} - I_{y7}) \quad A-40$$

Adding equations A-11 and A-23 and subtracting A-15 and A-17 gives,

$$D(X)_{xx} = \frac{K_y}{2}(I_{y1} + I_{y13} - I_{y5} - I_{y7}) \quad A-41$$

Adding equations A-19 and A-21 and subtracting A-18 and A-20 gives,

$$D(X)_{xy} = \frac{K_y \cos^2 \frac{\pi}{4}}{4}(I_{y9} + I_{y11} - I_{y10} - I_{y12}) = \frac{K_y}{2}(I_{y9} + I_{y11} - I_{y10} - I_{y12}) \quad A-42$$

Adding equations A-29 and A-33 and subtracting A-27 and A-31 gives,

$$D(X)_{xx} = \frac{K_y}{4\cos\lambda\sin\lambda}(I_{y19} + I_{y23} - I_{y17} - I_{y21}) \quad A-43$$

Finally, adding equations A-28 and A-32 and subtracting A-30 and A-34 gives,

$$D(X)_{yy} = \frac{K_y}{4\cos\lambda\sin\lambda}(I_{y18} + I_{y22} - I_{y20} - I_{y24}) \quad A-44$$

Similar equations can be developed for the Y axis error terms.

APPENDIX B: ISA Accelerometer Equation Solutions

Equations 3-19 to 3-24 were used to solve for the misalignment angles, bias, and scale factors of all three accelerometers mounted in the Inertial Sensor Assembly used in the gravimeter system. Each equation corresponds to the acceleration output generated by each Q-Flex accelerometer in the given position of the 6 position test. By adding together all 6 equations, it is a simple matter to solve for the bias terms.

$$B_x = \frac{1}{6k_x} [a_{x1} + a_{x2} + a_{x3} + a_{x4} + a_{x5} + a_{x6}] \quad B-1$$

$$B_y = \frac{1}{6k_y} [a_{y1} + a_{y2} + a_{y3} + a_{y4} + a_{y5} + a_{y6}] \quad B-2$$

$$B_z = \frac{1}{6k_z} [a_{z1} + a_{z2} + a_{z3} + a_{z4} + a_{z5} + a_{z6}] \quad B-3$$

All of the a_{mn} terms are known, being the measured accelerometer outputs for axis m in position n . That leaves only the scale factor terms, k_m , unknown on the right hand side. To solve for the scale factor, the following intermediate relationships are defined.

$$A_1 = \frac{a_{x2} - a_{x1}}{2g} = k_x \cos\alpha_x \quad A_4 = \frac{a_{x6} - a_{x4}}{2g} = k_x \cos\alpha_y \quad A_7 = \frac{a_{x5} - a_{x3}}{2g} = k_x \cos\alpha_x \quad B-4$$

$$A_2 = \frac{a_{y2} - a_{y1}}{2g} = k_y \cos\beta_z \quad A_5 = \frac{a_{y6} - a_{y4}}{2g} = k_y \cos\beta_y \quad A_8 = \frac{a_{y5} - a_{y3}}{2g} = k_y \cos\beta_x \quad \text{B-5}$$

$$A_3 = \frac{a_{z2} - a_{z1}}{2g} = k_z \cos\gamma_z \quad A_6 = \frac{a_{z6} - a_{z4}}{2g} = k_z \cos\gamma_y \quad A_9 = \frac{a_{z5} - a_{z3}}{2g} = k_z \cos\gamma_x \quad \text{B-6}$$

Equations B-4 relate to the X accelerometer, equations B-5 to the Y accelerometer and B-6 to the Z accelerometer. Using these relationships, and the fact that the root squared sums of the direction cosines must be one, the scale factors can be determined.

$$k_x = \sqrt{A_1^2 + A_4^2 + A_7^2} \quad \text{B-7}$$

$$k_y = \sqrt{A_2^2 + A_5^2 + A_8^2} \quad \text{B-8}$$

$$k_z = \sqrt{A_3^2 + A_6^2 + A_9^2} \quad \text{B-9}$$

Now that the scale factors are known, they can be substituted back into equations B-1 to B-3 to solve for the accelerometer bias terms. The direction cosines can also now be solved for by using equations B-4 to B-6.

$$\cos\alpha_x = \frac{A_1}{k_x} \quad \cos\alpha_y = \frac{A_4}{k_x} \quad \cos\alpha_z = \frac{A_7}{k_x} \quad \text{B-10}$$

$$\cos\beta_x = \frac{A_8}{k_y} \quad \cos\beta_y = \frac{A_5}{k_y} \quad \cos\beta_z = \frac{A_2}{k_y} \quad \text{B-11}$$

$$\cos\gamma_x = \frac{A_9}{k_z} \quad \cos\gamma_y = \frac{A_6}{k_z} \quad \cos\gamma_z = \frac{A_3}{k_z} \quad \text{B-12}$$

APPENDIX C: ISA Gyroscope Equation Solutions

Equations 3-33 to 3-48 were used to solve for the misalignment angles, bias, g-sensitive terms and scale factors of the X axis of Gyro 020 mounted in the Inertial Sensor Assembly used in the gravimeter system. Each equation corresponds to the torquer current supplied to the CSG-2 in the given position of the 16 position test. By adding or subtracting together combinations of these equations, all of the unknowns can be solved for explicitly. First, some intermediate relationships are set-up,

$$k_1 \cos \alpha_1 = \frac{I_1 + I_7 - I_3 - I_5}{4\omega_e \cos \lambda} \quad \text{C-1}$$

$$k_1 \cos \beta_1 = \frac{I_2 + I_8 - I_4 - I_6}{4\omega_e \cos \lambda} \quad \text{C-2}$$

$$k_1 \cos \gamma_1 = \frac{I_{13} + I_{15} - I_{11} - I_9}{4\omega_e \cos \lambda} \quad \text{C-3}$$

Once again using the fact that the root squared sums of the direction cosines must be equal to 1, the torquer scale factor can be determined.

$$k_1 = \sqrt{(k_1 \cos \alpha_1)^2 + (k_1 \cos \beta_1)^2 + (k_1 \cos \gamma_1)^2} \quad \text{C-4}$$

Using equation C-4, all of the direction cosines of the misalignments can be determined from the relationships in equations C-1 to C-3.

The mass unbalance term can be isolated by adding equations 3-44 and 3-46 and subtracting 3-42 and 3-48 to obtain,

$$MU_1 = \frac{I_{12} + I_{14} - I_{10} - I_{16}}{4k_1} - \omega_e \sin \lambda \cos \beta_1 \quad C-5$$

Similarly, the spin axis g sensitive drift term is isolated by adding equations 3-43 and 3-45 and subtracting 3-41 and 3-47 to obtain,

$$D(X)_{s1} = \frac{I_9 + I_{15} - I_{11} - I_{13}}{4k_1} + \omega_e \sin \lambda \cos \alpha_1 \quad C-6$$

The quadrature term is solved for slightly differently, by adding equations 3-33 to 3-36 and subtracting 3-37 to 3-40 to obtain,

$$Q_1 = \frac{I_1 + I_2 + I_3 + I_4 - I_5 - I_6 - I_7 - I_8}{8k_1} + \omega_e \sin \lambda \cos \gamma_1 \quad C-7$$

Finally, the bias is obtained simply by adding all 16 equations

$$B_1 = \frac{1}{16k_1} \sum_{n=1}^{16} I_n \quad C-8$$

Equations for the Y axis of gyro 020 and for both axes of gyro 001 are very similar to equations C-1 to C-8 and are developed using the same philosophy.

APPENDIX D: Dividing Head Gravity Variation

Consider the situation in which an accelerometer is mounted on a dividing head and is nominally sensing a 1 g input. The dividing head is then rotated through an angle θ , so that its input is now only $g - \Delta g$. From Figure D-1, the following relationship is evident.

$$g \cos \theta = g - \Delta g \quad \text{D-1}$$

$$\therefore \cos \theta = 1 - \frac{\Delta g}{g} \quad \text{D-2}$$

Using equation D-2, it is possible to determine by what angle the dividing head must be rotated for the accelerometer to undergo a given change in input. For instance, for a 7.5 μg change, θ would be

$$\theta = \cos^{-1} \left(1 - \frac{7.5 \times 10^{-6}}{1} \right) = 0.2219^\circ \approx 800'' \quad \text{D-3}$$

Similarly, for a 0.75 μg change, θ would have to be 252".

APPENDIX E: 11 Position Test

The following data file was transferred from the PDP-11 system. It shows the data output format from LOGDAT for the first four positions of an 11 position test.

Data File Name: POS33593A.DAT

Data Collected From Datron 3.3.

AN 11 POSITION TEST OF CSG-2, S/N 020, USING SGL ELECTRONICS
THE TEST USES THE STANDARD POSITIONS DEFINED BY DREO
SEE DATA FILE [7,7]POS11.MTL FOR FURTHER DETAILS
NOTE THAT THIS TEST INVOLVED A SHUTDOWN AFTER
POS33493A.DAT, BUT THAT THE HEATERS WERE ALWAYS ON.

Meter #3 Status:	Voltmeter.	Last Reading:	+0.0000499E-01 V
Range:	100 mVolts	Maths:	Normal
Function:	DC-Volts	B=	+0.0000000E+00
Filter:	Filter	C=	+0.0000000E+00
Delay:	Internal		
Ratio:	Normal Mode	Max/Min:	Normal
Trigger:	Internal		
Average:	Rolling		

Meter #4 Status:	Voltmeter.	Last Reading:	+0.0000757E-01 V
Range:	100 mVolts	Maths:	Normal
Function:	DC-Volts	B=	+0.0000000E+00
Filter:	Filter	C=	+0.0000000E+00
Delay:	Internal		
Ratio:	Normal Mode	Max/Min:	Normal
Trigger:	Internal		

Average: Rolling

MOTION TABLE STATUS PRIOR TO DATA ACQUISITION:

TABLE AXIS	Azimuth (1)	Tilt (2)
CONTROL MODE	POSITION	OFF
POSITION	+90.0018 Degrees	+180.0023 Degrees
RATE	+0.0006 Deg/S	+0.0000 Deg/S
SAMPLE PERIOD	1.0 Seconds	100.0 Seconds

Current Temperature = 68.42 Degrees Celsius

Data acquisition starting DATE and TIME: 01-DEC-93 10:29:41

Data Rate = 2.1127 Hz

Ext. Block Size = 450

--> Samples = 10

Test Duration = 0:35:30

SAMPLE NUMBER	TIME [MINUTES]	MULTIMETER # 3	MULTIMETER # 4
1	3.6166	-0.34473072E-02VC	-0.23466436E-02VC
2	7.2167	-0.34477771E-02VC	-0.23462870E-02VC
3	10.8167	-0.34488042E-02VC	-0.23454689E-02VC
4	14.4166	-0.34458247E-02VC	-0.23452481E-02VC
5	18.0167	-0.34450447E-02VC	-0.23448730E-02VC
6	21.6166	-0.34457596E-02VC	-0.23471370E-02VC
7	25.2167	-0.34482620E-02VC	-0.23452406E-02VC
8	28.8167	-0.34455799E-02VC	-0.23449768E-02VC
9	31.4166	-0.34466645E-02VC	-0.23459639E-02VC
10	34.0167	-0.34470788E-02VC	-0.23457982E-02VC

Rows of Data = 10

Maximum Values	-0.00344504	-0.00234487
----------------	-------------	-------------

Minimum Values	-0.00344880	-0.00234714
----------------	-------------	-------------

Range (P-P)	0.00000376	0.00000226
-------------	------------	------------

Means (Averages)	-0.00344681	-0.00234576
------------------	-------------	-------------

Standard Deviation	0.000001250	0.000000746
--------------------	-------------	-------------

% Deviation	0.036268565%	0.031820257%
-------------	--------------	--------------

% P-P Deviation	0.109072825%	0.096516305%
-----------------	--------------	--------------

Data acquisition completion DATE and TIME: 01-DEC-93 11:05:43

MOTION TABLE STATUS PRIOR TO DATA ACQUISITION:

TABLE AXIS	Azimuth (1)	Tilt (2)
CONTROL MODE	POSITION	OFF
POSITION	+0.0018 Degrees	+180.0033 Degrees
RATE	+0.0006 Deg/S	+0.0000 Deg/S
SAMPLE PERIOD	1.0 Seconds	100.0 Seconds

Current Temperature = 68.42 Degrees Celsius

Data acquisition starting DATE and TIME: 01-DEC-93 10:29:41

Data Rate = 2.1127 Hz

Ext. Block Size = 450

--> Samples = 10

Test Duration = 0:35:30

SAMPLE NUMBER	TIME [MINUTES]	MULTIMETER # 3	MULTIMETER # 4
1	3.6166	-0.14803087E-03VC	-0.56404095E-02VC
2	7.2167	-0.14830562E-03VC	-0.56405429E-02VC
3	10.8167	-0.14944929E-03VC	-0.56396711E-02VC
4	14.4167	-0.14854236E-03VC	-0.56391178E-02VC
5	18.0167	-0.14892487E-03VC	-0.56388514E-02VC
6	21.6166	-0.15194929E-03VC	-0.56403861E-02VC
7	25.2000	-0.14938420E-03VC	-0.56390649E-02VC
8	28.8000	-0.14875329E-03VC	-0.56392753E-02VC
9	32.4000	-0.14958951E-03VC	-0.56397111E-02VC
10	36.0000	-0.15055287E-03VC	-0.56407025E-02VC

Rows of Data = 10

Maximum Values	-0.00014803	-0.00563885
----------------	-------------	-------------

Minimum Values	-0.00015195	-0.00564070
----------------	-------------	-------------

Range (P-P)	0.00000392	0.00000185
-------------	------------	------------

Means (Averages)	-0.00014935	-0.00563977
------------------	-------------	-------------

Standard Deviation	0.000001169	0.000000690
--------------------	-------------	-------------

% Deviation	0.782493679%	0.012227200%
-------------	--------------	--------------

% P-P Deviation	2.623681982%	0.032822439%
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Data acquisition completion DATE and TIME: 01-DEC-93 12:02:45

MOTION TABLE STATUS PRIOR TO DATA ACQUISITION:

TABLE AXIS	Azimuth (1)	Tilt (2)
CONTROL MODE	POSITION	OFF
POSITION	+270.0017 Degrees	+180.0030 Degrees
RATE	+0.0006 Deg/S	+0.0000 Deg/S
SAMPLE PERIOD	1.0 Seconds	100.0 Seconds

Current Temperature = 68.42 Degrees Celsius

Data acquisition starting DATE and TIME: 01-DEC-93 10:29:41

Data Rate = 2.1127 Hz

Ext. Block Size = 450

--> Samples = 10

Test Duration = 0:35:30

SAMPLE NUMBER	TIME [MINUTES]	MULTIMETER # 3	MULTIMETER # 4
1	3.6167	0.31511834E-02VC	-0.23544292E-02VC
2	7.2167	0.31509674E-02VC	-0.23554932E-02VC
3	10.8167	0.31525641E-02VC	-0.23556110E-02VC
4	14.4000	0.31513328E-02VC	-0.23566622E-02VC
5	18.0000	0.31528240E-02VC	-0.23563256E-02VC
6	21.6000	0.31532534E-02VC	-0.23557092E-02VC
7	25.2000	0.31528622E-02VC	-0.23559052E-02VC
8	28.8000	0.31520019E-02VC	-0.23570621E-02VC
9	32.4000	0.31529418E-02VC	-0.23567828E-02VC
10	36.0000	0.31530426E-02VC	-0.23558568E-02VC

Rows of Data = 10

Maximum Values 0.00315325 -0.00235443

Minimum Values 0.00315097 -0.00235706

Range (P-P) 0.00000229 0.00000263

Means (Averages) 0.00315230 -0.00235598

Standard Deviation 0.000000855 0.000000765

% Deviation 0.027124125% 0.032487193%

% P-P Deviation 0.072517130% 0.111753271%

Data acquisition completion DATE and TIME: 01-DEC-93 12:59:47

MOTION TABLE STATUS PRIOR TO DATA ACQUISITION:

TABLE AXIS	Azimuth (1)	Tilt (2)
CONTROL MODE	POSITION	OFF
POSITION	+180.0016 Degrees	+180.0033 Degrees
RATE	+0.0006 Deg/S	+0.0000 Deg/S
SAMPLE PERIOD	1.0 Seconds	100.0 Seconds

Current Temperature = 68.42 Degrees Celsius

Data acquisition starting DATE and TIME: 01-DEC-93 10:29:41

Data Rate = 2.1127 Hz

Ext. Block Size = 450

--> Samples = 10

Test Duration = 0:35:30

SAMPLE NUMBER	TIME [MINUTES]	MULTIMETER # 3	MULTIMETER # 4
1	3.6167	-0.14332582E-03VC	0.93885044E-03VC
2	7.2167	-0.14237991E-03VC	0.93974136E-03VC
3	10.8167	-0.14188484E-03VC	0.93983344E-03VC
4	14.4167	-0.14213273E-03VC	0.93939222E-03VC
5	18.0167	-0.14357558E-03VC	0.93916542E-03VC
6	21.6167	-0.14186656E-03VC	0.93928851E-03VC
7	25.2000	-0.14215640E-03VC	0.93880238E-03VC
8	28.8000	-0.14331051E-03VC	0.93916624E-03VC
9	32.4167	-0.14168993E-03VC	0.93947569E-03VC
10	36.0167	-0.14248098E-03VC	0.93914036E-03VC

Rows of Data = 10

Maximum Values -0.00014169 0.00093983

Minimum Values -0.00014358 0.00093880

Range (P-P) 0.00000189 0.00000103

Means (Averages) -0.00014248 0.00093929

Standard Deviation 0.000000683 0.000000338

% Deviation 0.479199521% 0.035981465%

% P-P Deviation 1.323441972% 0.109771369%

Data acquisition completion DATE and TIME: 01-DEC-93 13:56:49

APPENDIX F: Geographical Survey of Motion Table

A GAX gyro compass was used to establish a bearing line in the Navlab to allow for alignment of the motion table to the geographic frame. Using the GAX in combination with Wild theodolyte, an angle between the true North line and a reference object was determined. The reference object used was a target selected on a large steel girder tower, located approximately 300 metres away from the brass plug in the lab. The tower was clearly visible by opening a garage door. The lab survey is shown in Figure F-1. Note

Figure F-1: Survey of the Contraves Motion Table in the Navlab

that when sighting on the reference object, it is important to allow the lab temperature to equalise with the outside temperature. If this is not done, thermal gradients can cause distortion of the line of sight to the tower. Furthermore, this procedure cannot be done on hot summer days, since the heat radiated from the pavement outside the Navlab will cause shimmer and make it difficult to accurately sight on the reference object. If necessary, a target on the lab wall can also be used as a reference object, but the short baseline makes this method significantly less accurate.

The angle between true North and the bearing line to the tower target, measured clockwise from the North line, was determined to be $231^{\circ}40'08.49'' \pm 8''$. The sources of error for this measurement are:

A.	Pointing error = 0.005m @ 300m	=	3.44"
B.	Centering error = 0.001m @ 300m	=	0.69"
C.	Seeks North (gyro)	=	6.00"
D.	"E" Constant Factor	=	2.24"
E.	T2 Theodolyte	=	2.45"

The pointing error relates to trying to centre the theodolyte on the reference target (which has a diameter of 5 mm) from a distance of 300 metres. The centering error is the effect over 300 meters of attempting to position the theodolyte over the brass plug (diameter 1 mm). Errors C and D both result from inaccuracies in the GAX itself, while error E is due to internal errors in the T2 theodolyte. From these five error sources, the total error is determined to be

$$\Delta_1 = \sqrt{A^2 + B^2 + C^2 + D^2 + E^2} = 7.71''$$

F-1

For the wall target, the bearing line was determined to be $69^{\circ}11'32'' \pm 37''$. Since the baseline is only 8 metres, so the error now becomes,

A.	Pointing error = 0.001m @ 8m	=	25.78"
B.	Centring error = 0.001m @ 8m	=	25.78"
C.	Seeks North (gyro)	=	6.00"
D.	"E" Constant Factor	=	2.24"
E.	T2 Theodolyte	=	2.45"

$$\Delta_2 = \sqrt{A^2 + B^2 + C^2 + D^2 + E^2} = 37.11'' \quad \text{F-2}$$

It is clear that with the wall target in the lab, the largest source of error is the short baseline length, while with the tower target, the GAX gyro errors are more significant. The tower was chosen since it was the furthest object away which could be seen through the garage door. This analysis shows that there would be no advantage to having an object further away, since it would not decrease the error significantly.

APPENDIX G: Misalignment of Reference Mirrors

Consider Figure G-1. The plane of the mirror is defined by the normal to the mirror surface. For the purposes of these calculations, we will consider the vector which points into the mirror plane. This can be interpreted as the line of sight looking into the mirror during autocollimation.

Consider now Figure G-2. This shows the system co-ordinate frame X_s, Y_s, Z_s , and the relationship of the X mirror vector to this frame. The X_s, Y_s, Z_s frame is attached to the IGS; in the home position of the CMT, the X_s, Y_s and Z_s axes correspond with the true North, East and Down directions respectively. However, as the table rotates, the X_s, Y_s, Z_s frame rotates with the table. The theodolite used for autocollimation is located at the position indicated by T2. The line of sight of the theodolite points South, which in this case is the negative X_s direction. Using the theodolite, the azimuth and tilt angles necessary for autocollimation can be measured (note that the CMT is not moved during these measurements, but is kept aligned with the geographic frame). Therefore, the co-ordinates of the X mirror vector can be expressed in the X_s, Y_s, Z_s frame by using the following relationships.

$$X_m = r \cos(\theta) \sin(\phi)$$

G-1

$$X_y = r \sin(\theta) \sin(\phi)$$

G-2

$$X_z = -r \cos(\phi)$$

G-3

where θ is the azimuth angle reading of the theodolite and ϕ is the tilt reading of the theodolite. Using the mean theodolite readings recorded during testing, the following relationships can be established.

T2 azimuth reading necessary for autocollimation on X mirror = $\theta = 179^\circ 59' 42.79''$

T2 tilt reading necessary for autocollimation on X mirror = $\phi = 89^\circ 54' 3.92''$

Therefore, the X mirror vector is

$$\begin{bmatrix} X_{xx} \\ X_{yx} \\ X_{zx} \end{bmatrix} = \begin{bmatrix} -0.999998506 \\ 0.0000833907 \\ -0.001726302 \end{bmatrix} \quad \text{G-4}$$

Consider now Figure G-3. The CMT has been rotated by 90° in azimuth so that the theodolite is viewing the Y mirror. That is, the theodolite is still pointing south in the geographic frame, however it is now sighting along the Y_z axis. Therefore, the Y mirror vector can be expressed using the following relationships.

$$Y_{xx} = r \cos(\theta) \sin(\phi) \quad \text{G-5}$$

$$Y_{yx} = r \sin(\theta) \sin(\phi) \quad \text{G-6}$$

$$Y_{zx} = -r \cos(\phi) \quad \text{G-7}$$

where θ is the azimuth angle reading of the theodolite minus 90° and ϕ is the tilt reading of the theodolite. Using the mean theodolite readings,

T2 azimuth reading necessary for autocollimation on Y mirror = $180^{\circ} 00' 08.32''$

Therefore, $\theta = 180^{\circ} 00' 08.32'' - 90^{\circ} = 90^{\circ} 00' 08.32''$

T2 tilt reading necessary for autocollimation on Y mirror $= \phi = 90^{\circ} 00' 57.6''$

Therefore, the Y mirror vector is

$$\begin{bmatrix} Y_x \\ Y_y \\ Y_z \end{bmatrix} = \begin{bmatrix} -0.0000403595 \\ 0.999999960 \\ 0.000279276 \end{bmatrix} \quad \text{G-8}$$

Finally, consider Figure G-4. The CMT has been rotated by 90° in tilt so that the theodolite is viewing the Z mirror. The theodolite is now pointed along the Z_x axis. Therefore, the Z mirror vector can be expressed using the following relationships.

$$Z_x = r \cos(\theta) \sin(\phi) \quad \text{G-9}$$

$$Z_y = r \cos(\phi) \quad \text{G-10}$$

$$Z_z = r \sin(\theta) \sin(\phi) \quad \text{G-11}$$

where θ is the azimuth angle reading of the theodolite minus 90° and ϕ is the tilt reading of the theodolite. Using the mean theodolite readings,

T2 azimuth reading necessary for autocollimation on X mirror = $180^{\circ} 03' 49.25''$

Therefore, $\theta = 180^{\circ} 03' 49.25'' - 90^{\circ} = 90^{\circ} 03' 49.25''$

T2 tilt reading necessary for autocollimation on X mirror $= \phi = 89^{\circ} 54' 11.17''$

Therefore, the Z mirror vector is

$$\begin{bmatrix} Z_x \\ Z_y \\ Z_z \end{bmatrix} = \begin{bmatrix} -0.001111456 \\ 0.001691153 \\ 0.999997952 \end{bmatrix} \quad \text{G-12}$$

Using the vector dot product, it is possible to define the angle between any two of these vectors.

$$N_{xy} = \arccos\left(\frac{X \cdot Y}{|X||Y|}\right) = 89^\circ 59' 34.57'' \quad \text{G-13}$$

$$N_{xz} = \arccos\left(\frac{X \cdot Z}{|X||Z|}\right) = 90^\circ 02' 06.79'' \quad \text{G-14}$$

$$N_{yz} = \arccos\left(\frac{Y \cdot Z}{|Y||Z|}\right) = 89^\circ 53' 13.55'' \quad \text{G-15}$$

END

17-1/1-95

FIN