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Damage Tolerance Based Life Extension of Turbine Discs

A PFM Approach

*A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the Degree of
Master of Engineering*

Gwyneth Gould

November 1987

Department of Mechanical and Aeronautical Engineering,
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Abstract

The safe life philosophy that has been traditionally used to determine the life of aircraft engine turbine discs is very conservative. One study has estimated that up to 80% of the discs are being retired prematurely. In recent years the application of a damage tolerance approach and the use of probabilistic fracture mechanics (PFM) principles for aircraft component life expectancy has generated considerable interest. Whilst this method shows great promise its reliability is heavily dependent on the ability to establish Safe Inspection Intervals (SII). In order to predict SII's and at the same time incorporate nondestructive inspection (NDI) uncertainties, a PFM computer simulation program was written to take into account, in particular, the probability of detecting a crack, the consequences of missing a crack, and the inherent variation of the crack growth rate.

A case study carried out for General Electric J85 CAN40 fifth stage compressor discs flown in the Canadian Forces CT 114 Tutor jet trainer has shown that the predicted SII varies with the NDI technique used. If the ultrasonic C-scan, with a 90% Probability of Detection (POD) value of 1.6 mm, was used for NDI inspection then the PFM analysis predicted a SII of 880 hours compared to 524 hours for a liquid penetrant inspection (LPI) technique with a 90% POD value of 3.3 mm.

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A significant amount of experimental work was carried out at AMDU. Major R A Raoux was Project Co-ordinator and Dr A K Koul was Project Sponsor. Major Drummond and subsequently Captain Dora Masters co-ordinated meetings between ourselves at NRC in Ottawa and the Canadian Forces Base at Trenton. John McCanna did much of the LPI and ECI work while John Stewart worked on the microfocus X-ray technique. At NRC the results from the ultrasonic C-scan were obtained by Pierre Stoute, a fourth year student, and Nick Bellinger, a co-ordinator between Carleton University and DND contract work at NRC. The final experimental work (mounting over 500 bolt hole specimens, looking for fatigue cracks, prying open about 220 specimens, and measuring the fatigue crack length) was done by Pierre, Nick, and myself. Many thanks to Ray Dainty for lessons on the SEM.

Dave Morphy also generously gave up time helping me find my way about the laboratory, ordering supplies, and showing me how to use certain equipment. I, of course, answered the phone for him! During the analysis, the IBM 370 vm/cms system did an excellent job though I was grateful for the occasional human back up at the NRC computer center.

On a more personal level, I wish to thank Professor M C de Malherbe who provided the opportunity to work in this field and with Raj Thamburaj continued to encourage me. Many thanks also to Dr Ashok Koul, who in particular thrashed out details for this probabilistic fracture mechanics analysis for J85 CAN 40 compressor discs. The final acknowledgement must be made to family: to parents who allowed their children to pursue interests of their choice, and to a husband who set up the inter-computer communications system.

Nomenclature

a_d	dysfunction crack length
a_i	initial crack length
actual	The actual crack length is the crack length measured by the SEM.
AFML	Air Force Materials Laboratory
AFWAL	Air Force Wright Aeronautical Laboratories
AGARD	Advisory Group for Aerospace Research and Development
AIAA	American Institute of Aeronautics and Astronautics
AMDU	Aerospace Maintenance Development Unit
ASIP	Aircraft Structural Integrity Program
ASME	American Society of Mechanical Engineers
ASTM	American Society for Testing and Materials
BIP	Born Inversion Procedure
CA	Combined Analysis
CF	Canadian Forces
CSCAN	See Table 4.2
detected	The detected crack length is the crack length indicated by an NDI technique.
DFM	Deterministic Fracture Mechanics
dysfunction	The condition where crack growth rate may become rapid causing eventual failure. (Also abbreviated to dys)
EC2	See Table 4.2
EC5	See Table 4.2
ECI	Eddy Current Inspection (Also EC)
ECP	Electric Current Perturbation
EDM	Electron Discharge Machined (flaws)
EIFS	Equivalent Initial Flaw Size
ENSIP	Engine Structural Integrity Program
FAA	Federal Aviation Authority or Failure Analysis Associates.
false call	If an NDI technique detects a crack where there is no crack, then this is called a false call.
FEM	Finite Element Method
FPI	Fluorescence Penetrant Inspection
GE	General Electric

IFQ	Initial Fatigue Quality
initial	Generally taken to be the condition where a crack is barely detectable. (Also abbreviated to init)
L45	See Table 4.2
L60	See Table 4.2
$L(\beta_0, \beta_1)$	Loglogistics Distribution with two parameters
$L(\lambda, \zeta)$	Lognormal Distribution with mean and standard deviation
LCF	Low Cycle Fatigue
LPI	Liquid Penetrant Inspection
MPI	Magnetic Penetrant Inspection
N_d	Number of cycles to dysfunction
N_i	Number of cycles to initiation
$N(\mu, \sigma)$	Normal Distribution with mean and standard deviation
NAE	National Aeronautical Establishment
NASA	National Aeronautics and Space Administration
NATO	North Atlantic Treaty Organization
NDE	Nondestructive Evaluation
NDI	Nondestructive Inspection
NDT	Nondestructive Testing
NRC	National Research Council
OPM	Optimized Probability Method
PFM	Probabilistic Fracture Mechanics
POD	Probability of Detection
PND	Probability of Nondetection
RFC	Retirement For Cause
RI	Radially Inward
RI	Range Interval Method (Also RIM)
RND	Random Number
RO	Radially Outward
SAE	Society of Automotive Engineers
SEM	Scanning Electron Microscope
SII	Safe Inspection Interval
SME	Society of Manufacturing Engineers
TSN	Time Since New
TSO	Time Since Overhaul
TTCI	Time To Crack Initiation
USAF	United States Air Force
$W(\eta, \beta)$	Weibull Distribution with characteristic value and slope

Contents

Abstract	iii
Acknowledgements	iv
Nomenclature	v
Contents	vii
List of Tables	xi
List of Figures	xiii
1 INTRODUCTION	1
1.1 Background for the Lifting of Aircraft Engine Components and The Damage Tolerance Philosophy	1
1.2 Problem Identification and Objectives	2
2 LITERATURE OVERVIEW OF PFM BASED LIFING	8
2.1 Traditional Safe Life Approach	8
2.2 Damage Tolerance Lifting	9
2.2.1 DFM Approach	9
2.2.2 PFM Approach	9
2.2.3 CA Approach	12
2.3 Initial Flaw Sizes	13
2.3.1 Single Values and Distributions of Initial Flaw Sizes	14
2.3.2 Initial Flaw Sizes Represented by TFCI	15
2.3.3 Initial Flaw Sizes Represented by EIFS	17
2.3.4 Initial Flaw Distributions Based on an NDI Database	18
2.4 Nondestructive Inspection in Relation to Damage Tolerance	20
2.4.1 Background	20
Design Approaches for NDI Methodologies	21

	Comprehensive Experimentally Based NDI Programs	23
2.4.2	NDI Methods Available for Detecting Fatigue Cracks	25
	Liquid Penetrant Inspection	26
	Ultrasonic Inspection	26
	Eddy Current Inspection (ECI)	28
	Magnetic Particle Inspection	31
	Radiographic Inspection	32
	Electric Current Perturbation (ECP)	32
	Other Advanced NDI Methods	33
2.5	Reliability of NDI Techniques	34
2.5.1	Estimation of POD Data	35
2.5.2	POD Data Analysis	36
	Binomial Curve and Confidence Bounds	36
	Loglogistic Curve and Confidence Bounds	37
2.5.3	Pertinent POD Results from the Literature and the Influence of The Human Factor	38
2.6	Crack Growth Rate Variability	40
2.7	Conclusions from the Literature Search	42
3	EXPERIMENTAL MATERIALS AND METHODS	53
3.1	J85 CAN40 Compressor Discs	53
3.2	NDI Techniques Used	53
3.2.1	Production LPI at Orenda	53
3.2.2	Laboratory LPI at AMDU (Trenton)	54
3.2.3	Eddy Current Inspection (ECI)	55
3.2.4	Microfocus X-Ray Radiography	57
3.2.5	The Ultrasonic C-scan	57
3.3	Destructive Optical and Radiographic Examination of Cracked Bolt Holes	59
3.3.1	Prying Open the Fatigue Cracks	59
3.3.2	Optical Assessment of the Bolt Holes	59
3.3.3	Examination of Fracture Surfaces	60
4	PFM SIMULATION OF SII FOR J85 CAN40/15 COMPRESSOR DISCS	71
4.1	An Overview of the DISK2 Simulation Program	72
4.1.1	Random Number Generation	72
4.1.2	Initial Conditions	73
4.1.3	Actual vs. NDI Measured Crack Length	75

4.1.4	Inspection Uncertainty	75
4.1.5	Distribution of Crack Lengths Missed by NDI	75
4.1.6	Controlled Random Crack Growth	76
4.1.7	Calculation of Safe Inspection Interval	77
4.2	Determination of the 0.1% Safe Inspection Interval	77
4.2.1	Lognormal SII Distribution	78
4.2.2	Weibull SII Distribution	78
4.3	Stability and Convergence	79
5	RESULTS AND ANALYSIS	87
5.1	NDI Data and Its Analysis	87
5.1.1	POD Analysis of the Bolt Hole Cracks	88
	The Loglogistics POD Curves	89
	Confidence Bounds on the Loglogistic POD Curves	89
5.1.2	Analysis of the Cracks Missed by NDI Techniques	91
	Lognormal Distribution of Cracks Missed	91
5.1.3	Correlation Between Actual and NDI Indicated Crack Lengths	92
5.1.4	Comments About The NDI Techniques	94
5.2	Sensitivity Analyses for the Predicted 0.1% SII	95
5.2.1	Effect of Initial Conditions on the 0.1% SII	95
5.2.2	Effect of NDI Inspection Uncertainty on the 0.1% SII	96
5.2.3	Effect of Crack Growth Rate on the 0.1% SII	97
6	DISCUSSION	119
6.1	Sensitivity and Reliability of Various NDI Techniques	119
6.2	Comparison of Confidence Bounds on Binomial and Loglogistic POD Curves	121
6.3	Simulation Model in Relation to In-Service Behaviour of Components	122
6.3.1	Comparison of In-Service Inspection Data with Fractographic Data	122
6.3.2	Comparison of In-Service vs Simulated Crack Growth Rate	124
6.4	The Distribution of Safe Inspection Intervals	124
6.4.1	Theoretical Distribution of SII's	124
6.4.2	Comparison of Lognormal and Weibull Distributions of SII's	125
6.5	Factors That Most Affect the Safe Inspection Interval	125
6.6	Deterministic, Combined Analysis, and Probabilistic Based SII's	126
6.6.1	SII by DFM	126
6.6.2	SII by TSO	126
6.6.3	SII using the N_i and N_d Weibull Lines	126

6.6.4	Comparison of SII's Based on Deterministic and Probabilistic Methods	127
7	CONCLUSIONS AND FUTURE WORK	138
7.1	Conclusions	138
7.2	Recommendations for Future Work	141
	Bibliography	143
	Appendices	
A	RELEVANT STATISTICS	153
A.1	Classical Approach To Estimating Parameters from Observational Data	153
A.2	Distributions	153
A.3	Regression Analysis	159
A.4	Confidence Bounds	160
B	COMPUTER PROGRAMS	162
C	EXPERIMENTAL RAW DATA	163
D	A TYPICAL DISK2 COMPUTER SIMULATION RUN	175
D.1	Typical Input for a DISK2 Run	175
D.2	Typical Output for a DISK2 Run	175

List of Tables

2.1	Maximum Number of Cracks that can be Missed given the Total Number of Cracks Inspected for 95% and 50% Confidence. [Rummel]	43
3.1	The ten J85 CAN40 Fifth Stage Compressor Discs chosen to generate the NDI Databases.	61
3.2	Summary of NDI Techniques used and the Institutions that performed the NDI.	61
4.1	Linear Regression Results for Crack Growth Rates.	80
4.2	Crack Growth Rates and their Parameters.	80
4.3	Summary for Stability and Convergence.	80
5.1	Actual Cracked and Uncracked Bolt Hole Data in the Ten Discs.	98
5.2	Summary of Cracked Bolt Hole Data for each NDI Technique.	98
5.3	Number of Cracks Detected by each NDI Technique.	99
5.4	Number of Cracks Missed by each NDI Technique.	99
5.5	Number of False Calls for each NDI Technique.	99
5.6	Cracks Detected by Different NDI Techniques and the Smallest Crack Detected.	100
5.7	Linear Regression Results for Loglogistic POD Curves.	100
5.8	Constants for Confidence Bounds on the Loglogistic POD Curves using F and Normal Distributions.	100
5.9	Loglogistic Parameters for the POD Curves for each NDI Techniques.	100
5.10	Cracks Missed by Various NDI Techniques and the Largest Crack Missed.	101
5.11	Lognormal Analyses of Missed Cracks.	101
5.12	Understanding the Distribution of Crack Lengths Missed.	101
5.13	Correlation between the Actual Crack Length and the NDI Inspected Crack Length.	102

5.14	Actual Crack Length Distribution given an NDI Inspected Crack Length of 0.8 mm in Length.	102
5.15	NDI Inspected Crack Length Distribution for a Known Crack Length of 4.5 mm.	102
5.16	Effect of Initial Conditions on the 0.1% SII.	103
5.17	Effect of NDI Inaccuracy on the 0.1% SII, when Estimating the Actual Crack Length.	103
5.18	Comparison of 0.1% SII for Different NDI Techniques in terms of POD Curves, Distributions of Cracks Missed, and 95% Confidence POD Curves.	103
5.19	Effect of Crack Growth Rate on the 0.1% SII.	103
6.1	Comparison of Mean and 95% Confidence Values for the Binomial and Loglogistic POD Curves.	128
6.2	Summary of Total Number of Bolt Hole Cracks found in Retired Fifth Stage J85 CAN40 Compressor Discs.	128
6.3	The Crack Lengths found in Retired Fifth Stage J85 CAN40 Compressor Discs.	129
6.4	Lognormal Parameters for Crack Distributions, for TSN Intervals, in Retired Fifth Stage J85 CAN40 Compressor Discs.	129
6.5	Comparison of In-Service vs Simulated Crack Growth Rates.	129

List of Figures

1.1	Damage Tolerance Concept showing SII. When a crack is found, the disc is retired. [Harris et al, 1981 with special acknowledgement to Dr. WH Reimann]	4
1.2	Deterministic vs Probabilistic Approach. [Harris et al, 1981]	4
1.3	The Perfect Crack – inspectable, slow to propagate, and good for another 500 years.	5
1.4	A J85 CAN40 Fifth Stage Compressor Disc. Fatigue cracks initiate at the bolt holes and propagate radially inwards.	5
1.5	Canadian Built CT-114 Tutor. A two seater jet trainer for the Canadian Air Force with one J85 CAN40 engine.	6
1.6	Orenda built General Electric J85 CAN40 turbojet.	6
1.7	The J85 Engine Compressor Section. The fifth stage compressor disc and spacer are highlighted.	7
1.8	The attachment of the fifth stage disc with two spacers using a stud, a washer, and two locknuts.	7
2.1	Different Defect Distributions for Initial Crack Sizes. [Gallagher et al, 1984]	44
2.2	General Shape of Distribution of Initial Crack Sizes. [Gallagher et al, 1984]	44
2.3	Sample Initial Flaw Distributions. [Jouris and Shaeffer, 1980] [Raussi and Tiainen]	45
2.4	Definition of Equivalent Initial Flaw Size (EIFS). [Rudd and Gray]	46
2.5	EIFS Distribution for A-7D Fastener Holes. [Rudd and Gray]	46
2.6	Model of TTCI and Back Extrapolation to EIFS. [Yang, Manning, and Garver]	47
2.7	Initial Flaw Distribution based on an NDI Database. [Tang]	48
2.8	Probability of Nondetection Curves. [Harris, 1977]	48

2.9	Probability of Detection Curves and Different Types of Confidence Bounds. [Yee et al, 1976] [Berens and Hovey, 1981]	49
2.10	The Variation of the Probability of Detection for a Given Crack Length.	50
2.11	The Human Factor in the Inspection Cycle. [Taylor]	50
2.12	ENSIP Summary of Nondestructive Inspection. [Taylor, 1985]	51
2.13	ECI POD Results for Aircraft Engines. [King, Cowie, and Reimann, 1985]	51
2.14	FPI POD Results for Compressor Discs. [King, Cowie, and Reimann, 1985]	51
2.15	Preliminary POD Results for Compressor Discs. [Raizenne et al, 1985]	52
2.16	Representative Profile Mission for an Advanced Turbine Engine. [Schwartz and Annis, Pratt and Whitney]	52
3.1	The Nortec RECHII I Rotating Eddy Current Hole Inspection Instrument with a Hand Held Scanner. The probe rotates at 3000 rpm. [Nortec]	62
3.2	The Eddy Current Differential Type Probe Consists of a Send and a Receive Coil. [Nortec]	63
3.3	Typical Eddy Current Output Display.	63
3.4	Drawing of a Real-Time Microfocus X-Ray System. [Ridge HOMX]	64
3.5	The Ultrasonic C-Scan at NAE. The compressor disc was positioned in the water tank and crack maps were produced by the plotter.	64
3.6	Dimensions for the J8 CAN40 Fifth Stage Compressor Disc.	65
3.7	Compressor Disc Section Containing Bolt Hole and Fatigue Crack.	66
3.8	Section of Compressor Disc showing Bolt Hole Fatigue Cracks Pried Open.	66
3.9	Types of Initiation Sites.	67
3.10	Different Crack Lengths.	69
3.11	The Boundary between the Fatigue Crack and the Pried Open Area.	70
4.1	Flow Diagram for DISK2 Simulation Program.	81

4.2	Initial Conditions of Constant Crack Length and Constant Time Since New (TSN).	82
4.3	Time To Crack Initiation (TTCI) Distribution at Constant Crack Length.	82
4.4	Flow Diagram for Simulating Uncertainty of Inspection.	83
4.5	Illustration of Crack Growth Rate Scatter.	83
4.6	Different Crack Growth Rates graphically depicting Least Square Results.	84
4.7	Typical Probability Plot Output showing the 0.1% Safe Inspection Interval (SII).	85
4.8	The Stability and Convergence of the 0.1% Safe Inspection Interval (Maximum Likelihood Method).	86
5.1	Comparison of NDI Techniques in terms of the Number of Detected and Missed Cracks, and the Number of False Calls.	104
5.2	Histograms of Detected and Total Number of Cracks for each NDI Technique.	105
5.3	Probability of Detection Curves for each NDI Technique.	107
5.4	Comparison of 90% POD Values for each NDI Technique.	109
5.5	Histograms of Crack Lengths Missed for each NDI Technique.	110
5.6	Lognormal Analysis of Crack Lengths Missed for each NDI Technique.	112
5.7	Distributions of Crack Lengths Missed by each NDI Technique.	114
5.8	Comparison of the Median Crack Length Missed by each NDI Technique.	109
5.9	The Correlation between the NDI Measured and the Actual Crack Length.	116
5.10	Distributions of Actual Crack Lengths, for various NDI Techniques, given an NDI Measured Crack Length of 0.8 mm.	118
6.1	Example of 95% Confidence Bounds on the Binomial POD Curve using the Range Interval Method and the Optimized Probability Method (OPM). [Stoute]	130
6.2	FEM Stress Analysis Results for the Fifth Stage J85 CAN40 Compressor Disc. [Kaprzynski]	131
6.3	Histograms of Crack Lengths in Retired Fifth Stage J85 CAN40 Compressor Discs, for various TSN Intervals.	132

6.4	Distributions of Crack Lengths of Retired Fifth Stage J85 CAN40 Compressor Discs for various TSN Values.	133
6.5	Average Number of Cracks Per Disc with respect to Time Since New (TSN).	134
6.6	Comparison of Lognormal and Weibull Distributions for 0.1% SII's.	135
6.7	Weibull Analysis showing the GE N_i and N_d (1983) Dis- tributions.	137
6.8	Comparison of SII's Calculated using Deterministic and Probabilistic Approaches.	137
D.1	Typical Input for a DISK2 Run.	177
D.2	Typical Output for a DISK2 Run.	178

Chapter 1

INTRODUCTION

1.1 Background for the Lifing of Aircraft Engine Components and The Damage Tolerance Philosophy

Flaws in turbine engine components such as the compressor discs, turbine discs, spacers and turbine blades are known to exist in the "as manufactured" condition. As nondestructive inspection (NDI) equipment becomes more and more sophisticated it is possible to detect an increasingly smaller size of flaws in these components. A component cannot simply be rejected because of the presence of a flaw in it. It is therefore imperative to decide as to what flaw size can safely be tolerated by a component under severe service conditions in order to calculate its safe operating life without causing catastrophic failure during service.

The safe life of a turbine engine disc is determined by calculating the number of high strain low cycle fatigue cycles required to grow an existing flaw to a detectable crack length in 1 component out of a total population of 1000 discs [40] [41] [87]. In other words when a batch of 1000 discs accumulates this predetermined number of cycles or hours, then the entire population of 1000 discs is retired even though only 1 disc contains a crack of this length. A detectable crack length of 0.8 mm is commonly chosen since this is a 90% probability of detection value for state-of-the-art NDI techniques. Unfortunately this can mean that as many as 80% of the discs can have lives in excess of the prescribed minimum life remaining in them [40].

A damage tolerance philosophy towards design and maintenance proposes that marked reductions in the overall life cycle management costs would accrue by allowing the damage to develop in each component and retiring them on an individual basis. Damage tolerance concepts assume that flaws exist in parts as

manufactured and these may be located in the most highly stressed regions or hot-spots of the component. It is further assumed that these flaws will grow in service, at a rate governed by the local stress distribution around the flaw, the operating temperature, and the environment [40].

The general objective of the damage tolerance approach is to establish expected crack growth rates under realistic operating conditions and to implement NDI¹ procedures with an appropriate level of sensitivity, reliability, and frequency to ensure that cracks can be detected and, if feasible, monitored in service to a point where the risk of rapid or unstable crack propagation becomes severe [10] [41]. Figure 1.1 illustrates the principle of the damage tolerance approach by showing that a component can continue to be used in service for another Safe Inspection Interval (SII) if the component satisfies the inspection procedure.

In a Deterministic Fracture Mechanics (DFM) method, worst case values are often used to determine the SII for a component, for example the *upper bound* crack growth rate in Figure 1.2. However recent efforts have concentrated on the development of Probabilistic Fracture Mechanics (PFM) strategies for determining the SII's. The PFM approach tries to get away from the conservatism of the DFM principles and instead of using worst case values for different variables, the PFM approach uses realistic distributions in fracture mechanics calculations, for example the *distribution* of crack growth rates in Figure 1.2. Ideally, of course, when applying damage tolerance to components the crack growth rate should be slow and the critical crack length should be long so that only infrequent inspections need to be made. Thus, "The Perfect Crack" — inspectable, slow to propagate, and good for another 500 years! (Figure 1.3) [80]

1.2 Problem Identification and Objectives

The Canadian Air Force was experiencing problems with the compressor section of a gas turbine aircraft engine. In particular the fifth stage compressor discs from the J85 CAN40/15 engines used in the Tutor CT-114 trainer and CF-5 aircraft respectively were cracking in service. Figure 1.4 is a photograph of one of the retired fifth stage discs from the Tutor (Figure 1.5) used for this case study.

The basic J85 turbojet is a General Electric gas turbine which incorporates an 8 stage axial flow compressor, an annular combustion chamber, and a two stage reaction turbine. The J85's are used in 38 countries for many kinds of aircraft and continued operational use is planned for at least 25 years. The J85 CAN40 is about 1 m in length, 0.5 m in diameter, and weighs 173 kg (Figure 1.6). The CF5

¹Abbreviations throughout the thesis are defined alphabetically in the nomenclature.

uses two J85 CAN15's, the afterburner version of the J85 CAN40. The compressor section is shown in Figure 1.7 highlighting the fifth stage disc and spacer. Torque is transmitted using bolts rather than a shaft and Figure 1.8 shows two spacers attached to a disc with a bolt, a washer, and two locknuts. The fifth stage disc has 40 bolt holes around the circumference which act as stress concentration sites and initiate low cycle fatigue (LCF) cracks. LCF cracks in the J85 CAN40/15 compressor discs initiate from the bolt holes and grow radially inward. When the crack length reaches a critical value, fast fracture and failure occurs.

Most discs are removed from service before any crack can be detected. However, as a crack can take several thousand flying hours to grow to its critical length, it may be possible to permit some discs with undetected cracks to remain in service for a finite period of time, or at least until a fatigue crack is detected, without affecting safety yet reducing cost and increasing efficiency. When pursuing a damage tolerance based lifing or maintenance approach it becomes increasingly apparent that its implementation relies heavily on the reliability of the NDI technique used for inspecting the components. Therefore, the first question to be answered is "How reliable is the nondestructive inspection procedure used?" Secondly, it is important to determine how many hours a component should be kept in service before inspecting it. This is called the safe inspection interval (SII) or a return-to-service interval. A SII can be predicted on the basis of DFM or PFM principles.

The three major objectives of this study were:

1. To obtain a sufficient experimental NDI database for bolt hole LCF cracks by destructively measuring crack lengths using fractography and comparing the results with nondestructive inspection results for several NDI techniques.
2. To use probabilistic fracture mechanics (PFM) based damage tolerance concepts for predicting safe inspection intervals (SII) for turbine discs using J85 CAN40/15 fifth stage compressor discs as a case study.
3. To quantify the influence of NDI uncertainties on SII predictions.

Figure 1.1: Damage Tolerance Concept showing SII. When a crack is found, the disc is retired. [Harris et al, 1981 with special acknowledgement to Dr. WH Reimann]

Figure 1.2: Deterministic vs Probabilistic Approach. [Harris et al, 1981]

Figure 1.3: The Perfect Crack – inspectable, slow to propagate, and good for another 500 years. [Ramsden]

Figure 1.4: A J85 CAN40 Fifth Stage Compressor Disc. Fatigue cracks initiate at the bolt holes and propagate radially inwards.

Figure 1.5: Canadian Built CT-114 Tutor. A two seater jet trainer for the Canadian Air Force with one J85 CAN40 engine.

Figure 1.6: Orenda built General Electric J85 CAN40 turbojet.

Figure 1.7: The J85 Engine Compressor Section.
The fifth stage compressor disc and spacer are highlighted.

Figure 1.8: The attachment of the fifth stage disc with two spacers using a stud, a washer, and two locknuts.

Chapter 2

LITERATURE OVERVIEW OF PFM BASED LIFING

Fong [31] states that there is a generation gap between traditional thinking and probabilistic methods, but suggests that there needn't be since each have safety factors, the one based on judgment and other on probabilities. He says that there is considerable room for engineering judgment to convert technical information into a risk assessment, and that judgment can be exercised through the choice of a probability distribution function in order to represent the design parameter.

2.1 Traditional Safe Life Approach

In North America, the safe life of an engine disc is defined as the number of LCF cycles required for 1 out of 1000 discs to initiate a crack of 0.8 mm (1/32 inches) in length [11] [39] [87]. The implication is that since one removes the other 999 discs when the 1 has reached safe life, that in fact 99.9% of the discs have remaining useful life.

For Rolls Royce the accepted method of establishing design life for turbine discs is by cyclic spin testing of full size components. In common with the lifing philosophy adopted in other countries, component safe life is based on a first crack criterion. Recently, however components have been run to burst, and the design life assumed to be 2/3 the number of cycles to failure [29]. Two benefits result since firstly, many components spend their lives in the crack growth region, not in crack initiation, and secondly, most disc components can tolerate cracks significantly larger than the 0.75/0.8 mm safe life limit.

2.2 Damage Tolerance Lifting

An international conference called by NATO in April 1985 was the culmination of exploratory work done in the past ten years concerning damage tolerance concepts for critical engine components. Several case studies, including a Pratt and Whitney F100 engine flown in the F-15 and F16 fighters (1980) [40], and 49 retired TF-33 third stage turbine discs (1984) [7] with a full-scale validation [41], concluded that damage tolerance maintenance was cost effective. One of the results of these studies was MIL-STD-1783 (USAF) entitled "A Military Standard for the Engine Structural Integrity Program (ENSIP)" dealing specifically with damage tolerance for aircraft engine components [64].

2.2.1 DFM Approach

Fracture mechanics has been used for some time in airframe design but only sparingly on the engine side. One of the difficulties has been that because of high stresses, the critical crack sizes for growth are small, and thus crack detection capabilities must be high. Besuner and Tetelman derive a formula for the total number of cycles to failure, N_f , based on the Paris crack growth equation:

$$N_f = \int_{a_i}^{a_f} \frac{da}{C(\Delta K)^n} \quad (2.1)$$

$$\text{where } \Delta K = \Delta \sigma \sqrt{\pi \lambda a} \quad (2.2)$$

$$\text{and } N_f = \frac{(n/2 - 1)^{-1}}{C(\Delta \sigma \sqrt{\pi \lambda})^n} \left[\left(\frac{1}{a_i} \right)^{n/2-1} - \left(\frac{1}{a_f} \right)^{n/2-1} \right] \quad (2.3)$$

In this formula worst case values are used. Mom [66] has summarized more complicated formulas for life prediction during a period 1972-1982. He notes that components in aircraft engines operate under a combination of complex loading and temperature cycling which makes life prediction extremely difficult. Both fatigue and creep contributions need to be taken into account and he summarizes four approaches. He also noted that life extension possibilities of gas turbine engine components received much attention in the AGARD Structures and Materials Panel activities and that damage tolerance lifting concepts is a considerable step forward with respect to the conventional safe life philosophy.

2.2.2 PFM Approach

Jouris [49] compared the deterministic and probabilistic approaches and concluded that the probabilistic approach is a powerful tool and should be used and developed

further. The probabilistic fracture mechanics (PFM) damage tolerance approach is similar to the DFM approach in many respects. The method uses a deterministic equation to compute residual LCF life of a cracked component from known values of input parameters such as $\Delta\sigma$, ΔK , a_i , and a_c . These input parameters can be treated as random variables with known or specified distributions. For example, instead of a specified crack length, a , the probability of occurrence of various flaw sizes $P(a)$ is specified. Life analysis processes can be broken down into the following four steps:

- The establishment of initial flaw sizes.
- The inspection procedure for flaw detection.
- The development of fatigue crack growth models.
- The definition of failure criteria.

Most of these techniques use Monte Carlo simulation, within a computer program, for generating distribution functions to solve complex statistical problems.

Harris et al [40] conducted a probabilistic fracture mechanics study of an F100 engine. The initial crack sizes were assumed to follow a lognormal distribution and after the computer program simulated an inspection of these cracks, those cracks that were detected were allowed to grow. Finally, by comparing the predicted residual life of the component with the component safe inspection interval it was possible to ascertain the probability of an in-service failure occurring.

Cook et al [22] describe a damage tolerant design system similar to MIL-A-83444 applicable to cold section engine disc materials. The damage integration package interrelates the following: criteria (eg. bolt holes), initial flaw characteristics, engine duty cycle, material crack growth model, stress intensity factor prediction and crack growth algorithm methods, and cumulative damage prediction model and cracked disc residual strength requirements. The output evaluates a disc design relative to its meeting crack growth requirements and plots the remaining useful disc life.

Along these same lines, King et al [54] summarizes the damage tolerance requirements that are now applied to USAF engine development and acquisition programs, ENSIP [64]. Part of the damage tolerance approach is as follows:

Components are designed so that the safety limit exceeds two times the required inspection interval. The safety limit or residual life is the time for assumed initial flaws to grow and cause failure.

This standard also states that initial flaws shall be assumed to exist, and flaw size should be based on the intrinsic material defect distribution and NDI methods used during manufacture. A surface flaw of 0.8 mm should be assumed if a liquid penetrant inspection technique is used, or 0.4 mm if an eddy current inspection technique is used.

King et al mentions the importance of design detail on damage tolerance by giving an example of a fifth stage disc made of high strength titanium. The web was strengthened to resist the loading of the rim spacers which heat up and thermally expand faster than the bolted structural backbone of the compressor. Overall weight increased by 4 pounds with a corresponding threefold increase in permissible flaw size.

Besuner and Tetelman [16] present an example of the application of probabilistic fracture mechanics. The residual life, N_{PR} , in a turbine rotor is assumed to depend on the applied stress, $\Delta\sigma$, the material crack growth rate parameter, C , and the flaw size a . The variation in each of these parameters was assumed lognormal so that:

$$\log C = N(\mu_C, s_C) \quad (2.4)$$

where $\log C$ is a normal random variable with mean μ_C and standard deviation s_C , and likewise for the other two parameters. The life distribution may be obtained from an equation of the form:

$$\log N_{PR} = N(\mu_N, s_N) \quad (2.5)$$

$$\text{where } \mu_N = \log B + p\mu_C + q\mu_{\Delta\sigma} + r\mu_a \quad (2.6)$$

$$s_N = p^2 s_C^2 + q^2 s_{\Delta\sigma}^2 + r^2 s_a^2 \quad (2.7)$$

and the number of cycles at a given failure probability α is simply:

$$N_{PR} = 10^{(\mu_N + K_\alpha s_N)} \quad (2.8)$$

where K_α is the standard normal deviate.

Yang et al [100] have also proposed a methodology to determine fatigue reliability in gas turbine engines based on the probabilistic approach and taking into account all the statistical variables in the damage tolerance system. The statistical

uncertainty of the initial fatigue quality was characterized by a TTCI distribution (TTCI is explained in the section on Initial Flaw Distributions). Both lognormal and Weibull distributions were used as below:

$$TTCI = L(\lambda, \zeta) \quad (2.9)$$

$$TTCI = W(\eta, \beta) \quad (2.10)$$

The variation in crack growth was taken into account by randomizing the Paris fatigue crack growth rate equation. The equation is multiplied by a variable X , a lognormal distribution with a mean of 1, and a standard deviation equal to the variation in C (See the section on Crack Growth Rate Variability).

The cracks were inspected using the logistic distribution to represent the POD values so that:

$$POD = \text{Loglogistics}(\beta_0, \beta_1) \quad (2.11)$$

Any cracks that were not detected were added to the initial flaw distribution for the second service interval. By assuming that the components were subject to controlled maintenance inspections, it was possible to determine the cumulative probability of failure. The results showed that the design life of gas turbine engine discs can be extended considerably with a high level of reliability [69] [100]. Additionally, the third stage of a TF-33 jet engine turbine disc was used to demonstrate that in-service maintenance inspections can significantly reduce the probability of failure in fatigue critical components.

Two cautions concerning the PFM approach should be pointed out:

1. There is a subtle influence in the choice of a distribution function for a given set of data. This will be reflected in the fatigue life prediction [31].
2. The difference between the fitted distribution and the real data often occurs in the tails of the distribution. This can lead to the inability of standard life distributions to predict early failures [88].

One noticeable advantage of the PFM approach is the rational definition of a safety factor - it can quantify the impact of uncertainties.

2.2.3 CA Approach

A combined analysis approach was developed by FAA in order to combine field experience with laboratory experiments and analytical techniques. The total fatigue

life of a turbine disc is assumed to be the sum of all the cycles which are required to initiate and propagate a crack to its critical length. Based on the analysis of the disc data and the results from some spin pit tests, the following model was proposed in Reference [7]:

$$N_f = N_i + N_p \quad (2.12)$$

$$= \frac{C_i}{\sigma^6} + \frac{C_p}{\sigma^{304}} (1.5 + \ln a_c)^{.585} \quad (2.13)$$

where in the most general sense, the input variables are firstly C_i , C_p , K_c , and a_{insp}/a (crack length inspection uncertainty). These vary randomly for each of the 10 bolt holes in the disc. Secondly, σ and N_{insp}/N (cycle counting uncertainty) which vary randomly from disc to disc. The predicted distribution of the disc time to failure was in excellent agreement with that obtained from operational discs.

Combined analysis is useful when key engineering factors are unknown and field experience alone cannot be used to make adequate predictions [81].

2.3 Initial Flaw Sizes

The safe inspection interval in a damage tolerance approach is influenced by the crack length assumed at the beginning of a usage period since firstly, the damage tolerance approach assumes that cracks are present in all critical locations, and secondly, the rate of crack growth depends on the crack length. The damage tolerance approach seeks to ensure that these cracks will not grow to critical size without being detected by calculating the time interval over which a crack will grow from its initial size to its critical size. This time period is then typically split into three or four equal inspection intervals. Thus the operator has several chances to detect the crack prior to it attaining its critical length. As the rate of crack growth increases with crack size it could be argued that, from a safety standpoint, initial crack lengths longer than those anticipated in service should be assumed. However, from an economic standpoint, the degree of conservatism must be limited so that realistic inspection intervals can be selected.

The distribution of crack lengths can be considered to consist of a composite of several distributions (Figure 2.1) [32]. The distribution of inherent defects in the material shows that the number of defects in the material increases to infinity as the crack size tends to zero and this is an intuitive model. These inherent flaws, assumed not to grow appreciably in service, can be characterized by a single crack length, a_c , an upper bound on the distribution.

The largest defect that could remain undetected after final inspection of cracks introduced during fabrication is a_0 . This crack length is used to determine service life or the need for in-service inspection. Cracks smaller than a_0 will propagate in service and other cracks will initiate. The largest crack that can remain undetected is designated a_{NDI} and becomes the initial crack size for the next usage period. a_{NDI} should be a crack length small enough so that it does not propagate to critical size, a_{cr} , during a period of operation before a scheduled inspection. Many distributions have been used to characterize initial flaw sizes, as is discussed later, but the general shape is as shown in Figure 2.2.

2.3.1 Single Values and Distributions of Initial Flaw Sizes

The Military Standard, MIL-STD-1783 (USAF) specifies that initial flaws shall be assumed to exist as a result of intrinsic material defects and, manufacturing and processing operations. It is recommended that the initial design and sizing of the components, as well as in-service maintenance, be based on a 0.8 mm surface flaw if LPI is used [64]. The USAF standard also states that initial flaw sizes, different from the assumed flaw specifications or a distribution of initial flaw sizes, can be used if sufficient initial quality data with respect to suitable NDI techniques has been developed.

In some computer analyses, especially those involving finite element analysis, different shapes and sizes of flaw have been assumed in order to see the effect of the initial defect size on the life of the component. In one case an elliptic surface crack was assumed to exist at four different parts of the rim slot in an aircraft jet engine rotor disc [107]. Other case studies are documented in [24] and [105].

Several studies were also carried out where the initial flaw size was characterized by a distribution rather than a single value. Jouris and Shaffer (1980) wrote that the two most plausible distributions for initial flaw depth were the exponential and the lognormal because they have the general tailshape which is characteristic of the flaw depth random variable. The truncated exponential (Figure 2.3 (a)) is given by the density function equation:

$$f(x) = \begin{cases} \frac{1}{\lambda} e^{-\frac{(x-\tau)}{\lambda}} & x \geq \tau \\ 0 & \text{otherwise} \end{cases} \quad (2.14)$$

where τ is the threshold parameter (assigned by the user) in order to eliminate having an infinite number of 'cracks' of length 0 [50], and λ is the mean flaw depth.

The three parameter lognormal density function (Figure 2.3 (b)) is given by

the equation:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma(x-\tau)} \exp\left[-\frac{1}{2}\left(\frac{\ln(x-\tau)-\mu}{\sigma}\right)^2\right] \quad (2.15)$$

where τ is defined as above and μ and σ are parameters.

A computer program was developed [51] for both the exponential and lognormal distributions above to use flaw depth as input data and output a 'factor'. The distribution with the lowest 'factor' fit the data best. An example using depth of flaws in welds in a pressure vessel, determined by radiography showed that while the exponential fit the data points better, they were both acceptable.

The initial defect size $a(0)$ was also characterized by Yang and Chen (1984) using a lognormal distribution as above with the location parameter $\tau = 0$ (ie. a two parameter lognormal) and the probability density function $f(x)$ equal to $f_{a(0)}(x)$ where $a(0)$ is the reference crack size. They note that this distribution can be used if the crack sizes are larger than the reference crack size $a(0)$ ie. the cracks should start to propagate after the component is introduced into service. If the cracks do not propagate upon entering service, then a time to crack initiation distribution should be used as well, because those cracks that are less than $a(0)$ in length undergo an incubation period.

Raussi and Tiainen (1980) analyzed the imperfection size distribution of weld defects appearing in nuclear power plant pipes. The best fitting curve, out of 7 linear regression choices each using 3 different functions, was of the form $a + b/x^2$ where x was the imperfection size and a and b were coefficients (Figure 2.3 (c)). The authors reason this distribution probably differs from the more usual exponential distribution and complementary error function distribution because the defects analyzed were all weld imperfections.

2.3.2 Initial Flaw Sizes Represented by TTCI

It was apparently during 1971 that fatigue life was considered to consist of initiation as well as propagation [42], and it was proposed that the fatigue process could be divided into stages of time to initiation and time to failure. A definition of time to initiation (TTCI) is: *"The time required to produce an identifiable crack for the inspection or observational technique being employed"*.

Unfortunately the definition has one meaning for the engineer using NDI, and a quite different meaning for the scientist using SEM. For gas turbine compressor discs, the TTCI is often defined as the time (number of hours or number of cycles) to initiate a crack approximately 0.8 mm (about 0.03 inches) long [99] [29] [39] [61]. Traditionally, gas turbine compressor discs are typically retired at a time when

1 in 1000 could be expected to have actually initiated a short (0.8mm) fatigue crack [39]. It should also be noted here that the time to crack initiation is often referred to as the low cycle fatigue life in gas turbine engine literature.

The advantages of a TTCI approach are:

- The crack size at crack initiation is physically observable.
- Test data can be obtained from coupon specimens and full scale components directly.
- Linear fracture mechanics for fatigue crack propagation can be applied after crack initiation.

The disadvantage, however, is that the TTCI is a function of several variables, such as stress level, loading spectra, manufacturing processes, etc. Thus, if any one of the influencing variables changes, eg. stress level or loading spectra, test data may have to be generated again, since the functional relation between TTCI and these influencing variables has not been established to date. In practice, the TTCI approach is often used as an input for initial flaw sizes because this is the one most familiar to engineers.

Yang and Chen (1985) represented the initial flaw sizes by the TTCI distribution alone. The authors state that this is applicable to some engine materials, such as titanium, Waspoly, Astroloy. However, since a reference crack size, a_0 , at crack initiation can be defined to be very small because of improved NDI capabilities, there may be other cases where cracks start to propagate immediately after the component is introduced into service causing the TTCI to be zero. And consequently the initial flaw sizes should perhaps be represented by two populations: one defined by the distribution of the time to crack initiation and the other by that of the initial flaw size.

An example for the third stage turbine disc of a TE-33 gas turbine engine was given using a reference crack size a_0 of 0.76 mm (0.03 inches) based on available test results. In this paper the TTCI was assumed to follow a lognormal distribution and the median life and coefficient of variation of the TTCI were 4000 hours and 75% respectively [100].

Traditionally, a lognormal distribution has been used to describe the statistical distribution of the TTCI, a Weibull is more consistent with the fatigue wear out process because the fatigue failure rate increases monotonically rather than initially increasing and eventually decreasing as in the case of the lognormal [104]. The three parameter Weibull distribution is as follows:

$$F_{TTCI}(t) = 1 - e^{-\left(\frac{t-t_0}{\eta}\right)^\beta} \quad (2.16)$$

where η and β are parameters and methods of estimating them from test data are available. ϵ is the lower bound and depends on the definition of crack size a_0 at crack initiation.

The two parameter Weibull, where ϵ is zero, has also been used to represent time to crack initiation as a random variable in a paper about fatigue sensitive aircraft structures [105]. In an example given, where the material of the critical component is aluminium, the initial crack size at crack initiation was $a_0 = 1.02$ mm (.04 inches) and in the Weibull curve $\alpha = 4$ and $\beta = 30000$ hours (Figure 2.5 (b)).

2.3.3 Initial Flaw Sizes Represented by EIFS

Some flaws are detectable but most "initial flaws" are not detectable by any NDI technique. Thus, the initial flaw size is usually obtained from the results of laboratory fatigue tests. When detectable flaws are observed during fatigue tests, they are extrapolated backwards using a suitable crack propagation law to estimate their "equivalent" initial flaw size (EIFS). The backward extrapolation has to be validated using fractographic results [104] [85] [32].

The advantage of the EIFS approach is that, once the initial flaw size distribution is established, the fatigue crack propagation under various service loading spectra can be predicted analytically without further experimental tests. Furthermore, this approach can be used to study the effect of various factors, such as drilling techniques, manufacturing processes, etc., on for example, the fastener hole quality.

The EIFS method has been very well explained by Rudd and Gray (1978) and quoted by the Damage Tolerant Design Handbook [32] as an example. The method is presented for fastener holes since this is a most prevalent source of cracking.

"Consider a piece of structure with a fastener hole containing the defect of characteristic dimension l (Figure 2.4) This defect results in fatigue crack initiation and propagation when subjected to some known load history. Upon failure of the structure, a fractographic examination of the fracture surface is performed to obtain as much of the crack growth curve as possible. Analytical crack propagation analyses are performed until there is good agreement between the analytical prediction and the fractographic test data. The initial crack length (crack length when the load history is first applied), a_i , of the analytical crack growth curve that correlates best with the fractographic test data is defined

as the equivalent initial quality. Hence, a_i is said to be the analytical equivalent of the actual defect of characteristic dimension l if each results in a crack size a_e after N_e cycles of the same load history have been applied. Hence, fastener holes that contain actual crack lengths less than a_e after N_e cycles have been applied are of better quality than those that contain actual crack lengths equal to or greater than a_e after N_e cycles. "

Rudd and Gray show that the EIFS can be found by fractography alone. The EIFS method was applied to specimens from an A-7D production aircraft wing in order to establish the manufacturing quality a_i . The selected stress spectrums marked the fracture surfaces well making it possible to fractographically determine the equivalent initial flaw size for each crack examined. The range of EIFS was found to be 0.0038 to 0.0559 mm (0.00015 to 0.0022 inches) and is shown graphically in Figure 2.5 (a) and (b). The EIFS method was checked using other results from a full scale fatigue test of an A-7A wing [85].

In the same paper for a different aircraft it was not possible using optical fractography to determine the crack length at time zero. Therefore, the EIFS distribution was determined from combined fractographic and crack propagation analyses as is the case with most analyses.

Yang, Manning, and Gerner [104] illustrate the same method as [85] and [32] to establish EIFS but using different distributions. The EIFS distribution was represented by a Weibull-compatible distribution (Figure 2.5 (a) and (b)). The Kolmogorov-Smirnov test for goodness of fit indicated that the Weibull-compatible fit the test results better than some other statistical distributions, such as the normal, lognormal, Weibull, Pearson, and Johnson [91]. The constants in the crack growth equation are summarized in reference [104] on pages 12 through 16. Figure 2.6 shows the predicted distribution of the equivalent initial flaw sizes and the observed distribution of cracks sizes after a time t . It was shown that the correlation between the predicted crack size distribution at time t , extrapolated from the EIFS distribution, and the observed crack sizes at time t was excellent.

The Johnson S_U distribution was also used to represent the EIFS. The EIFS distribution was based on data from an F-4 study.

2.3.4 Initial Flaw Distributions Based on an NDI Database

In an example Tang (1973) shows that the initial flaw distribution can be deduced from an NDI database. Two graphs are required:

- i. Actual crack length measured crack length.

ii. POD Curve.

See Figure 2.7 (a) and (b). In the first graph, it was observed that a linear trend fit the data approximately and gives a good first order approximation on the calibration characteristic for most NDI devices with various levels of detectability. The error can be characterized by a normal random distribution with zero mean and standard deviation σ^* .

From Bayes' Theorem of two events, Tang derives:

$$f_1(c_a) = kP(c_a)f_0(c_a) \quad (2.17)$$

where f_1 = distribution of length of detected cracks (assumed lognormal)

f_0 = distribution of length of all cracks

P = probability of detection

k = constant = 1 in this case

and c_a = actual crack depth

Tang notes that $f_1(c_a)$ is a measure of the relative frequency of the actual depths of those cracks that have been detected, that its mean and standard deviation are given from the first graph by the following:

$$\bar{x}(c_a) = \alpha + \beta\bar{x}(c_m) \quad (2.18)$$

$$\sigma(c_a) = \beta^2\sigma(c_m) + (\sigma^*)^2 \quad (2.19)$$

where c_m = measured crack depth

If the distribution of detected cracks is known and the POD curve is known from NDI and fractography, then $f_0(c_a)$, can be determined and the mean and standard deviation of the distribution of all cracks estimated.

Harris (1977) presents an analysis procedure to quantify a distribution of pre-service cracks or a distribution of in-service cracks. The distribution of *pre-service* cracks is based on the distribution of crack like defects due to fabrication and the probability of nondetection of a crack. More useful perhaps to us is the distribution of *in-service* cracks which is based on the distribution of cracks after some fatigue crack growth and the probability of nondetection as below:

$$\left(\begin{array}{c} \text{Distribution of} \\ \text{Cracks} \end{array} \right) \times \left(\begin{array}{c} \text{Probability of} \\ \text{Nondetection} \end{array} \right) \Rightarrow \left(\begin{array}{c} \text{Distribution of} \\ \text{In - Service Cracks} \end{array} \right)$$

The number and size of flaws introduced during fabrication was assumed to follow a lognormal distribution rather than an exponential. The former seemed more reasonable because the long 'tail' on the lognormal would be expected to be more representative of actual conditions [38]. The probability of nondetection should be nearly unity for small defects and asymptotically approach zero with increasing crack length. Three candidate functions were considered: lognormal, Weibull, and gamma. Probability of Nondetection curves, using data as reported by [94] are shown in Figure 2.8 along with least squares curve fits for the three equations. Although all three fit the data adequately, the lognormal results appeared to be the most reasonable and were used in the rest of the analysis in Harris' paper.

2.4 Nondestructive Inspection in Relation to Damage Tolerance

2.4.1 Background

The Metals Behaviour Branch of the Air Force Materials Laboratory (now AFWAL MLLN) has been conducting in-house research and development activities in damage tolerance, sometimes referred to as Retirement For Cause (RFC), since 1972. Pratt and Whitney Aircraft Group began extensive research and development programs under corporate, IR & D, and Government contract sponsorship in 1972 to identify and develop the applied fracture mechanics and NDI technologies necessary to realize the RFC concept [20]. Cargill also states that they were convinced that while the life analysis technologies for RFC were obtainable, implementation of an RFC maintenance concept hinged upon the ability to conduct NDI of candidate components.

This paper [20] presents a roadmap composed of feasibility, design, manufacturing and implementation programs, where Government, industry and research institutions all take part. In particular the RFC activity is being applied to gas turbine engines: the F100, TF30, J79 and TF41. The F100 engine being first in 1985.

It was the introduction of linear elastic fracture mechanics in engineering design that has forced nondestructive inspection technology development during the past 15 years. Introduction of linear elastic fracture mechanics caused refocus of nondestructive inspection practices from the traditional "*How small a flaw can be detected?*" to "*How large a flaw might be missed?*" W H Lewis identified a need for a guidelines document to provide a common baseline for demonstrating flaw detection capabilities in aircraft production in 1973 [86]. Finally in February 1976

the third draft of 'Recommended Practice for a Demonstration of Nondestructive Evaluation Reliability on Aircraft Production Parts' was published. Rummel states that the published results constitute a consensus of some of the most distinguished members of the aerospace community.

The purpose of the document was to demonstrate the capability of various NDI methods by defining the limiting flaw size which could be detected with a given probability of detection and with a given percent confidence in that probability. The intent was not to address the subject of NDI flaw size resolution.

Meanwhile in Britain a similar move towards the implementation of a damage tolerant design philosophy was taking place. Stone [92] says that while a great deal of effort was expended in the past by the air authorities and until recently aircraft were designed on a safe-life basis, nevertheless, despite every care, it was inevitable that defects would from time to time be found to be present in service. Their primary objective was to detect the defect although an ability to size it or characterize it in some other way was seen to be clearly useful. Demands for more data on detectability limits became more apparent when a new set of UK Fatigue and Damage Tolerance Requirements were being prepared [92].

It became increasingly obvious to all concerned with damage tolerant maintenance and design that the data to characterize NDI systems was simply not available and would have to be generated. Programs that collected this data are discussed in a following section.

The most definitive work to date is probably the USAF Damage Tolerant Design Handbook: Guidelines for the Analysis and Design of Damage Tolerant Aircraft Structures [32] put out by AFWAL in May 1984 covering a final report period of three and a half years. This handbook supports the USAF Airplane Damage Tolerance Requirements contained in MIL-A-83444 [32] and provides specific background data and justification for the detailed requirements of the specification. Although in general for metallic aircraft structures, the handbook was found to be useful when using the MIL-STD-1783 [64] which deals specifically with damage tolerance in the area of aircraft engine components.

Design Approaches for NDI Methodologies

The main purpose of a nondestructive inspection methodology is to make an accept-reject decision concerning a component on the basis of the results of a suitable set of nondestructive measurements. The ideal objective is to reject all parts that will fail in service and accept all parts that will survive in service (ie. perform satisfactorily according to some appropriate definition). This ideal objective is unattainable because of various kinds of randomness entering into the

overall NDI process. Clearly, in the real world situation we are forced to consider a less demanding objective, eg., to reject most of the components that are going to fail in service and to accept most of the components that are going to survive in service.

To provide some historical perspective, Richardson and Buckley [83] define, three phases in the development of NDI methodologies, namely, zero defects, deterministic, and probabilistic.

(a) Zero Defects:

Here, the ideal concept states that if any defect is present the part will be rejected. However, in practice this means that if any detectable defect is present, the part will be rejected and conversely if no detectable defect is present the part will be accepted. The difficulty with this approach is that as the techniques of NDI measurement evolve, ever smaller defects become detectable. While this leads to smaller false-acceptance rates, it also leads eventually to uneconomically high false-rejection rates.

It is clear that the determination of the optimal decision procedure depends on some knowledge of the failure process, a kind of knowledge that is ignored in this approach.

(b) Deterministic:

In this approach, an effort was made to rectify some of the deficiencies in the zero defect approach by the introduction of a deterministic model of failure which, in the case of crack defects, involves the use of fracture mechanics with a worst-case model of the stress environments. In ideal terms, the procedure here is to determine the size and shape of the crack from NDI measurements and determine whether or not such a crack would propagate to failure in the worst-case stress environments.

Thus untenably high false-rejection rates are avoided when increasingly sensitive NDI measurement systems are employed. However to avoid a significant probability of failure due to statistical fluctuation, even when the deterministic theory predicts no failure, it is necessary to introduce some kind of safety factor. A difficulty with this concept is that in the absence of probabilistic models, there is no rational procedure for determining what the safety factor should be except perhaps through the analysis of masses of field data.

(c) Probabilistic:

The probabilistic approach, the third phase in the evolution of NDI methodologies, attempts to derive an optimal accept-reject decision procedure based on several probabilistic models. More precisely these models represent nondestructive inspection measurements and failure processes as well as the frequency of occurrence of various defects.

Richardson and Buckley [83] suggest that the probability of failure can be found by inputting three probabilistic models:

1. A nondestructive measurement model yielding the probability of detection given the state of the dominant defect.
2. A failure model yielding the probability of failure given the dominant defect.
3. A statistical model characterizing the distribution of the dominant defect.

The probability of failure can be shown as two distributions, firstly the population that will survive, and secondly, the population that will fail. Thus a decision point can be chosen based on the degree of safety required.

Comprehensive Experimentally Based NDI Programs

In the 1970's most agreed that nondestructive inspection was a working arm for damage tolerance methodologies. Unfortunately very little was known about NDI capabilities and in particular state-of-the-art NDI capabilities involving a random selection of personnel, procedures and equipment. Thus many countries initiated programs to gather information and evaluate NDI capabilities in order to implement a damage tolerance design philosophy.

In the United Kingdom, initial attempts merely to collect suitable specimens were not very successful and it was concluded that the chances of mounting anything more than a very modest exercise were not very great [92]. However, Stone notes that the growing implementation of a damage tolerant design philosophy, and in particular the drafting of a new UK Fatigue and Damage Tolerance Requirements for Military Aircraft, would inevitably make these demands more severe. Thus some sort of reliability programme had to be carried out but in the short term it would only be practical to produce idealized 'base-line' data using laboratory data.

(a) NASA Program

In October 1976, NASA published a final report assessing the reliability of NDI data (Yee et al.) [106]. A comprehensive computer program

was written to calculate the probability of flaw detection at several confidence levels by binomial distribution.

The authors noted that there were four possible outcomes in an NDI investigation (discussed later under NDI reliability) but that developing a straightforward statistical method for handling the quadrinomial event was beyond the scope of the report. Detection and nondetection of known defects were found to be best described by a binomial distribution as opposed to a Chi-square, Normal, or Poisson distribution.

There are several methods to divide data for the binomial method but the confidence level is particularly sensitive to the amount of data in each. Thus, in order to obtain a more consistent confidence level, a new method called the Optimized Probability Method (OPM) was developed (explained in the section on POD curve analysis).

Two of the reports conclusions are as follows:

- Binomial statistics are recommended for analyzing NDI reliability data.
- The OPM method of data accumulation is preferred over the range interval or the overlapping sixty point scheme for computing POD.

(b) USAF "*Have Cracks*" Program

The second comprehensive experimentally based NDI program, final report 1978, was undertaken by USAF at an estimated cost of about \$1 million (Lewis et al.) [60]. The purpose was to investigate the reliability of NDI, and because testing was done all over the place, the program was commonly referred to as "Have Cracks Will Travel".

It was shown that with the present state-of-the-art NDI capabilities, and of course this is constantly changing, that the requirements placed on inspection in MIL-A-83444 were quite unrealistic.

The program used a range of real structural components containing fatigue cracks at features such as fastener holes and great care was taken in establishing confidence levels. Most of the major NDI techniques were employed, except magnetic techniques since no steel specimens were used. The following is an extract from the General Conclusions [60] page 12-2:

"Of foremost importance is the realization that the previously established 90-95 percent reliability criteria (90 percent probability of detection with a 95 percent confidence bound) cannot

be obtained with normal inspection methods. With the exception of fluorescent penetrant inspection, the NDI techniques employed in the program demonstrated considerable difficulty achieving a 50 percent probability of detection for a 1/2-inch crack size with a 95 percent confidence level. It should be pointed out, however, that all conclusions contained in this report are based on the data obtained during the NDI Reliability program which used the NDI techniques and equipment currently available at the Air Force installations visited.

The state-of-art is constantly changing in NDI technology. Inspection reliability improvements resulting from recent refinements in NDI methods, presently undergoing service evaluation were not included in this study. There was a limited opportunity to evaluate the effectiveness of semi-automatic eddy current and ultrasonic equipment, beginning at the thirteenth base visit. Unfortunately, due to the late incorporation of semi-automated equipment into the program, the data quantity restricted determination of a reasonable confidence level for this equipment, but the mean crack detection levels obtained from using this type of equipment indicate that 90-95 percent reliability criteria may be possible at crack sizes considerable smaller than 1/2 inch."

It was also noted that a distinctly higher level of success in flaw detection was achieved at one particular depot and this was attributed solely to individual proficiency.

Thus, although current NDI could not support the damage tolerant design philosophy, in the long run with semi-automated equipment, experienced personnel, and computers to capture and present data, the capability of NDI would be sufficient for damage tolerant design and maintenance.

2.4.2 NDI Methods Available for Detecting Fatigue Cracks

An NDI method should be chosen in a manner such that it is well suited to detect the type of flaw that is expected to be present in the component. This section briefly describes the most commonly used NDI techniques [32], their theory of operation, the types of flaw that can be detected, their advantages, and their limitations.

NDI applicable to engine disc bolt holes is specifically mentioned since low cycle fatigue (LCF) cracks generally initiate from the bolt holes in gas turbine engine discs. The most accurate NDI method for detecting these cracks is said to be the eddy current method although ultrasonics and the electric current perturbation methods have also been investigated. Many of the studies were carried out on retired engine discs.

Liquid Penetrant Inspection

LPI is a nondestructive method for finding discontinuities that are open to the surface of parts fabricated from essentially nonporous materials. When the penetrant is applied it seeps or is drawn into various minute surface openings. After the excess amount of penetrant is removed, a developer can be applied to highlight the flaws under ultraviolet light. Indications of flaws can be found regardless of size, configuration, internal structure, or chemical composition of the workpiece and regardless of the orientation of the flaw with respect to the workpiece.

LPI is a relatively simple and inexpensive NDI method that can be applied to a broad range of materials. However, only surface flaws can be detected using this method and surface coatings and surface roughness can adversely affect the results.

Ultrasonic Inspection

Ultrasonic inspection uses high frequency sound waves as a probing medium to detect subsurface as well as surface flaws. The sound waves travel through the part with attendant energy loss and are reflected at material-flaw interfaces. Ultrasonic inspection devices detect flaws by monitoring one or more of the following:

- (a) Reflection of energy from interfaces or discontinuities with the metal;
- (b) Time of transit of a sound wave through the test piece;
- (c) Attenuation of the beams by absorption and scattering within the test piece.

It is one of the most widely used NDI methods for detecting cracks, laminations, shrinkage cavities, bursts, flakes, pores, bonding faults, inclusions, bond characteristics and extent of corrosion in a wide variety of components. The penetrating power of ultrasonic waves allows the detection of flaws deep within a part.

The advantages of the ultrasonic method are that it provides greater accuracy than other NDI methods in determining the position of internal flaws, estimating

their size, and characterizing their orientation, shape and nature. In addition, very small flaws can be detected using this technique. One of the major constraints of the technique is that if the gain is set too high, it is at the expense of many false indications. Other limitations include the requirement for experienced technicians, the difficulty in developing inspection procedures, the need for reference standards for equipment calibration, and the physical limitation of the hardware. In particular, the couplants are required to provide effective transfer of ultrasonic wave energy between transducers and the material. Lastly, inspection results of very thin components can be questionable.

In order to pursue RFC in aircraft gas turbine engine discs, cracks having lengths of 0.127 mm to 0.254 mm must be reliably detected. A hybrid technique consisting of an initial optical inspection for screening (since most bolt holes contain no fatigue cracks) followed by an ultrasonic inspection for quantitative crack parameters has been proposed [3]. The system is currently in the detailed design stage of development.

The engine disc will be loaded onto a rotary table and then moved into position so that the optical scanning probe can be inserted into a bolt hole to locate the flaw. The conceptual design is based on the requirements for the engine discs of the F100 engine which has seven different diameters and depths for fan discs, compressor disc, and turbine discs. However, visibility may be affected by several parameters including the surface finish of the bolt holes, the compressive stress on the crack, and the type of illumination employed. After the locations of all crack-like flaws have been determined using the optical detection system, a detailed examination of each location technique will provide the overall length of the crack including any subsurface details. It will also provide the depth of the crack as a function of position along the length.

The resolution of the image produced by this technique is chiefly determined by the focal spot size of the ultrasonic beam. At a frequency of 10MHz the wavelength in water is 150 micrometers. The length of the crack can be measured to this accuracy or slightly better. If it proves necessary, the ultrasonic frequency can be raised to provide better accuracy. To obtain accurate results for the depth off the crack it is anticipated that some calibration techniques will have to be employed using cracks of known depth.

A second NDI technique for ultrasonic inspection of bolt holes is the Born Inversion Procedure (BIP). While the scattered elastic waves from a flaw inspected by standard ultrasonics contains a great deal of information concerning the size, shape, orientation and composition of the flaw, the mathematical nature of the inversion problem makes this technique one of the most difficult to quantify. BIP

seeks to infer the flaw size from an ultrasonic flaw reflection based on the Born scattering approximation [21]. The technique is difficult to automate but work is continuing to eliminate the limitations. The technique was applied to a number of spherical internal flaw types ranging from voids to inclusions with sizes from 350 to 1200 microns in diameter. The procedure can estimate flaw size within roughly 50% which is probably the best obtainable given the limited transducer bandwidths and the single physical model required by an automated technique.

Eddy Current Inspection (ECI)

The principles of electromagnetic induction are used in eddy current inspections to detect surface and near surface flaws in electrically conductive materials. When an electrically conductive material is subjected to an alternating magnetic field, small circulating electric currents are generated in the material. In practice, eddy currents are induced in the part to be inspected with a coil carrying an alternating current. The induced eddy currents generate their own magnetic field which interacts with the magnetic field of the exciting coil and changes the impedance of the exciting coil. By measuring the impedance of the exciting coil, or a separate indicating coil, the inspector can infer the presence of flaws in the material.

An important use of the eddy current method has been in the detection of fatigue cracks around fastener holes after the cracks have grown beyond the fastener head. Special bolt hole probes have also been devised for use after the fastener has been removed for locating cracks emanating from the wall of the fastener hole. This inspection procedure has been automated to remove operator influence, speed inspections, and produce a permanent inspection record.

Advantages are that contact with the specimen is not needed, and therefore specimen clean up after the inspection is not required. It is generally faster than LPI or radiography. Eddy current methods can detect both surface and subsurface flaws, but the depth below the material surface is limited to approximately 6mm. Another disadvantage is that eddy currents are influenced by many material variables and false indications can easily be caused by sensitivity to part geometry, edge effects and permeability variations. Lastly, EC requires well trained operators and reference standards. The US Air Force Materials Laboratory investigated the viability of quantifying NDI results based on the relationships between eddy current signals and flaw dimensions. The work was carried out from a retirement-for-cause viewpoint and turbine discs with radially cracked bolt holes were inspected by a semiautomatic eddy current technique to illustrate the procedure [45]. As an initial effort, electron discharge machined (EDM) flaws were used as a model for fatigue cracks since the actual fatigue cracks in components were found to have

initiated at many sites and in different planes. The EDM notches were machined in two basic shapes, rectangular and semielliptical, with aspect ratios varying from 0.28 to 7.16 mm in depth, 0.76 to 19.18 mm in length and 0.13 to .18 mm in width. The findings of the program demonstrated that relatively well behaved functional relationships exist between notches and various characteristics of the related eddy current signal, and it was reasoned that these relationships would hold true for actual fatigue cracks as well. This technique successfully detected fatigue cracks of 0.76 by 1.02 mm and EDM notches of 0.25 by 0.38 mm. The crack size, the type of eddy current system, and the sensitivity level of the system were all observed to affect the eddy current/flaw dimension relationships. In general the signal characteristic increased monotonically (primarily logarithmic in nature) with an increase in notch size. Failure Analysis Associates (FAA) in conjunction with the US Air Force Materials Laboratory (AFWAL/MLLP) conducted a comprehensive ECI program on 49 randomly selected retired TF-33 third stage turbine discs where each disc had ten bolt holes and each bolt hole was inspected by five independent inspections using both conventional and high-resolution eddy current NDI equipment [7]. The major objective of this study was to estimate the state-of-the-art capability of nondestructively detecting and evaluating fatigue damage in turbine discs.

After the eddy current inspections were completed, the bolt holes were optically inspected and measured. Then they were electropolished and two plastic replicas were made of each bolt hole and the surface crack lengths were measured using an optical microscope. Finally a number of bolt holes were sectioned to obtain the specific crack depth. Results of 725 destructive sectioning planes were documented for 56 surface crack indications. Only three inspection results were used in the investigation because the other inspection data could not distinguish between radially inward (RI) and radially outward (RO) cracks within the bolt holes. The results were analyzed in two ways. The first approach, also called the conventional approach, used multiple cracks and multiple eddy current signals, whereas only the maximum replica crack length and the maximum eddy current signal for each half bolt hole was used in the second approach.

In the case of the conventional approach, the eddy current information was converted from the number of consecutive probe turns to crack lengths by knowing the approximate spot size of the particular probe. For conventional detection/nondetection counting analysis in this case, the eddy current signal was counted as a crack indication if it exceeded a predetermined signal value and was not counted if it was less than the predetermined value. This counting analysis was used to generate the reliability of the 'indication'. Reliability was analyzed

in terms of crack indication, location, length and position agreement. Obviously, eddy current reliability decreases if the inspector specifies the location, length, and position of the crack within the bolt hole. For example if just the location is required, the high resolution inspection reliability decreases from 82% to 71% for cracks between 1.3 and 2.5 mm long.

The second method of analysis accounted for the uncertainty in the inspection signal by establishing the distribution of maximum eddy current crack length for the maximum real crack length in a half bolt hole volume. Nonlinear regression analysis was used to correlate the maximum real crack length to both the number of probe turns and voltage amplitude for the maximum eddy current signal recorded within the corresponding bolt hole. For the high resolution results and for cracks less than 2.5 mm, there was a strong correlation between crack length and voltage amplitude. Several trends could be seen for the high resolution inspection uncertainty which was similar for the other two inspections. The probability of detection increased as the real crack size increased, and the distribution of apparent to actual crack sizes was tighter as the cracks became larger. Furthermore, for small crack lengths (0.025 to 0.25 mm), the eddy current signal tended to oversize the real crack length more so than for larger real crack lengths on a percentage basis.

In conclusion, the inspection reliability of three eddy current inspections was found to be adequate for an RFC procedure whose critical crack size is similar to the bolt holes in the TF-33 third stage disc. The actual pre-inspection material quality defect distribution was determined from the results of the destructive section. There were fewer uncracked RI bolt holes than uncracked RO bolt holes. 90% of RI cracks are less than 10.2 mm while 90% of RO cracks are less than .5 mm. The apparent crack distributions showed that the high resolution inspection identified a smaller percentage of the bolt holes uncracked than did the outside laboratory inspections. The high resolution inspection and the outside laboratory inspections identified 48% and 63% of the bolt holes being uncracked respectively for the RI location and 86% and 95% respectively for the RO location. A third study published recently [72] deals with eddy current inspection of sleeved bolt holes. In engine disc overhaul, a cracked region in a bolt hole may be repaired by drilling out the flawed region, leaving an oversized bolt hole. These oversized bolt holes are further repaired by press fitting a sleeve to proper size. Since the sleeve is not in metallurgical contact with the engine disc, it is not subject to the stress loading of the disc. Consequently, new in-service crack formation and growth occur in the disc region adjacent to the sleeve and not in the sleeve itself.

* In this study, the inspection of the subsurface region by eddy current techniques

was complicated by two major factors. First, the flaw size of interest (0.38 to 0.76 mm) was smaller than, or comparable to, the sleeve wall thickness. This factor tended to obscure the crack details. Secondly, the electrical conductivity of the stainless steel sleeve material was higher than the disc material. Thus the sleeve shielded the electromagnetic fields from the region of interest unless sufficiently low frequencies were employed. However, low frequencies imply longer skin depths and lower sensitivity than the optimum frequencies used for surface crack detection.

After an analysis of the sleeved bolt hole problem, theoretical results suggest that a sensitivity of 1×10^{-5} would be required in the instrumentation for the reliable detection of subsurface cracks in the present geometry. The sensitivity of the state-of-the-art instruments is only about 1×10^{-3} . Therefore effort is underway towards developing an improved eddy current system that would provide the required sensitivity for the sleeved bolt hole inspection.

Recently, Chimenti and Moran (1985) analyzed eddy current data from a standard test using an inversion theory. The results using machined slots and a conventional air-core eddy current coil were quite good. Real crack lengths versus derived crack lengths from the eddy current signals were in good agreement in the size range of 0.5 to 2 mm.

Magnetic Particle Inspection

Magnetic particle inspection is effective in the detection of surface and near surface flaws in ferromagnetic parts. The inspection is accomplished by inducing a magnetic field in the part and applying either a dry magnetic powder or a liquid suspension of iron particles on the surface being inspected. Defects in the part cause local bipolar perturbations in the magnetic field which attract the magnetic particles, producing visible indications by colour contrast or by fluorescence under 'black light'. The magnetically held particles form the outline of the discontinuity and generally indicate its location, size, shape, and extent to an experienced inspector.

The main advantages of MPI are that it is relatively fast and inexpensive, and elaborate precleaning procedures are not necessary. The disadvantages are that this method can only be used on ferromagnetic materials which include most of the iron, nickel and cobalt alloys. The material must often be de-magnetized and post-cleaned to remove the clinging magnetic particles, and thin coatings of paint such as plating adversely affect the sensitivity of this inspection technique. Non-ferromagnetic materials which cannot be inspected by this method include aluminium, magnesium, copper, and titanium alloys and austenitic stainless steels.

Radiographic Inspection

Radiographic NDI is based on the differential absorption of penetrating radiation by the structure being inspected. In conventional radiography, the component is bombarded by a beam of X-rays and the portion of the radiation that is not absorbed by the component impinges on a sheet of film. The unabsorbed radiation exposes the film emulsion similar to the way light exposes film in photography. Development of the film produces an image that is a two-dimensional 'shadow picture' of the entire volume of the component. Variations in density, thickness, and composition of the component can cause variations in the intensity of the unabsorbed radiation and appear as variations in shades of gray in the developed film.

The main advantage of radiographic inspection is that it has the capability to reveal the internal defects in materials and components, for example structural weaknesses, assembly errors, or voids and other material anomalies. The main disadvantages of this technique are that it is expensive, slow, and strict controls are required, due to the hazards of exposure to X-rays, to prevent biological damage to the inspectors. Also radiography is not sensitive to detecting certain types of flaws, for example cracks which are not parallel to the radiation beam and minute discontinuities such as inclusions in material.

Electric Current Perturbation (ECP)

ECP is an electromagnetic method currently under development primarily for increased crack detection reliability, but also partially for quantitative NDI. The basis of the method is to induce a relatively uniform electric current sheet in the surface of the metal with a large coil and detect changes in normally small orthogonal components of the magnetic field with a small, properly orientated detection coil [21].

The capability of the ECP technique was evaluated for detecting and characterizing low cycle fatigue cracks in tiebolt holes of TF-33 third stage turbine discs [19]. Examinations were performed on one hundred tiebolt holes in ten scrap discs. The ECP probe utilized a noncontacting induction method for establishing current flow in the tiebolt hole. The scanning system incorporated a motor for driving the probe in a circumferential direction and a manual positioning arrangement to allow precise axial probe positioning. ECP records were acquired using a clockwise circumferential scan procedure in which the first scan was located at a distance of .64 mm from the top (serial side) surface of the disc hole and successive scans were located at .64 mm increments for a total of 32 scans per hole. Data

was recorded on a 2-channel strip chart recorder. Two data imaging approaches, the A-scan and the C-scan, were evaluated since they are more useful and easier to interpret than strip chart recordings. Preliminary results showed that there were correlations to relate signal characteristics from unknown cracks to those of known cracks.

The ECP method is capable of detecting fatigue cracks measuring 0.3 mm long by 0.15 mm deep. The minimum detectable crack size based on extrapolation and with the present probe would be 0.25 mm by 0.08 mm. Both experimental data and theoretical data show that good potential exists to differentiate between small, closely spaced cracks and single cracks. In a more recent report [21], using a breadboard probe and scanner, cracks as small as 250 micron in surface length were detected in a complex geometry of a turbine disc.

Comparisons have also been made between simple features of the ECP signals from an actual fatigue crack and those generated by an analytical model. Theory predicts an approximately linear relationship between signal amplitude and interfacial area of the crack. Experimental results show very close agreement with theory, thus indicating that crack interfacial area can be estimated by measurement of the ECP signal amplitude [21].

Other Advanced NDI Methods

(a) Computed Tomography (CT)

More broadly classified as radiography, the instrument consists of a high energy source which emits a thin, fan-shaped beam which passes through a selected cross-section of the component being inspected. Then an x-ray detector array measures the flux of x-ray photons passing through given line elements in the component. Mechanical rotation and translation stages move the component through the required angles for the acquisition of an adequate amount of data for a tomographic reconstruction. In the engine area, this method is currently being used to inspect blades and in particular cooling holes [21].

(b) Thermal Wave Imaging Techniques

The image is achieved by measurement, by one of several means, of the surface thermal properties of a component. A medium power modulated laser causes periodic heating of a small region some 20 micron in diameter at the surface being tested. The heat diffuses into the specimen interior, and the periodic heating of the surface results in a time dependent temperature fluctuation of the small volume of air directly

above the heated zone. A second low power laser passes through the tiny volume of heated air and is optically deflected by an amount depending on the air's temperature. The amplitude of that temperature excursion and its phase relative to the heating pulse will be functions of the thermal properties of the surface: heat conductivity and heat capacity. A crack like defect introduces a spatially dependent conductivity [21].

The utility of thermal methods for discrete defects is now well established, and fatigue cracks in bolt holes have been imaged thermally. However, because of the amount of time this technique currently requires to produce such a high resolution image of a crack, its major application, it is felt, will be as a tool for quantification of superficial defects once they have been detected by other means.

(c) Acoustic Microscope

A radio frequency tone burst is sent to a transducer, the resulting ultrasound emerging through a highly convergent acoustic lens into a couplant fluid. By arranging for the acoustic focal point to coincide with the surface to be examined, a microscopic image of near subsurface detail can be obtained. The microscope will generate leaky surface acoustic waves whose penetration allows imaging of subsurface defects. As the surface waves intercept a defect and reradiate into the fluid, returning, to the transducer, the perturbed sound wave appears as a defect signal in the device's electronics. Scanning the lens over the surface of interest yields a two-dimensional image. It is mentioned that this technique is complementary to a conventional C-scan [21].

Reliable detection of 0.4 mm planar flaws are clearly visible. For engine materials, where the critical flaw size can be very small, this NDI technique of leaky surface waves offers high sensitivity and an image useful for NDI.

2.5 Reliability of NDI Techniques

There are two steps involved in assessing the reliability of an NDI technique. Firstly the NDI technique either detects or misses a crack, and secondly, a crack length value is obtained from the NDI reading which will be an approximation of the real crack length. The first step can have four possible outcomes:

- i. Crack exists and NDI technique detects a crack (*Correct*).

- ii. Crack exists and NDI technique misses the crack (*This requires POD analysis*).
- iii. No crack exists and no detection by NDI (*Correct*).
- iv. No crack exists but NDI detects a crack (*False Call*).

The outcome of Numbers i and iii are correct, but numbers ii and iv introduce uncertainties in the NDI results. Number ii has the most serious consequences and the problem is handled graphically and statistically by plotting "Probability of Detection (POD) vs Crack Length" graphs where POD is defined as *the proportion of cracks that will be detected by the NDI system*. Unfortunately the same crack length will not always have exactly the same POD value and in order to quantify the statistical distribution, a confidence bound, normally 95% is shown on the POD curve as well [64].

In instances where NDI detects a crack which does not exist (number iv above), safety is not compromised, although a damage tolerance maintenance plan may be jeopardized due to the uneconomical replacement of components which do not actually need to be replaced. Because there is no crack-length associated with a nonexistent crack, a distribution curve cannot be plotted and therefore a number or a percentage is used to specify the number of false calls ie. number of nonexistent cracks detected by NDI [76].

2.5.1 Estimation of POD Data

(a) NDI Capability at One Crack Length

This category of experiments can be viewed as a method for demonstrating that an NDI system can detect at least a given percentage of cracks of a specified length with a specified confidence limit. For example this approach could be used to show that there is 95% confidence that at least 90% of all cracks of length x mm will be found. These experiments have limited use as information is provided only at the crack length inspected.

(b) Estimation of POD with One Observation per Crack

The majority of experiments fall in this category. Many components covering a range of crack lengths are inspected once and the results are used to estimate the POD as a function of crack length with confidence bounds. Since there are generally not a sufficient number of cracks of the same length, the crack lengths are grouped into intervals of crack

length. It is assumed that all cracks within a specified interval have approximately the same POD.

(c) Estimation of the POD with Multiple Observations per Crack

This category of experiment resulted from a large NDI reliability program performed by USAF, the "Have Cracks Will Travel" program [60], where sections of aircraft were transported to depots and inspected for cracks by personnel using various NDI systems. This method of collecting data yielded an estimate of a POD for each individual crack and demonstrated clearly that not all cracks of the same length had the same POD.

2.5.2 POD Data Analysis

In class (a) of the previous section, the binomial method (discussed later) is used to specify the confidence bounds. Traditionally the binomial distribution has been used to specify the confidence bounds for POD values for class (b) data, since this distribution has been found to have the optimum fit compared to other curves such as the Chi-square, Normal, and Poisson distributions [106]. The problem with the binomial distribution is that the confidence bounds are significantly influenced by the method for assigning cracks to an interval [32]. In addition, the "Have Cracks" program [106] of class (c) data demonstrated that cracks of the same length could have different POD values and therefore a binomial distribution was not suitable for representing the POD values. Berens and Hovey [14] investigated statistical methods for estimating crack detection probabilities, where the POD curve was the mean of the detection probabilities. They concluded that the loglogistics curve, out of six other distributions tried, most adequately fit the data [15].

Binomial Curve and Confidence Bounds

The probability, P , of r detections is given by:

$$P(r) = \frac{n!}{(n-r)!r!} p^r (1-p)^{n-r} \quad (2.20)$$

where n = total no of cracks of length a , or the number
in the crack length interval with mean crack length a

r = no of cracks detected

P = probability of detection

The maximum unbiased likelihood estimate of P is:

$$\hat{P} = \frac{r}{n} \quad (2.21)$$

However, it is unlikely that P will equal $\hat{P}$ so that a lower confidence interval estimate is calculated which yields a range of P values which are consistent with the observed number of detections. For a given POD and confidence required, an equation can be solved for the minimum number of detections required for a total number of cracks. Solutions are given in statistics books [67], or relevant papers [32]. The solution for the 90% POD/95% Confidence and the 90% POD/50% Confidence is shown in Table 2.1.

In a class (b) experiment, data is grouped into crack lengths so that there is sufficient POD data for analysis. There are several methods by which crack length intervals can be formed. These include:

- i. Range Interval Method: The crack length intervals are defined with equal lengths across the range of data. Although the method appears very logical, the extremely erratic behaviour of the confidence bounds can result from the small sample size in some of the intervals (Figure 2.9 (a)).
- ii. Constant Sample Size: In order to avoid the problems from variable sample size, this method varies the length of the interval to maintain constant sample size. The two types, non-overlapping and overlapping are self-explanatory.
- iii. Optimized Probability Method (OPM): This method was developed by Yee et al [106] when analyzing the NASA results in 1976. In essence the sample size is increased by adding cumulatively across the intervals from smallest to largest or vice-versa. The confidence bound is better behaved [70] (See Figure 2.9 (b)) but at the expense of unknown statistical validity [13].

Confidence on the binomial distribution is most easily obtained using graphs found in many statistic books [67] [8] [36] [37]. For more accurate results, Rumel [86] presents concise equations and calculations can be carried out straightforwardly using a table of F values. An overview, particularly concerned with the statistical treatment of inspection reliability, can be found elsewhere [71].

Loglogistic Curve and Confidence Bounds

It has been demonstrated that cracks of the same length do not necessarily have the same POD [60] (See Figure 2.10) and therefore the binomial distribution is

not always suitable for specifying the POD values. From the literature it can be concluded that a loglogistics curve was the most suitable distribution [14] [32]. Figure 2.9 (c) shows the results from eddy current inspections of fastener holes with 60 inspections per crack.

The functional form of the loglogistics curve is as follows:

$$POD(a) = \frac{\exp(\beta_0 + \beta_1 \ln a)}{1 + \exp(\beta_0 + \beta_1 \ln a)} \quad (2.22)$$

where POD = Probability of Detection (P_i)

a = crack length (a_i)

β_0 = location parameter

β_1 = slope parameter

For linear regression analysis and given n pairs of (a_i, P_i) data, the loglogistics curve is rearranged to the form:

$$Y_i = \alpha + \beta X_i \quad (2.23)$$

$$\text{where } Y_i = \ln \left(\frac{P_i}{1 - P_i} \right)$$

$$X_i = \ln a_i$$

and the two parameters α and β are found as the constant and slope respectively from the linear regression. When no cracks or all cracks are detected P_i can be defined as $\frac{1}{n+1}$ and $\frac{n}{n+1}$ respectively.

The loglogistic curve represents the mean POD for a crack length. Standard statistical regression methods can be used to find confidence bounds on the linear regression line and transformed back to the POD curve. Figure 2.9 (c) illustrates a loglogistic curve with a 95% lower confidence bound fitted. Berens and Hovey [13] note that many points lie below the 95% confidence bound. They consider this discrepancy to be reasonable since the confidence bound is on the average detection probability at each crack length and not on the population of all detection probabilities at a crack length.

2.5.3 Pertinent POD Results from the Literature and the Influence of The Human Factor

Rolls Royce [95] suggest that radiography was the NDI method with the worst record of 'defect misses' in the "Have Cracks" program simply because the 'evidence' was available after the event. It is shown that reliability increases as the

number of independent inspections increase (Figure 2.11). In order to eliminate or at least minimize the problem, Rolls Royce has embarked on a "No Eyes" NDI programme. In particular, improvements have been made in the area of ultrasonic inspection of discs. Also improved reliability in the automation of eddy current inspection has been accomplished by Reluxtrol by using a computer program and better probes on fan discs [95].

Preliminary NDI results for general aero engine parts for the ENSIP programme are summarized in Figure 2.12. The results were obtained from American military engines. NDI requirements are implemented on fracture critical parts during manufacture and field/depot inspection to protect safety. A flaw size assumption of less than 0.76 mm (0.030 inches) surface length requires implementation of enhanced NDI ie. eddy current. The ability of ECI to reliably detect surface flaws down to a 0.13 mm (0.005 inch) depth has been demonstrated in several applications (Figure 2.13) [53]. In general FPI is specified for areas requiring 0.76 mm (0.03 inch) surface length or larger. The ability of current FPI processes in common use today to reliably detect 0.8 mm (0.03 inch) flaws is not clear. It is clear however that FPI can be significantly improved depending on personnel training, equipment, and procedures (See Figure 2.14). The aim within the manufacturer and the user communities is to implement the best FPI process because FPI will remain the most widely used method to inspect large proportions of crack like damage [53].

There are many POD curves and results of 90% POD/95% CL for various applications. One of the most complete (and general) lists of 90/95 limits are contained in the USAF Damage Tolerance Handbook [32] which among other results, summarizes the results of the NASA program [106]. POD's for bolt hole cracks are also given specifically.

A FAA report concerning eddy current inspection of 49 retired TF-33 third stage turbine discs did not present the information in conventional POD curves [7]. However, it is mentioned that the probability of nondetection is 50% for a crack range 1.0 to 1.5 mm and 10% for a crack range of 2.5 to 5.0 mm. The results of a preliminary study on J85 CAN40 fifth and seventh stage compressor discs are shown in Figure 2.15. The 95% confidence bound was obtained using the binomial overlapping interval method [79]. The actual data showed that 3 out of 6 cracks were detected at crack lengths 0.5 to 1.0 mm and that 7 out of 7 cracks were detected in the 2.0 to 2.5 mm crack interval. The conclusions of this preliminary investigation were that the LPI employed by AMDU was very successful in detecting fatigue cracks greater than 1.0 mm, but more data was needed for detailed analysis. In addition, the POD curve represented a tightly

controlled exercise in crack detection and as such it was not truly representative of the sensitivity of the NDI technique used.

2.6 Crack Growth Rate Variability

Crack growth rate scatter can be taken into account either by using conservative worst case (deterministic) crack growth equations or by characterizing the scatter using probability density functions to describe parameters in a crack growth rate equation.

Many deterministic crack growth rate models have been postulated. Hoepfner and Krupp [42] wrote a detailed discussion of the development of fatigue crack growth equations in 1974 and listed 33 relationships! They take the form of three general relationships where crack growth rate as a function of the stress intensity is the most preferred relationship. Schütz [89] also summarizes several new methods in terms of their accuracy with respect to test programs and concluded that none of the new models had reliably shown convincing improvements compared with Miner's Rule or the Forman model.

More recently complex models have been developed. Larsen and Nicholas [58] attempt to define the key parameters that influence crack growth in turbine components. Figure 2.16 presents a full range of operational activities that may occur in a given mission. These stresses can be broken down and characterized by frequency, stress ratio, temperature, and sustained load hold-time, and an analytical model is used to predict crack growth rate as a function of the primary variables. The general shape of the crack growth rate curve was modelled by the Modified Sigmoidal Equation (developed by General Electric) and also by the hyperbolic sine equation (developed by Pratt and Whitney). With the exception of overload effects, the linear cumulative damage modeling was found to work well, and provided an extremely powerful tool.

Schwartz and Annis [90] comment that understanding the cumulative effects of varying load sequences, such as overloads, is essential in crack growth life analysis. They present an empirically based model which is broken into three parts: segregation, interpolation, and computer algorithm. Firstly, a mission profile, generated from a flight recorder carried in an aircraft (Figure 2.16), is broken down into elemental load events. Then the elemental damage is characterized using a crack growth rate model, in this case the hyperbolic sine, and finally a total mission life time is determined by integrating cycle by cycle. Verification of the model using test specimens demonstrated accurate prediction of fatigue crack growth in IN100 and Waspaloy. Other models have also been mentioned by Hurchalla et al [44],

and Cruse and Meyer [24].

Randomization of crack propagation is one of the the most straightforward ways to take into account the scatter in fatigue crack growth data using probabilistic concepts. This method proposes that the statistical variability in crack growth rate can be accounted for by randomizing the Paris crack growth rate equation as follows:

$$\frac{da}{dN} = XC(\Delta K)^n \quad (2.24)$$

where X is a positive random process taking values around unity. Thus the deterministic fracture mechanics model represents the central tendency of the crack growth behaviour, and the statistical variability is taken care of by the random process X . The variability in terms of standard deviation can be obtained using linear regression analysis on a da/dN vs ΔK graph [101] [102] [103]. Statistical variation in crack growth rate due to material variability, component geometry, and stress intensity factor, etc. are all taken into account by one random variable [100]. A specific example for the third stage turbine disc of a TF-33 jet engine was worked out to demonstrate this analysis [100].

Ichikawa et al [46] considered randomizing both C and/or n in the Paris crack growth equation. They also examined:

- i. The inter-specimen variability where C and/or n are assumed to be constant within a specimen but to have specimen-to-specimen variability. From the experiments, $\log C$ and n were found to be normal distributions.
- ii. The intra-specimen variability which assumes C is a random variable within a specimen, and n is usually deterministic.

They concluded that both methods were acceptable. Akita (1986) [6] also discusses this method. Palmberg [73] based his model upon Yang's et al. but used a general deterministic crack growth rate relation instead of the power law relation.

Besuner and Tetelman [16] use the Paris crack growth equation but input distributions for the random variables C and $\Delta\sigma$. The tool is a computer program that uses Monte Carlo simulation. Rau and Besuner [81] give an example of growth of fatigue cracks in turbine discs. C_p was a parameter (treated as a random variable) which defined the variation in crack growth at a given σ caused by geometric or metallurgical variables. The cumulative probability distribution of C_p was assumed to be lognormal.

Becher and Pedersen [12] of the Reactor Engineering Department in the Danish Atomic Energy Commission considered a variation of crack growth rate for a given ΔK for reactors. The probability density function $f(da/dN)$ was taken to be normal

in the logarithmic scale. Whittaker and Saunders [98] investigated random crack growth rates for aircraft structures. Finally, in a more recent paper, Ghonem and Dore [33] theoretically estimate crack growth rate scatter based on the assumption that the crack propagation process could be formulated in terms of a particular discontinuous Markovian process.

2.7 Conclusions from the Literature Search

It was concluded that the PFM analysis to predict a SII for the life extension of gas turbine discs should include the following variables:

- Initial Flaw Distribution. A TTCI distribution as suggested in the literature, was considered since the exact crack lengths were unknown.
- NDI Inspection Uncertainty. From the literature, the loglogistics curve used by Berens and Hovey seemed to be the best choice for representing the POD data.
- Random Crack Growth Rate. It was found that Yang et al presented the most straightforward method to introduce scatter into crack growth rate equation by randomizing it. This method was particularly useful in this case since the variability to be introduced was already known from experimental laboratory da/dN vs ΔK graphs.

It was also thought that since the NDI experimental work included NDI inspection of bolt holes as well as prying them open, that the variability of the actual crack length compared to the inspected crack length should be included in the analysis.

Output from the computer program should be in the form of a distribution of safe inspection intervals where the SII was calculated as the number of hours required for the crack to grow from an initial crack length to some dysfunction length at which time it should be removed from service. Monte Carlo simulation techniques would be used to represent the statistical randomness of the variables.

Table 2.1: Maximum Number of Cracks that can be Missed given the Total Number of Cracks Inspected for 95% and 50% Confidence. [Rummel]

No Failures	Confidence Level							
	50	60	70	80	90	95	99	99.9
0	7	9	12	16	22	29	51	66
1	17	20	24	29	38	46	72	89
2	27	31	36	42	52	61	90	108
3	37	42	47	54	65	76	106	126
4	47	52	58	66	78	89	122	143
5	57	63	70	78	91	103	137	159
6	67	73	81	90	104	116	152	175
7	77	84	91	101	116	129	167	190
8	87	94	102	113	128	142	181	205
9	97	105	113	124	140	154	195	220
10	107	115	124	135	152	167	209	235
11	117	125	135	146	164	179	222	249
12	127	136	145	157	175	191	236	263
13	137	146	156	169	187	203	249	277
14	147	156	167	180	199	215	262	291
15	157	167	177	191	210	227	275	305
16	167	177	188	202	222	239	288	318
17	177	187	199	213	233	251	301	332
18	187	197	209	224	245	263	314	345
19	197	208	220	234	256	275	327	358
20	207	218	230	245	267	286	340	372

Figure 2.1: Different Defect Distributions for Initial Crack Sizes. [Gallagher et al, 1984]

Figure 2.2: General Shape of Distribution of Initial Crack Sizes. [Gallagher et al, 1984]

(a) Truncated Exponential

(b) Three Parameter Lognormal

(c) Imperfection Size Distribution

Figure 2.3: Sample Initial Flaw Distributions.
[Jouris and Shaeffer, 1980] [Raussi and Tiainen]

Figure 2.4: Definition of Equivalent Initial Flaw Size (EIFS). [Rudd and Gray]

(a) Probability density of occurrence of A-7D EIFS.

(b) Cumulative probability of occurrence of A-7D EIFS.

Figure 2.5: EIFS Distribution for A-7D Fastener Holes. [Rudd and Gray]

(a) Model showing TTCI and EIFS.

(b) Three Parameter Weibull Distribution for TTCI.

(c) Weibull Compatible Distribution for EIFS.

Figure 2.6: Model of TTCI and Back Extrapolation to EIFS. [Yang, Manning, and Garver]

(a) Actual vs Measured Crack Depth

(b) POD Graph

Figure 2.7: Initial Flaw Distribution based on an NDI Database. [Tang]

Figure 2.8: Probability of Nondetection Curves. [Harris, 1977]

(a) Binomial Analysis -
Confidence Bounds
for Range Interval
Method.

(b) Binomial Analysis -
Confidence Bounds
for Optimized
Probability Method.

(c) Loglogistics Curve -
with confidence bound
linear regression.

**Figure 2.9: Probability of Detection Curves and
Different Types of Confidence Bounds. [Yee et
al, 1976] [Berens and Hovey, 1981]**

Figure 2.10: The Variation of the Probability of Detection for a Given Crack Length.

Fluorescent penetrant inspection Lower 95% confidence limit

Figure 2.11: The Human Factor in the Inspection Cycle. [Taylor]

ENSIP NDT

Preliminary conclusions — Aero engine parts

	Average	Best
Magnetic inspection POD	60% — .300" crack	65% — .250"
Penetrant inspection	90% — .220"	90% — .175"
Eddy current	90% — .090"	90% — .030"
Ultrasonic	80% — .375"	90% — .180"

● Average results obtained from majority of technicians

● Best results obtained from top 10% technicians

Figure 2.12: ENSIP Summary of Nondestructive Inspection. [Taylor, 1985]

Figure 2.13: ECI POD Results for Aircraft Engines. [King, Cowie, and Reimann, 1985]

Figure 2.14: FPI POD Results for Compressor Discs. [King, Cowie, and Reimann, 1985]

inches	LEL
0.01	.25
0.02	.51
0.03	.76
0.04	1.02
0.05	1.27
0.06	1.52
0.07	1.78
0.08	2.03
0.09	2.29
.10	2.54
.20	5.08
.30	7.62
.40	10.16
.50	12.70
.60	15.2
.70	17.8
.80	20.3
.90	22.9
1.00	25.4

Figure 2.15: Preliminary POD Results for Compressor Discs. [Raizenne et al, 1985]

Figure 2.16: Representative Profile Mission for an Advanced Turbine Engine. [Schwartz and Annis, Pratt and Whitney]

Chapter 3

EXPERIMENTAL MATERIALS AND METHODS

3.1 J85 CAN40 Compressor Discs

Ten J85 CAN40 fifth stage compressor discs containing a large number of radially inward (and some outward) bolt hole LCF cracks were used for generating a POD database for various NDI techniques (Table 3.1). The discs had been manufactured from AM355 material, a precipitation hardened martensitic stainless steel, and a detailed drawing of the J85 CAN40 fifth stage compressor disc is shown in Figure 3.6.

Each bolt hole (40 per disc) was inspected for radial cracks to determine the location of the crack and its indicated crack length (if the method permitted). Subsequent to the NDI examination, the cracked bolt holes were pried open at NAE and the actual crack length was measured with the help of SEM.

3.2 NDI Techniques Used

The NDI techniques used are summarized in Table 3.2. The ten discs were inspected by six NDI techniques at three institutions: Orenda, AMDU, and NAE. The six NDI techniques include LPI, eddy current, and the ultrasonic C-scan.

3.2.1 Production LPI at Orenda

The liquid penetrant inspection (LPI) process, also abbreviated FPI if a fluorescent dye is used, is intended for the detection of discontinuities open to the surface such as cracks. Not all penetrant sensitivity levels, materials and process methods are applicable to all inspection requirements. The sensitivity level should be adequate for the intended purpose of the inspection.

54

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The first NDI of the discs was performed by Orenda under the maintenance contract for these engines. Crack maps were drawn showing the position and length of each crack detected and sent with the discs to NAE for further analysis. The procedure was as follows [74]:

Type I - fluorescent dye

Method B - post emulsifiable, lipophilic (oil soluble)

Group 5/6

- i. Apply ARDROX 987 penetrant - dwell time 30 minutes
- ii. Emulsify in ARDROX 9PR8-M, dwell time 1.5 mins $\pm$ 15secs
- iii. Hydrowash - 3 minutes maximum
- iv. Warm air dry
- v. Apply ARDROX D-499C, nonaqueous developer - dwell time 10 mins minimum
- vi. Inspect

3.2.2 Laboratory LPI at AMDU (Trenton)

The discs were sent to AMDU for laboratory LPI. The inspection was carried out by experienced NDI technicians. The method followed by the two technicians was in accordance with the Military Standard MIL-STD-6866 of 30 September 1985 [65]. One technician used a penetrant dwell time of 45 minutes whereas the other used a dwell time of 60 minutes. The dwell time is the total time that a penetrant (in other cases an emulsifier or remover, or a developer) spends in contact with the component [65]. Two dwell times were used because AMDU noticed that more cracks could be seen when the penetrant dwell time was increased.

The maintenance or overhaul inspection of turbine engine critical components should be done only with the following penetrant system [65]:

Type I - fluorescent dye

Method D - post-emulsified hydrophilic process

Sensitivity Level - high 3 or ultrahigh 4

The disc surfaces were pre-cleaned by various degreasing procedures followed by a solvent rinse in PR551. The entire surface of the disc was covered with the penetrant ARDROX 985-P2-E using a green fluorescent dip. The discs were then left to dry for a dwell time of 45 minutes by the first inspector and 1 hour by the

second inspector. Excess penetrant removal also known as emulsifier application was carried out by using a hydrophilic post-emulsified penetrant system that requires a water prerinse, application of the hydrophilic emulsifier and then a water postrinse. The discs were then placed in an electric air dryer at 40 degC where heated air was blown across their surfaces for 15 to 20 minutes. The water on the surface evaporates while the liquid penetrant remains in the cracks. The discs were put into a cloud chamber full of white powder (developer) and a shot of air was applied to completely cover the discs. Following this, each disc was taken into a dark room and specific crack areas were developed more precisely using 2 sprays to alternatively develop and clean a specific area of interest. The cleaner ARDROX 9PR551 was used with a clean cloth to wipe across the cracks in order to remove the penetrant which seeps out of the cracks. Then when the developer is sprayed on, ARDROX 9D6, a white powder, the cracks appear brighter as fluorescent areas shine in the black light. The inspection was performed under a black light with minimum intensity 800 microwatts per sq cm at the part surface. Black light gives off electromagnetic radiation in the near ultraviolet range ie. wavelengths from 320 to 400 micrometers. The cracks appear fluorescent and all areas of fluorescence (front and back of the disc) were examined. The crack lengths were measured with digital readout vernier callipers considered to be accurate to ± 0.02 mm. All ten discs were inspected on both sides by the experienced technicians. They each listed their crack measurements (for both sides) for each bolt hole which contained a crack as far as they could determine. Finally, post inspection cleaning of the discs was done using the same solvent rinse as before, ie. ARDROX PR551.

The inspectors noted that crack lengths measured on each side of the discs were different in some cases. In addition when a crack was neither a through crack nor a corner crack, it did not appear on either surface. However, it was felt that a crack such as this could be seen by looking at the surface of the bolt hole. (Ref. private communication with John McCanna etc at AMDU, Trenton)

3.2.3 Eddy Current Inspection (ECI)

AMDU also carried out eddy current inspection on all bolt holes of the ten J85 CAN40 fifth stage compressor discs using two inspectors.

The Nortec RECHII I, a rotating eddy current hole inspection instrument with a 3/16 inch diameter probe fitted to a small light hand held scanner was used to carry out ECI on the compressor disc bolt holes. This portable unit operates at a test frequency of 500 kHz and uses a reflector type differential probe that spins at 3000 rpm [68]. A bright CRT displays the signals from one complete revolution. Both size and location can be provided by the display where crack size

is approximated by the signal amplitude. Position is referenced by an index mark on the scanner and the CRT graticule is marked in degrees (See Figure 3.1). The RECHII I uses a reflection type differential probe which consists of a send coil, A, and a differential receiver coil, B (See Figure 3.2). The study of RECHII wave form interpretation is complex, but two general shapes are helpful (Figure 3.3):

- A sine wave with a short wavelength represents a narrow, deep flaw.
- A sine wave with a long wavelength represents a wide, shallow flaw.

A probe size of 3/16 inch was selected and teflon tape wrapped around the probe so that clearance would be between .004 and .010 inches. Any further clearance results in more noise and loss of sensitivity while less clearance hinders probe movement. A reference hole, of the same material and heat treatment as the test part, was chosen to be a bolt hole without a crack as indicated by LPI for calibration purposes. The equipment was used with the following settings:

FILTER=OFF Selects a high pass filter to reduce unwanted low frequency components of the signal.

REJECT=OFF A linear reject used to eliminate unwanted baseline noise without affecting the amplitude of a flaw signal. A video or positive signal can be displayed instead of the full signal display by using the reject control which removes all signal display below its adjusted level.

STEP GAIN=2 or 5 Selects gain required for the desired flaw signal amplitude. Step gain is in calibrated steps and variable gain is also available. One inspector used a gain setting of 2, the other used a gain setting of 5. Gain can be compared to sensitivity and consequently the more sensitive gain setting of 5 detect more apparent cracks.

When a good bolt hole, ie. a bolt hole without a crack, is inspected the signal output is just noise. When a bolt hole with a crack is inspected a sine wave rising above and below the noise can be seen sometimes clearly but sometimes it can be hazy. The location of the crack can be seen by the signal position on the screen between 0 and 360 degrees. If the probe is held at 90 degrees to its previous position, the sine curve will shift 90 degrees. The first inspector used a step gain setting of 2 and the second inspector used a more sensitive gain setting of 5..

3.2.4 Microfocus X-Ray Radiography

Attempts were also made to detect the bolt hole cracks with the recently acquired microfocus X-ray facility at AMDU. A microfocus x-ray source is an x-ray tube with an electron focal spot smaller than 0.1 mm. In practice, focal spots from 0.002 to 0.25 mm have proven to be the most useful for real-time systems, while spots from 0.025 to 0.075 mm have proven satisfactory for film techniques using moderate magnification levels.

The equipment that was used to carry out NDI on the compressor discs was set up as shown in Figure 3.4. MIXIS, the Microfocus Integrated X-ray System, combines the Ridge HOMX (High Output Microfocus X-ray) generating system with a test part manipulator, radiation shielding system, and imaging system to provide a real-time x-ray imaging capability [84]. The operator's control console, shielded from radiation, was in a room next to the system at AMDU, and a window allowed visibility between the two rooms. For archival purposes, hardcopies can be the form of film x-rays, video hardcopies, or video tapes [62].

Lead was used to cover the bolt hole being inspected such that the crack was not *whited out*. The compressor disc was then placed in a jig which could move vertically, horizontally, and rotate. The main body of the disc appeared grey, while the outer annulus of the disc being thicker (Figure 3.6), appeared darker whereas the bolt hole appeared lighter. A combination of 24 mA and 68 kV produced a good picture. Greater thicknesses of material could be analyzed by decreasing the wavelength i.e. increasing the penetrating power by increasing the voltage to about 160 kV and correspondingly decreasing the amount of radiation energy to 5 mA. The disc could be manipulated using the jig and *zoomed* towards the x-ray tube so that a magnified image of the crack in the disc could be seen on the screen.

The changing thickness of the compressor disc cross-section along the crack path proved to be a major limitation for this technique. It was felt that sufficient contrast between the crack and the surrounding material could not be obtained to clearly show the crack. In practice only cracks of 5 mm or greater were thought to be seen with any degree of confidence and therefore it was decided not to inspect the discs using this method.

3.2.5 The Ultrasonic C-scan

After receiving the discs from AMDU inspections, an ultrasonic C-scan technique, developed by NAE for bolt hole crack detection, was used to inspect all discs [30]. The ultrasonic C-scan uses a transducer (or probe) to transmit sound waves onto the surface of a disc and records any reflections coming back from

the cracks. The scanning mode used to present the data is the C-scan. In this mode, the transducer is synchronized with a recording instrument such that its movement across the surface of the test specimen is transferred to the motion of the recording pen on a chart recorder. In this way a two dimensional image of defects showing the planar extent of internal damage is generated [30].

The compressor disc was mounted inside a water bath where it could be rotated freely. A 10 MHz probe was positioned at an angle to the disc to send and receive signals. As the disc turned 1 revolution the probe took readings, and then moved radially inwards such that the whole disc was inspected in concentric circles. A sinusoidal potential was induced in the probe (electrical input pulse) which deformed the probe and led to a transmitted ultrasonic wave. The electrical input pulse appeared on an oscilloscope screen, with time as its horizontal axis, at a constant position. When the ultrasonic wave was at or near a crack, a pulse or a number of pulses may appear on the screen at a later time due to a reflected, scattered or dispersed sound wave. This pulse appeared at a later time since the initial wave travels from the probe to the defect and then back to the probe. Thresholds were set to eliminate the scatter caused by any surface defects, resulting in the formation of a map of the disc, showing the bolt holes and any associated cracks.

A calibration procedure was established using two discs by comparing actual crack lengths (by prying open the cracks and measuring the crack lengths with a SEM) with the image of the crack obtained on the C-scan crack map. It was found that using a 10 MHz probe:

- Crack lengths could be measured directly from the ultrasonic C-scan image.
- Cracks less than 1.0 mm in length could not be measured accurately.
- Cracks as small as 0.5 mm were detected.

The most time consuming part of the C-scan process was the setting up procedure. For the best results, the threshold level and the probe angle as well as other variables, must be set as carefully as possible. Once this is done however, the other discs can be inspected simply by replacing the inspected disc with a new one and running the probe in concentric circles (computer controlled) covering the entire area of the disc and producing a crack map (Figure 3.5). The crack lengths were then measured from the crack maps and noted for both sides of each bolt hole.

3.3 Destructive Optical and Fractographic Examination of Cracked Bolt Holes

After the 3 methods of NDI, namely LPI, ECI, and the Ultrasonic C-scan inspection, had been completed for the ten discs, the results were verified by destructive methods. Each bolt hole and its surrounding area which could contain a crack was cut out by first machining two concentric circles with diameters either side of the bolt holes and secondly, by sawing the annulus into strips so that each strip contained one bolt hole (See Figure 3.7).

Those bolt holes that were thought to have cracks as indicated by the C-scan results were pried open while the rest were cold mounted and polished. The mounted bolt hole specimens were then examined using an optical microscope at a magnification of 500 in order to determine whether in fact there was a crack. If a crack was thought to be seen, the crack was pried open and the fracture surface examined with the SEM. If no crack could be seen the mounts were broken open and remounted on the other side to examine for corner cracks. As before the mounted specimens, after polishing, were inspected and those with cracks were pried open. Those specimens where cracks were not seen were assumed to contain no cracks.

3.3.1 Prying Open the Fatigue Cracks

As can be seen from Figure 3.8 (a), the specimens were sawed from the bolt hole radially outward. Then, the end of the fatigue crack having been roughly marked using a microscope, a V-notch was sawed so that the point of V would fall just short of the end of the fatigue crack. Finally, the specimen was placed horizontally and sideways in a vice and tightened so that as the two extreme ends came together the crack was forced open from the bolt hole. Care was taken to ensure that the fracture surfaces would not rub and thus would not be damaged.

If a radially outward crack was indicated, the specimen was sawed across the bolt hole avoiding both fatigue cracks, and V-notches cut as mentioned above for both cracks indicated (See Figure 3.8 (b)).

3.3.2 Optical Assessment of the Bolt Holes

The specimens, a typical one shown in Figure 3.7, were cold mounted with the surface to be examined face down. It was found that the bolt hole surface did not sit flat in the mount (See Figure 3.6) because of the change in disc thickness from the bolt hole towards the disc centre. This proved inconvenient for automatic

polishing. Therefore some aluminium foil was wrapped around the thinner end of the bolt hole specimens such that they should sit flat inside the mounts.

After the specimen were dry, the mounting cups were removed and the specimens were marked with the disc letter, bolt hole number, and serial or nonserial side by using an electric engraver. All specimens were polished following the standard procedures using SiC paper with 240, 400, and 600 grit size. The specimens were finally polished on a 6 μm wheel and the polished bolt hole specimens were examined using an optical microscope. The specimens were then pried open or remounted on the other side depending on whether a fatigue crack was thought to be present.

3.3.3 Examination of Fracture Surfaces

The fracture surfaces of each crack was examined to determine the initiation sites and the maximum crack length was measured. The fracture surface was mounted horizontally with the crack growing away along the y axis. Examples of corner, edge, side, and multiple initiation sites are shown in Figure 3.9 (a), (b), (c), and (d).

Crack lengths were measured at 500 magnification by placing the crosshairs on the maximum crack extent and noting the vernier reading on the y axis to 4 decimal places in inches. Then without moving the x axis, the y axis knob was turned until the crosshairs were placed at the edge of the bolt hole where the crack had initiated, and again a vernier reading was taken. The two readings were subtracted and converted to millimeters and the accuracy of the crack length measurement technique was 0.01 mm. Some different crack lengths are illustrated in Figure 3.10: A typical crack of length 2.2 mm, (a); a shorter crack, (b), which is not through thickness; and a longer crack, (c), extending radially inward in the disc into the narrower section of the disc.

The end of the cracks could be clearly seen as shown in Figure 3.11 where striations typical of fatigue cracks can be seen in the lower part of the photograph, and ductile dimples, in the upper part indicating the overload that occurred when the specimen was pried apart.

Table 3.1: The ten J85 CAN40 Fifth Stage Compressor Discs chosen to generate the NDI Databases.

LETTER DESIGNATION	DATE	SERIAL NUMBER	TSN
A	20 Oct 82	P18649-14	UNKNOWN
B	07 Mar 84	11J200-278	3840.1
C	17 Apr 83	P18649-19	4036.4
D	01 May 84	P18521-8	4159.5
E	28 Apr 84	11J200-263	3929.2
F	14 Apr 83	P18521-19	3402.5
G	07 Jun 83	11J200-102	UNKNOWN
H	02 May 84	11J200-260	4319.5
I	01 May 84	P18649-1	4421.7
J	03 Mar 83	P18521-10	4398.6

Table 3.2: Summary of NDI Techniques used and the Institutions that performed the NDI.

INSTITUTION PERFORMING NDI	NDI TECHNIQUE	
	Description	Abbreviation
Hawker Siddeley Canada Inc Orenda (Maintenance Division) Toronto	Production LPI	ORENDA
Aerospace Maintenance Development Unit (AMDU) NDTB - Development CFB Trenton	Laboratory LPI (45 mins dwell time) (60 mins dwell time)	L45 L60
	Eddy Current (gain setting 2) (gain setting 5)	EC2 EC5
National Research Council (NRC) National Aeronautical Establishment (NAE) Structures and Materials Lab Ottawa	Ultrasonics (C-scan representation)	CSCAN

Figure 3.1: The Nortec RECHII I Rotating Eddy Current Hole Inspection Instrument with a Hand Held Scanner. The probe rotates at 3000 rpm. [Nortec]

Figure 3.2: The Eddy Current Differential Type Probe Consists of a Send and a Receive Coil.
[Nortec]

Figure 3.3: Typical Eddy Current Output Display.

Figure 3.4: Drawing of a Real-Time Microfocus X-Ray System. [Ridge HOMX]

Figure 3.5: The Ultrasonic C-Scan at NAE. The compressor disc was positioned in the water tank and crack maps were produced by the plotter.

Figure 3.6: Dimensions for the J85 CAN40 Fifth Stage Compressor Disc.

Figure 3.7: Compressor Disc Section Containing Bolt Hole and Fatigue Crack.

(a) Bolt hole with radially inward fatigue crack

(b) Bolt hole with radially inward and outward fatigue cracks

Figure 3.8: Section of Compressor Disc showing Bolt Hole Fatigue Cracks Pried Open.

(a) Edge Initiation Crack
(0.66 mm)

(b) Corner Initiation Cracks
(0.75 mm max)

Figure 3.9: Types of Initiation Sites.

(c) Side Initiation Crack (rare)
(0.47 by 0.34 mm)

(d) Multiple Initiation Cracks
(0.26 mm max)

Figure 3.9: Types of Initiation Site Cont.

(a) Average Crack Length at about 4000 hours (2.2 mm)

(b) Shorter Corner Crack (0.91 mm)

(c) Longer than Average Crack Length (4.7 mm)

Figure 3.10: Different Crack Lengths.

- (c) Ductile dimples representing overload in the pried open section

- (a) The Boundar

- (b) Striations representing fatigue crack growth

Figure 3.11: The Boundary between the Fatigue Crack and the Pried Open Area.

Chapter 4

PFM SIMULATION OF SII FOR J85 CAN40/15 COMPRESSOR DISCS

In order to implement a PFM based damage tolerance maintenance methodology for LCF life limited aero engine gas turbine discs, it is necessary to predict some sort of a safe inspection interval with an associated degree of confidence. In general a PFM based damage tolerance maintenance methodology hinges on three areas of expertise. These include:

- i. Stress analysis of components and the prediction of stress intensity factors operative ahead of the cracks in critical locations and the determination of a critical or a dysfunction crack length.
- ii. Crack growth rate prediction in components at critical locations.
- iii. Accurate nondestructive inspection (NDI) capabilities.

In regard to Item (i) above, stress intensity factors and a critical crack length of a component can be determined through finite element analysis techniques using typical engine mission profiles. In Item (ii) above, crack growth rate data always contains an inherent scatter and if this scatter can be characterized statistically then the growth of a given crack can also be predicted in a statistical manner. In Item (iii) above, a component must be removed from service prior to a crack reaching a critical or a dysfunction ² size and the component must be inspected frequently enough to avoid catastrophic failure. However, in all NDI techniques, the probability of detection (POD) of a crack varies as a function of crack length and component geometry. The POD data simply depicts the number of cracks that would be detected or missed for a given crack length and NDI technique at

²See the nomenclature for a definition.

a specified confidence level. It is therefore imperative to make decisions regarding the impact of a missed or a detected crack of a specified size on the safe operational life of a component. Besides, the missed cracks will also have to be sized using certain realistic assumptions. To account for the scatter in crack growth rates and NDI uncertainties, statistically significant amounts of data on crack growth rates and POD of cracks varying as a function of crack length are therefore required. It is the purpose of this section to compile a computer programme that simulates the growth of in-service cracks in turbine engine discs assuming that the location of a crack and the stress intensity factors varying as a function of crack length are known. In compiling this program it was further assumed that significant amounts of data on crack growth rates and POD of cracks are available right from the onset of the PFM analysis.

Thus the DISK2 computer program was written which attempts to predict the SII by modelling the initial distribution of crack lengths in the disc, carrying out NDI, allowing controlled random crack growth, and removing the disc when a crack reaches dysfunction length or after a specific time of crack growth. The flow diagram for this program is shown in Figure 4.1. The process is repeated for many discs, for example 5000, to simulate many possibilities. The associated confidence for the SII's is calculated by fitting a distribution to the SII's and choosing a 0.1% value. In other words at this 0.1% SII only one in one thousand discs will contain a crack of dysfunction length.

A typical DISK2 computer run with sample input and output is documented in Appendix D.

4.1 An Overview of the DISK2 Simulation Program

4.1.1 Random Number Generation

One of the main tasks in a Monte Carlo simulation is the generation of random numbers from prescribed probability distributions. Most computers have the capability to generate uniform random numbers between 0 and 1 either directly or through a library. In the DISK2 program, uniform random numbers were generated using the IMSL function GGUBFS(DSEED) where DSEED is the seed required to start the process. Thus at the start of the program a seed (in double precision) must be input eg. 123457.d0 and each call to this function throughout the DISK2 program produces a subsequent random number.

Random numbers from a normal distribution (FUNCTION RNORM) were

generated in pairs using an algorithm defined by Box and Muller (1958) found in [9] on page 284, and summarized in Appendix A.

Random variables for most other distributions can be found by using one of the above two distributions known as R_U and R_N where R_U is a random value from the uniform distribution over the interval (0,1) and R_N is a random value from the normal distribution with mean, $\mu = 0$ and standard deviation, $\sigma = 1$. Equations to find random variables from a lognormal or Weibull curve are given in Appendix A and a more comprehensive list is given in [37] p242-3.

4.1.2 Initial Conditions

The initial conditions that need to be input at the start of the DISK2 simulation program are crack length in mm and TSN in hours. They can be entered either as constants or as distributions. If a distribution is chosen the computer program randomly chooses values from the specified distribution (refer to Appendix A for exact equations). If we are dealing with life extension ie. life after safe life and if the safe life is 2285 hours, then we need to input the distribution of crack lengths in the discs at this constant TSN. Since the distributions of lengths is often unknown, a distribution of times to reach a particular crack length (eg. TTCI) or a constant crack length (eg. one that is likely to be missed by NDI) could be used instead (SUBROUTINE INIT). Several initial conditions for the fifth stage J85 CAN40 compressor discs were used and are described below.

(a) Constant Crack Length and TSN

A single value of crack size with a corresponding number of hours the disc had been in service ie. Time Since New (TSN) was chosen from the TTCI distribution explained below.

$$a_{initial} = 0.8 \text{ mm} \quad (4.1)$$

$$TSN_{initial} = 2000 \text{ hours} \quad (4.2)$$

where 5% of the discs contain cracks of 0.8 mm at 2000 hours. At 3000 hours and 4000 hours, 40% and 90% of discs respectively have 0.8 mm cracks whereas at 1000 hours, 0.1% of the discs have cracks of 0.8 mm in length. There is no justification for the particular values chosen except that 0.8 mm is the crack length at initiation generally defined as a smallest crack length detectable by NDI and 2000 hours is approximately the safe life which has varied from 3000 hours to 1777 hours in the past 15 years and is currently 2285 hours [96] (Figure 4.2).

(b) TTCI Distribution at Constant Crack Length

A Time To Crack Initiation (TTCI) distribution, or the number of hours for a crack to initiate and reach a measurable crack length, is often used in engineering because the actual distribution of crack length for a particular TSN is unknown and hard to find. The GE (1983) N_i line for a 0.8 mm crack was used as the TTCI. It is a Weibull distribution of TSN's with a characteristic value, η , and slope, β , as follows:

$$\eta = 3000 \text{ hours} \quad (4.3)$$

$$\beta = 5.0 \quad (4.4)$$

Thus the initial crack length is 0.8 mm and a corresponding initial TSN is randomly chosen from this distribution (See Figure 4.3).

(c) Distribution of Crack Lengths at constant TSN

This choice of initial conditions was used in two ways: firstly as a worst case scenario, and secondly to compare the crack growth in the simulation model with crack growth when the component flies in the aircraft.

Several methods for deducing an initial flaw distribution from an NDI data base are proposed in the literature (See Chapter 2). In the case of the J85 CAN40 compressor discs, if we assumed there was no data available about the flaw size in the material we could propose a worst case scenario based on NDI. We could assume a distribution of cracks which the NDI technique could not detect ie. assume a crack length distribution equal to the distribution of cracks missed by a particular NDI technique. This is probably not the case, but it would be a worst case approximation of the distribution. The initial TSN could be taken as 2285 hours which was the safe life for these components at time of writing.

This choice of initial conditions was also used for distributions of crack lengths in retired 5th stage J85 CAN40 compressor discs as detected by LPI for a given TSN range. As discussed later, the reason for this was to compare the simulated crack growth rate with the in service crack growth rate. Note that bolt holes for which LPI detected no crack were assumed to have a crack length of less than the smallest detected crack length.

4.1.3 Actual vs. NDI Measured Crack Length

Four of the six NDI techniques estimated a crack length when a crack was detected. The results were compared with the real crack length as measured by the SEM using linear regression analysis. One piece of information extracted from this analysis was the distribution of actual crack lengths given that an NDI technique detected a crack of 0.8 mm in length (See Figure 5.10). It is clear from Figure 5.10 that the C-Scan is not only more accurate than the Orenda LPI technique but also has less scatter.

4.1.4 Inspection Uncertainty

Inspection uncertainty is taken into account by using a predetermined POD curve and programming the computer to perform NDI. Given the initial crack length, the corresponding POD value is calculated using the loglogistic equation with parameters that need to be input for each NDI technique. The chance of detecting or missing a crack is simulated by the computer by generating a uniform random number between the values of 0 and 1 and comparing it with the calculated POD value (See Figure 4.4). If the random number generated is less than the POD for a particular crack length, then the crack is detected; if it is greater then the crack is missed (FUNCTION FOUND).

Loglogistic POD curves were determined for all 6 NDI techniques and used in the DISK2 program as a filter to simulate the probability of detection of a crack as a function of its length. Both the mean lines and the 95% confidence bounds were used. The POD curves and confidence bounds will be shown later.

4.1.5 Distribution of Crack Lengths Missed by NDI

If the crack is not detected by NDI, then the disc is assumed to have a crack, the length of which is randomly chosen from the distribution of crack lengths missed by the NDI technique. This data should be available to the user from experimental NDI-POD data. SUBROUTINE AIDIST produces a random crack length using a distribution having specified a mean and a standard deviation using the experimental NDI-POD data.

Distributions of cracks missed were represented by lognormal distributions and the probability density functions are shown in Figure 5.7.

4.1.6 Controlled Random Crack Growth

The scatter in crack growth is taken into account simply by randomizing the crack growth equation as proposed by Professor J.N. Yang [102] [103] so that:

$$\frac{da}{dN} = XC(\Delta K)^n \quad (4.5)$$

where X is a normal random lognormal distribution with a median value equal to unity.

Taking logs:

$$\log \frac{da}{dN} = Z + \log C + n \log \Delta K \quad (4.6)$$

where Z is a normal random variable with zero mean and a standard deviation equal to the standard deviation of the log of the crack growth (Figure 4.5). In this model, both 'C' and 'n' are assumed to be constant within a specimen but to have specimen-to-specimen variability although this may not be the case [6]. The crack growth rate in mm/hour based on mission profiles and striation counting [97] is obtained from the following:

$$\frac{da}{dN} \left(\frac{\text{mm}}{\text{hour}} \right) = \frac{da}{dN} \left(\frac{\text{m}}{\text{cycle}} \right) \times 5 \left(\frac{\text{cycles}}{\text{hour}} \right) \times 1000 \left(\frac{\text{mm}}{\text{m}} \right) \quad (4.7)$$

Thus values for C , n , and the variation in terms of standard deviation need to be input into FUNCTION A0VAR for each crack growth rate condition.

The relationship between the crack length and the stress intensity factor also must be known. If the stress distribution is known from finite element analysis of the disc, the following equation can be used to find ΔK operative ahead of a crack tip at a given value:

$$\Delta K = \Delta \sigma \sqrt{\pi a} \quad (4.8)$$

In this case several analyses have been performed based on FEM [52] and striation spacing [96]. Combining these results and using the knowledge that:

$$\Delta K \propto \sqrt{a} \quad (4.9)$$

we can substitute two realistic points (0.5,30) and (4.5,63) for crack length (mm) and ΔK (MPa $\sqrt{\text{m}}$) to define the relationship.

Crack growth rates in the 5th stage J85 CAN40 compressor discs were determined both by laboratory experiments at NRC [5] and by spin pit testing in Toronto [43]. The laboratory experiments were carried out under three sets of conditions and the da/dN vs ΔK graphs are shown in Figure 4.6. Standard linear regression analysis (results in Table 4.1) was used to obtain C and n values for

the Paris crack growth equation and the conditional standard deviation $s_{Y/x}$ calculated in order to take into account the randomness of crack growth. Figure 4.6 shows a mean solid line through the data points and two parallel dashed lines at ± 1 standard deviation from the mean line.

The spin pit testing results assumed a constant crack growth rate of 0.00312 mm/hour over a crack length range of 1.0 mm to 6.0 mm. By various extrapolations, C and n values were determined and an average standard deviation was used.

Table 4.2 shows the three parameters, C, n and the conditional standard deviation $s_{Y/x}$, that were input into the computer program to obtain different crack growth rates and different variations. The crack growth rates for an average ΔK value are given in the Table both in m/cycle and mm/hour for comparison.

4.1.7 Calculation of Safe Inspection Interval

In the DISK2 program, the initial (or safe life) conditions and the final (or dysfunction) conditions each consist of a crack length in mm and an associated TSN in hours. Thus given (a_{init}, TSN_{init}) and (a_{dys}, TSN_{dys}) , the safe inspection interval is determined by subtraction:

$$SII = TSN_{dysfunction} - TSN_{initial} \quad (4.10)$$

4.2 Determination of the 0.1% Safe Inspection Interval

The DISK2 program outputs as many SII's as specified by the user, for example 5000. Thus it was necessary to carry out some analysis in order to characterize the SII's and if a distribution of the SII's could be drawn then a safety factor could be introduced by using the 0.1% SII ie. a safe inspection interval where 1 in 1000 cracks will reach dysfunction length before the discs are removed from service.

In some cases it was possible to determine the theoretical distribution of SII's, depending on the input distributions, but in many cases this was not a viable option and histograms were drawn to assess the probable distribution. In practice either the lognormal or the Weibull distribution were found to adequately represent the SII's.

Two computer programs were written to organize the SII's into manageable form. They not only calculate the parameters for the distributions but also produce probability plots so that by visual examination an evaluation could be made of the assumed probability model. If an assumed model is correct, the plotted

points will tend to fall in a straight line. An added advantage is that estimates of the percentiles of the distribution can be read off the graph (See Figure 4.7). With a computer program the 0.1% SII is calculated using the parameters of the distribution which in turn have been determined using the method of least squares and/or the maximum likelihood method.

In the rare cases where the SII was zero or negative because an initial crack length was already longer than the dysfunction crack length, the SII's were not included in the analyses.

4.2.1 Lognormal SII Distribution

The statistical details for probability plotting are given in Appendix A. The SII values are sorted numerically and given a probability plotting position using for example Hazen's equation of $\left(\frac{i-0.5}{N}\right) \times 100\%$ where i is the order number and N is the total number [8].

The only difficulty encountered was the plotting of the cumulative probability scale using a computer which is linear. The problem was solved using an algorithm given in a handbook [2] which converted cumulative probabilities into standard normal deviates which occur on a linear scale.

The parameters for the lognormal distribution, λ and ζ , and hence the 0.1% SII were calculated using least squares and maximum likelihood but the maximum likelihood estimates were used for comparison since they are often more accurate for large databases (Hoel 1962) found in [8]. Program listings can be found in Appendix B.

4.2.2 Weibull SII Distribution

The Weibull cumulative distribution function can be transformed so that the distribution will plot in a straight line and its parameters are easily calculated. Aircraft engine manufacturers have found Weibull analysis very useful and have developed methods for analyses [1]. In the Weibull computer program data is sorted and ranked according to a GEC Handbook [27]. The relevant equations are summarized in Appendix A and explained in more detail in references [35] and [56].

The parameters for the Weibull distribution, η and β , were determined using the method of least squares and hence the 0.1% SII could be calculated. Computer listings can be found in Appendix B.

4.3 Stability and Convergence

When random numbers are generated a seed is always required to start off the algorithm. Secondly when simulating a distribution a sufficient number of iterations are required so that the distribution is adequately represented. The time and cost of a computer simulation increases as the number of iterations increases and therefore this stability and convergence test was carried out firstly to make sure the solution converged and secondly to find the 'sufficient' numbers of iterations required for this particular analysis.

This analysis was carried out for safe inspection intervals for varying conditions. Results are presented only for one case. Figure 4.8 shows the results of maximum likelihood estimates for the 0.1% SII for five different seeds [37] and for the number of simulations varying from 10 to 5000. As was expected the variation for SII's at 50 simulations (786 to 941 hours) was much larger than the variation at 5000 simulations (880 to 889 hours).

Table 4.3 is a summary of the average of the 0.1% SII's with increasing number of simulations for both the least squares and the maximum likelihood method. The results show that:

- i. Least square estimates are more accurate when a small number of simulations is used (eg. 10).
- ii. Maximum likelihood estimates are not only more accurate but converge better when a large number of simulations are required (eg. more than 100).

2

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Table 4.1: Linear Regression Results for Crack Growth Rates.

CRACK GROWTH RATE CONDITIONS (lab work)	LINEAR REGRESSION $\log(da/dN)$ vs $\log(\Delta K)$			
	N	INTERCEPT	SLOPE	$s_{Y/X}$
high time, high temp	135	-11.8841	3.3230	.1265
medium time, high temp	139	-11.6694	3.1619	.0711
high time, room temp	130	-11.3975	2.9289	.1088

Table 4.2: Crack Growth Rates and their Parameters.

CRACK GROWTH RATE CONDITIONS	CRACK GROWTH RATE PARAMETERS AND VARIABILITY			CRACK GROWTH RATES IN	
	C	n	$s_{Y/X}$	m/cycle ($\times 10^{-7}$)	mm/hour ^d
high time, high temp	1.31×10^{-12}	3.32	.1265	3.75 ^b	.00188 ^b
medium time, high temp	2.14×10^{-12}	3.16	.0711	3.34 ^b	.00167 ^b
high time, room temp	4.05×10^{-12}	2.93	.1088	2.65 ^b	.00133 ^b
spin pit test	6.46×10^{-14a}	4.32 ^a	1021 ^a	6.24 ^c	.00312 ^c

^a deduced

^b assuming $\Delta K = 44 \text{ MPa}\sqrt{\text{m}}$

^c valid over crack length range $1.0 \leq a \leq 6.0 \text{ mm}$

^d crack growth rate: $\frac{m}{\text{cycle}} \times 5 \frac{\text{cycles}}{\text{hour}} \times 10^3 \frac{\text{mm}}{m} = \frac{\text{mm}}{\text{hour}}$

Table 4.3: Summary for Stability and Convergence.

NUMBER OF SIMULATIONS	0.1% SAFE INSPECTION INTERVAL					
	LEAST SQUARES			MAX LIKELIHOOD		
	mean	st dev	range	mean	st dev	range
10	813.2	90.73	233	997.0	85.93	200
50	783.4	69.11	169	857.0	66.84	155
100	812.0	44.83	119	856.4	44.10	116
200	865.0	19.58	43	891.0	19.39	44
500	869.8	19.51	49	882.4	20.49	51
1000	748.4	12.58	28	880.8	13.00	29
2000	793.6	11.63	32	880.4	12.16	33
5000	856.0	4.18	11	883.4	3.51	9

DAMAGE TOLERANCE BASED PFM ANALYSIS
A METHODOLOGY FOR PREDICTING A SAFE INSPECTION INTERVAL

A computer program flow diagram that predicts safe inspection interval taking into account the variation in crack growth rate and the reliability of NDI.

Figure 4.1: Flow Diagram for DISK2 Simulation Program.

Figure 4.2: Initial Conditions of Constant Crack Length and Constant Time Since New (TSN).

Figure 4.3: Time To Crack Initiation (TTCI) Distribution at Constant Crack Length.

Figure 4.4: Flow Diagram for Simulating Uncertainty of Inspection.

Figure 4.5: Illustration of Crack Growth Rate Scatter.

Figure 4.6: Different Crack Growth Rates graphically depicting Least Square Results.

Lognormal Analysis for Safe Inspection Interval

Figure 4.7: Typical Probability Plot Output showing the 0.1% Safe Inspection Interval (SII).

Figure 4.8: The Stability and Convergence of the 0.1% Safe Inspection Interval (Maximum Likelihood Method).

Chapter 5

RESULTS AND ANALYSIS

5.1 NDI Data and Its Analysis

Ten 5th stage compressor discs from the General Electric J85 CAN40 engine used to carry out NDI analyses were removed from service during the period Oct. 1982 through May 1984 with an average TSN of approximately 4000 hours (Table 3.1). The NDI techniques that were used are referred to by abbreviated names explained as follows (Table 3.2):

- The data from the three LPI techniques are referred to in the abbreviated forms as ORENDA, L45, and L60. ORENDA refers to the place where the discs were initially inspected whereas L45 and L60 refer to the inspections carried out at AMDU with 45 and 60 minute dwell times respectively.
- The data from the two eddy current techniques are abbreviated as EC2 and EC5 where numbers 2 and 5 refer to the respective gain settings and where a gain setting of 5 is more sensitive than 2. As already stated no crack lengths were measured in this technique.
- Ultrasonic C-scan data is abbreviated as CSCAN.

Appendix C contains the raw NDI data for the ten discs examined. The NDI techniques are listed across the page and the crack length or indication as detected by the NDI technique is also given. On the extreme right, the actual crack length as measured by the SEM technique is recorded. Table 5.1 summarizes the number of cracked and uncracked bolt holes for each disc. 205 out of a total population of 400 bolt holes were found to contain cracks. Table 5.2 compares the NDI results in terms of the number of cracks detected or missed, and the number of false calls for the individual NDI technique. The results are also reproduced in the form of histograms in Figure 5.1.

The Eddy Current technique with a gain setting 5 detected the maximum number of cracks, 187 out of 205 (90%), but it also had the highest incidence of false calls, 73 in total. The LPI techniques, on the other hand, detected about 50% of the cracks with an average of about 5 false calls. The C-scan appeared to give the best results since 80% of the cracks were detected with only 4 false calls. Tables 5.3, 5.4, and 5.5 detail the above information for the individual disc. The overall results for the individual discs also followed a similar pattern to the above results. For example in disc E, Orinda LPI detected 11 out of 24 cracks whereas the C-scan detected 22 cracks, and the number of false calls for each technique were 0 and 1 respectively.

Some interesting results are the following:

- The longest crack missed was 4.43 mm and was missed by EC2 (Table 5.10).
- The smallest crack detected was .09 mm and was detected by EC5 (Table 5.6).

5.1.1 POD Analysis of the Bolt Hole Cracks

A good indication of what the POD curves for each NDI technique will look like (before any assumptions or analyses are carried out) is given by the histograms in Figure 5.2 (a)-(f). The histograms show the number of cracks detected and the total number of cracks in a particular crack length interval eg. 0.5 to 1.0 mm for each NDI technique. The percentage of cracks detected and the smallest crack detected for each technique is summarized in Table 5.6. Since the NDI data generated in the present experiment conformed to Type II experiments, in other words each crack was inspected only once, crack lengths had to be grouped together to calculate the probability of detection data points. Several crack length intervals from 0.2 mm to 1.0 mm were tried and acceptable POD results were obtained with a crack length interval of 0.5 mm, Figure 5.3. The observed points plotted in Figure 5.3 are in fact *binomial points*.

The methods for analyzing the POD data have already been described in chapter 2. Drawing a straight line through the POD data at lower crack lengths would as suggested by Packman [8] provide an excellent fit for the C-scan results but this method would prove inadequate for other NDI results because of the 'scatter' problems. The scatter in POD results arises from small cracks detected and a few larger cracks being missed by ECI and LPI techniques. A loglogistics curve was thus fitted through all NDI data as suggested by Berens and Hovey [13] [14] and USAF [32]. The probability of detection data as a function of crack lengths did not increase in 'leaps and bounds', as would be represented by a binomial distribution, but rather it increased smoothly.

The Loglogistics POD Curves

The two parameters for the loglogistic curve in each case was obtained straightforwardly by transforming the axes and performing linear regression (See Appendix A). When no crack was detected or all cracks were detected within a crack length interval, POD values of 0 and 1 are obtained. This creates problems in the analysis since the loglogistic curve asymptotes at these values. However, Berens and Hovey [13] suggest that:

$$POD = \begin{cases} \frac{1}{n+1} & \text{for } POD=0 \\ \frac{n}{n+1} & \text{for } POD=1 \end{cases} \quad (5.1)$$

where n represents the number of data points. In our case, reasonable fits were obtained by plotting the data on the transformed axes and choosing n to be the number of histogram bars with data.

Linear regression was only carried out up to and including the data points where the POD remained constant at 100%. This can be treated as a natural censoring method since the inclusion of very long cracks, which are all detectable, in the analysis would skew the curve and such a curve would not represent the true values of POD for the smaller crack lengths. The results of the linear regression for the loglogistic POD curve for each NDI techniques are shown in Table 5.7. The parameters for the loglogistics POD curve can be directly written down since the two parameters, β_0 and β_1 , represent the intercept and slope respectively. Figures 5.3 (a)-(f) show the fitted loglogistics curves through the POD data for each NDI technique.

Confidence Bounds on the Loglogistic POD Curves

For reliability and design purposes it is useful to characterize an NDI technique by one POD point and in damage tolerance analysis the 90/95 value is chosen for various calculations, ie. the crack length at 90% probability of detection with 95% confidence. Thus it has become customary to plot a 95% confidence bound along with the mean POD curve. Although the following only deals with some of the possible confidence bounds on a loglogistic curve, the discussion (Chapter 5) compares the different confidence bounds on a binomial and a loglogistic POD curve in more detail.

Three distributions were used to calculate the confidence bounds on all loglogistic POD curves. These included the F distribution, the normal distribution, and the Student t distribution. It is worth noting that the F distribution confidence bound is narrower at $(\bar{x}, \bar{y})$ since this is the mean point.

The F distribution does not assume any underlying distribution of the variation of a variable but is based on the correlation between two variables. The bounds are hyperbolas and valid for the entire regression line. Since the linear regression line must always pass through this point, this narrowness is also reflected on the loglogistics curve after it has been transformed back (Figure 5.3). The F values required are obtained from Tables and specific values are listed in Table 5.8.

In another trial the probability of detection was assumed to vary normally for a given crack length and assuming constant variance (ie. the same variance for any crack length), the 95% confidence bounds are calculated following:

$$POD_{95\%} = POD_{mean} \pm 1.96s_{Y/x} \quad (5.2)$$

where $s_{Y/x}$ is the conditional standard variance and 1.96 is the normal standard variate for 95% confidence. As expected the confidence bound is also a loglogistic curve with the same slope, β_0 , but different intercept, β_1 (Table 5.9). This line is also plotted on the POD curves for each NDI technique (Figure 5.3).

Assuming a Student t distribution of POD at a given crack length calculations for confidence were identical to the normal distribution except that the normal standard deviate 1.96 was replaced by some t value which varies with the number of data points (Appendix A has more detail). The t values for each NDI technique are listed in Table 5.8 and the corresponding loglogistic parameters are presented in Table 5.9. The resulting 95% confidence bound in all cases was only negligibly wider than a normal distribution line.

The 50% lower confidence bound was also calculated for the loglogistic POD curve for comparison purposes since 90/50 is a suggested alternative for damage tolerance calculations in the literature, since the 90/95 limit often leads to overly conservative estimates of the inspection interval. If the probability of detection is assumed to be normal for a given crack length, the 50% confidence is given in Table 5.9. The 50% confidence bounds are about half way between the mean and the 95% confidence bound loglogistic POD curves.

The above analysis clearly indicates that confidence bounds vary widely with the procedure followed to establish these limits. In order to chart the sensitivity of the NDI techniques investigated a 90% POD value was used for comparison purposes. Figure 5.4 illustrates the 90% POD values for a mean loglogistics curve for each NDI technique. The C-scan was able to detect cracks of 1.6 mm with 90% reliability and the LPI techniques were able to detect cracks of 3.3, 4.0, and 4.5 mm respectively with 90% probability.

5.1.2 Analysis of the Cracks Missed by NDI Techniques

For each NDI technique, the cracks that were missed by the NDI technique were histogrammed and plotted using the actual crack lengths as determined by SEM. The POD program allows the user to choose the crack length interval and in Figure 5.5 it is 0.25 mm. These histograms give a rough distribution of the crack lengths missed and where there are a sufficient number of cracks missed, such as for the LPI techniques, it can be seen that the slope could be represented by a lognormal distribution.

Table 5.10 shows the longest crack missed by each technique which is a measure of the reliability of the technique. The largest crack missed by the C Scan was 1.48 mm and this was attributed to an operator error. The Orenda LPI method missed a crack length of about 4 mm. The AMDU L45 operator missed the longest crack of 4.33 mm whereas the L60 operator was consistently good and the largest crack missed was 3.69 mm.

The LPI methods missed about 40%-50% of the total number of cracks found. The C Scan missed only 20% of the cracks. The less sensitive eddy currents technique had results similar to LPI whereas, the more sensitive eddy current technique missed only 9% of the total cracks.

Lognormal Distribution of Cracks Missed

Several distributions for representing crack lengths missed are suggested in the literature eg. exponential or lognormal distributions. Theoretically as the crack length decreases to zero, the number of "cracks" tends to infinity. However in practice one is interested in real cracks and while the smallest crack length detected by the SEM was .09 mm, nothing less than this length could be identified as a crack. It can be seen from the histograms (Figure 5.5) that a lognormal distribution would adequately represent the distribution of cracks missed by each NDI technique. To check this further the crack lengths were plotted on cumulative lognormal probability plots (Figure 5.6) and a straight line relationship suggests that a lognormal distribution can be approximated to the present case.

The parameters for the two-parameter lognormal were calculated using both linear regression and maximum likelihood methods (Table 5.11). The means, λ , for the two methods were very similar and the standard deviations, ζ , for the least squares were slightly wider than that for maximum likelihood method. The parameters, λ and ζ , can be better understood from Table 5.12 which shows the median (or most common) crack length missed by each NDI Technique along with the 95% lower and upper confidence bounds (See also Figure 5.8). Thus, the most

probable crack length the C-scan will miss is approximately 0.5 mm whereas it is approximately 0.8 and 0.9 mm for the LPI techniques.

It is also noticeable that if a technique was not consistently reliable ie. if the technique or the personnel occasionally missed a large crack, it severely effects the POD curve as well as the missed cracks distribution. Compare, for example, Orenda LPI and EC2: the most likely crack length that they will miss (the median) is approximately the same, 0.92 and 0.9 mm. However, EC2 missed a couple of larger cracks which caused the standard deviation ζ to be larger than for Orenda LPI (Table 5.11). Consequently, the upper 95% bound on the median for Orenda is 4.46 mm compared to 5.51 mm for EC2 (Table 5.12). EC2 was the worst NDI technique in terms of the distribution of cracks that were missed even though it consistently detected small cracks. The C-scan, with the smallest median, 0.51 mm, and the smallest standard deviation which resulted in a 95% upper confidence bound on the median of 2.2 mm, was the best NDI technique. In other words NDI Techniques with smaller medians or means and smaller standard deviations are preferable.

5.1.3 Correlation Between Actual and NDI Indicated Crack Lengths

Linear regression analysis between actual and NDI indicated crack lengths was carried out on the results from four of six NDI techniques examined. This obviously excluded the eddy current techniques since they did not estimate a crack length but simply indicated whether a crack was present or not. Least square analysis was carried out to determine:

- (a) The predicted NDI indicated crack length for each NDI technique given the actual crack length.
- (b) The actual crack length and associated confidence bounds for a given NDI indicated crack length.

The linear regression results in terms of slope, intercept, and conditional standard deviation, $s_{y/x}$, are given in Table 5.13. Perfect results would give an intercept=0, slope=1, and $s_{y/x}=0$. Graphically, the NDI indicated or measured Crack Length vs Actual Crack Length relations for each NDI technique are plotted in Figure 5.9 (a)-(d). The, $y = mx + c$, best least squares line is shown and assuming constant variance, the upper and lower bounds of one conditional standard deviation are also shown. These bounds contain 63% of the points if a normal distribution is assumed.

An example of a calculation to find the actual crack length and the associated confidence given the NDI crack length is shown below and the results are tabulated in Table 5.14. Using Orenda's results and the appropriate linear regression results, we calculate that the mean actual crack length:

$$\begin{aligned}\bar{a}_{actual} &= \text{slope} \times a_{NDI} + \text{intercept} \\ &= .5672 \times 0.8 + 1.0986 \\ &= 1.55 \text{ mm}\end{aligned}\tag{5.3}$$

Furthermore, the lower and upper 95% confidence bounds for the for the actual mean crack length are calculated by assuming a normal distribution where:

$$\begin{aligned}\langle \bar{a}_{actual} \rangle_{95\%} &= \bar{a}_{actual} \pm 1.96 \times s_{Y/X} \\ &= 1.55 \pm 1.96 \times .9549 \\ &= -.32 \text{ and } 3.42 \text{ mm}\end{aligned}\tag{5.4}$$

It is not possible to have a negative crack length, but theoretically this is a solution since the normal distribution extends from negative to positive infinity. The statistics simply give a working guideline for damage tolerance calculations.

Given an NDI detected crack length of 0.8 mm, the actual crack length distribution was found. The ORENDAs and L45LPI techniques were found to have about the same mean (1.55 mm) but the scatter in the ORENDAs results was much greater giving an upper 95% confidence bounds of 3.42 mm compared to 2.90 mm. The third LPI technique, L60 proved better with a mean of 1.27 mm and even less scatter. The CSCAN had the advantage of overestimating the crack length (conservative) so that given a detected crack length of 0.8 mm, the actual crack length was 0.65 mm. The scatter was about the same as L45 and L60, Table 5.14. From the linear regression results of Actual vs NDI Indicated Crack Length it is evident that:

- The LPI techniques tended to underestimate the actual crack lengths.
- The ultrasonic C-scan tended to overestimate the actual crack lengths.

In order to check the prediction of NDI techniques for longer cracks, NDI indicated crack lengths were calculated for an actual measured crack of 4.5 mm length. Using the same basic equations with the appropriate linear regression results, the mean and 95% confidence bounds for the NDI indicated crack length

are shown in Table 5.15. The general conclusions are the same as above but in this case it is even more clear that the order of preference for NDI is first the C-scan, and secondly the LPI techniques in the order of L60, L45 and lastly Orenda. This can be seen directly from Table 5.15 where first, the C-scan NDI mean crack length is the closest to the actual crack length and Orenda's was the farthest, and secondly, the conditional standard deviation is the smallest for the C-scan and largest for Orenda.

5.1.4 Comments About The NDI Techniques

Some general comments about each NDI technique are as follows:

- **ORENDA:** LPI carried out at Orenda did not appear to be significantly hampered by the production environment and the consistent detection of cracks of a particular size showed clearly in the analysis to be an attribute. The largest crack missed was 3.98 mm and the smallest crack detected was 0.50 mm. The correlation between the actual crack length and the NDI measured crack length was the worst of all the techniques.
- **L45:** This LPI technique carried out in a laboratory environment clearly showed the effect of carelessness. The results showed that although the technique was capable of detecting most small cracks, it missed a few large ones and the POD curve and distribution of cracks missed were significantly affected by this carelessness.
- **L60:** This LPI technique probably gives the best results as can reasonably be achieved by LPI. The technique and the personnel involved not only consistently detected small cracks but did not miss any large cracks. Overall this was the second best NDI technique. The crack length at 90% POD was 3.3 mm.
- **EC2:** This eddy current technique at a less sensitive setting was comparable to L45 in that it was not consistent. This may have been due to personnel interpretation or the technique itself since the gain setting was probably too low.
- **EC5:** This NDI technique was the most sensitive but was probably set too sensitive since 73 bolt holes said to contain cracks in fact did not. The indications were attributed to porosities, out of round holes, and interior defects. Thus the POD curve was excellent but at the expense of a high number of false calls.

A disadvantage with the eddy current techniques used in this experiment was that no relation could be found between the crack length and the signal amplitude and therefore only an indication that a crack exists rather than the crack length was noted for each bolt hole.

- **CSCAN:** Overall, this ultrasonic technique with results output in the form of a C-scan representation was considered to be the best NDI technique. 80% of all the cracks were detected with only 4 false calls. An additional advantage was a diagrammatic computer output so that a permanent record of each crack length was available and could be checked. Cracks above 1 mm could be sized and while cracks less than this length could not be sized accurately, they were detected consistently as small as 0.5 mm. The crack length at 90% POD was 1.6 mm.

5.2 Sensitivity Analyses for the Predicted 0.1% SII

A sensitivity analysis was carried out in order to determine the effect of different input conditions on the 0.1% SII output result. One input condition at a time was varied while the rest were kept constant. All variables held constant had default values as used in the typical DISK2 run documented in Appendix D.

5.2.1 Effect of Initial Conditions on the 0.1% SII

The types of initial conditions are discussed in an earlier section and summarized with the results in Table 5.1. These runs assumed that all the cracks were detected and therefore no NDI uncertainty was introduced. In the case where an initial crack length was greater than dysfunction, the disc was removed from further analysis. eg. LPI at Orenda showed that 5 out of 5000 cracks could be more than 4.5 mm. The results show that there is negligible difference in the 0.1% SII when using a constant value of 2000 hours instead of a TTCI distribution. This is because the SII is calculated by subtracting the initial TSN from the final TSN, and an encouraging result since both the predicted 0.1% SII's of 1006 and 1015 hours respectively are close and conservative to the value that would be predicted by using the GEC(1983) N_i and N_d lines at 0.1% which gives $2285-1067=1218$ hours.

Secondly, when a distribution of cracks missed is used as an initial condition, the 0.1% SII is markedly reduced if the NDI technique does not consistently detect cracks. For example:

- If the initial crack lengths at safe life are represented by the distribution of cracks missed by LPI Orenda as compared to the Ultrasonic C-Scan, the 0.1% SII drops from 848 to 311 hours.
- EC2 has the worst 0.1% SII because the distribution has a large standard deviation, despite the fact that the mean crack length of the distribution is quite acceptable.

5.2.2 Effect of NDI Inspection Uncertainty on the 0.1% SII

(a) Inaccurate Measurement of Crack Length

If an NDI technique detects a crack and measures it to be 0.8 mm, the actual crack length depends on the accuracy of the NDI technique. This section investigates the effect of this inaccuracy on the 0.1% SII.

The most significant observation was that if a distribution is used for a crack length as opposed to a mean value, regardless of whether the new mean is an underestimate or overestimate, the 0.1% SII seems to be lowered. Thus, even using the C-Scan results, where a mean of 0.65 mm should predict a longer 0.1% SII than a mean of 0.8 mm (as in the standard case), nevertheless the 0.1% SII is almost halved from 1015 hours to 596 hours. The low value for the 0.1% SII can be attributed to the scatter created in the SII distribution (.4059 as opposed to .1873) since the median for the C-Scan is actually lower than if no distribution was used (Table 5.17).

Likewise, since the LPI methods generally underestimated the crack lengths, the actual crack length was even longer than predicted by NDI, and the time to grow to dysfunction less. Thus the 0.1% SII's are even less than predicted using the C-Scan results which overestimated the actual crack length.

Finally, excellent correlation coefficients of 0.99 indicated that the SII's did follow a lognormal distribution

(b) Different POD Curves and Different Distributions of Cracks Missed

In the computer program, if a crack is not detected by NDI according to the POD curve, then a crack length is chosen from a distribution of crack lengths that may be missed by an NDI technique. Firstly, only the POD curve is varied for different NDI techniques, then only

the distribution of cracks missed is varied, and finally both are varied together.

When the POD curve is varied, contrary to what might be expected, it appears to have very little effect on the end result. The 0.1% SII values range from 842 hours to 931 hours (See Table 5.18). On the other hand, when the POD curve is held the same while the distributions of cracks missed are varied, a significant change in the 0.1% SII is noticed. Likewise, if both the POD curve and the distribution of cracks missed are varied, the 0.1% SII is affected. These are probably the most realistic results of the three in that if a particular NDI technique is being used to inspect the discs then if a crack is not detected, the crack length assumed to have been missed should come from the distribution of cracks missed by that particular NDI technique. The best LPI technique showed a 0.1% SII of 524 hours while 880 hours was predicted for the C-scan (See Table 5.18).

(c) 95% Confidence Bound on the POD Curves

As in the third case in (b), both the POD curve and the distribution of cracks missed were varied together. However, a 95% confidence bound on the POD curve was used instead of the mean. The 0.1% SII's were not significantly different and only dropped slightly in all cases, for example 387 to 347 hours, or 880 to 858 hours (See Table 5.18).

It should be noted that all values less than 600 hours in Table 5.18 have correlation coefficients of 0.96 or less compared to the others at 0.98 and above. The distributions were found to be Weibull rather than lognormal but the general conclusions remain valid and the results are useful for trends if not numerically accurate.

5.2.3 Effect of Crack Growth Rate on the 0.1% SII

Different crack growth rates have a significant effect on the 0.1% SII.

- The spin pit test shows a 0.1% SII of half the most severe traditional laboratory crack growth tests (ie. 588 hours compared to 1015 hours). However, it should be strongly noted that unorthodox calculations were performed in order to use the spin pit data in the simulation program.
- If the TSN hours on the disc or the temperature at which the laboratory experiment is carried out is increased, the 0.1% SII decreases by about 30% (ie. 1456 to 1015 hours and 1530 to 1015 hours respectively) (See Table 5.19).

Table 5.1: Actual Cracked and Uncracked Bolt Hole Data in the Ten Discs.

DISC	NO. OF CRACKS	NO NOT CRACKED	TOTAL NO. OF BOLT HOLES
A	19	21	40
B	12	28	40
C	8	32	40
D	30	10	40
E	24	16	40
F	16	24	40
G	21	19	40
H	23	17	40
I	31	9	40
J	21	19	40
TOTAL	205	195	400

Table 5.2: Summary of Cracked Bolt Hole Data for each NDI Technique.

CRACK DATA	NDI TECHNIQUE					
	LPI			EDDY CURRENT		ULTRASONIC
	ORENDA	L45	L60	EC2	EC5	CSCAN
Detected						
No	92	94	120	115	187	165
%	45%	46%	59%	56%	91%	80%
Missed						
No	113	111	85	90	18	40
%	55%	54%	41%	44%	9%	20%
Total						
No	205	205	205	205	205	205
%	100%	100%	100%	100%	100%	100%
False Calls						
No	3	4	8	27	73	4
%	1.5%	2%	4%	13%	36%	2%

Table 5.3: Number of Cracks Detected by each NDI Technique.

DISC	NDI TECHNIQUE					
	LPI			EDDY CURRENT		ULTRASONIC
	ORENDA	L45	L60	EC2	EC5	CSCAN
A	7	5	11	12	18	19
B	8	6	6	12	11	10
C	5	6	6	1	6	6
D	8	6	16	17	28	21
E	11	12	15	14	21	22
F	7	8	8	12	13	11
G	13	10	11	8	18	14
H	12	18	20	4 A	22	18
I	13	16	18	22	30	23
J	8	7	9	13	20	21
TOTAL	92	94	120	115	187	165

Table 5.4: Number of Cracks Missed by each NDI Technique.

DISC	NDI TECHNIQUE					
	LPI			EDDY CURRENT		ULTRASONIC
	ORENDA	L45	L60	EC2	EC5	CSCAN
A	12	14	8	7	1	0
B	4	6	6	0	1	2
C	3	2	2	7	2	2
D	22	24	14	13	2	9
E	13	12	9	10	3	2
F	9	8	8	4	3	3
G	8	11	10	13	3	7
H	11	5	3	19	1	5
I	18	15	13	9	1	8
J	13	14	12	8	1	0
TOTAL	113	111	85	90	18	40

Table 5.5: Number of False Calls for each NDI Technique.

DISC	NDI TECHNIQUE					
	LPI			EDDY CURRENT		ULTRASONIC
	ORENDA	L45	L60	EC2	EC5	CSCAN
A	1	0	0	4	16	1
B	1	1	2	8	7	2
C	0	1	0	0	5	0
D	0	0	0	0	7	0
E	1	1	2	1	4	0
F	0	0	1	8	7	1
G	0	0	1	0	6	0
H	0	1	1	0	8	0
I	0	0	1	3	6	0
J	0	0	0	3	7	0
TOTAL	3	4	8	27	73	4

Table 5.6: Cracks Detected by Different NDI Techniques and the Smallest Crack Detected.

100

NDI TECHNIQUE	NO OF CRACKS DETECTED	% OF CRACKS DETECTED TOTAL=205 (%)	SMALLEST CRACK DETECTED (mm)
ORENDA	92	44.9	.50
L45	94	45.9	.55
L60	120	58.5	.55
EC2	115	56.1	.30
EC5	187	91.2	.09
CSCAN	165	80.5	.17

Table 5.7: Linear Regression Results for Loglogistic POD Curves.

NDI TECHNIQUE	N	LINEAR REGRESSION		
		INTERCEPT	SLOPE	$s_{y/x}$
ORENDA	9	- .826	2.145	.8714
L45	10	-.754	1.965	.7572
L60	9	-.133	1.933	.4818
EC2	10	-.495	1.549	.6069
EC5	7	1.885	0.956	.6115
CSCAN	4	1.253	2.048	.2868

Table 5.8: Constants for Confidence Bounds on the Loglogistic POD Curves using F and Normal Distributions.

NDI TECHNIQUE	NO OF DATA POINTS		CONSTANT FOR CONFIDENCE BOUND	
	N	N-2	$F_{2,N-2}^{95\%}$	$t_{975,N-2}^{95\%}$
ORENDA	9	7	4.74	2.365
L45	10	8	4.46	2.306
L60	9	7	4.74	2.365
EC2	10	8	4.46	2.306
EC5	7	5	5.79	2.571
CSCAN	4	2	19.00	4.303

Table 5.9: Loglogistic Parameters for the POD Curves for each NDI Techniques.

NDI TECHNIQUE	LOGLOGISTIC PARAMETERS				
	MEAN LINE		CONFIDENCE BOUND		
			Normal Distr		t Distr
			95%	50%	95%
	β_0	β_1	β_0	β_0	β_0
ORENDA	-.827	2.145	-2.534	-1.413	-2.887
L45	-.754	1.965	-2.238	-1.265	-2.501
L60	-.133	1.933	-1.244	-0.458	-1.273
EC2	-.495	1.549	-1.685	-0.905	-1.895
EC5	1.885	0.956	0.686	1.472	0.312
CSCAN	1.253	2.048	0.691	1.060	0.019

Table 5.10: Cracks Missed by Various NDI Techniques and the Largest Crack Missed.

NDI TECHNIQUE	NO OF CRACKS MISSED	% OF CRACKS MISSED TOTAL=205 (%)	LARGEST CRACK MISSED (mm)
ORENDA	113	55.1	3.98
L45	111	54.2	4.33
L60	85	41.5	3.69
EC2	90	43.9	4.43
EC5	18	8.8	2.7
CSCAN	40	19.5	1.48

Table 5.11: Lognormal Analyses of Missed Cracks.

NDI TECHNIQUE	CORRELATION COEFFICIENT	LEAST SQUARES		MAXIMUM LIKELIHOOD	
		λ (mm)	ζ (mm)	λ (mm)	ζ (mm)
ORENDA	.970	-0.08	.8032	-0.08	.7338
L45	.975	-0.06	.8227	-0.06	.7550
L60	.984	-0.24	.8381	-0.24	.7692
EC2	.992	-0.10	.9221	-0.10	.8552
EC5	.973	-0.87	.9103	-0.87	.7428
CSCAN	.984	-0.68	.7438	-0.68	.6579

Table 5.12: Understanding the Distribution of Crack Lengths Missed.

NDI TECHNIQUE	CRACK LENGTHS MISSED		
	MEDIAN	95% BOUNDS	
		LOWER	UPPER
ORENDA	.92	.19	4.46
L45	.94	.19	4.72
L60	.79	.15	4.07
EC2	.90	.15	5.51
EC5	.42	.07	2.49
CSCAN	.51	.12	2.18

Table 5.13: Correlation between the Actual Crack Length and the NDI Inspected Crack Length.

NDI TECHNIQUE	NDI TECHNIQUE					
	NDI vs Actual			Actual vs NDI		
	Slope (mm)	Intercept (mm)	$s_{y/x}$ (mm)	Slope (mm)	Intercept (mm)	$s_{y/x}$ (mm)
ORENDA	1.0109	.2443	1.2748	.5672	1.0986	.9549
L45	1.0282	-.4645	0.7895	.7686	0.9461	.6826
L60	1.0200	-.2108	0.6705	.8087	0.6225	.5971
CSCAN	0.8435	.6257	0.6257	.9212	-.0886	.6538

Table 5.14: Actual Crack Length Distribution given an NDI Inspected Crack Length of 0.8 mm in Length.

NDI TECHNIQUE	NDI CRACK LENGTH	ACTUAL CRACK LENGTH (using linear regression)				CONCLUSION in terms of CRACK LENGTH
		MEAN	ST DEV	95% CONFIDENCE		
				Lower	Upper	
ORENDA	0.8	1.55	.9549	-.32	3.42	LPI underestimates
L45	0.8	1.56	.6826	.22	2.90	LPI underestimates
L60	0.8	1.27	.5971	.10	2.44	LPI underestimates
CSCAN	0.8	0.65	.6538	-.63	1.93	CSCAN overestimates

Table 5.15: NDI Inspected Crack Length Distribution for a Known Crack Length of 4.5 mm.

NDI TECHNIQUE	ACTUAL CRACK LENGTH	NDI CRACK LENGTHII (using linear regression)				CONCLUSION about NDI TECHNIQUE
		MEAN	ST DEV	95% CONFIDENCE		
				LOWER	UPPER	
	(mm)	(mm)	(mm)	(mm)	(mm)	
ORENDA	4.5	4.77	1.2748	2.27	7.27	Worst
L45	4.5	4.16	.7895	2.61	5.71	Medium
L60	4.5	4.38	.6705	3.07	5.69	Better
CSCAN	4.5	4.44	.6257	3.21	5.67	Best

Table 5.16: Effect of Initial Conditions on the 0.1% SII.

INITIAL CONDITIONS	SAFE INSPECTION INTERVAL <i>Lognormal Distribution</i>					
	No of Discs	λ (hrs)	ζ (hrs)	0.1% (hrs)	Median (hrs)	CC
0.8 and 2000	5000	7.50	.1895	1006	1807	.9740
0.8 and TTCI	5000	7.50	.1873	1015	1811	.9745
Cracks missed by NDI and Safe Life = 2285 hrs						
ORENDA	4995	7.32	.5097	311	1503	.9452
L45	4990	7.30	.5211	295	1475	.9520
L60	4995	7.43	.4580	411	1690	.9647
EC2	4979	7.31	.5872	243	1489	.9310
EC5	5000	7.78	.3383	839	2387	.9729
CSCAN	5000	7.71	.3131	880	2232	.9808

Table 5.17: Effect of NDI Inaccuracy on the 0.1% SII, when Estimating the Actual Crack Length.

NDI TECHNIQUE USED TO DETERMINE ACTUAL CRACK LENGTH	SAFE INSPECTION INTERVAL (SII)					
	No of Discs	λ (hrs)	ζ (hrs)	0.1% (hrs)	Median (hrs)	CC
None	5000	7.50	.1873	1015	1811	.9745
ORENDA	5000	6.95	.5100	216	1045	.9962
L45	5000	6.94	.3801	320	1035	.9942
L60	5000	7.15	.3506	433	1279	.9912
CSCAN	5000	7.65	.4059	596	2091	.9917

Table 5.18: Comparison of 0.1% SII for Different NDI Techniques in terms of POD Curves, Distributions of Cracks Missed, and 95% Confidence POD Curves.

NDI TECHNIQUE	0.1% SAFE INSPECTION INTERVAL				
	ASSUME ALL CRACKS DETECTED	EFFECT OF ...			
		POD CURVE	CRACKS MISSED	BOTH	BOTH AT 95% POD
ORENDA	1015	842	599	387	347
L45	1015	843	593	369	321
L60	1015	852	690	524	467
EC2	1015	844	519	355	273
EC5	1015	931	838	899	827
CSCAN	1015	880	880	880	858

Table 5.19: Effect of Crack Growth Rate on the 0.1% SII.

CRACK GROWTH RATE CONDITIONS	SAFE INSPECTION INTERVAL <i>(Lognormal Distribution)</i>				
	λ (hrs)	ζ (hrs)	0.1% (hrs)	Median (hrs)	CC
High time, high temp	7.50	.1873	1015	1811	.9745
Medium time, high temp	7.61	.1059	1456	2019	.9409
High time, room temp	7.83	.1623	1530	2527	.9499
Spin pit test	6.81	.1417	588	911	.9641

Figure 5.1: Comparison of NDI Techniques in terms of the Number of Detected and Missed Cracks, and the Number of False Calls.

Histogram Showing the Total Number of Cracks and the Detected Number of Cracks per Interval

Histogram Showing the Total Number of Cracks and the Detected Number of Cracks per Interval

Histogram Showing the Total Number of Cracks and the Detected Number of Cracks per Interval

Figure 5.2: Histograms of Detected and Total Number of Cracks for each NDI Technique.

Histogram Showing the Total Number of Cracks and the Detected Number of Cracks per Interval

Histogram Showing the Total Number of Cracks and the Detected Number of Cracks per Interval

Histogram Showing the Total Number of Cracks and the Detected Number of Cracks per Interval

Figure 5.2: Cont.

**Figure 5.3: Probability of Detection Curves for
each NDI Technique.**

POD Curve for Eddy-Current at AMPU
for Gas Turbine Disc Bolt Holes

POD Curve for Eddy-Current at AMPU
for Gas Turbine Disc Bolt Holes

POD Curve for C-Scan at NAE
for Gas Turbine Disc Bolt Holes

Figure 5.3: Cont.

Figure 5.4: Comparison of 90% POD Values for each NDI Technique.

Figure 5.8: Comparison of the Median Crack Length Missed by each NDI Technique.

Histogram of Crack Lengths Missed
for Gas Turbine Disc Bolt Holes

Histogram of Crack Lengths Missed
for Gas Turbine Disc Bolt Holes

Histogram of Crack Lengths Missed
for Gas Turbine Disc Bolt Holes

Figure 5.5: Histograms of Crack Lengths Missed
for each NDI Technique.

Histogram of Crack Lengths Missed
for Gas Turbine Disc Bolt Holes

Histogram of Crack Lengths Missed
for Gas Turbine Disc Bolt Holes

Histogram of Crack Lengths Missed
for Gas Turbine Disc Bolt Holes

Figure 5.5: Cont.

Lognormal Analysis

for Crack Lengths Missed by NDI

(a) ORENDA

Lognormal Analysis

for Crack Lengths Missed by NDI

(b) L45

Lognormal Analysis

for Crack Lengths Missed by NDI

(c) L60

Figure 5.6: Lognormal Analysis of Crack Lengths Missed for each NDI Technique.

Lognormal Analysis
for Crack Lengths Missed by NDI

(d) EC2

Lognormal Analysis
for Crack Lengths Missed by NDI

(e) EC5

Lognormal Analysis
for Crack Lengths Missed by NDI

(f) CSCAN

Figure 5.6: Cont.

Figure 5.7: Distributions of Crack Lengths Missed by each NDI Technique.

Figure 5.7: Cont.

Figure 5.9: The Correlation between the NDI Measured and the Actual Crack Length.

Figure 5.9: Cont.

Figure 5.10: Distributions of Actual Crack Lengths, for various NDI Techniques, given an NDI Measured Crack Length of 0.8 mm.

Chapter 6

DISCUSSION

This chapter is divided into two parts. The first part discusses the experimental NDI results including the confidence bounds on the POD curves. The second section is concerned with the prediction of a safe inspection interval using the DISK2 PFM computer simulation programme. The PFM simulation results are compared with in-service data, the sensitivity analysis is discussed, and the factors that most significantly affect the SII are pointed out. Lastly, a comparison is made between deterministic and probabilistic based safe inspection interval predictions.

6.1 Sensitivity and Reliability of Various NDI Techniques

The 90% POD values for ORENDA, L45, L60, EC2, EC5, and CSCAN are 4.0, 4.5, 3.3, 5.7, 1.4, and 1.6 mm respectively (Figure 5.4). Remembering that the eddy current technique with gain setting 5 had an unsuitably large number of false calls, the C-Scan followed by LPI with the 60 minute dwell time were the best overall techniques. The 90% POD value is a commonly chosen value in the literature and has been used here for assessing the reliability of the NDI techniques since in practical terms for design, the crack length is too long if 100% reliable detection is chosen. The NDI techniques were compared with each other using the 90% POD value with no associated confidence because the results vary greatly depending on the distribution assumed for the confidence bounds. If the distribution through the actual POD data is a cumulative binomial (ie. discrete), then for a large population, the confidence on the POD data points can be approximated by a Gaussian distribution by virtue of the central limit theorem. On the other hand, if a distribution such as a loglogistics curve is fit through the POD data points, then confidence on this mean curve can be determined by assuming that the POD for a crack length varies according to an F-distribution, a normal

distribution or a Student t distribution: The loglogistic curve was used because it is a continuous function unlike the cumulative binomial, and therefore the POD increases continuously with crack length as one would expect realistically rather than in steps. All three distributions used to find confidence bounds on the POD curve were "suitable" but the normal distribution was used for comparison purposes since the F distribution did not always reach 90% POD and the distribution was very similar to the normal distribution.

The 90/95 crack length values for ORENDA, L45, L60, EC2, EC5, and CSCAN are 9.0, 9.2, 5.5, 12.0, 5.0, and 2.1 mm respectively and the 90/50 values are 6.8, 7.1, 4.5, 9.0, 3.0, and 1.8 mm respectively. The 90/95 limit implies that the lower bound on the mean 90% POD value is above this limit with 95% confidence. Likewise, the lower bound on the mean 90% POD value has a lower value of the 90/50 limit with 50% confidence.

The POD curve was significantly worsened if a larger crack was unexpectedly missed. Therefore a desirable NDI technique is not only one which is able to detect small crack lengths but also one in which cracks of a given length and above are consistently detected. Again, C-scan was considered the best NDI technique.

There may be a source of error in the results because all the potential cracks were not pried open. If all the potential cracks were pried open and cracks were found, the confidence in the NDI database would be improved even if the POD data point was lowered for small cracks.

Although the C-scan technique was the best overall technique, two improvements are possible. Firstly, a more sensitive transducer, eg. 100 MHz probe rather than the 10 MHz probe used, operating at its optimum power would be able to more accurately detect smaller cracks since it produces a narrower beam. Secondly, the C-scan equipment has the ability to magnify certain areas of the disc and this would allow more accurate measurement of small cracks. The eddy current NDI technique has been used widely and successfully for detecting small LCF fatigue cracks. Unfortunately this was not our experience and much time would need to be spent optimizing the gain setting as well as developing a procedure for converting signal amplitude to crack length before this NDI technique would be a viable alternative. The LPI technique is limited in its success by its method since the crack length is generally measured by eye using a vernier. It may be significant that LPI at AMDU predicted actual crack lengths more accurately than LPI at Orenda. Apart from the technique this may be due to the measuring instrument since AMDU used a digital read out vernier whereas Orenda's results were given in terms of $1/16^{th}$ or $1/32^{th}$ and rarely less than $1/32^{th}$ inch = 0.8 mm.

6.2 Comparison of Confidence Bounds on Binomial and Loglogistic POD Curves

An example of confidence bounds on the binomial distribution is shown in Figure 6.1 and can be compared with the loglogistic curves and their confidence bounds in Figure 5.3. Note that the observed data points in both cases are the same and were calculated as binomial points using the range interval method.

The following observations about confidence bounds on the binomial POD curve were made:

- The confidence bounds using the range interval method fluctuate with crack length.
- The confidence bounds using the Optimized Probability Method (OPM) increase continuously but the meaning of the results is subtly different. These points were calculated by cumulatively adding from the largest crack length to the smallest crack length. This means, for example, that a crack of 4.5 mm *and longer* has an 80% chance of detection as opposed to a crack length of say *between* 4.0 and 5.0 mm having this chance of detection. This of course is OK provided one realizes the difference.

Although confidence bounds on a binomial are often approximated using graphs (as used in above) which can be found in many books [37] [8] [36], Rummel [86] summarizes a more exact equation method used below. Table 6.1 gives a comparison of the loglogistic curve with confidence bounds calculated using two different distributions (the normal distribution and the F distribution), and the binomial curve also with confidence bounds using two different methods (range interval and OPM).

Two observations were obvious when the binomial and the loglogistic POD curves were compared:

- As the actual observed POD increases from 60% to 100% and again 100%, the binomial means are exact whereas the loglogistic means approximate the value, 65% as compared to 60%, and asymptote towards 100% ie. 92% and 98%.
- The confidence bounds on the loglogistic curve are more realistic since they increase (50%, 85% and 97%) with increasing observed POD whereas the confidence bounds on the binomial often depend primarily on sample size and in this case decrease from 87% to 64% as the crack grows longer from 2.0 to 4.5 mm.

6.3 Simulation Model in Relation to In-Service Behaviour of Components

The initial conditions, ie. distribution of crack lengths when the disc is retired at safe life, were not available, but both the TTCI distribution for 0.8 mm and distributions of cracks lengths missed by the 6 NDI techniques were considered conservative estimates. It was also possible to model nondestructive inspection very accurately using POD curves based on sufficient experimental work carried out.

The weakness in the simulation program was in the modelling of the crack growth rate. Laboratory experiments gave results for parameters C and n [5] while spin pit testing suggested a constant crack growth rate between 1 and 6 mm [43]. Preliminary finite element analysis showed that ΔK remained constant over a crack length interval when defined as a 2D problem, and increased with increasing crack length when defined as a 3D problem (See Figure 6.2) [52].

Lastly, of course, the fatigue cycles that a disc undergoes has to be converted to flying hours using a mission severity factor. More recently aircraft are being fitted with monitors which find a count number based on engine rpm and thus life consumed can be determined depending on actual user missions [75]. For this analysis, a conversion of 5 cycles per flying hour was used as calculated from fractographic examination of striations and analysis of mission profiles [97]. The controlled random crack growth rate simulated was compared with in-service crack growth rate of cracks when the discs were flying using the distributions of crack lengths for various TSN's. Despite the approximations the simulated crack growth rate was found to give realistic results as seen below.

6.3.1 Comparison of In-Service Inspection Data with Fractographic Data

The maintenance contractor for the J85 CAN40 engines, Orenda, carried out routine LPI on compressor discs and spacers recording the position and length of cracks detected and the TSN. Data collected from 1976 to the end of 1984 included 275 fifth stage J85 CAN40 compressor discs with TSN's from 12.2 to 5500 hours. Cracks were detected in 103 out of a total of 275 retired discs (See Table 6.2). The 103 discs with various TSN's and the 553 cracks of various crack lengths can be correlated in two ways:

- i. Distribution of TSN's for specific crack lengths, which was done by the author in a previous report.

- ii. Distribution of crack lengths for a specific TSN interval, presented here in order that the simulated crack growth could be compared with the actual crack growth experienced by the discs in service (Table 6.3).

Distributions of crack lengths were analyzed for 4 TSN ranges, 3000-3500, 3500-4000, 4000-4500, and 4500-5000 hours since there was insufficient data elsewhere. (Figure 6.3) shows histograms of the cracks detected within these TSN ranges. It was assumed that since an LPI method was used to detect the cracks that many cracks less than some reliably detectable crack length would exist in the discs. Therefore, the analysis assumed that bolt holes where no cracks were detected had cracks of less than detectable length. This seemed to imply that the distribution of crack lengths would be exponential or a Weibull with characteristic value less than unity, however, after analyses were completed the lognormal distribution correlated better with the data than the Weibull. The major percentiles and parameters for the lognormal distribution are in Table 6.4 and Figure 6.4 shows how the crack lengths increase as the TSN increases.

Specific comparisons between in-service inspection data and fractographic data are as follows:

- Most discs were retired in the 4500-5000 TSN range (there were 82 discs in this range compared to 38, 27, or 16 discs respectively in the other TSN ranges) and therefore this data may have more significance. As expected, the ten discs destructively examined had an average TSN of 4063 hours.
- The longest crack detected by in-service LPI in the 3500-4000 hours range was 1/2 inch (12.7 mm) whereas the longest crack detected in the next interval 4000-4500 hours was 5/8 inch (15.9 mm). The longest crack found fractographically in the ten discs which had an average of 4063 hours was 8.34 mm.
- The crack length distribution (lognormal) for a particular TSN or range interval of TSN using crack lengths detected in-service using LPI was very similar to the crack length distribution when destructive methods were used to measure the crack lengths.

Lastly, the entire simulation is based on one crack initiating from one bolt hole in a disc. However, we need to remember that in practice the number of cracks per disc increases with the time the disc remains in service. As the TSN increase from 3000 to 5000 hours, the number of cracks increase from 1 crack per disc to

almost 5 cracks per disc (Figure 6.5). The value of 3.7 cracks per disc in the TSN range 5000-5500 hours can be regarded with suspicion since the value is from only 2 cracked discs compared to say the 4000-4500 hour range which had 57 cracked discs (Table 6.2).

6.3.2 Comparison of In-Service vs Simulated Crack Growth Rate

Four crack growth rates were used in the simulation and are referred to as slowest, medium slow, medium fast, and fastest respectively. Also 4 distributions of crack lengths with means of 3250, 3750, 4240, and 4750 TSN hours respectively were used. The distributions of crack lengths were input as initial condition, allowed to grow for 500, 1000, or 1500 hours as required. Then the output of distribution of crack lengths from the simulation model was compared to the actual distribution of crack lengths. Table 6.5 is a summary of the results giving the overall conclusion that real crack growth rate is somewhere between the fastest and slowest crack growth rates used in the simulation and in fact favours the slower crack growth rates meaning that the simulation produces conservative results.

Studying the results more closely and bearing in mind that the program uses a ΔK which continuously increases for increasing crack length, one could come to the conclusion that ΔK is too small for the small cracks and too large for the longer cracks, and this in turn may imply that ΔK remains constant over a certain crack length interval.

As an aside, unorthodox methods were used to predict a distribution of cracks lengths at 2750 hours (an approximate safe life for the discs) giving a lognormal distribution with parameters $L(-5.04, 2.45)$ and median 0.007 mm. Defying the laws of crack growth for small cracks and treating the distribution as an EIFS distribution, the crack lengths were allowed to grow for 1000 hours and 2000 hours respectively. In both cases the slowest crack growth rate was not quite slow enough, which may mean that the crack growth rate simulated is too fast or that the actual distribution of crack lengths at safe life is smaller or that in fact the analysis is erroneous to begin with.

6.4 The Distribution of Safe Inspection Intervals

6.4.1 Theoretical Distribution of SII's

In the very simplest simulation, all cracks were assumed detectable, thus any filtering by the POD curve or substitution of a crack length from a distribution

of cracks missed did not need to be taken into account. Secondly the initial crack length was constant and thirdly the initial TSN was constant so that the only variability taken into account was the crack growth rate.

From the randomized equation of crack growth rate we have:

$$SII = \int_{N_i}^{N_d} dN \quad (6.1)$$

$$= \int_{a_i}^{a_d} \frac{da}{XC(\Delta K)^n} \quad (6.2)$$

where X is a random number lognormally distributed and ΔK is a function of the square root of the crack length. Expanding we get the the following:

$$SII = \frac{\text{constant}}{X} \left[\left(\frac{1}{a_i} \right)^{\frac{n}{2}-1} - \left(\frac{1}{a_d} \right)^{\frac{n}{2}-1} \right] \quad (6.3)$$

Since a_i and a_d for this simplest case are constants, X is lognormal, and the quotients of lognormals are lognormal, the SII will be a lognormal distribution.

6.4.2 Comparison of Lognormal and Weibull Distributions of SII's

The lognormal distribution adequately represented the safe inspection interval's in most cases. However, in some cases, for example, when the "distributions of cracks missed" for the *worse* NDI techniques (ORENDA, L45, L60, and EC2) were used if a crack was not detected, the Weibull was found to be a better fit. On the other hand, Figure 6.6 illustrates that when an LPI technique (L60) was used for inspection, the SII's were found to fit a Weibull distribution better than a lognormal, whereas when the C-scan technique was used, the SII's were characterized better by a lognormal distribution.

6.5 Factors That Most Affect the Safe Inspection Interval

In order to improve the SII most cost effectively, it is important to know not only how each variable affects the SII, but which factors affect it most. To carry out a sensitivity analysis all variables must be kept the same except the one which is being varied to see its effect on the final result.

By performing a sensitivity analysis, it was possible to discover whether particular variables in the life prediction process affected the 0.1% SII significantly or

insignificantly. The crack growth rates and distributions of crack lengths missed (used either as initial conditions or for crack lengths when NDI did not detect crack) were found to significantly affect the 0.1% SII by as much as 50%.

On the other hand, the POD curves used to simulate nondestructive inspection, or a 95% confidence bound on the POD curve, were found to have almost no effect on the 0.1% SII, for example at most 10%.

6.6 Deterministic, Combined Analysis, and Probabilistic Based SII's

6.6.1 SII by DFM

A safe inspection interval can be calculated to determine the remaining propagation life assuming an initial flaw size just large enough to have been missed, and an upper bound crack growth rate. Using results as stated in Reference [55] and [96], the safe inspection interval was calculated as follows:

$$\begin{aligned} \text{SII} &= \frac{a_d - a_i}{C(\Delta K)^n} \\ &= \frac{4.5 - 1.75}{1.53 \times 10^{-10} \times (45)^{2.3}} \end{aligned} \quad (6.4)$$

which gives 2833 cycles or 567 hours since 5 major throttle excursions occur per flight hour for the J85 CAN40 fifth stage compressor discs.

6.6.2 SII by TSO

The SII can be determined using field experience if Weibull analysis is performed by plotting the Time Since Overhaul (TSO) of discs cracked to dysfunction. After a t_0 correction [1], the 0.135% cumulative probability value is 556 hours [55]. Since the discs were replaced with new ones if a crack was detected, this means that crack growth during the TSO period for this analysis was from an undetectable crack length to a maximum crack length of dysfunction. This is a SII calculated by combined analysis based on PFM.

6.6.3 SII using the N_i and N_d Weibull Lines

Weibull analysis carried out on data provided by Orenda [25] [26] showed that the GE N_d line could be represented by considering failure of a disc to be when the maximum crack length in a disc reached dysfunction length. Insufficient data

prevented the same calculations for the N_i line, however TSN-TSO data for cracks to dysfunction length gave a line a little to the right of the N_i line which was proper since this represented time to detection rather than time to initiation. See Figure 6.7.

The SII could be calculated as follows:

$$\begin{aligned} \text{SII} &= \frac{0.1\%N_d - 0.1\%N_i}{\text{SafetyFactor}} \\ &= \frac{2285 - 1067}{2} \end{aligned} \quad (6.5)$$

which is approximately 600 hours if a safety factor of 2 and GE (1983) values are assumed. A SII based on NDI could be calculated if a 0.1% time to detection was used instead of the time to initiation.

6.6.4 Comparison of SII's Based on Deterministic and Probabilistic Methods

One of most useful features when calculating a SII based on a probabilistic method is that the risk can be quantified. Using Weibull analysis the 0.1% SII for a crack to grow from 0.8 mm to 4.5 mm was found to be 1218 hours. Then assuming a safety factor of 2, the actual SII was determined as 609 hours. In the probabilistic analysis, if all the cracks are detected, a crack grows from 0.8 mm to 4.5 mm at 0.1% in 1015 hours taking into account the variation in crack growth rate and this is conservative of 1218 hours.

Further, if we take into account the NDI technique being used, and therefore the probability of detecting a crack as well as the distribution of cracks that may have been missed, then the 0.1% SII changes according to the NDI technique. LPI at Orenda, LPI at AMDU (60 min dwell time) and C-Scan at NAE estimated 0.1% SII's of 387, 524, and 880 hours respectively (See Figure 6.8). Thus in effect the safety factor can increase or decrease depending on the accuracy of the NDI technique used.

A totally deterministic calculation uses worst case constants for the variables. One example as above suggests a SII of 567 hours. The manufacturer recommended a SII of 555 hours.

Table 6.1: Comparison of Mean and 95% Confidence Values for the Binomial and Loglogistic POD Curves.

EXAMPLE USING C-SCAN RESULTS	CRACK LENGTH (mm)		
	0.5-1.0	1.5-2.0	4.0-4.5
<i>Observations</i>			
No of cracks detected	25	27	6
Total no of cracks	42	27	6
Observed POD (Total for OPM)	60% (184)	100% (106)	100% (17)
<i>Loglogistics</i>			
Mean	65%	92%	98%
95% Confidence (normal distr)	50%	85%	97%
95% Confidence (F distr)	64%	80%	85%
<i>Binomial</i>			
Mean	60%	100%	100%
95% Confidence (range)	43%	87%	64%
95% Confidence (OPM)	52%	96%	79%

Table 6.2: Summary of Total Number of Bolt Hole Cracks found in Retired Fifth Stage J85 CAN40 Compressor Discs.

TSN (hours)	TOTAL NO OF CRACKS	NO OF CRACKED DISCS	TOTAL NO OF DISCS
0-500	0		37
500-1000	0		52
-1500	0		3
-2000	0		10
-2500	0		0
-3000	4	2	11
-3500	27	8	23
-4000	132	23	38
-4500	302	57	82
-5000	77	11	16
-5500	11	2	3
-6000			0
TOTAL	553	103	275

Table 6.3: The Crack Lengths found in Retired Fifth Stage J85 CAN40 Compressor Discs.

TSN (hours)	CRACK LENGTH (mm) and (inches)									
	1.6 $\frac{1}{16}$	3.2 $\frac{1}{8}$	4.8 $\frac{3}{16}$	6.4 $\frac{1}{4}$	7.9 $\frac{5}{16}$	9.5 $\frac{3}{8}$	11.1 $\frac{7}{16}$	12.7 $\frac{1}{2}$	14.3 $\frac{9}{16}$	15.9 $\frac{5}{8}$
0-500										
500-1000										
-1500										
-2000										
-2500										
-3000			2	1	1					
-3500	7	9	7	4						
-4000	33	61	18	12	3	1	3	1		
-4500	75	120	44	35	10	7	5	4	1	1
-5000	23	38	8	6			1	1		
-5500	1	2	5	2			1			
-6000										
TOTAL	139	230	84	60	14	8	10	6	1	1

Table 6.4: Lognormal Parameters for Crack Distributions, for TSN Intervals, in Retired Fifth Stage J85 CAN40 Compressor Discs.

TSN INTERVALS (hours)	PERCENTILES			PARAMETERS		CRACKS LARGER THAN DYSFUNCTION
	10 th (mm)	50 th (mm)	90 th (mm)	λ (mm)	ζ (mm)	
3000-3500	.026	.16	0.94	-1.83	1.44	0.7%
3500-4000	.12	.50	2.0	-0.69	1.13	2.0%
4000-4500	.15	.55	2.2	-0.60	1.07	2.2%
4500-5000	.25	.74	2.3	-0.30	0.89	2.3%

Table 6.5: Comparison of In-Service vs Simulated Crack Growth Rates.

DURATION OF CRACK GROWTH (hours)	TSN ON DISCS		BEST FITTING CRACK GROWTH RATE CURVE
	FROM (hours)	TO (hours)	
500	3250	3750	fastest
500	3750	4250	slowest
500	4250	4750	slowest
1000	3250	4250	medium slow
1500	3250	4750	slowest

Figure 6.1: Example of 95% Confidence Bounds on the Binomial POD Curve using the Range Interval Method and the Optimized Probability Method (ORM). [Stoute]

Figure 6.2: FEM Stress Analysis Results for the Fifth Stage J85 CAN40 Compressor Disc. [Kaprzynski]

Figure 6.3: Histograms of Crack Lengths in Retired Fifth Stage J85 CAN40 Compressor Discs, for various TSN Intervals.

Figure 6.4: Distributions of Crack Lengths of Retired Fifth Stage J85 CAN40 Compressor Discs for various TSN Values.

Figure 6.5: Average Number of Cracks Per Disc with respect to Time Since New (TSN).

Lognormal Analysis for Safe Inspection Interval

Weibull Analysis for Safe Inspection Interval

Figure 6.6: Comparison of Lognormal and Weibull Distributions for 0.1% SII's.

Weibull Analysis

for Safe Inspection Interval

Lognormal Analysis

for Safe Inspection Interval

Figure 6.6: Cont.

Figure 6.7: Weibull Analysis showing the GE N_i and N_d (1983) Distributions.

Figure 6.8: Comparison of SII's Calculated using Deterministic and Probabilistic Approaches.

Chapter 7

CONCLUSIONS AND FUTURE WORK

7.1 Conclusions

Turbine disc safe inspection intervals (SII) for a damage tolerance approach to maintenance were predicted using probabilistic fracture mechanics (PFM) principles. The effect of nondestructive inspection (NDI) uncertainties on the 0.1% SII was also shown. The fifth stage J85 CAN40 compressor discs used in the CT-114 Tutor Aircraft were chosen for a case study since this component is particularly prone to low cycle fatigue crack growth from its bolt holes.

Experimental work involved NDI of ten retired discs, each with forty bolt holes using 6 different NDI techniques, and the destructive examination of the fatigue cracks. The results, incorporating the work of three institutions are thought to be a significant contribution for the implementation of damage tolerance maintenance procedures which rely heavily on the inspection capabilities of various NDI techniques. The following observations were made about the experimental work:

1. Different NDI Techniques.

- The ultrasonic C-scan NDI technique produced the "best" results detecting 80% of all cracks with 4 false calls, and a 90% POD value of 1.6 mm.
- The three LPI techniques varied greatly, but the AMDU LPI technique with a 60 minute penetrant dwell time was considered overall the second best technique. The 90% POD value was 3.3 mm. The most significant single reason that this technique was second best even though other techniques could detect smaller crack lengths, was the consistency of both the technique and the personnel in detecting cracks.

- The microfocus X-ray radiography technique was rejected since it was an expensive process and gave results comparable to the LPI techniques.
 - The eddy current method, although potentially a very capable technique in this case did not produce very satisfactory results. The gain setting of 5 appeared to be too sensitive (73 false calls), and the gain setting of 2 was probably not sensitive enough. Another disadvantage in this case was that no crack lengths were estimated.
2. Carelessly missing a larger crack severely affected the capability of a technique. The eddy current technique at a gain setting of 2 (EC2) consistently detected smaller crack lengths than the LPI technique with a 60 minute penetrant dwell time (L60) - *smallest detected crack lengths were 0.30 and 0.55 mm respectively*. However EC2 missed a couple of large crack lengths whereas L60 did not - *largest crack lengths missed were 4.45 and 3.69 mm respectively*. The results show that L60 had a 90% POD value of 3.3 mm compared with EC2 which had a 90% POD value of 4.5 mm. The conclusion drawn is that it may be better to use a technique which consistently detects cracks up to a certain crack length rather than one that can detect very small cracks. Also, it is important that the machine and/or the personnel are thorough. One way of achieving higher reliability is simply to carry out the same inspection twice.
 3. Analyses to determine the correlation between the fractographically measured crack lengths and the NDI indicated crack lengths showed that:
 - LPI techniques tended to underestimate the actual crack length. (Perhaps this is because the maximum crack length is often in the center of the component rather than at either edge.)
 - The C-scan tended to overestimate the actual crack length.
 4. The POD curve characterizes the ability of a technique to detect cracks. It has been demonstrated that not all cracks of the same length have the same detection probability and this is one reason why the traditional binomial distribution was not used to represent the POD curve.
 - A loglogistics POD curve was found to adequately fit the observed data. The loglogistic parameters can be straightforwardly found by transforming the data and carrying out linear regression. The POD is

a function of the crack length, 'a', as follows:

$$POD = \frac{\exp(\beta_0 + \beta_1 \ln a)}{1 + \exp(\beta_0 + \beta_1 \ln a)}$$

where β_0 and β_1 are constants. Confidence bounds on the mean POD values were also found using standard statistical methods.

- The loglogistic curve which increases smoothly in a slanted S-shape and which is asymptotical at zero and 1, probably represents the true situation more accurately than the cumulative binomial which increases in steps as the crack length increases.
 - Confidence bounds on the loglogistic curve indicated the variability of the mean POD value and did not fluctuate with small sample sizes as in the case of the range interval binomial method.
 - Confidence bounds obtained by the optimized probability method have unknown statistical value in that the correlation between the POD value and the crack length is unknown. However these curves are well behaved and a comparison using an LPI technique shows that *'There was an 62% chance that a crack of length 2.0 mm and longer will be detected.'*, rather than the conventional *'There was an 55% chance that a crack of length 2.0 mm will be detected.'*
5. The two parameter lognormal adequately represented the distribution of cracks missed for each NDI technique.

A computer program using Monte Carlo simulation techniques was written in order to predict safe inspection intervals based on a probabilistic fracture mechanics approach. Initial conditions of crack length and TSN, in constant or distribution form, are used at the beginning of the program. NDI is then performed by the computer using a POD curve and if a crack is not detected, then a crack length is randomly chosen from a distribution of cracks that could be missed by that particular NDI technique. The crack is then allowed to grow in a controlled random fashion according to a randomized Paris crack growth equation. Finally, the disc is removed from service when the crack reaches dysfunction length or after a specified number of hours. Some comments about the program follow:

- A weakness in the program was the incomplete knowledge of the relationship between the crack length and the stress intensity although preliminary FEM had been carried out.

- Analysis showed that the simulated computer model crack growth rates compared well with the in-service crack growth rates.
- The distribution of safe inspection intervals was mostly lognormal and occasionally Weibull depending on the input distributions.
- Sufficient stability and convergence of the 0.1% SII was achieved with 5000 simulations.

The SII was affected by the NDI technique used for inspection when applying damage tolerance maintenance. The 0.1% SII's were 387 hours, 369 hours, and 524 hours respectively when the three LPI techniques were used compared to 880 hours when the ultrasonic C-scan NDI technique was used. A deterministic approach suggested a safe inspection interval of 567 hours but the values are difficult to compare since in the deterministic case the safety factor is unknown whereas in the probabilistic analysis, the failure rate is one in a thousand. The manufacturer recommended 555 hours.

7.2 Recommendations for Future Work

1. It would be useful to know the distribution of crack lengths in discs with a specific TSN, particularly at the safe life TSN. Unfortunately there is little real data available for this range of TSN but perhaps it could be extrapolated by using the crack growth model backwards to obtain some suitable equivalent initial quality size (EIFS) distribution.
2. The relationship between ΔK and the crack length is only approximately known particularly since preliminary analysis has been contradictory. It is not sure how certain factors, for example the changing thickness of the disc, affect the results. When the crack grows from the bolt hole, the disc has constant thickness for about 2 mm, then suddenly becomes thinner, and thereafter increases slightly in thickness towards the center of the disc. It has been suggested that a more rigorous FEM analysis needs to be carried out in order to obtain a better understanding of the stresses in the fifth stage J85 CAN40 compressor discs, and stress intensity factors at the crack tip [96] [5].
3. It has also been suggested that the use of corner cracked test pieces, rather than compact tension specimens, result in a better correlation between actual and predicted crack growth as applied to gas turbine components. Since this

simulation used results from compact tension specimens, it may be of interest to carry out crack growth experiments using the corner cracked test pieces.

4. The PFM computer simulation program as it stands predicts a safe inspection interval based on a crack length at which the discs should be withdrawn from service and one NDI inspection. It would be interesting and useful to modify the program in order to accommodate multiple NDI inspections at a fixed SII and to see how many discs have crack lengths larger than "dysfunction" length without being detected and withdrawn from service. Obviously, the shorter the SII, the more the chance of detection and the less the chance of failure. The SII could be predicted based on the failure rate tolerated so that safety is balanced with cost and efficiency.
5. The last recommendation for future work to improve the prediction of a SII concerns a cost analysis since in the end this is often the deciding factor. The most economical way to maintain aircraft engine components could be obtained by juggling the cost of inspection, the cost of replacement, and the cost of failure in order to determine the SII for the minimum cost. For fast growing cracks replacement with no SII may be the most economical.

Bibliography

- [1] Abernethy Dr RB, Breneman JE, Medlin CH, and Reinman GL. *Weibull Analysis Handbook*. AFWAL-TR-83-2079, Pratt and Whitney Aircraft, United Technologies Corporation, November 1983.
- [2] Abramowitz M; and Stegun IA. *Handbook of Mathematical Functions*. Page 933.
- [3] Addison RC. *An Enhancement for the Ultrasonic Test Bed To Inspect Engine Disk Bolt Holes*. Review of Progress in Quantitative Nondestructive Evaluation, Vol I. Editors Thompson DO and Chimenti DE, 1982, pp97-105.
- [4] AGARD-CP-393. *Damage Tolerance Concepts for Critical Engine Components*. North Atlantic Treaty Organization. Papers presented at the 60th Meeting of the Structures and Materials Panel in San Antonio, Texas, USA on 22-26 April 1985.
- [5] Aker GF. *A Damage Tolerance Approach for The Life Extension of J85-CAN40/15 Compressor Discs*. Masters Thesis, Carleton University, Ottawa, September 1986.
- [6] Akita S and Ichikawa M. *A New Model for Scatter of Fatigue Crack Growth Rate*. Int J of Fracture 32: R3-R6 (1986).
- [7] Allison DE, Besuner PM, Eischen JW, Hopkins SW, Rau CA Jr, Robinson JN, and Wachob HF. *Cost /Risk Analysis for Disk Retirement*. Vol I, Failure Analysis Associates, February 1984. AFWAL-TR-83-4089.
- [8] Ang AH-S and Tang WH. *Probability Concepts in Engineering Planning and Design*. Vol I, John Wiley and Sons, 1975.
- [9] Ang AH-S and Tang WH. *Probability Concepts in Engineering Planning and Design*. Vol II, John Wiley and Sons, 1984.

- [10] Annis CG, Cargill JS, Harris JA Jr, and Van Wanderham MC. *Engine Component Retirement for Cause: A Nondestructive Evaluation (NDE) and Fracture Mechanics Based Maintenance Concept*. J of Metals, July 1981.
- [11] Annis CG Jr, Van Wanderham MC, Harris JA Jr, and Sims DL. *Gas Turbine Engine Disk Retirement-for-Cause: An Application of Fracture Mechanics and NDE*. ASME 80-GT-127, 1980.
- [12] Becher PE and Pedersen A. *Application of Statistical Linear Elastic Fracture Mechanics To Pressure Vessel Reliability Analysis*. Nuclear Eng and Des 27 (1974) 413-425.
- [13] Berens AP and Hovey PW. *Characterization of NDE Reliability*. Review of Progress in Nondestructive Evaluation, Vol I. Edited by Thompson DO and Chimenti DE, Plenum Press NY, 1982.
- [14] Berens AP and Hovey PW. *Statistical Methods for Estimating Crack Detection Probabilities*. Probabilistic Fracture Mechanics and Fatigue Methods. ASTM-STP-798. Edited by Bloom and Ekvall, 1983. pp79-84.
- [15] Berens AP and Hovey PW. *Evaluating POD/CL Characterizations of NDE Reliability*. Review of Progress in Nondestructive Evaluation, Vol 2A. Edited by Thompson DO and Chimenti DE, 1983, pp53-68.
- [16] Besuner PM and Tetelman AS. *Probabilistic Fracture Mechanics*. Nuclear Eng and Des 43 (1977) 99-114.
- [17] Blom AF. *Relevance of short fatigue crack growth data for durability and damage tolerance analyses of aircraft*. ICAF Document No 1536, FFA TN 1986-23, The Aeronautical Research Institute Of Sweden, Stockholm 1986.
- [18] Bloom JM and Ekvall JC, editors. *Probabilistic Fracture Mechanics and Fatigue Methods: Applications for Structural Design and Maintenance*. Papers presented at symposium in St Louis, Missouri, October 1981. ASTM STP 798, 1983.
- [19] Burkhardt GL, Teller CM, Beissuner RE, and Barton JR. *Demonstration of the Electric Current Perturbation Technique for Fatigue Crack Characterization in TF-39 Turbine Disk Tiebolt Holes*. AFML/LLP, Wright-Patterson AFB, Ohio, November 1979. [AD-A128819]

- [20] Cargill JS, Thompson DO, and Chimenti DE. *Retirement For Cause Inspection System Design*. Review of Progress in Quantitative Nondestructive Evaluation, Vol I, 1981.
- [21] Chimenti DE and Moran TJ. *Advanced Quantitative Methods for Nondestructive Evaluation*. AGARD-CP-393, April 1985.
- [22] Cook CH, Spaeth CE, Hunter DT, and Hill RJ. *Damage Tolerant Design of Turbine Engine Disks*. ASME 82-GT-311, 1982.
- [23] Cruse TA and Gallagher JP (Editors). *Fatigue Life Technology*. Papers presented at the 22nd Annual International Gas Turbine Conference, Philadelphia, Pennsylvania, 27-31 March 1977.
- [24] Cruse TA and Meyer TG. *A Cumulative Fatigue Damage Model for Gas Turbine Engine Disks Subjected to Complex Mission Loading*. ASME 78-WA/GT-14, 1978.
- [25] Data. *All compressor discs and spacers for J85 CAN40 engine rotor serial numbers, time since new (TSN), and whether or not the disc had any cracks are documented.* 1976 to the end of 1984, Orenda of Hawker Siddeley.
- [26] Data. *All cracked Fifth Stage Compressor Discs for the J85 CAN40 Engine*. For each disc the number of cracks, a description of each and the total life of the component in hours (TSN) is given. 1977 to the end of 1984, Orenda of Hawker Siddeley.
- [27] Daub WJ. *Weibull Analysis for Aircraft Engine Components*. General Electric (J85 Project).
- [28] Eddy Current paper from John McCanna, NDTB Development, AMDU, CFB Trenton.
- [29] Evans WJ, Smith MEF, and Williams CHH. *Disc Fatigue Life Predictions for Gas Turbine Engines*. AGARD-CP-368, May-June 1984.
- [30] Fahr A and Charlesworth AM. *An Introduction to the Ultrasonic C-Scan Inspection of Advanced Composite Materials*. NRC/NAE LTR-ST-1602. 18 November 1986.
- [31] Fong JT. *Uncertainties in Fatigue Life Prediction and a Rational Definition of Safety Factors*. Nuclear Eng and Des 51 (1978) 45-54.

- [32] Gallagher JP, Griessler FJ, Berens AP, and Engle RM Jr. *USAF Damage Tolerant Design Handbook: Guidelines for the Analysis and Design of Damage Tolerant Aircraft Structures*. AFWAL-TR-82-3073. Final Report, March 1984, Air Force Wright Aeronautical Laboratories, Wright Patterson Air Force Base, Ohio.
- [33] Ghonem H and Dore S. *Probabilistic Description of Fatigue Crack Growth in Polycrystalline Solids*. Eng Frac Mech, Vol 21, No 6, pp 1151-1168, 1985.
- [34] Gould GM. *Life Predictions for Aircraft Engine Components*. Weibull Analysis on Fifth Stage Compressor Discs using J85 CAN40 Engine Data to Predict Crack Propagation Rate. Project Report, Carleton University, 15 April 1985.
- [35] Gould GM and Koul AK. *Computer Documentation for Three Weibull Analysis Programmes, A Programme for Predicting a Safe Inspection Interval using Probabilistic Fracture Mechanics Principles, and a Programme for Plotting Histograms*. NRC/NAE ST-428, September 1986.
- [36] Guenther WC. *Concepts of Statistical Interference*. McGraw-Hill, 1965.
- [37] Hahn GJ and Shapiro SS. *Statistical Models in Engineering*. John Wiley and Sons, 1967.
- [38] Harris DO. *A Means of Assessing the Effects of NDE on the Reliability of Cyclically Loaded Structures*. Materials Evaluation, July 1977.
- [39] Harris JA Jr, Annis CJ Jr, Van Wanderham MC, and Sims DL. *Engine Component Retirement For Cause*. Conference in Service of High Temperature Parts, The Netherlands, October 1981.
- [40] Harris JA Jr, Sims DL, and Annis CG Jr. *Concept Definition: Retirement for Cause of F100 Rotor Components*. AFWAL-TR-80-4118, 1980. Microfiche [AD A095584]
- [41] Hill RJ, Reimann WH, and Ogg JS. *A Retirement-for-Cause Study of an Engine Turbine Disk*. AFWAL-TR-81-2094, November 1981.
- [42] Hoepfner DW and Krupp WE. *Prediction of Component Life by Application of Fatigue Crack Growth Knowledge*. Eng Frac Mech, 1974, Vol 6, pp 47-70.

- [43] Hull DA and Hoepfner DW. *The Structural Integrity of Gas Turbine Engine Discs - Spin Pit Fatigue Crack Growth Studies*. Progress Report for September 1984 to March 1985, Dept. of Mech Engineering, University of Toronto, Toronto, Ontario, M5S 1A4.
- [44] Hurchalla J, Johnson HE, and Wallace RM. *Fracture Mechanics LCF Life Prediction System With Application To An Advanced Gas Turbine Alloy*. AIAA Paper No 75-1311, AIAA/SAE 11th Propulsion Conference, Anaheim, California, 29 September-1 October, 1975.
- [45] Hyzak JM, Allison JE, and Reimann WH. *Development of Quantitative NDI for Retirement-for-Cause*. Air Force Materials Lab, Wright-Patterson AFB, OH. AFML-TR-78-198, February 1979.
- [46] Ichikawa M, Takamatsu T, and Nakamura T. *Theoretical and Experimental Investigation of some Fundamental Problems in Probabilistic Fracture Mechanics*. 4th International Conference on Structural Safety and Reliability, ICAF Doc No 1516, pp 111-423 to 111-427. Micossar 1985.
- [47] *Jane's All the World's Aircraft*. Jane's Publishing Company Ltd., 1964-65, p16 and p505.
- [48] Jimboh K, Aono H, and Kawashima T. *Ferris Wheel Testing - Low Cycle Fatigue Test of Gas Turbine Engine Discs*. ASME 80-GT-1, 1980.
- [49] Jouris GM. *Some Comments on the Probabilistic Approach to Safety Analysis*. Nuclear Eng and Des 50 (1978) 169-172.
- [50] Jouris GM and Shaffer DH. *A Procedure for Estimating the Probability of Flaw Nondetection*. Nuclear Eng and Des 48 (1978) 517-521.
- [51] Jouris GM and Shaffer DH. *Methodology for Inferring Initial Flaw Distribution*. Nuclear Eng and Des 60 (1980) 43-47.
- [52] Kaprzynski JJ. *Determination of Stress Intensity Factors in a J85 CAN40 Fifth Stage Compressor Disc by Finite Element Analysis*. NRC/NAE Unpublished Work, March 1984.
- [53] King TT, Cowie WD, and Reimann WH. *Damage Tolerance Design Concepts for Military Engines*. AGARD-CP-393, April 1985.
- [54] King TT, Hurchalla J, and Nethaway DH. *United States Air Force Engine Damage Tolerance Requirements*. AIAA/SAE/ASME/ASEE 21st Joint Propulsion Conference, Monterey, California, July 1985.

- [55] Koul AK, Thamburaj R, Raizenne MD, Wallace W, and De Malherbe MC. *Practical Experience with Damage Tolerance Based Life Extension of Turbine Engine Components*. National Research Council/National Aeronautical Establishment. NRC/NAE LTR-ST-1537, April 1985.
- [56] Koul AK, Thamburaj R, Schofield J, and Wallace W. *Weibull Analysis in Turbine Engine Component Lifing*. National Research Council/National Aeronautical Establishment. NRC/NAE LTR-ST-1528, 23 August 1985.
- [57] Koul AK, Wallace W, and Thamburaj R. *Problems and Possibilities for Life Extension in Gas Turbine Components*. National Research Council/National Aeronautical Establishment. NRC/NAE LTR-ST-1503, May 1984.
- [58] Larsen JM and Nicholas T. *Cumulative Damage Modeling of Fatigue Crack Growth*. AGARD-CP-368, May-June 1984.
- [59] Lemon GH. *A Problem in the Distribution of Maximum Flaw Length After Inspection*. Technometrics, Vol 16, No 4, November 1974.
- [60] Lewis WH, Dodd BD, Sproat WH, and Hamilton JM. *Reliability of Non-destructive Inspections*. July 1974-December 1978 Final Report No. SA-ALC/MEE 76-6-38-1, US Air Force, Performing Organization Lockheed-Georgia Co., Controlling Office San Antonio Air Logistics Centre, Kelly Air Force Base, Texas, 1978. [AD-A072097]
- [61] Meece CE and Spaeth CE. *Damage Tolerant Design: An Approach to Reduce the Life Cycle Cost of Gas Turbine Engine Disks*. AIAA/SAE/ASME 15th Joint Propulsion Conference, Las Vegas, Nevada, 18-20 June 1979.
- [62] Microfocus Radiography papers from John Stewart, Non Destructive Testing Building (NDTB) Development, Aerospace Maintenance Development Unit (AMDU), Canadian Forces Base (CFB) Trenton.
- [63] MIL-A-83444 Military Specification. *Airplane Damage Tolerance Requirements*. USAF 1974.
- [64] MIL-STD-1783 Military Standard. *Engine Structural Integrity Program (ENSIP)*. 30 November 1984, Dept of Defense, Washington DC, 20360.
- [65] MIL-STD-6866 Military Standard. *Inspection, Liquid Penetrant*. 30 September 1985, Dept of Defense, Washington DC, 20402, (AFWAL/MLSE).

- [66] Mom AJA. *Overview of the AGARD SMP Activities on Turbine Engine Materials Technology in the 1972-1982 Period*. AGARD-CP-368, May-June 1984.
- [67] Natrella MG. *Experimental Statistics*. Handbook 91, National Bureau of Standards, 1963.
- [68] Nortec RECHII I. *Rotating Eddy Current Hole Inspection Instrument*. Operating Manual, 1981. Revised 1982..9, 421 N Quay, Kennewick, Washington 99336.
- [69] Olmabe H, Funatogawa O, and Yang JN. *Monte Carlo Simulation for Probabilistic Fatigue Analysis of Jet Engine Disks*. 4th International Conference on Structural Safety and Reliability, ICAF Doc No 1518, Micossar 1985.
- [70] Packman PF. *Advanced Nondestructive Inspection Techniques as applied to Fracture Mechanics Design for Turbine Engine Components*. Fatigue Life Technology edited by Cruse TA and Gallagher JP, 1977, pp95-116.
- [71] Packman PF, Klima SJ, Davies RL, Malpani J, Moyzis J, Walker W, Yee BGW, and Johnson DP. *Reliability of Flaw Detection by Nondestructive Inspection*. ASME Metals Handbook, Vol II, Nondestructive Inspection and Quality Control, 1978, pp414-424.
- [72] Palanisamy R and Lakin KJ. *Eddy Current Detection of Subsurface Cracks in Sleeved Bolt Holes*. ZUX-SME-IQ-83-950, 1983.
- [73] Palmberg B, Blom AF, and Eggwertz S. *Probabilistic Damage Tolerance Analysis of Aircraft Structures*. Stockholm FFA TN 1985-68, ICAF Doc No 1528, 1985.
- [74] Personal Communication with Orenda.
- [75] Private Communication, General Electric Company, Aircraft Engine Group, 85L118/118A, Nov 1982, 8-STAGE/-21.
- [76] Private Communication with A P Berens in May 1987.
- [77] Private Communication with Dr R Thamburaj dealing with the JS5 CAN15/40 Component Improvement Program, November 1986.
- [78] *Quest for the Best*. Structures and Materials Laboratory, National Aeronautical Establishment, National Research Council, Canada.

- [79] Raizenne MD, Thamburaj R, Koul AK, and Doswell W. *J85-CAN-40 Gas Turbine Engine Compressor Disk Retirement-for Cause: An Application of Quantitative NDI*. NRC/NAE LTR-ST-1529, 18 February 1985.
- [80] Ramsden JM. *The Inspectable Structure*. Flight International, 30 August 1986, p116.
- [81] Rau CA Jr and Besuner PM. *Risk Analysis by Probabilistic Fracture Mechanics*. Product Engineering, October 1979.
- [82] Raussi J and Tiainen OJA. *The Effect of Inspection and Repair on the Size Distributions of the Weld Imperfections in Nuclear Power Plant Pipes*. Nuclear Eng and Des 60 (1980) 395-399.
- [83] Richardson JM and Buckley MJ. *Overview of Probabilistic Failure Prediction and Accept-Reject Decisions*. Review of Progress in Nondestructive Evaluation, Vol I, 1981. Edited by Thompson and Chimenti.
- [84] Ridge HOMX. *High Output Microfocus X-Ray Inspection System*. HOMX-160 Brochure.
- [85] Rudd JL and Gray TD. *Quantification of Fastener-Hole Quality*. J Aircraft, Vol 15, No 3, March 1978.
- [86] Rummel WD. *Recommended Practice for a Demonstration of Nondestructive Evaluation (NDE) Reliability of Aircraft Production Parts*. Materials Evaluation 40, August 1982: 922-932.
- [87] Sattar SA and Sundt CV. *Gas Turbine Engine Disk Cyclic Life Prediction*. J Aircraft, Vol 12, No 4, April 1975, pp360-365.
- [88] Saunders SC. *The Problems of Estimating a Fatigue Service Life With a Low Probability of Failure*. Eng Frac Mech 1976, Vol 8, pp205-215.
- [89] Schütz Dr-Ing W. *Calculation Methods for Fatigue Life and Crack Propagation*. Chapter 4 in AGARD-AG-231, January 1978, pp45-76.
- [90] Schwartz BJ and Annis CG Jr. *A Synergistic Fracture Mechanics Approach to Fatigue Life Evaluation*. Eng Frac Mech, Vol 18, No 4, pp815-826, 1983.
- [91] Shinozuka M. *Initial Quality Representation*. Durability Methods Development, AFFDL-TR-79-3118, Air Force Flight Dynamics Laboratory, September 1979.

- [92] Stone DEW. *Non-destructive Inspection and the Implementation of a Damage Tolerant Design Philosophy*. Technical Memorandum Structures 982. Royal Aircraft Establishment, February 1982.
- [93] Stoute P. *Evaluation of NDI of General Electric Compressor Disc Bolt Holes*. Project, Carleton University, 6 April 1987.
- [94] Tang WH. *Probabilistic Updating of Flaw Information*. J Testing and Evaluation. JTEVA Vol 1, No 6, November 1973, pp459-467.
- [95] Taylor RG. *Limitations of Manual NDT Systems and the 'No Eyes' Concept*. AGARD-CP-393, April 1985.
- [96] Thamburaj R and de Malherbe MC. *Service Induced Degradation and Life Extension Possibilities for CF Engine Components*. Dept of Mech and Aero Engineering, Carleton University, Ottawa, Ontario, K1S 5B6. Interim Progress Report, April 1985.
- [97] Thamburaj R, Koul AK, Wallace W, and de Malherbe MC. *Life Extension Concepts for Canadian Forces Aircraft Engine Components: J85 CAN40 Compressor and Turbine Materials*. NRC/NAE LTR-ST-1585, 5 May 1986.
- [98] Whittaker IC and Saunders SC. *Exploratory Development on Application of Reliability Analysis to Aircraft Structures considering Interaction of Cumulative Fatigue Damage and Ultimate Strength*. AFML-TR-72-283, January 1983.
- [99] Yang JN and Chen S. *An Exploratory Study of Retirement-For-Cause for Gas Turbine Engine Components*. 20th Joint Propulsion Conference, 11-13 June 1984, Cincinnati Ohio. AIAA-84-0850.
- [100] Yang JN and Chen S. *Fatigue Reliability of Gas Turbine Engine Components under Scheduled Inspection Maintenance*. J Aircraft, Vol 22, No 5, May 1985, pp415-422.
- [101] Yang JN and Donath RC. *Statistical Crack Propagation in Fastener Holes Under Spectrum Loading*. J Aircraft, Vol 20, No 12, December 1983, pp1028-1032.
- [102] Yang JN, Donath RC, Salivar GC. *Statistical Fatigue Crack Propagation of IN100 at Elevated Temperatures*. ASME International Conference on Advances in Life Prediction Methods, April 1983, pp 241-247.

- [103] Yang JN, Hsi WH, Manning SD, and Rudd SL. *Stochastic Crack Propagation in Fastener Holes*. J Aircraft, Vol 22, No 9, September 1985, pp 810-817.
- [104] Yang JN, Manning SD, and Garver WR. *Durability Analysis Methodology Development*. Durability Methods Development, Vol V, AFFDL-TR-79-3118, September 1979.
- [105] Yang JN and Trapp WJ. *Reliability Analysis of Aircraft Structures under Random Loading and Periodic Inspection*. AIAA Journal, Vol 12, December 1974, pp1623-1630.
- [106] Yee BGW, Chang FH, Couchman JC, Lemon GH, and Packman PF. *Assessment of NDE Reliability Data*. General Dynamics, Fort Worth, Texas, July 1974-September 1976. NASA-CR-134991, Final Report, October 1976.
- [107] Zako M, Kawashima T, Aono H, Jimboh K, Ohnabe H, and Miyoshi T. *Residual Life Prediction of Jet Engine Rotor Disks*. Design of Fatigue and Fracture Resistant Structures, ASTM STP 761, 1982, pp408-423.

Appendix A

RELEVANT STATISTICS

Most of the following has been taken from [8], [9], [36], [37], and [67].

A.1 Classical Approach To Estimating Parameters from Observational Data

If

$$\text{population mean} = \mu \quad (\text{A.1})$$

$$\text{population variance} = \sigma^2 \quad (\text{A.2})$$

and

$$\text{sample mean} = \bar{x} \quad (\text{A.3})$$

$$\text{sample variance} = s^2 \quad (\text{A.4})$$

then

$$\bar{x} = \frac{1}{n} \sum x_i \quad (\text{A.5})$$

$$s_x^2 = \frac{1}{n-1} \left[\sum x_i^2 - \frac{(\sum x_i)^2}{n} \right] \quad (\text{A.6})$$

and similarly,

$$\bar{y} = \frac{1}{n} \sum y_i \quad (\text{A.7})$$

$$s_y^2 = \frac{1}{n-1} \left[\sum y_i^2 - \frac{(\sum y_i)^2}{n} \right] \quad (\text{A.8})$$

A.2 Distributions

Normal Distribution

Abbreviated as $N(\mu, \sigma)$.

PDF

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{x-\mu}{\sigma} \right)^2 \right] \quad (\text{A } 9)$$

for

$$-\infty < x < \infty$$

$$-\infty < \mu < \infty$$

$$\sigma > 0$$

Procedure to Find Parameters

$$\text{mean } \mu = \bar{x} \quad (\text{A } 10)$$

$$\text{standard deviation } \sigma = s \quad (\text{A } 11)$$

Procedure to Obtain Random Values

A computer algorithm, using random values from a uniform distribution over the interval (0,1), was used to generate random values from a normal distribution given a mean and standard deviation.

Box and Muller (1958) have shown that if u_1 and u_2 are two independent standard uniformly distributed random numbers, then a pair of independent random numbers from a normal distribution $N(\mu, \sigma)$ may be generated by the following (Reference Ang and Tang, Volume I, page 285).

$$x_1 = \mu + \sigma \sqrt{-2 \ln u_1} \cos 2\pi u_2 \quad (\text{A } 12)$$

$$x_2 = \mu + \sigma \sqrt{-2 \ln u_1} \sin 2\pi u_2 \quad (\text{A } 13)$$

Lognormal

Abbreviated as $L(\lambda, \zeta)$

PDF

$$f(x) = \frac{1}{\sqrt{2\pi}\zeta x} \exp \left[-\frac{1}{2} \left(\frac{\ln x - \lambda}{\zeta} \right)^2 \right] \quad (\text{A } 14)$$

for

$$0 \leq x < \infty$$

$$-\infty < \lambda < \infty$$

$$\zeta > 0$$

Different Procedures to Find Parameters

a. Graphically

Data following a lognormal distribution will plot as a straight line on lognormal probability paper. Lognormal probability paper has a log x axis and a cumulative probability y axis. The y axis plotting position is determined as follows:

$$\begin{aligned} & \frac{(i-0.5)}{N} \times 100\% \quad \text{Hazen(1930)} \\ \text{or} \quad & \frac{i}{N+1} \times 100\% \quad \text{Gumbel(1954)} \end{aligned} \quad (\text{A.15})$$

where i = Position of the i^{th} value after all the
values have been placed in increasing order
 N = Total number of observations.

After the values have been plotted, and a line passed through the points, the mean and standard deviation can be determined where:

$$\lambda = \ln(50^{\text{th}} \text{ percentile}) \quad (\text{A.16})$$

$$\zeta = \frac{2}{5} [\ln(90^{\text{th}} \text{ percentile}) - \ln(10^{\text{th}} \text{ percentile})] \quad (\text{A.17})$$

For practical engineering decisions, visual graphical solutions are often desirable to interpret the information given in the form of statistical parameters. The cumulative probability axis can be produced on a computer by converting the cumulative probabilities into standard normal deviates, which is a linear scale, by integration, tables, or an algorithm.

In this case an algorithm (Reference, Abramowitz and Stegun, page 933) was used as follows:
Given a cumulative probability, p ,

$$\text{where } 0 < p \leq 0.5 \quad (\text{A.18})$$

the standard normal deviate, x_p , can be calculated:

$$x_p = t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \epsilon(p) \quad (\text{A.19})$$

$$\text{where } t = \sqrt{\ln \frac{1}{p^2}} \quad (\text{A.20})$$

$$\begin{array}{ll} c_0 = 2.515517 & d_1 = 1.432788 \\ c_1 = 0.802853 & d_2 = 0.189269 \\ c_2 = 0.010328 & d_3 = 0.001308 \end{array}$$

and the error

$$|\epsilon(p)| < 4.5 \times 10^{-4} \quad (\text{A.21})$$

b. Maximum Likelihood Estimates

$$\lambda = \frac{\sum \ln x_i}{n} \quad (\text{A.22})$$

$$\zeta^2 = \frac{1}{n} \sum (\ln x_i - \lambda)^2 \quad (\text{A.23})$$

c. Lognormal Parameters from Point Estimators

For a lognormal, the expected mean and variance are:

$$E(X) = \exp\left(\lambda + \frac{1}{2}\zeta^2\right) \quad (\text{A.24})$$

$$\text{Var}(X) = E^2(X)(e^{\zeta^2} - 1) \quad (\text{A.25})$$

Thus, having calculated the sample mean and variance using classical methods for point estimates, the parameters of the lognormal can be determined

$$\lambda = \ln \bar{x} - \frac{1}{2}\zeta^2 \quad (\text{A.26})$$

$$\zeta^2 = \ln\left(\frac{\zeta^2}{\bar{x}^2} + 1\right) \quad (\text{A.27})$$

$$\text{where } \bar{x} = E(X) \quad (\text{A.28})$$

$$\zeta^2 = \text{Var}(X) \quad (\text{A.29})$$

Procedure to Obtain Random Values

$$x_i = e^{\sigma R_N + \mu} \quad (\text{A.30})$$

where R_N is a random value from a normal distribution with $\mu = 0$ and $\sigma = 1$.

Confidence Required

$$\text{median} = e^\lambda \quad (\text{A.31})$$

$$0.1\% = e^{\lambda - 3.0905\zeta} \quad (\text{A.32})$$

where -3.0905 is the standard normal deviate for 0.1%.

Loglogistic Distribution

The Equation

Abbreviated as $L(\beta_0, \beta_1)$.

The loglogistic function, shaped like a tilted S with asymptotes at 0 and 1, has been found appropriate in many instances involving a binary dependent variable. The expected value of y , which can be called the probability, p , can be calculated as follows:

$$p = \frac{\exp(\beta_0 + \beta_1 \ln a)}{1 + \exp(\beta_0 + \beta_1 \ln a)} \quad (\text{A.33})$$

Procedure to Find Parameters

The function is easily linearized and values can be transformed using:

$$p' = \ln \left(\frac{p}{1-p} \right) \quad (\text{A.34})$$

$$a' = \ln x \quad (\text{A.35})$$

Note

$$\begin{aligned} p = 0 & \text{ can be approximated by } p' = \frac{1}{n+1} \\ p = 1 & \text{ can be approximated by } p' = \frac{n}{n+1} \end{aligned} \quad (\text{A.36})$$

The loglogistic parameters, β_0 and β_1 , can be found using regression analysis and the equation

$$p' = \beta_0 + \beta_1 a' \quad (\text{A.37})$$

Binomial Distribution

CDF

$$F_X(j) = \sum_{i=0}^j \binom{n}{i} p^i (1-p)^{n-i} \quad j = 0, 1, \dots, n \quad (\text{A.38})$$

where i = number of occurrences

n = number of trials

and

$$\binom{n}{i} = \frac{n!}{i!(n-i)!} \quad (\text{A.39})$$

Estimation of Parameters

$$E(X) = np \quad (\text{A.40})$$

$$\text{Var}(X) = np(1-p) \quad (\text{A.41})$$

Maximum Likelihood Estimator

$$\hat{p} = \frac{1}{n} \sum_{i=1}^n X_i \quad (\text{A.42})$$

Confidence for p

There are equations as in Ang and Tang, Volume I, page 253, but normally a graph is used where the variance of $\hat{p}$ decreases as the sample size n increases.

Weibull

Abbreviated as $W(\eta, \beta)$.

PDF

$$f(t; \eta; \sigma) = \begin{cases} \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1} \exp \left[- \left(\frac{t}{\eta}\right)^{\beta} \right] & t \geq 0, \eta > 0, \beta > 0 \\ 0 & \text{elsewhere} \end{cases} \quad (\text{A.43})$$

CDF

$$F(t; \eta; \sigma) = 1 - \exp \left[- \left(\frac{t}{\eta}\right)^{\beta} \right] \quad t \geq 0 \quad (\text{A.44})$$

Procedure to Find Parameters

The Weibull distribution can be transformed so that:

$$y' = \ln \ln \left[\frac{1}{1 - F(t)} \right] \quad (\text{A.45})$$

$$t' = \ln t \quad (\text{A.46})$$

The Weibull parameters, η and β , are determined using the regression analysis equation:

$$y' = -\beta \ln \eta + \beta t' \quad (\text{A.47})$$

so that

$$\text{slope} = \beta \quad (\text{A.48})$$

$$\text{intercept} = -\beta \ln \eta \quad (\text{A.49})$$

The graph is read using the cumulative probability values, $F(t) \times 100\%$, and the natural log of t . Detailed calculations are shown in Dodd and specifically for this case in an NRC ST-428 by the author. These references also specify a preferred method for determining $F(t)$ using order numbers and ranking.

Procedure to Obtain Random Values

$$y' = \eta \left[\ln \left(\frac{1}{1 - R_U} \right) \right]^{\frac{1}{\beta}} \quad (\text{A.50})$$

where R_U is a random value from a uniform distribution over the interval (0,1).

A.3 Regression Analysis

Linear Regression with Constant Variance

A least squares regression line, in particular the regression of Y on X :

$$E(Y/x) = a + bx \quad (\text{A.51})$$

can be obtained by minimizing the sum of the squared errors between an observed value y_i and the predicted value, $\hat{y}_i$. The slope, b , and intercept, a , can be calculated as follows:

$$b = \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2} \quad (\text{A.52})$$

$$a = \bar{y} - b \bar{x} \quad (\text{A.53})$$

For a given x value, the dispersion of y values can be characterized by the conditional variance, $s_{Y/x}^2$ where:

$$s_{Y/x}^2 = \frac{1}{n-2} \left[\sum (y_i - \bar{y})^2 - \beta^2 \sum (x_i - \bar{x})^2 \right] \quad (\text{A.54})$$

$$= \frac{1}{n-2} \left[\sum y^2 - \frac{(\sum y)^2}{n} - \beta^2 \left(\sum x^2 - \frac{(\sum x)^2}{n} \right) \right] \quad (\text{A.55})$$

The correlation coefficient, a measure of the strength of the linear relationship between the two variables x and y , can be estimated by:

$$\rho = \frac{1}{n-1} \left[\frac{\sum x_i y_i - n \bar{x} \bar{y}}{s_x s_y} \right] \quad (\text{A.56})$$

where a ρ close to ± 1 may indicate a strong linear relationship and $\rho \simeq 0$ may indicate a lack of linear relationship.

A.4 Confidence Bounds

Consider a $(1 - \alpha) \times 100\%$ confidence interval as a range within which we are $(1 - \alpha) \times 100\%$ sure that the true parameter is contained.

Confidence Interval for a Single Point on the Regression Line

If we assume that Y has a normal distribution about the regression line $E(Y/x)$ for all values of x , then the regression values will be t distributed.

The Student t distribution is a symmetric distribution $(-\infty$ to $+\infty)$ that approaches the normal distribution as its parameters (ie. degrees of freedom or number of points, n) approaches infinity. It is a distribution of one parameter. In regression analysis two degrees of freedom are removed since the slope and intercept have been calculated in order to determine $\hat{y}$. The confidence bounds on the predicted value are as follows:

$$(\hat{y}_i)_{1-\alpha} = \hat{y}_i \pm t_{1-\frac{\alpha}{2}, n-2} s_{Y/x} \quad (\text{A.57})$$

where $\hat{y}_i$ = y value predicted from the
regression line given x_i

$$s_{Y/x}^2 = \text{conditional variance} \\ (\text{variance assumed constant})$$

Note: If variance is not assumed constant, use s_{Y_i} instead of $s_{Y/x}$ (Neter and Wasserman, page 65) where,

$$s_{Y_i}^2 = s_{Y/x}^2 \left[\frac{1}{n} + \frac{(x_i - \bar{x})^2}{\sum (x_i - \bar{x})^2} \right] \quad (\text{A.58})$$

Confidence intervals will decrease with increasing n and the t distribution will approach the normal distribution. Thus for large n

$$(\hat{y}_i)_{1-\alpha} = \hat{y}_i \pm k_{1-\frac{\alpha}{2}} s_{Y/x} \quad (\text{A.59})$$

This equation is also useful as a first approximation since the standard normal deviate is a constant for a given confidence level. For example, for a confidence of 95%:

$$1 - \alpha = .95 \quad (\text{A.60})$$

$$\alpha = .05 \quad (\text{A.61})$$

$$\frac{\alpha}{2} = .025 \quad (\text{A.62})$$

$$1 - \frac{\alpha}{2} = .975 \quad (\text{A.63})$$

and from tables $k_{1-\frac{\alpha}{2}} = 1.960$.

Confidence Bounds for the Regression Line as a Whole

To obtain a confidence band for the entire regression line, all possible combinations of slope and intercept need to be considered. The boundaries of the region are hyperbolas with the equation:

$$(\hat{y}_i)_{1-\alpha} = \hat{y}_i \pm \sqrt{2F_{1-\alpha, 2, n-2}} s_{Y_i} \quad (\text{A.64})$$

$$\text{where } s_{Y_i} = s_{Y/x} \left[\frac{1}{n} + \frac{(x_i - \bar{x})^2}{\sum (x_i - \bar{x})^2} \right] \quad (\text{A.65})$$

The F distribution is the combined distribution of two distributions and has two parameters, the numerator degrees of freedom and the denominator degrees of freedom. Confidence bounds are calculated for various x values, plotted and joined together.

Tables for t values, k values, or F values for various confidence levels can be found in standard statistic books, for example Natrella, Ang and Tang, or Hahn and Shapiro.

Appendix B

COMPUTER PROGRAMS

Many useful computer programs were written and are available from the author. These include the lognormal analysis and plots, the POD analysis and plots, the Weibull plots, the histograms and distribution plots as well as some linear regression analysis.

Appendix C

EXPERIMENTAL RAW DATA

All the bolt holes are listed for each of the retired ten fifth stage J85 CAN40 turbine discs used for the experimental work. The crack lengths found by each NDI technique are shown as well as the crack lengths as measured using SEM. In the case of the eddy current inspection technique, cracks are only indicated since it was not possible to determine the crack lengths. The last page lists the bolt holes and the crack lengths where cracks were detected growing radially outward from the bolt holes.

Disc A - P18649-14

Bolt Hole Number	U ECI - 1 2 5	Serial Side (Back)				Non-Serial Side (Front)				Actual
		AMDU LPI (45 min)	AMDU LPI (1 hr)	Orenda LPI	NAE C-Scan	AMDU LPI (45 min)	AMDU LPI (1 hr)	Orenda LPI	NAE C-Scan	
1	* +									
2	* +		2.01	3.20	2.20			0.28	1.70	1.54
3										
4										
5	* +				1.70				0.60	1.43
6	* +	1.43	4.23	1.6	2.70	3.41	3.56		3.10	3.06
7	* +									
8	* +	0.03	1.89	3.20	2.80	2.22	2.26		2.10	2.20
9	* +				0.60				1.10	1.14
10	* +									
11	* +									
12	* +				0.60				0.90	.17
13	* +									
14	* +									
15	* +				1.00				1.10	0.98
16	* +									
17	* +		2.13	1.6	2.30				1.90	2.31
18	* +				0.90					
19	* +									
20	* +								2.20	1.33
21	* +									
22	* +								1.70	0.94
23	* +									
24	* +									
25	* +									
26	* +	1.20	1.52	3.2	1.70				2.20	1.75
27	* +	1.41	1.74	1.6	2.00	1.43	2.04		2.30	1.95
28	* +									
29	* +									
30	* +	1.24	1.87	3.2	2.10		1.93		1.20	2.32
31	* +									
32	* +									
33	* +									
34	* +		0.35		1.60		0.40		0.50	0.90
35	* +		0.62		1.20				0.80	1.21
36	* +				1.40		0.88		1.70	1.50
37	* +				0.70				1.10	0.89
38	* +				1.10				1.30	1.11
39	* +									
40	* +				0.90		0.79		0.60	0.82

Disc F - P18521-19

Bolt Hole Number	Serial Side (Back)					Non-Serial Side (Front)					Actual
	AMDU ECI	q	AMDU LPI (45 min)	AMDU LPI (1 hr)	Orenda LPI	NAE C-Scan q	AMDU LPI (45 min)	AMDU LPI (1 hr)	Orenda LPI	NAE C-Scan q	
1	*	+			1.32	2.85	3.99	3.85	3.2	4.35	3.97
2	*										
3											
4											.30
5											
6											
7		+									
8	*	+								1.05	
9											
10	*	+	4.22	3.07	4.0	3.70	3.62	3.57	4.8	2.70	4.44
11											
12	*	+	4.00	4.25	3.2	2.50	3.81	4.56	3.2	3.60	4.70
13	*	+	4.18	3.61		3.60	3.40	3.84	3.2	3.05	3.92
14											
15											.13
16											
17		+	1.28								.33
18	*	+				1.65		1.41		2.90	1.88
19								2.40			
20											
21	*	+									
22											
23	*	+			4.0	2.55	4.25	4.53	1.6	4.20	4.20
24	*	+				2.10	1.97			3.00	2.33
25	*										0.60
26											
27											
28	*	+				2.45				3.50	2.20
29	*	+				1.60		1.97	1.6	2.80	2.31
30	*										
31											
32											
33	*	+									
34	*	+									
35		+									.25
36	*	+									
37	*	+									
38	*	+	3.57	3.33	3.2	2.90	2.89	3.83	3.6	2.10	3.45
39											
40	*	+				3.30				2.30	2.45

Disc G - P21014-15 (11J200-102)

Bolt Hole Number	Serial Side (Back)					Non-Serial Side (Front)					Actual		
	AMDU ECI	1	AMDU LPI	AMDU LPI	Orenda LPI	NAE C-Scan	1	AMDU LPI	AMDU LPI	Orenda LPI		NAE C-Scan	1
	2	5	(45 min)	(1 hr)				(45 min)	(1 hr)				
1													
2		+			1.6						1.75		0.75
3		+											.45
4													.20
5	*	+									1.70		1.26
6				0.69									
7													.47
8		+			1.6	1.90					1.60		0.91
9													
10		+			1.6						1.20		0.68
11		+			1.6								0.39
12		+			1.6								0.53
13		-			6.4	2.95	3.05	3.13			3.15		2.97
14	*	+	2.05	4.09	6.4	5.45	5.75	5.47			5.55		5.42
15		+											
16													
17	*	+	0.97	2.80	6.4	2.15	1.63	1.72			4.20		1.99
18		+									.90		1.24
19													
20	*	+	1.13	1.11		1.20		0.35			4.05		1.50
21		+	3.98	3.93	6.4	3.50	4.34	3.66					3.37
22													
23		+											
24		+		0.66				1.03					1.13
25	*	+	4.76	4.39	6.4	5.45	4.60	4.60			4.25		5.19
26													
27	*	+	2.19	1.91	4.8	3.90	3.16	3.17			2.95		2.83
28													
29													
30			0.32	0.49				2.70					0.57
31		-											
32	*	+	3.09		6.4	3.65	3.00	2.71			2.90		2.89
33		+											
34		+											
35		+											
36													
37													
38													
39													
40	*	+	4.04	3.87	6.4	2.80	4.48	2.67			3.70		3.97

Disc J - P18521-10

Bolt Hole Number	Serial Side (Back)					Non-Serial Side (Front)					Actual
	AMDUECI		AMDULPI	AMDULPI	OrendaLPI	NAEC-Scan	AMDULPI	AMDULPI	OrendaLPI	NAEC-Scan	
	2	5	(45 min)	(1 hr)			(45 min)	(1 hr)			
1											
2	*	+	0.88	1.27		1.85		0.33		1.20	
3		+				.60					
4											
5		+				.8					
6		+				.6					
7	*	+				1.60				1.60	
8		+									
9		+				1.50					
10		+	3.27	3.25		3.40	3.52	3.50		3.30	
11											
12	*	+				1.70	1.94	1.62		2.60	
13											
14											
15		+				2.15	1.24	1.22		2.75	
16											
17						1.05					
18											
19	*	+									
20		+	2.00	1.97		2.60	0.54			2.55	
21											
22						1.45				1.60	
23		+									
24											
25	*	+				.70				1.50	
26	*	+				1.95		0.89		1.65	
27	*	+				.60				.65	
28		+									
29		+									
30	*	+		3.34		2.20		0.69		2.10	
31	*	+				2.70				1.50	
32	*	+				1.10				2.10	
33	*	+		1.71		2.35	0.15	0.59		2.30	
34	*	+				1.15					
35	*	+				1.70					
36											
37	*										
38	*	+									
39		+									
40	*	+	3.73	3.59		4.20	4.81	6.02		4.75	

Radially Outward Cracks

Bolt Hole Number	Serial Side				Non-Serial Side				Actual
	AMDU LPI (45 min)	AMDU LPI (1 hr)	Orenda LPI	NAE C-Scan ¶	AMDU LPI (45 min)	AMDU LPI (1 hr)	Orenda LPI	NAE C-Scan ¶	
B1				1.60					0.25
B2				1.60					
B11				1.00					1.26
C16				1.05					0.41
C27				0.7					0.10
E4				1.40					
E13							0.60		
E15		0.32							
E20				1.70					
E21				1.60					
E35				0.70					
E39				2.30					
F25				1.00					
G13									0.59
G21						1.03			
H6	0.57	0.75							
I15								0.70	
I27				1.00					
J12					0.32				
J17				1.05					

Appendix D

A TYPICAL DISK2 COMPUTER SIMULATION RUN

D.1 Typical Input for a DISK2 Run

The interactive input required and typical conditions are summarized in Figure D.1 which includes the number of discs or simulations required; the seed or starting number for the random number generators in double precision; the initial conditions; crack growth rate data; and the end conditions in terms of crack length or number of hours. If not all the cracks are assumed to be detected, then a probability of detection curve and a distribution of cracks missed must also be input.

D.2 Typical Output for a DISK2 Run

One of the DISK2 output files was deliberately commented so that the simulation process could be understood more clearly. Two sample discs are shown in Figure D.2. In the first case the crack was detected and the crack length from the initial conditions was used in the remainder of the analysis. In the second case, where the crack was not found, a random crack length was generated from the lognormal distribution of cracks missed which itself uses a normal distribution $N(0,1)$ (Figure D.2).

Crack growth rate varies lognormally but for ease of calculation this was transformed so that a random number could be chosen from a normal distribution with a mean of zero and a standard deviation depending on the original scatter in the crack growth data.

The crack is grown in steps of 100 hours and if the difference between the ΔK 's is more than $2 \text{ MPa}\sqrt{\text{m}}$, a new ΔK is calculated based on the new crack length. Thus the crack length, ΔK , and the crack growth rate in mm/hour are shown in Figure D.2 for each iteration.

The disc is removed from service according to the end conditions required which in this case were that the crack reached dysfunction length of 4.5 mm. The corresponding number of hours on the disc are also recorded. The Safe Inspection Interval (SII), 1374 hours and 1699 hours for disc 1

and disc 8 respectively, was calculated simply as the difference between the initial and dysfunction TSN values.

Typical Distribution Plot showing 0.1% SII

Stability and convergence tests showed that 5000 simulations were required for adequate representation of the variables. Therefore some analysis was required in order to choose an adequate safe inspection interval. Using the safety factor common to aircraft engine components ie. to choose a SII such that 1 in 1000 reach this condition, then we need the 0.1% SII

The distribution of SII's varied depending on the input conditions, however in this "typical" case the SII's were found to follow a lognormal distribution. Figure 4.7 shows a cumulative probability plot for this lognormal distribution. The 5000 SII's are plotted, the lognormal parameters shown were calculated using the maximum likelihood method, and the resulting 0.1% SII of 880 hours is also given.

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<i>Input Variables</i>	<i>Description</i>	<i>Values</i>
Number of Simulations	Integer	5000
Seed for Random Numbers	Double Precision	123457 D0
Initial Safe Life Conditions	$T S N_{init} - TTCI$ $a_{init} = 0.8 \text{ mm}$	Weibull Distribution $\eta = 3000 \text{ hours}$ $\beta = 5$
POD Curve	C-Scan	Loglogistics Curve $\beta_0 = 1.2530$ $\beta_1 = 2.0475$
Distribution of Cracks Missed	C-Scan	Lognormal Curve Median = 0.51 mm $\lambda = -0.68$ $\zeta = 0.7438$
Crack Growth Rate	high time, high temp	Paris Equation $C = 1.31 \times 10^{-12}$ $n = 3.22$ $\sigma_{Y/x} = 0.1265$
End Conditions	Dysfunction Crack Length	$a_{dye} = 4.5 \text{ mm}$

Figure D.1: **Typical Input for a DISK2 Run.**

DISK NO 1

INITIAL SAFE LIFE VALUES (A.TSN)

	0.80	3829	
1 ESTABLISH WHETHER CRACK IS DETECTED OR NOT			
FOUND (RND <= POD)	0.260711	0.689143	
2 CRACK GROWTH			
NORMAL DISTRIBUTION (MEAN,SD)	0.000	0.127	
RANDOM NUMBER (RNORM)	0.125		
VARIABLE STRESS INTENSITY (A,DELTAK)	0.80	33.62	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.001037		
VARIABLE STRESS INTENSITY (A,DELTAK)	1.00	35.88	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.001268		
VARIABLE STRESS INTENSITY (A,DELTAK)	1.26	38.50	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.001601		
VARIABLE STRESS INTENSITY (A,DELTAK)	1.58	41.57	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.002066		
VARIABLE STRESS INTENSITY (A,DELTAK)	1.99	45.22	(a) Crack Found
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.002733		
VARIABLE STRESS INTENSITY (A,DELTAK)	2.26	47.48	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.003214		
VARIABLE STRESS INTENSITY (A,DELTAK)	2.59	50.01	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.003818		
VARIABLE STRESS INTENSITY (A,DELTAK)	2.97	52.86	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.004589		
VARIABLE STRESS INTENSITY (A,DELTAK)	3.43	56.09	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.005589		
VARIABLE STRESS INTENSITY (A,DELTAK)	3.99	59.79	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.006909		
DYSFUNCTION VALUES (A.TSN)	4.50	5203	
HOURS (INITIAL SAFE LIFE TO DYS)			
(IE. SAFE INSPECTION INTERVAL, SII)	1	1374	

DISK NO 8

INITIAL SAFE LIFE VALUES (A.TSN)

	0.80	2420	
1 ESTABLISH WHETHER CRACK IS DETECTED OR NOT			
NOT FOUND (RND > POD)	0.898330	0.689143	
NORMAL DISTRIBUTION (MEAN,SD)	0.000	1.000	
LOGNORMAL DISTRIBUTION (MEAN,SIGNAL)	-0.680	0.744	
RANDOM CRACK LENGTH MISSED(MM)	0.79		
2 CRACK GROWTH			
NORMAL DISTRIBUTION (MEAN,SD)	0.000	0.127	
RANDOM NUMBER (RNORM)	0.032		
VARIABLE STRESS INTENSITY (A,DELTAK)	0.79	33.53	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.000818		
VARIABLE STRESS INTENSITY (A,DELTAK)	1.04	36.22	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.001057		
VARIABLE STRESS INTENSITY (A,DELTAK)	1.25	38.40	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.001283		
VARIABLE STRESS INTENSITY (A,DELTAK)	1.50	40.88	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.001580		
VARIABLE STRESS INTENSITY (A,DELTAK)	1.82	43.75	(b) Crack Not Found
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.001979		
VARIABLE STRESS INTENSITY (A,DELTAK)	2.22	47.09	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.002527		
VARIABLE STRESS INTENSITY (A,DELTAK)	2.47	49.11	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.002905		
VARIABLE STRESS INTENSITY (A,DELTAK)	2.76	51.33	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.003364		
VARIABLE STRESS INTENSITY (A,DELTAK)	3.10	53.79	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.003928		
VARIABLE STRESS INTENSITY (A,DELTAK)	3.49	56.52	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.004631		
VARIABLE STRESS INTENSITY (A,DELTAK)	3.95	59.58	
CRACK GROWTH RATE (A_VAR) IN MM/HR	0.005517		
DYSFUNCTION VALUES (A.TSN)	4.50	4119	
HOURS (INITIAL SAFE LIFE TO DYS)			
(IE. SAFE INSPECTION INTERVAL, SII)	8	1699	

Figure D.2: Typical Output for a DISK2 Run.

END

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FIN