# Subitizing, Finger Gnosis, and Finger Agility as Precursors to the Representation of Number

by

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ABSTRACT

The goal for this program of research was to determine whether the ability to represent number is facilitated by (1) the ability to represent small numerosities (indexed by subitizing), (2) the ability to mentally represent one's fingers (indexed by finger gnosis), and (3) finger agility, as proposed by Butterworth (1999).

In a series of three experimental studies, the concurrent relation between the precursors and number system knowledge and calculation skill in Grade 1 children (Studies 1 & 2), and the longitudinal relation between the precursors and performance on tasks designed to specifically assess numerical representations: number comparison and number-line estimation from Grade 1 to Grade 2 (Study 3) were examined. Subitizing was related to number system knowledge and calculation skill (Studies 1 & 2), but not to tasks assessing the representation of number (Study 3). Finger gnosis was related to both number system knowledge and calculation skill (Studies 1 & 2) and, moreover, to measures of the representation of number: the distance effect in number comparison and the linearity and slope of children's estimates. Children who had more distinct mental representations of their fingers, as measured by finger gnosis, also had more distinct representations of number, as measured by a reduced numerical distance effect in number comparison and more accurate number-line estimations. Finger agility was related to calculation skill, but not to number system knowledge (Study 2).

Second, in a pair of synthetic studies, the relation between finger gnosis and number representations was further examined. In Study 4, a novel view of this relation was proposed, the redeployment view, according to which one of the functional circuits

originally evolved for finger representation has since been redeployed to support the representation of number and now serves both uses. In Study 5, imaging results from multiple domains were used to investigate and propose a mechanism responsible for the phylogenetic impact of finger representation on the development of number representations (Study 5).

This work constitutes the first complete test of Butterworth's theory and provides support for elements of the model in typically developing children in Grades 1 -2.

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1. Penner-Wilger, M., Fast, L., LeFevre, J., Smith-Chant, B. L., Skwarchuk, S., Kamawar, D., & Bisanz, J. (2007). The foundations of numeracy: Subitizing, finger gnosia, and fine-motor ability. In D. S. McNamara & J. G. Trafton (Eds.), *Proceedings of the 29th Annual Cognitive Science Society* (pp. 1385-1390). Austin, TX: Cognitive Science Society.
2. Penner-Wilger, M., Fast, L., LeFevre, J., Smith-Chant, B. L., Skwarchuk, S., Kamawar, D., & Bisanz, J. (2009a). *Precursors to numeracy: Subitizing, finger gnosia, and fine-motor ability*. Manuscript submitted for publication.
3. Penner-Wilger, M., Fast, L., LeFevre, J., Smith-Chant, B. L., Skwarchuk, S., Kamawar, D., & Bisanz, J. (2009b). *Subitizing, finger gnosis, and the representation of number*. Manuscript accepted for publication.
4. Penner-Wilger, M., & Anderson, M.L. (2008). An alternative view of the relation between finger gnosis and math ability: Redeployment of finger representations for the representation of number. In B.C. Love, K. McRae & V.M. Sloutsky (Eds.), *Proceedings of the 30th Annual Cognitive Science Society* (pp. 1647–1652). Austin, TX: Cognitive Science Society.
5. Penner-Wilger, M., & Anderson, M.L. (2009). *The relation between finger gnosis and mathematical ability: Can we attribute function to cortical structure with cross-domain modeling?* Manuscript submitted for publication.

# CHAPTER 1: INTRODUCTION AND BACKGROUND

Number is "among the most abstract ideas which the human mind is capable of forming" (Adam Smith as cited in Butterworth, 1999, p. 4). Given the complexity, how do humans develop a mental representation of number? More specifically, in this dissertation I address the following research question: Which precursor abilities facilitate the development of number representations?

The organization of this dissertation is as follows. First, in Chapter 1, I provide an overview of the field of numerical cognition as it pertains to my research question. Next, in Chapter 2, I outline the method for my experimental studies (Studies 1 – 3). In Chapter 3, I provide an overview of the five studies that form the core of my dissertation. For each study, I explain the motivation, summarize the results, and describe how the results will address the central question of which precursor abilities facilitate the development of number representations in humans. All five studies are presented as Appendices to this document. Study 2, as it appears in Appendix B, combines the work of Studies 1 and 2. Study 5, as it appears in Appendix E, combines the work of Studies 4 and 5. Thus, Study 1 is a subset of the information in Study 2 and, likewise, Study 4 is a subset of the information in Study 5.

## What is a Number?

Within and across the disciplines in which researchers have a vested interest in understanding humans' numerical abilities, there is confusion in the basic terminology. This situation is often the case in interdisciplinary study. As a result, it was important throughout this work to carefully choose and define terms. I start here, as it were, at the beginning. What is a number? Importantly, a *number* is an abstract semantic concept of discrete magnitude – *how many*. Numbers are shared concepts, thus, the number [3] in my mind is the same as the number [3] in your mind. Moreover, concepts bear entailment. For example, the concept [3] entails "more than [2]". Thus, numbers, so defined, have cardinality – they represent numerosities.

Different types of *number representations,* however, are posited to account for humans' numerical abilities, including: preverbal representations that are non-symbolic and various symbolic representations. There is controversy over the form of non-symbolic representations. Butterworth and colleagues (Butterworth, 1999; Zorzi & Butterworth, 1999) assert that we have discrete representations of number that indicate "how many". To avoid confusion with other definitions of number, Butterworth (1999) uses instead the term *numerosity*, to refer to the number of things in a collection. In contrast to Butterworth and colleagues, Dehaene (Dehaene, 1997) asserts that we have analogue representations of number that indicate "how much". To this point, the distinction between *how much* and *how many* has not garnered a great deal of attention in the field.

Precursor Abilities that Facilitate the Development of Number Representations

How do we develop representations of number? Butterworth (1999) posits that the ability to represent number is built on three precursor abilities: (a) the ability to represent small numerosities, (b) the ability to mentally represent one's fingers, and (c) the ability to functionally use one's fingers (i.e., finger agility). Moreover, Butterworth asserts that each of these abilities is *necessary* for the normal development of number representations.

*Number Module*

Butterworth (1999) hypothesizes we have a *number module*, (i.e., a domain-specific, informationally-encapsulated, cognitive mechanism) that allows us to categorize aspects of the world in terms of numerosities (i.e., the number of items in a collection). This number module is at the core of our numerical abilities. Butterworth asserts that the number module developed phylogenetically, over the course of evolution, with the genome containing instructions for specialized neural circuits with an innate sensitivity to numerosities that, moreover, operate automatically. The number module fails to develop normally in some individuals (Defective Number Module Hypothesis, Butterworth, 1999, 2005), leading to widespread numerical deficits, that is, dyscalculia. *Dyscalculia* refers to disorders of numeracy development (i.e., the numerical counterpart to dyslexia) and is characterized by slow, difficult, and atypical ways of dealing with number (Butterworth & Reigosa, 2007). Support for the number module is derived from animal studies, infant studies, studies of dyscalculia, and brain imaging research (for reviews see Butterworth, 1999; Brannon, 2005). The number module has a limited capacity, however, extending only to numerosities of four or five. Butterworth posits that we extend our numerical representation using cognitive tools.

*Cognitive tools*

Cognitive tools have been developed and transmitted to extend the limited capacity of the number module. Categories of cognitive tools include body-part representations (e.g., use of fingers and other body parts as portable manipulatives), external representations (e.g., calculators), linguistic representations (e.g., counting words), and numerals. Among the most important cognitive tools (in Butterworth's view) are the 10 fingers. Fingers are used in the course of development to represent numerosities and to perform counting and arithmetic procedures across cultures and without formal instruction (Butterworth, 1999). The use of fingers, as representational tools, extends the limited capacity of the number module. Indeed, Butterworth hypothesizes, that "without the ability to attach number representations to the neural representations of fingers...the numbers themselves will never have a normal representation" (p. 226). Thus, where finger representation fails to develop normally this deficit is posited to have serious effects on the development of numerical skills.

The prevailing view is that symbolic number representations "acquire their meaning by being mapped onto non-symbolic representations of numerical magnitude" (Ansari, 2008, p. 278). Butterworth posits that the fingers provide this link between numerosity representations (associated with the number module) and number words. Fayol and Seron (2005), likewise hypothesize that fingers may be the "missing link" between non-symbolic representations of number and number words. Number words (e.g., one, two, three) are cognitive tools that provide a sequence of numerosities, ordered by size, that build up from the limited capacity number module into a full numerosity system. Thus, the functional ability to use one's fingers may be critical to developing numerical representations, because the fingers help to link non-symbolic number knowledge with the (learned) symbolic system.

## Formalization of Butterworth's Theory

In summary, Butterworth posits that the ability to represent number is built on three abilities: (a) the ability to represent small numerosities, (b) the ability to mentally represent one's fingers, and (c) the ability to functionally use one's fingers (i.e., finger agility). Moreover, Butterworth asserts that each of these skills is *necessary* for the normal development of number representations. In this dissertation, Butterworth's view is formalized into a testable model (shown in Figure 1 below). The experimental studies (Studies 1, 2, & 3) constitute the first complete tests of Butterworth's model. Butterworth posits that the core component facilitating the development of number representation is the ability to categorize the world in terms of numerosities. This core ability is indexed in the formalized model by *subitizing*, the ability to enumerate small sets without counting. Butterworth also posits a crucial role for fingers, both as representational and procedural tools. The representation role of fingers is indexed in the model by *finger gnosis*, the ability to mentally represent one's fingers. The procedural role of fingers is indexed in the model by *finger agility*. According to the model shown in Figure 1, each of these abilities contributes independently to the development of number representations.

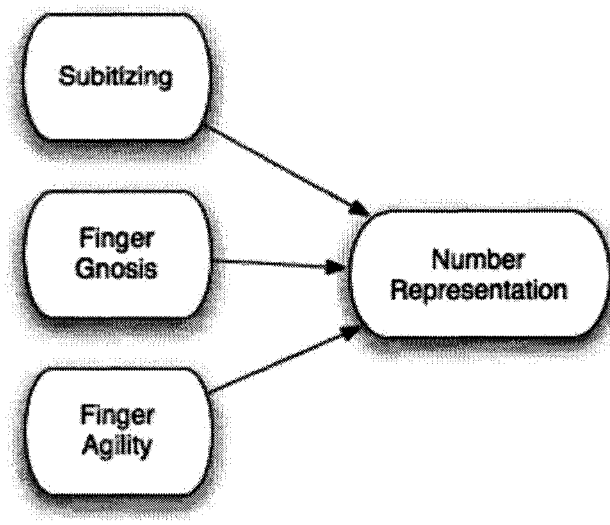


*Figure 1.* Formalized model of Butterworth (1999).

*Subitizing*

Subitizing is the ability to enumerate small sets of items without counting (Mandler & Shebo, 1982). The subitizing range – the number of items that can be enumerated without counting- is generally considered to be 1-3 items for children (Trick, Enns, & Brodeur, 1996). Subitizing ability is seen in newborns as well as other species (for a review see Dehaene, 1992). Beniot, Lehalle, and Jouen (2004) point to the importance of subitizing in the mapping of number words to numerosities. Both Butterworth (1999, 2005) and Dehaene (1997) posit subitizing as the core of our numerical representations: Butterworth as an index of the *number module*, and Dehaene as an index of *number sense*.

Views on subitizing fall into three categories: (1) numerical, (2) general non-numerical, and (3) perceptual views (Noël, Rousselle, & Mussolin, 2005). On the *numerical* view, subitizing relies on an innate, domain specific, neural system for processing numerosities. Butterworth's number module falls under this view, as does Dehaene's (1997) number sense. Gallistel and Gelman's (1992) accumulator model is also an example of the numerical view. For Butterworth the output of subitizing is discrete—how many-- whereas for Dehaene, as well as Gallistel and Gelman, the output is analogue –how much.

On the *general non-numerical* view, subitizing relies on domain-general cognitive capacities. The object file (Simon, 1997) and FINST (Trick & Pylyshyn, 1994) models would fall under this general cognitive view, wherein more general mechanisms related to multiple target tracking, such as spatial reference tokens, are used. On this view, the representations are discrete. On the *perceptual* view, perceptual non-numerical cues that naturally co-vary with number are represented and discriminated rather than number (Mix, Huttenlocher, & Levine, 2002). Representations are, thus, necessarily analogue.

Moreover, not all researchers agree that subitizing is a distinct process. Some argue that subitizing instead reflects really fast counting (Boles, Phillips & Givens, 2007). For example, Peterson and Simon (2000) propose that subitizing is a learned recognition process. In an ACT-R model the authors show that the empirical patterns associated with subitizing (i.e., shallow RT slopes for 1-3 items with increasing slopes for 4 or more items) could derive from a counting system, wherein learning the patterns associated with 1-3 items was fast and accurate.

The relation between subitizing and numerical ability has not yet been examined in a typically-developing population. Willburger et al. (2008) found that children with dyscalculia were slower to perform rapid automatized naming of 1-4 items than normally-achieving children (see also van der Sluis et al., 2003, who reported a similar result). Other researchers have noted a trend towards poorer subitizing in studies of children with dyscalculia and arithmetic learning disabilities. Koontz and Berch (1996) noted that children with arithmetic learning disabilities appeared to count the number of items in displays of 2 – 3 dots; this observation was supported with response time data. In summary, although Butterworth's theory posits a critical role for subitizing in the development of numerical abilities this position has yet to be strongly supported in either dyscalculic or typically-developing populations. In a series of experimental studies (Studies 1 – 3), I examine the relation between subitizing and numerical outcomes.

*Finger Gnosis*

Finger gnosis is the presence of an intact finger schema or ability to mentally represent one's fingers as distinct (from one another). Neuropsychological tests have been designed to assess the presence of finger gnosis, or its absence—finger agnosia—in neuropsychological populations. In one common test (Baron, 2004), the examiner shields the participant's hand from view and lightly touches one or more fingers. The participant is asked to identify which fingers were touched. Finger agnosia is one of a constellation of symptoms in Gerstmann's syndrome, along with acalculia, agraphia, and left-right disorientation. Gerstmann (1940) identified finger agnosia as the core deficit of the syndrome. Butterworth (1999) posits that finger representations extend the limited capacity of the innate numerosity system and, moreover, 'bridge the gap' from numerosity representations to more abstract number words. Fayol and Seron (2005) concur, stating that, unlike linguistic representations "finger representations exhibit an iconic relation to numerosities, since they preserve the one-to-one matching relation between the represented set and the fingers used to represent it" (p. 16). Thus, the fingers seem ideally suited as a cognitive tool to aid in the development of symbolic representations of number ontogenetically, in the course of development.

Two prevailing views exist on the relation between finger gnosis and the representation of number (though a third is proposed in Study 4). On the *localizationist* view, finger gnosis is related to numerical abilities because the two abilities are supported by neighboring brain regions, and these regions tend to have correlated developmental trajectories. On this view, there is no direct causal link between the representation of fingers and numbers (Dehaene et al., 2003). In contrast, on the *functional* view, finger gnosis and numerical abilities are related because the fingers are used to represent quantities and perform counting and arithmetic procedures. As a result, the representation of numbers and of fingers becomes entwined (Butterworth, 1999).

A relation between finger gnosis and mathematics ability has been shown in typically-developing populations of children. Fayol, Barrouillet and Marinthe (1998) discovered that a set of neuropsychological tests, including tests of finger gnosis, were the best longitudinal predictor of Grade 1 children's math scores. This finding was confirmed by Noël (2005), who demonstrated that children's finger gnosis scores predicted accuracy and fluency on a variety of mathematical tests, both concurrently in Grade 1 and longitudinally one year later. Thus, evidence exists for a relation between finger gnosis and math, in isolation from the other precursor abilities investigated in this dissertation. In a series of experimental studies (Studies 1-3), I investigate the joint and independent contribution of each of the precursors to numeracy outcomes. In Studies 4 and 5, a novel view of the relation between finger gnosis and numerical representations is developed.

### *Finger Agility*

Given the importance of the functional use of fingers in the course of numerical development, and the precise accurate control required for finger counting, finger agility is posited to be important for the development of numerical representations (Butterworth, 1999). Finger agility can be assessed using a finger tapping test, where the participant taps a key with their index finger as many times as possible during a given time interval (Gilandas, Touyz, Beumont, & Greenberg, 1984).

Barnes et al. (2005) investigated the link between fine motor ability and math in children with Spina Bifida. Spina Bifida is a neurodevelopmental disorder that produces, among others, deficits in fine motor ability. Moreover, children with Spina Bifida show math difficulties early in development and these difficulties persist into adulthood. Barnes et al. found that fine motor ability, measured with the Purdue Pegboard task, was correlated with skill in multi-digit calculation in a combined group of 120 children aged 8 – 16 years (60 with Spina Bifida and 60 age- and grade-matched controls). In a similarly-combined group of younger children (aged 36 months old) fine motor ability measured with the Visual Motor Integration test (Beery & Beery, 2004) predicted significant unique variance in children's nascent quantitative skills including: object counting, finger counting, quantitative vocabulary, and counting concepts. These results support a link between fine motor ability and counting and calculation skill, though the relation may be driven by the combination of typically developing and atypically developing populations. Moreover, the relation between finger agility and number system knowledge has not been examined.

## Time Course for the Impact of Precursors to Numerical Representations

What is the time course for the impact of the precursors to numerical representations; do they assert their effects ontogenetically (over the course of development) and/or phylogenetically (over the course of evolution)? Here, I summarize the prevailing views on the time course for the impact of (a) the ability to represent small numerosities, (b) the ability to mentally represent one's fingers, and (c) finger agility, on the development of number representations.

### *Subitizing*

The ability to represent small numerosities is hypothesized to facilitate the development of number phylogenetically (Ansari, 2008; Butterworth, 1999; Dehaene & Cohen, 2007). Evidence for this position comes from studies with non-human primates. In a review, Ansari concludes "the similarity in the performances of monkeys and humans suggests there is a high degree of phylogenetic continuity in basic representations of numerical magnitude" (p. 279). Likewise, the numerical abilities of young infants support this view (Ansari, 2008; Butterworth, 1999; Dehaene, 1992). Thus, subitizing has a long evolutionary history and appears to support the development of the representation of number phylogenetically, rather than in the course of individual development.

*Finger Gnosis and Finger Agility*

The ability to mentally represent one's fingers, and the finger agility required to functionally use one's fingers, is hypothesized to facilitate the development of number ontogenetically (Butterworth, 1999; Fayol & Seron, 2005). On this view, fingers serve as a cognitive tool to extend the limited-capacity number module and provide a link between non-symbolic and symbolic representations of number. Butterworth (1999, p. 213) speculates, "that somehow in the child's brain the representation of numbers is very closely linked to representations of hand shapes" with experienced use of the fingers reinforcing this link. This leaves open the possibility that the link between the ability to represent one's fingers and the representation of number does not rest on the experienced use of the fingers to represent numerosities and perform counting procedures. This possibility is explored in Studies 4 and 5, where I propose that finger gnosis facilitates the development of number phylogenetically.

Dissertation Work

In this research project, I examined whether the ability to represent number is built on three abilities: (a) the ability to represent small numerosities, (b) the ability to mentally represent one's fingers, and (c) finger agility. This work took two forms: experimental and synthetic. In Studies 1 and 2, the relation between mathematical skill and the posited precursors of subitizing, finger gnosis, and finger agility was examined. In Study 3, the relation between tasks designed to assess numerical representations and the hypothesized precursors was examined to more specifically assess the precursors' role. All experimental work was done using data from the Count Me In longitudinal project, in which we examined the precursors of mathematical skill in primary school children (project website: www.carleton.ca/cmi). The finger gnosis task was selected, designed and included in the Count Me In study for the purpose of my ongoing research into the neuropsychological precursors of numerical representations. Finally, a novel view of the relationship between finger gnosis and number representations is proposed along with supporting evidence (Studies 4 and 5). The research program is summarized in Table 1.

*Table 1.* List of studies.

| Study | **Appendix** | **Contribution** |
|---|---|---|
| **1** | A | Experimental work: Concurrent relation between precursors and number system knowledge and calculation skills |
| **2** | B | Experimental work: Relation between different precursor skill profiles and math outcomes both concurrently and longitudinally |
| **3** | C | Experimental work: Longitudinal relation between precursors and numerical representation tasks |
| **4** | D | Synthetic work: Development of novel hypothesis on the relation between finger and number representations |
| **5** | E | Synthetic work: Investigation of the shared working between finger and number representations |

# CHAPTER 2: METHODS

## Method for Experimental Work (Studies 1 to 3)

All data for the experimental studies reported in this dissertation were collected as part of the Count Me In longitudinal project. The Count Me In project was a four-year research project investigating the precursors of mathematical skill in primary school children, and was funded by the Canadian Social Sciences and Humanities Research Council (SSHRC). Canadian children from eight schools in three cities: Ottawa, and Peterborough, Ontario and Winnipeg, Manitoba, participated in the Count Me In project annually from 2004 to 2007 ($N = 456$). The project design was both longitudinal and cross-sectional, with two cohorts: Cohort 1, started in 2004 (students in Senior Kindergarten, Grade 1, or Grade 2) and continued for four years and Cohort 2 started in 2005 (students in Junior Kindergarten, Senior Kindergarten, or Grade 1) and continued for three years. Students were assessed on a number of mathematical tasks (e.g., counting, arithmetic, number system knowledge, number comparison, estimation), cognitive and neuropsychological tasks (e.g., Corsi blocks task, digit span, processing speed, finger gnosis, finger tapping), and standardized tests of numeracy skills (i.e., KeyMath and Woodcock-Johnson), literacy skills (i.e., Peabody Picture Vocabulary Test, Comprehensive Test of Phonological Processing, Woodcock Reading Mastery), and general cognitive skills (i.e., Cognitive Intelligence Test).

For the experimental studies in this dissertation, Grade 1 children were selected from the second year of Cohort 2 (2006) and followed for one year into Grade 2. The Grade 1 group was selected for two reasons (1) Grade 1 represents the transition from informal to formal math skills, and (2) 2006 was the first year that data was available for all the precursor tasks of interest (subitizing, finger gnosis, and finger tapping).

The longitudinal design from Grade 1 to Grade 2 captures interesting and relevant changes in children's mathematical knowledge. For example, the numeration and calculation expectations increase notably from Grade 1 to Grade 2 in Ontario (Ontario Ministry of Education, 2005). For numeration skill, children are expected to be able to read, represent, compare and order numbers to 50 by the end of Grade 1 and 100 by the end of Grade 2. Children are expected to gain conceptual understanding including one-to-one correspondence and conservation of number in Grade 1 and equality, commutativity and the property of zero in addition and subtraction in Grade 2. Children in Grade 1 are expected to be able to estimate the number of items in a set, in Grade 2 children are expected to estimate the location of a number on a number line. For calculation skills, children in Grade 1 are expected to perform single-digit addition and subtraction using concrete materials. Children in Grade 2 are expected to perform single-digit addition and subtraction mentally and to perform two-digit addition and subtraction with materials.

### *Participants*

Grade 1 children ($N$ = 144) who participated in 2006 were selected. The testing sessions took place in the late Spring, by which time the children had participated in nine months of mathematical instruction. The children (71 girls and 73 boys) ranged in age from 5 to 7 years old (M = 6 years 10 months). Based on the hand they used for writing, 128 (89%) of the children were right handed and 16 (11%) were left handed.

For Study 2, 112 of the 144 children from Study 1 participated in Grade 2. Of these 112 children, 10 did not have complete data and were not included in the longitudinal analyses, leaving 102 children in the longitudinal analyses.

For Study 3, data were missing for 12 of the 112 children on both the tasks of interest: number comparison and number-line estimation. Thus, the analyses are based on the 100 children who had complete Grade 1 data and who also had data for at least one Grade 2 outcome measure.

## *Procedure*

Most children completed the computer measures in one half-hour session and the rest of the measures in a separate half-hour session on a different day. Some children completed all of the measures in a single session of approximately one hour.

All of the computer tasks were presented using software developed specifically for this project. For the computer tasks, children initiated the trials themselves by pressing the spacebar. Response times were measured from the point at which the stimuli appeared, until the experimenter pressed the stop-timer key (using a separate keyboard) when the child spoke their response. The experimenter then typed in the child's response.

## *Materials*

### *Subitizing (Studies 1 – 3)*

On each trial the computer displayed a set of 1 to 6 circular red target objects. The children were instructed to respond with the number of objects, out loud, as quickly as possible. The child initiated each trial by pressing the space bar. Targets remained on the display until the experimenter entered the child's response. There were 18 trials, preceded by two practice trials of two and seven objects. Half of the trials were within the subitizing range (1 to 3), and half in the counting range (4 to 6).

*Finger Gnosis (Studies 1 – 3)*

The Finger Gnosis measure is based on one designed by Noël (2005). Ten trials were conducted on each hand, beginning with the dominant hand. In each trial, two fingers were lightly touched below the first knuckle. The child's view of the touches was obstructed with a cloth cover raised from the child's wrist. After the cloth cover was lowered, the child pointed to the two fingers that had been touched.

*Finger Agility (Studies 1 and 2)*

We developed a computer-game version of the Finger Tapping Test (Baron, 2004). The game was presented as a canal-digging exercise, where the longer the canal, the bigger the fish that will swim in it. Each tap on the space bar was a 'dig of the shovel', and increased the length of the canal, with an indicator of the maximum length from any previous trials. Children were instructed and encouraged to tap as fast as they could. Timing began upon the first press of the space bar and continued for 10 seconds (Baron, 2004), until an animated fish appeared. The tapping score determined the type of fish, with a prized killer whale animation appearing for scores over 50 taps. Tapping scores were collected for three trials on each hand, beginning with the dominant hand.

*Digit Recognition/Next Number (Studies 1 and 2)*

The Digit Recognition and Next Number tasks were performed on the computer. Each of these tasks had 18 trials of increasing difficulty. The task ended if the child made three sequential errors. In each trial, a number was displayed on the screen. The first half of the trials were under 100, then increased in difficulty through the hundreds, thousands and for Next Number, into the ten and hundred thousands, ending at 407,276. For Digit Recognition, the experimenter asked the child, "What number is that?" Later during the testing session, for the Next Number task, the child was asked to respond with the number "that comes next when counting." Responses that were spoken as digits–for example, "one oh oh" instead of "one hundred"–were marked as errors.

*Place Value (Study 1)*

Knowledge of place value was measured through a test designed for this study. Each page of the testing easel displayed an image of a set of proportionately sized unit, ten, hundred or thousand blocks. Above the blocks was a two- to four-digit number with one digit underlined. The child was asked to identify whether the number of blocks was consistent with the underlined number. A consistent trial would show the number <u>3</u>52 with an image of 3 hundred-unit blocks. An inconsistent trial would show the number <u>3</u>52 with an image of 3 ten-unit blocks. Two example trials and two practice trials precede the twelve test trials.

*Addition (Studies 1 – 3)*

Addition accuracy and latencies were measured on the computer. 16 trials of single-digit sums were displayed on the computer screen. In Grade 1, all sums were less than 10. In Grade 2, all sums were greater than ten. This task has a stop condition of five sequential errors and trials timed out if the child did not respond within 20 seconds. The child initiated each trial by pressing the 'GO' button. When the child spoke their answer, the experimenter pressed a key to stop the timer and typed in their response.

*KeyMath Numeration (Studies 1 – 3)*

Children completed the Numeration subtest of a multi-domain math achievement test, the KeyMath Test-Revised (Connolly, 2000). This task is individually administered. It covers concepts such as quantity, order, and place value (on later items). Most of the items in the range for these children require knowledge of the symbolic number system. The reported alternate-form reliability coefficient for the grade-scaled Numeration subtest is .75 (Connolly, 2000). Connolly provides a split-half reliability coefficient of .81 for spring Grade 2.

*Woodcock-Johnson Calculation Skill (Studies 1 – 3)*

Children completed the calculation subtest of the Woodcock Johnson Psycho-Educational Battery - Revised (Woodcock & Johnson, 1989). This calculation measure involves all four operations (addition, subtraction, multiplication, and division), although most of the questions that were attempted by the children in the present study involved addition or subtraction only. Children were stopped after six incorrect answers, or when they indicated to the experimenter that they did not know how to solve any of the remaining questions. This test has a median reliability of .85 and a one-year test-retest correlation of .89 for Grades 2 through 4 (Woodcock & Johnson, 1989). The WJ-R manual cites the split-half reliability for six year olds as .93, *SEM(W)*=5.7 (*N*=309).

*Processing Speed (Studies 1 – 3)*

To assess processing speed, we implemented a computer-based simple choice reaction time task (Petrill, Luo, Thompson, & Detterman, 2001). Two types of stimuli (an X or an O) were displayed for 1 second, preceded by a half second fixation point.

Children were instructed to press the key corresponding to the target letter shown on the screen. The display then cleared and the next trial began automatically 1 second later. There were 24 trials. The child positioned the index finger of their dominant hand on the keyboard key with an 'X' sticker (the 'X' key) and their middle finger on the key above it labeled 'O' (the 'D' key). Left-handed children used similar stickers on the right side of the keyboard.

*Vocabulary (Studies 1 – 3)*

Receptive language was measured using the Peabody Picture Vocabulary Test–Third Edition, form B (Dunn & Dunn, 1997). It was included primarily as a measure of verbal, non-mathematical knowledge. Dunn and Dunn cite the split-half reliability coefficient for Form B for seven year olds as .95.

*Number Comparison (Study 3)*

The number comparison task was designed based on the numerical condition from Landerl, Bevan, and Butterworth (2004). The child was shown two numerals on the screen (range from 1 – 9) and was asked, "Which number is more than the other number?" Children indicated their response by pressing a yellow key on the side that was more (z on the left or . on the right). For each trial, there was a 500 ms delay prior to the stimulus presentation. Stimuli were displayed until the child responded or until a 3s maximum was reached.

Stimuli varied on two dimensions, physical size (large vs. small font) and numerical size. For congruent trials, the number that was larger numerically was also larger physically. For incongruent trials, the number that was larger numerically was smaller physically. Numerical distance was defined with small splits as a distance of 1 (e.g., 2 3) and large splits as a distance of 5 (e.g., 2 7). Half of the trials were congruent and half were incongruent. There were 40 trials preceded by two practice trials. The stimuli pairs were taken from Landerl et al. (2004) with 24 trials of the six small-split combinations (1-2, 2-3, 3-4, 6-7, 7-8, 8-9) and 16 trials of the four large-split combinations (1-6, 2-7, 3-8, 4-9). Two pseudo-random orders were created.

*Number-Line Estimation (Study 3)*

Number-line estimation was measured using a computerized test of numerical estimation skill (Siegler & Opfer, 2003) adapted to have a game-like context. In this game, children were shown a number between 1 and 1000 and used the mouse to position a red car at the appropriate spot on a number line starting at 0 and ending at 1000. Children initiated each trial by pressing a Go button. Next, the target number was displayed at the top of the screen and the cursor transformed into the image of a car. Children moved to their selected position on the line and clicked the cursor. The computer recorded the location and the solution latency.

Following a training session with three trials at 0, 500 and 1000, the experimenter launched the main Estimation task. The instructions before the task began were based on the script used by Siegler and Booth (J. Booth, personal communication, February 21, 2006). We added the phrase "as quickly as you can" to encourage children to estimate. For each trial, the instruction area at the top of the screen displayed the message “Click where this number goes” followed by the target number displayed in the same large red font as the starting ‘0’ and ending ‘1000’ below the target line. The experimenter would read the instruction and number aloud, switching to just reading the number aloud once the child demonstrated proficiency at the task.

Order of the 25 trials was randomized separately for each child. The stimuli were chosen based on Laski and Siegler (2007) and were balanced with four targets between 0 and 100, four between 900 and 1000, two targets from each other decade and distances matched from the endpoints. The targets were: 6, 994, 18, 982, 59, 991, 97, 903, 124, 876, 165, 835, 211, 789, 239, 761, 344, 656, 383, 617, 420, 580, 458, 542, and 500.

# CHAPTER 3: OVERVIEW OF STUDIES

## Program of Research

This chapter provides an overview of the motivations, findings, and conclusions for each of the five studies that comprise this dissertation. A general discussion can be found in Chapter 4 and the complete studies can be found in Appendices A – E. This program of research includes: formalization of Butterworth's theory into a testable model (Chapter 1), a series of experimental studies designed to test the model (Studies 1 – 3), and synthetic work on the relation between finger gnosis and the representation of number (Studies 4 & 5). The program of research is significant to the field of mathematical cognition, in that it provides a first test of Butterworth's theory, examining both the precursor abilities that facilitate the development of number representations and the form of number representations. The program of research is also significant to the broader fields of cognitive science and education. For cognitive science, this interdisciplinary work on the form and development of number representations is relevant and has been sited as widely as artificial intelligence and robotics (Anderson, 2008). Moreover, the methodology of cross-domain modeling, of which Study 5 provides a example, is advocated as the future of cognitive neuroscience research (Anderson, 2007a,b,c; Bergeron, 2008; Cabeza & Nyberg, 2000). For education and developmental psychology, far less is known about the precursors to numeracy (compared to reading). The findings from this research program provide evidence of the precursors and different pathways to numeracy, which can inform the assessment of children at risk of math difficulties and the development of interventions.

## Study 1: Are the Precursors (Subitizing, Finger Gnosis, Finger Agility) Related to Numeracy Skill?

### *Motivation*

The motivation for Study 1 was to undertake the first complete test of the formalized Butterworth model. The primary research goal was to determine if subitizing, finger gnosis, and finger agility are correlated with children's math achievement. Specific research questions included: Do subitizing, finger gnosis, and finger agility reflect separate abilities? Do subitizing, finger gnosis, and finger agility jointly predict number system knowledge and calculation skill? Do subitizing, finger gnosis, and finger agility each independently predict number system knowledge and calculation skill?

### *Summary of Results*

Butterworth (1999; 2005) proposed that several component abilities support our numerical representations and processes: an innate capacity to represent small numerosities (indexed by subitizing), finger agility (indexed here by finger tapping), and the ability to mentally represent one's fingers (indexed by finger gnosis). In Study 1, I evaluated the predictive power of these component abilities in the development of number system knowledge and calculation skills in Grade 1 children. No significant correlations were found among subitizing, finger gnosis, and finger agility, consistent with the view that these measures reflect separate abilities. Each component ability was found to be a significant unique predictor of number system knowledge, which in turn was related to calculation skill. Finger gnosis was related to calculation skill indirectly through number system knowledge. In contrast, subitizing predicted calculation skill both directly and indirectly through number system knowledge. The results of Study 1 support Butterworth's view of the foundations of numeracy and have implications for the early identification of children at risk of math difficulties. For the detailed results, please refer to Appendix A.

*Conclusion*

In the first test of the formalized model of Butterworth's theory, we determined that the three precursor abilities (i.e., subitizing, finger gnosis, and finger agility) were each independently related to children's numerical abilities. This finding is an important first step in determining the precursor abilities that facilitate the development of number, in that the results provide support for both Butterworth's view and for the formalized model.

Though suggestive, the results from Study 1 are less than satisfying for at least three reasons. First, the relation between the precursors and the math outcome variables was examined concurrently, rather than longitudinally. This is a necessary consequence of examining the results in the first year of administration of the precursor tasks.

Second, given Butterworth's view that the precursor abilities combine to produce a strong representation of number, the interaction terms (i.e., subitizing x finger gnosis, subitizing x finger agility, subitizing x finger gnosis x finger agility) or composite terms were expected to account for significant unique variance in the math outcome measures. This prediction was not borne out. In Study 2, one possible explanation for why the composite/interaction terms did not predict math outcomes was investigated; there may be different developmental paths to numeracy, essentially different ways to be good at math, or different combinations of skills that produce a favorable outcome. This possibility would not be consistent with Butterworth's view, given the necessary roles for subitizing, and for fingers as representational and procedural tools in the development of normal representations of number. It would, however, be of greater value for the remediation of math disabilities if individuals could compensate for weaknesses in one or more proposed precursor skills with strengths in another. Thus, further investigation was of interest for theoretical reasons, to determine the necessity of each precursor, and for practical reasons, to determine possible interventions for those with math disabilities.

Third, the outcome variables used in Study 1 involve a variety of mathematical skills and, as such, are not specific to numerical representations. For Study 3, tasks designed to more specifically assess numerical representation were included in the Count Me In Study and are examined as outcome variables longitudinally from Grade 1 - 2.

## Study 2: Are there Different Pathways to Numeracy?

### *Motivation*

In Study 2, I addressed one of the unanswered questions from Study 1 by investigating the possibility that different skill combinations could result in a favorable math outcome. To this end, I identified groups of children based on characteristic skill profiles across subitizing, finger gnosis, and finger agility using cluster analysis. Specific research questions included: Can different subgroups of children be identified based on subitizing, finger gnosis, and finger agility scores? And, if so, do the sub-groups have different math outcomes concurrently and longitudinally?

*Summary of Results*

First, number-system knowledge and calculation skill were predicted from the precursor abilities, concurrently in Grade 1. Subitizing and finger gnosis were related to number system knowledge and all three precursors were related to calculation skill[1]. Second, three sub-groups of children were identified based on characteristic skill profiles across subitizing, finger gnosis, and finger tapping using cluster analysis. The *good-all-around* group had strong subitizing ability, and relatively strong finger gnosis, and tapping ability. The *not-so-good* group had moderate subitizing and finger gnosis ability, paired with relatively weak tapping ability. The *non-subitizer* group had weak subitizing ability, paired with relatively moderate finger gnosis ability, and strong tapping ability. We found that the good-all-around group had significantly better number system knowledge and calculation skills than the other two groups both concurrently in Grade 1 and longitudinally in Grade 2. No significant differences were found in the math outcomes of the not-so-good and non-subitizer groups. For the detailed results, please refer to Appendix B.

*Conclusion*

The longitudinal results from Study 2 further address the question of which precursor abilities facilitate the development of number representations. Students with strong subitizing, finger gnosis and finger agility had better numeracy outcomes both concurrently in Grade 1 and longitudinally in Grade 2.

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[1] Note: For Study 1 the finger gnosis and finger agility measures were based solely on the non-dominant hand, on the view that the non-dominant hand provided a more neuropsychologically pure measure. In Studies 2 and 3 the dependent measures were revised to include both dominant and non-dominant hand information, based on both a reviewer's comment on Study 1 and prospectus committee feedback. This change affected the observed pattern of results for finger agility from Study 1 to 2.

The outcome variables used in Study 1 and Study 2, however, involve a variety of mathematical skills and, as such, are not specific to numerical representations. For Study 3, tasks designed to more specifically assess numerical representation were included in the Count Me In Study and are examined as outcome variables. Given the conclusion from Study 2, that subitizing and finger gnosis facilitate number representation whereas finger agility facilitates numerical procedures, converging evidence for a relation between both subitizing and finger gnosis and numerical representation tasks is required to assert a relation more specifically between the precursor abilities and the representation of number.

## Study 3: Are Subitizing and Finger Gnosis related to Number Representations?

### *Motivation*

In Study 2 (Penner-Wilger et al., 2009a) subitizing and finger gnosis were found to be related to mathematical skills including number system knowledge and calculation skill. Mathematical skill, however, involves a complex set of cognitive abilities, so why this relation exists is still an open question. I posit that subitizing and finger gnosis predict mathematical skill because these abilities form the building blocks of numerical representations (Butterworth, 1999; Penner-Wilger et al., 2009a). The motivation for Study 3 was to address the unanswered question from Studies 1 and 2 of whether the precursor abilities are related to performance on tasks designed to assess numerical representation and, moreover, to assess the relation longitudinally.

In a review of information processing deficits in dyscalculia and arithmetic learning disability, Butterworth and Reigosa (2007), assert that mathematical difficulties stem in part from slower and less efficient processing of numerical information, specifically the estimation and comparison of numerosities. In Study 3, I examined subitizing and finger gnosis as predictors of two tasks designed to assess numerical representations -- number-line estimation and number comparison. Specific research questions included: Do subitizing and finger gnosis jointly predict number-line estimation and number comparison performance? Do subitizing and finger gnosis each independently predict number-line estimation and number comparison performance? Additionally, group differences in number comparison and estimation performance were investigated for the subgroups from Study 2, with the goal of distinguishing the not-so-good and non-subitizer groups.

### *Summary of Results*

What precursor abilities form the building blocks of numerical representations? Two abilities were investigated: the ability to mentally represent small numerosities, indexed by subitizing (Butterworth, 1999; Dehaene, 1997), and the ability to mentally represent one's fingers, indexed by finger gnosis (Butterworth, 1999; Penner-Wilger & Anderson, 2008). The longitudinal relation between these abilities in Grade 1 and tasks assessing numerical representation in Grade 2—symbolic number comparison and number-line estimation—were investigated. Canadian children ($N = 100$) participated as part of the Count Me In longitudinal study. Finger gnosis in Grade 1 was related to children's symbolic distance effect in number comparison and to both the linearity and slope of children's estimates in Grade 2. Subitizing in Grade 1 was related only to the overall comparison latency in Grade 2. Thus, children with better finger gnosis scores had lower symbolic distance effects and more accurate estimates, reflecting a more precise mapping between numerals and their associated magnitude. The distance effect distinguished between the non-subitizer and not-so-good groups from Study 2; the non-subitizer group had a greater distance effect. For the detailed results, please refer to Appendix C.

### *Conclusion*

Finger gnosis was related to measures of numerical representation. Despite converging evidence that finger gnosis is related to math achievement and, more specifically, to tasks assessing numerical representations, what is not addressed from the findings of Studies 1 - 3 is *why and how* finger gnosis facilitates the development of number representations. That is, even if Butterworth is correct about the relation between finger gnosis and math he may still be incorrect about the mechanism. In Study 4, I critically evaluate the existing views on the relation between finger gnosis and the representation of number and propose a novel hypothesis of this relation that is consistent with existing neuroscience research.

## Study 4: Why is Finger Gnosis Related to the Representation of Number?

### *Motivation*

In Study 4, I build on the empirical findings from Studies 1 to 3 that finger gnosis, the ability to mentally represent one's fingers, is related to children's math achievement and representation of number. I present a novel view- the *redeployment* view - that explains the relation between finger gnosis and the representation of number as phylogentically determined. Moreover, rather than a predictive relation, a partial identity relation is instead proposed -- part of the neural circuitry is shared across both finger and number representation.

*Summary of Results*

On the proposed *redeployment* view, finger gnosis is related to math ability because part of the functional complex for number representation overlaps with the functional complex for finger representation. Thus, finger and number share a common neural resource, a shared working, which supports both sorts of representation. The comorbidity of finger agnosia and acalculia as well as the relation between finger gnosis and number in normally-developing children arise because these representations share a neural circuit. One possible shared working in these two very different functional complexes could be a register—a series of switches that can be activated independently of each other. Consider that both numerical values, and a memory of individual finger state, could be stored in a register. Thus, our position is that a shared working, such as a finger register, evolved to support our "finger sense" and was redeployed by a number representation circuit able to benefit from a component of that abstract structure.

The redeployment view of the relation between fingers and number is not a localizationist view. On the redeployment view, finger and number representations are not just neighboring neural functions on a correlated developmental trajectory; rather, they share a common neural substrate forming part of the neural complex supporting each function. Nor is the redeployment view a functional view, because the connection between finger and number does not rest on the experienced use of the actual fingers to represent numerosities and perform arithmetic procedures. Instead, the redeployment view is a phylogenetic view, wherein a part of the functional complex that supports finger representation has been redeployed, in the course of evolution, to support the representation of number, and now serves both uses. For the detailed results, please refer to Appendix D.

*Conclusion*

Study 4 addresses the relation between finger gnosis and the representation of number. In Butterworth's (1999) functionalist view, the ability to mentally represent one's fingers facilitates the development of numerical representations ontogenetically, in the course of a child's development. Children use their fingers to represent quantities and perform counting and arithmetic and through this process the neural representation of numbers comes to be attached to neural representation of fingers. In contrast, in Penner-Wilger and Anderson's (2008) redeployment view, the ability to mentally represent one's fingers facilitates the development of numerical representations phylogenetically, in the course of evolution.

In Study 4 one possible shared working between finger gnosis and number representation was proposed—a register. Though plausible, I did not have any data to recommend this shared working over any other. In Study 5, additional evidence is gathered to modify or refine my hypothesis of the shared function between finger and number representation.

# Study 5: How are Finger and Number Representations Related: What is the Shared Working?

*Motivation*

In Study 4, I proposed that finger representations and number representations were related, not ontogenetically due to the use of fingers to represent number, but phylogenetically due to a shared working. I submitted, based on consideration of the two uses, a register as a possible shared working. In Study 5, I investigated the shared working more systematically using a cross-domain methodology (as advocated by Anderson, 2007a,b,c; Bergeron, 2008; Cabeza & Nyberg, 2000).

*Summary of Results*

Imaging results from multiple domains were used to further bolster investigation of the shared working between finger and number representations. To this end I (1) identified the brain area of interest – a region within the left precentral gyrus, (2) identified, across domains, other cognitive uses that the area of interest supports, and (3) looked across these cognitive uses to ascertain the shared working of the area of interest. The final result of this process is a proposed shared working – an array of pointers - that can be tested empirically and will allow for the further elaboration of the redeployment view of the relation between finger and number representations. For the detailed results, please refer to Appendix E.

*Conclusion*

Based on the cross-domain imaging results, I proposed a shared working between finger and number representation – an array of pointers—that allows for both ordered storage and mapping between representational forms. This shared working suggests a novel decomposition (and candidate implementation) of number representation. Currently, we are implementing a model of number representation that makes use of the proposed array of pointers structure (Penner-Wilger, Anderson, & Stewart, in preparation). Moreover, this methodology of investigating overlapping functional complexes rather than modeling in isolation is expected to drive the field of cognitive science forward.

# CHAPTER 4: MAIN FINDINGS AND GENERAL DISCUSSION

The goal for this program of research was to determine whether the ability to represent number is facilitated by the ability to represent small numerosities (indexed by subitizing), the ability to mentally represent one's fingers (indexed by finger gnosis), and finger agility. This research question was addressed in a series of three experimental studies examining the relation between the precursors and number system knowledge and calculation skill (Studies 1 & 2) and performance on tasks designed to specifically assess numerical representations (Study 3). Based on the results from the experimental work, the relation between finger gnosis and number representations was further investigated in two synthetic studies examining evidence supporting an ontogenetic versus phylogenetic view of the relation between finger and number representations (Study 4), and proposing a mechanism responsible for the phylogenetic impact of finger representation on the development of number representations (Study 5).

## Subitizing

Subitizing was related to numeracy skills including number system knowledge and calculation skill (Studies 1 & 2). Subitizing, however, was not related to tasks assessing the symbolic representation of number: symbolic number comparison and number-line estimation (i.e., with Arabic digits). This finding is inconsistent with the views of both Butterworth (1999) and Dehaene (1997). Ansari (2008; Holloway & Ansari, 2008) suggests that, rather than being built on non-symbolic representations, symbolic representations are built on separate underlying representations. On Ansari's view, a relation would not be expected between subitizing (indexing non-symbolic representations) and the symbolic number representation tasks as used in Study 3. In contrast, subitizing would be predicted to relate to performance on non-symbolic versions of our tasks. This is an interesting empirical question for further study. Regardless, our findings in Study 3 are consistent with the view that symbolic representations are distinct from non-symbolic representations. Consistent with Butterworth (1999, 2005) and Dehaene (1997; Dehaene & Cohen, 2007), we hypothesize that subitizing impacts the development of numeracy phylogenetically, given the wealth of evidence in non-human species and pre-verbal infants (for a review see Brannon, 2005; Dehaene, 1997).

## Finger Gnosis

Finger gnosis was related to numeracy skills including number system knowledge and calculation skill (Studies 1 & 2). Finger gnosis was also related to measures of the symbolic representation of number: the distance effect in number comparison and the linearity and slope of children's estimates. Children who had more distinct mental representations of their fingers, as measured by better finger gnosis scores, also had more distinct representations of number, as measured by a reduced numerical distance effect in number comparison and more accurate number-line estimations. It has been proposed that fingers facilitate the mapping between symbolic and non-symbolic representations of number (Butterworth, 1999; Fayol & Seron, 2005). Based on Ansari's view, and consistent with the findings of Study 3 (Penner-Wilger et al., 2009b), symbolic representations are not mapped directly onto non-symbolic representations of number (Ansari, 2008; Holloway & Ansari, 2008). A mapping must occur, however, between representational forms to allow us to recognize common content (e.g., the numerosity three) in different representational vehicles (e.g., three, 3, OOO), though consistent with Ansari it need not be from non-symbolic to symbolic. Rather, both symbolic and non-symbolic representations of number could map directly onto our number concepts.

In Studies 4 & 5, a phylogenetic view of the relation between finger gnosis and number representation was proposed. On the *redeployment view*, one of the functional circuits originally evolved for finger representation has since been redeployed to support the representation of number and now serves both uses. Thus, the relation between finger and number representations is one of partial identity, rather than functional and predictive. The form of the shared working in Study 5, an array of pointers, supports both the storage of ordered elements and mapping between discrete representational forms. This mechanism could explain the ability to recognize common content in different vehicles. Indeed, consistent with this hypothesis, activation in the ROI associated with both finger and number representation was also was shown in a task requiring the recognition of common content (letters) in different representational vehicles (upper- vs. lowercase; Bunge et al., 2001) compared to recognition in the same vehicles (i.e., A a vs. A A). Moreover, Venkatraman, Ansari, and Chee (2005) found activation in the ROI for both symbolic and non-symbolic arithmetic, consistent with the view that the shared working between number and finger representation supports the mapping between representational forms.

Finger Agility

Finger agility was found to be related to calculation skill, but not to number system knowledge (Study 2). The finding that finger agility was related to calculation skill is consistent with the view that the fingers are used to perform arithmetic procedures. Thus, finger agility appears to index the ability to use the fingers as procedural tools. The measure of finger agility used in this research is also a measure of fine motor speed (Baron, 2004). Children who could more quickly use their fingers to perform arithmetic procedures may be in a better position to form strong representations of arithmetic facts, as both the operands could remain in memory allowing children to link the operands and answer (Geary, 1993).

On Butterworth's (1999) view, finger agility is also necessary for building representations of number and for linking non-symbolic and symbolic representations of number, as precision is needed to use the fingers to represent numerosities. At least two possibilities exist for why the relation between finger agility and numeracy skills was confined to calculation. First, it is possible that in our typically-developing population, most if not all children had sufficient agility to use their fingers to represent numerosities – thus a relation was not evident. Second, on the redeployment view, the relation between finger and number representations does not result from the use of fingers to represent numerosities. Thus, on the redeployment view no relation between finger agility and number representation would be predicted.

In summary, evidence was found for a relation between finger agility and calculation skill. Finger agility was hypothesized to impact calculation skill ontogenetically, as children use their fingers to perform arithmetic procedures. However, finger agility is not sufficient for the development of strong calculation skills, as evidenced in Study 3. The group with strong finger agility but poor subitizing (non-subitizer group) had poor outcomes on number system knowledge and calculation skill both concurrently and longitudinally.

## Summary

In response to the research question that guided this research, I found a different pattern of results from what Butterworth (1999) predicted. I found evidence that number system knowledge (e.g., counting, ordering, numeral recognition, sequencing, and place value) and calculation skill are both facilitated by the ability to represent small numerosities and the ability to mentally represent one's fingers. Calculation skill was also facilitated by finger agility.

I also found evidence that the ability to mentally represent one's fingers facilitated the representation of number. The ability to represent small numerosities, however, did not appear to facilitate the representation of number more specifically. This finding may reflect a distinction between symbolic and non-symbolic representations, which is inconsistent with Butterworth's view (1999). Thus, the strongest relation between a precursor and number representation was found for finger gnosis - the ability to mentally represent one's fingers. In contrast to Butterworth's ontogenetic view of the relation between finger and number representations, I outlined a phylogenetic view in Studies 4 and 5 that was consistent with findings across cognitive disciplines. There is no existing empirical evidence to recommend one view over the other. However, the mechanism outlined in Study 5 works equally well to map symbolic representations onto non-symbolic, the prevailing view, or to map different representational forms onto concepts, consistent with Holloway and Ansari (2008) and the results of Study 3.

## Implications

This program of research contributes to the field of cognitive science in four ways. First, increased understanding of the form and development of number representations will benefit the field of numerical cognition and cognitive science more generally. Compared to literacy, considerably less is known about the precursors of numeracy. Though correlational, the findings from Studies 1 – 3 suggest alternative relations between the precursors and number representations than those proposed by Butterworth (1999). Thus, the current results should inform and guide further research on the development of number representations.

Second, greater understanding of the precursors that facilitate number representations and the different skill profiles that lead to positive numeracy outcomes will benefit the fields of education and developmental psychology, aiding in the assessment of numeracy in children and the development of math curricula. Of particular relevance to the assessment of math difficulties, in Study 3, performance on the number comparison task differentiated our three skill groups. The distance effect was the only math measure to differentiate the non-subitizers, who showed a greater distance effect and appeared to have a specific number difficulty, from the not-so-good group, who appeared to have more global non-verbal difficulties. Overall comparison RT differentiated the lower skilled groups, who were slower to make comparisons overall, from the good-all-around group and showed a trend towards differentiating all three groups. Thus, between the two dependent measures, distance effect in RT and overall comparison RT, the number comparison task can be used to differentiate among the three groups. The number comparison task is fast and easy to administer and has been integrated into tests for dyscalculia (Butterworth, 2003). The results from Study 3 provide converging support for the use of number comparison as an assessment tool and point to which measures to use to distinguish between math specific and more global non-verbal difficulties. For intervention, it remains an open empirical question whether training of the precursor skills will improve math outcomes. Gracia-Bafalluy and Noël (2008) report a finger gnosis training study with Grade 1 students where they conclude that finger gnosis training improves both finger gnosis and numeracy outcomes. Though upon closer examination the results are less than compelling, there exists the possibility that training on any or all precursors will improve children's math outcomes.

Third, development of a new view of the relation between finger representation and number representation has generated testable hypotheses, as shown in Studies 4 and 5, that are expected to drive the study of numerical cognition in new and fruitful directions. The results of the synthetic work, development of the redeployment view and proposal for a shared working, have generated additional interdisciplinary work currently underway. Current projects following from my dissertation work include (1) imaging work to distinguish between the redeployment and functionalist view and (2) computational modeling work to implement a representation of number supported by our proposed shared working.

Finally, the theoretical approach and methodology used in Study 5 -- examining functions across rather than within cognitive domains is advocated as essential in the quest to understand human cognition and should reflect the future of cognitive neuroscience (Anderson, 2007a, b, c; Bergeron, 2008; Cabeza & Nyberg, 2000). In conclusion, this program of research applies methodologies and results from the cognitive sciences with the ultimate goal of understanding how humans represent number. This program of research has already proven fruitful in terms of both experimental and synthetic work (see Penner-Wilger & Anderson, 2008; Penner-Wilger et al., 2007; Penner-Wilger et al., 2008) and the results of continued work are expected to be of broad interest within cognitive science.

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APPENDIX A

Study 1

Penner-Wilger, M., Fast, L., LeFevre, J., Smith-Chant, B. L., Skwarchuk, S., Kamawar, D., & Bisanz, J. (2007). The foundations of numeracy: Subitizing, finger gnosia, and fine-motor ability. In D. S. McNamara & J. G. Trafton (Eds.), *Proceedings of the 29th Annual Cognitive Science Society* (pp. 1385-1390). Austin, TX: Cognitive Science Society.

The Foundations of Numeracy: Subitizing, Finger Gnosia, and Fine Motor Ability

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## Abstract

Butterworth (1999; 2005) proposed that several component abilities support our numerical representations and processes: an innate capacity to represent small numerosities (indexed by subitizing), fine motor ability (indexed here by finger tapping), and the ability to mentally represent one's fingers (indexed by finger gnosia). In the current paper, we evaluated the predictive power of these component abilities in the development of numeration and calculation skills in Grade 1 children (N = 146). Each component ability was found to be a significant unique predictor of number system knowledge, which in turn was related to calculation skill. Finger gnosia was related to calculation skill indirectly through number system knowledge. In contrast, subitizing predicted calculation skill both directly and indirectly through number system knowledge. Our results support Butterworth's view of the foundations of numeracy and have implications for the early identification of children at risk of math difficulties.

The Foundations of Numeracy: Subitizing, Finger Gnosia, and Fine Motor Ability

Mathematical skill is vital in our society. Poor skill in basic mathematics has been shown to have a greater negative effect on employment opportunities and job retention than poor literacy skills (Bynner & Parsons, 1997). In comparison to literacy, however, less is known about the precursor abilities that support math. Butterworth (1999; 2005) proposed that three component abilities support our numerical representations and processes: an innate capacity to represent small numerosities, the ability to functionally use one's fingers, and the ability to mentally represent one's fingers. In the current paper, we evaluate the component abilities underlying the development of numeration and calculation skills.

In Butterworth's view of the development of numerical abilities, the core component is the ability to "categorize the world in terms of numerosities—the number of things in a collection" (1999, p. 6). This ability, he posits, can be indexed by *subitizing*, that is, the ability to quickly enumerate the items in a set without counting. Subitizing generally applies only to sets of up to 3 or 4 items (Mandler & Shebo, 1982). To date, no one has examined the specific relation between subitizing ability and math skill.

To extend our core numerical abilities, Butterworth (1999) asserts, we construct concrete and abstract numerical representations using fingers, number words, and numerals. Across cultures, fingers are used to represent numerosities. The representations of fingers, thus, become tied to the representation of numerosities. Butterworth's position is that children's representation of number beyond the subitizing range hinges on the ability to mentally represent one's fingers, that is, *finger gnosia*. Children use their fingers not only to represent numerosities, but also for counting and arithmetic. Fingers serve as portable manipulatives, providing a bridge from concrete to abstract representations of quantity and of operations. Thus, the fine motor ability necessary to perform such functions may be important for the development of counting and calculation.

### *Subitizing*

Butterworth (1999; 2005) suggested that the core component upon which all other math abilities are built is an innate ability to recognize, represent, and manipulate cardinal values, termed *numerosity*. Deficits are posited to lead to a plethora of math difficulties. In accordance with Butterworth's view, Benoit, Lehalle, and Jouen (2004) concluded that subitizing is a necessary component for the mapping of number words to numerosities, as subitizing "allows the child to grasp the whole and the elements at the same time" (p. 21). The ability to predict math performance based on subitizing ability, however, has not yet been examined.

Koontz and Berch (1996) evaluated children's speed in enumerating sets of items. The authors found that in contrast to normally-achieving children, children with dyscalculia appeared to count the number of dots in a display of 1 – 3 such that their response times increased with each additional dot. These results and similar results from a dyscalculia study by Landerl and colleagues (Landerl, Bevan, & Butterworth, 2004) suggest that a relation will exist between subitizing speed and math skill.

*Mental Representation of Fingers*

Consistent with Butterworth's position that children's representation of number hinges on the ability to mentally represent one's fingers, young children of every culture use their fingers to represent quantities and for counting (Butterworth, 1999). As a result of children's use of their fingers to represent numerosity, he posits that fingers come to be associated with numerosities. Fayol and Seron (2005), likewise posit that fingers may be the "missing link" between preverbal representations of number and number words. In support of the importance of fingers in math development, both finger agnosia and fine motor deficits have been linked to math deficits in groups with neuropsychological disorders (Barnes, Smith-Chant, & Landry, 2005). Finger gnosia performance has been found to predict children's math performance both concurrently and longitudinally (Fayol, Barrouillet, & Marinthe, 1998; Noël, 2005).

Fayol, Barrouillet, and Marinthe (1998) examined the predictive power of finger-related measures on math performance in a longitudinal study of 5- and 6-year-old children ($N = 177$). Neuropsychological tests of perceptuo-tactile performance, including tests for simultagnosia, digital gnosia, digital discrimination, and graphisthesia, were given in June preceding entry into Grade 1. Developmental tests, including the Lozenge Drawing Test and Goodenough's Person Drawing Test, were also given during this session. These tests were combined to derive a neuropsychological score and a developmental score, which were used to predict math outcome both concurrently and longitudinally eight months later in Grade 1. Math outcome was measured by children's performance on tests including: write down all numbers you know, completing sequences of numbers, identifying cardinal values, and verbal word problems. The neuropsychological score was the best predictor of mathematical scores in preschool and Grade 1 ($r$ = .49 and .46 respectively). This evidence supports a role for fingers in math development.

Noël (2005) examined the predictive power of finger gnosia on numerical tasks in a longitudinal study of children from the beginning of Grade 1 to the end of Grade 2 (6 – 7 year olds, $N$ = 41). Noël refined the finger gnosia task, eliminating the confound of number labels for identifying the fingers. Children were given the finger gnosia test in Grade 1 and again in Grade 2. Numerical tasks were given in Grade 2 and included: magnitude comparison (range 1- 9, with dots and with Arabic digits), number transcoding (write dictated number in Arabic digits, range 2- to 3-digit numbers), subitizing (range 1-8), finger counting (identifying conventional and unconventional patterns of raised fingers), and single-digit addition. A numerical error factor was created from accuracy measures (comparison, subitizing, number transcoding, finger counting, and addition), and a numerical speed factor was created from response time measures (comparison, finger counting). Finger gnosia scores in Grade 1 were correlated with both the numerical error factor (-.48) and the numerical speed factor (-.30). Finger gnosia scores in Grade 2 were correlated with the numerical error factor (-.36). The predictive power of finger gnosia was specific to numerical skills and was not correlated with reading skill. Thus, even once the confound of number labels is removed, finger gnosia remains a good predictor of children's numerical abilities in the early grades. In summary, existing research supports the view that children's mental representation of fingers is related to the development of mathematical skill.

### *Functional Role of Fingers*

Consistent with Butterworth's position that fingers play a functional role in the development of numeracy, fine motor deficits are associated with math disabilities. Barnes et al. (2005) investigated the link between fine motor ability and math in children with Spina Bifida. Spina Bifida is a neurodevelopmental disorder that produces, among others, deficits in fine motor ability. Moreover, children with Spina Bifida show math difficulties early in development and these difficulties persist into adulthood.

In a group of 120 children aged 8 – 16 years (60 with Spina Bifida and 60 age- and grade-matched controls) Barnes et al. found that fine motor ability, measured with the Purdue Pegboard task, was correlated with skill in multi-digit calculation, accounting for 28 % of the variance. In a group of younger children (aged 36 months old) fine motor ability measured with the Visual Motor Integration test (Beery & Beery, 2004) predicted significant unique variance in children's nascent quantitative skills including: object counting, finger counting, quantitative vocabulary, and counting concepts. Thus, there is a link between fine motor skill and a variety of numerical tasks that is robust across measures and development.

### *Present Research*

What component abilities underlie the development of numeration and calculation skills? Butterworth posits a role for three component skills: subitizing, finger gnosia, and fine motor ability. In the current paper we investigated the joint and independent contributions of each component ability to the numeration and calculation skills of children in Grade 1. Numeration skills include counting, ordering, recognizing numerals, sequencing, and place value. These tasks make use of the representations of number, which are posited to be formed based on subitizing ability and extended via finger gnosia, and of the procedures facilitated by fine motor ability. We hypothesized that these component abilities make independent contributions to the development of numerical skill. We further hypothesized that the component abilities relate to calculation skill indirectly via numeration. In contrast, on the view that subitizing forms the core of all numerical skills, we hypothesized that subitizing may also relate to calculation skills directly.

## Method

### *Participants*

Grade 1 children ($N$ = 146) were selected from an ongoing longitudinal study. The testing sessions took place in the late Spring, by which time the children had participated in nine months of mathematical instruction. The children (71 girls and 75 boys) ranged in age from 5 to 7 years old ($M$ = 82 months).

Socio-economic status (as measured by parent education levels) was relatively high across all of the schools in the sample. Seventy-three percent of the participants' parents provided education level information, of these, only 19% did not hold university or college degrees, while 25% held post-graduate degrees.

*Procedure*

Most children completed the computer measures in one half-hour session and the rest of the measures in a separate half-hour session on a different day. Some children completed all of the measures in a single session of approximately an hour.

All of the computer tasks were presented using software developed specifically for this project. For the computer tasks, children initiated the trials themselves by pressing the spacebar. Response times were measured from the point at which the stimuli appeared, until the experimenter pressed the stop-timer key (using a separate keyboard) when the child spoke their response. The experimenter then typed in the child's response.

*Materials*

*Subitizing.* On each trial the computer displayed a set of 1 to 6 circular red target objects. The children were instructed to respond with the number of objects, out loud, as quickly as possible. The child initiated each trial by pressing the space bar. To promote accuracy, the targets remained on the display until the child's response was entered by the experimenter. There were 18 trials, preceded by two practice trials of two and seven objects. Half of the trials were within the subitizing range (1 to 3), and half in the counting range (4 to 6). Although the subitizing range is often defined as from 1 to 4 objects (Trick & Pylyshyn, 1994), previous research with children in the Grade 1 age range shows accuracy and speed declined for four items (Trick, Enns, & Brodeur, 1996).

The measure of interest for the present research is the response time (RT) slope as a function of set size. To compensate for the variability in the response times of this small number of trials, median response times were calculated for each set size (1 – 3 items) for each child. The best-fitting regression line through these medians was calculated for each child. The slope values were used as the dependent measure.

*Finger gnosia.* The finger gnosia measure is based on one designed by Noël (2005). Ten trials were conducted on each hand, beginning with the dominant hand. In each trial, two fingers were lightly touched below the first knuckle. The child's view of the touches was obstructed with a cloth cover raised from the child's wrist. After the cloth cover was lowered, the child pointed to the two fingers that had been touched. A point was awarded for each correct identification of a touched finger in a trial, with a maximum of 20 points per hand. The score for the non-dominant hand was used as the dependent measure.

*Finger tapping.* To isolate fine motor ability from visual-spatial performance, we developed a computer-game version of the Finger Tapping Test (Baron, 2004). The game was presented as a canal-digging exercise, where the longer the canal, the bigger the fish that will swim in it. Each tap on the space bar was a 'dig of the shovel', and increased the length of the canal, with an indicator of the maximum length from any previous trials. Children were instructed and encouraged to tap as fast as they could. Timing began upon the first press of the space bar and continued for 10 seconds (Baron, 2004), until an animated fish appeared. The tapping score determined the type of fish, with a prized killer whale animation appearing for scores over 50 taps. Tapping scores were collected for three trials on each hand, beginning with the dominant hand. The maximum number of taps achieved across the three non-dominant hand trials was used as the dependent measure.

*Digit recognition/Next number.* The Digit Recognition and Next Number tasks were performed on the computer. Each of these tasks had 18 trials of increasing difficulty. The task ended if the child made three sequential errors. In each trial, a number was displayed on the screen. The first half of the trials were under 100, then increased in difficulty through the hundreds, thousands and for Next Number, into the ten and hundred thousands, ending at 407,276. For Digit Recognition, the experimenter asked the child, "What number is that?" Later during the testing session, for the Next Number task, the child was asked to respond with the number "that comes next when counting." Responses that were spoken as digits–for example, "one oh oh" instead of "one hundred"–were marked as errors. The total number of correct responses was used as the dependent measure.

*Place value.* Knowledge of place value was measured through a test designed for this study. Each page of the testing easel displayed an image of a set of proportionately sized unit, ten, hundred or thousand blocks. Above the blocks was a two- to four-digit number with one digit underlined. The child was asked to identify whether the number of blocks was consistent with the underlined number. A consistent trial would show the number <u>3</u>52 with an image of 3 hundred-unit blocks. An inconsistent trial would show the number <u>3</u>52 with an image of 3 ten-unit blocks. Two example trials and two practice trials precede the twelve test trials. The task taps children's understanding of the values associated with the location of digits in numbers. The total number of correct trials was used as the dependent measure.

*Numeration.* Concepts such as quantity, order, and place value were measured with the Numeration subtest of a multi-domain diagnostic test, the KeyMath Test-Revised, Form B (Connolly, 2000).

*Calculation skill.* Mathematical skill was assessed with the Calculation subtest from the Woodcock-Johnson Psycho-Educational Battery—Revised (Woodcock & Johnson, 1989). This subtest begins with small mathematical problems in both horizontal and vertical formats. The problems progress in difficulty and include addition, subtraction, and multiplication.

*Processing speed.* To assess processing speed, we implemented a computer-based simple choice reaction time task (Petrill, Luo, Thompson, & Detterman, 2001). Two types of stimuli (an X or an O) were displayed for 1 second, preceded by a half second fixation point. The display then cleared and the next trial began automatically 1 second later. There were 24 trials. The child positioned the index finger of their dominant hand on the keyboard key with an 'X' sticker (the 'X' key) and their middle finger on the key above it labeled 'O' (the 'D' key). Left-handed children used similar stickers on the right side of the keyboard. The median response time for pressing the correct key in response to the stimuli was used as the dependent measure.

*Vocabulary.* Receptive language was measured using the Peabody Picture Vocabulary Test–Third Edition (Dunn & Dunn, 1997). It was included primarily as a measure of verbal, non-mathematical knowledge.

# Results

Descriptive statistics for each measure are shown in Table 1. Correlations among the measures are shown in Table 2. Notably, there were no significant correlations among the three precursor abilities, subitizing, finger tapping, and finger gnosia, supporting Butterworth's view of separate component abilities (Butterworth, 1999). Subitizing and finger gnosia were correlated with both number system knowledge and calculation skill. Finger tapping was correlated with number system knowledge.

*Table 1.* Descriptive information ($N = 146$)

| Measure | Max | Mean | SD |
|---|---|---|---|
| Subitizing Slope[4] | | 100.8 | 129.4 |
| Finger Gnosia[2] | 20 | 15.0 | 2.4 |
| Finger Tapping | | 38.8 | 5.5 |
| Calculation Skill[1] | | 99.8 | 15.1 |
| Processing Speed[4] | | 781.2 | 209.4 |
| Vocabulary[1] | | 109.5 | 11.5 |
| Number System Knowledge [Component Measures] | | | |
| Digit Recognition[2] | 18 | 14.5 | 2.6 |
| Next Number[2] | 18 | 10.2 | 3.8 |
| Place Value[2] | 12 | 7.2 | 2.1 |
| Numeration[3] | 17 | 12.6 | 3.7 |

*[1] Standardized score; [2] Number correct; [3] Scaled Score; [4] Milliseconds*

*Table 2.* Correlations among measures ($N = 146$).

| Task | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1. Gender[1] | — | | | | | | |
| 2. Vocabulary | .13 | — | | | | | |
| 3. Processing Speed | -.08 | -.06 | — | | | | |
| 4. Finger Gnosia | -.15 | .10 | -.16 | — | | | |
| 5. Tapping | -.01 | .06 | -.11 | .06 | — | | |
| 6. Subitizing | -.03 | -.05 | -.10 | -.10 | .04 | — | |
| 7. Number System | .23** | .43** | -.23** | .27** | .18* | -.30** | — |
| 8. Calculation Skill | .16** | .23** | -.10 | .19** | .12 | -.31** | .61** |

[1] Gender coding: Female = 1 and Male = 2. Significance levels: *p<.05, **p<.01.

### *Data Reduction*

Tasks that index number system knowledge (i.e. digit recognition, next number, place value, and KeyMath numeration) were entered into a principal components factor analysis with varimax rotation. A one-factor solution emerged that accounted for 66% of the variance among the measures with the following loadings: digit recognition (.89), next number (.86), place value (.59), and KeyMath numeration (.87). Factor scores were saved and used as the Number System Knowledge measure in the subsequent regressions.

### *Regressions*

To test the hypothesis that the finger tasks and subitizing would each predict number system knowledge, a regression was conducted with the number system knowledge factor as the dependent variable and subitizing, finger gnosia, and finger tapping as predictors. In both this and the following regression, gender, receptive vocabulary, and processing speed were included as control variables. The pattern of significant predictors is shown on the left of Figure 1. Consistent with Butterworth's view, each precursor ability was a significant unique predictor of number system knowledge. Overall, the model accounted for 36.4 % of variance in number system knowledge.

Next, a second regression was conducted with calculation skill as the dependent variable, and with number system knowledge and the three precursor abilities as the predictors. As shown in Figure 1, number system knowledge was related to calculation skill. Finger gnosia was related to calculation skill indirectly through number system knowledge. This finding is consistent with Butterworth's view that the ability to mentally represent fingers is an integral building block for the representation of numbers beyond the subitizing range. In contrast, subitizing predicted calculation skill both directly and indirectly through number system knowledge. This finding is consistent with Butterworth's view that the capacity to represent small numerosities is the core ability upon which all other mathematical skills are built. Overall, the model accounted for 36.0 % of variance in calculation skill.

In summary, as predicted, all component skills (subitizing, finger gnosia, and finger tapping) contributed uniquely to number system knowledge. As predicted, finger gnosia was indirectly related to calculation skill. Finger tapping, however, was not related to calculation skill. This finding may reflect that our operational definition of fine-motor skill as indexed by finger tapping was not ideal. In contrast, subitizing exerted influence on calculation skill beyond that accounted for by number system knowledge.

## Discussion

In the current paper we investigated the joint and independent contributions of each component ability (subitizing, finger gnosia, and fine motor ability) to the numeration and calculation skills of children in Grade 1. No correlations were found among the component abilities, consistent with the view that each reflects a separate ability. Further, each component ability was found to predict significant unique variance in children's number system knowledge.

Subitizing was correlated with both number system knowledge and calculation skill. Thus, children able to enumerate 1 - 3 items without counting performed better in mathematics. Butterworth (1999) proposes that this is the core ability upon which math is built. Indeed, both young infants and many non-human animals exhibit the ability to distinguish small sets of items based on magnitude (for a review see Butterworth, 1999; Dehaene, 1997). Despite the evidence supporting subitizing as a developmental and evolutionary precursor to mathematics, our findings are the first that we are aware of to show a correlation between subitizing and mathematical ability in a typically-developing population.

Finger-based representations of magnitude are posited to extend the representation of quantity beyond the subitizing range. Finger gnosia was correlated with both number system knowledge and calculation skill. Thus, children able to use their fingers as representational tools performed better in mathematics. Butterworth (1999) proposes a functional relation between finger gnosia and math; the two are related because children use their fingers to represent quantities. Alternative neural views of the relation between finger gnosia and math have also been proposed (see Anderson & Penner-Wilger, 2007; Dehaene, et al. 2003). Anderson and Penner-Wilger suggest that the two tasks share a common underlying neural circuit, which though originally purposed for finger representation has been redeployed for the representation of magnitude.

Children with greater finger agility, as measured by finger tapping, would be in a better position to use their fingers to perform counting and arithmetic procedures. We found that finger tapping was correlated with number system knowledge. The numeration component of number system knowledge involved counting and simple arithmetic problems.

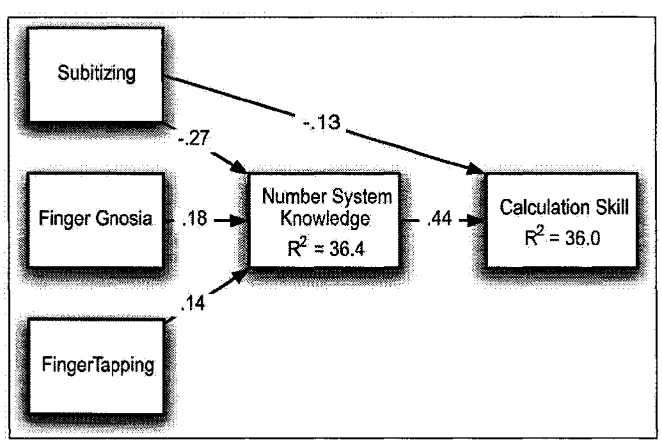


*Figure 1.* Regression model with semi-partial regression coefficients.

## Conclusion

What component abilities underlie the development of mathematical skills? Overall, the results of the present study suggest independent contributions for the ability to enumerate small sets of items, fine motor ability, and the ability to mentally represent one's fingers. These predictors hold promise for the early identification of children with math difficulties. Mathematical skills form a complex mosaic, however, and the component abilities investigated constitute only a portion of the answer to why some children are better at math than others. In the reported work we examined the components concurrent with mathematical skill in Grade 1; we are currently investigating whether these predictors hold longitudinally.

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APPENDIX B

Study 2

Penner-Wilger, M., Fast, L., LeFevre, J., Smith-Chant, B. L., Skwarchuk, S., Kamawar, D., & Bisanz, J. (2009). Precursors to numeracy: Subitizing, finger gnosis, and fine-motor ability. Manuscript submitted for publication.

Precursors to Numeracy: Subitizing, Finger Gnosis, and Finger Agility

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AUTHOR NOTE

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## Abstract

Butterworth (1999, 2005) proposed that several component abilities support our numerical representations and processes: an innate capacity to represent small numerosities (indexed by subitizing), the ability to mentally represent one's fingers (indexed by finger gnosis), and finger agility (indexed here by finger tapping). In the current paper, we evaluated the predictive power of these component abilities in the development of number system knowledge and calculation skills in Grade 1 children (N = 144). We found that subitizing and finger gnosis were related to children's number system knowledge and that all three component abilities were related to children's calculation skill. Using cluster analysis, we found that students with strong subitizing, finger gnosis and finger agility had better numeracy outcomes both concurrently in Grade 1 and longitudinally in Grade 2. Our results support Butterworth's view of the foundations of numeracy and have implications for the early identification of children at risk of math difficulties.

# Precursors to Numeracy: Subitizing, Finger Gnosis, and Finger Agility

Numeracy is vital in our society. Poor numeracy skills have been shown to have a greater negative effect on employment opportunities and job retention than poor literacy skills (Bynner & Parsons, 1997). In comparison to literacy, however, less is known about the precursor abilities that support numeracy. Butterworth (1999, 2005) proposed that three component abilities support our numerical representations and processes: an innate capacity to represent small numerosities, the ability to mentally represent one's fingers, and the ability to functionally use one's fingers (i.e., finger agility). Moreover, Butterworth asserts that each of these skills is *necessary* for the normal development of number representations and numeracy skills. In the current paper, we evaluate the component abilities underlying the development of numeracy skills.

## *Precursor Abilities that Facilitate the Development of Numeracy*

How do we develop the representations of number that form the basis for numeracy? Butterworth (1999) hypothesizes we have a *number module*, a domain-specific, cognitive module that allows us to categorize aspects of the world in terms of numerosities (i.e., the number of items in a collection). This number module is at the core of our numerical abilities and forms our non-symbolic representation of number. Support for the number module is derived from animal studies, infant studies, studies of dyscalculia, and brain imaging research (for reviews see Brannon, 2005; Butterworth, 1999). The number module is indexed by subitizing, the ability to quickly enumerate items in a set without counting. The number module has a limited capacity, however, extending only to numerosities of four or five. Butterworth posits that we extend our non-symbolic numerosity representations using cognitive tools. Examples of such cognitive tools include: use of fingers, number words, numerals, and calculators. One of the most important cognitive tools, in Butterworth's (1999) view, are the ten fingers. Fingers are used in the course of development to represent numerosities and to perform counting and arithmetic procedures across cultures and without formal instruction (Butterworth, 1999). Indeed, Butterworth hypothesizes, that "without the ability to attach number representations to the neural representations of fingers…the numbers themselves will never have a normal representation" (p. 226). Thus, the use of fingers as cognitive tools extends the limited capacity of the number module.

The prevailing view of numerical development is that symbolic number representations (i.e., number words and numerals) "acquire their meaning by being mapped onto non-symbolic representations of numerical magnitude" (Ansari, 2008, p. 278). Butterworth posits that the fingers facilitate this mapping between non-symbolic numerosity representations (associated with the number module) and symbolic number words. Number words (e.g., one, two, three …) are cognitive tools that provide a sequence of numerosities, ordered by size, that build up from the limited capacity number module into a full numerosity system. Fayol and Seron (2005) likewise hypothesize that fingers may be the "missing link" between preverbal representations of number and number words, stating that, unlike linguistic representations "finger representations exhibit an iconic relation to numerosities, since they preserve the one-to-one matching relation between the represented set and the fingers used to represent it" (p. 16). Thus, the fingers seem ideally suited as a cognitive tool to aid in the development of symbolic representations of number.

Two digital (i.e., finger related) abilities are proposed to be of importance in the development of number representations and numeracy skills. First, given the importance of the functional use of the fingers in the course of numerical development, and the precise accurate control required to use the fingers for representing numerosities and for finger counting and arithmetic procedures, finger *agility* is hypothesized to be crucial for the development of numerical representations. Second, given the importance of finger representations as a link between non-symbolic and symbolic representations of number, the ability to mentally represent one's fingers is also hypothesized to be crucial. Both the finger agility required to functionally use the fingers and the representational ability are proposed as precursors to number representations and numeracy skills.

In summary, Butterworth (1999) posits that the core component facilitating the development of number representation is the ability to categorize the world in terms of numerosities. This core ability is indexed by *subitizing*, the ability to enumerate small sets without counting. Butterworth also posits a crucial role for fingers, both as representational and procedural tools. The representation role of fingers is indexed by *finger gnosis*, the ability to mentally represent one's fingers. The procedural role of fingers is indexed in the current study by *finger agility*.

### *Subitizing*

Subitizing is the ability to enumerate small sets of items without counting (Mandler & Shebo, 1982). The subitizing range – the number of items that can be enumerated without counting- is generally considered to be one to three items for children ranging up to four or five items for adults (Trick, Enns, & Brodeur, 1996). In accordance with Butterworth's view on the importance of subitizing, Benoit, Lehalle, and Jouen (2004) concluded that subitizing is a necessary component for the mapping of number words to numerosities, as subitizing "allows the child to grasp the whole and the elements at the same time" (p. 21). The relation between subitizing and numerical ability has not yet been examined in a typically-developing population, however, researchers have noted a trend towards poorer subitizing in studies of children with dyscalculia and arithmetic learning disabilities.

Willburger et al. (2008) found that children with dyscalculia were slower to perform rapid automatized naming of 1-4 items than normally-achieving children (see also van der Sluis et al., 2003, who reported a similar result). Koontz and Berch (1996) noted that children with arithmetic learning disabilities appeared to count the number of items in displays of two to three dots; this observation was supported with response time data. In summary, although Butterworth's theory posits a critical role for subitizing in the development of numerical abilities this position has yet to be supported in typically-developing populations.

*Finger Gnosis*

Finger gnosis is the presence of an intact finger schema or ability to mentally represent one's fingers as distinct (from one another). Neuropsychological tests have been designed to assess the presence of finger gnosis, or its absence—finger agnosia—in neuropsychological populations. In one common test (Baron, 2004), the examiner shields the participant's hand from view and lightly touches one or more fingers. The participant is asked to identify which fingers were touched. Finger agnosia is one of a constellation of symptoms in Gerstmann's syndrome, along with acalculia, agraphia, and left-right disorientation. Gerstmann (1940) identified finger agnosia as the core deficit of the syndrome. Butterworth (1999) posits that finger representations extend the limited capacity of the innate numerosity system and, moreover, 'bridge the gap' from numerosity representations to more abstract number words.

Fayol, Barrouillet, and Marinthe (1998) examined the predictive power of finger-related measures on math performance in a longitudinal study of 5- and 6-year-old children ($N = 177$). Neuropsychological tests of perceptuo-tactile performance, including tests for simultagnosia, digital gnosis, digital discrimination, and graphisthesia, were given in June preceding entry into Grade 1. Developmental tests, including the Lozenge Drawing Test and Goodenough's Person Drawing Test, were also given during this session. These tests were combined to derive a neuropsychological score and a developmental score, which were used to predict math outcome both concurrently and longitudinally eight months later in Grade 1. Math outcome was measured by children's performance on tests including: write down all numbers you know, completing sequences of numbers, identifying cardinal values, and verbal word problems. The neuropsychological score was the best predictor of mathematical scores in preschool and Grade 1 ($r = .49$ and $.46$ respectively). This evidence supports a role for fingers in math development.

Noël (2005) examined the predictive power of finger gnosis on numerical tasks in a longitudinal study of children from the beginning of Grade 1 to the end of Grade 2 (6 – 7 year olds, $N = 41$). Noël refined the finger gnosis task, eliminating the confound of number labels for identifying the fingers. Children were given the finger gnosis test in Grade 1 and again in Grade 2. Numerical tasks were given in Grade 2 and included: magnitude comparison (range 1- 9, with dots and with Arabic digits), number transcoding (write dictated number in Arabic digits, range 2- to 3-digit numbers), subitizing (range 1-8), finger counting (identifying conventional and unconventional patterns of raised fingers), and single-digit addition. A numerical error factor was created from accuracy measures (comparison, subitizing, number transcoding, finger counting, and addition), and a numerical speed factor was created from response time measures (comparison, finger counting). Finger gnosis scores in Grade 1 were correlated with both the numerical error factor (-.48) and the numerical speed factor (-.30). Finger gnosis scores in Grade 2 were correlated with the numerical error factor (-.36). The predictive power of finger gnosis was specific to numerical skills and was not correlated with reading skill. Thus, even once the confound of number labels is removed, finger gnosis remains a good predictor of children's numerical abilities in the early grades. In summary, existing research supports the view that children's mental representation of fingers is related to the development of mathematical skill. However, the independent and joint role of finger gnosis has yet to be examined in concert with subitizing and finger agility.

*Finger Agility*

Finger agility can be assessed using a finger tapping test, where the participant taps a key with their index finger as many times as possible during a given time interval (Gilandas, Touyz, Beumont, & Greenberg, 1984). As the relation between fingers and number is a functional one, in Butterworth's view, the precise motor control required to use the fingers to represent numerosities and perform counting and arithmetic procedures is vital for the development of numeracy (but see Penner-Wilger & Anderson, 2008, for an alternate view). Consistent with Butterworth's position that the fingers play a functional role in the development of numeracy, fine motor deficits are associated with math disabilities.

Barnes et al. (2005) investigated the link between fine motor ability and math in children with Spina Bifida. Spina Bifida is a neurodevelopmental disorder that produces, among others, deficits in fine motor ability. Moreover, children with Spina Bifida show math difficulties early in development and these difficulties persist into adulthood. Barnes et al. found that fine motor ability, measured with the Purdue Pegboard task, was correlated with skill in multi-digit calculation in a combined group of 120 children aged 8 – 16 years (60 with Spina Bifida and 60 age- and grade-matched controls). In a similarly-combined group of younger children (aged 36 months old) fine motor ability measured with the Visual Motor Integration test (Beery & Beery, 2004) predicted significant unique variance in children's nascent quantitative skills including: object counting, finger counting, quantitative vocabulary, and counting concepts. These results support a link between fine motor ability and counting and calculation skill, though the relation may be driven by the combination of typically developing and atypically developing populations. Moreover, the relation between finger agility and number system knowledge has not been examined.

*Present Research*

What component abilities underlie the development of numeration and calculation skills? Butterworth posits a role for three component skills: subitizing, finger gnosis, and finger agility. In the current paper we investigated the joint and independent contributions of each component ability to the number system knowledge and calculation skills of children in Grade 1. We also investigated the concurrent and longitudinal numeracy outcomes of children based on characteristic skill profiles across the component skills using cluster analysis. Number system knowledge includes counting, ordering, recognizing numerals, sequencing, and place value. These tasks make use of the representations of number, which are posited to be formed based on subitizing ability and extended via finger gnosis, and at least for counting and arithmetic, of the procedures facilitated by finger agility. We hypothesized that subitizing, finger gnosis and finger agility make independent contributions to the development of numerical skill: with subitzing and finger gnosis related to both number system knowledge and calculation skill and finger agility related to calculation skill. Finger agility was not expected to relate to number system knowledge in this typically developing population, as most, if not all, children were expected to have sufficient finger agility to use their fingers to represent quantities. Finger agility was expected to relate to calculation skill, however, given the role of fingers as procedural tools to perform arithmetic.

# Method

*Participants*

Grade 1 children (N = 144) were selected from an ongoing longitudinal study. The testing sessions took place in the late Spring, by which time the children had participated in nine months of mathematical instruction. The children (71 girls and 73 boys) ranged in age from 5 to 7 years old (M = 6 years 10 months). The following spring, 112 of the children now in Grade 2 participated. Of these 112 children, 10 did not have complete data and were removed from the longitudinal analyses, leaving 102 children in the longitudinal analyses.

Socio-economic status (as measured by parent education levels) was relatively high across all of the schools in the sample. Eighty-three percent of the participants' parents provided education level information, of these, 58.5% of parents held university degrees or higher.

*Procedure*

Most children completed the computer measures in one half-hour session and the rest of the measures in a separate half-hour session on a different day. Some children completed all of the measures in a single session of approximately one hour.

All of the computer tasks were presented using software developed specifically for this project. For the computer tasks, children initiated the trials themselves by pressing the spacebar. Response times were measured from the point at which the stimuli appeared, until the experimenter pressed the stop-timer key (using a separate keyboard) when the child spoke their response. The experimenter then typed in the child's response.

*Materials*

*Subitizing.* On each trial the computer displayed a set of 1 to 6 circular red target objects. The children were instructed to respond with the number of objects, out loud, as quickly as possible. The child initiated each trial by pressing the space bar. To promote accuracy, the targets remained on the display until the child's response was entered by the experimenter. There were 18 trials, preceded by two practice trials of two and seven objects. Half of the trials were within the subitizing range (1 to 3), and half in the counting range (4 to 6).

The measure of interest for the present research is the response time (RT) slope as a function of set size. To compensate for the variability in the response times of this small number of trials, median response times were calculated for each set size (1 – 3 items) for each child. The best-fitting regression line through these medians was calculated for each child. The slope values were used as the dependent measure.

*Finger gnosis.* The finger gnosis measure is based on one designed by Noël (2005). Ten trials were conducted on each hand, beginning with the dominant hand. In each trial, two fingers were lightly touched below the first knuckle. The child's view of the touches was obstructed with a cloth cover raised from the child's wrist. After the cloth cover was lowered, the child pointed to the two fingers that had been touched. A point was awarded for each correct identification of a touched finger in a trial, with a maximum of 20 points per hand. The total score across both hands was used as the dependent measure, with a maximum score of 40.

*Finger tapping.* To assess finger agility, we developed a computer-game version of the Finger Tapping Test (Baron, 2004). The game was presented as a canal-digging exercise, where the longer the canal, the bigger the fish that will swim in it. Each tap on the space bar was a 'dig of the shovel', and increased the length of the canal, with an indicator of the maximum length from any previous trials. Children were instructed and encouraged to tap as fast as they could. Timing began upon the first press of the space bar and continued for 10 seconds (Baron, 2004), until an animated fish appeared. The tapping score determined the type of fish, with a prized killer whale animation appearing for scores over 50 taps. Tapping scores were collected for three trials on each hand, beginning with the dominant hand. The sum of the maximum number of taps achieved across the three trials for both the dominant and non-dominant hand was used as the dependent measure (i.e., maximum non-dominant hand + maximum dominant hand score).

*Digit recognition/Next number.* The digit recognition and next number tasks were performed on the computer. Each of these tasks had 18 trials of increasing difficulty. The task ended if the child made three sequential errors. In each trial, a number was displayed in Arabic digits on the screen. The first half of the trials were under 100, then increased in difficulty through the hundreds, thousands and for next number, into the ten and hundred thousands, ending at 407,276. For digit recognition, the experimenter asked the child, "What number is that?" Later during the testing session, for the next number task, the child was asked to respond with the number "that comes next when counting." Responses that were spoken as digits–for example, "one oh oh" instead of "one hundred"–were marked as errors. The total number of correct responses was used as the dependent measure.

*KeyMath Numeration.* Concepts such as quantity, order, and place value were measured with the Numeration subtest of a multi-domain diagnostic test, the KeyMath Test-Revised, Form B (Connolly, 2000). Connolly provides a split-half reliability coefficient of .82 for the spring Grade 1 norming sample and .81 for spring Grade 2.

*Woodcock-Johnson Calculation.* Mathematical skill was assessed with the Calculation subtest from the Woodcock-Johnson Psycho-Educational Battery—Revised (Woodcock & Johnson, 1989). This subtest begins with small mathematical problems in both horizontal and vertical formats. The problems progress in difficulty and include addition, subtraction, and multiplication. The WJ-R manual cites the split-half reliability for six year olds as .93, *SEM(W)*=5.7 (*N*=309).

*Addition.* Addition accuracy and latencies were measured on the computer. Sixteen trials of single-digit sums were displayed on the computer screen. In Grade 1, the sums were always less than 10; in Grade 2, the sums were greater than ten. This task has a stop condition of five sequential errors and trials timed out if the child did not respond within 20 seconds. The child initiated each trial by pressing the 'GO' button. When the child spoke their answer, the experimenter pressed a key to stop the timer and typed in their response. Each child's median addition latency was computed from their correct trials. Two children did not have the minimum 50% correct trials to compute a median latency, therefore the group mean was substituted for these scores in the factor analysis.

*Processing speed.* To assess processing speed, we implemented a computer-based simple choice reaction time task (Petrill, Luo, Thompson, & Detterman, 2001). Two types of stimuli (an X or an O) were displayed for 1 second, preceded by a half second fixation point. Children were instructed to press the key corresponding to the target letter shown on the screen. The display then cleared and the next trial began automatically 1 second later. There were 24 trials. The child positioned the index finger of their dominant hand on the keyboard key with an 'X' sticker (the 'X' key) and their middle finger on the key above it labeled 'O' (the 'D' key). Left-handed children used similar stickers on the right side of the keyboard. The median response time for pressing the correct key in response to the stimuli was used as the dependent measure.

*Vocabulary.* Receptive language was measured using the Peabody Picture Vocabulary Test–Third Edition, Form B (Dunn & Dunn, 1997). It was included primarily as a measure of verbal, non-mathematical knowledge. Dunn and Dunn cite the split-half reliability coefficient for Form B for seven year olds as .95.

# Results

Descriptive statistics for each measure in Grade 1 are shown in Table 1. All results are significant at $p < .05$ unless otherwise noted.

## *Data Reduction*

Tasks that index number system knowledge (i.e., digit recognition, next number, and the KeyMath numeration subtest) were entered into a principal components factor analysis with varimax rotation. A one-factor solution emerged that accounted for 79.7% of the variance among the measures with the following loadings: digit recognition (.91), next number (.89), and KeyMath numeration (.88). Factor scores were saved and used as the Number System Knowledge measure in the subsequent regressions.

Measures that index calculation skill (i.e., addition accuracy, addition latency, and the Woodcock-Johnson calculation subtest) were entered into a principal components factor analysis with varimax rotation. A one-factor solution emerged that accounted for 66.2% of the variance among the measures with the following loadings: addition accuracy (.77), addition latency (-.81), and Woodcock-Johnson calculation (.85). Factor scores were saved and used as the Calculation Skill measure in the subsequent regressions.

*Table 1.* Descriptive information for Grade 1 measures (N = 144)

| Measure | Max. Score | M | SD |
|---|---|---|---|
| Subitizing Slope[1] | | 98 | 126 |
| Finger Gnosis[2] | 40 | 31.1 | 4.0 |
| Finger Tapping | | 82.3 | 10.1 |
| Vocabulary[3] | | 109.6 | 11.6 |
| Processing Speed[1] | | 783 | 210 |
| | Number System Knowledge | | |
| Digit Recognition[2] | 18 | 14.5 | 2.6 |
| Next Number[2] | 18 | 10.2 | 3.8 |
| KeyMath Numeration[4] | 17 | 12.6 | 3.7 |
| | Calculation Skill | | |
| Addition Accuracy[2] | 16 | 13.9 | 2.1 |
| Addition Latency[1] | | 4058 | 1831 |
| WJ Calculation[3] | | 99.7 | 15.1 |

[3] Standardized score; [2] Number correct; [4] Grade-Scaled Score; [1] Milliseconds

*Table 2.* Intercorrelations among Grade 1 measures (N = 144)

| | 1. | 2. | 3. | 4. | 5. | 6. |
|---|---|---|---|---|---|---|
| 1. Subitizing | | | | | | |
| 2. Finger Gnosis | -.10 | | | | | |
| 3. Finger Tapping | .01 | .09 | | | | |
| 4. Vocabulary | -.06 | .15 | .05 | | | |
| 5. Processing Speed | -.09 | .17* | .18* | .07 | | |
| 6. Number System | -.27** | .31** | .17* | .44** | -.25** | |
| 7. Calculation Skill | -.23** | .35** | .25** | .27** | -.21* | .71** |

Significance levels: $*p < .05$, $**p < .01$

*Are the Component Abilities Related to Numeracy Skills?*

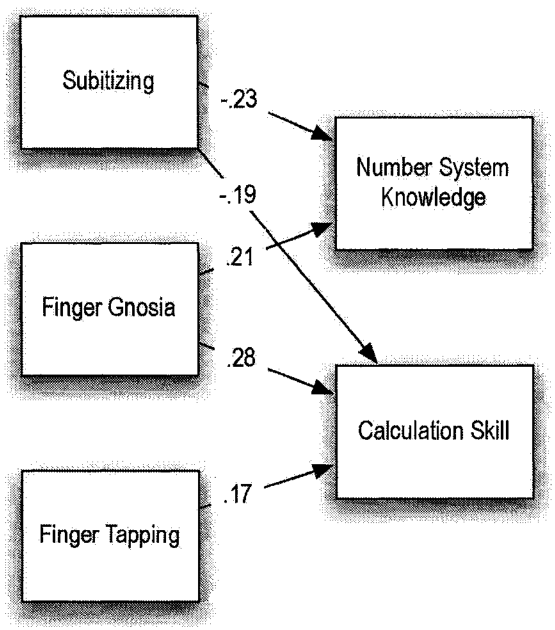
To determine whether the component abilities reflect separate abilities and whether the component abilities are related to numeracy skills, correlational analyses were performed. Partial correlations among the measures, with gender, vocabulary, and processing speed included as control variables, are shown in Table 2. Notably, there were no significant correlations among the three component abilities, subitizing, finger gnosis, and finger tapping, supporting the view that they represent separate abilities. As predicted, subitizing and finger gnosis skill were significantly correlated with number system knowledge. As also predicted, subitizing, finger gnosis and finger tapping were each correlated with calculation skill. Note that the relation between subitizing slope and the outcome measures is negative because higher slope values reflect poorer subitizing ability, whereas higher number system knowledge and calculation skill values reflect better performance. In summary, the component abilities were each related to numeracy skills.

*Do the Component Abilities Jointly and Independently Predict Numeracy Skills?*

To determine whether the component abilities predict numeracy skills, both jointly and independently, regressions were performed. First, number system knowledge was predicted from the component abilities, subitizing, finger gnosis, and finger tapping. In both this and the following regression, gender, vocabulary, and processing speed were included as control variables. Based on reviewer comments, chronological age was investigated as a control, but was not related to performance on the tasks of interest and is therefore not included in the current analyses. As shown in Figure 1, subitizing and finger gnosis predicted number system knowledge, accounting jointly for 12.6% of the 38.8% of variance accounted for by the entire model. Subitizing and finger gnosis accounted uniquely for 5.2% and 4.6% of variance, respectively.

Second, calculation skill was predicted from the component abilities: subitizing, finger gnosis, and finger tapping. As shown in Figure 1, the component abilities each predicted calculation skill, accounting jointly for 16.3% of the 28.6% of variance accounted for by the entire model. Subitizing, finger gnosis, and finger tapping accounted uniquely for 3.7%, 7.6%, and 3.0% of variance, respectively.

*Figure 1.* Regression models with semi-partial regression coefficients. Only significant paths shown.




Are the component abilities related to numeracy skills? In summary, we found that subitizing and finger gnosis were related to number system knowledge and that all three component abilities were related to calculation skill. On Butterworth's (1999) view, however, the abilities combine to produce strong numeracy skills. One way to test this hypothesis is to examine whether the interaction terms (i.e., subitizing x finger gnosis, subitizing x finger agility, subitizing x finger gnosis x finger agility) or composite terms account for significant unique variance in the numeracy outcome measures (i.e., number system knowledge and calculation skill). Interaction terms in a first set of regression analyses and composite terms in a second set were added in a final block to determine if any term accounted for additional significant unique variance in the outcome measures.

In contrast to expectations, no interaction or composite terms reached significance for number system knowledge or calculation skill. One possible explanation for why the composite/interaction terms did not predict numeracy outcomes is that there may be different developmental paths to numeracy or different combinations of skills that produce a favorable outcome. This possibility would not be consistent with Butterworth's (1999) view, given the necessary roles for subitizing, and for fingers as representational and procedural tools in the development of numeracy.

*Can we Find Sub-Groups of Children, Based on the Component Skills, with Different Numeracy Outcomes?*

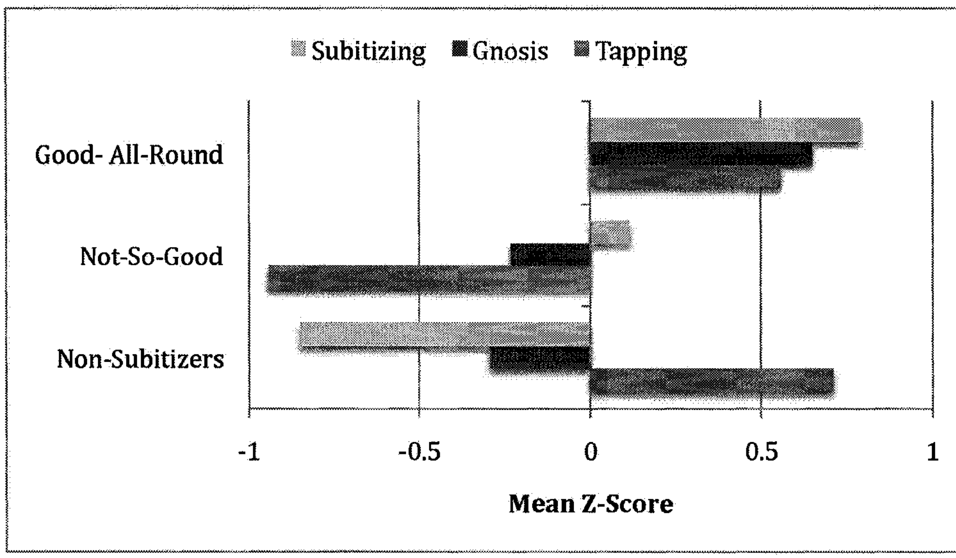
To investigate the possibility that different combinations of the component skills produce favorable numeracy outcomes, we first identified groups of children based on characteristic skill profiles across subitizing, finger gnosis, and finger tapping using cluster analysis. Using the two-step cluster procedure in SPSS 15 with the log likelihood clustering algorithm, a three-cluster solution emerged. Mean subitizing, finger gnosis, and finger tapping z-scores are shown by cluster in Figure 2. Groups did not differ on receptive vocabulary or on processing speed.

A first cluster ($n$ = 43) has strong subitizing ability, and relatively strong finger gnosis, and tapping ability, compared to the other groups, as shown in Figure 2. We labeled this cluster the *good-all-around* group. Children in this cluster are expected to have positive numeracy outcomes as they have strong subitizing ability (mean slope = -1ms) paired with the ability to use their fingers as representational and procedural tools.

A second cluster ($n$ = 57) has moderate subitizing and finger gnosis ability, paired with relatively weak tapping ability, as shown in Figure 2. We labeled this group the *not-so-good* group. Children in this cluster may be able to draw on their moderate subitizing (mean slope = 86 ms) and finger gnosis abilities as representational tools to develop numeracy skills. However, this group is expected to have poorer numeracy outcomes than the good-all-around group.

A third cluster ($n$ = 44) has weak subitizing ability, paired with relatively moderate finger gnosis ability, and strong tapping ability, as shown in Figure 2. We labeled this cluster the *non-subitizer* group; children in this cluster appear to be counting to enumerate 1-3 dots, as evidenced by their mean subitizing slope of 210 ms. Children in this cluster are expected to have poor numeracy outcomes as a result of their poor subitizing ability. This group's strong finger agility may allow children to use their fingers as procedural tools (e.g., to perform counting procedures), but their moderate-to-poor finger gnosis ability may reduce their ability to compensate.

*Figure 2.* Component z-scores by cluster. Note subitizing is reverse coded so that positive scores for all measures indicate better performance.



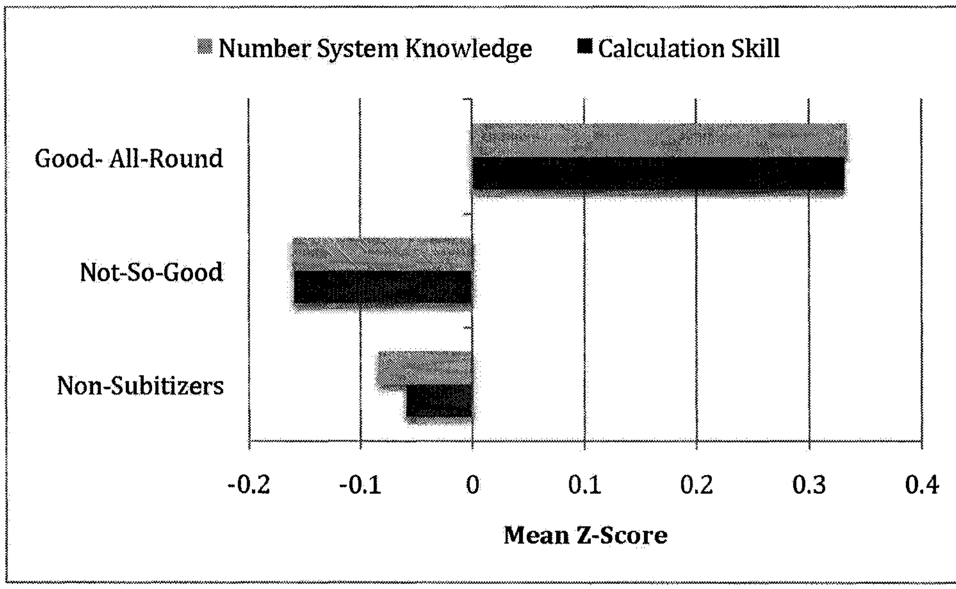

*Do the Sub-Groups have Different Numeracy Outcomes both Concurrently and Longitudinally?*

To investigate whether the sub-groups (good-all-around, not-so-good, and non-subitizers) have different numeracy outcomes, two orthogonal contrasts were used to examine group differences. The first contrasted the good-all-around group with the other two groups. It was hypothesized that the good-all-around group would perform better on both concurrent and longitudinal tests, as this group of students had strengths across all precursor abilities. The second contrasted the not-so-good and non-subitizer groups. There was no clear hypothesis for the second contrast. On Butterworth's view, the non-subitizer group would be expected to have the poorest numeracy outcomes, especially in number system knowledge, as this group lacks the core numerical ability. It is possible, however, that the non-subitizer group could draw on their strength in finger agility, bolstering counting and arithmetic skills. In comparison, the not-so-good group did not have any relative strengths to draw on. Thus, we had a strong prediction for the first contrast – the good-all-around group was predicted to have better numeracy outcomes than the other two groups. On Butterworth's view (1999), for the second contrast, the non-subitizer group was expected to have poorer numeracy outcomes than the not-so-good group.

Children's number system knowledge and calculation skill were examined concurrently, in Grade 1, and longitudinally, in Grade 2. Longitudinal measures were available for 102 of the 144 children (26 good-all-around, 47 not-so-good, and 29 non-subitizers). In Grade 2, all measures were the same with the exception of addition, where larger single-digit sums were used (as noted in the method section). Z-scores were computed for each factor (Grade 1 and Grade 2 number system knowledge and calculation skill) by taking the mean z-score of each factor measure (note: addition RT was reverse coded) and these mean z-scores were used as the dependent measure.

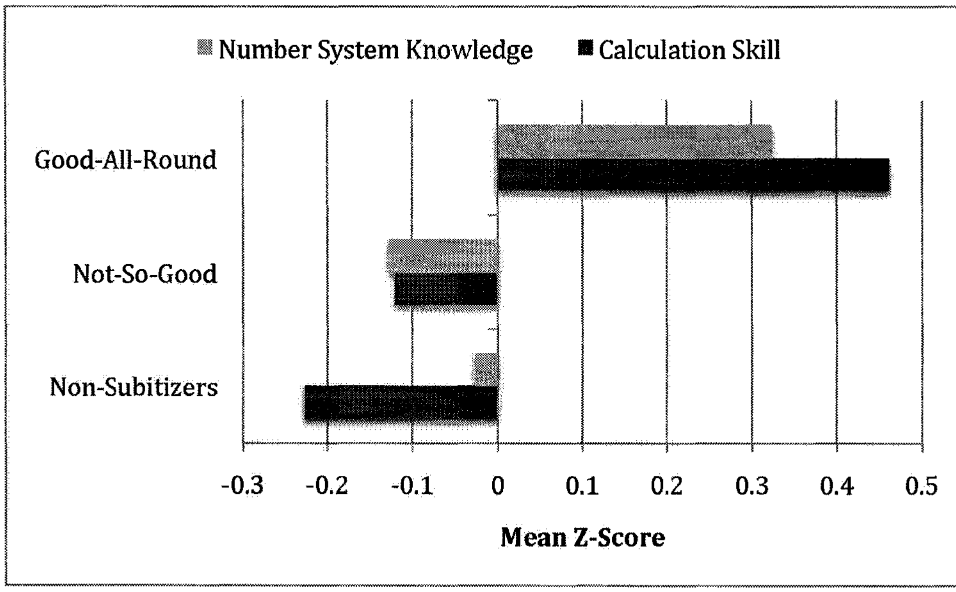
*Concurrent analyses.* Concurrent numeracy outcomes by group are shown in Figure 3. As predicted, the good-all-around group performed better on concurrent tests of number system knowledge (mean z-score of .33) and calculation skill (.33) than the not-so-good (-.16 and -.16, respectively) and non-subitizer groups (-.08 and -.06, respectively), $t$ (141) = 2.85 and $t$ (141) = 3.17, respectively. There was no significant difference between the not-so-good and non-subitizer groups on concurrent tests, contrary to Butterworth's (1999) view that non-subitizers would have poorer numeracy outcomes.

*Figure 3.* Concurrent math outcomes by cluster.




*Longitudinal analyses.* Longitudinal numeracy outcomes by group are shown in Figure 4. The same pattern of results was found longitudinally. As predicted, the good-all-around group performed better on tests of number system knowledge (mean z-score of .32) and calculation skill (.46) than the not-so-good (-.13 and -.12, respectively) and non-subitizer groups (-.03 and -.23, respectively), $t$ (99) = 2.132 and $t$ (99) = 3.727, respectively. Again, there was no significant difference between the not-so-good and non-subitizer groups on longitudinal tests, contrary to Butterworth's (1999) view that non-subitizers would have poorer numeracy outcomes.

*Figure 4.* Longitudinal math outcomes by cluster.




In summary, the sub-groups did have different numeracy outcomes. As hypothesized, the good-all-around group had significantly better numeracy outcomes both concurrently and longitudinally than the other two groups. The not-so-good and non-subitizer groups did not have different numeracy outcomes, in contrast to what was expected based on Butterworth's view.

## Discussion

In the current paper we investigated whether subitizing, finger gnosis, and finger agility are related to children's numeracy skills. First, in a series of correlations and regressions we found that subitizing and finger gnosis were related to children's number system knowledge and that all three component abilities were related to children's calculation skill. Second, using cluster analyses and planned comparisons we found that students with strong subitizing, finger gnosis and finger agility had better numeracy outcomes both concurrently in Grade 1 and longitudinally in Grade 2.

Butterworth (1999) proposes that subitizing is the core ability upon which number representations and numeracy skills are built. Consistent with this view, we found that subitizing was correlated with both number system knowledge and calculation skill. Thus, children able to enumerate 1 - 3 items without counting had better numeracy skills. Benoit, Lehalle, and Jouen (2004) also hypothesize subitizing is a necessary precursor to numeracy, aiding the mapping of number words to numerosities, Despite the evidence supporting subitizing as a developmental and evolutionary precursor to numeracy (for a review see Butterworth, 1999; Dehaene, 1997), our findings are the first that we are aware of to show a correlation between subitizing and numeracy skills in a typically-developing population.

Finger-based representations are posited to extend the representation of number beyond the subitizing range by providing a means of mapping non-symbolic representations of number onto symbolic representations (i.e., counting words and numerals; Butterworth, 1999; Fayol & Seron, 2005). Consistent with this view, finger gnosis was correlated with both number system knowledge and calculation skill. Thus, children able to use their fingers as representational tools had better numeracy skills. Butterworth (1999) proposes a functional relation between finger gnosis and numeracy; the two are related because children use their fingers to represent numerosities. Alternative neural views of the relation between finger gnosis and math have also been proposed (see Penner-Wilger & Anderson, 2008; Dehaene, et al. 2003). For example, in Penner-Wilger and Anderson's (2008) redeployment view, the two tasks share a common underlying neural circuit, which though originally evolved for finger representation has been redeployed for the representation of number. In contrast, Dehaene et al. (2003) propose that finger gnosis and math share a similar developmental trajectory due solely to implementation in neighboring regions in the parietal lobe. Thus, although the results of the current study add to the converging evidence of a relation between finger gnosis and math, there is controversy over the form of the relation.

Children with greater finger agility, as measured by finger tapping, would be in a better position to use their fingers to perform counting and arithmetic procedures. Consistent with this view, we found that finger tapping was correlated with calculation skill. Children with strong finger agility paired with poor subitizing ability (i.e., non-subitizers group), however, had poor calculation skill suggesting that finger agility alone is not sufficient to build strong calculation skill.

Children with strengths in all three component abilities have better numeracy outcomes. The results of the cluster analysis and planned comparisons are consistent with Butterworth's (1999) view that each component ability is a *necessary* precursor to the development of strong numeracy skills. The current research is significant in that it provides a first test of the precursors to numeracy proposed by Butterworth (1999). The results of this research have theoretical implications for the increased understanding of the form and development of number representations. The results of this research also have practical implications, including a greater understanding of the precursors that support the development of numeracy skills and of the different ability profiles that lead to positive numeracy outcomes. As a result, these findings will be of benefit in the assessment of numeracy in children and the development of math curricula that capitalizes on children's early numerical abilities.

## Conclusion

What component abilities underlie the development of numeracy skills? Overall, the results of the present study suggest independent contributions for the ability to enumerate small sets of items, the ability to mentally represent one's fingers, and finger agility. These predictors hold promise for the early identification of children with math difficulties. Mathematical skills form a complex mosaic, however, and the component abilities investigated constitute only a portion of the answer to why some children are better at math than others.

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APPENDIX C

Study 3

Penner-Wilger, M., Fast, L., LeFevre, J., Smith-Chant, B. L., Skwarchuk, S., Kamawar, D., & Bisanz, J. (2009). *Subitizing, finger gnosis, and the representation of number.* Manuscript accepted for publication.

Subitizing, Finger Gnosis, and the Representation of Number

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AUTHOR NOTE

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## Abstract

*What precursor abilities form the building blocks of numerical representations?* Two abilities were investigated: the ability to mentally represent small numerosities, indexed by subitizing speed (Butterworth, 1999; Dehaene, 1997), and the ability to mentally represent one's fingers, indexed by finger gnosis (Anderson & Penner-Wilger, 2007; Butterworth, 1999). We examined the longitudinal relation between these abilities in Grade 1 and tasks assessing numerical representation in Grade 2—symbolic number comparison and number-line estimation. Canadian children (*N*=100) participated as part of the Count Me In longitudinal study. Finger gnosis in Grade 1 was related to children's symbolic distance effect in number comparison and to both the linearity and slope of children's estimates in Grade 2. Subitizing in Grade 1 was related only to the overall comparison latency in Grade 2. Thus, children with better finger gnosis scores had lower symbolic distance effects and more accurate estimates, reflecting a more precise mapping between numerals and their associated magnitude.

Subitizing, Finger Gnosis, and the Representation of Number

Penner-Wilger et al. (2007, 2009) found that *subitizing*, the ability to quickly enumerate small sets without counting, and *finger gnosis*, the ability to mentally represent one's fingers, were related to children's number system knowledge and calculation skill in Grade 1. We hypothesized that this relation occurred because subitizing and finger gnosis facilitated the development of number representations (Butterworth, 1999). In the current paper, we test this hypothesis by examining the relation between the precursors, subitizing and finger gnosis, in Grade 1 and tests designed to assess the strength of numerical representations: magnitude comparison and number-line estimation in Grade 2.

Subitizing is a developmentally and evolutionarily primary numerical ability that is seen both in infants as well as other species (for a review see Dehaene, 1992). Benoit, Lehalle, and Jouen (2004) assert that subitizing is a necessary component for the mapping of number words to numerosities, as subitizing "allows the child to grasp the whole and the elements at the same time" (p. 21). In Butterworth's (1999, 2005) theory of numeracy development, subitizing is an index of our (exact) numerosity representations and forms the core numerical ability upon which all others are built. Dehaene's theory (1997) is similar in that subitizing plays a core role, but he asserts that subitizing forms the core of our (inexact) quantity representations. In summary, although there is disagreement on the form of non-symbolic representations of number (exact vs. inexact), there is agreement that subitizing indexes these representations.

How are non-symbolic representations of number related to more abstract symbolic representations of number? The prevailing view is that symbolic representations of number (number words, numerals, etc.) acquire meaning by being mapped onto non-symbolic representations (Brannon, 2005; Butterworth, 1999, Dehaene, 1997; Diester & Nieder, 2007; Verguts & Fias, 2004; for a review see Ansari, 2008). In contrast, Ansari (2008; Holloway & Ansari, 2008) suggests that symbolic and non-symbolic representations may be distinct. Regardless of the form of the relation between symbolic and non-symbolic representations of number, humans are able recognize common representational content in different vehicles (e.g., dots, number words, numerals, etc.). Thus, even if non-symbolic and symbolic representations are not built upon one another, both forms must be linked to the semantic representation of number.

Finger gnosis is hypothesized to support the mapping of symbolic and non-symbolic representations of number – increasing the range and precision of numerical representations. Fayol and Seron (2005) propose that the fingers are well suited as a tool to link non-symbolic and symbolic representations of number, as, unlike linguistic representations "finger representations exhibit an iconic relation to numerosities, since they preserve the one-to-one matching relation between the represented set and the fingers used to represent it" (p. 16). Butterworth (1999) likewise hypothesizes that children's use of fingers to represent numerosities in the course of numerical development helps to 'bridge the gap' from numerosity representations to more abstract number words. Thus, the ability to mentally represent one's fingers is thought to aid in the mapping of non-symbolic representations (indexed by subitizing) onto symbolic representations of number, building up a full number system.

Three views exist on the relation between finger gnosis and the representation of number. On the *localizationist* view, finger gnosis is related to numerical abilities because the two abilities are supported by neighboring brain regions, and these regions tend to have correlated developmental trajectories. On this view, there is no direct causal link between the representation of finger and number (Dehaene et al., 2003). In contrast, on the *functional* view, finger gnosis and numerical abilities are related because the fingers are used to represent quantities and perform counting and arithmetic procedures. As a result, the representation of numbers and of fingers becomes entwined (Butterworth, 1999). On the *redeployment* view, finger gnosis is related to math ability because part of the functional complex for number representation overlaps with the functional complex for finger representation. On this view, finger and number share a common neural resource that supports both representations (Penner-Wilger & Anderson, 2008). In summary, despite the distinct mechanisms proposed for the link between finger and number representation, each view hypothesizes a relation between finger gnosis and numerical representations.

Numerical comparison and numerical estimation have been proposed as indices of the strength of number representations: (Butterworth & Reigosa, 2007; Holloway and Ansari, 2008). Butterworth and Reigosa (2007), assert that mathematical difficulties stem in part from slower and less efficient processing of numerical information, specifically the estimation and comparison of numerosities. In the current paper we investigate the relations among subitizing, finger gnosis, number comparison and estimation to determine if the precursors are related to tasks designed to assess numerical representations.

*Number Comparison*

Number comparison involves recognition and judgment of the magnitude of numerosities, and is used as an index of the semantic representation of number (McCloskey, 1992). Multiple forms of number comparison tasks exist: using symbolic and non-symbolic stimuli, as well as tasks designed to induce Stroop-like effects. Two robust findings in magnitude comparison are the (1) the *distance effect* – large splits (e.g., 2 vs. 7) are judged faster than small splits (e.g., 2 vs. 3) and (2) the *congruity effect* – in a Stroop paradigm, trials with congruent stimuli are responded to more quickly than incongruent trials (See Noël et al., 2005 for a review of effects in magnitude comparison).

*Distance effect.* The distance effect is hypothesized to reflect the mapping between external and mental representations of number, with larger distance effects reflecting noisier mappings (Dehaene, Dehaene-Lambertz, & Cohen, 1998; Holloway & Ansari, 2008). Both adults and children show a distance effect, but the effect is attenuated in adults (Dehaene, Dehaene-Lambertz, & Cohen, 1998; Duncan & McFarland, 1980; Noël et al., 2005). Larger distance effects are thought to reflect less-distinct representations of numerical magnitude (Holloway & Ansari, 2008).

Holloway and Ansari (2008) examined the concurrent relation between the distance effect and math achievement in typically developing 6- to 8-year-old children. Both non-symbolic and symbolic number comparison tasks were used. They found that the symbolic distance effect (in response time), but not the non-symbolic, was related to both math fluency and calculation skill. Further analyses showed that this relation held for both math measures for the 6-yr-olds, for math fluency for the 7-yr-olds, and was not significant for the 8-yr-olds.

Rousselle and Noël (2007) examined symbolic and non-symbolic numerical comparison in typically achieving and math disabled second-grade children. They found that the math disabled group performed more slowly on the symbolic comparison task than the typically achieving group. No group differences were found in non-symbolic comparison.

*Congruity effect.* In a common numerical Stroop paradigm, stimuli vary along two dimensions: numerical size (e.g., 2 vs. 9), and physical size (i.e., small and large font). In one task children are asked to select the numerically largest number, in the other children are asked to select the physically largest number. For each task, a trial could be congruent (i.e., numerical size and physical size give consistent information), or incongruent (i.e., numerical size and physical size give conflicting information). The congruity effect is calculated as performance on incongruent trials – neutral trials. For the physical comparison task, the congruity effect reflects interference by the irrelevant magnitude information, with larger congruity effects indicating greater interference. For the numerical comparison task, the congruity effect reflects interference by the irrelevant physical information. The congruity effect in the physical comparison task is hypothesized to index automatic activation of the semantic representation (i.e., magnitude) of numerals (Landerl, Bevan, & Butterworth, 2004).

Girelli, Lucangeli, and Butterworth (2000) used a Stroop paradigm to examine, in typically developing students, the development of automaticity of accessing numerical magnitude from Grade 1 to adulthood. The authors did not find a robust congruity effect in Grade 1 or Grade 3 children, but did find an effect for Grade 5 children and for adults. These findings suggest that either the irrelevant magnitude information was not accessed in younger children or that the accessing of irrelevant magnitude information was not completed in time to interfere with the task.

Differences in the congruity effect have been examined between typically achieving and math disabled groups. Rousselle and Noël (2007) used a Stroop paradigm to assess group differences in automatic activation between math disabled and typically achieving Grade 2 students. Both groups showed a congruity effect, suggesting the irrelevant magnitude information was automatically activated. No group differences, however, were found in the congruity effect. Landerl et al. (2004) compared the performance of 8- and 9-year-old children with dyscalculia with that of age-matched controls. The authors found that dyscalculics were slower than controls on the numerical comparison but not on the physical comparison. A congruency effect was found only in the numerical comparison task, suggesting that the irrelevant physical dimension caused interference, but that numerical magnitude was not automatically accessed. The lack of congruency effect for the dyscalculic group is consistent with the findings of Koontz and Berch (1996), that 10-year-old children with math disabilities do not access the meaning of numerals fast enough to create interference by irrelevant numerical information. However, the typically-achieving students did show a congruity effect.

In summary, the congruity effect has been found in both typically achieving and math disabled students in Grade 2 (Rousselle & Noël, 2007) as well as in typically achieving students in Grade 5 (age 10 years) and higher (Girelli et al., 2000; Koontz & Berch, 1996). Though examined across multiple studies, group differences in the congruity effect were found only in one study (Koontz & Berch, 1996), suggesting that group differences in math ability do not result from differences in the automatic activation of the numerical meaning of numerals.

*Number-Line Estimation*

Estimation "is a process of translating between alternative quantitative representations, at least one of which is inexact." (Siegler & Booth, 2005, p. 198). Number-line estimation, more specifically, is hypothesized to provide direct information about representations of numerical magnitude (Siegler & Booth, 2005). Multiple dependent measures are used in number-line estimation tasks including the fit of the linear function between target and response (i.e., $R^2$) and the slope of the linear function. Siegler and colleagues assert that the linearity of children's estimates is an index of the quality of their numerical representations, with more linear estimates reflecting better representations (Siegler & Booth, 2004).

In earlier grades and for larger-scale number lines, the relation between the targets and children's estimates is best fit by a logarithmic function. The shift from log to linear representations happens between Kindergarten and Grade 2 for 0-100 number lines and between Grade 2 and Grade 6 for 0-1000 number lines (Siegler & Booth, 2004; Siegler & Opfer, 2003). The linearity of children's estimates has been found to correlate with concurrent math achievement for children in Kindergarten through grade four (Booth & Siegler, 2006; Siegler & Booth, 2004).

*Predictions: Subitizing, Finger Gnosis, and Numerical Representations*

*Subitizing.* On Butterworth's view (1999, 2005), subitizing will predict both number comparison and estimation performance, as subitzing forms the core numerical ability upon which all others are built. On Dehaene's view (1997), subitizing will also predict both number comparison and estimation performance, as quantity representations are required in both tasks. On Ansari's view (Ansari, 2008; Holloway & Ansari, 2008), subitzing will not predict performance on either task, as both tasks used in the current paper are symbolic and Ansari proposes that symbolic and non-symbolic representations of number are not strongly related.

*Finger gnosis.* On Butterworth's view (1999), finger gnosis will predict both number comparison and estimation performance, as finger gnosis facilitates the mapping of non-symbolic representations onto symbolic representations of number. On Dehaene's view (Dehaene et al., 2003), finger gnosis may also predict both number comparison and estimation performance, but only due to the shared developmental trajectory of the brain regions involved in the representation of finger and number. On Penner-Wilger and Anderson's view (2008), finger gnosis will predict both number comparison and estimation performance, as all three tasks make use of a common underlying neural resource, which originally evolved as part of the functional complex supporting the representation of fingers and has since been redeployed as part of the functional complex supporting the representation of number, serving both uses.

*The current research.* The primary goal of this paper is to examine the longitudinal relation between subitizing, finger gnosis, and tasks designed to assess numerical representations; a secondary goal is to examine the concurrent relation between numerical representation tasks and math outcome measures. To this end, we assessed children's subitzing and finger gnosis in Grade 1, and their performance on number comparison, number-line estimation, and standardized math outcome measures (including KeyMath Numeration subtest and Woodcock-Johnson Calculation subtest) in Grade 2. We hypothesized, that finger gnosis would predict comparison and estimation performance. Both Butterworth (1999) and Dehaene (1997) would hypothesize that subitizing would predict comparison and estimation performance. Consistent with these views, we previously found that subitizing predicted concurrent performance on number system knowledge and calculation skill in Grade 1 (Penner-Wilger et al., 2007, 2009). Based on the view of Holloway and Ansari (2008), however, subitizing would not predict performance on the symbolic comparison and estimation tasks used in the current experiment. The participants in the current study are the same as in Penner-Wilger et al. (2007; 2009), where we examined number system knowledge and calculation skill concurrently in Grade 1. Here we extend that work to determine if subitizing and finger gnosis are longitudinal predictors of tasks assessing number representations in Grade 2.

## Method

### *Participants*

Grade 1 children ($N = 148$) who participated in the *Count Me In* longitudinal study in 2006, were selected for the current paper. Children were recruited for the Count

Me In study from seven schools in three Canadian cities: Peterborough, ON, Ottawa, ON, and Winnipeg, MB. The testing sessions occurred in May or June each year. Of the 148 children, 112 participated the following year, in Grade 2. Data were missing for 12 of the 112 children on both of the tasks of interest: number comparison and number-line estimation. Thus, the present analyses are based on the 100 children (51 boys, 49 girls, age range in Grade 1 of 5:7 to 7:4, mean age of 6:10, years:months) who had complete Grade 1 data and who also had data for at least one Grade 2 outcome measure.

Socio-economic status (as measured by parent education levels) was relatively high across all of the schools in the sample. Ninety-eight percent of the participants' parents provided education level information, of these, 1% did not have a high school diploma, 18% held a high school diploma, 21% held a college or vocational diploma, and 59% of parents held undergraduate or graduate university degrees.

*Procedure*

Most children completed the computer measures in one half-hour session and the rest of the measures in a separate half-hour session on a different day. Some children completed all of the measures in a single session of approximately an hour.

All of the computer tasks were presented using software developed specifically for this project. For the computer tasks, children initiated the trials themselves by pressing the spacebar. Response times were measured from the point at which the stimuli appeared, until the experimenter pressed the stop-timer key (using a separate keyboard) when the child spoke their response. The experimenter then typed in the child's response.

*Materials*

*Subitizing.* Children were shown arrays of one to six dots and were asked to state 'how many' dots as quickly and accurately as possible. Children initiated the trial by pressing the space bar. The trial ended when the child stated the numerosity of the set and the experimenter pressed a key and then typed in the response. There were three trials of each array size, each in a different random pattern. Two practice sets (of 1 and 7 dots) were included as the first two trials. The median subitizing latency was computed from the latencies of the nine trials showing one, two, and three dots.

*Finger gnosis.* The Finger Gnosis measure is based on one designed by Noël (2005). Ten trials were conducted on each hand, beginning with the dominant hand. In each trial, two fingers were lightly touched below the first knuckle. The child's view of the touches was obstructed with a cloth cover raised from the child's wrist. After the cloth cover was lowered, the child pointed to the two fingers that had been touched. A point was awarded for each correct identification of a touched finger in a trial, with a maximum of 20 points per hand. The total score across both hands was used as the dependent measure, with a maximum score of 40.

*Processing speed.* To assess processing speed, we implemented a computer-based simple choice reaction time task (Petrill, Luo, Thompson, & Detterman, 2001). Two types of stimuli (an X or an O) were displayed for 1 second, preceded by a half second fixation point. Children were instructed to press the key corresponding to the target letter shown on the screen. The display then cleared and the next trial began automatically 1 second later. There were 24 trials. The child positioned the index finger of their dominant hand on the keyboard key with an 'X' sticker (the 'X' key) and their middle finger on the key above it labeled 'O' (the 'D' key). Left-handed children used similar stickers on the right side of the keyboard. The median response time for pressing the correct key in response to the stimuli was used as the dependent measure.

*Vocabulary.* Receptive language was measured using the Peabody Picture Vocabulary Test–Third Edition, Form B (Dunn & Dunn, 1997). It was included primarily as a measure of verbal, non-mathematical knowledge. Dunn and Dunn cite the split-half reliability coefficient for Form B for seven year olds as .95.

*KeyMath Numeration.* Children completed the Numeration subtest of a multi-domain math achievement test, the KeyMath Test-Revised (Connolly, 2000). This task is individually administered. It covers concepts such as quantity, order, and place value (on later items). Most of the items in the range for these children require knowledge of the symbolic number system. The reported alternate form reliability coefficient for the grade-scaled Numeration subtest is .75 (Connolly, 2000). Connolly provides a split-half reliability coefficient of .81 for spring Grade 2.

*Woodcock-Johnson Calculation.* Children completed the calculation subtest of the Woodcock Johnson Psycho-Educational Battery - Revised (Woodcock & Johnson, 1989). This calculation measure involves all four operations (addition, subtraction, multiplication, and division), although most of the questions that were attempted by the children in the present study involved addition or subtraction only. Children were stopped after six incorrect answers, or when they indicated to the experimenter that they did not know how to solve any of the remaining questions. This test has a median reliability of

.85 and a one-year test-retest correlation of .89 for Grades 2 through 4 (Woodcock & Johnson, 1989). The WJ-R manual cites the split-half reliability for six year olds as .928, *SEM(W)*=5.7 (*N*=309).

*Addition fluency.* Addition accuracy and latencies were measured on the computer. Sixteen trials of single-digit sums were displayed on the computer screen. In Grade 2, the sums were greater than ten. This task has a stop condition of five sequential errors and trials timed out if the child did not respond within 20 seconds. The child initiated each trial by pressing the 'GO' button. When the child spoke their answer, the experimenter pressed a key to stop the timer and typed in their response. Each child's median addition latency was computed from their correct trials.

*Number comparison.* The number comparison task was designed based on the numerical condition from Landerl, Bevan, and Butterworth (2004). The child was shown two numerals on the screen (range from 1 – 9) and was asked, "Which number is more than the other number?" Children indicated their response by pressing a yellow key on the side that was more (z on the left or . on the right). For each trial, there was a 500 ms delay prior to the stimulus presentation. Stimuli were displayed until the child responded or until a 3 s maximum was reached.

Stimuli varied on two dimensions, physical size (large vs. small font) and numerical size. For congruent trials, the number that was larger numerically was also larger physically. For incongruent trials, the number that was larger numerically was smaller physically. Numerical distance was defined with small splits as a distance of 1 (e.g., 2  3) and large splits as a distance of 5 (e.g., 2  7). Half of the trials were congruent and half were incongruent. There were 40 trials preceded by two practice trials. The stimuli pairs were taken from Landerl et al. (2004) with 24 trials of the six small-split combinations (1-2, 2-3, 3-4, 6-7, 7-8, 8-9) and 16 trials of the four large-split combinations (1-6, 2-7, 3-8, 4-9). Two pseudo-random orders were created.

Three dependent variables were computed: overall response time for correct trials, overall accuracy, and distance effect calculated as in Holloway and Ansari (2008) to capture the increase in RT from large to small splits controlling for individual differences in RT ([small split RT – large split RT]/large split RT). Note: The congruity effect of interest (incongruent – neutral in the *physical* condition) cannot be calculated in the current study, as the physical condition was not included due to testing time constraints. As a result, the interference of the irrelevant magnitude information (indexing the automatic activation of the numerical meaning of numerals) cannot be assessed.

*Number-line estimation.* Number-line estimation was measured using a computerized test of numerical estimation skill (Siegler & Opfer, 2003) adapted to have a game-like context. In this game, children were shown a number between 1 and 1000 and used the mouse to position a red car at the appropriate spot on a number line starting at 0 and ending at 1000. Children initiated each trial by pressing a Go button. Next, the target number was displayed at the top of the screen and the cursor transformed into the image of a car. Children moved to their selected position on the line and clicked the cursor. The computer recorded the location and the solution latency.

Following a training session with three trials at 0, 500 and 1000, the experimenter launched the main Estimation task. The instructions before the task began were based on the script used by Siegler and Booth (J. Booth, personal communication, February 21, 2006). We added the phrase "as quickly as you can" to encourage children to estimate. For each trial, the instruction area at the top of the screen displayed the message “Click where this number goes” followed by the target number displayed in the same large red font as the starting ‘0’ and ending ‘1000’ below the target line. The experimenter would read the instruction and number aloud, switching to just reading the number aloud once the child demonstrated proficiency at the task.

Order of the 25 trials was randomized separately for each child. The stimuli were chosen based on Laski and Siegler (2007) and were balanced with four targets between 0 and 100, four between 900 and 1000, two targets from each other decade and distances matched from the endpoints. The targets were: 6, 994, 18, 982, 59, 991, 97, 903, 124, 876, 165, 835, 211, 789, 239, 761, 344, 656, 383, 617, 420, 580, 458, 542, and 500.

Regression was used to calculate the relation between the actual values of the presented numbers and the locations that the child chose for those numbers on the number line. Perfect performance would occur if, for each presented number, it was placed at the appropriate location (e.g., 500 at the centre of the line, etc.). Various indices can be used based on these fits. In previous work, Siegler and colleagues have used the mean $R^2$ values from the linear regressions (regressing line location on actual number) as the index of fit. Larger $R^2$ values are indicative of a close correspondence between the number line locations and the presented numbers. An alternative index is the slope of the regression line. Perfect placement of each number on the line would result in a slope of

1.0. Random placement would result in slopes close to zero. Both linearity (i.e., $R^2$) and slope were used in the current paper.

## Results

Descriptive statistics for each measure are shown in Table 1. All results are significant at $p < .05$ unless otherwise noted. One-way ANOVA was used to test for gender effects in all dependent variables. Significant effects of gender were found for finger gnosis scores, and both estimation measures (slope and linearity). For finger gnosis, girls had significantly higher scores than boys ($M = 31.7$, $SD = 3.4$ vs. $M = 29.9$, $SD = 4.1$), $F(1, 98) = 5.96$, $MSE = 14.07$. For number-line estimation, boys had significantly higher scores than girls, for both slope ($M = .60$, $SD = .23$ vs. $M = .49$, $SD = .19$) and linearity ($M = .77$, $SD = .24$ vs. $M = .60$, $SD = .26$), $F(1, 97) = 7.27$, $MSE = .04$ and $F(1, 98) = 10.46$, $MSE = .06$, respectively. As a result of these differences, gender was controlled for all regressions.

### *Distance Effect in Number Comparison*

A significant distance effect was found in both accuracy and RT. Participants were more accurate comparing numbers with splits of five ($M = 90\%$, $SD = 13$) than numbers with splits of one ($M = 84\%$, $SD = 13$), $F(1, 85) = 53.10$, $MSE = 28.76$. Participants were faster comparing numbers with splits of five ($M = 1150$ ms, $SD = 262$) than numbers with splits of one ($M = 1278$ ms, $SD = 269$), $F(1, 85) = 49.35$, $MSE = 14335$. The distance effect in accuracy was not correlated with subitizing or finger gnosis, and is not discussed further in this paper.

*Table 1.* Descriptive information for all measures.

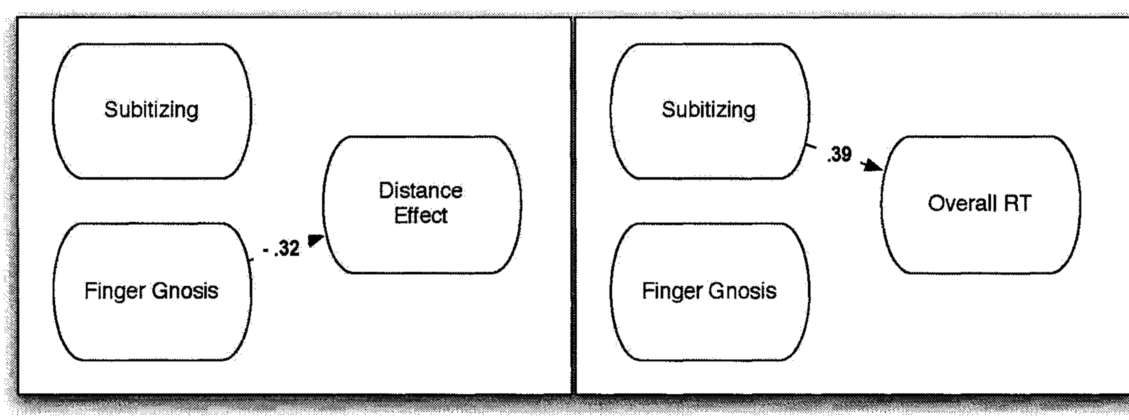
| Measure | M | SD |
|---|---|---|
| Subitizing [1] | 1261 | 199 |
| Finger Gnosis[2] | 30.7 | 3.8 |
| Processing Speed[1] | 653 | 109 |
| Vocabulary[3] | 110 | 11 |
| Distance Effect[1] | .12 | .14 |
| Overall Comparison RT[1] | 1243 | 240 |
| Comparison Accuracy[5] | 81.1 | 18.6 |
| Estimation Slope | .54 | .21 |
| Estimation Linearity | .69 | .26 |
| KeyMath Numeration[4] | 12.3 | 3.2 |
| WJ Calculation[3] | 100 | 14 |
| Addition Fluency[1] | 3773 | 1208 |

[1] Milliseconds; [2] Number correct; [3] Standardized score; [4] Grade-Scaled Score; [5] Percent correct

### *Do Subitizing and Finger Gnosis Jointly and Independently Predict Number Comparison?*

To determine whether subitizing and finger gnosis predict magnitude comparison performance, both jointly and independently, regressions were performed. First, the symbolic distance effect in Grade 2 was predicted from subitizing and finger gnosis in Grade 1. In both this and the following regressions, gender, processing speed, and receptive vocabulary were included as control variables. Based on reviewer comments, chronological age was investigated as a control, but was not related to performance on the tasks of interest and is therefore not included in the current analyses. As shown in Figure 1 (left panel), only finger gnosis significantly predicted the symbolic distance effect, accounting uniquely for 10% out of the 12% of variance accounted for by the entire model.

*Figure 1.* Regression models predicting magnitude comparison performance from the precursor abilities, showing semi-partial regression coefficients. Only significant paths shown.

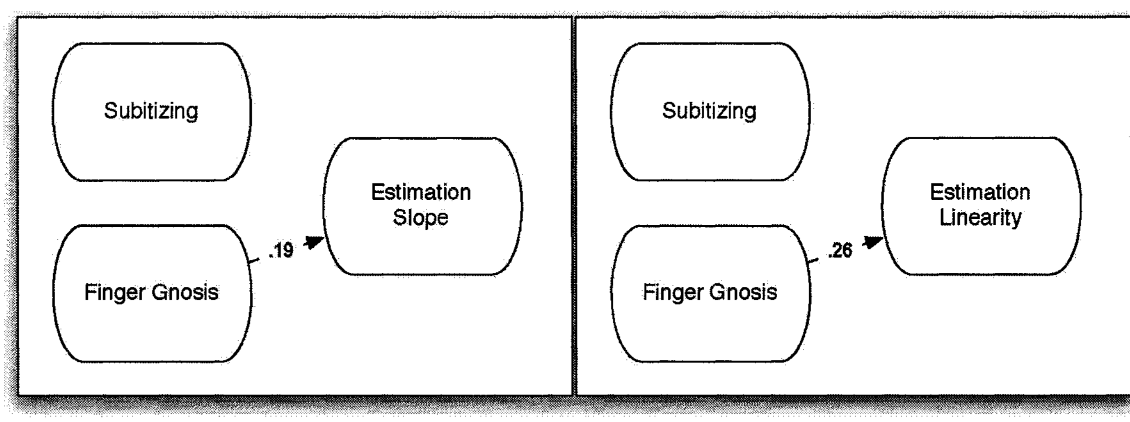



Second, overall RT in Grade 2 was predicted from subitizing and finger gnosis in Grade 1. As shown in Figure 1 (right panel), only subitizing significantly predicted the overall RT, accounting uniquely for 15% out of the 26% of variance accounted for by the entire model.

*Do Subitizing and Finger Gnosis Jointly and Independently Predict Number-Line Estimation?*

To determine whether subitizing and finger gnosis predict performance of number-line estimation, both jointly and independently, regressions were performed. First, the estimation slope in Grade 2 was predicted from subitizing and finger gnosis in Grade 1. As shown in Figure 2 (left panel), only finger gnosis significantly predicted estimation slope, accounting uniquely for 4% out of the 22% of variance accounted for by the entire model.

*Figure 2.* Regression models predicting number-line estimation performance with semi-partial regression coefficients. Only significant paths shown.




Second, as shown in Figure 2 (right panel), finger gnosis also predicted estimation linearity, accounting uniquely for 7% out of the 36% of variance accounted for by the entire model.

*Are Number Comparison and Estimation Related to Math Skills?*

To determine whether number comparison and estimation were related to concurrent math outcomes in Grade 2 partial correlations were performed controlling for gender, vocabulary, and processing speed. Correlations are shown in Table 2.

*Number comparison.* Contrary to Holloway and Ansari (2008), we did not find that the symbolic distance effect was correlated with performance on the KeyMath Numeration subtest, Woodcock-Johnson Calculation subtest, or addition fluency. This finding is surprising given that the same age group was investigated in both studies, the studies had a similar number of participants ($N = 87$ vs. 99 in the current paper), and one of the outcome measures, the Woodcock-Johnson calculation subtest, was the same. The same calculation was used for the distance effect ([small split RT – large split RT]/large split RT) and a similar number of trials were used in computing the small and large splits, (24 per split for Holloway and Ansari compared to 20 per split in the current paper). There were, however, task differences – we employed a Stroop paradigm whereas Holloway and Ansari did not. Also, Holloway and Ansari used the mean of splits 1 and 2 for small splits and the mean of splits 5 and 6 for large splits. In contrast, in the current paper, we defined small splits as splits of 1 and large splits as splits of 5. Thus, it is possible that the relation between the symbolic distance effect and math outcomes is not robust across task differences.

*Number-line estimation.* Consistent with Siegler and Booth (2004), we found that the linearity of children's estimates as well as the slope, were correlated with performance on the KeyMath Numeration subtest, Woodcock-Johnson Calculation subtest, and Addition RT.

*Table 2.* Intercorrelations among Grade 2 measures.

| | 1. | 2. | 3. | 4. | 5. | 6. |
|---|---|---|---|---|---|---|
| 1. Distance Effect | | | | | | |
| 2. Comparison RT | -.12 | | | | | |
| 3. Estimation slope | -.05 | -.15 | | | | |
| 4. Estimation linear. | -.08 | -.09 | .86** | | | |
| 5. KM Numeration | -.12 | -.04 | .39** | .39** | | |
| 6. WJ Calculation | -.18 | -.02 | .32** | .38** | .41** | |
| 7. Addition Fluency | .19 | .08 | -.25* | -.29* | -.31* | -.62** |

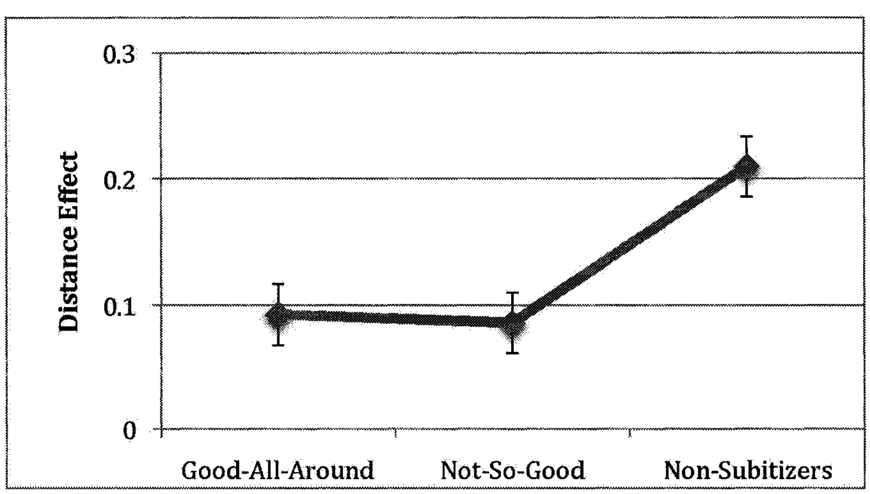
Significance levels: $*p < .05$, $**p < .01$

*Do the Sub-Groups Identified Previously Based on Subitizing, Finger Gnosis and Finger Agility have Different Outcomes Longitudinally on Numerical Representation Tasks?*

In Penner-Wilger et al. (2009), we identified groups of children (good-all-around, not-so-good, and non-subitizers) based on characteristic skill profiles in Grade 1 across subitizing, finger gnosis, and finger agility using cluster analysis. This same grouping of students is used in the current paper. In Penner-Wilger et al., we did not find any differences in the concurrent or longitudinal math outcomes of the not-so-good and non-subitizers subgroups, contrary to our hypothesis that the non-subitizers would have impaired number representations. Here we investigate whether the sub-groups from Penner-Wilger et al. have different outcomes on tasks designed to assess numerical representations one year later. Two orthogonal contrasts were used to examine group differences. The first contrasted the good-all-around group with the other two groups. The second contrasted the not-so-good and non-subitizer groups.

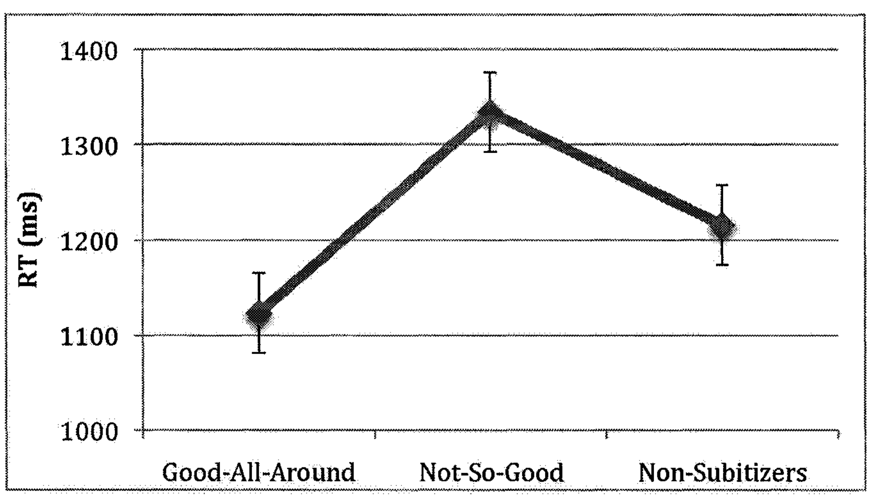
*Number comparison.* As shown in Figure 3, the non-subitizer group showed a greater distance effect ($M = .21$) than the not-so-good group ($M = .09$), $t$ (83) = 3.53. This is the only significant difference between the not-so-good and non-subitizer groups, and is consistent with our prediction that the non-subitizer group would have impaired numerical representations. The greater distance effect is evidence that the non-subitizer group had less distinct representations of number, consistent with the predictions of Carey (2001; LeCorre & Carey, 2007) that subitizing is crucial for the formation of (distinct) integer representations. The not-so-good group did not show a greater distance effect, revealing the first observed difference between the two lower-skilled groups.

*Figure 3.* Symbolic distance effect by cluster.



As shown in Figure 4, the good-all-around group was faster overall to make comparisons ($M = 772$ ms) than the not-so-good ($M = 966$ ms) and non-subitizer groups ($M = 844$ ms), $t(77) = 2.63$. The non-subitizers also showed a trend towards faster comparisons than the not-so-good group ($p = .05$). There were no differences between groups in accuracy. Thus, consistent with the findings of Landerl et al. (2004) that overall comparison RT distinguished between dyscalculic and typically achieving children, overall RT distinguished the three groups, with the not-so-good group showing the slowest RTs.

*Figure 4.* Overall comparison RT by cluster.




*Estimation.* The good-all-around group had estimates that were more linear (mean $R^2 = .78$) than the not-so-good and non-subitizers groups ($M = .65$ and $.67$ respectively), reflecting better estimation, $t(97) = 2.12$. The good-all-around group also showed a trend towards slope values closer to one, again reflecting better estimation, $t(96) = 1.96$, $p =$

.05. The second contrast, not-so-good versus non-subitizers, did not approach significance for either dependent measure. Thus, as with the number system knowledge and calculation skill measures used in Penner-Wilger et al. (2009) no differences were found in estimation skill between the not-so-good and non-subitizer groups.

In summary, the distance effect distinguished the non-subitizers from the not-so-good group. The different findings for these two groups may explain why we do not see a relation between the distance effect and math outcomes, as the two groups are indistinguishable on math outcomes both concurrently and longitudinally (Penner-Wilger et al., 2009). Moreover, they point to a distinction between the two lower-skilled groups. The non-subitizers are not slower overall in number comparisons but show a greater distance effect, whereas the not-so-good group are slower overall but do not show a larger distance effect. This distinction suggests that the non-subitizer group has a very specific numerical difficulty. To test this hypothesis, group differences were examined on additional measures that were given as part of the longitudinal study. One-way ANOVA revealed no significant group differences in processing speed, receptive vocabulary (PPVT), or on performance in the Elision subtest of the Comprehensive Test of Phonological Processing (CTOPP; Wagner et al., 1999), or Word Identification subtest of the Woodcock Reading Mastery (Woodcock, 1998). However, group differences were found on the Cognitive Intelligence Test (CIT; Gardner, 1990), a non-verbal reasoning test, $F$ (2, 98) = 4.23, $MSE$ = 217.26. Post-hoc tests revealed that the not-so-good group had poorer CIT scores (age-scaled score of 96.7) than the good-all-around group ($M$ = 107.0), suggesting that the not-so-good group has a more generalized non-verbal learning difficulty. Individuals with non-verbal learning disabilities often have math difficulties (Rourke, 1993). Hence, it appears that our non-subitizer group does show a specific numerical difficulty consistent with Butterworth's view, whereas the not-so-good group has more generalized non-verbal difficulty.

## Discussion

The primary goal of this paper was to examine the longitudinal relation between subitizing, finger gnosis, and tasks designed to assess numerical representations. We found that finger gnosis in Grade 1 was related to children's symbolic distance effect in magnitude comparison and to both the linearity and slope of children's estimates in Grade 2. Children with better finger gnosis scores had lower symbolic distance effects, reflecting a more precise mapping between numerals and their associated magnitude. The relation between the symbolic distance effect and finger gnosis is consistent with the view that finger gnosis facilitates the mapping between non-symbolic (magnitude) representations and symbolic representations. Children with better finger gnosis scores also had more precise estimates, again reflecting a more precise mapping between numerals and their associated magnitude.

Subitizing in Grade 1 was related only to the overall comparison latency in Grade 2. This finding is not consistent with the predictions of Butterworth (1999) or Dehaene (1997), whereby subitizing was hypothesized to relate to the symbolic distance effect and to estimation. Holloway and Ansari (2008) hypothesize that symbolic and non-symbolic representations are distinct, contrary to prevailing views that symbolic representations are built on non-symbolic representations. The pattern of results may, therefore, be quite different if non-symbolic versions of comparison and estimation tasks were used. Further work will explore the relation between the precursors and non-symbolic representations.

A secondary goal was to examine the concurrent relation between numerical representation tasks and math outcome measures. The linearity and slope of children's estimates were related to all investigated math outcomes including: KeyMath numeration subtest, Woodcock-Johnson Calculation subtest, and addition fluency. In contrast, the symbolic distance effect was not related to any of the investigated math outcomes.

In conclusion, finger gnosis was related to all indexes of the symbolic representation of number. This finding may reflect a developmental phenomenon whereby the mental representations of fingers and of number become linked functionally, through the practiced use of fingers to represent numerosities (Butterworth, 1999). Alternatively, the relation between finger and number representations may be one of identity, wherein the relation reflects a shared underlying representational form (Penner-Wilger & Anderson, 2008).

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APPENDIX D

Study 4

Penner-Wilger, M., & Anderson, M.L. (2008). An alternative view of the relation between finger gnosis and math ability: Redeployment of finger representations for the representation of number. In B.C. Love, K. McRae & V.M. Sloutsky (Eds.), *Proceedings of the 30th Annual Cognitive Science Society* (pp. 1647–1652). Austin, TX: Cognitive Science Society.

An Alternative View of the Relation between Finger Gnosis and Math Ability:

Redeployment of Finger Representations for the Representation of Number

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Abstract

This paper elaborates a novel hypothesis regarding the observed predictive relation between finger gnosis and mathematical ability. In brief, we suggest that these two cognitive phenomena have overlapping neural substrates, as the result of the re-use ("redeployment") of part of the finger gnosis circuit for the purpose of representing number. We offer some background on the relation and current explanations for it; an outline of our alternate hypothesis; some evidence supporting redeployment over current views; and a plan for further research.

An Alternative View of the Relation between Finger Gnosis and Math Ability:

Redeployment of Finger Representations for the Representation of Number

Finger gnosis, alternatively called finger recognition or finger localization, is the presence of an intact finger schema or "finger sense". A variety of neuropsychological tests have been designed to assess the presence of finger gnosis, or its absence—finger agnosia—in neuropsychological populations. In one common test (Baron, 2004), the examiner shields the participant's hand from view and lightly touches one or more fingers. The participant is asked to identify which fingers were touched. Finger gnosis tests have been used by neuropsychologists to provide an indication of parietal lobe damage, especially to the left angular gyrus (Gilandas et al., 1984). Finger agnosia is one of a constellation of symptoms in Gerstmann's syndrome, along with acalculia, agraphia, and left-right disorientation. Gerstmann (1940) identified finger agnosia or the loss of "finger sense" as the core deficit of the syndrome.

Perhaps surprisingly, recent research is demonstrating links between finger gnosis and mathematics ability in neuropsychologically-normal children. For instance, Fayol, Barrouillet and Marinthe (1998) discovered that a set of neuropsychological tests, including tests of finger gnosis, were the best longitudinal predictor of Grade 1 children's math scores. This finding was confirmed by Noël (2005), who demonstrated that children's finger gnosis scores predicted accuracy and fluency on a variety of mathematical tests, both concurrently in Grade 1 and longitudinally one year later. More recent results show that finger gnosis scores predict Grade 1 children's performance on *all* standard mathematical tests including number system knowledge and calculation

(Penner-Wilger et al., 2007), and also predict Grade 1–5 children's performance on tasks assessing numerical representations including number-line estimation and magnitude comparison, concurrently (Penner-Wilger et al., 2008). Thus, the relation between finger gnosis and math in normal children is robust across means of assessing mathematics ability.

## *Current Views*

Though a clear relation has been demonstrated between finger gnosis and math, what remains unclear is why such a relation exists. Here, we outline the two prevailing views, one functional and the other localizationist, and briefly review some of the evidence for these views.

### *Functionalist View*

On the functionalist view, finger gnosis is related to math ability because fingers are used almost universally in the course of math development to represent quantities and perform counting and arithmetic procedures (Butterworth, 1999). Thus, the representations of fingers and of numbers become linked developmentally. According to this view, the comorbidity of finger agnosia and acalculia as well as the relation between finger gnosis and math in normally-developing children arise functionally, because the representation of numbers is not only co-located with, but also linked to, the representation of fingers. Importantly, on the functionalist view there is a direct causal link between finger gnosis and math ability and, moreover, this link is formed experientially in the course of normal development.

*Localizationist View*

On the localizationist view, finger gnosis is related to math ability because the two abilities are supported by neighboring brain regions in the parietal lobe (Dehaene et al., 2003). The comorbidity of finger agnosia and acalculia, as seen in Gerstmann's syndrome, is explained as arising from common vascularization to the associated parietal areas, with damage typically affecting both areas. The relation between finger gnosis and math in normally developing children is a reflection of the correlated developmental trajectories of neighboring brain regions. Consistent with the localizationist view, Simon et al. (2002) found regions in the intraparietal sulcus activated for calculation-only, calculation and language, manual tasks (i.e., pointing), and visuospatial tasks. Importantly, on the localizationist view, there is no *direct* causal link between finger gnosis and math ability.

*Empirical Support*

Empirical support for each of these views has been mixed. The *functional* view predicts that facility in finger use (e.g., finger agility/fine-motor ability) should predict math ability as strongly as finger gnosis. Thus, children who can more easily use their fingers would form a stronger association between finger and number. Despite children's considerable variability in finger agility, this prediction is not borne out. In contrast to finger gnosis, finger agility is uncorrelated or only weakly correlated with all standard math tasks (Penner-Wilger et al., 2007). The *localizationalist* view predicts that, like finger gnosis, all co-located functions such as left-right orientation and graphia, should be equally well correlated with math ability. This prediction, however, is also not borne out.

In contrast with finger gnosis, these co-located functions are uncorrelated or only weakly correlated with math ability (Noël, 2005).

## *Redeployment of Finger Representations*

In Anderson and Penner-Wilger (2007) we briefly outlined an alternative view of the relation between fingers and math that used as its base the massive redeployment hypothesis of brain evolution (Anderson, 2007a). Here we further develop the redeployment view of the relation between fingers and number and outline some predictions and supporting evidence. Finally, we suggest further empirical work to test the redeployment view.

### *The Massive Redeployment Hypothesis*

The massive redeployment hypothesis (MRH) is both a theory about the functional topography of the cortex, and an account of how it got that way. According to MRH, neural circuits evolved for one use are frequently *exapted* for later uses, while retaining their original functional role. That is, the process of cognitive evolution is analogous to component re-use in software engineering. Components originally developed to serve a specific purpose are frequently re-used in later software packages. The new software may serve a purpose very different from the software for which the component was originally designed, but may nevertheless require some of the same low-level computational functions (e.g., sorting). Thus, efficient development dictates re-use of existing components where possible. Note that in such re-use, the component just does whatever it does (e.g., sorts lists) for all the software packages into which it has been integrated, even if that computational function serves a very different high-level purpose in each individual case.

The end result of such re-use in the brain is a functional architecture such that brain areas are typically recruited to support many different functions across cognitive domains. Preliminary investigations—generally involving mining a large collection of brain imaging experiments—have uncovered evidence for four specific predictions made by MRH. First, any given brain area is typically redeployed in support of many cognitive functions, and such redeployment will not respect traditional domain boundaries (that is, brain areas are not domain-restricted entities). Second, differences in domain functions will be accounted for primarily by differences in the way brain areas cooperate with one another, rather than by differences in which brain areas are used in each domain. Third, more recently evolved cognitive functions will utilize more, and more widely scattered brain areas. And fourth, evolutionarily older brain areas will be deployed in more cognitive functions. See (Anderson 2007a; 2007b; 2007c; in press) for details of the methods and results.

*Alternate Uses for the Finger Circuit*

In line with the general findings of MRH, we propose that one of the neural circuits integrated into the functional complex supporting finger gnosis is *also* part of the functional complex supporting the representation of number. That is, one of the functional circuits originally evolved for finger representation has since been redeployed to support the representation of number and now serves both functions.

Note that one should expect redeployment only in cases where different functional complexes could plausibly benefit from incorporating the same low-level functional circuits. What are the possible functional overlaps in these two very different functional complexes? Consider from a computational perspective that one foundational element in any calculating circuit is a register for storing the number(s) to be manipulated. Such a register is typically implemented as a series of switches that can be independently activated. Likewise, at least one way to implement the ability to know whether and which fingers have been touched (and other aspects of a general "finger sense") would be with such a register of independent switches. Such a finger register—one part of the functional complex supporting finger gnosis—would be a candidate for redeployment in any later-developing complex with functional elements able to take advantage of a component with this abstract functional structure. Our suggestion is that the number representation complex did just that.

In order to properly understand this suggestion, it is crucial to make a distinction between the representation itself and the representation *consumer* (Millikan, 1984). The content of a representation depends both on the intrinsic properties of the representation (e.g., which switches are on, and which are off), but also on the details of the mechanism that treats the representation as having significance. For a simple example, consider the following representation: 1001. Depending on the context, and on the assumptions of the interpreter, that representation can be taken to have the same content as the English phrase "one thousand and one" or as the Arabic numeral 9. It could conceivably also have alphabetic, numerological, or iconographic content, or be an instruction set for a Turing machine. The point is: we are suggesting *only* that this particular representational resource may be shared between the finger-gnosis complex and the number-representation complex. The meaning of whatever representation was stored in that resource would depend on the representation consumer, and presumably would vary greatly depending on the needs of the functional complexes incorporating the resource. Indeed, even within each complex the specific content could change with context. For instance, switches could be interpreted as an ordered binary set in one context, (1001 = 9) but as a simple, non-ordered additive set in another (1001 = 2). Importantly, then, it is no part of our suggestion that the capacity of our number representation component is limited to (or is exactly) $2^{10}$—the capacity of 10 ordered binary switches. Our capacity could be larger or smaller, depending on the details of the different representation consumers that have developed to work in concert with a register of the sort proposed.

On the redeployment view, finger gnosis is related to math ability because part of the functional complex for number representation overlaps with the functional complex for finger representation. Thus, finger and number share a common neural resource that supports both sorts of representation. The comorbidity of finger agnosia and acalculia as well as the relation between finger gnosis and math in normally-developing children arise from the shared neural circuit used for both representations.

The redeployment view of the relation between fingers and number is not a localizationist view. On the redeployment view, finger and number representations are not just neighboring neural functions on a correlated developmental trajectory; rather, they share a common neural substrate forming part of the neural complex supporting each function. Nor is the redeployment view a functional view. Importantly, on the redeployment view the connection between finger and number does *not* rest on the experienced use of the actual fingers to represent numerosities and perform arithmetic procedures, though it might suggest reasons we find it natural to use the fingers in this way (as well as natural ways to use the fingers; see *Evidence for Prediction 3*, below).

Before moving on to some supporting evidence for the view, we should pause here to admit that if one were modeling the finger gnosis complex in isolation it is unlikely that a register-like implementation for one of its components would leap out as the obvious choice. In fact, one of the important general implications of MRH is that one should *not* model functional complexes in isolation, but should consider what other complexes may also be using the same neural substrates. The effect of this change in methodology is often to suggest novel decompositions (and candidate implementations) of cognitive functions, of which the current hypothesis is one specific example. Of course, whether such speculations are fruitful (or *merely* novel) remains to be determined, both in this specific case, and as a general approach. But there is reason to be hopeful.

*Evidence and Predictions*

If the relation between finger gnosis and math arises because part of the neural circuit responsible for the representation of fingers has been redeployed in support of the representation of number then the following predictions should be borne out:

> (1) Brain regions associated with the representation of fingers should be activated during tasks requiring the representation of number.

(2) Damage/disruption of the neural substrate should affect both finger gnosis and tasks requiring the representation of number.

(3) There should be measurable interference between tasks involving finger gnosis and tasks involving number representation, insofar as these would be competing for the same neural resource.

(4) Individuals *without* finger agnosia who could not or did not use their fingers to represent quantities during development, should nevertheless show activation in the finger circuit during tasks requiring the representation of number.

*Evidence for Prediction 1*

Brain regions associated with the representation of fingers are activated during tasks requiring the representation of number. Prediction one would differentiate redeployment from localization, given adequate precision; however it does not differentiate the redeployment view from the functional view, except in very young children. On both the functional and redeployment views the representation of fingers and numbers are linked, with the key difference being the experiential requirement in the functional view.

Zago et al. (2001) found activation of a finger-representation circuit in the left parietal lobe during adults' performance of basic arithmetic. Increased activation was observed in the premotor strip at the coordinates for finger representation during performance of single-digit multiplication compared to a digit reading condition. Andres, Seron, and Oliver (2007), using transcranial magntic stimulation over the left M1 hand area to measure changes in corticospinal excitability, found that hand motor circuits were activated during adults' number processing in a dot counting task. Both sets of authors speculated that the activation might represent a developmental trace consistent with the functional view. The findings, however, are equally consistent with the redeployment view that part of the circuit responsible for the representation of fingers was redeployed in the representation of number.

Across a variety of number and finger tasks, functional imaging studies have shown overlapping activation in parietal regions (see Andres, Seron, & Oliver, 2007). Thus, the finding that brain regions associated with the representation of number and fingers are co-activated is robust, consistent with the functional and redeployment views. It remains possible, however, that future increases in the accuracy of functional imaging will eventually produce evidence favoring the localizationist view.

*Evidence for Prediction 2*

Damage/disruption affects both finger gnosis and tasks requiring the representation of number. Prediction two is again inconsistent with the localizationist view, yet it does not differentiate between redeployment and functionalism. Studies where disruption was induced using either repetitive transcranial magnetic stimulation (rTMS) or direct cortical stimulation provide converging evidence that disruption in the left angular gyrus affects both finger gnosis and tasks requiring the representation of number.

Rusconi, Walsh, and Butterworth (2005) used rTMS applied to parietal sites to determine if there was a common neural substrate between number and fingers. In a series of experiments, they found that rTMS over the left angular gyrus disrupted both magnitude comparison and finger gnosis in adults. Roux et al. (2003) using direct cortical stimulation also found a site in the left angular gyrus that produced both acalculia and finger agnosia. Thus, consistent with the redeployment and functional views, stimulation of the left angular gyrus across methods has been found to disrupt finger gnosis along with number comparison and calculation.

*Evidence for Prediction 3*

There should be measurable interference between tasks involving finger gnosis and tasks involving number representation, insofar as these would be competing for the same neural resource. The predictions here are somewhat subtle and surprising. If two tasks are using the same representational resource, they will interfere with one another *only* when the representation as it stands in the resource is *inaccurate* when read by the representation consumers in one or both domains. The obvious way to cause such a situation is to inject noise into the system. For instance, electrical stimulation of the fingers (but not of various locations on the forearm) might impact performance on mathematical tasks, if the stimulation caused the register to contain activation that represented a number *different* from the one stored there by the mathematical complex.

Finding such electrical-stimulation based interference would not be accounted for by the localizationist view, but is once again likely to be consistent with the functional view. Once the finger and number representations become linked, there would be every reason to suppose that tasks utilizing one would interfere with tasks utilizing the other. Note this does suggest the possibility of a *developmental* interference test. For the functionalist, there should be a time before the intertwining has occurred, and thus a time before one would expect interference. Unfortunately, the *failure* to find a pre-interference period would not be strong evidence against the functionalist, as there are always multiple possible explanations for the lack of an effect. Likewise, the *discovery* of a pre-interference period wouldn't necessarily be strong evidence against the redeployment view, since even genetically determined cognitive structures and relationships unfold over time. Thus, any evidence along these lines is unlikely to be definitive, however interesting and suggestive.

That's the story for electrical stimulations producing inaccurate representations. But if by chance a finger stimulus produced activation *consistent* with the standing number representation, there would be no interference. This is relevant because counting on the fingers, during which process one successively touches fingers, is a real-world instance of a finger stimulation task that by design produces representations in the shared register that can provide accurate information to consumers in both complexes. This realization suggests a further implication. Although the details of the procedure one can use to count on one's fingers—which fingers are touched in which order with what meaning—are theoretically arbitrary, the set of such procedures that can produce representations that would be accurate in both domains would be constrained by the representation consumers in both domains; not every procedure will produce representations compatible with the available consumers. Thus, on the redeployment view, there should exist a set of *self-interfering* finger-based counting procedures. The complementary implication is that there should be a set of procedures that are more natural and/or easier to acquire, insofar as they produce representations consistent with existing consumers.

Discovering self-interfering counting procedures would seem to count against both the localizationist view and the functionalist view. If the intertwining of representations is the result of experience, then there need be no *a priori* limit on the nature of the procedure that would cause the intertwining; and no consistent procedure that produced intertwining could be self-interfering. Likewise with the complementary implication: if *any* such procedure could be learned, then there is no specific theoretical reason that one should be easier than another.

Of course, in the event of any such discoveries, the functionalist would have available many plausible extra-theoretical elaborations (e.g., the measurable performance deficiencies of different procedures, or limits on which are easily learnable, could be the result of body mechanics, competing social conventions, and the like), and the advantage of redeployment on this issue might be limited to an explanatory parsimony. Moreover, the prediction is complicated by the difficult question of whether, and how easily, one can train arbitrary representation consumers in the numerical domain to work with finger-based counting procedures. So perhaps even evidence along these lines is unlikely to be definitive.

Still, it seems an intriguing line of research that could push both models in new directions. For instance, the discovery of a counting procedure that didn't just reduce performance, but resulted in systematic errors consistent with the mismatch between the procedure and the inferred properties of a representation consumer, could be strong evidence for the redeployment view.

In general, which outcomes in interference experiments would, and would not, be consistent with the functionalist view depend in part on the nature and timeline of the hypothesized intertwining of representations that occurs during development. The strongest interpretation of intertwining is that number representations and the finger sense become inextricably linked, even coming to share the same neural resources. If this is the hypothesis, then perhaps redeployment and functionalism cannot be distinguished based on interference studies, since they posit the same underlying neural relationship, and differ only with respect to the account of how that relationship came about. However, there are weaker versions of intertwining that are consistent with the original model. For instance, one might expect that number representations would come to depend on the finger sense, but not the reverse. Such a model might be somewhat more plausible developmentally speaking, as although it may be typical to use the fingers whenever one is doing mathematics, it would certainly be atypical to think of mathematics whenever one is using the fingers. At the very least, the various possibilities to be entertained in designing interference experiments suggest the need for a clarification of the functionalist model.

*Evidence for Prediction 4*

Individuals *without* finger agnosia who could not or did not use their fingers to represent quantities during development, will nevertheless show activation in the finger circuit during tasks requiring the representation of number. Prediction four is the key difference in distinguishing the redeployment view from the functional view. Special populations may play a crucial role in testing this prediction.

If the relation between fingers and number is a functional one, then the ability to functionally use ones' fingers to represent numerosities and perform counting and arithmetic procedures would be a crucial element in the development of numeracy. We have already found that finger gnosis in a typically-developing population of children is more highly correlated with numeracy and calculation skills than is finger agility. This finding is apparently at odds with the functional view, but is consistent with the redeployment view that the connection between fingers and number does not rest on the experiential use of fingers to represent number.

Children with Spina Bifida have both finger agnosia and poor finger agility co-morbid with significant mathematical difficulties (Bannister & Tew, 1991; Barnes, Smith-Chant, & Landry, 2005). This finding has been taken as evidence for a functional role of fingers in mathematical development, as children with Spina Bifida would have difficulty using their fingers to form a functional/developmental link with number. However, as this population also has disrupted finger gnosis, the finding is likewise consistent with the redeployment view.

In contrast, children with developmental coordination disorder (DCD) have poor finger agility, but most have preserved finger gnosis (Cermak & Larson, 2001; Hamilton, 2002). Thus, children with DCD are ideally suited as a population with which to test the redeployment view against the functional view. Approximately 6% of children meet the criteria for DCD outlined in the DSM-IV. On the redeployment view, we predict that children with DCD and preserved finger gnosis will show activation in the finger circuit (left angular gyrus) during tasks requiring the representation of number such as magnitude comparison. We are currently designing an imaging experiment to test this prediction in a population of children with DCD.

As a first indication of support for the redeployment view, DCD is not generally comorbid with mathematical difficulties (Cermak & Larson, 2001). Thus, despite motor problems limiting the ability to use the fingers to represent numerosities, the representation of number appears unaffected in children with DCD. This finding is consistent with the redeployment view, but on its face presents difficulties for the functional view. On the redeployment view, children with DCD might be expected to show some deficits with arithmetic, given a functional role of the fingers in the development of counting and arithmetic *procedures*, but such deficits would not be expected to impact numerical *representation*. On the functional view, the use of fingers is crucial for the development of *both* numerical representations and arithmetic procedures. Hence, on the functional view children with DCD would be expected to show widespread math disabilities as seen in children with Spina Bifida.

*Summary of Available Evidence*

In summary, two of the four predictions (1 & 2) that arise from the redeployment view are well supported by empirical evidence, and there is suggestive evidence for the fourth. The third remains to be investigated. The two supported predictions are inconsistent with the localizationist view that finger gnosis is related to math ability because the two abilities are supported by neighboring brain regions. However, support for the redeployment view could not be distinguished from that for the functional view on the basis of evidence for predictions one and two (and possibly not on the basis of any evidence for three). Thus, support for prediction four would be the crucial evidence to conclude that the relation between fingers and number is not functional. We provided evidence consistent with prediction four and outlined further empirical tests of the redeployment view.

## Conclusion

This paper elaborated a novel hypothesis regarding the observed predictive relation between finger gnosis and mathematical ability. In brief, we suggested that these two cognitive capacities have overlapping neural substrates, as the result of the re-use ("redeployment") of part of the finger gnosis circuit for the purpose of representing number. We offered some background on the relation and current explanations for it; an outline of our alternate hypothesis; some evidence supporting redeployment over current views; and a plan for further research.

It is important to reiterate that on the redeployment view, the neural circuitry shared between finger gnosis and number representation forms only one part of the functional complex necessary for number representation. In MRH, existing neural circuits are redeployed for new purposes and combined to support new capacities. Along with the neural circuit shared with finger gnosis, additional neural circuits (with additional abstract functional capacities) are expected to combine in support of the capacity for number representation.

One capacity expected to play a role in the representation of number is *subitizing*, an evolutionarily-primary ability to distinguish the numerosities of small sets quickly without counting, with supporting neural circuitry in the horizontal segment of the intraparietal sulcus (Dehaene & Cohen, 2007). It may be that the functional capacity redeployed from finger representation forms the digital representation of number (how many), whereas the functional capacity redeployed in subitizing forms the analog representation of number (how much).

Despite some overlap between our view and Dehaene and Cohen's Neuronal Recycling Hypothesis, it is worth noting a significant difference: they posit that neural circuits originally evolved for subitizing were re-used in the course of *cultural* development for a sufficiently similar function in arithmetic, possibly to the decrement of the original function (Dehaene & Cohen, 2007). In contrast, MRH posits that neural circuits originally evolved for one use were re-used in the course of *evolution* for the exact same function in another use, with *no decrement* of the original function.

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APPENDIX E

Study 5

Penner-Wilger, M., & Anderson, M.L. (2009). The relation between finger gnosis and mathematical ability: Can we attribute function to cortical structure with cross-domain modeling? Manuscript submitted for publication.

The Relation between Finger Gnosis and Mathematical Ability: Can we Attribute Function to Cortical Structure with Cross-Domain Modeling?

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Abstract

This paper elaborates a novel hypothesis regarding the observed relation between finger gnosis and mathematical ability. In brief, we suggest that these two cognitive phenomena have overlapping neural substrates, as the result of the re-use ("redeployment") of part of the finger gnosis circuit for the purpose of representing number. In the current paper, imaging results from multiple cognitive domains were used to investigate the shared working between finger and number representation. To this end we (1) identified the brain area of interest – a region within the left precentral gyrus, (2) identified, across several cognitive domains, other cognitive uses that the area of interest supports, and (3) looked across these cognitive uses to ascertain the shared working of the area of interest. The final result of this process is a proposed shared working – an array of pointers – that can be tested empirically and will allow for further elaboration of the redeployment view of the relation between finger and number representations. This work is significant not just for understanding the relationship between finger gnosis and math, but because cross-domain modeling is a departure from the typical practice in cognitive neuroscience. We hope that the current essay can serve as a provisional model for a new, more integrative approach to functional localization.

# The Relation between Finger Gnosis and Mathematical Ability: Can we Attribute Function to Cortical Structure with Cross-Domain Modeling?

## Section 1: The Relation between Finger Gnosis and Math

Finger gnosis, alternatively called finger recognition or finger localization, is the presence of an intact finger schema or "finger sense" (Gerstmann, 1940). A variety of neuropsychological tests have been designed to assess the presence of finger gnosis, or its absence—finger agnosia—in neuropsychological populations. In one common test (Baron, 2004), the examiner shields the participant's hand from view and lightly touches one or more fingers. The participant is asked to identify which fingers were touched. Finger gnosis tests have been used by neuropsychologists to provide an indication of parietal lobe damage, especially to the left angular gyrus (Gilandas et al., 1984). Finger agnosia is one of a constellation of symptoms in Gerstmann's syndrome, along with acalculia, agraphia, and left-right disorientation. Gerstmann (1940) identified finger agnosia or the loss of "finger sense" as the core deficit of the syndrome.

Perhaps surprisingly, recent research is demonstrating links between finger gnosis and mathematics ability in neuropsychologically-normal children. For instance, Fayol, Barrouillet and Marinthe (1998) discovered that a set of neuropsychological tests, including tests of finger gnosis, were the best longitudinal predictor of Grade 1 children's math scores. This finding was confirmed by Noël (2005), who demonstrated that children's finger gnosis scores predicted accuracy and fluency on a variety of mathematical tests, both concurrently in Grade 1 and longitudinally one year later.

Penner-Wilger et al. (2007, 2009a) found that finger gnosis was related to children's number system knowledge and calculation skill concurrently in Grade 1. Moreover, Penner-Wilger et al. (2009b) found that finger gnosis in Grade 1 predicted performance on tasks designed to assess number representations – number comparison and estimation – in Grade 2. Thus, there is converging evidence for a relation between finger gnosis and math ability in both selected and typically-developing populations and evidence to suggest that this relation is mediated by number representations.

## *Current Views*

Though a clear relation has been demonstrated between finger gnosis and math, what remains unclear is why such a relation exists. Here, we outline the two prevailing views, one functionalist and the other localizationist, and briefly review some of the evidence for these views.

### *Functionalist View*

On the functionalist view, finger gnosis is related to math ability because fingers are used almost universally in the course of math development to represent quantities and perform counting and arithmetic procedures (Butterworth, 1999). Thus, the representations of fingers and of numbers become linked developmentally. According to this view, the comorbidity of finger agnosia and acalculia as well as the relation between finger gnosis and math in normally-developing children arise functionally, because the representation of numbers is not only co-located with, but also linked to, the representation of fingers. The functionalist view predicts that facility in finger use (e.g., finger agility/fine-motor ability) should also predict math ability. Thus, children who can more easily use their fingers would form a stronger association between finger and number. Despite children's considerable variability in finger agility, this prediction is not borne out. In contrast to finger gnosis, finger agility is uncorrelated or only weakly correlated with all standard math tasks (Penner-Wilger et al., 2007, 2009a). Importantly, on the functionalist view there is a direct causal link between finger gnosis and math ability and, moreover, this link is formed experientially in the course of normal development.

*Localizationist View*

On the localizationist view, finger gnosis is related to math ability because the two abilities are supported by neighboring brain regions in the parietal lobe (Dehaene et al., 2003). The comorbidity of finger agnosia and acalculia, as seen in Gerstmann's syndrome, is explained as arising from common vascularization to the associated parietal areas, with damage typically affecting both areas. The relation between finger gnosis and math in typically-developing children is a reflection of the correlated developmental trajectories of neighboring brain regions. Consistent with the localizationist view, Simon et al. (2002) found regions in the intraparietal sulcus activated for calculation-only, calculation and language, manual tasks (i.e., pointing), and visuospatial tasks. The localizationalist view predicts that, like finger gnosis, all co-located functions such as left-right orientation and graphia, should be equally well correlated with math ability. This prediction, however, is not borne out. In contrast to finger gnosis, these co-located functions are uncorrelated or only weakly correlated with math ability (Noël, 2005).

Importantly, on the localizationist view, there is no direct causal link between finger gnosis and math ability.

*Redeployment View*

In Penner-Wilger and Anderson (2008), we briefly outlined an alternative view of the relation between fingers and math that used as its base the massive redeployment hypothesis of brain evolution (Anderson, 2007a, b, c). Here we further develop the *redeployment view* of the relation between fingers and number.

The massive redeployment hypothesis (MRH) is both a theory about the functional topography of the cortex, and an account of how it got that way. According to MRH, neural circuits evolved for one *use* are frequently exapted for later uses, while retaining their original operation, or *cognitive working*[2]. That is, the process of cognitive evolution is analogous to component re-use in software engineering. Components originally developed to serve a specific purpose are frequently re-used in later software packages. The new software may serve a purpose very different from the software for which the component was originally designed, but may nevertheless require some of the same low-level computational workings (e.g., sorting). Thus, efficient development dictates re-use of existing components where possible. Note that in such re-use, the component just does whatever it does (e.g., sorts lists) for all the software packages into which it has been integrated, even if that computational working serves a very different high-level purpose, or use, in each individual case.

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[2] For clarity, a new agreed upon terminology is adopted for this paper, given that the terms *function* and *role* were used inconsistently (and in fact used by different theorists in opposite ways) within the cognitive science literature (Bergeron, 2008). The term *cognitive working* is used in place of the term *function* to refer to a low-level operation performed by a small, anatomically distinct brain, and *cognitive use* will be used in place of the term *role* to refer to higher-level cognitive functions.

The end result of such re-use in the brain is a functional architecture such that brain areas are typically recruited to support many different uses across cognitive domains. Preliminary investigations—generally involving data-mining a large collection of brain imaging experiments—have uncovered evidence for four specific predictions made by MRH. First, any given brain area is typically redeployed in support of many cognitive uses, and such redeployment will not respect traditional domain boundaries (that is, brain areas are not domain-restricted entities). Second, differences in domain uses will be accounted for primarily by differences in the way brain areas cooperate with one another, rather than by differences in which brain areas are used in each domain. Third, more recently evolved cognitive uses will utilize more, and more widely scattered brain areas. And fourth, evolutionarily older brain areas will be deployed in more cognitive uses. See (Anderson 2007a, b, c, 2008) for details of the methods and results.

*Redeployment of Finger Representations to Support Number Representations*

In line with the general findings of MRH, we propose that one of the neural circuits integrated into the functional complex supporting finger gnosis is also part of the functional complex supporting the representation of number. That is, one of the functional circuits originally evolved for finger representation has since been redeployed to support the representation of number and now serves both uses. In summary, on the redeployment view, finger gnosis is related to math ability because part of the functional complex for number representation overlaps with the functional complex for finger representation. Thus, finger and number share a common neural resource--a shared working that supports both sorts of representation. The comorbidity of finger agnosia and acalculia as well as the relation between finger gnosis and math in normally-developing children arise from the shared neural circuit used for both representations.

The redeployment view of the relation between fingers and number is not a localizationist view. On the redeployment view, finger and number representations are not just neighboring neural functions on a correlated developmental trajectory; rather, they share a common neural substrate forming part of the neural complex supporting each function. Nor is the redeployment view a functionalist view. Importantly, on the redeployment view the connection between finger and number does not rest on the experienced use of the actual fingers to represent numerosities and perform arithmetic procedures, though it might suggest reasons we find it natural to use the fingers in this way.

In sum, the primary claim of the redeployment view is that the functional complexes supporting finger and number representation share some neural circuitry. The next section outlines some specific predictions arising from this claim, and evaluates existing empirical evidence for those predictions. But a neural overlap raises the obvious question: what is that shared circuit *doing* for the different functional complexes of which it is a part? According to MRH, redeployment is not random, but is a process constrained by both functional demands and metabolic constraints. In particular, one should expect redeployment only in cases where different functional complexes could plausibly benefit from incorporating the same low-level working. What are the possible functional overlaps in these two very different functional complexes? In Section 3 we broaden the scope of our investigation to include 65 other studies, across several cognitive domains, where post-subtraction analysis also shows activation of the neural circuit of interest. One of the methodological implications of MRH is that the attribution of function to structure should be guided by a consideration of the broadest possible range of studies indicating that the region supports the experimental task. We follow this method in investigating the shared working between finger and number representation, and discuss a specific proposal for that working.

## Section 2: Evidence and Predictions for the Redeployment View

If the relation between finger gnosis and math arises because part of the neural circuit responsible for the representation of fingers has been redeployed in support of the representation of number and now supports both uses then the following predictions should be borne out:

(1) Brain regions associated with the representation of fingers should be activated during tasks requiring the representation of number.

(2) Damage/disruption of the neural substrate should affect both finger gnosis and tasks requiring the representation of number.

(3) There should be measurable interference between tasks involving finger gnosis and tasks involving number representation, insofar as these would be competing for the same neural resource.

(4) Individuals without finger *agnosia* who could not or did not use their fingers to represent quantities during development, should nevertheless show activation in the finger circuit during tasks requiring the representation of number.

*Evidence for Prediction 1*

Brain regions associated with the representation of fingers are activated during tasks requiring the representation of number. Prediction 1 would differentiate redeployment from localization, given adequate precision; however it does not differentiate the redeployment view from the functionalist view, except in very young children. On both the functionalist and redeployment views, the representation of fingers and numbers are linked, with the key difference being the experiential requirement in the functionalist view.

As discussed further in Section 3, there is a strong empirical support for Prediction 1 (Dehaene et al., 1996; de Jong et al., 1996; Gobel et al., 2004; Jancke et al., 2000; Kuhtz-Bushbeck et al., 2003; Liu et al., 2006; Numminen et al., 2004; Pesenti et al., 2000; Pinel, Piazza, LeBihan, & Dehaene, 2004; Venkatraman, Ansari, & Chee, 2005). Zago et al. (2001) found activation of a finger-representation circuit in the left parietal lobe during adults' performance of basic arithmetic. Increased activation was observed in the premotor strip at the coordinates for finger representation during multiplication performance compared to a digit reading condition. Andres, Seron, and Oliver (2007), using transcranial magntic stimulation over the left M1 hand area to measure changes in corticospinal excitability, found that hand motor circuits were activated during adults' number processing in a dot counting task. Both sets of authors speculated that the activation might represent a developmental trace consistent with the functionalist view. The findings, however, are equally consistent with the redeployment view that part of the circuit responsible for the representation of fingers was redeployed in the representation of number.

In summary, across a variety of number and finger tasks, functional imaging studies have shown overlapping activation in parietal regions (see Andres, Seron, & Oliver, 2007). Thus, the finding that brain regions associated with the representation of number and fingers are co-activated is robust, consistent with the functionalist and redeployment views. It remains possible, however, that future increases in the accuracy of functional imaging will eventually produce evidence favoring the localizationist view.

*Evidence for Prediction 2*

Damage/disruption affects both finger gnosis and tasks requiring the representation of number. Prediction 2 is again inconsistent with the localizationist view, yet it does not differentiate between the redeployment and functionalist views. Studies where disruption was induced using either repetitive transcranial magnetic stimulation (rTMS) or direct cortical stimulation provide converging evidence that disruption in the left angular gyrus affects both finger gnosis and tasks requiring the representation of number.

Rusconi, Walsh, and Butterworth (2005) used rTMS applied to parietal sites to determine if there was a common neural substrate between number and fingers. In a series of experiments, they found that rTMS over the left angular gyrus disrupted both magnitude comparison and finger gnosis in adults. Roux et al. (2003) using direct cortical stimulation also found a site in the left angular gyrus that produced both acalculia and finger agnosia. Thus, consistent with the redeployment and functionalist views, stimulation of the left angular gyrus across methods has been found to disrupt finger gnosis along with number comparison and calculation.

### *Evidence for Prediction 3*

Prediction 3 is that there should be measurable interference between tasks involving finger gnosis and tasks involving number representation, as these would be competing for the same neural resource. If two tasks are using the same representational resource, they will interfere with one another only when the representation for one use is incongruent with the representation for the other concurrent use. Two methods would allow for the testing of this hypothesis: a dual task paradigm or injection of noise into the system. As an example of noise injection, electrical stimulation of the fingers (but not of various locations on the forearm) might impact performance on mathematical tasks. Finding interference effects would not be accounted for by the localizationist view, but is once again likely to be consistent with the functionalist view. Once the finger and number representations become linked, there would be every reason to suppose that tasks utilizing one would interfere with tasks utilizing the other. To our knowledge, neither of these interference methodologies has been tested with finger and number representations.

### *Evidence for Prediction 4*

Prediction 4 is that individuals without finger agnosia who could not or did not use their fingers to represent quantities during development, will nevertheless show activation in the finger circuit during tasks requiring the representation of number. Prediction 4 is the key in distinguishing the redeployment view from the functionalist view. Special populations may play a crucial role in testing this prediction.

If the relation between fingers and number is a functional one, then the ability to functionally use ones' fingers to represent numerosities and perform counting and arithmetic procedures would be a necessary element in the development of numeracy. We have already found that finger gnosis in a sample of normally-achieving children is more highly correlated with numeracy and calculation skills than is finger agility. This finding is apparently at odds with the functionalist view, but is consistent with the redeployment view that the connection between fingers and number does not rest on the experiential use of fingers to represent number.

Children with Spina Bifida have both finger agnosia and poor finger agility co-morbid with significant mathematical difficulties (Bannister & Tew, 1991; Barnes, Smith-Chant, & Landry, 2005). This finding has been taken as evidence for a functional role of fingers in mathematical development, as children with Spina Bifida would have difficulty using their fingers to form a functional/developmental link with number. However, as this population also has disrupted finger gnosis, the finding is likewise consistent with the redeployment view.

In contrast, children with developmental coordination disorder (DCD) have poor finger agility, but most have preserved finger gnosis (Cermak & Larson, 2001; Hamilton, 2002). Thus, children with DCD are ideally suited as a population with which to test the redeployment view against the functionalist view. Approximately 6% of children meet the criteria for DCD outlined in the DSM-IV. On the redeployment view, we predict that children with DCD and preserved finger gnosis will show activation in the finger circuit during tasks requiring the representation of number such as magnitude comparison. We are currently designing an imaging experiment to test this prediction in a population of children with DCD.

As a first indication of support for the redeployment view, DCD is not generally comorbid with mathematical difficulties (Cermak & Larson, 2001). Thus, despite motor problems limiting the ability to use the fingers to represent numerosities, the representation of number appears unaffected in children with DCD. This finding is consistent with the redeployment view, but presents difficulties for the functionalist view. On the redeployment view, children with DCD might be expected to show some deficits with arithmetic, given a functional role for the fingers in the development of counting and arithmetic procedures, but such deficits would not be expected to impact numerical representation. On the functionalist view, the use of fingers is necessary for the development of both numerical representations and arithmetic procedures. Hence, on the functionalist view, children with DCD would be expected to show widespread math disabilities as seen in children with Spina Bifida.

*Summary of Available Evidence*

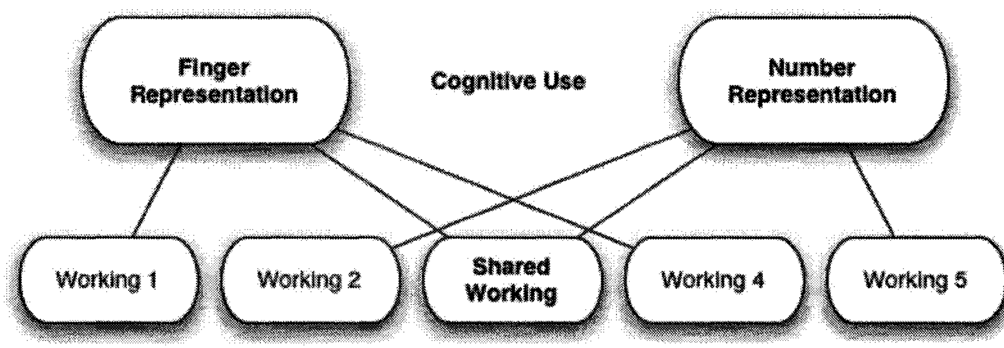
In summary, two of the four predictions that arise from the redeployment view are well supported by empirical evidence: (1) brain regions associated with the representation of fingers are activated during tasks requiring the representation of number and (2) damage/disruption of the neural substrate affects both finger gnosis and tasks requiring the representation of number. There is suggestive evidence for Prediction 4, that individuals without finger agnosia who could not or did not use their fingers to represent quantities during development, will nevertheless show activation in the finger circuit during tasks requiring the representation of number. Prediction 3, that there should be measureable interference between tasks involving finger gnosis and tasks involving number representation has yet to be systematically investigated. The two supported predictions are inconsistent with the localizationist view. However, support for the redeployment view could not be distinguished from that for the functionalist view on the basis of evidence for Predictions 1 – 3. Thus, support for Prediction 4 would be the crucial evidence to conclude that the relation between fingers and number is not functional. We provided evidence consistent with Prediction 4 and outlined further empirical tests of the redeployment view.

## Section 3: Investigation of the Shared Working between Finger and Number Representations

We now turn to investigating the crucial question that a shared neural circuit raises: what is that shared circuit *doing* for the different functional complexes of which it is a part? What is the working of this circuit that allows it to support tasks is such apparently different cognitive domains?

First we discuss a few terms that will help make the analysis more precise. *Workings* represent low-level operations that are performed by small, anatomically distinct brain areas (Bergeron, 2008). As such, workings are neither consciously available nor describable at the higher level of psychological vocabulary. Therefore, in contrast to the current practice in cognitive neuroscience, workings should be described using domain-independent vocabulary, which is currently, though not necessarily, drawn from information processing theory (Anderson, 2007b; what Bergeron calls operation specific terms). Multiple workings, in concert, contribute to higher-level cognitive *uses*. A typical brain area will contribute to many cognitive uses, across domains, but perform the same working across uses (Anderson, 2007a, b, c). Both Anderson (2007a,b,c) and Bergeron (2008) advocate for the determination of shared cognitive workings within and across domains as a method to advance our understanding of high-level cognition and to achieve the interdisciplinary goals of cognitive science.

*Figure 1.* Illustration of redeployment view of the relation between finger and number representations.




Two tenets of MRH (Anderson, 2007a, b, c) are of particular relevance to the goals of this section. First, each brain area is typically redeployed in support of multiple cognitive uses both within and across domain boundaries. Second, redeployed areas have the same working in each of the functional complexes they support. Anderson's view straddles the middle ground between localization and holism in that, although he claims that parts of the brain are specialized (i.e., they always perform the same working), this specialization is at the lower-order level of cognitive workings (e.g., computations or transformations) rather than that of higher order cognitive uses. Anderson (2007b, p. 339) uses the analogy of "finding the right letter to go into a box on a (multidimensional) crossword puzzle" to describe the task of determining a shared cognitive working. Thus, knowing the many cognitive uses that a brain area supports will help to determine what that brain area does (i.e., its cognitive working within the many anatomico-functional complexes).

For example, Hubbard et al. (2005) hypothesize a cognitive working - a computational transformation for spatial updating (i.e., the working) within the parietal sulcus; this cognitive working is also thought to play a role in another *cognitive use* – shifting attention along the mental number line. It is hypothesized that the *SNARC effect* – the finding that smaller numbers are responded to faster with the left hand and larger numbers are responded to faster with the right hand - arises as a result of this shared working. Multiple cognitive workings, often in disparate parts of the brain, work in concert within in a variety of functional complexes. Thus, in contrast to localization views, on MRH, most neural circuits are not domain specific.

The methodology of looking across domain boundaries to determine the working of a brain area is not common in cognitive neuroscience; activations are generally attributed to processes specific to the domain under investigation (Cabeza & Nyberg, 2000). Cabeza and Nyberg conclude, in a review of 275 imaging studies, "it would be useful to systematically compare functional neuroimaging data in different cognitive domains and to develop general theories that account for the involvement of brain regions in a variety of cognitive tasks" (Cabeza & Nyberg, 2000, p. 31). Tettamanti and Weniger (2006) do just that, by examining activation of Broca's area in a variety of cross-domain tasks with the purpose of determining the shared working. Though generally held as a language area, responsible for phonological processing and language production, Broca's area is also activated in non-linguistic domains such as object manipulation, action perception, and music (Tettamanti & Weniger, 2006). Tettamanti and Weniger examined the existing imaging data, across domains, in an effort to determine the cognitive working. The authors concluded that the cognitive working "of Broca's area may be to process hierarchical structures in a wide range of functional domains" (p. 491). Thus, the investigation of Broca's area illustrates how looking across domain boundaries for shared workings "can generate better, more accurate, and more fruitful descriptions of high-level cognitive domains and their relations" (Anderson, 2007b, p 343).

In Penner-Wilger and Anderson (2008), one possible shared working between finger and number representation was suggested—a register. A register is a representational form comprised of an ordered series of elements that can be independently activated. Different patterns of activation across the register reflect different representational content. The suggestion of a register, however, was based solely on the two domains of interest: finger and number. In the current paper, imaging results from multiple domains are used to more systematically investigate the shared working. The first step in this process was to identify the brain area of interest. The second step was to identify, across domains, other cognitive uses that the area of interest supports. The third step was to look across tasks and domains to ascertain the shared working of the area of interest. The final result of this process was a proposed shared working that can be tested empirically (e.g., via interference studies), will allow for the further elaboration of the redeployment view, and will generate a better, more accurate, and more fruitful description of the nature of our representation of number and its relation to other domains.

*Step 1: Identify the Brain Region of Interest*

The brain region of interest is taken from the PET study of Zago et al. (2001), who reported activation in the left precentral gyrus at the coordinates for finger representation during adults' performance of both single-digit and multi-digit multiplication. This same area within the left precentral gyrus was also activated during single-digit multiplication (Dehaene et al., 1996), single-digit addition (de Jong et al, 1996; Pesenti et al., 2000), number comparison (Dehaene et al., 1996; Pinel, Piazza, LeBihan, & Dehaene, 2004), and symbolic and non-symbolic exact and approximate addition with dots (Venkatraman, Ansari, & Chee, 2005). Zago et al. had four conditions: rest, read, retrieve, and compute. The *read* condition involved reading pairs of Arabic digits, which were composed of zeros and ones. The *retrieve* condition involved solving single-digit multiplication problems from 2 x 2 to 5 x 6. The *compute* condition involved solving two-digit by two-digit multiplication problems with a product less than 1000. Stimuli were presented visually in all conditions and participants responded verbally. Participants were six right-handed male French students (mean age 21 years).

The foci of significant activation in the precentral gyrus are shown in Table 1 for each subtraction of neural activation in the region of interest as measured indirectly by fMRI signal, reflecting blood oxygen level, for one condition minus another condition. The most relevant subtraction for the present purposes is the conjunction analysis of compute and retrieve conditions minus the read condition, as number representations are accessed for both single and multi-digit arithmetic. Thus, the conjunction analysis isolates the common neural activation for the two arithmetic tasks over and above activation associated with the control (i.e., read) condition. The area of interest, however, is significantly activated in all three comparisons. Thus the region of interest (ROI) for investigation of the shared working between finger and number representation is taken to be within the left precentral gyrus centered on coordinates (-42, 0, 38).

*Table 1.* Areas of activation in the precentral gyrus, taken from Zago et al. (2001). Shaded row is the region of interest.

| **X Y Z Coordinates (mm)** | | | **Volume (cm$^3$)** | **Subtraction** |
|---|---|---|---|---|
| -44 | -2 | 38 | 0.3 | **Retrieve - Read** |
| -40 | 0 | 36 | 3.8 | **Compute - Read** |
| -42 | 0 | 38 | 2.4 | **Compute & Retrieve - Read** |

*Step 2: Identify, Across Domains, Other Cognitive Uses that the Region of Interest Supports*

Identification, across domains, of other cognitive uses that the ROI supports was accomplished using (1) existing neuroscience results of shared brain areas in number and finger representation from Step 1, and (2) the Action-Grounded Cognition Lab's (www.agcognition.org) database of *post-subtraction* activations for 2164 studies from 692 journal articles, including all 156 qualifying articles reported in the *Journal of Cognitive Neuroscience* from 1996-2006 (Anderson, Brumbaugh, & Suben, 2008). All studies in the database were conducted with healthy adult participants. Areas of activation recorded in the database reflect greater neural activation in the noted region for a given condition compared to a baseline or other comparison as noted (i.e. post-subtraction activations as reported in the original papers).

The output of the database search included the following information for experiments reporting post-subtraction activation with a center inside the ROI within the left precentral gyrus: publication citation, domain (i.e., action, cognition, emotion, interoception, perception, etc.) and sub-domain (e.g., attention, language, memory, etc.) based on the BrainMap database classification system (Laird, Lancaster, & Fox, 2005), imaging method, Talairach coordinates and Brodmann area of each recorded activation, relative placement within the Brodmann area, and the subtraction used to generate the results (Anderson, Brumbaugh, & Suben, 2008). Finding that our region of interest was activated in tasks across domain boundaries would provide further support for MRH. Investigation of the output of the database search at the levels of domain, sub-domain, and the relevant tasks and subtractions used will be necessary to guide and constrain identification of a proposed shared working.

*Results*

The results of the database search provided 65 studies and 80 subtractions showing post-subtraction activation within the region of interest in the left precentral gyrus. Of the subtractions, 11 were in the domain of action, 60 in cognition, 2 in emotion, and 7 in perception. Within the domain of action, four were in the sub-domain of execution, five in inhibition, and one in each of imagination and preparation. Within the domain of cognition, 20 were in the sub-domain of attention (including four in visual attention), 16 in language (one in orthography, two in phonology, nine in semantics, and four in speech), 2 in mathematics, and 20 in memory (seven in explicit memory and ten in working memory) and one in each of time and theory of mind. Within the domain of perception, two were in audition, one in somesthesis, and four in vision. Thus, consistent with Anderson's (2007b) MRH, the region of interest was activated in varied cognitive uses across domains.

*Step 3: Look Across Cognitive Uses to Ascertain the Shared Working of the Region of Interest*

Given the variety of domains, sub-domains, tasks, and subtractions that showed activation in our ROI in the left precentral gyrus, the challenge was to glean the underlying shared working. The output of such an endeavor should be a low-level cognitive working, described in domain-neutral vocabulary, which the different cognitive uses could plausibly benefit from incorporating.

The region of interest was activated in expected tasks including number comparison (Gobel et al., 2004; Liu et al., 2006) and mental representation of fingers (Kuhtz-Bushbeck et al., 2003; Jancke et al., 2000; Numminen et al., 2004), confirming that this area is involved in both cognitive uses. In examining the variety of cognitive uses that share common activation in the ROI, three additional themes emerged: generation (e.g., generate items in a given category), inhibition (e.g., incongruent Stroop condition, anti-saccade, response inhibition), and order (e.g., n-back task, performing memorized sequences of saccades, judging alphabetical or sequential order). Further examination of tasks within each theme was undertaken as a means to both guide and constrain the proposed shared working.

In nine papers (10 subtractions), generation tasks showed activation in the ROI. Increased activation was found when participants covertly generated words within a given category, compared to rest (Frankenstein, Richter, Mcintyre, & Remy, 2001; Smith et al., 2001; Tremblay & Gracco, 2006; Vitali et al., 2005) or compared to listing numbers (Pihlajamaki et al., 2000). Increased activation was found for generation of related verbs when participants were shown nouns, relative to fixation or rest (Drobyshevsky, Baumann & Schneider, 2006; Hamzei et al., 2001) and also found when participants produced verbs, compared to nouns, in the context of short phrases or sentences (Shapiro, Moo, & Caramazza, 2006). Increased activation was found for the generation of neutral words relative to emotional words (Cato et al., 2004). Thus, word generation is one cognitive use associated with our shared working of interest. Word generation requires, among other things, representation of the category of items to be generated, mapping between category and items, and some means of keeping track of items already generated.

In thirteen papers (16 subtractions), inhibition tasks showed activation in the ROI. Increased activation was found in task switching conditions compared to task repetition (Cools, Clark, & Robbins, 2004; Dove et al., 2000; Luks et al., 2002). Increased activation was also found in task switching, where the meaning of the cue switched, compared to cue switching, where two cues indicated the same action (Brass & Cramon, 2004) or to direct mapping between cue and task (Dassonville et al., 2001). Increased activation was found for the incongruent compared to neutral condition in Stroop paradigms (Liu et al., 2006; Milham et al., 2001; Norris et al., 2002; Ruff et al., 2001). Increased activation was also found for antisaccades compared to controlled saccades or fixation – in the antisaccade task participants are required to direct their gaze to a mirror-symmetrical location in the opposite visual field of the target, requiring response inhibition (Chikazoe et al., 2007; Connolly et al., 2000) as well as for anti-pointing (Connolly et al., 2000). Thus, task switching/response inhibition is one cognitive use associated with our shared working of interest. Task switching requires, among other things, representation of the response sets (tasks) and some means of mapping cues to response sets.

In six papers (10 subtractions), order tasks showed activation in the ROI. Executing saccades to a sequence of memorized locations, compared to rest (Heide et al., 2001; Petit et al., 1996), correct hits in deciding whether the first and last letter of a word were in alphabetical order compared to misses (Henson et al., 2005), in an order-memory task following presentation of five letters compared to an item-memory task (Marshuetz et al., 2000), in spatial and non-spatial n-back tests, where participants are asked to recall items presented n-items previously compared to indicating which object changed luminance or recalling items presented 0-back (Owen et al., 1998; Ragland et al., 2002). Thus, storage and recall of order information is one cognitive use associated with our shared working of interest. Order tasks require, among other things, some means of representing information in an ordered form.

In summary, the cognitive uses for our ROI include: finger representation, number representation, category representation, task representation, and order representation. Given the brain area of interest here, and the cognitive uses the area supports, our goal is to look across uses to identify the shared working.

*Proposal for a Shared Working*

Given the shared uses from the database search, the specifications for a shared working are: that it allows for ordered storage of discrete representations and for mapping between representational forms. Does our initial proposal of a register (Penner-Wilger & Anderson, 2008) meet these updated requirements? A register does provide ordered storage (the reason for our initial proposal). A register, however, does not provide a means for mapping. One possibility is that our ROI performs storage and another shared region performs mapping, in which case a register would be a plausible shared working. Another possibility is that our selected ROI did not pick out a small, anatomically-distinct brain area and, therefore, is large enough to encompass two or more workings, one of which could again plausibly be a register. The third possibility, is that our ROI did successfully pick out a small, anatomically-distinct brain area that performs one working but that this working is more complex than our original proposal and performs both the storage and the mapping. This final possibility is addressed for the duration of the paper.

One computational unit that could implement both the ordered storage and mapping requirements is an array of pointers. An array is an ordered group, meeting the requirements for ordered storage. A pointer is a data structure that designates a memory location and can indicate different data types. The added functionality in this proposed working comes not from the array, as our proposed register could easily have instead been described as an array, but from what the array contains. An array of pointers allows for storage and access of ordered elements, which are able to point to – or index – representations or locations in memory, allowing for mapping between different representational forms. Thus, an array of pointers would allow for the ordered storage of different types of information and would facilitate the mapping between representations.

In finger representation, an array of pointers could hold distinct ordered representations for each finger. In number representation, an array of pointers could hold representations of discrete numbers, ordered by magnitude, across different representational forms: non-symbolic and number words, numerals, etc. This area would be expected to be activated for number comparison, which evidence suggests it is, but also for numerical estimation tasks – as estimation involves translating between alternative representations. In generation tasks, an array of pointers could store category items and map from categories to items. In task switching, the array could point to the different task demands and map cues to response sets. In order tasks, as in number and finger representation, the array could store ordered information (e.g., alphabet, sequence of movements, etc.). Thus, each cognitive use could benefit from a shared working in the form of an array of pointers.

In summary, the results of the cross-domain investigation of a shared working lead to further specification of the proposed shared working, from a register (Penner-Wilger & Anderson, 2008) to an array of pointers. The function of pointers is similar to one of the physical functions of the fingers. It would thus not be implausible to suppose that this basic function of the fingers was supported by a brain mechanism wherein representational content is determined by the object being indexed, rather than the state of the indicator *per se*. An array of pointers—one part of the functional complex supporting finger gnosis—would be a candidate for redeployment in any later-developing complex with functional elements able to take advantage of a component with this abstract functional structure. Our suggestion is that the number representation complex did just that. Before moving on to some supporting evidence for the view, we should pause here to admit that if one were modeling the finger gnosis complex in isolation it is unlikely that a array of pointers implementation for one of its components would leap out as the obvious choice. In fact, one of the important general implications of MRH is that one should not model functional complexes in isolation, but should consider what other complexes may also be using the same neural substrates. The effect of this change in methodology is often to suggest novel decompositions (and candidate implementations) of cognitive functions, of which the current hypothesis is one specific example.

How can the same working, in this case an array of pointers, be used to support diverse types of representations across uses? For our proposed working, the representational power is associated both with the properties of the array, including (1) which ordered elements in the array are bound (i.e. pointing to or indexing something) and which are not (i.e., are free), and (2) the representational flexibility inherent in the structure of the array, as well as with the properties of the pointers, including (3) what the pointers are indexing (i.e., bound to). Consider by analogy playing an F-major chord with the right hand. In this case there would be three pointers bound (indexing notes). For the second property, imagine "+" to indicate a bound pointer, and "-" an unbound one. The following configurations are equally possible (and not the only) candidates for the array configuration: +++--; +-++-. (The latter might be more convenient if the next cord to be played were F-minor.) Finally, the index content would be the notes played, F,A,C, respectively, in the standard (root) position, but A,C,F and C,F,A equally comprise an F-major chord in its various inversions. The full content of the representation depends on all three properties. Across different uses, tasks, and contexts, the same working can be used to represent vastly different content by altering any one of these properties of the array or pointers. This distinction is more broadly captured by the distinction between the representation itself and the representation consumer (Millikan, 1984). Depending on how the representation consumer is tuned, it might be sensitive to whether there are bound elements (yes or no); how many bound elements there are (3); the particular ordering of the bound elements (and there is great flexibility here, depending on how the order is exploited for content); the individual index content of the pointers (e.g., A, C, F); the overall unordered content (F-Major); the overall ordered content (e.g., F-major in 1st inversion).

Thus, the same working, an array of pointers, could support different forms of numerical representation (e.g., Arabic digits, number words, etc.). Doing so would require binding of the indexed content of the pointers to different locations. Other properties of the array, however, could remain constant. This rebinding would incur costs associated with switching the indexed location as well as reading the new value. This same working could also support diverse representations such as fingers, items in a category, response sets, or an ordered list of spatial locations. Switching between item classes would again require binding of the indexed content of the pointers to different locations, but would also likely involve differences in the other properties of the array such as the number of elements in the array and number of bound elements. Precise modeling of the various steps involved in different sorts of task/representation switching will allow for specific reaction-time predictions in each case (e.g. which kinds of switching will take more, and which less time; and also how the time needed will change with variations in the number of items being tracked). We are currently developing such a model (Penner-Wilger, Anderson and Stewart, in preparation) to be used in experiments designed to test the proposal made here.

## Conclusion

This paper elaborated a novel hypothesis regarding the observed predictive relation between finger gnosis and mathematical ability. In brief, we suggested that these two cognitive capacities have overlapping neural substrates, as the result of the re-use ("redeployment") of part of the finger gnosis circuit for the purpose of representing number. The results of cross-domain imaging studies were used to further specify the shared working between finger and number representations. We outlined predictions of the redeployment view, assessed existing empirical evidence, and outlined a plan for further research.

The findings from the cross-domain search lead us to propose to a shared working between the two tasks – an array of pointers. This shared working suggests a novel decomposition (and candidate implementation) of number representation. Currently, we are implementing a model of number representation that makes use of the proposed array of pointers structure. Moreover, this methodology of investigating overlapping functional complexes rather than modeling in isolation is expected to drive the field of cognitive science forward.

Some have taken the redeployment view to be neo-nativist, which it is not. Neo-nativism is characterized by a certain type of domain specificity; within specific domains there are innate specialized systems for building representations and processes (Keil, 1999). First, the massive redeployment hypothesis (MRH; Anderson, 2007a, b, c), which forms the basis of the redeployment view, argues against domain specificity in the functional organization of the brain. Further evidence against domain specificity was found in the current results of cross-domain modeling. Second, on the redeployment view, the representations of finger and number need not be innate representations. The proposal is simply that over evolutionary time redeployment has happened; specifically at least one working that supports the representation of fingers has been redeployed. It is the structure supporting finger representations, proposed herein to be an array of pointers, which is redeployed, rather than the representation itself. The redeployment view does not argue against a role for experience in the acquisition of finger and number representations nor does it rule out other types of redeployment such as Dehaene's neuronal recycling hypothesis (2004; Dehaene & Cohen, 2007). Moreover, MRH is neutral about the genetic mechanisms involved in redeployment. For example, both the environment and the body are typically inherited; nothing in MRH requires that any given instance of redeployment be determined entirely by genetic factors outside of environmental influences. In summary, MRH is not neo-nativist, nor does it disentangle nativist and empiricist views.

It is important to reiterate that on the redeployment view, the neural circuitry shared between finger gnosis and number representation forms only one part of the functional complex necessary for number representation. In MRH, existing neural circuits are redeployed for new uses and combined to support new capacities. Along with the neural circuit shared with finger gnosis, additional neural circuits (with additional abstract functional capacities) are expected to combine in support of the capacity for number representation.

One capacity expected to play a role in the representation of number is subitizing, an evolutionarily-primary ability to distinguish the numerosities of small sets quickly without counting, with supporting neural circuitry in the horizontal segment of the intraparietal sulcus (Dehaene & Cohen, 2007). It may be that the functional capacity redeployed from finger representation forms the digital representation of number (how many), whereas the functional capacity redeployed in subitizing forms the analog representation of number (how much).

Dehaene (Dehaene, 2004; Dehaene & Cohen, 2007) outlines a view that, like MRH (Anderson, 2007a, b, c), takes as its base the mechanism of exaptation – whereby features (or functions) that evolved by selection for one purpose were later adapted to a new purpose. The explanatory goal of the neuronal recycling theory is to determine how humans acquire cultural tools such as reading and arithmetic (Dehaene, 2004; Dehaene &

Cohen, 2007). There are five notable differences between MRH (Anderson, 2007a, b, c) and Dehaene's neuronal recycling hypothesis (Dehaene, 2004; Dehaene & Cohen, 2007).

First, the scope of these two views is different. The MRH is a broader theory of the functional organization of the human cortex. The neuronal recycling hypothesis is a theory of how humans specifically acquire cultural tools such as reading and arithmetic. Although there is some overlap, as both views have been used to explain the development of elements of mathematical skill, the explanatory scope of two views is quite different. Indeed, the first and only comparisons of these two views are contained within the papers that form this thesis, because the two views have not previously been used to explain phenomena within the same domain.

Second, the time course of the mechanisms in the two views is different. In MRH, neural circuits originally evolved for one use were re-used in the course of evolution. In contrast both to the mechanism of exaptation and to the MRH, in the neuronal recycling hypothesis, neural circuits are adapted to serve a new purpose in the course of an individual's development, rather than occurring in the course of evolution. Thus, the re-use in MRH happens over the course of evolution whereas in the neuronal recycling hypothesis re-use happens over the course of development. The two views need not necessarily be in conflict, because both evolutionary and developmental mechanisms could reasonably contribute to novel cognitive functions. Dehaene asserts that cultural tools such as reading and arithmetic have not be around long enough for the human brain to have evolved to support them. However, in the redeployment view, part of the neural circuitry that evolved to support the representation of fingers has been re-used to support our representation of number, which does have a long evolutionary history (Dehaene, 2004). Thus the two views are explaining cognitive functions (number representation versus arithmetic) with very different evolutionary histories, as appropriate for the timeline of the mechanisms outlined in each view.

Third, the level of description of these two views is different. MRH describes re-use at the lower level of cognitive workings (e.g., registers). In contrast, the neuronal recycling hypothesis describes re-use at the higher level of cognitive uses (e.g., object recognition). This distinction (lower versus higher level cognitive functions) may be the crux of the different predictions/implications for the original and novel functions between the two views.

Fourth, the relation between the original and novel functions is different between these two views. In MRH re-used neural circuits perform exactly the same working, but in a novel cognitive use. In the neuronal recycling hypothesis, neural circuits are re-used for a sufficiently similar, but not exactly the same, function. Again, this difference may result from the different levels of description, working versus use, across the two views.

Fifth, the impact on re-used functions is different between the two views. In MRH, re-use may result in interference -- that is, a decrement under, and only under, specific conditions -- between cognitive uses incorporating the same working (in the same brain region). However, MRH predicts no global decrement in the original function. In the neuronal recycling hypothesis, re-use can globally impact the original function: “the invasion of an evolutionary older circuit by a new cultural tool may have a measureable cost…the evolutionary older competence may be reduced or even lost" (Dehaene, 2004, p 150). This difference in implications for the original function across views is an empirical question. MRH clearly predicts no global decrement to the original function, so observed global decrements would support the neuronal recycling hypothesis.

In summary, although the MRH and the neuronal recycling hypothesis may appear to be conflicting views of the development of mathematical skills, they need not be. It is reasonable to expect that both evolutionary and developmental mechanisms play a role in the acquisition of mathematics. Differences between the implications and predictions of the two views stem in part from the level of description that re-use is considered at. When viewed at the lower level of cognitive workings, the original and novel functions Dehaene proposes are likely to remain constant, as in MRH. It is important to note that the scope of the MRH and the neuronal recycling hypothesis is quite different. Although both views have been used to explain elements of mathematical skill, the neuronal recycling hypothesis is a theory of the acquisition of cultural tools whereas MRH is a theory of the functional evolution of the cortex. Hence, the views may be seen by some as competing for explanatory power in the current domain, but they do not compete more broadly.

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