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EVANESCENT-WAVE AMPLIFICATION
IN OPTICAL-WAVEGUIDE DEVICES

by

Robert Ian MacDonald, B.Sc.(Eng.), M.Eng.

A thesis submitted to the Faculty of
Graduate Studies in partial fulfillment
of the requirements for the degree of

Doctor of Philosophy

Carleton University
Ottawa, Ontario

April 1975

The undersigned hereby recommend to the
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this thesis

Evanescent-Wave Amplification
in Optical-Waveguide Devices

submitted by Robert Ian MacDonald, B.Sc.(Eng.),
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ments for the degree of
Doctor of Philosophy.

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ABSTRACT

Optical devices employing the principle of evanescent-wave gain form an interesting new class of active optical elements with potential applications in integrated-optics apparatus and in optical communications. Three representative evanescent-wave devices are studied in this thesis: two different optical amplifiers and a distributed-feedback laser. Computational methods are developed for determining operational characteristics of these devices from their physical parameters. On the basis of the computational results, conclusions are drawn concerning the possible uses of such devices.

The two evanescent-wave amplifiers considered can be used to amplify signals carried on optical waveguide and, potentially, as mode-filters and modulators. Amplifiers employing evanescent-wave gain are easier to construct than other types of optical-waveguide amplifiers because there is no requirement for an active core-region in the waveguide in which they are formed. The analyses and the results of the calculations show that, if adequate control can be maintained on certain waveguide parameters, these amplifiers can have gain comparable to other types of optical-waveguide amplifiers.

The unique modal gain properties of evanescent-wave amplifiers and devices based on them are thus shown to be of practical importance.

The oscillator that is studied employs a corrugated-waveguide structure to provide distributed feedback in an evanescent-wave amplifier. Methods are developed for determining the coupling and radiation loss coefficients for a general waveguide corrugation. These are employed, along with the methods developed for determining evanescent-wave gain, to calculate the threshold gains for oscillation on hybrid modes for a device of this type that has been studied experimentally. The published experimental data agrees with the computed results, in that the hybrid modes of lowest threshold are those that were observed to oscillate. This finding supports the analytical methods used throughout the thesis to determine the properties of evanescent-wave devices.

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CHAPTER I

INTRODUCTION

1.1 GENERAL

This thesis reports analytical studies of three optical devices that employ the principle of evanescent-wave amplification (1-6). Two of them are amplifiers of bound optical radiation. These are hypothetical devices that serve to illustrate the analytical methods used to determine evanescent-wave gain, and the properties that such amplifiers would be likely to exhibit. The third device is an optical oscillator that has been reported in the literature⁽⁶⁾. The threshold gains for oscillation on the various modes of this oscillator are determined analytically, and computed results are compared with the reported experimental data. The analytical methods used throughout the thesis are justified in that the modes of lowest calculated threshold gain are, in fact, those that were observed to oscillate.

1.1.1 Evanescent-Wave Devices

All three of the evanescent-wave devices considered embody optical quantum amplifiers. They are thus members of the class of active optical elements for which "laser" is now the general term. They are distinguished from other laser amplifiers and laser oscillators in that the electromagnetic

waves they amplify do not interact directly with an active material. Amplification occurs instead as a result of interactions involving an evanescent field that is associated with the signal wave because it is bound to a dielectric waveguiding structure. This structure is arranged so that there are regions in it that have optical gain, but are external to the waveguiding core. The bound wave interacts with these regions through the evanescent fields that result from bound propagation in a dielectric waveguide.

Dielectric waveguiding structures consist essentially of a central core made of a dielectric with a high permittivity, surrounded by dielectrics of lower permittivity. Electromagnetic radiation that propagates in the core can be totally reflected at the interfaces between that region and the surrounding regions if the angle of its incidence on the boundary is sufficiently high. In such circumstances, the radiation cannot escape from the core and thus is bound to the dielectric structure. The radiation propagates almost entirely in the core. (A portion of the power is carried in the surrounding medium adjacent to the core. This is discussed below.) Radiation can propagate in such a structure by various modes that correspond essentially to different numbers of reflections of the bound radiation at the core boundary, per unit length of the waveguide. Propagation on a high-order

mode, for example, entails many reflections back and forth across the core to achieve a given progress along the waveguide. Propagation on lower-order modes involves fewer such reflections to achieve the same progress, and in consequence the propagation velocity is higher for the lower-order modes.

Evanescent waves arise whenever electromagnetic radiation is reflected at an interface between the dielectric material in which it propagates and another material of lower permittivity, and the angle of incidence is greater than the critical angle for total internal reflection. Under these conditions, the radiation is reflected back into the region of higher permittivity without loss; hence the name total internal reflection for the phenomenon. Although all the power is reflected, there is an electromagnetic field established in the region of lower permittivity. This field is not associated with a wave that propagates away from the boundary in this region, but rather with a certain portion of the reflected power that travels parallel to the boundary. This power is distributed in a thin layer that extends only a short way from the boundary into the external dielectric. Since the reflection at the interface is total, all this power is eventually returned to the reflected wave, after travelling for a short distance along the boundary. The field in the external region is called an evanescent field because it must vanish at large

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distances from the boundary between the two dielectric regions.

Interactions of the evanescent field with the external region can result in the loss of power in the evanescent portion of the reflected wave. The reflection is then not total at the boundary, even though the conditions for total internal reflection are fulfilled. A common example of this is the phenomenon of frustrated total internal reflection. To demonstrate this phenomenon, a second region of high permittivity is brought near to the first so that the two are separated only by a narrow region of low permittivity. The evanescent field penetrates into the second high permittivity region, and is converted there to a wave that propagates away from the boundary. Since this wave carries power away from the evanescent field, the initial reflection is no longer total, and hence the name frustrated total internal reflection is appropriate. The electric fields of this phenomenon are analogous to wave functions in the phenomenon of tunnelling in quantum mechanics. Power can also be lost in the evanescent field if the low permittivity region is lossy. In such a case, though there is no transmitted wave carrying power away from the interface, the power lost by the portion of the reflected wave that travels along the boundary in the lossy medium results in a reflection that is no longer total.

By analogy to the lossy external dielectric, one can consider the case where there is bulk gain in this region. In this case, power would be added to the portion of the reflected wave that travels along the boundary and the reflection would result in power gain rather than loss. If a waveguide were to be constructed in this way, each time a bound mode is reflected at the core boundary, the power in the mode would increase. The result would be an amplification as the bound wave propagates in the waveguide. It is possible to create the effect of bulk gain in a dielectric material by stimulated emission. Thus, if the external dielectric of a waveguide were to contain a suitable laser material, and pumping were provided, then bound waves of an appropriate frequency to obtain gain from this material would be amplified as they propagate in the waveguide. This is the principle of evanescent-wave amplification.

Evanescent-wave devices were first constructed in 1968⁽¹⁾. These devices took the form of a passive glass waveguiding core surrounded by a neodymium-doped glass cladding, pumped by an external flashtube. Both an amplifier and, using external feedback-mirrors, an oscillator, were produced. The modal characteristics of evanescent-wave optical amplifiers were first analyzed in the dielectric-slab waveguide configuration in 1971⁽²⁾ in a paper which noted that the effect of a

cladding medium with gain on the modes of such a waveguide can be obtained from first-order perturbation theory. In a subsequent paper⁽³⁾, the case was analyzed wherein the active medium surrounding the core-slab exhibited gain because of excited states that were established by the evanescent fields of another bound mode, the pump mode, that propagates in the structure. It was shown in this work that energy can be transferred from the pump mode to a signal mode by the interaction of the evanescent fields of the two modes with the active cladding medium adjacent to the core-slab in which the two modes are guided. It was noted that the gain that can be delivered to the signal mode is obtained more efficiently when pumping is by a bound mode than when externally-incident radiation is used.

These analyses of gain in optical dielectric-slab waveguides were performed under the simplifying assumption that the waveguides were symmetric, that is, that the cladding materials on both sides of the core were identical both in refractive index and in gain properties. An analysis of the more realistic asymmetric case has also been performed⁽⁴⁾. In this analysis, the effect of a parasitic absorption occurring in one cladding layer was considered and several unusual properties of mode-pumped evanescent-wave amplifiers were noted.

Distributed-feedback oscillators with evanescent-wave gain have been reported^(5,6). In one of these, distributed feedback was provided by corrugation in the interface between the core and one of the cladding media⁽⁶⁾. This device is of particular interest because of the ease of its construction. An analysis of this oscillator, and the computation of the threshold gains for lasing on its various modes, forms a major objective of this thesis.

The analytical results for the two evanescent-wave amplifiers reported in the thesis represent an extension of work reported earlier by K.O. Hill, A. Watanabe and the author⁽³⁾. The development of the analytic methods used is given in detail to outline the perturbation-theory technique basic to the analysis of the evanescent-wave oscillator. In turn, the experimental verification of the results for the oscillator lends support to the conclusions reached, on the basis of the calculations, about the amplifiers.

1.1.2 Optical Communications and Integrated Optics

The rapid development of guided-wave optical communications⁽⁷⁾ and the level of current interest in that field ensure that any new optical-waveguide device comes under immediate scrutiny for communications purposes. Optical devices working by the principle of evanescent-wave amplification may have applications in a future optical communications system. It is

not possible to evaluate the communications potential of evanescent-wave devices in detail on the basis of the results obtained for those examples studied in the thesis. Nevertheless, some applications do become apparent during the course of the analyses presented here, and these are discussed as they arise.

The activity in optical communications has led to a complementary activity in the field of integrated optics⁽⁸⁾. Many different devices can be fabricated from dielectric waveguide that takes the form of flat strips of a guiding core-dielectric deposited on a dielectric substrate⁽⁹⁻¹⁴⁾. Such waveguide is comparatively convenient to manufacture because many of the required techniques are similar to those used in the manufacture of semiconductor integrated circuits. Because it is possible to construct optical devices that consist of many different elements fabricated at the same time, on a single substrate, the field has come to be called integrated optics.

In general, the waveguides used in integrated optics are thin-film dielectric waveguides that are much wider than the thickness of the core-region. They are formed on a substrate that is usually planar. They thus approximate closely the dielectric-slab waveguide structure. In Fig. 1-1 is shown a cut-away view of a typical integrated optics waveguide structure.

FIG. 1-1. Integrated optics waveguiding structure.

Integral amplifiers and oscillators are desirable for integrated optics apparatus because of the difficulty of coupling power into and out of optical waveguide with good efficiency. Waveguide amplifiers that provide gain in their core-regions have been proposed for use as integrated oscillators and amplifiers (15-17).

Evanescent-wave active devices may have advantages over those with gain in their core-regions. Since the active medium is external to the waveguiding core, such devices can be constructed without regard to the necessity of incorporating the laser material into the core. The properties of the core, in particular its bulk loss mechanisms, and the smoothness of the boundary between the core and cladding are important in determining the quality of the passive waveguide in which the active device is constructed. It is therefore beneficial to have the possibility of making the core by means that provide the best control of these properties and are not compromised by the necessity of introducing an active material. Since the laser material in planar evanescent-wave devices is contained in one of the cladding regions, it has no direct influence on the construction of the core. In fact, the dielectric region that provides the gain need not play any role in maintaining the structural form of the device. It is even possible to have a liquid laser-dye as the active dielectric region of a planar

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evanescent-wave amplifier. Such a device consists of a low-permittivity substrate with a high-permittivity core-region on its surface, and the entire structure immersed in the laser dye, which can be flowing to eliminate the problems of dye-bleaching in the pumped area. Considerable design freedom is gained by the use of evanescent-wave active devices in integrated optics.

All the analyses of evanescent-wave devices given in this thesis assume the slab-waveguide configuration. These analyses are therefore directly applicable to the implementation of these devices in integrated optics. Furthermore, manufacturing considerations make the slab-waveguide form of evanescent-wave devices the easiest to construct in practice. This is especially true in the case of the evanescent-wave distributed-feedback oscillator. The feedback structure of this device can be formed by relatively simple techniques on a plane surface, whereas its construction on a curved surface, such as the core-boundary of an optical-fibre waveguide, would be much more difficult. While it is true that the optical waveguides used for communication will likely be of circular cross-section, the convenience of integrated optics makes it likely that planar waveguide will be used for most of the processing of the signal. The analysis of evanescent-wave devices in the planar configuration is considerably simpler than in the circular, and gives a clearer physical insight into the

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operation of the devices. The analysis in the slab-waveguide structure is thus addressed to the requirements of foreseeable optical communications systems.

1.2 THESIS ORGANIZATION

The three evanescent-wave devices that are examined in this thesis are closely related to each other. All are amplifiers that provide gain to optical radiation propagating as bound modes of a dielectric waveguiding structure. One amplifier is provided with positive feedback sufficient to cause oscillation; the others are not. Since, however, the characteristics and uses of these devices are not the same, and many of the details of analyzing their properties differ, it is appropriate to treat each of them separately, borrowing in each case what is appropriate from the others. Accordingly, each device is treated in a separate Chapter. The organization of these Chapters is outlined in this section.

Chapter 2: The Externally-Pumped Evanescent-Wave Amplifier

The simplest device that is studied is treated in this Chapter, and the methods used are given in detail, for they are also used in subsequent Chapters. This simplest evanescent-wave device is an amplifier that consists of a dielectric-slab optical waveguiding structure in which one cladding region is provided with optical gain. This region contains a lasing material, for example, a laser dye that is pumped by optical

radiation. The pumping radiation is incident from outside the waveguiding structure, and penetrates it in a direction transverse to the propagation direction for bound modes. Signal radiation, of an appropriate frequency to induce stimulated emission in the pumped laser material, can be amplified as it propagates as a bound mode of the structure. Amplification occurs through the evanescent fields of the bound wave, which extend into the pumped active material.

The modal gain coefficients of such a device are determined using a perturbation-theory method. This method requires knowledge of the modal field-distributions in the waveguide, which in general, is an asymmetric structure. Expressions for the modal field distributions of such a structure are developed in Appendix A. The gain coefficients for the amplifier are expressed in terms of the gain-coefficient ratio (GCR) which is the ratio of the modal gain coefficient to the gain coefficient for a free wave propagating in the pumped active material. The GCR is independent of the length of the amplifier and the intensity of the pump radiation, and therefore is used to study how the properties of the amplifier depend on the parameters of the waveguiding structure.

The dependence of modal GCR on the difference between the refractive indices of the core and the active and passive sides, and on the thickness of the core, are presented for a

1
"standard amplifier", which is a hypothetical device whose parameters are chosen to be representative of a practical configuration. The effects of parasitic losses, such as might be caused by bulk absorption or scattering in the core and the two cladding regions, are determined.

The feasibility of constructing the "standard amplifier" is examined, and it is found that, for this particular configuration, the required tolerances on refractive indices may prevent the attainment of high efficiencies, but that moderate efficiencies appear practical, and in view of this, other advantages of the evanescent-wave amplifier may predominate. The evanescent-wave amplifier is easier to manufacture than other types of waveguide amplifiers because the lasing medium and the guiding medium are separate. Furthermore, there exist mode suppression effects in the evanescent-wave amplifier that are not available in other types because the evanescent-wave device has its highest gain on the highest-order, rather than the lowest-order, propagation mode of the waveguide. These effects may be of advantage in matching the amplifier to particular modes of a passive waveguide, and so constructing an active optical mode filter. Finally, the actual gain of the modes of an amplifier of this type with a specified pump power are shown as a function of amplifier length, for subsequent comparison with the second type of evanescent-wave amplifier that is studied in Chapter 3.

Chapter 3: The Bound-Mode Pumped Evanescent-Wave Amplifier

The efficiency of evanescent-wave amplifiers can be improved if the pumping radiation is made to propagate as a bound mode of the waveguide rather than simply to pass transversely through it. When the pump radiation is bound, its power is confined to nearly the same regions of the waveguide in which the signal power travels. The transfer of power between the pump and signal radiation that is mediated by the amplifier structure thus takes place with best efficiency. The gain properties of the evanescent-wave amplifier with bound-mode pumping depend on the orders of both the pump and the signal modes. The same perturbation theory approach as is used in Chapter 2 is employed to obtain expressions for calculating measures related to the gain available from the structure for each pump-mode, signal-mode pair.

The pump-mode is attenuated as it propagates because it loses power to the signal mode. For this reason, the signal-mode gain is not constant along the length of the amplifier, and it is not possible to define a gain coefficient. To characterize the gain properties in a general way, two quantities are defined: the Net Gain Coefficient (NGC) and the Interaction Length (IL). The NGC is the natural logarithm of the gain that would be available from an infinitely long amplifier. It serves to indicate the dependence of the gain properties of the device on structure parameters. The IL is

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the distance that the pump and signal modes must travel together in the device to produce a change in the signal-mode irradiance equal to half that which would occur if they were to propagate to infinity. The IL thus relates the NGC to a finite version of the device.

The dependences of NGC and IL on the refractive index differences between the core and the active and passive sides, and on the core thickness, are calculated for the mode-pairs of the standard amplifier that is defined in Chapter 2. The effects of parasitic losses are also considered. In general, it is found that the best combinations of pump and signal modes are those with similar field distributions, i.e. of the same order. The mode-filtering capabilities of the evanescent-wave amplifier in the bound-mode pumped configuration are discussed. A direct comparison is made between the actual gains of a bound-mode pumped amplifier and an externally-pumped amplifier.

Chapter 4: The Passive-Core Corrugated-Waveguide Laser

In this Chapter, an evanescent-wave device is studied for which experimental data are available. This is an oscillator consisting of a transversely-pumped evanescent-wave amplifier provided with distributed-feedback (DFB) by second-order diffraction from a corrugation in one waveguide wall. The parameters of the waveguide structure and the corrugation, and the wavelengths of the oscillations, were accurately deter-

mined for this device. These data provide the basis for a direct comparison between experimental and theoretical results for an evanescent-wave laser oscillator.

Perturbation theory is used in conjunction with the wave equation for modal propagation in the device, as in previous Chapters, to determine the effects of the perturbations in the passive waveguiding structure. In this case, the perturbations consist of the corrugation as well as the presence of a cladding region with gain. Coupled-mode equations are obtained that relate the field-amplitude distributions of the two contradirectional modes that are coupled together to cause oscillation. In corrugated-waveguide DFB lasers, these modes may not be of the same order, in which case the oscillation is said to be on a "hybrid" mode. The wavelengths on which the experimental device oscillated could be identified unambiguously with the wavelengths of a particular group of hybrid modes, in which one of the modes was always of the highest order that the structure allowed. By solving the coupled-mode equations for the threshold gain conditions of the possible hybrid modes, a comparison is made to establish whether the hybrid modes of lowest threshold for oscillation were, in fact, those that were observed to oscillate. In order to obtain calculated threshold gain coefficients that can be compared with the experimental results, the coupled-mode equations are

solved for a DFB laser structure that is pumped only in a central portion of its corrugated length, and is therefore lossy at the ends. Development and solution of the coupled-mode equations are treated in detail in Appendices B and C, respectively.

Methods are developed in the course of the analysis for computing coupling coefficients corresponding to the various orders of a general corrugation waveform, and for determining the loss coefficients corresponding to the effects of low-order coupling into radiated modes by the corrugation. These are given in detail in Appendices D and E. These methods are general, and can also be applied in the analysis of other types of corrugated-waveguide lasers. The approach used to obtain coupling coefficients makes clear why hybrid-mode oscillations are particularly characteristic of corrugated-waveguide DFB devices.

The results of threshold calculations for various oscillator configurations show, in general, that the dominant factors in determining which hybrid modes have the lowest thresholds for oscillation are the length of the passive laser structure, the proportion of it that is pumped, and therefore provides gain, and the corrugation waveform, which determines the relative strengths of the coupling coefficient and the radiation loss coefficient. In particular, it is found that

the hybrid modes that were observed to oscillate in the experimental device are indeed those with the lowest computed thresholds for oscillation.

Comments are made on the utility of the analyses given in this Chapter for the design of evanescent-wave DFB lasers, and on the uses to which such lasers might be put.

Chapter 5: Conclusions

The analytical and computational results for the three evanescent-wave optical devices studied are reviewed. The importance of the devices and the results obtained are discussed in the context of the development of integrated optics and optical communications. Possible future developments are outlined, and suggestions are made for further work.

CHAPTER 2

THE EXTERNALLY-PUMPED EVANESCENT-WAVE AMPLIFIER

2.1 INTRODUCTION

The principles of evanescent-wave amplification are best illustrated by considering the simplest device to employ them. This is the externally-pumped evanescent-wave amplifier, which amplifies an optical signal propagating as a bound mode of a dielectric waveguide. In such an amplifier, there is a gain coefficient associated with each bound mode of propagation. Expressions giving these modal gain coefficients in terms of the properties of the amplifier structure are derived in this Chapter. The method that is used to develop these expressions is given in some detail, since it underlies the results of later Chapters as well. The results of numerical calculations of parameters related to the modal gain coefficients of a practically-possible externally-pumped evanescent-wave amplifier are presented. Some conclusions based on these calculations are drawn about the properties and potential uses of the device.

The externally-pumped evanescent-wave amplifier to be studied consists of a dielectric waveguiding structure in which one cladding region is provided with optical gain. This

region contains a laser material pumped by optical radiation incident, in this case, from outside the waveguide, so that the pumping radiation penetrates the waveguiding structure in a direction transverse to the guided propagation direction. Signal radiation of an appropriate frequency to induce stimulated emission in the pumped laser material, can be amplified as it propagates in a bound mode of the structure, through the interaction of its evanescent field with the region of optical gain.

The distribution of dielectric constant that forms the waveguiding structure basic to evanescent-wave devices is asymmetric in the general case. A solution of Maxwell's equations that gives the bound and unbound modal field distributions for propagation in such a structure is developed in Appendix A. The notation set forth in this solution is used in this and subsequent Chapters.

2.2 GAIN COEFFICIENTS OF THE EXTERNALLY-PUMPED EVANESCENT-WAVE AMPLIFIER

The gain of an externally-pumped, lossless, symmetric evanescent-wave amplifier was determined by Hill, Watanabe and Chambers⁽²⁾, using a perturbation method. That analysis is extended in this section to include asymmetric waveguiding structures, and the effects of parasitic losses are also considered.

2.2.1 The Gain Perturbation

It is noted in Appendix A that the transverse field distributions $E_{y;n}(z)$ corresponding to the modes of propagation of a passive dielectric waveguide satisfy a wave equation that is formally identical to the Schrodinger wave-equation:

$$\frac{\partial^2 E_{y;n}(z)}{\partial z^2} + [\epsilon'(z)\mu\omega^2 - k_{x;n}^2]E_{y;n}(z) = 0. \quad (2-1)$$

In this equation, $\epsilon'(z)$ is the distribution of dielectric constant that forms the waveguiding structure, and is analogous to the potential-energy function of the Hamiltonian. The squares of the modal-propagation constants, $k_{x;n}$, are eigenvalues analogous to the energy eigenvalues of the Schrodinger equation. Minor changes in the waveguide structure defined by $\epsilon'(z)$ therefore produce small changes that can be determined by first-order perturbation theory, in the eigenvalues $k_{x;n}^2$.

The dielectric constant distribution $\epsilon(z)$ forming an optical waveguide in which there is gain takes the form:

$$\epsilon(z) = \epsilon'(z) + j\epsilon''(z). \quad (2-2)$$

In Eq. 2-2, $\epsilon'(z)$ is the dielectric constant distribution that forms the passive waveguide, and $j\epsilon''(z)$ represents the distribution of gain in this structure. The presence of a positive imaginary component of dielectric constant to represent gain is clarified by the following demonstration.

Consider a parallel-plate capacitor with a capacitance in vacuo of C_0 . When this capacitor is filled with a substance of dielectric constant, ϵ , the charge Q stored in it as a function of applied voltage V is $Q = C_0 \frac{\epsilon}{\epsilon_0} V$ where ϵ_0 is the dielectric constant of the vacuum. (We assume the dielectric constant to be independent of voltage.) The current that flows as a result of the application of a sinusoidal voltage $V = V_0 e^{j\omega t}$ to the capacitor is therefore

$$\frac{dQ}{dt} = j \omega C_0 \frac{\epsilon}{\epsilon_0} V. \quad (2-3)$$

If ϵ is complex, of the form,

$$\epsilon = \epsilon' + j\epsilon'', \quad (2-4)$$

then the current is given by

$$I = j \omega \frac{\epsilon'}{\epsilon_0} C_0 V - \omega \frac{\epsilon''}{\epsilon_0} C_0 V. \quad (2-5)$$

The first term on the right-hand side of Eq. 2-5 represents a current in quadrature with the voltage, which does not result in the loss of real power. If ϵ'' is negative, the second term on the right-hand side, however, indicates a resistive component in the equivalent circuit for the capacitor, with a parallel resistance of $\epsilon_0 / \omega \epsilon'' C_0$. This component causes power loss. A lossy dielectric is therefore represented by a dielectric constant of the form $\epsilon' + j\epsilon''$, where ϵ'' is negative.

If the dielectric constant of the material in the capacitor were to be represented by a complex number with a positive rather than a negative imaginary component, the resistance representing "loss" would have a negative value, and would correspond instead to power gain induced by the capacitor. We therefore can represent as having dielectric constants of the form $\epsilon' + j\epsilon''$ those materials in the amplifier structures under consideration that yield energy to, rather than absorb energy from, the electric field. The dielectric constant distribution that forms the evanescent-wave amplifier structure can thus be represented by

$$\epsilon(z) = \epsilon'(z) + j\epsilon''(z). \quad (2-6)$$

The distribution of the real component of dielectric constant, $\epsilon'(z)$, shown in Fig. 2-1, is typical of a passive slab-waveguide structure suitable for use as an evanescent-wave amplifier. The distribution of the imaginary component of dielectric constant $\epsilon''(z)$ shown in that figure represents a gain distribution that could be introduced into the passive waveguiding structure to provide gain by evanescent-wave interactions. In this case, gain is provided to only one of the two evanescent fields of the bound wave. Only one cladding region of the slab waveguide, denoted region 3, is represented as active because this configuration seems a likely one for a

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FIG. 2-1. Distribution of the real and imaginary components of dielectric constant in a typical evanescent-wave amplifier structure.

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practically-realizable device. The other cladding side, region 1, would probably need to be a passive substrate upon which the core, region 2, is formed. The refractive index difference between the core and cladding materials is smaller on the side with gain than it is on the passive side of this structure. Such a configuration is desirable in evanescent-wave devices because it results in an evanescent field on the active side that is relatively weakly bound to the core. Under these conditions, a larger proportion of the modal power is carried in the evanescent wave on the active side, thus interacting with the medium providing gain, than is carried on the passive side, which provides no gain. Losses may occur in all three regions of the structure, provided only that any loss mechanisms in the active cladding material do not overcome the gain mechanism. Such losses are represented by a negative component $j\epsilon''(z)$ of dielectric constant in the lossy region.

2.2.2 Propagation of Signal Modes

If the imaginary component of dielectric constant $j\epsilon''(z)$ that represents the gain (and loss) distribution is small in magnitude compared to the real component $\epsilon'(z)$ representing the passive structure, it can be treated as being responsible for a perturbation of the properties of the passive structure.

In Appendix A, the propagation constant eigenvalues $k_{x;n}^2$ and electric field distributions $E_{y;n}(z)$ corresponding to modal propagation of signal radiation as bound waves in the passive waveguiding structure are obtained. According to first-order perturbation theory, the effect of a small change in the passive dielectric constant distribution is to add a perturbation δ_n to the eigenvalue $k_{x;n}^2$, so that the perturbed eigenvalue can be written

$$(k'_{x;n})^2 = k_{x;n}^2 + \delta_n \quad (2-7)$$

where the perturbation in the eigenvalue is given by

$$\delta_n = \mu\omega^2 \int_{-\infty}^{\infty} E_{y;n}(z) j\epsilon''(z) E_{y;n}^*(z) dz \quad (2-8)$$

Only TE modes are considered in the development that follows. The analysis can be carried out for TM modes by similar methods. It has been pointed out⁽¹⁸⁾ that, in this case, an extra term appears in the perturbation of the propagation constant, and that the term corresponding to the inner product in Eq. 2-8 is modified.

The distribution of the imaginary component of dielectric constant, $j\epsilon''(z)$, must be determined in order to evaluate the perturbation. If the relation between the intensity of

the pumping radiation and the imaginary portion of the dielectric constant is known, the distribution $j\epsilon''(z)$ can be calculated from the distribution of pumping intensity. In all cases, this relation is monotonically increasing. For the laser-dye active media to be considered in subsequent sections, the imaginary component of the dielectric constant is directly proportional to the pump-field intensity⁽³⁾. High pumping intensity is therefore desired in the active cladding near the core, where the evanescent fields of the signal wave are strongest.

Ideally, the distribution of gain in the active region should match the distribution of the evanescent field intensity of the signal. Since the pumping radiation is absorbed in the active material, the best configuration for the externally-pumped amplifier has the pump radiation incident from the passive side of the structure, so that the greatest pumping intensity is at the interface of the active region with the core. This arrangement may not be convenient in practice. Furthermore, dye concentrations great enough to give pump-radiation absorption coefficients as large as the damping constants for the evanescent fields are not possible because non-radiative processes interfere with the lasing transition at very high concentrations. In a practical configuration,

the dye concentration is low enough that the pump radiation is not strongly absorbed, and the radiation can be incident from the active side of the structure. We therefore consider an externally-pumped evanescent-wave amplifier in which the pumping radiation intensity is approximately constant over the distance to which the evanescent fields of the signal interact strongly with the active cladding medium, and do not specify from which side the radiation is incident.

By performing the integral in Eq. 2-8 using the modal field distributions given by Eq. A-15, we obtain the perturbation in the propagation constant eigenvalue for the n th mode caused by the presence of the gain in the structure. The result is

$$\delta_n = \mu \omega^2 A_n^2 \epsilon_3'' \sin^2(-k_{z;n} d + \phi_n) \left(\frac{j}{2k_{3;n}} \right). \quad (2-9)$$

The modal field distributions $\mathcal{E}_{y;n}(x, z)$ are functions of both x and z , and are related to the normalized transverse field distributions $E_{y;n}(z)$ by

$$\mathcal{E}_{y;n}(z) = A_{0;n} E_{y;n}(z) e^{-j k_{x;n} x} \quad (2-10)$$

where $A_{0;n}$ is the modal amplitude at $x=0$. The perturbation caused by the presence of gain results in the alteration of the propagation constant $k_{x;n}$ to $k'_{x;n}$ where the perturbed

propagation constant $k'_{x;n}$ is given by Eq. 2-7. If δ_n is small, as is required by perturbation theory, we can write the axial variation of the fields approximately as

$$e^{-jk'_{x;n}x} \approx e^{-jk_{x;n}(1+\delta_n/2k_{x;n}^2)x} \quad (2-11)$$

By substituting Eq. 2-11 into Eq. 2-10, we obtain the perturbed modal field distributions

$$\tilde{E}_{y;n}(x,z) = \tilde{E}_{y;n}(0,z) e^{(\alpha_{G;n}/2)x} \quad (2-12)$$

where

$$\alpha_{G;n} = -j \frac{\delta_n}{k_{x;n}} \quad (2-13)$$

From Eqs. 2-9 and 2-11, it can be seen that $\alpha_{G;n}$ is a positive quantity if ϵ''_3 is positive and therefore the amplitude of the signal-wave increases as it propagates in the x-direction.

The term $\alpha_{G;n}$ is the modal intensity-gain coefficient for the bound wave.

It is convenient to relate the gain coefficient for a bound wave to the gain coefficient for a plane wave propagating in a medium with the same complex dielectric constant as the active cladding region. The bulk intensity-gain coefficient α_{03} for a plane wave in a dielectric medium with dielectric constant $\epsilon' + j\epsilon''$ is

$$\alpha_{03} = \left[\frac{k_{03} \epsilon_3''}{\epsilon_3'} \right], \quad (2-14)$$

where

$$k_{03} = \omega \sqrt{\mu \epsilon_3'}$$

and $\epsilon_3'' \ll \epsilon_3'$. (This has been required already by perturbation theory.) The process of optical pumping induces an imaginary component of dielectric constant, measured at the frequency of the signal radiation, in the active cladding region. This component is positive and, in the case of the laser dyes with which we will be concerned, is proportional to the electric field intensity of the pumping radiation. We can therefore express the imaginary component of the dielectric constant that is induced in the active cladding region by the pumping radiation as

$$\epsilon_3''(z) = I_p(z) x_3, \quad (2-15)$$

where

$I_p(z)$ is the distribution of the pumping radiation intensity transverse to the propagation axis;
 x_3 is the proportionality constant between the imaginary component of the dielectric constant of the active region, and the pumping field intensity, multiplied by the refractive index of the active medium at the pump wavelength.

It should be noted that the pumping intensity is assumed to be uniform in the axial direction of the amplifier as well as constant over the range of the transverse co-ordinate z in which evanescent-wave interactions take place. The radiation intensity I_p is related to the power P of the pumping radiation by the Poynting relation

$$P = \sqrt{\frac{\epsilon_0}{\mu}} \iint_{-\infty}^{\infty} I_p \, dz dy. \quad (2-16)$$

Since the intensity of the pumping radiation is uniform over the pumped region in the axial direction, we have from Eqs. 2-14, 2-15 and 2-16

$$\alpha_{03} = \frac{2\pi\chi_3}{\lambda_p \epsilon_0 n_3^2} \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{P}{Lw}, \quad (2-17)$$

where λ_p is the wavelength of the pumping radiation in vacuo;
 L is the length of the pumped region;
 w is the width of the pumped region.

There may be loss mechanisms in the active material that operate at the signal frequency so that, in the absence of any pumping, this material would be lossy. These losses are, of course, overcome by the gain when the material is pumped. Nevertheless, they may have an effect on the properties of the amplifier because they constitute a component of

the imaginary part of the dielectric constant of the active material that is independent of the intensity of the pumping radiation. Such losses merely cause a constant reduction in the bulk gain coefficient of the active material, so that we may write the effective gain coefficient for this material in the form

$$\alpha_{03}^{\text{eff}} = \alpha_{03} - \alpha_{03}^{\text{loss}} \quad (2-18)$$

where α_{03}^{eff} is the effective gain coefficient

and $\alpha_{03}^{\text{loss}} = \frac{k_{03} \epsilon_3''}{\epsilon_3}$ where $j\epsilon_3''$ is the negative imaginary component of the active material in the absence of pumping.

From Eqs. 2-13 and 2-18, we obtain the modal gain coefficient in the form

$$\alpha_{G;n} = \frac{k_{03}^2 A_n^2 \sin^2(-k_{z;n} d + \phi_n)}{2K_{3;n} k_{x;n}} \alpha_{03}^{\text{eff}} \quad (2-19)$$

We have thus used perturbation theory to determine the modal gain coefficients in terms of the waveguide properties and the bulk gain coefficient induced in the active region by pumping.

2.2.3 Parasitic Bulk Losses

So far, the effects of bulk losses that are present in the passive dielectrics have not been discussed. By saying that the

active material has bulk gain, i.e. a positive imaginary component of dielectric constant, we imply that those processes leading to gain dominate those processes leading to loss in this material. But loss processes also occur in the passive cladding and in the waveguiding core itself. Since there are no processes leading to gain in these regions, these losses give rise to a parasitic loss in the amplifier. Such parasitic losses cause an additional perturbation $\delta_{L;n}$ in the propagation constant eigenvalue, given by Eq. 2-8. The evaluation of the integral yields the perturbation due to these losses in the form

$$\delta_{L;n} = -\mu\omega^2 A_n^2 j \left\{ \epsilon_2'' \left[d + \frac{\sin(2k_{z;n}d)\cos(2k_{z;n}d)}{2k_{z;n}} \right] + \epsilon_1'' \frac{\sin^2(k_{z;n}d + \phi_n)}{2K_{1;n}} \right\} \quad (2-20)$$

where ϵ_2'' and ϵ_1'' are the magnitudes of the negative imaginary components of dielectric constant in regions 2 and 1 of the waveguide structure. These represent the losses.

We define a modal attenuation coefficient, $\alpha_{L;n}$, by

$$\alpha_{L;n} = j \frac{\delta_{L;n}}{k_{x;n}} \quad (2-21)$$

The bulk attenuation coefficients α_{02} and α_{01} of the core and passive cladding respectively are quantities that can be

measured directly. It is therefore convenient to use relations similar to Eq. 2-14 to express the modal attenuation coefficients as

$$\alpha_{L;n} = \frac{A_n^2}{k_{x;n}} \left[\alpha_{02} k_{02} \left\{ d - \frac{\sin(2k_{z;n}d) \cos(2k_{z;n}d)}{2k_{z;n}} \right\} + \frac{\alpha_{01} k_{01} \sin^2(k_{z;n}d + \varphi_n)}{2K_{1;n}} \right] \quad (2-22)$$

When the effects of loss are included, the intensity-gain coefficient $\alpha_{I;n}$ for the n th mode is given by

$$\alpha_{I;n} = \alpha_{G;n} - \alpha_{L;n} \quad (2-23)$$

The actual gain $G_n(L)$ for propagation on the n th mode of an externally-pumped evanescent-wave amplifier of length L is

$$G_n(L) = e^{\alpha_{I;n}L} \quad (2-24)$$

We have developed expressions by which the modal gain coefficients of an externally-pumped evanescent-wave amplifier may be calculated. These gain coefficients depend on the core thickness and the refractive indices of the three regions of the passive dielectric-slab waveguide that forms the basic structure of the amplifier, on the wavelength of the signal radiation, and on the gain and loss properties of the materials used to construct the device. In the next section, some com-

puted values are presented for a typical amplifier of this type.

2.3 NUMERICAL CALCULATIONS FOR A TYPICAL EXTERNALLY-PUMPED EVANESCENT-WAVE AMPLIFIER

Computed values characterizing the gain of an externally-pumped evanescent-wave amplifier are presented in this section. Since the gain depends on several independent parameters of the amplifier structure, it is advantageous to examine the effects of these parameters in a systematic way. For the purposes of the calculations, we define a "standard amplifier" that serves as a base-configuration. The calculations presented in this section show the effect on the gain of the standard amplifier of varying each of its parameters while all the other parameters remain constant.

2.3.1 The Standard Amplifier

The refractive index distribution of the standard amplifier is shown in Fig. 2-1. The values of its various parameters are shown in Table 2-1. The standard amplifier has been chosen to represent a practical device. One cladding side is passive to represent a passive substrate. The other cladding material has properties representative of a Rhodamine 6-G laser-dye solution. The pumping power is 10kW, which is typical of nitrogen lasers that may be used to pump such a

TABLE 2-1. Parameters of the Standard Amplifier
for Calculations on the Externally-
Pumped Evanescent-Wave Amplifier.

	Refractive Index	Thickness (μ)	Proportion- ality Constant χ	Absorption Coefficient
Passive Cladding (Region 1)	1.4	-	0	0
Core (Region 2)	1.5	5	0	0
Active Cladding (Region 3)	1.495	-	2.79×10^{-29}	0

Pumped Length: 2cm

Pumped Width: 5 μ

Pump Power: 10kW

Signal Wavelength: 630nm

device. The width of the pumped region is 5μ , which is the width of the diffraction-limited spot produced by focussing the pumping radiation on the device with an $f/8$ cylindrical lens. The thickness of the waveguiding core and the distribution of the passive component of dielectric constant have been chosen so that the standard amplifier supports ~~only~~ two modes of propagation at the signal wavelength. This choice is made to show the dependence of gain on mode-order while, at the same time, demonstrating the properties of the device in configurations supporting only small numbers of modes. The inter-mode coupling induced by the presence of the gain perturbation limits the bandwidth of multimode evanescent-wave amplifiers because the propagation-time through the amplifier is different for each mode. It is thus anticipated that configurations supporting the least possible number of modes will be preferred. The signal wavelength chosen for calculations with the standard amplifier is 630nm, which is in the region where Rhodamine 6-G laser-dye exhibits optical gain.

The standard amplifier defined in Table 2-1 for the study of the externally-pumped evanescent-wave amplifier is also used in Chapter 3 in the study of the bound-wave pumped evanescent-wave amplifier configuration. A direct comparison can therefore be made between numerical results for the gain of both types of evanescent-wave amplifiers.

2.3.2 Gain Coefficient Ratio (GCR)

In order to determine the effect that the parameters of the passive waveguiding structure basic to the externally-pumped evanescent-wave amplifier have on its gain, it is convenient to define a measure of the gain that is independent of the length and the active component of the amplifier structure. The gain-coefficient ratio (GCR) was introduced by Hill et al.⁽²⁾ to serve as this measure. The GCR is defined to be the ratio of the modal gain coefficient α_n to the bulk gain coefficient α_{03} for the signal radiation if it were to propagate as an unbound, plane wave in the pumped laser material itself, ignoring any loss mechanisms that may occur in this material to offset the potential gain available from stimulated emission. If all the losses in the device are ignored, the GCR is independent of the particular properties of the material in the cladding region that provides the gain, and of the pumping radiation. It is therefore a suitable measure of gain for comparing the properties of amplifiers with different passive waveguiding structures. It is also convenient to compare the gain of a particular structure with loss to that of the same structure without it, in terms of the GCR.

When there are no losses in the amplifier structure, the GCR has a maximum possible value of unity; this value is attained for each mode when it is at cut-off at the core interface with the active medium. This can be seen as follows.

The mode designated n is at cut-off on the active side when its guided propagation constant $k_{x;n}$ is equal to the propagation constant k_{03} of a plane wave at the signal frequency propagating in the material of the active region. In this case, the transverse component of the propagation constant in the active region $k_{3;n}$ is zero, so that the electric field amplitude of the mode is constant in the transverse (z) direction throughout the active medium. (The transverse component $k_{3;n}$ of the propagation constant is in this case a damping constant, since the field is evanescent.) Remembering that the modal transverse field distributions $E_{y;n}(z)$ are normalized, we obtain from Eq. 2-8 the result $\delta_n = j\mu\epsilon_3''\omega^2$ for this case. We have therefore from Eq. 2-15 that $\alpha_{G;n} = \omega^2\mu\epsilon_3''/k_{03}$. By using Eq. 2-14, we can show that $\alpha_{G;n}/\alpha_{03} = 1$. The significance of this result is that the gain coefficient for a mode at cut-off on the active side is the same as if the signal radiation were to propagate as an unbound wave in the active medium. Since the cladding regions have been assumed to be extended infinitely in the transverse direction, a mode that is bound to the core at the passive-side interface, but at cut-off at the active-side interface, carries a proportion of its power that is infinitesimally smaller than unity in the active-side evanescent wave. The guided propagation constant for such a mode is $k_{x;n} = k_{03}$. Propagation on the mode is thus indistinguishable from the propagation of a

plane wave in the active medium. It follows by a similar argument that a mode at cut-off on the passive side carries effectively none of its power in the evanescent wave that interacts with the region providing the gain, and therefore has no gain.

2.3.3 The Effects of Passive-Side Parameters

The dependence of the GCR of the standard amplifier on the difference between the refractive indices of the core and the passive side dielectric medium, $(n_2 - n_1)$, is plotted in Fig. 2-2. For this plot, the index difference $(n_2 - n_1)$ is varied by allowing n_1 to increase from its value in the standard amplifier to cut-off on the passive side, for each mode. The gain coefficient ratios of each of the two modes that propagate are shown for the cases where the passive side is lossless, and where the passive side bulk absorption coefficient is 10cm^{-1} and 100cm^{-1} . In this and all subsequent figures, mode numbers are enclosed in parentheses. Values of the bulk absorption coefficients are noted on the figures, in units of cm^{-1} .

We examine first the curves corresponding to zero bulk absorption. The GCR for both modes saturates at large values of index difference $(n_2 - n_1)$. This occurs because the penetration of the evanescent field into medium 1 becomes small for large values of $(n_2 - n_1)$, so that most of the modal power is

carried in the core and the active region. Saturation occurs when further reduction, by decreasing $(n_2 - n_1)$, of the proportion of the power propagating in the passive region produces an insignificant change in the power distribution in the other two regions. The saturation values for the standard amplifier are $GCR \approx 0.160$ for mode (2) and $GCR \approx 0.027$ for mode (1). The saturation value for mode (2) is larger than that for mode (1) because mode (2) is the less strongly bound. The penetration of the evanescent fields into both cladding regions and hence the proportion of power propagating in the active cladding region is greater for mode (2) than for mode (1) if both modes are far from cut-off on the passive side.

As $(n_2 - n_1)$ decreases, the GCR for both modes also decreases, falling rapidly toward a value of zero for each mode when $(n_2 - n_1)$ reaches the value for modal cut-off on the passive side. Since the cut-off value of the index difference is greater for mode (2) than it is for mode (1), the two curves must cross at some point near cut-off, and a condition exists for $(n_2 - n_1)$ that produces a higher value of GCR for mode (1) than for mode (2). The GCR approaches zero as modal cut-off is approached because the proportion of power carried in the active-side evanescent-wave goes to zero at passive-side cut-off.

We now consider the effects of bulk absorption in the material of the passive cladding. Such loss causes only a

small reduction in GCR when the mode is far from cut-off. As passive-side cut-off is approached, however, the effect of passive-side absorption becomes more pronounced, leading eventually to negative values of GCR, corresponding to net loss, rather than gain, in the amplifier. This occurs when a sufficient proportion of the modal power propagates in the evanescent wave, on the lossy passive side to overcome the gain obtained by the evanescent wave on the active side. At passive-side cut-off, the amplifier is lossy, with a loss coefficient equal to the absorption coefficient of the passive medium, since effectively all the modal power propagates in that medium.

The values of bulk absorption-loss coefficient for which the GCR is plotted in Fig. 2-3 have been chosen to demonstrate that passive side absorption has little effect on gain provided that the mode being amplified is far from cut-off. The absorption values chosen are very much greater than would normally be considered for the substrate of an optical waveguide. Even an absorption coefficient of 100cm^{-1} in the passive side only reduces the GCR by approximately 10% from the lossless case for the standard-amplifier structure. The reason for the insensitivity of the gain to the passive-side absorption is that, in the standard amplifier, very little power is carried by the passive substrate, and so only a small proportion of the power is subject to a large absorption

FIG. 2-3. Gain coefficient ratio as a function of the refractive index of the active side, for the standard amplifier with no parasitic losses, and with parasitic absorption of the signal radiation in the active side corresponding to a reduction of the bulk gain coefficient of that side by 0.5cm^{-1} and 5cm^{-1} .

there. As cut-off is approached on the passive side, the effect of the passive-side absorption becomes more pronounced because of the increased proportion of the power in that region.

2.3.4 The Effects of Active-Side Parameters

The dependence of GCR on the difference $(n_2 - n_3)$ between the refractive indices of the core and the active cladding medium is shown in Fig. 2-3. For this plot, $(n_2 - n_3)$ is decreased from twice that of the standard amplifier structure to cut-off for each mode by increasing the value of n_3 . The effect of the presence of bulk absorptions of $\alpha_{03}^{\text{loss}} = 0.5\text{cm}^{-1}$ and 5cm^{-1} in the active side is also shown. These losses act to reduce the bulk gain coefficient of the active medium as calculated from the pumping intensity.

We consider first the lossless case. The GCR's of the three modes that propagate over this range of values for $(n_2 - n_3)$ are seen to increase with reducing active-side refractive index difference, to reach a value of unity for each mode at cut-off. The GCR increases as the active-side index difference is reduced because the modal fields are rendered less strongly bound on the active side. Thus, a larger proportion of the modal power propagates on the evanescent wave in the active material. At cut-off on the active side, the modal gain coefficient equals the bulk gain coefficient of the active medium because all the power is carried in the active region.

as has been discussed in Section 2.3.2. The GCR is unity at cut-off.

The introduction of bulk absorption into the active medium, as well as bulk gain, has only the effect of reducing the net bulk gain coefficient of the region. Since the GCR has been defined as the ratio of the actual modal gain coefficient to the bulk gain coefficient neglecting losses, the introduction of active-side absorption merely reduces the GCR for all modes by a constant factor, regardless of the value of $(n_2 - n_3)$. We have assumed, of course, that there are no losses in the other two regions of the waveguiding structure. At cut-off on the active side, the modal GCR has a value equal to the factor by which the bulk gain coefficient of the active region has been reduced by the introduction of the loss mechanism. The bulk gain coefficient, calculated from Eq. 2-17, for the active medium of the standard amplifier, ignoring loss is $\alpha_{03} = 5.333 \text{ cm}^{-1}$. The introduction of a bulk absorption of $\alpha_{03}^{\text{loss}} = 0.5 \text{ cm}^{-1}$ therefore reduces the GCR by a factor of 0.906, and the introduction of a bulk absorption of $\alpha_{03}^{\text{loss}} = 5 \text{ cm}^{-1}$ reduces it by a factor of 0.062. At cut-off, these are the actual values of GCR for these absorptions, since in the lossless case, GCR equals unity.

2.3.5 The Effects of Core Parameters

The effect on the value of modal GCR of reducing the core thickness $2d$ from a value twice that of the standard amplifier to cut-off for each mode is shown in Fig. 2-4. Thickness has been chosen, rather than core refractive index, as the core-parameter of most interest because the effects of core index changes are essentially the same as the effects of simultaneous, identical changes in both active and passive cladding indices. We have already studied such changes individually. The dependence of modal GCR on core thickness is also shown, over the same thickness range, for two cases in which the core is lossy. The two core bulk loss coefficients, 0.01cm^{-1} and 0.1cm^{-1} , are considered.

In the lossless case, the GCR of each of the four modes that propagate over the thickness range under consideration rises with reducing core thickness to reach a value of unity at cut-off. When cut-off results from reducing the core thickness, it always occurs on the side with the higher refractive index, which in the case of the standard amplifier, is the active side. The GCR therefore reaches unity for the reasons outlined in Section 2.3.2.

For modes that are not very close to cut-off, the effect of absorption in the core medium is much more pronounced than that of absorption in the cladding media. The introduction of an absorption of 0.1cm^{-1} in the core region of the

FIG. 2-4. Gain coefficient ratio as a function of the thickness of the waveguiding core of the standard amplifier, with no parasitic losses, and with parasitic absorption of the signal radiation in the core corresponding to $\alpha_{02}=0.01\text{cm}^{-1}$ and 0.1cm^{-1} .

standard amplifier reduces the GCR of mode (1) by a factor of 3. By contrast, it was shown in the previous section that the introduction of an absorption of 100cm^{-1} into the passive cladding region of the standard amplifier only reduced the GCR of mode (1) by about 10%. The core loss has much more influence on GCR than the cladding loss because a much larger proportion of the power of the modes of the standard amplifier flows in the core than in either cladding.

2.4 CONCLUSIONS

In this Chapter, perturbation theory was employed to obtain expressions for the gain coefficients of the bound modes of a dielectric-slab waveguide with evanescent-wave gain, induced by external pumping. These expressions were used to evaluate numerically the gain coefficient ratios for the modes of an externally-pumped evanescent-wave optical amplifier; the gain coefficient ratio being defined as the ratio of the modal gain coefficient to the bulk gain coefficient of the active region of the amplifier. Variation of the modal gain properties of the amplifier, indicated by the gain coefficient ratio, were examined as the parameters of the waveguide structure were varied. The properties of the device were seen to be determined by the distribution of signal power among the guided wave in the core region and the two evanescent waves in

the cladding regions of the waveguide, and by the distribution of bulk gain and loss in the waveguide structure. In this section, we will indicate possible applications for the externally-pumped evanescent-wave amplifier that are suggested by its calculated properties. We also present gain calculations for a particular amplifier configuration for comparison with the gain of the second type of evanescent-wave amplifier that is discussed in the following Chapter.

2.4.1 Power Amplification

It is interesting to compare the evanescent-wave amplifier configuration with the more customary waveguide amplifier configuration, which has the gain present in the core. In this context, we examine first the obvious application of the externally-pumped evanescent-wave amplifier to amplify the power of signal radiation as it passes through the device.

The proportion of modal power that propagates in the core of a dielectric waveguide is higher than that which travels in the evanescent waves unless the mode is near cut-off. Any advantages of evanescent-wave devices over amplifiers with core gain are therefore offset to some degree by the constraint that the mode on which the signal propagates must be kept near cut-off to obtain high gain. To keep a particular mode near cut-off requires close manufacturing tolerances on the waveguide structure as well as good frequency stability in

the signal radiation. Since the laser sources contemplated for generating the signal radiation are generally very stable, we consider here only the tolerances that must be met in the construction of the waveguide. These are estimated from the numerical results that have been given, and hence are particular to the structural design that was chosen for the standard amplifier. These tolerance estimates indicate, however, the order of difficulty of the manufacturing problems to be expected.

The maximum possible value of modal gain coefficient in either type of waveguide amplifier is, of course, the value of the bulk gain coefficient of the active region. The results of this Chapter have shown that the gain coefficient ratio of the externally-pumped evanescent-wave amplifier approaches unity, which corresponds to this maximum possible gain, as the mode on which the signal propagates reaches cut-off on the active side. The critical parameters of the waveguide are the core-thickness and the difference between the refractive indices of the core and the active cladding region. The tolerances on these parameters can be estimated from Figs. 2-3 and 2-4. From Fig. 2-3, we see that the gain coefficient of mode (3), which has the highest gain for any fixed value of $(n_2 - n_3)$ for which it propagates, is reduced by more than 50% if the value of the refractive index difference is as much as

0.0004 larger than the cut-off value, and by 80% if the difference increases to 0.0025 larger than the cut-off value. Close control of the refractive index difference between active side and core is thus seen to be essential to ensure that an acceptable proportion of the bulk gain coefficient is achieved. By contrast, when the gain is in the core, the only requirement for high percentages of the bulk gain coefficient to be achieved by the modal gain coefficients is simple; the refractive index difference between the core and both cladding regions must be made as large as is consistent with other requirements. If, however, gain coefficients of between 20% and 40% of the bulk gain coefficient are satisfactory in the evanescent-wave device, then a control of the value of n_3 to within ± 0.001 about the nominal value suffices. Since the bulk gain coefficients of pumped laser materials can be quite high, an amplifier with limited efficiency could well be useful because of the other advantages it offers. The tolerances required on the refractive indices of the structure are therefore stringent, but within the bounds of practicality.

Fig. 2-4 shows that the thickness of the core must be within 0.1μ of the cut-off value to obtain 50% of the bulk gain, and must be controlled to within $\pm 0.5\mu$ to obtain between 20% and 40% of the bulk gain for mode (3). The cut-off value for this mode is approximately 6.2μ . Such a control of core

thickness is within the capabilities of many of the current techniques for laying down thin, uniform films of dielectrics⁽¹⁹⁾. We conclude therefore that it is practically possible to obtain useful gain with externally-pumped evanescent-wave amplifiers. This conclusion is supported by the observation of laser oscillation in a distributed feedback laser in which gain was provided by an externally-pumped evanescent-wave amplifier⁽⁶⁾. We note also that gain has been reported in evanescent-wave amplifiers manufactured in cylindrical waveguide⁽¹⁾.

2.4.2 Mode Filtering

In the evanescent-wave amplifier, the highest gain is obtained for the highest-order mode. This feature suggests a possible application for the device to which waveguide amplifiers that provide gain in their core regions cannot be put, since they always offer the highest gain on the lowest-order mode. The actual order of the mode of highest gain can be controlled in the evanescent-wave amplifier simply by controlling the number of modes that propagate in the structure. By this means, an externally-pumped evanescent-wave amplifier might be matched to a particular mode of a passive slab-waveguide in order to act as a mode-filter, providing more gain to the mode to which it is matched than to any other. The possibility arises of using such a mode-filter to emphasize, perhaps in order to detect signals on it, the power in a particular mode of a passive slab-waveguide.

The mode-filtering properties of the transversely-pumped evanescent-wave amplifier can be improved by judicious use of loss in the core region of the waveguiding structure. We note in Fig. 2-4 that the gain coefficient of mode (4) in the standard amplifier structure, with a core thickness of 10μ , is three times that of mode (3), which has the next-highest gain. This gain advantage of mode (4) is enhanced by the introduction of loss in the core. When $\alpha_{02}=0.1\text{cm}^{-1}$, the gain coefficient of mode (4) remains nearly unchanged, while that of mode (3) is reduced to a fifth of that of mode (4), and modes (2) and (1) are actually attenuated. It is possible to arrange the structure so that all the modes except the highest are lossy. One such condition is reached in the standard amplifier with a 9μ thickness when the bulk absorption coefficient of the core is 0.5cm^{-1} . In this case, four modes propagate of which the first three are lossy and the fourth has gain. If this device is 2cm long, the numerical power gain of mode (4) is 40.7, while the power of mode (3) is reduced by a factor of 0.8. The highest-order mode is thus separated in gain by nearly 17dB from the other modes. Increasing the amplifier length while maintaining the pump power constant reduces the gain available to mode (4), but increases the attenuation of the other modes, and also the separation. When the length is 4cm, mode (4) has

a power gain of 22.5, while that of mode (3) is reduced to 0.33. Mode (4) is isolated by 18.3dB in this case.

A similar internal mode-suppression phenomenon occurs, of course, in waveguide amplifiers with gain in their cores rather than evanescent-wave gain. In these amplifiers, lossy passive cladding regions can be employed to suppress the gain of high-order modes relative to that of the lowest-order mode. In such amplifiers, however, the technique would be less useful. The distribution of power among the three dielectric regions of the waveguide changes less between modes of low order than between modes of high order. The difference in gain between low-order modes is thus smaller than that between high-order modes, irrespective of whether the gain arises from evanescent-wave interactions or from interactions in the core. The lowest-order mode cannot, therefore, be isolated to the same degree in the amplifier with gain in its core, to which the highest-order mode can be isolated in the evanescent-wave amplifier.

The mode-filtering possibilities of the evanescent-wave amplifier could be particularly useful in suppressing spurious modes excited within the amplifier itself. Perturbation theory indicates that mode-coupling results from the presence of the gain perturbation in the waveguide structure. Thus, the amplifier inevitably injects power from each mode into the other

modes. If, however, all the propagating modes into which power is coupled are lossy, then the power leaving the amplifier in the spurious modes is reduced. The spurious modes, in this case, are all those of order lower than the mode on which the signal should propagate.

2.4.3 Gain of the Externally-Pumped Evanescent-Wave Amplifier

In the previous sections of this Chapter, the gain properties of the externally-pumped evanescent-wave amplifier were discussed only in terms of the gain coefficient ratio. The GCR is a convenient measure for examining the dependence of gain on the parameters of the amplifier, but it does not give a direct indication of the actual gain that is obtained from a specific device. In this section, the modal gains of the standard amplifier are determined for amplifiers of given lengths. These are calculated using Eq. 2-24. The results that are presented serve as an example of the gain that can be expected from what as, hopefully, a representative externally-pumped evanescent-wave amplifier.

In Fig. 2-5, the gain of the standard amplifier is shown as a function of amplifier length for the two modes that propagate in this particular dielectric waveguide. Gain values are shown for the lossless amplifier and for amplifiers with losses in each of the three dielectric regions of the waveguiding structure. In all cases, the pumping intensity is

FIG. 2-5. Gain of the externally-pumped standard amplifier as a function of amplifier length.

————— Loss in the passive cladding,
 $\alpha_{01}=10\text{cm}^{-1}$
 - - - - - Loss in the core, $\alpha_{02}=0.01\text{cm}^{-1}$
 - - - - - Loss component in gain region,
 $\alpha_{03}^{\text{loss}}=0.5\text{cm}^{-1}$
 No losses:

uniform over the entire device length, and the total pump power incident on the device is maintained constant. The pumping intensity thus decreases as the length increases.

When the amplifier structure is lossless, the gain of each mode is independent of the length of the amplifier. This is because the decreased pumping intensity of longer amplifiers is exactly compensated by the increased gain that results from propagation over a larger distance in the amplifier. When the effects of losses are included in the calculations, the gain decreases with increasing amplifier length. This decrease in gain is a consequence of decreased pumping intensity. When the pump intensity decreases, the bulk gain coefficient α_{03} and hence the modal gain coefficients $\alpha_{G;n}$ decrease. The loss coefficient $\alpha_{03}^{\text{loss}}$ and the modal loss coefficients $\alpha_{L;n}$ remain constant, however, so that as the amplifier length increases the loss tends to dominate the gain in Eqs. 2-18 and 2-23.

The highest gain occurs for the highest-order mode of the two that propagate, for the same reasons that the GCR is highest for this mode. The modal gains for the lossless structure and the structures with loss in the passive cladding and the core are all very similar for the loss coefficient values and the length range considered. When there is a fixed loss component in the active medium, however, the second order mode loses gain rapidly with length. This results from the

relatively large influence of loss in the active region on the gain of the highest-order mode, which has a large proportion of its power propagating in this region.

The results of gain calculations have shown that with a 10kW pump source, a gain of about 5 can be obtained at 630nm by the standard amplifier. Larger gains can be obtained by adjusting the structure so that the highest-order mode is nearer cut-off. We have seen that a gain of 40 can be obtained on the 4th mode of a standard amplifier with a 9μ core. Manufacturing tolerances ultimately limit the fraction of the bulk gain of the active medium that can be achieved. The development of externally-pumped evanescent-wave amplifiers for practical purposes clearly requires the use of lasing materials that yield the highest possible bulk gain for moderate pumping intensities. Since the study reported here is concerned with the analysis of the properties of evanescent-wave devices that relate to their structures rather than their materials, the pursuit of improved externally-pumped evanescent-wave amplifiers is beyond its scope. Instead, in the next Chapter, a different type of evanescent-wave amplifier, with a more efficient arrangement for pumping, is considered. The pump power requirement can be much lower in this amplifier for a given lasing material.

CHAPTER 3

THE BOUND-MODE PUMPED EVANESCENT-WAVE AMPLIFIER

3.1 INTRODUCTION

Improved pumping efficiency can be obtained in evanescent-wave amplifiers by using bound rather than externally-incident radiation for pumping. If the pumping radiation propagates on a bound mode, it is confined to nearly the same regions of the dielectric waveguide in which the bound signal radiation propagates. The interaction mediated by the amplifier structure between the pump and signal radiation thus occurs with best efficiency. In this Chapter, we examine the signal-wave modal gains for an evanescent-wave amplifier pumped by radiation propagating on bound modes. The analytical methods used, and some of the calculations, have been reported in the literature by K.O. Hill, the author, and A. Watanabe⁽⁴⁾. This previous work is extended here to include the effects of parasitic losses in all regions of the amplifier structure.

The evanescent-wave amplifier to be considered is the same typical "standard amplifier" structure that was discussed in the previous Chapter. In this case, the pumping radiation of frequency ω_p propagates as a bound mode, designated mode m , of the waveguide. The method of coupling radiation into the waveguide is not discussed here; the problem of coupling optical

radiation into dielectric waveguides has been treated extensively in the literature^(10,20-23). The signal radiation, of frequency $\omega_s < \omega_p$, also propagates as a bound mode, designated mode n. In order to characterize the gain properties in as general a way as possible, we define two characteristic measures of the gain of bound-mode pumped evanescent-wave amplifiers: the net gain coefficient (NGC) and the interaction length (IL). These measures of the gain are independent of the length of the amplifier, and thus are useful in comparing the effects of the various structure parameters on gain. Expressions are developed for NGC and IL by means of perturbation theory and values are calculated for the standard amplifier to show the effects of changes in the structure parameters. Finally, by calculating the actual gains of amplifiers of specified length, a direct comparison is made between the gains of an externally-pumped and a bound-mode pumped evanescent-wave amplifier.

3.2 THE GAIN COEFFICIENTS OF MODE-PAIRS

The gain coefficient for a signal mode is determined by the distribution of the imaginary component of dielectric constant in the waveguide at the signal frequency. As in the previous Chapter, this distribution can be considered to be responsible for a perturbation of the properties of the passive waveguiding structure, and the gain coefficient obtained from perturbation theory. When the pump radiation pro-

pagates as a bound mode, the z -direction distribution of the pumping intensity is not constant but is determined by the transverse field distribution of the pump mode. The z -direction distribution of the gain perturbation thus depends on the order of the pump mode. Furthermore, since energy must be removed from the pump mode as it propagates, in order to achieve pumping, the mode is necessarily attenuated in the x -direction. The gain perturbation for the bound-mode pumped amplifier is therefore not constant as was assumed in the previous Chapter, but is a function both of x and of z .

The bulk attenuation in the active region at the pump frequency is a constant, negative, imaginary perturbation of the dielectric constant of this region. The pump mode attenuation coefficients can therefore be obtained by perturbation theory in the same way as gain and attenuation coefficients for the signal mode were obtained in the previous Chapter. With the pump mode attenuation coefficients thus determined, the field distributions of the pump mode can be expressed as functions of x and z . These field distributions determine the pumping intensity, and hence the gain perturbation at the signal frequency throughout the structure. By the use of perturbation theory for a second time, the gain obtained by the signal mode as a result of the perturbation induced by the pumping can be determined, for each pump-mode, signal-mode combination.

3.2.1 Propagation of the Pump Mode

The modal attenuation coefficient $\alpha_{3;m}^p$ that arises from bulk absorption of the pump mode in the gain region is given by an equation of exactly the form of Eq. 2-19. This is evident because a perturbation in the dielectric constant associated with bulk loss only differs in sign from a perturbation associated with bulk gain. We write the equation as

$$\alpha_{3;m}^p = k_{03}^p \alpha_{03}^p (A_m^p)^2 \frac{\sin^2(-k_{z;m}^p d + \phi_m^p)}{2k_{3;m}^p k_{x;m}^p} \quad (3.1)$$

where

- k_{03}^p is the propagation constant for a free wave at the pump frequency in region 3, ignoring loss;
- α_{03}^p is the bulk attenuation coefficient of region 3 at the pump frequency.

The modal quantities in Eq. 3-1 are those pertaining to propagation on the m th mode. The index p is used to identify quantities associated with the pump mode, as opposed to the signal mode, for which the index s is used. The pump mode may also suffer losses in regions 1 and 2 of the waveguide structure. We assume such losses to be entirely parasitic; they do not result in gain to the signal mode. These losses give rise to a modal attenuation coefficient $\alpha_{2;m}^p$ given by Eq. 2-22, with variables appropriate to the pump mode. The total attenuation coefficient α_m^p for the pump mode is therefore

$$\alpha_m^p = \alpha_{3;m}^p + \alpha_{2;m}^p \quad (3-2)$$

By using Eq. 3-1 and Eq. 2-22 as it can be written for the pump mode, we can express Eq. 3-2 in a form that allows the computation of the pump-mode attenuation coefficient from parameters that are conveniently measurable. These are the physical dimensions and the refractive indices of the waveguiding structure, and the attenuation coefficients for the pump radiation in each of the three dielectric regions of that structure, α_{01}^p , α_{02}^p and α_{03}^p . The equation is

$$\alpha_m^p = \frac{(A_m^p)^2}{k_{x;p}^p} \left\{ \alpha_{03}^p k_{03}^p \frac{\sin^2(-k_{z;m}^p + \phi_m^p)}{2k_{3;m}^p} + \alpha_{02}^p k_{02}^p \left[d - \frac{\sin(2k_{z;m}^p d) \cos(2k_{z;m}^p d)}{2k_{z;m}^p} \right] + \alpha_{01}^p k_{01}^p \frac{\sin^2(k_{z;m}^p + \phi_m^p)}{2k_{1;m}^p} \right\} \quad (3-3)$$

where k_{01}^p , k_{02}^p and k_{03}^p are the propagation constants of the pump radiation propagating as a free wave in each of the three dielectric materials that make up the waveguide structure, ignoring losses.

The power $P_m^P(x)$ per unit length in the y -direction that is carried by pump mode m in the perturbed waveguide is given by the Poynting relation

$$P_m^P(x) = \frac{1}{2} \frac{k_{x;m}^P}{\omega_p} \int_{-\infty}^{\infty} \left| \mathcal{E}_{y;m}^{'P}(x,z) \right|^2 dz \quad (3-4)$$

where $\mathcal{E}_{y;m}^{'P}(x,z)$ is the perturbed field distribution of the pump mode as given by Eq. 2-12. The pump power that is put into the amplifier is the power carried by the pump mode across the plane $x=0$, written $P_m^P(0)$. From Eqs. 2-10 and 3-4 we obtain an expression for the amplitude of the pump mode at $x=0$, A_{0m}^P , in terms of the pump power put into the device

$$A_{0m}^P = \sqrt{\frac{2\omega_p \mu P_m^P(0)}{k_{x;m}^P}} \quad (3-5)$$

The use of this result in Eq. 2-12, together with the relation

$$\mathcal{E}_{y;m}^{'P}(0,z) = A_{0m}^P E_{y;m}^{'P}(z) \quad (3-6)$$

gives the perturbed field distribution in the form

$$\mathcal{E}_{y;m}^{'P}(x,z) = \sqrt{\frac{2\omega_p \mu P_m^P(0)}{k_{x;m}^P}} e^{-\alpha_m^P x/2} E_{y;m}^{'P}(z) \quad (3-7)$$

The spatial distribution of the gain induced in the structure by bound-mode optical pumping can be determined from this distribution of pump-field amplitude.

3.2.2 Propagation of the Signal Mode

The m th pump mode induces a positive imaginary component $\epsilon''_{3;m}(x,z)$ in the dielectric constant of the laser material present in region 3 of the waveguide. The distribution of this component is proportional, for the laser dyes we consider, to the local intensity of the pump-mode radiation. From Eqs. 2-15 and 3-7, we obtain an expression for this component of the dielectric constant of region 3 in terms of the pump-mode characteristics:

$$\epsilon''_{3;m}(x,z) = \frac{2\omega_p \mu_p^p \exp[-\alpha_m^p x/2]}{k_{x;m}^p} \left\{ x_3 |E_{y;m}(z)|^2_{z<-d} \right\} \quad (3-8)$$

By substituting the expression given by Eq. 3-8 for the perturbation $\epsilon''_{3;m}(x,z)$ into Eq. 2-8, performing the overlap integral to obtain the propagation-constant perturbation, and then making use of Eq. 2-13, we obtain an expression for the gain coefficient $\alpha_{m,n}^s$ of the mode pair (m,n) consisting of pump mode m and signal mode n :

$$\alpha_{m,n}^s(x) = \frac{\mu_s^2 \omega_s^2 \omega_p^p \mu_m^p (A_m^p)^2 (A_n^s)^2 x_s}{k_{x;n}^s k_{x;m}^p} \left[\frac{\sin^2(-k_{z;m}^p d + \phi_m^p) \sin^2(-k_{z;n}^s d + \phi_n^s)}{(k_{3;m}^p + k_{3;n}^s)} \right] e^{-\alpha_m^p x} \quad (3-9)$$

The gain coefficient of the signal mode depends on the distance x , from the plane $x=0$ where the modes are launched that the pump and signal modes have travelled along the waveguide. In the absence of parasitic losses at the frequency of the signal radiation, the signal mode gains intensity as it propagates along the waveguide in direct proportion to the exponent of the integral of $\alpha_{m,n}^s(x)$ over this distance. The gain $G_{m,n}(L)$ of an amplifier of length L is thus given by

$$G_{m,n}(L) = \exp \left[\int_0^L \alpha_{m,n}^s(x) dx \right]. \quad (3-10)$$

In all three regions of the dielectric waveguiding structure, there may also be parasitic losses that are effective at the signal frequency. Absorption of the signal radiation gives rise to a signal mode attenuation coefficient $\alpha_{\ell,n}^s$ that is given by Eq. 3-3, with variables appropriate to the signal mode. In this case, the gain of an amplifier of length L is given by

$$G_{m,n}(L) = \exp \left[\int_0^L \alpha_{m,n}^s(x) dx - \alpha_{\ell,n}^s L \right] \quad (3-11)$$

Both the gain and the gain coefficient depend on position along the x -axis of the amplifier. It is convenient to define parameters that are independent of x to characterize the gain

for the purpose of examining the influences that the structure parameters have on it. Such parameters are defined in the next section.

3.2.3 Net Gain Coefficient and Interaction Length

2. There is no single parameter, corresponding to the gain coefficient ratio that was used in the previous Chapter, by which the gain of the bound-mode pumped amplifier can be characterized in a manner that is independent of the amplifier length and the pumping intensity. The reason is that the bulk gain coefficient of the active medium at the signal frequency is a function of position along the x-axis of the amplifier. It therefore cannot be taken as a reference gain-coefficient for the gain coefficients of the signal modes. In order to have a measure of gain independent of amplifier length or position in the structure, we define the two quantities Net Gain Coefficient (NGC) and Interaction Length (IL). These measures permit comparisons to be made between the gain properties of different waveguiding structures, under certain restrictions regarding losses.

The NGC is defined to be the magnitude to which the argument of the exponential form on the right-hand side of Eq. 3-10 tends as $L \rightarrow \infty$. From Eqs. 3-9 and 3-10, we obtain an expression yielding the NGC for the pump-mode, signal-mode pair (m,n) in terms of the characteristics of the modes:

$$NGC_{m,n} = \frac{\mu^2 \omega_s^2 P_m^P (A_m^P)^2 (A_n^S)^2 \chi_3 \sin^2(-k_{z;m}^P d + \phi_m^P) \sin^2(-k_{z;n}^S d + \phi_n^S)}{\alpha_{m,x;n}^P k_{x;n}^S k_{x;m}^P (K_{3;m}^P + K_{3;n}^S)} \quad (3-12)$$

The NGC is defined only for amplifier structures in which there are no losses at the signal frequency in any region. If the signal mode suffers any absorption, an amplifier of infinite length can never provide gain, but is always lossy. This is evident from Eq. 3-11. As $L \rightarrow \infty$, the argument of the exponent tends to a negative value, since the second term of the argument is negative, and decreases without limit, while the first term tends to a finite positive value. When signal-mode losses are considered, therefore, the actual gain $G_{mn}(L)$ given by Eq. 3-11 must be determined. This requires the assumption of an amplifier length L . With the exception of cases where signal-mode loss must be considered, the NGC is used for comparing the effects that the various parameters of the waveguiding structure have on the gain of the mode-pairs. The numerical calculations given in the following sections therefore chiefly concern the NGC, and its associated parameter, the interaction length.

The signal-mode gain associated with the NGC can not be achieved in practice, since the modes would have to propagate to infinity in the structure. In order to determine the relation between the NGC and the practically possible gain of an

amplifier structure, we also define an interaction length (IL). The IL is the distance that both signal and pump modes must propagate together in the waveguide in order that the change in the signal-mode intensity is equal to one-half the change that would occur were they to propagate to infinity. This definition and Eq. 3-10 give

$$\exp \left[\int_0^{IL_{m,n}} \alpha_{m,n}^s(x) dx \right] = \frac{e^{NGC_{m,n+1}}}{2} \quad (3-14)$$

Solving Eq. 3-14 for the interaction length, we obtain

$$IL_{m,n} = - \frac{1}{\alpha_m^p} \ln \left[1 - \frac{1}{NGC} \ln \left(\frac{e^{NGC_{m,n+1}}}{2} \right) \right] \quad (3-15)$$

The interaction length depends inversely on the attenuation coefficient of the pump mode and logarithmically on an expression dependent solely on the NGC. The logarithmic factor is weak. Since the NGC itself is dependent inversely on the attenuation coefficient of the pump mode, the dominant dependence is on the pump-mode order-number, and occurs through the pump-mode attenuation coefficient. This can be understood on physical grounds. The distribution of pumping intensity along the propagation axis of the amplifier depends on the pump-mode absorption coefficient. The interaction length is more directly related to the distribution of gain than to the integrated effect of that gain.

3.3 NUMERICAL CALCULATIONS FOR A TYPICAL BOUND-MODE PUMPED EVANESCENT-WAVE AMPLIFIER

Computed values of NGC and IL are presented in this section to show the dependence of the gain of mode-pairs of the bound-mode pumped amplifier on the structural parameters of the waveguide and on parasitic absorption of the pump mode. To show the effect of signal-mode absorption, we also present calculations of the gain of an amplifier of specified length under various conditions of signal-mode absorption.

The basic amplifier structure considered is that of the standard amplifier discussed in Section 2.3.1. Because of dispersion in the dielectric materials of the waveguide, the refractive indices are different at the pump and signal frequencies. We assume for the calculations that the refractive index is larger by 0.005 for the pump mode than for the signal mode in all regions of the structure. The values of the parameters of the standard amplifier as used for calculations concerning bound-mode pumping are presented in Table 3-1.

The pump-power requirements of the bound-mode pumped amplifier are not the same as those of the externally-pumped amplifier. Since the energy of the bound pump mode is confined to propagate in a very narrow region, the intensity of the pumping radiation can be very high, even for moderate pump powers. This advantage is somewhat offset by the difficulty

TABLE 3.1. Characteristics of the Standard Amplifier

CLADDING REGION 1	THICKNESS (μm)	PROPORTIONALITY CONSTANT $\times$ (FmV^{-2})	REFRACTIVE INDEX	BULK ABSORPTION COEFFICIENT α cm^{-1}	AT SIGNAL WAVELENGTH AT PUMP WAVELENGTH
	-	-	1.4000 1.4050	0 0	
CORE REGION 2	5	-	1.5000 1.5050	0 0	AT SIGNAL WAVELENGTH AT PUMP WAVELENGTH
CLADDING REGION 3	-	2.79×10^{-29}	1.4950 1.5000	0 10	AT SIGNAL WAVELENGTH AT PUMP WAVELENGTH

SIGNAL WAVELENGTH = $0.63 \mu\text{m}$

PUMP WAVELENGTH = $0.59 \mu\text{m}$

PUMP MODE POWER = $2 \times 10^5 \text{ W/m}$

of coupling the pumping radiation efficiently into the optical waveguide. Available techniques of coupling require close control of the beam profile of the incident radiation and do not yield high efficiencies. The necessity for controlling the beam profile means that only lasers operated in their fundamental modes of resonance are suitable as pumping sources. These generally are considerably less powerful than the lasers that were considered as sources for the externally-pumped evanescent-wave amplifier in the previous Chapter.

We consider a laser of 0.1 watt output power coupled into the waveguide with 50% efficiency to excite the pump mode. Such a source could be, for example, a moderate-size argon-ion laser operating on a single line. We assume that essentially all the pump power propagates in the core or in layers of the cladding regions contiguous to the core and of a thickness equal to half the core thickness. The width of the amplifier structure in the y-direction is assumed to be 25mm. This value is of the order of 10 times the width of the z-direction structure of the waveguide and is therefore consistent with the assumption that there is no field variation in the y-direction, which was made in Appendix A. It is assumed that the pump intensity is uniform in the y-direction. The pump-mode power per unit length in the y-direction for this arrangement is $2 \times 10^5 \text{ W/m}$.

The influences on gain of the refractive indices of the two cladding media and of the thickness of the waveguiding core are studied in the following sections. The behaviour of calculated values of NGC and IL is examined as each of these three parameters is varied. Calculations are presented for each structure parameter first in the lossless case, and then to show the effect of parasitic absorption in the region of the structure characterized by the parameter under consideration. We adopt the convention that combinations of pump mode and signal mode are denoted (m,n) where m is the order of the pump mode and n is the order of the signal mode.

3.3.1 The Effects of Passive-Side Parameters

In Fig. 3-1, we show the dependence of the NGC of various mode pairs on the difference $(n_2 - n_1)$ between the core and the passive-side refractive indices. The value of n_1 is increased from the standard amplifier value to cut-off for a mode of each pair. Since there are two pump and two signal modes supported in this range of $(n_2 - n_1)$, there are four possible pump-mode, signal-mode combinations. The four curves of NGC fall into two groups. The NGC values of mode pairs containing the same signal mode are close, indicating that dependence on the pump-mode order is weak. The NGC values for mode pairs containing the same pump-mode order differ more widely, indicating a stronger dependence on signal-mode order. If the signal

FIG. 3-1. Variation of net gain coefficient as a function of the passive-side refractive index, for the standard amplifier with no parasitic losses.

mode order is held constant, the NGC decreases with increasing pump-mode order, whereas if the pump-mode order is constant, the NGC increases with increasing signal-mode order. The NGC increases with signal-mode order for the same reasons outlined in the previous Chapter to explain the increase in GCR with signal-mode order. The reversed dependence of NGC on pump-mode order can also be readily understood. As the pump-mode order increases, a greater proportion of the pump power travels in the cladding regions. The attenuation coefficient of the pump mode increases because of the absorption in the active side. An increased pump-mode attenuation coefficient results in a reduced NGC, as can be seen from Eq. 3-12.

The NGC saturates for large values of $(n_2 - n_1)$ for the same reasons that the GCR of the externally-pumped amplifier saturates under the same conditions: the field distributions stabilize for large refractive index differences. As the refractive index difference between core and passive side decreases, the higher-order modes approach and reach cut-off, followed by the lower-order modes. Whenever a mode closely approaches cut-off, the NGC's of the mode pairs associated with it drop sharply. Because the signal wavelength is larger than the pump wavelength, the signal mode reaches cut-off first in those combinations where both signal and pump mode are of the same order. When cut-off for a mode pair occurs on the

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signal mode first, the NGC of the pair tends to zero as cut-off is approached because the normalization constant A_n^s of the signal mode tends to zero at cut-off. Of the four mode pairs, only in one, the pair (2,1), does the pump mode cut-off before the signal mode. In this case, the NGC does not tend to zero at cut-off, but to a finite value NGC_c given by Eq. 3-12 in the limit as $K_{1;m}^p \rightarrow 0$; $k_{x;m}^p \rightarrow k_{01}^p$ and $(-k_{z;m}^p d + \phi_m^p) \rightarrow \frac{\pi}{2}$:

$$NGC_c \rightarrow \frac{2\mu^2 \omega_s^2 \omega_p^p (A_n^s)^2 \chi_3 \sin^2(-k_{z;n}^s d + \phi_n^s)}{k_{x;n}^s (1 + K_{3;n}^s / K_{3;m}^p)} \quad (3-16)$$

The finite NGC at pump-mode cut-off implies that the decrease in the pump-mode attenuation coefficient as cut-off is approached exactly compensates for the decrease in amplitude of the pump-mode field distribution and the consequent reduction in gain. This is not the case if the passive side is lossy, as will be discussed later.

The interaction length is plotted against $(n_2 - n_1)$ in Fig. 3-2. As with the NGC, the values fall into two groups. In contrast to the NGC, however, the IL is more sensitive to pump-mode order than to signal-mode order. The IL increases with increasing signal-mode order, and decreases with increasing pump-mode order. Its dependence on mode order is the same as that for NGC. This dependence demonstrates that the

FIG. 3-2. Variation of interaction length, in metres, as a function of the passive-side refractive index, for the standard amplifier with no parasitic losses.

dominant factor in the expression for IL given by Eq. 3-15 is the pump-mode absorption coefficient, as was stated in Section 3.2.3. If the pump mode is held constant, the slight rise in IL with signal-mode order comes from the logarithmic factor of Eq. 3-15, and is minor even though the rise in NGC is relatively large, as can be seen by comparing Fig. 3-1. If the signal-mode is held constant and the pump mode order increases, however, there is a much larger decrease in IL, resulting from the increased pump-mode absorption.

The IL tends to a constant value at large values of $(n_2 - n_1)$ for the same reason as the NGC and the GCR: the field distribution in the waveguide is not much affected by further changes in index difference. As cut-off is approached by reducing $(n_2 - n_1)$, the IL rises sharply. In the case of mode-pairs for which the NGC approaches zero at cut-off, the IL approaches a finite value given by the limiting value of Eq. 3-15 as $\text{NGC} \rightarrow 0$:

$$IL_{m,n} \rightarrow \ln \left(\frac{2}{\alpha_{m;\text{cut-off}}^p} \right) \quad (3-17)$$

where $\alpha_{m;\text{cut-off}}^p$ is the value of $\alpha_{3;m}^p$ at signal-mode cut-off. In the other case, where the pump-mode reaches cut-off first, the IL approaches infinity at cut-off because the pump-mode attenuation coefficient approaches zero while NGC approaches

a constant. All the pump power is carried in the lossless passive cladding, in this case.

In Fig. 3-3, we show a plot of NGC versus IL for the data presented in Figs. 3-1 and 3-2. This plot makes clear the behaviour of NGC and IL as cut-off is approached on the passive side. For all the mode-pairs except (2,1), the signal mode cuts off first. The interaction lengths approach their respective asymptotic values and the NGC values decrease without limit as cut-off is approached. In the case of mode pair (2,1), the pump mode controls cut-off; the NGC therefore tends to a finite value and the interaction length increases without limit as cut-off is approached, as was established above.

Plots of the type shown in Fig. 3-3 are useful in establishing the best mode pair on which to operate the amplifier. Clearly a large NGC and a small IL are desired. For the structure under consideration, the mode pair (2,2) offers the best compromise.

We now consider the effects of parasitic absorption of the pump mode in the passive cladding medium. This has its greatest effect on both IL and NGC when the passive side carries a large proportion of the pump-mode power. Such a condition only occurs when the pump mode is near cut-off on the passive side. In Fig. 3-4, we show the dependence of NGC on $(n_2 - n_1)$ near cut-off. The horizontal axis of this plot is expanded by

FIG. 3-3. Net gain coefficient as a function of interaction length, as the passive-side refractive index varies, from the data of the previous two figures. The structure is that of the standard amplifier, and there are no parasitic losses.

FIG. 3-4. Net gain coefficient near cut-off as a function of passive-side refractive index, for the standard amplifier with parasitic absorption of the pump radiation in the passive side corresponding to $\alpha_{01}^p = 0$, and 10 cm^{-1} .

INTERACTION LENGTH

a factor of 10 by comparison with Fig. 3-1. Values are shown for the case of a lossless passive cladding which indicate clearly the finite NGC of mode pair (2,1) at pump-mode cut-off. The effect of the introduction of loss that results in a bulk attenuation coefficient $\alpha_{01}^P = 10\text{cm}^{-1}$ in the passive cladding is to lower the NGC by an amount that increases as cut-off is approached. The NGC of the mode pair (2,1) does not tend to a finite value as cut-off is approached, but goes to zero. This results because, when α_{01}^P is not equal to zero in Eq. 3-3, the value of NGC given by Eq. 3-12 tends to zero in the limit given earlier. When the passive side attenuates the pump mode, the loss of pump-mode amplitude as cut-off is approached is not made up by a sufficiently reduced attenuation coefficient.

In Fig. 3-5, we show the dependence of IL on $(n_2 - n_1)$ for mode pairs (2,2) and (1,1) over the same range as in the previous figure, and for a number of different passive-side absorption coefficients. In this case, only two mode pairs are shown because the dependence on signal mode is weak. The IL tends asymptotically to finite values at cut-off for these mode pairs because the signal mode cuts off first. For some passive-side absorption coefficients, the interaction length exhibits a peak value that occurs at higher values of $(n_2 - n_1)$ as the bulk absorption coefficient increases. This behaviour reflects the dependence of the pump-mode attenuation coefficient

FIG. 3-5. Interaction length as a function of passive-side refractive index, for the standard amplifier with parasitic absorption of the pump radiation in the passive side corresponding to $\alpha_{01}^p = 0, 0.1, 0.2, 1$ and 10cm^{-1} .

on the absorption coefficients of both the passive and the active sides. An examination of Eq. 3-3 shows that if α_{01}^P is less than α_{03}^P a minimum value for α_m^P can be reached by properly choosing the propagation constants $k_{z;m}^P$, $k_{1;m}^P$ and $k_{3;m}^P$, which is accomplished in this case by varying the refractive index of the passive side.

In Fig. 3-6, we show a plot of NGC versus IL for the case where the passive side has bulk absorption equal to that of the active side (10cm^{-1}). This plot should be compared with Fig. 3-3. A dramatic difference is apparent in the behaviour near cut-off. The mode pairs now all approach finite interaction lengths as cut-off is approached. The IL of mode pair (2,1) approaches a finite value because, when there is passive-side absorption, the NGC approaches zero as the pump-mode approaches cut-off, and the pump-mode attenuation coefficient approaches α_{01}^P . The relative values of the interaction lengths at cut-off are altered also. The cut-off value of IL for mode pair (1,1) is less than that for (2,1) and (2,2), contrary to the lossless case. Clearly, the effect of passive-side absorption of the pump mode is considerable near passive-side cut-off.

3.3.2 The Effects of Active-Side Parameters

The dependence of NGC on the refractive index difference ($n_2 - n_3$) between the core and the active cladding is shown

FIG. 3-6. Net gain coefficient as a function of interaction length, as the refractive index of the passive side varies, for the standard amplifier with parasitic absorption of the pump radiation on the passive cladding region corresponding to $\alpha_p = 10 \text{ cm}^{-1}$.

in Fig. 5-7. This plot was obtained by allowing n_3 to increase from twice its value in the standard amplifier to cut-off for a mode of each combination. Three signal and three pump modes propagate in this range, so that nine combinations exist. The NGC is again seen to be insensitive to pump-mode order-number, and strongly dependent on signal-mode order-number.

In contrast to the dependence of NGC on passive-side refractive index difference, the NGC's of all mode pairs decrease slowly with increasing values of $(n_2 - n_3)$. There is no saturation here. As the active-side index difference increases, the penetration of the evanescent fields into the active material decreases, so that all the NGC's would approach zero if $(n_2 - n_3)$ were to increase without limit. Since it is n_3 that is varied in this plot, the largest attainable index difference is $(n_2 - 1)$.

Cut-off occurs in the active cladding as a result of decreasing $(n_2 - n_3)$. This results in cut-off behaviour different from that observed previously with passive-side cut-off. As the pump-mode approaches cut-off on the active side, its evanescent field on that side carries a larger proportion of the modal power, so that the pumping power is effectively increased. This results in a rise in NGC. After a certain point, however, cut-off is closely enough approached that the decreasing pump-mode normalization constant, A_m^p , becomes more

FIG. 3-7. Net gain coefficient as a function of the active-side refractive index difference, for the standard amplifier with no parasitic losses.

influential than the increasing proportion of pump power in the evanescent field, and the NGC drops sharply, reaching zero at cut-off in all cases. The peak value of NGC that is reached increases with signal mode-order, and decreases with pump mode-order. Whenever the pump mode-order exceeds that of the signal, the approach to the peak value is curtailed by pump-mode cut-off and there results a strong decrease in the peak value with increasing pump mode-order for a given signal mode. When signal-mode cut-off occurs first, the peak values reached by the NGC show only the mild dependence on pump-mode order that was noted previously. Thus, the peak NGC for mode pairs (1,3), (2,3) and (3,3) all have values of approximately 0.5, while the peak values for modes (3,1), (3,2) and (3,3) are widely separated.

Interaction length is plotted against $(n_2 - n_3)$ in Fig. 3-8. As in the previous section, we note that IL is essentially independent of signal mode-order. The IL increases with increasing active-side index difference because less power is carried in the active cladding and hence the pump-mode attenuation coefficient is reduced. For mode pairs in which signal-mode cut-off on the active side is approached first, the interaction length goes to a finite cut-off value given by Eq. 3-17. In those cases where the pump mode cuts off first, the interaction length approaches a value of $\ln(2)$ times the reciprocal of the active-side bulk absorption coefficient.

FIG. 3-8. Interaction length, in metres, as a function of the active-side refractive index, for the standard amplifier with no parasitic losses.

The plot of NGC versus IL is illustrated in Fig. 3-9. This shows the finite values to which the interaction lengths go at cut-off. It is clear from this plot that the three mode-pairs for which the pump-mode cuts off first, mode pairs (3,1), (3,2) and (2,1), all have a common interaction length at cut-off. Again, mode pair (1,1) offers the best compromise of NGC and IL.

When there is a component of bulk absorption in the active cladding region that results from a parasitic loss-mechanism, by which the pump radiation is attenuated without pumping the laser material, it increases the effective bulk absorption coefficient of the active cladding. It therefore has the effect of increasing the modal absorption coefficients for the pump radiation by a fixed amount independent of the structure parameters. In a plot of NGC versus active-side index difference, the NGC values of all mode pairs, and for all values of $(n_2 - n_3)$ are reduced by the same factor by which the pump-mode absorption coefficients are increased, as can be seen from Eq. 3-12. The effect on interaction length is also nearly uniform because of the small influence of the logarithmic factor in Eq. 3-15. Because the effect of a parasitic absorption of the pump radiation in the active material is easily understood, no plots of NGC or IL are presented for this case.

FIG. 3-9. Net gain coefficient as a function of interaction length as the active-side refractive index varies, for the standard amplifier with no parasitic losses.

3.3.3 The Effects of Core Parameters

In Fig. 3-10, we show the relation between NGC and the core thickness of the waveguide, $2d$, for the entire set of mode combinations possible in the standard amplifier over the thickness range $0 < 2d < 10\mu$. Four signal and four pump modes propagate in this range, making a total of 16 mode pairs. In general, the NGC rises with decreasing core thickness to reach a peak just before one mode of the pair cuts off. At cut-off, the NGC drops rapidly to zero. This behaviour reflects the changing distribution of power in the three regions of the amplifier as the core thickness is varied. It is similar to the dependence of NGC on the active-side refractive index difference because cut-off brought about by decreasing the core thickness always occurs on the active side in the standard amplifier, which is asymmetric.

The highest peak values of NGC are reached when the lowest-order pump mode propagates in the amplifier. The peak values, in general, fall into two groups. When the pump mode-order is lower than or equal to that of the signal mode, the highest peak values are reached. These are only weakly dependent on both the pump and the signal mode-order. When the signal mode-order is lower than that of the pump, the peak values of NGC are relatively low, and depend strongly on the signal mode-order. These relationships imply that the peak

FIG. 3-10: Net gain coefficient as a function of the thickness of the waveguiding core of the standard amplifier with no parasitic losses.

value of the NGC is largest when the damping constant of the pump-mode evanescent field, $K_{3;m}^p$, determines the extent of the overlap of the pump and signal fields in the gain region, and a large proportion of signal power propagates in that region.

The interaction lengths are plotted in Fig. 3-11 over the same range of core thickness. These decrease with decreasing core thickness because the pump-mode attenuation coefficient rises as cut-off is approached on the active side. As was noted in the previous section, the asymptotic values of IL for mode pairs in which the signal mode is the first to reach cut-off are given by the limit of Eq. 3-15 as cut-off is approached. All combinations in which the pump mode cuts off first have a common asymptotic value of IL, set by the bulk absorption coefficient of the material of the active region.

In Fig. 3-12, the relation between NGC and IL is shown. The two classes of mode pairs, distinguished by which mode reaches cut-off first, are clearly differentiated. Those mode pairs for which the signal mode reaches cut-off first offer the best compromises between NGC and IL. Matched mode-pairs are the best combinations of all. The mode pairs for which the pump mode reaches cut-off first form an untidy group on this plot, but it is evident that their behaviour near cut-off is not the same as that of the other mode combinations, in that they tend to a common IL at cut-off, as discussed above.

FIG. 3-11. Interaction length in metres and a function of the thickness of the waveguiding core of the standard amplifier with no parasitic losses.

FIG. 3-12. Net gain coefficient as a function of interaction length as the thickness of the core varies, for the standard amplifier with no parasitic losses.

We now consider the influence of parasitic absorption of the pump mode in the core-region. To obtain a clear illustration, it is useful to consider absorptions much stronger than would be likely to occur in practice in the cores of optical waveguides. The calculations presented for the lossless and the lossy amplifiers then show two extreme conditions between which the situations normally encountered may be found.

The variation of NGC with core thickness when there is a bulk absorption of 1cm^{-1} in the core is shown in Fig. 3-13. The general features of this plot are similar to those of Fig. 3-9 for the lossless case. The NGC of each mode-pair rises with decreasing core thickness to a peak near the cut-off thickness, and drops rapidly thereafter. Although the peak value reached by each mode pair is only slightly reduced from the lossless case, the NGC values for thicker cores are very much reduced, so the plot of Fig. 3-13 covers a much wider range of NGC than does Fig. 3-9. The reduced values of NGC result from the increased attenuation coefficient of the pump mode. As the thickness of the lossy core increases, greater proportions of the pump mode power are confined to propagate in it, thus raising the pump mode attenuation coefficient. Another difference from the lossless case is that the mode pairs which reach the highest peak values of NGC are not the same. The lossy-core amplifier has the highest NGC values occurring for matched-mode combinations. We also note that

FIG. 3-13. Net gain coefficient as a function of the thickness of the waveguiding core of the standard amplifier with parasitic absorption of the pump radiation in the core corresponding to $\alpha_{02}^p = 1\text{cm}^{-1}$.

the dependence of NGC on pump mode-order is reversed between the lossless and lossy-core cases for those mode pairs in which the pump mode-order is less than the signal mode-order. The combinations showing highest NGC in the lossy case are those in which the sum of the pump and signal mode-orders is highest, rather than those containing low-order pump modes. The advantage of a weakly-bound pump mode for avoiding attenuation in the core appears to outweigh the advantage of a strongly-bound pump mode for avoiding attenuation in the active cladding region.

The plot of IL versus core thickness is shown in Fig. 3-14. The only major difference between the two cases is that the range of values for IL is much less when the core is lossy than when it is lossless. This results from the stability that the presence of core-loss gives to the pump-mode attenuation coefficient. Any attenuation that may be avoided in the core by a less strongly bound mode is offset by increased attenuation in the active cladding.

The variation of NGC with IL is shown in Fig. 3-15. The IL has been plotted on a linear rather than a logarithmic scale in this figure to show details that would be obscured on a logarithmic scale because of the crowding due to the narrow range of IL. The best compromises between NGC and IL are not altered by the presence of absorption in the core in spite of the difference in the NGC values resulting from this attenuation.

FIG. 3-14. Interaction length as a function of the thickness of the waveguiding core of the standard amplifier with parasitic absorption of the pump radiation in the core corresponding to $\alpha_{02}^p = 1\text{cm}^{-1}$.

FIG. 3-15. Net gain coefficient as a function of interaction length, as the core-thickness varies, for the standard amplifier with parasitic absorption of the pump radiation in the core corresponding to $\alpha_{02}^p = 1 \text{ cm}^{-1}$.

3.4. CONCLUSIONS

In this Chapter, perturbation theory was used to study the gain of evanescent-wave amplifiers that are pumped by a bound mode. Since no gain coefficients can be defined for such amplifiers, the gain was characterized by two measures. One of these, the NGC, relates to the gain that would be obtained from the amplifier if it were infinitely long. The other, the IL, is the actual length of the amplifier structure required to produce a defined gain. Values of these two measures of the gain were calculated for the various possible combinations of pump-mode and signal-mode order as each of the parameters of the standard amplifier varied from its nominal value, following the same strategy as was used in the previous Chapter.

In general, it was found that the dependence of NGC on the structure parameters is similar to that of the GCR defined in the previous Chapter, provided that neither pump nor signal mode is near cut-off. When the NGC is related to the gain to be obtained from an actual, finite, device, the IL must be considered. This also varies with the structure parameters, so that it is not to be expected that the gain properties of a finite-length amplifier with bound-mode pumping will resemble those of a transversely-pumped amplifier.

The behaviour of NGC and IL near cut-off is conditioned by whether the pump or the signal mode cuts off first. The best mode-combinations, giving high NGC and short IL, are obtained near cut-off on the active side for mode pairs in which the pump-mode is more strongly bound than the signal-mode. Of these, the best combinations are those in which the field distributions of pump and signal modes are most similar. Since the wavelength of the signal-radiation is longer than that of the pump-radiation, these conditions are best met when the pump-mode and the signal-mode are of the same order. The effect of parasitic absorption of the pump-mode is, in general, to decrease the NGC and the IL. The asymptotic behaviour of these characteristics as cut-off is approached can be altered by pump-mode absorption in the passive side.

In the following sections of this Chapter, a comparison is made between the bound-mode pumped and the externally-pumped configurations of the evanescent-wave amplifier. In this context, the structural tolerances required to obtain high gain, and the relation between the actual gain of an amplifier and its length for a given pump power, are examined. Finally, the possible uses of bound-mode pumped active mode-filters, analogous to the filters discussed in Chapter 2, are examined.

3.4.1 Relaxed Structural Tolerances with Bound-Mode Pumping

One advantage of bound-mode pumping over external pumping is that, under certain circumstances, the close structural tolerances that are required to keep the gain acceptably high can be relaxed somewhat. It was demonstrated in the previous Chapter that the signal mode must be kept as close as possible to cut-off on the active side in the externally-pumped amplifier because the modal gain falls off quickly as the signal is more strongly confined to the waveguiding core. To keep the mode near cut-off required close structural tolerances, particularly on the value of the difference between the refractive indices of the core and the active side. Like the GCR of the externally-pumped amplifier, the NGC of the mode-combinations of the bound-mode pumped amplifier reach peak values as cut-off is approached on the active side. The NGC, however, falls off relatively slowly from this peak value as the pump and signal modes are forced farther from cut-off by increasing the active side index difference ($n_2 - n_3$). This behaviour is illustrated in Fig. 3-7. Since the mode combination (1,1) offers the best combination of short interaction length and high net gain coefficient, we will consider this combination in particular. The figure shows that the value of ($n_2 - n_3$) need be specified only to within ± 0.004 on

a nominal value of 0.005 if the NGC of mode-pair (1,1) is constrained to be not less than 20% of its peak value. This is a significantly wider tolerance than the ± 0.001 that is required to stay within 20% of the maximum possible gain coefficient ratio in the externally-pumped amplifier. We have, however, only considered the NGC, which is representative of a device of finite length only if that length is several times the interaction length. In such cases, the gain of the amplifier approaches the gain of an amplifier of infinite length. We see from Fig. 3-8 that, over the range of $(n_2 - n_3)$ specified above, the interaction length varies from about 1mm to about 10cm. Thus, the actual gain of a finite-length amplifier drops more rapidly as $(n_2 - n_3)$ increases than the NGC indicates unless the amplifier is of the order of 1 metre long. Since the only access to the amplifier that is required is at the ends, it may not be impractical to employ such lengths in the construction of bound-mode pumped evanescent-wave amplifiers. It is therefore possible that amplifiers of this type can be constructed in configurations that require less stringent tolerances on refractive index than the externally-pumped amplifiers.

The reason for the relatively weak dependence of NGC on $(n_2 - n_3)$ is that the NGC is determined in part by the field distributions of both the pump-mode and the signal-mode. One

mode is always farther from cut-off than the other, and hence its field distribution is more stable against changes in the structure. The NGC thus varies less with changes in the structure than the GCR of Chapter 2, which depends only on the field distribution of a single mode.

A small relaxation of the tolerance on core-thickness also arises for long bound-mode pumped amplifiers. We do not discuss it in detail because the core-thickness tolerance does not seem to present a difficult manufacturing problem.

3.4.2 Gain of the Bound-Mode Pumped Evanescent-Wave Amplifier

To provide an insight into the properties of a bound-mode pumped evanescent-wave amplifier of finite length and to demonstrate the effects of parasitic absorption of the signal-mode as well as the pump-mode, we show in Figs. 3-16a and b the actual gain of the standard-amplifier structure as a function of its length. The pump-mode is assumed to be excited by a laser of 0.1 watt output power in the manner described in Section 3.3. Values are shown for the four mode-combinations that the standard amplifier allows, both under lossless conditions and when there is parasitic loss in each of the three regions of the dielectric waveguiding structure. The parasitic bulk-loss coefficients are assumed to be the same at both pump and signal frequencies. These results also serve as the basis for a comparison between the gain of the bound-mode

FIG. 3-16a. Gain of the bound-mode pumped amplifier as a function of amplifier length.

—— Gain assuming no losses other than the pump-mode absorption in the active region that is required for pumping of 10cm^{-1} .

----- Gain assuming a parasitic component of bulk loss in the active cladding region of 0.5cm^{-1} at both pump and signal frequencies.

FIG. 3-16b. Gain of the bound-mode pumped amplifier as a function of amplifier length.

———— Gain assuming a parasitic loss in the passive region corresponding to a bulk loss coefficient of 10cm^{-1} at both frequencies.

----- Gain assuming a parasitic loss in the core corresponding to a bulk loss coefficient of 0.01cm^{-1} at both frequencies.

pumped amplifier and that of the externally-pumped amplifier.

The gain of an amplifier of zero length is, of course, unity. Gain values for amplifier lengths of less than 1cm are omitted in Figs. 3-16a and b to avoid confusion near the origin. We consider first the gain of the lossless amplifier, shown in Fig. 3-16a. For each mode-combination, the gain rises with increasing amplifier length and eventually approaches a saturation value. The rate at which the gain approaches saturation depends mainly on the pump-mode order. Mode pairs in which the pump-mode is of order 2 closely approach the saturation gain at an amplifier length of about 3cm, whereas for mode-pairs in which the pump-mode order is 1, the saturation gain is not approached closely until the length reaches 20cm.

From Fig. 3-2, it can be seen that the interaction lengths of the standard amplifier depend chiefly on the pump-mode order. Mode pairs having pump-mode order 2 have interaction lengths of about 0.6cm, while those with pump-mode order 1 have interaction lengths of about 4cm. The results plotted in Fig. 3-16a thus show that the saturated gain is closely approached in the standard amplifier if the amplifier length is more than five times the interaction length. This saturation gain is, of course, the gain of an amplifier of infinite length, and is obtained by exponentiating the NGC. The highest saturated gain shown in Fig. 3-16a is obtained by mode pair (1,2) which also has the highest NGC, as can be seen in Fig. 3-1.

In a lossless standard-amplifier structure 1cm long, the externally-pumped evanescent-wave amplifier exhibited a gain of about 5, under the pumping conditions established in the previous Chapter. To compare the merits of the bound-mode pumped amplifier, we examine its gain in the lossless case when the length is also 1cm. At this length, Fig. 3-16a shows that the highest gain is obtained on the (2,2) mode pair rather than the (2,1) which is far from having reached its saturation gain. (It was noted in Section 3.3 that the (2,2) mode combination offers the best compromise between short interaction length and high NGC.) A gain of 1.184 is obtained. While this is less than the gain of 5 offered by the externally-pumped amplifier, it is obtained with a pump-source power of only 0.1 watt, as compared with 10kW for the externally-pumped amplifier. By examination of Eqs. 3-12 and 3-14, it can be seen that the logarithm of the gain of the bound-mode pumped amplifier is proportional to the power of the pump-mode, when there are no losses. The pump-power required to match the performance of the externally-pumped amplifier is thus only about 1 watt. The bound-mode pumped amplifier thus requires less power than the externally-pumped amplifier by a factor of 10^5 . The price of this drastic reduction in pump power is, of course, the necessity of coupling the pumping power into a bound mode.

The effects of parasitic loss on the gain of the standard amplifier are shown in Figs. 3-16a and b. In all cases, the gain is reduced by the presence of parasitic loss, and this reduction increases with increasing amplifier length, so that if it is sufficiently long, the "amplifier" becomes lossy. This is a consequence of the parasitic loss of signal radiation. If, however, the losses and the length are chosen correctly, it is possible to construct the amplifier so that only one signal-mode has gain, and the others are lossy. This raises the possibility of constructing active mode filters similar to those discussed in Chapter 2.

3.4.3 Mode-Filtering with the Bound-Mode Pumped Amplifier

The gain of a bound-mode pumped evanescent-wave amplifier constructed using the standard-amplifier waveguide configuration and with a parasitic loss coefficient of 0.01cm^{-1} in the core at both pump and signal frequencies is shown in Fig. 3-16b. In this amplifier, over a length of 20cm, only the mode combinations (1,2) and (2,2) have gain while the others are lossy at any length. By careful selection of the core-loss, it appears possible to construct a longer amplifier so that only one signal mode receives gain. Such a long bound-mode pumped amplifier might serve to carry signals from point to point with no loss and with minimal spread in transit-time.

The evanescent-wave amplifying principle has, so far, been considered only to amplify signals and for related applications such as mode-filtering. With the provision of feedback, evanescent-wave amplification can be used to construct optical oscillators. In fact, the most convenient evanescent-wave devices to study experimentally are the evanescent-wave oscillators because no signal need be coupled into them, and the properties of the output radiation can be used to deduce parameters of the oscillating modes. In the next Chapter, we derive the gain thresholds for oscillation in an evanescent-wave oscillator for which experimental results have been reported, and compare the computed results with the observed oscillations.

CHAPTER 4

THE PASSIVE-CORE CORRUGATED-WAVEGUIDE LASER

4.1 INTRODUCTION

Like any amplifier, the evanescent-wave optical amplifier can be made to oscillate by the provision of suitable feedback. The usual arrangement for providing feedback to optical amplifiers is to place reflectors at each end so that the output of the amplifier is reflected back into it. The space between the reflectors, containing the amplifier, thus forms an optical resonant cavity. Because of the difficulty of coupling power into optical waveguides, it is inconvenient to construct oscillators using optical waveguide amplifiers, such as the evanescent-wave devices, with reflectors external to the amplifier. Instead, feedback is best provided within the amplifier structure itself. The simplest method is to use the ends of the waveguide as reflectors. Optically-pumped waveguide lasers using this technique have been constructed^(1,23,24). It is difficult, however, to control the orientation of the end surfaces of such a small structure as an optical waveguide with sufficient precision for them to act as the end-reflectors of a resonant cavity. The most common use of waveguide end reflectors has been in injection-pumped diode lasers⁽²⁵⁾. Since these solid-state lasers are constructed

of single crystals, it is possible to ensure the correct alignment of the end-reflectors by making use of the crystallographic properties of the waveguide, for example, by cleaving the end-surfaces. It is not possible to use such techniques in waveguide amplifiers constructed of amorphous materials such as glass. An alternative method of providing feedback integral to a waveguiding structure is therefore of interest.

When the ends of the waveguide cannot easily be used as reflectors, it is preferable to provide feedback distributed along the propagation axis of the amplifier. Such feedback can arise from diffraction of the guided radiation by a structure in the waveguide that is periodic along the propagation axis. This structure can consist of a periodic variation of any of the parameters of the waveguide about its nominal value. At specific frequencies, depending on the period of the perturbation and the properties of the waveguide, such perturbations in the waveguide structure couple modes travelling in opposite directions. They thus provide narrow resonances similar to those of the more customary form of optical resonant cavity. Various distributed feedback (DFB) lasers have been reported. Most commonly feedback is provided by a spatially periodic perturbation of the gain of an optical amplifier^(17,26-28) or, as in the device with which we shall be concerned, by a perturbation of the thickness of the core of an optical waveguide amplifier.

For reasons concerning manufacture, it is often most convenient to use a periodic variation in core-thickness to provide contradirectional mode-coupling in a dielectric waveguide. Such a variation is manifest as corrugation in one or both walls of the core region. Accordingly, lasers making use of such structures are called corrugated-waveguide DFB lasers. Even though laser diodes can be manufactured using cleaved end-reflectors, the corrugated-waveguide structure has become important in their development. The DFB structure can give manufacturing advantages since it can be constructed by planar semiconductor techniques, and also it can be made to serve a dual purpose. As well as providing the primary feedback mechanism⁽²⁹⁻³⁴⁾, it can be employed to adjust the intensity distribution in the oscillating device to ensure that the intensity at the end surfaces is not high enough to cause damage to the crystal⁽³⁵⁾. This adjustment of the internal intensity distribution is possible because the corrugation can provide coupling of the bound modes into radiation modes of the waveguide, and thereby couple power out of the device. Coupling into radiation modes by a corrugation has also been suggested as the primary method for extracting radiation from the laser⁽³⁶⁾. Because of the interest in the corrugated-waveguide laser diode, methods for making high-quality corrugations on semiconductor crystal surfaces have

been developed^(36,37). Experiments have also been carried out with corrugated-waveguide dye-lasers, which present fewer manufacturing difficulties than semiconductor lasers, at present. These have been optically-pumped lasers, both with gain in the core region⁽³⁸⁻⁴¹⁾ and with evanescent-wave gain⁽⁶⁾. One notable property of these lasers, in contrast to other types of distributed feedback lasers, is that when more than one mode can propagate in the waveguide, oscillations occur in which modes of diverse order are coupled together. Such modes of oscillation are called "hybrid" modes^(41,6).

In this Chapter, we employ the methods developed in earlier Chapters to study the hybrid modes of a corrugated waveguide, evanescent-wave laser reported in the literature⁽⁶⁾. Since the salient feature of an evanescent-wave laser is that the core is passive, this device was called the "passive-core, corrugated-waveguide laser", in the original publication, and that name is adopted here. For brevity, the device will be referred to as "PCL" in what follows. The calculations that are reported in this Chapter were undertaken to obtain the threshold gains for lasing on each hybrid mode of the device and so to determine whether the hybrid modes that were observed to oscillate were, in fact, those whose threshold gain was computed to be lowest. In the course of the calculations, it was necessary to develop methods for calculating intermode coupling coefficients for corrugations of any waveshape, and

for accounting for the losses due to the coupling to leaky modes. These methods are described in this Chapter and in the associated Appendices, Appendices B through E. For clarity, we present here only the general outline of the methods used, the numerical results of calculations, and a discussion of their significance. The details of derivations are relegated to the Appendices.

It was possible to make full use of the accurately-known parameters of the experimental laser, described in (6), for threshold computations. It was found from these computations that the observed hybrid modes are, in fact, those with lowest threshold gain. The dominant factors in mode selection in the PCL are the leaky-wave losses, the length of the corrugated-waveguide structure, and the proportion of this length that is optically pumped.

4.2 HYBRID MODES OF CORRUGATED-WAVEGUIDE LASERS

In this section, coupled-mode theory is employed to obtain a dual-eigenvalue equation whose eigenvalues represent the threshold gain and the frequency of oscillation of the hybrid modes of corrugated-waveguide lasers. The methods used are general, but the development given here is specifically for the evanescent-wave laser-structure. Expressions for the intermode coupling coefficients, the hybrid-mode loss coefficients for coupling to radiation modes, and the hybrid-mode

gain coefficients are obtained from the coupled-mode analysis.

4.2.1. The Coupled-Mode Equations for Hybrid Modes

The corrugated-waveguide laser that we consider is modelled as a dielectric-slab waveguiding structure formed by the dielectric constant distribution $\epsilon(x,z)$. The propagation direction is x , and it is assumed that there is no y -variation in either the dielectric constants or the electromagnetic fields. The modes are assumed to be TE, since all of the modes of oscillation of the PCL were observed to be so polarized. The analysis for TM modes can be carried out by similar techniques.

The dielectric constant distribution $\epsilon(x,z)$ is made up of three parts, corresponding to the unperturbed slab-waveguide, the perturbation introduced into this structure by the presence of gain in one cladding region, and the perturbation introduced by the corrugation in one waveguide wall. The dielectric constant distribution thus takes the form

$$\epsilon(x,z) = \epsilon_0(z) + (\Delta\epsilon - j\epsilon'')S(x,z) + j\epsilon''L(z) \quad (4-1)$$

where

$\epsilon_0(z)$ is the unperturbed component of the dielectric constant distribution;

$\Delta\epsilon$ is the difference between the dielectric constants of the core and cladding at the corrugated side of the structure;

ϵ'' is the imaginary portion of the dielectric constant of the active cladding region and represents the gain in the active material in this region of the structure, which is assumed to be independent of position, as in the externally-pumped evanescent-wave amplifier;

$S(x,z)$ is a function used to define the regions in which the dielectric constant distribution is perturbed by the corrugation and takes the form

$$S(x,z) = \begin{cases} 1 & \text{for } d < z < d+R(x) \text{ if } R(x) > 0 \\ -1 & \text{for } d+R(x) < z < d \text{ if } R(x) < 0 \\ 0 & \text{otherwise} \end{cases}$$

where d is the z co-ordinate of the unperturbed waveguide wall;

$R(x)$ is a function representing the deviation of the corrugated waveguide wall from the unperturbed wall (the "corrugation function");

$L(x)$ represents the distribution of gain, and is of a form such that gain is present only in the cladding region of the waveguiding structure on the same side as the corrugation.

Since it is assumed that there is no gain or loss in the core, and hence no imaginary component of its dielectric constant, the difference between the imaginary components of

the dielectric constants of the core and the active cladding is $-j\epsilon''$. The complex difference between the dielectric constants is thus $(\Delta\epsilon - j\epsilon'')$, which is the factor that multiplies $S(x,z)$ in Eq. 4-1. The forms of the functions $\epsilon_0(z)$, $S(x,z)$ and $L(z)$ are illustrated in Fig. 4-1 for a simple passive-core corrugated waveguide laser in which the corrugation function $R(x)$ is of sinusoidal form.

At a certain wavelength, a dielectric waveguide may allow two modes of propagation such that the sum of the magnitudes of their axial propagation constants $k_{x,m}$ and $k_{x,n}$ is nearly equal to the magnitude of a wave vector that is associated with periodic corrugation of the waveguide wall. In this circumstance, energy can be coupled contradirectionally between one mode and the other. Such coupling gives rise to hybrid modes of oscillation. Perturbation theory can be used to obtain coupled-mode equations for the hybrid modes of the laser. Coupled-mode equations have been used extensively in the analysis of DFB laser structures (16,42-46). The general properties of coupled-mode solutions for the threshold conditions of DFB lasers are given in (47) and (48). To obtain the coupled-mode equations, the field distributions in the perturbed waveguide can be expressed in terms of the modal field distributions of the unperturbed structure. By using the orthogonality of the normalized transverse modal field

FIG. 4-1. The passive dielectric-constant distribution $\epsilon_0(z)$, the active dielectric-constant distribution $L(z)$, and the perturbation function $S(x,z)$ for a passive core corrugated waveguide laser.

distributions $E_{y;m}(z)$, and the slow-growth approximation for the field amplitudes, the coupled mode equations can be obtained in the following form:

$$\sqrt{|k_{x;m}|} \frac{\partial C_m}{\partial x} = -j K_{mn} (\Delta\epsilon - j\epsilon'') e^{j\Delta_{mn}x} \sqrt{|k_{x;n}|} C_n + \alpha_m \sqrt{|k_{x;m}|} C_m \quad (4-2)$$

$$\sqrt{|k_{x;n}|} \frac{\partial C_n}{\partial x} = j K_{mn}^* (\Delta\epsilon - j\epsilon'') e^{-j\Delta_{mn}x} \sqrt{|k_{x;m}|} C_m - \alpha_n \sqrt{|k_{x;n}|} C_n$$

where

$\Delta_m = (k_{x;m} - k_{x;n} - r\Lambda)$ and gives the offset of the hybrid-mode oscillation frequency from the Bragg condition;

Λ is the wavevector magnitude of the fundamental component of the corrugation;

r is the order of the component of the corrugation that provides the feedback;

C_m is the complex amplitude of the m th mode;

α_m is the amplitude-gain coefficient for the m th mode;

$K_{mn} [\Delta\epsilon - j\epsilon'']$ is the intermode coupling coefficient for coupling between modes m and n .

In Eq. 4-2, mode m is positive-going and mode n is negative-going. We use the convention that $k_{x;m} = -k_{x;n}$. Mode index m is thus a positive number and mode index n is a negative number. The derivation of the coupled-mode equations, Eqs. 4-2, from perturbation theory is given in detail in Appendix B.

The gain coefficient α_m is given by

$$\alpha_m = \frac{\epsilon'' \mu_0 \omega^2}{2 |k_{x;m}|} \int_{-\infty}^{\infty} E_{y;m}(z) L(z) E_{y;m}(z) dz \quad (4-3)$$

By using alpha to denote the amplitude-gain coefficient in the coupled-mode equations, we conform to the usual notation of the literature. See, for example (48). It should be noted, however, that in the previous two Chapters, the symbol alpha referred to the intensity-gain coefficient in conformity with the existing literature on evanescent-wave devices, e.g. (4). The intensity-gain coefficient is, of course, twice the amplitude-gain coefficient of the same wave. Although the subscripts and superscripts used in Chapters 2 and 3 to identify each specific intensity-gain coefficient ensure that there is no actual conflict of notation with the amplitude gain coefficients used in this Chapter, it is worthwhile noting the two different usages of the symbol alpha.

The factor K_{mn} in the coupling coefficient is defined by

$$K_{mn} = \frac{\mu_0 \omega^2}{2\sqrt{|\beta_m \beta_n|}} \int_{-\infty}^{\infty} E_{y;m}(z) G_r(z) e^{-j\theta_r(z)} E_{y;n}(z) dz \quad (4-4)$$

where the r th component of a Fourier expansion of the function $S(x, z)$ along the x direction has been expressed in the form

$$G_r(z) = \left\{ e^{j[r\Lambda x - \theta_r(z)]} + e^{-j[r\Lambda x - \theta_r(z)]} \right\} \quad (4-5)$$

The Fourier expansion of $S(x, z)$ that gives rise to the terms $G_r(z)$ and $\theta_r(z)$ is discussed in a later section.

Applying the transformations

$$\begin{aligned} \sqrt{k_{x;m}} C_m &= R \exp \left\{ j \left[\Delta_{mn} + (\alpha_m - \alpha_n) \right] \frac{x}{2} \right\} \\ \sqrt{k_{x;n}} C_n &= S \exp \left\{ -j \left[\Delta_{mn} - (\alpha_m - \alpha_n) \right] \frac{x}{2} \right\} \end{aligned} \quad (4-6)$$

to Eq. 4-2 leads to coupled-mode equations that have been discussed in the literature⁽⁴⁸⁾. The proportionality between the electric field intensity of a mode and the power it carries depends directly on the axial component of the modal propagation constant. This dependence represents the fact that energy is transported along the waveguide at different velocities by modes of different order. The symbols R and S thus

represent modal electric field amplitudes that are normalized with respect to the power carried by the modes. A single gain coefficient equal to the average of the two modal gain coefficients α_m and α_n appears in the transformed coupled-mode equations. This gain coefficient is defined to be the gain coefficient α_{mn} of the hybrid mode in which mode m is coupled to mode n . The hybrid mode is denoted m - n .

From the solutions to the coupled-mode equations, a dual-eigenvalue equation can be obtained, whose eigenvalues yield the threshold values of the gain coefficient α_{mn} and detuning parameter Δ_{mn} for the hybrid modes. These eigenvalues are found to depend only on the lengths of the passive laser structure and the pumped region within this structure, and the coupling and loss coefficients. In Appendix C, we present a solution of the coupled-mode equations and derive the eigenvalue equation. The eigenvalue equation can be solved by numerical techniques.

In this section, we have shown the way in which coupled-mode theory can be used to obtain coupled-mode equations for the hybrid modes of corrugated-waveguide structures and have given the forms of the appropriate gain and coupling coefficients. The details of the development are given in the appropriate Appendices. We now examine the gain and coupling coefficients of the hybrid modes.

4.2.2 The Corrugation Coupling Coefficient

The perturbation in the dielectric-constant distribution that is responsible for contradirectional mode-coupling has been represented by the function $S(x,z)$. This perturbation may be resolved into spatial Fourier components in each of two directions: parallel to the waveguide axis, and transverse to it. Contradirectional mode-coupling occurs when the magnitude of one of the Fourier component wavevectors in the axial direction is nearly equal to the sum of the absolute values of the axial propagation constants of the two coupled modes. The periodicity and the relatively large extent of the feedback structure in the axial direction result in a Fourier spectrum in this direction consisting of sharply defined lines. As a result, the range $\Delta(|k_{x;m}| + |k_{x;n}|)$ of axial propagation constants over which two contradirectional modes can be coupled is narrow. Coupling two modes, however, implies coupling both the axial and the transverse components of their modal wavevectors, $k_{x;m}$ and $k_{z;m}$. In the case of intramode coupling, there is no change in the transverse component $k_{z;m}$; but, in the case of intermode coupling, modes whose wavevectors have different transverse components must be coupled together. The Fourier spectrum of the coupling perturbation in the direction transverse to the waveguide axis is broad in most corrugated waveguide devices as a result of the relatively

small amplitude of the corrugation. The range of transverse wavevectors $k_{x;m}$ that can be coupled together by the perturbation is correspondingly large. Among the distributed-feedback lasers reported in the literature, the corrugated-waveguide lasers are distinguished by small transverse dimensions for the feedback structure, and for this reason, hybrid modes have been observed to be characteristic of corrugated-waveguide lasers.

In order to obtain an expression for the portion of the perturbation $S(x,z)$ that is responsible for contradirectional mode-coupling due to r th-order diffraction from the corrugation we perform a two dimensional Fourier transformation of $S(x,z)$, select the r th component in the x direction and re-transform this component. This procedure yields a function of period $2\pi/r\Lambda$ which can be considered to represent the effective corrugation. The procedure is written as

$$G_T(z') \cos[r\Lambda x' - \theta(z')] = \frac{1}{4} \iiint_{-\infty}^{\infty} df_x df_z dx$$

$$\int_d^{d+R(x)} dz \exp\{j2\pi[f_x(x'-x)$$

$$+ f_z(z'-z)]\} \frac{\Lambda}{2\pi} \left[\delta\left(f_x - \frac{r\Lambda}{2\pi}\right) + \delta\left(f_z + \frac{r\Lambda}{2\pi}\right) \right] \quad (4-7)$$

where f_x and f_z are spatial frequencies in the x and z directions, respectively, and x' , z' are the co-ordinates of the retransformed function. Performing the integrals in Eq. 4-7 leads to the following expressions for the effective perturbation when the diffraction corresponding to feedback is on the r th order.

$$G_r(z') \cos[r\Lambda x' - \theta(z')] = \frac{2}{T} \int_0^T \int_d^{d+R(x)} \delta(z' - z) \{ \cos(r\Lambda x) \cos(r\Lambda x' + \sin(r\Lambda x) \sin(r\Lambda x') \} dx dz \quad (4-8)$$

where $T=2\pi/\Lambda$. In what follows, the primes denoting the variables of the retransformed function are dropped. In the case of the sinusoidal corrugation shown in Fig. 4-1, it can be shown that, for even-order diffraction,

$$G_r(z) = \begin{cases} -\frac{1}{r\pi} \sqrt{1 - [(d-z/a)]^2} U_{r-1}[(d-z/a)] & \text{if } [(d-z/a)]^2 < 1 \\ 0 & \text{if } [(d-z/a)]^2 > 1 \end{cases} \quad (4-9)$$

$$\theta_r(z) = 0$$

where $U_{r-1}[(d-z/a)]$ is the Chebyshev polynomial of the second kind of order $r-1$. The details of the calculation are given in Appendix D. The result for odd-order diffraction is similar.

A comparison of the method given in this section for calculating coupling coefficients with a method that has been reported in the literature⁽⁵⁰⁾ for the special case of intramode coupling of weakly-bound modes by the first diffraction order of a sinusoidal corrugation shows agreement for this case.

Earlier methods of calculating corrugation coupling coefficients have generally involved assumptions that can be violated in the intermode coupling with which we are concerned in studying hybrid modes of oscillation. The assumption that the field amplitudes vary negligibly across the corrugation^(40,49,50) may not be justified when one of the mode pairs of the hybrid is strongly bound, for example, in coupling between the highest and lowest-order modes of a multi-mode waveguide. Alternatively, the approximation assuming strongly bound modes⁽⁵¹⁾ can be in error in the same situation. The assumption of a particular corrugation waveform, generally a square-wave, has often been made^(40,42,47,52). This, of course, limits the accuracy with which other corrugations can be handled. Even the subtle approximation that the deviations of the waveguide walls from parallelism are small⁽⁵³⁾ can be erroneous in a practical device.

The problem of determining numerically the coupling-coefficients corresponding to feedback on any order of the corrugation but the fundamental has apparently not yet been dealt with in the literature.

The method given in this section for determining corrugation coupling coefficients does not make any assumptions other than the assumption of small corrugation amplitude or refractive index difference on the corrugation side, which has been made earlier in order to apply perturbation theory. Feedback on any order of corrugations of any shape can be treated by this method.

4.2.3 The Gain Coefficient

In computing the modal gain coefficients from Eq. 4-3, it is convenient to express the imaginary part of the dielectric constant in terms of the bulk gain-coefficient of the active medium, α_{03} . Equation 4-3 then becomes

$$\alpha_m = \frac{k_{03}\alpha_{03}}{2|k_{x;m}|} \int_{-\infty}^{\infty} E_{y;m}(z)L(z)E_{y;m}(z)dz \quad (4-10)$$

where $k_{03} = 2\pi n_3/\lambda_{mn}$ and λ_{mn} is the free-space wavelength of the radiation generated by the hybrid-mode oscillation. The hybrid-mode gain coefficients are given by

$$\alpha_{mn} = \frac{\alpha_m + \alpha_n}{2} \quad (4-11)$$

We have not yet considered the effect of coupling to the modes of the continuum. The corrugation in the waveguide wall causes coupling into these modes as well as to the bound modes. If the strong contradirectional coupling that generates

the feedback required for laser action results from a high-order component of the corrugation waveform, then the lower-order components also cause strong contradirectional coupling, and this coupling may be to modes of the radiation continuum, rather than to bound modes. The radiation modes carry power away from the laser, and coupling to them therefore constitutes a loss mechanism. There results a reduction in the effective gain coefficient for the hybrid mode, so that the threshold-gain eigenvalues must be increased to make up for the loss to the radiation modes. The actual threshold gain-coefficient required for oscillation, including the effects of radiation loss, is α_{mn} and is given by

$$\alpha_{mn} = \alpha_{mn} + \sum_{s=1}^q \alpha_{mn;s}^L \quad (4-12)$$

where α_{mn} is the threshold gain-coefficient for the hybrid mode determined without considering losses;

$\alpha_{mn;s}^L$ is the hybrid-mode loss-coefficient for coupling into the radiation continuum by the s th-order component of the corrugation waveform;

q is the highest component-order of the corrugation that couples power into modes of the continuum.

Each modal loss-coefficient is individually combined with the corresponding modal gain-coefficient to obtain the effective modal gain-coefficient. The effective hybrid gain

coefficient α'_{mn} is the average of the individual effective modal gain coefficients, so that $\alpha_{mn;s}^L$ in Eq. 4-12 is seen to represent the average of the two modal losses $\alpha_{m;s}^L$ and $\alpha_{n;s}^L$ due to the sth-order component of the corrugation. The hybrid loss-coefficient $\alpha_{mn;s}^L$ is thus the simple average of the loss coefficients $\alpha_{m;s}^L$ and $\alpha_{n;s}^L$ for coupling from the bound modes into the continuum by the sth-order component of the corrugation.

The modal loss coefficient $\alpha_{m;s}^L$ is, in fact, a coupling coefficient for coupling from bound mode m to the radiation continuum. An expression similar to Eq. 4-4 can be obtained for the coupling to radiation modes by including the radiation modes in the expansion that represents the perturbed field-distributions in the waveguide, and developing the appropriate coupled-mode equation. Since a bound mode is coupled only to a narrow region of the radiation continuum⁽⁶⁷⁾, the power-loss coefficient can be obtained by relating the power gained by the group of radiation modes to the power lost by the bound mode. The amplitude loss-coefficient obtained in this manner is

$$\alpha_{m;s}^L \approx \pi |K_{mp}|^2 (\Delta\epsilon)^2 \quad (4-13)$$

where

$$K_{mp} = \frac{\mu\omega^2}{2\sqrt{|\rho k_{x;m}|}} \int_{-\infty}^{\infty} G_s(z) e^{-\theta_s(z)} E_{y;m}(z) E_{y;p}(z) dz \quad (4-14)$$

and p is the transverse component of the propagation constant of the leaky wave in the active cladding region. The derivation of Eqs. 4-13 and 4-14 is given in Appendix E.

When feedback is provided by the second-order component of the corrugation, coupling to the radiation modes occurs only as a result of the first-order component. This is the situation with which we shall be concerned in the PCL. The modal loss coefficients are simply given by the first term in the sum of Eq. 4-14 and can be obtained from Eq. 4-13 using Eq. 4-14. To evaluate them, it is necessary to know the appropriate values of p , the transverse component of the propagation constant of the leaky wave in the active cladding region. The axial components of the leaky-wave propagation constants are obtained by subtracting the magnitudes of the bound-wave propagation constants from the magnitudes of the wavevector corresponding to the first-order periodicity of the corrugation. From the axial component of the radiated-wave propagation constant, and the known frequency of the hybrid-mode oscillation, the transverse component can be determined. When feedback is on the second order of the corrugation, the radiation modes that are excited centre about the mode that propagates transversely to the propagation axis of the waveguide, and the leaky wave thus leaves the laser structure at right angles to the propagation axis.

4.3 CHARACTERISTICS OF THE PASSIVE-CORE CORRUGATED-WAVEGUIDE LASER

In this section, the construction of the PCL⁽⁶⁾ is reviewed and computed values of its hybrid-mode gain, coupling and loss coefficients are given. For comparison, the corresponding coefficients are also presented for a passive-core corrugated-waveguide laser of identical construction, but with a sinusoidal corrugation waveform of the same amplitude and period as that of the grating.

4.3.1 The Passive-Core Corrugated-Waveguide Laser

Because of the method used to construct it, the parameters of the waveguide structure forming the passive-core corrugated-waveguide laser reported in (6) were known or were accurately measurable. The waveguiding core of the dielectric structure, the passive cladding region and the corrugation were all provided by a diffraction grating replica. The resin in which the grating replica was cast had a higher refractive index than the quartz substrate that supported it, and so could serve as the waveguiding core. The grating itself formed the corrugation that gave distributed feedback. The active cladding region was provided by bathing the grating replica in a solution of rhodamine-6G laser dye in various alcohols, so that the corrugated surface of the resin waveguide was in contact with the dye solution which thus formed the active

cladding region. The properties of the corrugation and the refractive indices of the resin and quartz were accurately known from data supplied by the manufacturer. The authors of (6) measured the real component of the refractive index of the dye solution with an Abbé refractometer and the thickness of the resin core by an interferometric technique. The core thickness was also determined by the present author from the angles at which leaky waves were coupled out of the waveguide by the corrugation, and good agreement with the earlier results was obtained. The measured parameters of the PCL are given in Table 4-1, and the device itself is shown schematically in Fig. 4-2. The corrugation profile for the blazed diffraction grating is shown in Fig. 4-3. It should be noted that the corrugated waveguide structure was not pumped over its entire length, but over a relatively small region in the middle.

In the PCL structure, gain was provided by evanescent-wave amplification, and feedback arose from the second-order component of the corrugation waveform. The first-order component of the corrugation coupled power into a narrow range of modes of the continuum. These leaky modes were observed as radiation from the structure in directions nearly normal to the propagation axis of the waveguide. Oscillations were observed to occur in the PCL at a number of wavelengths that could be unambiguously identified as corresponding to a parti-

TABLE 4-1

CHARACTERISTICS OF THE PCL STRUCTURE

	Refractive Index		Thickness (μm)	Corrugation	
	Real	Imaginary		Period (nm)	Amplitude (nm)
Cladding Region 1	1.4575	0	---	---	---
Core Region 2	1.5216	0	6.85	416.67	71.5
Cladding Region 3	1.4719	2.17×10^{-2}	---	---	---

Corrugated Length = 3.0cm

Pumped Length = 0.75cm

FIG. 4-2. The passive-core, corrugated-waveguide laser structure.

OPTICAL PUMP BEAM

SUBSTRATE

CORRUGATED THIN FILM

DYE SOLUTION

FIG. 4-3. The corrugation function $R(x)$ of the passive-core corrugated-waveguide laser (PCL).

cular series of hybrid modes. In these series, each mode that the structure supported in the wavelength range over which gain was provided was coupled in turn to the highest-order mode. No oscillation was observed in which the highest-order mode was not involved.

In the following, computed values of the coupling, gain and radiation-loss coefficients are presented graphically by plotting the coefficient value as a function of hybrid-mode order at the corresponding hybrid-mode oscillation frequency. To do this, the hybrid modes are grouped into series in which each mode of order less than some common mode is coupled in turn to that common mode. The series are then arranged in sequence by common-mode order.

4.3.2 Calculation of the Corrugation Coupling Coefficient

The effective perturbation $G_r(z) \cos [r\Lambda x - \theta_r(z)]$ for feedback on the r th order of the corrugation waveform of the grating can be obtained by the methods of Section 4.2.2 in the forms

$$G_r(z) = \frac{1}{2} \sqrt{M_r^2(z) + N_r^2(z)} \quad (4-15)$$

$$\theta_r(z) = \arctan \left(\frac{M_r(z)}{N_r(z)} \right)$$

where

$$M_r(z) = \left\{ (-1)^r \cos \left[\frac{r\pi(z+d)}{ST} \right] - \cos \left[\frac{r\pi(z+d)}{QT} \right] \right\} \frac{1}{r\pi}$$

(4-16)

$$N_r(z) = \left\{ (-1)^r \sin \left[\frac{r\pi(z+d)}{ST} \right] + \sin \left[\frac{r\pi(z+d)}{QT} \right] \right\} \frac{1}{r\pi}$$

and Q, S are absolute values of the slopes of the sides of the corrugation as shown in Fig. 4-3. The details are given in Appendix D.

The coupling coefficients for each of the nine possible hybrid-mode series of the PCL were calculated using Eq. 4-15. The results of the calculations are shown in Fig. 4-4. For comparison, the coupling coefficients for a structure identical to the PCL but with a sinusoidal corrugation are also shown in the figure. In general, the coupling coefficients decrease with decreasing mode-order, reflecting the decreased penetration by the tightly-bound mode-fields into the corrugation region. It is noteworthy that in the case of the sinusoidal corrugation the coupling coefficients for hybrid modes in which two high-order propagation modes are coupled decrease with increasing mode-order. This behaviour results from the fact that the normalized transverse field-distributions $E_{y,m}(z)$ for weakly-bound modes are nearly constant throughout the corrugation region. Since $G_r(z)$ for the sinusoidal corrugation is an odd function if r is even, the integral in Eq. 4-4

FIG. 4-4. Coupling coefficients for the hybrid modes of the PCL structure (solid lines) and for the hybrid modes of the same structure with a sinusoidal corrugation (dashed lines). (The modal values have been joined by lines for convenience in viewing this and subsequent figures.)

can lead to low values for the coupling coefficients if $E_{y;m}(z)$ and $E_{y;n}(z)$ are nearly constant. No similar effect occurs for the case of the grating corrugation of the PCL because the integrand in Eq. 4-4 is not antisymmetric in this case. The dependence of coupling coefficient on hybrid-mode order for the grating corrugation is slightly reduced when both modes of the hybrid are of high order. This is a result of the decreased amplitudes of the normalized field-distributions in the corrugation region for modes near cut-off. It can be seen in Fig. 4-4 that the coupling coefficient for coupling by second-order diffraction from the corrugation is, in general, higher for the blazed grating corrugation than for the sinusoidal corrugation.

4.3.3 Calculation of the Gain and the Radiation Loss Coefficients

In order to calculate the gain coefficients of the hybrid modes of the PCL structure from Eqs. 4-10 and 4-11, a bulk gain-coefficient of $\alpha_{03}=1\text{cm}^{-1}$ was assumed. The actual hybrid gain-coefficients of the structure can therefore be obtained by multiplying the results of the calculations given in this section by the actual bulk gain-coefficient of region 3 of the PCL structure, if this is known. For the assumed bulk gain of 1cm^{-1} , the contribution of the active component of the corrugation to the coupling coefficients was of the order

of 10^{-4} of the contribution of the passive component. Consequently, the effect of the active component of the corrugation was ignored in the calculations. It will be seen that the experimental results of (6) are consistent with this assumption.

The calculated gain coefficients of the hybrid modes of the PCL structure, neglecting radiation loss, are shown in Fig. 4-5. The hybrid-mode gain coefficients decrease with decreasing mode-order for the hybrid series of each common mode-order because of the decreased penetration of the low-order modal field-distributions, which are the most tightly bound, into the active region of the waveguide.

Since the PCL operates by second-order feedback from the corrugation, the loss-coefficient for coupling to radiated modes is simply the first term in the sum of Eq. 4-12. Values of the PCL's hybrid-mode loss-coefficients α_{mn}^L , calculated by using Eqs. 4-13 and 4-14, are shown in Fig. 4-6. The radiation-loss coefficients are of the order of 0.1cm^{-1} . The bulk loss-coefficients due to absorption and scattering in the materials of the dielectric waveguide are much smaller than this. The effects of deviation from smoothness in the waveguide wall are dominated by the corrugation, so that coupling by the corrugation to radiated modes is the dominant loss-mechanism in the PCL.

FIG. 4-5. Gain coefficients for the hybrid modes of the PCL structure.

FIG. 4-6. Radiation loss coefficients for the hybrid modes of the PCL structure (solid lines) and the same structure with a sinusoidal corrugation (dashed lines).

The radiation-loss coefficients of the PCL shown in Fig. 4-6 decrease with decreasing mode-order because of decreased penetration of the tightly-bound modal field-distributions into the corrugation. There is also a slight reduction in slope for the highest-order hybrid of the highest-order series because of the reduced ~~field~~-amplitudes of the highest-order mode, which is near cut-off. This effect was also seen in the coupling coefficients.

For comparison, the loss coefficients for a sinusoidal corrugation are also shown in the figure. These are generally lower in value ~~than~~ the loss coefficients for the triangular corrugation profile of the grating. It is noteworthy that the loss coefficients for coupling to radiation modes by the sinusoidal corrugation are not markedly reduced for the high-order hybrids, as the coupling coefficients are for this type of corrugation. Although the radiation-loss coefficient is proportional to the coupling coefficient for coupling to the modes of the continuum, in this case, such coupling occurs only on an odd-order component of the corrugation. For the sinusoidal corrugation, the function $G_s(z)$ is an even function for odd orders, and so the integral in Eq. 4-14 does not tend to zero for weakly-bound modes as the integral in Eq. 4-4 does.

4.3.4 Calculation of the Bulk Threshold Gain for Hybrid Modes

The bulk threshold gain-coefficient eigenvalues for the hybrid modes of the PCL and of an identical laser, but with a sinusoidal corrugation, are presented in this section. These have been determined by solving the eigenvalue equation, Eq. C-21, for the ~~threshold~~ gain and detuning eigenvalues, using the numerical technique outlined in Appendix C, and values for the gain, coupling, and radiation-loss coefficients determined as in the previous sections. The threshold gains are expressed in terms of the bulk gain-coefficients required in the active cladding to initiate oscillation. The calculated values show that the length of the laser structure, the proportion of this length that is pumped, and the corrugation waveform, which determines the relative strength of the feedback and the loss mechanisms, are all important parameters in determining which hybrid modes have the lowest threshold gain for oscillation in the passive-core corrugated-waveguide structure. To demonstrate the effect that each parameter has on the hybrid-mode thresholds, we present the computed results in the following way in Figs. 4-7 to 4-10. First, the bulk threshold gain-coefficients in region 3 of the waveguide for hybrid modes of lasers that are pumped over their entire corrugated length are presented as a function of mode-orders of the hybrid modes.

FIG. 4-7. Bulk threshold gain coefficients of the hybrid modes of the PCL structure for various lengths of the structure. The lengths are given in cm. The dashed area is shown with expanded ordinate in Fig. 4-9.

FIG. 4-8. Bulk threshold gain coefficient of the 9th and 8th common-order hybrid series of the PCL, for various lengths of the structure, in cm.

3

4

OF/DE

FIG. 4-9. Bulk threshold gain coefficients for the hybrid modes of a structure identical to the PCL, but with a sinusoidal corrugation waveform, for various lengths of the structure in cm.

FIG. 4-10. Bulk threshold gain coefficients for the 9th and 8th common-order hybrid series of the PCL, for integral values of pumping-ratio between 1 and 7. The structure length is 3cm.

These gain coefficients have been calculated for various values of the length of the corrugated-waveguide laser structure to show how this length affects the relative threshold gains of the various hybrid modes. The results are presented both for the triangular corrugation-profile of the grating used to make the PCL and for the hypothetical identical laser with a sinusoidal corrugation, and a comparison is made between them to show the effect of corrugation waveform. Finally, the bulk threshold gain-coefficients for the PCL structure with a length equal to the length of the actual PCL laser of (22) are shown as a function of hybrid-mode order, for various ratios of the length of the structure to the pumped length in the centre. These results show that the proportion of the length that is pumped can be important in determining the hybrid-mode oscillation thresholds. The results show that the hybrid modes of lowest threshold gain in the PCL laser are indeed those that were reported to oscillate in (6).

The solutions of the eigenvalue equation Eq. C-21, actually yield $\alpha'_{mn;T}$, the threshold value of the effective hybrid-mode gain coefficient, which includes compensation for radiation losses. The threshold value $\alpha_{mn;T}$ for the actual hybrid-mode gain coefficient α_{mn} can be determined from Eq. 4-12 since the loss coefficient $\alpha_{mn;l}^L$ is known from the earlier calculation. Since the values of the hybrid gain coefficients

α_{mn} given in Fig. 4-5 were calculated on the assumption of unit bulk gain-coefficient in region 3, per centimetre, the actual bulk threshold gain-coefficient, $\alpha_{03,T}$, can be obtained by dividing the value of $\alpha_{mn,T}$ by the appropriate value of α_{mn} as given by the calculations leading to Fig. 4-5.

The dependence of the bulk threshold-gain coefficient of medium 3 on the hybrid-mode designation is shown in Fig. 4-7 for various lengths of the PCL structure. It is assumed in this case that the laser has uniform gain over its entire corrugated length. In general, the bulk threshold gain-coefficients increase as the propagation mode-orders of the hybrid decrease. This behaviour is a result of the reduction in gain and coupling for tightly-bound modes, due to the small penetration of their evanescent fields into the corrugation and the gain medium. Although the effects of radiation loss also are reduced for tightly-bound modes, the rise in threshold gain-coefficient associated with reduced coupling dominates the net threshold gain-coefficient as given by Eq. 4-12 at the laser lengths considered for Fig. 4-7.

The bulk threshold gain-coefficients of the 9th and 8th common mode-order series of the PCL structure with uniform pumping are shown again in Fig. 4-8, but with an expanded vertical scale. It can be seen from this plot that the order of the hybrid-mode oscillations of lowest threshold gain

depends on the length of the laser. For example, in a laser whose length is greater than 1.5cm, the modes of the 9th common-order series, down as far as 9-3, have lower thresholds than the modes of the 8th, and lower, common-order series. As shown by the arrows, for a laser of length 1cm, mode 8-8 has a lower threshold than mode 9-4; whereas, for a laser of length 1.5cm, mode 8-8 has a higher threshold than mode 9-3. If the PCL laser had been uniformly pumped along its length, we would be able to deduce from the observed hybrid mode oscillations, which occurred only on the modes of the 9th common-order series down to 9-3, that the length of the laser must not be less than about 1.5cm. Furthermore, since at lengths greater than about 2.5cm, the 9-2 hybrid mode, which was not observed, has a lower threshold than the 9-7 hybrid mode, which was, we would be able to conclude that the length could not be greater than about 2.5cm. In fact, the passive structure of the PCL was 3cm long, but it was not uniformly pumped. When the effects of non-uniform pumping are taken into consideration, we will see that the correct threshold relationships are obtained to agree with the observed hybrid-mode spectrum.

The bulk threshold gains of the hybrid modes of a passive-core corrugated waveguide laser identical to the PCL in structure, but with a sinusoidal corrugation, are shown in Fig. 4-9. The sinusoidal corrugation used for these calcula-

tions has the same amplitude and period as the grating used to make the PCL. By comparing Figs. 4-7 and 4-9, we see that the hybrid-mode threshold gains are higher for the sinusoidal corrugation than for the grating when the laser is short, i.e. less than 2cm in this case. The lower coupling coefficients of the sinusoidal corrugation are responsible for this higher threshold. When the laser is long, greater than 2cm in length, for this case, it can be seen that high-order hybrid modes can have lower thresholds in the structure with the sinusoidal corrugation than in the one with the triangular corrugation profile provided by the blazed grating. This results from the fact that the loss coefficients corresponding to coupling into radiated modes are also lower for the sinusoidal corrugation. The effect of loss to radiated modes is more pronounced in long lasers than in short because, for a given coupling coefficient, the hybrid threshold gain-coefficient eigenvalue α_{mn} decreases as the length of the laser increases, with the result that the hybrid loss-coefficient $\sum_{s=1}^q \alpha_{mn,s}^L$ begins to dominate in Eq. 4-12.

The passive-core corrugated waveguide laser that was reported in (6) was not pumped over its entire length, but in a relatively small region in the centre of the corrugated structure. To obtain threshold-gain values that can be compared with the observed hybrid-mode oscillations of the PCL, the

effect of non-uniform pumping must be considered. The corrugated-waveguide structure that formed the PCL was a circular grating of 3cm diameter. The pumping radiation was focussed on the grating surface in a line that was normal to the direction of the grooves of the grating, and about 0.75cm long. The pumping intensity was only approximately uniform over this length, so that the ends of the pumped region were not sharply defined. A possible error of ± 0.25 cm must therefore be assumed in the measurement of the pumped length. The ratio of the corrugated length to the pumped length, which we call the pumping ratio, R , thus had a value between $R=3$ and $R=6$ in the PCL.

The dependence of bulk threshold-gain on hybrid-mode order is shown in Fig. 4-10, for integral values of pumping ratio between $R=1$ and $R=7$. For all pumping ratios, all hybrid modes of the 8th common-order series have higher thresholds than the hybrid modes of the 9th common-order series. The absence of any observed hybrid oscillations in the PCL that did not include the 9th-order mode is thus confirmed by the computed results. This arrangement of oscillating modes can always occur, if the actual gain of the active material is not too high, for any value of pumping ratio up to $R=7$. The arrows drawn on Fig. 4-10 show that for pumping ratios below $R=2$ or above $R=5$, the 9-2 hybrid mode has a threshold lower than one

of the highest-order hybrids of the 9th common-order series. Since the 9-2 and 9-1 hybrid modes were not observed to oscillate, and the hybrid modes from 9-9 to 9-3 were, it can be concluded that if the analysis is to agree with the experimental results, the pumping ratio must be between $R=2$ and $R=5$. The measured length provides the limits $R=3$ and $R=6$, which overlap the calculated limits by a reasonable margin. We can conclude, therefore, that the hybrid-mode thresholds computed by the methods of this Chapter are in agreement with the observed results to the extent that the hybrid modes of lowest computed threshold are, in fact, those that are observed to oscillate.

An examination of Fig. 4-10 shows that the bulk gain of the active medium cannot have been less than about 10cm^{-1} nor more than about 20cm^{-1} ; otherwise, mode 9-7 would not have been observed in the first case, or mode 9-2 would have been in the second. The contribution of the active component of the corrugation to the coupling coefficient is of the order of 10^{-4} of the contribution of the passive component when the bulk gain coefficient of the active medium is assumed to be 1cm^{-1} . The actual contribution of the imaginary component of the corrugation at threshold to the coupling coefficient is thus of the order of 10^{-3} of the contribution of the real component. The assumption that the active component of the corrugation could be ignored is thus justified.

The computed limits of bulk threshold gain stated above for the PCL correspond to bulk gains of 3dB/mm and 16dB/mm in the active material. These values are in general agreement with the gain to be expected from the laser dyes used under the pumping conditions of the experiment.

4.4 CONCLUSIONS

Since the relative thresholds of the PCL deduced from experiment are in agreement with the computed thresholds, we conclude that the methods of this Chapter provide adequately accurate values of bulk threshold gain for this laser. These methods are quite general and may be expected to be valid for other lasers as well. These need not necessarily obtain gain by evanescent-wave interactions. Gain coefficients for waveguide lasers with gain anywhere in the waveguiding structure can be obtained by evaluating Eq. 4-3. All the other parameters of the structure are independent of the location of the active region in the waveguide.

These methods should be appropriate to the determination of the threshold gains of hybrid modes of corrugated-waveguide diode-lasers in particular. They also offer the possibility of determining the best corrugation waveforms, periodicities and waveguide configurations to use in constructing these devices, and of determining the effectiveness with which power is coupled out of them by the presence of a corrugation.

An interesting feature of non-uniformly-pumped corrugated waveguide lasers is discussed in the following section. This phenomenon may provide a method for improving the efficiency of these lasers, near threshold.

4.4.1 Efficiency of Non-Uniformly-Pumped Corrugated-Waveguide Lasers

One interesting feature of the threshold gains of the non-uniformly-pumped PCL is that the pumping efficiency is raised by decreasing the proportion of the corrugated structure that is pumped. In Fig. 4-11, the bulk threshold-gain of the 9-9 hybrid mode of the PCL is shown as a function of pumping ratio R . If the pumping is uniform, that is, $R=1$, the bulk threshold-gain is 6.45cm^{-1} . As the proportion of the corrugated length that is pumped decreases, the amount of power received by the laser structure also decreases for constant pump intensity. To maintain the power received by the laser constant, the intensity of the pump-radiation must rise in proportion to the pumping ratio R . Since the bulk gain-coefficient is proportional to the pump intensity, it too must rise in proportion to R . The rise in threshold gain that would result simply from maintaining the total pumping power constant as R increases is shown by the straight line in the figure. The actual rise in threshold gain is less than that required to maintain constant input power. This means that the input power needed to achieve threshold is reduced by reducing the proportion of the

FIG. 4-11. Bulk threshold gain coefficient of the hybrid mode 9-9 as a function of pumping ratio for the PCL. The length of the structure is 3cm.

corrugated length of the PCL structure that is pumped. A similar phenomenon occurs for all the hybrid modes.

The increase of pumping efficiency that results from increasing the pumping ratio occurs because the axial field-intensity distributions of the modes of the hybrid oscillation, $|C_m(x)|^2$ and $|C_n(x)|^2$ peak in the centre of the device. The power that is extracted from the active region by evanescent-wave interactions increases with the modal field intensity. Consequently, the concentration of the gain in the centre of the laser structure where the axial distributions of field-intensity are greatest provides the gain in the place where it is most effective.

Each hybrid oscillation in the passive-core corrugated-waveguide laser can occur on a number of longitudinal modes corresponding to different detuning parameters Δ_{mn} and different threshold gains. Each longitudinal mode also corresponds to a characteristic axial field-intensity distribution for the modes of the hybrid oscillation. In the device we have considered, the lowest threshold gain occurs for the first-order longitudinal mode, for which the field-intensity distributions are minimal at the ends of the laser, and rise to a maximum at the centre. The second-order longitudinal mode has field distributions that are minimal at the ends of the laser and at the centre, and two maxima, spaced symmetrically about the

centre. Each longitudinal mode has a number of intensity maxima in the axial field distributions that is equal to its order. The even-order longitudinal modes have field-intensity minima at the centre of the laser; the odd-order modes have maxima at the centre. Each longitudinal mode corresponds to a slightly different oscillation wavelength because of the different detuning. The detuning is so small, however, that for the PCL all the oscillations on longitudinal modes of low order occur at wavelengths within less than an Angstrom unit of the Bragg wavelength.

When the laser is uniformly pumped, the threshold gain increases with increasing longitudinal mode-order. Because only the first-order longitudinal-mode field-intensity distributions have a single maximum at the centre, concentrating the gain in the centre discriminates against longitudinal modes of order greater than the first. By matching the gain distribution to the field-intensity distribution of another mode, it would be possible to lower its threshold with respect to the other longitudinal modes in a similar way. If, by controlling the gain distribution in this way the longitudinal mode of lowest threshold can be selected at will, then a method of controlling the oscillation wavelength over a narrow range is provided. Although the range is likely to be very small in a practical device - in the PCL, the oscillation wavelength

varies by only one part in 10^6 between the first and second-order longitudinal modes - it could, in principle, be sufficient for wideband modulation of the output of the laser because of the very high frequency of the oscillation. Another possible application of longitudinal mode-control is in determining the properties of the output beam of semiconductor lasers from which the output is extracted by coupling to radiation modes. The distribution of intensity along the axis of such a laser determines the far-field beam pattern. Some control over the far-field pattern can thus be obtained by determining which longitudinal mode can oscillate. Longitudinal mode-control can also be obtained from asymmetric longitudinal gain distributions⁽¹⁶⁾. These give the additional possibility of unidirectional operation.

The analytical treatment of the passive-core corrugated-waveguide laser has provided some insights into the possible uses of the device. Since oscillators of the type reported in (6) are relatively simple to construct, it is likely that the development of evanescent-wave devices will proceed first by investigations of oscillators rather than the amplifiers discussed in the earlier Chapters. In the next Chapter, the potential applications of all the evanescent-wave devices that have been considered in the thesis are reviewed, and the properties of the devices are summarized.

CHAPTER 5

CONCLUSION

The properties of the evanescent-wave devices investigated in the thesis are reviewed in this Chapter. Some interesting features of these devices that are apparent from the analyses and calculations are examined. In conclusion a summary of the work reported in the thesis, and an outline of some possibilities for future research are given.

5.1 EVANESCENT-WAVE AMPLIFIERS

The evanescent-wave amplifiers that were studied were assumed to be constructed in asymmetric dielectric-slab waveguide, of which one cladding region is passive, to represent a passive substrate. Such a configuration would be suitable for evanescent-wave devices in integrated optics applications. The gain characteristics of modes of amplifiers constructed in this way are largely governed by the proportion of the modal power that propagates in the active cladding region, in comparison with that propagating in the core and the passive substrate. The general properties of these amplifiers can be estimated quite reliably on this basis. For example, the highest-order propagation modes have the highest gains because they are the least strongly bound to the core, and hence a

relatively large proportion of their power propagates in the active cladding. As another example, reducing the difference between the refractive indices of the core and the passive substrate reduces the gain of all modes. In this case, the distribution of modal power in the three regions of the waveguide is altered so that a smaller proportion of the power is carried in the active region. The effects of the presence of parasitic loss in each of the three regions of the waveguide can be understood in a similar way. Loss in the core-region severely reduces gain. Loss in the cladding regions is usually less problematic because the major portion of the power is carried in the core, except for modes that are very near cut-off.

Two optical pumping arrangements for evanescent-wave amplifiers were considered: external pumping and bound-mode pumping. These pumping schemes result in amplifiers whose properties differ from each other in detail. Most notably, an externally-pumped evanescent-wave amplifier, which is pumped by radiation passing transversely through the waveguide, requires much more pump power than a bound-mode pumped amplifier of the same length to provide a specified gain. In the bound-mode pumped amplifier, the pumping radiation propagates along with the signal as a bound mode of the waveguide, with the result that most of the pumping power is confined to those regions of the waveguide in which the signal propagates. The pump power is thus used very efficiently.

If high gain is to be obtained for a signal propagating on a particular mode of an evanescent-wave amplifier, then that mode must be kept as close as possible to cut-off on the active side. This maximizes the proportion of the modal power that propagates in the gain region. To keep the mode sufficiently close to cut-off that the gain coefficient is within even an order of magnitude of the maximum that would be obtained exactly at cut-off, a very close tolerance is required on the difference between the refractive indices of the core and the active cladding. This tolerance is relaxed somewhat in the bound-mode pumped amplifier by comparison with the externally-pumped amplifier because the gain of the bound-mode pumped amplifier is dependent on the field distributions of both the pump and the signal modes. The properties of this combination of two modes are more stable against changes in the structure than those of a single mode.

Because it is more efficient of the pump power, and because its manufacture may be less critical, the bound-mode pumped evanescent-wave amplifier appears to be the most promising. Its main disadvantage is the inconvenience of having to couple the pumping radiation into a bound mode of the waveguiding structure. This may pose difficult problems, however, because of the necessity to avoid coupling the signal

radiation out of the device at the point where the pump-radiation is coupled in. The relative simplicity of the external pumping arrangement makes it likely that this will be preferred in some applications.

The gain of bound-mode pumped evanescent-wave amplifiers is characterized by the net gain coefficient (NGC) and the interaction length (IL). The NGC is the natural logarithm of the gain available from an amplifier of infinite length, and the IL indicates the rapidity with which this gain is approached as the length of a finite amplifier increases. It is desirable to manufacture amplifiers with large NGC's and small IL's. In the amplifiers that were studied, the gain of a finite amplifier approaches the gain of an infinite amplifier closely when the amplifier length is greater than about five interaction lengths. The NGC and IL vary considerably with the orders of the pump and signal modes. Generally, the best combinations of high NGC and short IL are obtained when the mode orders of the pump and signal are equal, and as high as the structure allows. This is because the field distributions of matched pump and signal modes are the most similar.

Both the externally-pumped and the bound-mode pumped evanescent-wave amplifiers can act as active mode-filters. By the introduction of relatively large bulk losses into their core regions, these amplifiers can be constructed so that modes

whose order is less than a specified number are attenuated, while those of higher order are amplified. Because the difference in gain between the highest-order mode and the next-highest is greater than the difference in gain between any other two adjacent modes, the highest-order signal-mode can be isolated from the others quite effectively in such a device. Filters of this type could be used to isolate signals on particular modes of a waveguide. The principle could also be applied to suppress undesired hybrid-mode oscillations in evanescent-wave lasers.

The evanescent-wave amplifiers have been considered in the previous discussion only as devices that amplify a signal present on a bound wave that propagates in them. It is also possible to employ them as modulators by impressing the signal as a variation in pump intensity. This would provide a nonlinear modulation of the power in the signal mode. In the case of the externally-pumped amplifier, the intensity-gain coefficient $\alpha_{G;n}$ is directly proportional to the pump intensity. The gain of an amplifier of fixed length is thus exponentially dependent on pump intensity. The presence of loss does not alter this relationship. Similar results are obtained for the bound-mode pumped amplifier.

5.2 EVANESCENT-WAVE OSCILLATORS

The analysis of the evanescent-wave amplifiers forms an introduction to the more complex analysis of the evanescent-wave distributed feedback laser. The study of this device forms a major part of the thesis.

The evanescent-wave oscillator studied in the thesis is an externally-pumped evanescent-wave amplifier provided with feedback by second-order diffraction from a corrugation in the boundary between the core and the active region. Because experimental results are available for an oscillator of this type, a comparison between the results of calculations concerning an evanescent-wave device can be compared with experiment in this case, to verify the computational methods used.

Coupled-mode equations were formulated and solved to obtain threshold-gain eigenvalues for the hybrid modes of oscillation in the laser. The hybrid modes with lowest threshold gains were, in fact, those that were observed experimentally to oscillate. This result indicates that the methods used to calculate the thresholds are valid. One feature of general interest is that this calculation includes the determination of coupling coefficients and radiation-loss coefficients for corrugations of any waveshape, and on any order. The method of determining these coefficients is of wide applicability, and could be used in particular in the design of semiconductor and other corrugated-waveguide lasers.

The possibility exists of improving the pumping efficiency of distributed-feedback lasers at threshold by employing non-uniform longitudinal distributions of gain. When the pumping is concentrated near the centre of the corrugation, the longitudinal mode of first order is pumped more effectively than the other modes. The concentration of pumping at the centre is a partial matching of the longitudinal gain distribution in the laser to the field-distributions of the first-order longitudinal mode. By arranging the longitudinal distribution of gain correctly, it may be possible to exercise control over which longitudinal mode has the lowest threshold. Such techniques could be used to provide frequency-shift modulation in semiconductor lasers in which several independent junctions share the same corrugation in series. The gain distribution could be controlled in such devices by adjusting the current to each junction.

Some possible future research areas concerning evanescent-wave devices are considered in Section 5.4.

5.3 SUMMARY

Analytical methods for obtaining the gain coefficients and other gain parameters for externally-pumped and bound-mode pumped evanescent-wave amplifiers with parasitic losses were developed. These methods were used to determine the gain properties of hypothetical evanescent-wave amplifiers, and the potentially useful features of such amplifiers were discussed. The methods for calculating evanescent-wave gain were augmented with other analytical methods to determine the threshold gains for the hybrid modes of a passive-core corrugated waveguide

laser. These included the calculation of coupling coefficients and leaky-wave loss coefficients for a waveguide corrugation of arbitrary shape, on arbitrary order. Coupled-mode equations were formulated and solved for the hybrid-mode oscillations of a distributed-feedback laser pumped only in a central region. All these methods were confirmed by the agreement of the relative calculated threshold gains with the observed hybrid modes.

5.4 AREAS FOR FUTURE RESEARCH

Several possible applications of evanescent-wave amplifiers have been mentioned. The potential value of these devices as amplifiers of signals propagating as bound modes of optical waveguides is established by the results of the gain calculations. These results have been discussed in some detail in the Chapters devoted to evanescent-wave amplifiers. The details of their potential use as mode-filters, however, have not been investigated. The practicability of constructing such filters, and their usefulness, are matters for further study.

The gain calculations for the evanescent-wave amplifiers established that the gain of the highest-order mode is very sensitive to the proximity of that mode to cut-off. The structural tolerances of the waveguide must, in fact, be very strict if the amplifier is to approach its maximum gain, which occurs for the highest-order mode when it is just at cut-off

on the active side. The possibility arises of turning this difficulty to an advantage by employing some form of modification of the waveguide structure to modulate the gain of the amplifier in a controlled way. The gain of the evanescent-wave amplifier near cut-off can be extremely sensitive to the refractive-index difference between the core and the active cladding. One possible method of constructing a variable-gain amplifier might be to control the refractive index of one of the three regions of the dielectric waveguiding structure, preferably the core or the active cladding, by the electro-optic effect. Even relatively small refractive index variation could produce large variation in gain in such a device.

The advantage of evanescent-wave amplifiers over waveguide amplifiers with active core regions in such an application is that evanescent-wave amplifiers provide their highest gain on the highest-order mode. In any waveguide amplifier, the gain of this mode is the most sensitive to changes in the waveguide structure. By contrast, the active-core amplifiers give their highest gain on the lowest-order mode, which is the least sensitive to structural changes. Thus, in the evanescent-wave amplifier, high gain and high sensitivity to modulation are compatible qualities, whereas in the active-core amplifier, they are not. A second possible advantage of evanescent-wave amplifiers as modulation devices is that their highest-gain mode can be more widely separated in gain from the other modes

than is the case with the active-core amplifier. This property has been discussed in connection with mode-filtering. The gains of the various modes of a waveguide amplifier are altered by different amounts by waveguide structural changes. If all the high-gain modes have similar gain, as in the active-core amplifiers, then there will be a component of modulation non-linearity induced by the modulation of the signal on modes other than the desired one. In the evanescent-wave amplifier, this problem can be minimized by ensuring that only one mode has high gain. The evanescent-wave amplifiers thus show promise as optical modulators. Studies of these devices would be of interest.

A detailed experimental examination of evanescent-wave amplifiers is clearly required before their operational usefulness can be definitely established. It may, however, be advantageous to begin such experimental studies by developing the existing experimental work on the evanescent-wave oscillator. Evanescent-wave oscillators are more convenient to study experimentally than evanescent-wave amplifiers. There is no necessity to distinguish the output of the device from the input which can pose problems in optical experiments. It is possible to avoid the necessity of coupling into bound modes entirely, if the externally-pumped configuration is used. It

is therefore to be expected that the evanescent-wave oscillator is the device of immediate experimental interest.

The analytical methods developed in the thesis provide a basis for the design of more efficient corrugated-waveguide structures. One area for future research would be the experimental investigation of a device designed by these methods. It may be possible to achieve continuous-wave operation in a passive-core corrugated-waveguide dye-laser. Bound-mode pumping will almost certainly be required. When a corrugated waveguide laser structure is pumped by a bound mode, the pump intensity is not uniform because the pump mode suffers attenuation due to absorption in the active region, and possibly to coupling to leaky modes by the corrugation. This results in a non-uniform and non-symmetrical distribution of gain in the device. An analytic study to determine the effect of such a distribution on the threshold gains of the various longitudinal modes would be of interest.

The immediate future of evanescent-wave optical devices is closely coupled to the future of optical communications. Because this field is now so active, it seems likely that the actual worth of these devices will soon be tested. If they are of value to optical communications, they will surely become commonplace very rapidly. But if they are not, their unique properties among waveguide amplifiers make them interesting

devices in their own right. It is not unjustified to predict that optical elements operating by the principle of evanescent-wave amplification will assume some practical importance in the future.

REFERENCES

1. C.J. Koester, IEEE Journal of Quantum Electronics, QE-2, p. 580, 1966.
2. K.O. Hill, A. Watanabe, J.G. Chambers, Applied Optics, 11, p. 1952, 1972.
3. A. Watanabe, K.O. Hill, R.I. MacDonald, Canadian Journal of Physics, 51, p. 761, 1973.
4. K.O. Hill, R.I. MacDonald, A. Watanabe, Journal of the Optical Society of America, 64, p. 263, 1974.
5. K.O. Hill, A. Watanabe, Optics Communications, 5, p. 389, 1972.
6. K.O. Hill, A. Watanabe, Applied Optics, 12, p. 430, 1973.
7. R.L. Gallawa, IEEE Transactions on Communications, COM-23, p. 131, 1975.
8. H. Kogelnik, IEEE Transactions on Microwave Theory and Techniques, MTT-23, p. 2, 1975.
9. R. Schubert, J.H. Harris, IEEE Transactions on Microwave Theory and Techniques, MTT-16, p. 1048, 1968.
10. P.K. Tien, Applied Optics, 10, p. 2395, 1971.
11. F.A. Blum, D.W. Shaw, W.C. Holton, Applied Physics Letters, 25, p. 116, 1974.
12. H.F. Taylor, Journal of Applied Physics, 44, p. 3257, 1973.
13. C. Elachi, C. Yeh, Journal of Applied Physics, 44, p. 3146, 1973.
14. S.E. Miller, IEEE Journal of Quantum Electronics, QE-8, p. 199, 1972.
15. J.P. Wittke, RCA Review, 33, p. 674, 1972.

16. R. Shubert, Journal of Applied Physics, 45, p. 209, 1974.
17. J.E. Bjorkholm, C.V. Shank, IEEE Journal of Quantum Electronics, QE-8, p. 833, 1972.
18. W.H. Weber, S.L. McCarthy, G.W. Ford, Applied Optics, 13, p. 715, 1974.
19. M. Fogiel, Modern Microelectronics, Research and Education Association, New York, 1972, p. 109.
20. L.P. Boivin, Applied Optics, 13, p. 391, 1974.
21. H. Kogelnik, T.P. Sosnowski, Bell System Technical Journal, 49, p. 1602, 1970.
22. S.T. Peng, T. Tamir, H.L. Bestoni, Electronics Letters, 9, p. 150, 1973.
23. J. Stone, C.A. Burrus, Applied Optics, 13, p. 1256, 1974.
24. B.I. Miller, W.D. Johnston, Jr., Applied Physics Letters, 25, p. 216, 1974.
25. F.A. Blum, K.L. Lawley, W.C. Scott, W.C. Holton, Applied Physics Letters, 24, p. 430, 1974.
26. H. Kogelnik, C.V. Shank, Applied Physics Letters, 18, p. 152, 1971.
27. C.V. Shank, J.E. Bjorkholm, H. Kogelnik, Applied Physics Letters, 18, p. 395, 1971.
28. J.E. Bjorkholm, T.P. Sosnowski, C.V. Shank, Applied Physics Letters, 22, p. 132, 1972.
29. C.V. Shank, R.V. Schmidt, B.I. Miller, Applied Physics Letters, 25, p. 200, 1974.
30. D.R. Scifres, R.D. Burnham, W. Streifer, Applied Physics Letters, 25, p. 203, 1974.
31. M. Nakamura, H.W. Yen, A. Yariv, E. Garmire, S. Somekh, Applied Physics Letters, 23, p. 224, 1973.
32. H.M. Stoll, D.H. Selb, Applied Optics, 13, p. 1981, 1974.

16. R. Shubert, Journal of Applied Physics, 45, p. 209, 1974.
17. J.E. Bjorkholm, C.V. Shank, IEEE Journal of Quantum Electronics, QE-8, p. 833, 1972.
18. W.H. Weber, S.L. McCarthy, G.W. Ford, Applied Optics, 13, p. 715, 1974.
19. M. Fogiel, Modern Microelectronics, Research and Education Association, New York, 1972, p. 109.
20. L.P. Boivin, Applied Optics, 13, p. 391, 1974.
21. H. Kogelnik, T.P. Sosnowski, Bell System Technical Journal, 49, p. 1602, 1970.
22. S.T. Peng, T. Tamir, H.L. Bestoni, Electronics Letters, 9, p. 150, 1973.
23. J. Stone, C.A. Burrus, Applied Optics, 13, p. 1256, 1974.
24. B.I. Miller, W.D. Johnston, Jr., Applied Physics Letters, 25, p. 216, 1974.
25. F.A. Blum, K.L. Lawley, W.C. Scott, W.C. Holton, Applied Physics Letters, 24, p. 430, 1974.
26. H. Kogelnik, C.V. Shank, Applied Physics Letters, 18, p. 152, 1971.
27. C.V. Shank, J.E. Bjorkholm, H. Kogelnik, Applied Physics Letters, 18, p. 395, 1971.
28. J.E. Bjorkholm, T.P. Sosnowski, C.V. Shank, Applied Physics Letters, 22, p. 132, 1972.
29. C.V. Shank, R.V. Schmidt, B.I. Miller, Applied Physics Letters, 25, p. 200, 1974.
30. D.R. Scifres, R.D. Burnham, W. Streifer, Applied Physics Letters, 25, p. 203, 1974.
31. M. Nakamura, H.W. Yen, A. Yariv, E. Garmire, S. Somekh, Applied Physics Letters, 23, p. 224, 1973.
32. H.M. Stoll, D.H. Selb, Applied Optics, 13, p. 1981, 1974.

33. M. Nakamura, K. Aiki, J-I. Umeda, A. Yariv, H.W. Yen, T. Morikawa, Applied Physics Letters, 25, p. 487, 1974.
34. M. Nakamura, A. Yariv, H.W. Yen, S. Somekh, H.L. Garvin, Applied Physics Letters, 22, p. 515, 1973.
35. R.F. Kazarinov, R.A. Suris, Fizika i Tekhnika Poluprovodnikov, 6, p. 1359, 1972.
36. L. Comerford, P. Zory, Applied Physics Letters, 25, p. 208, 1974.
37. L. Yang, J.M. Ballantyne, Applied Physics Letters, 25, p. 67, 1974.
38. Y. Aoyagi, S. Namba, Applied Physics Letters, 24, p. 538, 1974.
39. S. Wang, S. Sheem, Applied Physics Letters, 22, p. 460, 1973.
40. D.P. Schinke, R.G. Smith, E.G. Spencer, M.F. Galvin, Applied Physics Letters, 21, p. 494, 1972.
41. P. Zory, Applied Physics Letters, 22, p. 125, 1973.
42. S. Wang, Journal of Applied Physics, 44, p. 767, 1973.
43. H. Stoll, A. Yariv, Optics Communications, 8, p. 5, 1973.
44. R.E. deWames, W.F. Hall, Applied Physics Letters, 23, p. 28, 1973.
45. A. Yariv, H.W. Yen, Optics Communications, 8, p. 10, 1974.
46. S.R. Chinn, P.L. Kelley, Optics Communications, 10, p. 123, 1974.
47. A. Yariv, IEEE Journal of Quantum Electronics, QE-9, p. 919, 1973.
48. H. Kogelnik, C.V. Shank, Journal of Applied Physics, 43, p. 2327, 1972.

49. D. Marcuse, Bell System Technical Journal, p. 3187, 1969.
50. D.C. Flanders, H. Kogelnik, R.V. Schmidt, G.V. Shank, Applied Physics Letters, 24, p. 194, 1974.
51. R.E. deWames, W.F. Hall, Journal of Applied Physics, 44, p. 3639, 1973.
52. S. Wang, IEEE Journal of Quantum Electronics, QE-10, p. 413, 1974.
53. D. Marcuse, Bell System Technical Journal, 52, p. 63, 1973.

APPENDIX A

MODES OF AN ASYMMETRIC DIELECTRIC-SLAB WAVEGUIDE

A.1 THE DIELECTRIC-SLAB WAVEGUIDING STRUCTURE

It is established in Section 1.1.2 that the dielectric-slab waveguide forms a practical basic structure for evanescent-wave devices. Such a structure is also the most convenient to deal with analytically. The results of computations for evanescent-wave devices based on slab-waveguides can be taken to indicate the results to be expected from calculations for other types of waveguide under analogous conditions. In this Appendix, we present solutions of the wave-equation for the bound and unbound modes of an asymmetric dielectric-slab waveguide. These solutions and the notation used in them are employed in the discussion of all the evanescent-wave devices treated in this work.

The dielectric-slab structure is shown schematically in Fig. A-1. A layer of a dielectric material, of thickness $2d$, is bordered on each side by a semi-infinite region of material of smaller dielectric constant. We refer to the central layer as the core region, and to the semi-infinite regions contiguous to it as the cladding regions of the waveguide. To aid in the notation, we number each region as shown in Fig. A-2;

FIG. A-1. Co-ordinate system for analysis of the evanescent-wave amplifier.

GUIDING REGION

PROPAGATION
DIRECTION

FIG. A-2.: Distribution of dielectric constant
in the asymmetric waveguiding
structure.

the core is region 2, the cladding regions are regions 1 and 3. In the development to be given, these numbers are used as subscripts to indicate the regions to which various terms refer. By convention, region 3 is always the active cladding region and region 1 the passive in evanescent-wave devices with only one active side. The co-ordinate system to be used is shown in Fig. A-1. The x-axis is the axis of propagation of bound waves. The boundaries between the regions of different dielectric constant are normal to the z-axis. There is no variation in the dielectric constants of the structure in the direction of the y-axis, and the structure is assumed to be infinitely extended in this direction.

A.2 BOUND MODES

Since the available experimental results for evanescent-wave devices concern only TE modes, we consider only this polarization. The solution can be carried out for TM modes by analogous methods. For TE modes, there is no electric-field variation in the y-direction and since there are no currents or charges in the waveguiding structure, there are no components of the electric field in the x or y directions. The wave equation for modes of this type is:

$$\frac{\partial^2 E_{y;n}(z)}{\partial z^2} + \left[\epsilon_i(z) \mu \omega^2 - k_{x;n}^2 \right] E_{y;n}(z) = 0 \quad (A-1)$$

where $\epsilon'(z)$ is a function representing the real dielectric constant distribution in the z -direction that forms the passive waveguiding structure. The transverse electric field-amplitude distributions $E_{y;n}(z)$ that form the fundamental solutions of Eq. A-1 are independent of y . In the x -direction, the fields vary as $e^{-jk_{x;n}x}$ where $k_{x;n}$ is the x -direction propagation constant of the bound wave and the subscript n denotes the mode of propagation. Equation A-1 has exactly the form of the one-dimensional Schrodinger equation, where $k_{x;n}^2$ plays the role of the energy eigenvalue, $E_{y;n}(z)$ that of the wave-function, and $\epsilon'(z)$ is analogous to the potential energy function of the Hamiltonian.

If bound modes of propagation exist in the structure, then the refractive indices of the three regions, n_1 , n_2 and n_3 , must be related by $n_1 < n_2$ and $n_3 < n_2$. For the asymmetric dielectric-slab structure under consideration, the propagation constant eigenvalues that satisfy Eq. A-1 are formally equivalent to the energy levels of a one-dimensional, asymmetric potential well with step-function sides. Similarly, the refractive indices n_1 , n_2 and n_3 of the core and cladding regions of the waveguide are formally analogous to the potentials in the three regions of the well.

The wave equation in the three regions of the structure can be written:

$$\frac{\partial^2 E_{y;n}}{\partial z^2} + [k_1^2 - k_{x;n}^2] E_{y;n} = 0 \quad z > d \quad (A-2a)$$

$$\frac{\partial^2 E_{y;n}}{\partial z^2} + [k_{x;n}^2 - k_2^2] E_{y;n} = 0 \quad -d < z < d \quad (A-2b)$$

$$\frac{\partial^2 E_{y;n}}{\partial z^2} + [k_3^2 - k_{x;n}^2] E_{y;n} = 0 \quad z < -d \quad (A-2c)$$

where

$$k_1^2 = \epsilon_1 \mu \omega^2 \quad (A-3a)$$

$$k_2^2 = \epsilon_2 \mu \omega^2 \quad (A-3b)$$

$$k_3^2 = \epsilon_3 \mu \omega^2 \quad (A-3c)$$

and, for conciseness, the z -dependence of the fields has not been written explicitly.

The solutions to Eq. A-2 have the form:

$$E_{y;n} = D_1 e^{K_{1;n}(z-d)} + B_1 e^{-K_{1;n}(z+d)} \quad z > d \quad (A-4a)$$

$$E_{y;n} = A_1 e^{jk_{z;n}(z)} + A_2 e^{-jk_{z;n}(z)} \quad -d < z < d \quad (A-4b)$$

$$E_{y;n} = B_2 e^{K_{3;n}(z+d)} + D_2 e^{-K_{3;n}(z+d)} \quad z < -d \quad (A-4c)$$

where

$$K_{1;n} = \sqrt{k_{x;n}^2 - k_1^2} \quad (A-5a)$$

$$k_{z;n} = \sqrt{k_2^2 - k_{x;n}^2} \quad (A-5b)$$

$$K_{3;n} = \sqrt{k_{x;n}^2 - k_3^2} \quad (A-5c)$$

Since the fields $E_{y;n}$ must be finite everywhere, it follows that $D_1 \neq D_2 = 0$. The solutions may thus be written in the form:

$$E_{y;n} = B_1 e^{-K_{1;n}(z-d)} \quad z > d \quad (A-6a)$$

$$E_{y;n} = A \sin(k_{z;n}z + \phi_n) \quad -d < z < d \quad (A-6b)$$

$$E_{y;n} = B_2 e^{K_{3;n}(z+d)} \quad z < -d \quad (A-6c)$$

We now apply the boundary condition requiring continuity of the electric field $E_{y;n}$ and its derivative (i.e. the continuity of $H_{y;n}$) at the boundary of the core region at $z=d$. Continuity of the electric field $E_{y;n}$ and Eq. A-6a give the result:

$$B_1 = A \sin(k_{z;n}d + \phi_n) \quad (A-7)$$

Continuity of the derivative of the electric field as expressed by Eq. A-6a gives:

$$-B_1 K_{1;n} = A k_{z;n} \cos(k_{z;n} d + \phi_n). \quad (A-8)$$

We divide Eq. A-7 by Eq. A-8 to eliminate the unknown constants A and B_1 , and obtain:

$$\frac{k_{z;n}}{K_{1;n}} = \tan(k_{z;n} d + \phi_n) \quad (A-9)$$

A similar procedure at the boundary $z=-d$ yields:

$$B_2 = A \sin(-k_{z;n} d + \phi_n) \quad (A-10)$$

and

$$\frac{k_{z;n}}{K_{3;n}} = \tan(-k_{z;n} d + \phi_n). \quad (A-11)$$

By manipulating Eqs. A-5, it can be shown that the propagation constants obey the relations:

$$K_{1;n} = \sqrt{k_2^2 - k_{z;n}^2 - k_1^2} \quad (A-12a)$$

$$K_{3;n} = \sqrt{k_2^2 - k_{z;n}^2 - k_3^2} \quad (A-12b)$$

$$k_{x;n} = \sqrt{k_2^2 - k_{z;n}^2} \quad (A-12c)$$

From Eqs. A-9 and A-12a, we obtain:

$$\tan^2(k_{z;n}d + \phi_n) = \frac{k_{z;n}^2}{(k_2^2 - k_{z;n}^2 - k_1^2)}, \quad (\text{A-13})$$

which can be written in the form

$$k_{z;n} = \sqrt{(k_2^2 - k_1^2)} \sin(k_{z;n}d + \phi_n), \quad (\text{A-14})$$

from which we see

$$\phi_n = \arcsin\left(\frac{k_{z;n}}{\sqrt{k_2^2 - k_1^2}}\right) - k_{z;n}d + N_1\pi. \quad (\text{A-15})$$

From Eqs. A-11 and A-12b, we obtain in a similar way

$$\phi_n = -\arcsin\left(\frac{k_{z;n}}{\sqrt{k_2^2 - k_3^2}}\right) + k_{z;n}d + N_2\pi. \quad (\text{A-16})$$

By eliminating ϕ_n from Eqs. A-16 and A-15, we obtain the transcendental eigenvalue equation:

$$k_{z;n} = \frac{1}{2d} \left[n\pi - \arcsin\left(\frac{k_{z;n}}{\sqrt{k_2^2 - k_1^2}}\right) - \arcsin\left(\frac{k_{z;n}}{\sqrt{k_2^2 - k_3^2}}\right) \right] \quad (\text{A-17})$$

In Eq. A-18, $n = N_1 \pm N_2$ where n is the mode index of the bound wave. The equation can be solved by numerical or graphical

methods to yield the finite set of propagation constant eigenvalues $k_{z;n}$ that the structure allows at the frequency ω of the radiation.

The propagation constants $K_{1;n}$ and $K_{3;n}$ may be considered as transverse components of the wavevector in regions 1 and 3. In fact, for evanescent waves, these propagation constants represent damping of the field amplitudes in the transverse direction rather than propagation in this direction, as is evident from Eq. A-6. These modal constants and the x-direction propagation constant $k_{x;n}$ which is the same for all three regions of the structure, can be obtained from Eq. A-12. The squares of the x-direction propagation constants are the eigenvalues in Eq. A-1.

With the time-harmonic dependence of the fields in the waveguide rendered implicit, the normalized bound-modes that propagate in the positive x-direction have field distributions given to within a constant phase factor by

$$E_{y;n}(x,z) = E_{y;n}(z)e^{-jk_{x;n}x}, \quad (A-18)$$

where

$$E_{y;n}(z) = A_n \sin(k_{z;n}d + \phi_n) e^{-K_{1;n}(z-d)}, \quad z < d \quad (A-19a)$$

$$E_{y;n}(z) = A_n \sin(k_{z;n}z + \phi_n), \quad -d < z < d \quad (A-19b)$$

$$E_{y;n}(z) = A_n \sin(-k_{z;n}d + \phi_n) e^{K_{3;n}(z+d)}, \quad z < -d, \quad (\text{A-19c})$$

and A_n is a normalizing constant determined by imposing the orthonormality condition:

$$\int_{-\infty}^{\infty} |E_{y;n}(z)|^2 dz = 1 \quad (\text{A-20})$$

on the transverse field-distribution. Performing the integral over the transverse field-distribution given by Eq. A-20 yields an expression for the normalizing constant of the n th mode:

$$A_n = \sqrt{\frac{2}{\frac{1}{K_{3;n}} + \frac{1}{K_{1;n}} + 2d}} \quad (\text{A-21})$$

It is noteworthy that the eigenvalue Eq. A-17 always has at least one solution if the waveguide is symmetric. If it is not symmetric, there may be no solution. Thus, for asymmetric waveguiding structures in which the core is very thin, or one of the cladding indices approaches that of the core too closely, there may not exist any bound modes.

A.3 CUT-OFF

A dielectric-slab waveguide supports the propagation of radiation of a particular frequency in a finite number of bound modes corresponding to the number of eigenvalues that exist for Eq. A-17. If the parameters of the waveguide are changed or if the frequency of the radiation is altered, the number of bound modes that the waveguide supports may change. Given a waveguide and a particular radiation frequency, then for each waveguide parameter there exists a value beyond which the parameter cannot be varied without altering the number of modes supported by the waveguide. This is termed the cut-off value for the mode that is added or removed. Cut-off may occur as a result of the value of any of the refractive indices of the structure or the thickness of the core.

When conditions are such that a mode is on the point of disappearing, that mode is said to be at cut-off. As cut-off of a bound mode is approached by varying the wavelength or some parameter of the waveguide, the mode becomes unbound to the waveguiding core. The value of the transverse propagation constant $K_{1;n}$ or $K_{3;n}$ in one of the cladding regions approaches zero, so that the field amplitude in that region no longer decays exponentially with distance from the core boundary. We refer to cut-off occurring on the cladding side in which this

happens. At cut-off, the axial propagation constant becomes equal to the propagation constant k_{01} or k_{03} for unbound radiation propagation in the cladding medium of the side on which cut-off occurs.

The electric field distribution of a mode at cut-off becomes constant in the cladding region where the cut-off occurs. The normalization condition expressed in Eq. A-20 therefore requires that the normalizing constant A_n approach zero at cut-off. The amplitude of the electric field of a mode at cut-off as given by Eq. A-19 thus approaches zero throughout the waveguide.

A.4 RADIATION MODES

As well as the bound modes that have been discussed, the wave equation is satisfied in the dielectric structure by field distributions corresponding to two types of unbound mode. Such modes are called radiation modes, or modes of the continuum. They are not bound to the waveguide structure, but form orthogonal sets which can be used to express field distributions that result when radiation in the core region is incident on a cladding boundary at more than the critical angle. In this case, the radiation is not returned to the core region by total internal reflection, as is the case with a bound mode, but escapes from the waveguiding structure. In this context, such modes are also called "leaky" modes.

Radiation modes that are unbound at both sides of the waveguide core correspond to solutions of the wave equation of the form:

$$E_{y;\rho} = D_1 e^{j\sigma_\rho z} + B_1 e^{-j\sigma_\rho z}, \quad d < z \quad (A-22a)$$

$$E_{y;\rho} = A_1 e^{jk_{z;\rho} z} + A_2 e^{-jk_{z;\rho} z}, \quad -d < z < d \quad (A-22b)$$

$$E_{y;\rho} = B_2 e^{j\rho z} + D_2 e^{-j\rho z}, \quad z < -d \quad (A-22c)$$

where

$$k_{z;\rho} = \sqrt{k_2^2 - k_{x;\rho}^2} \quad (A-23a)$$

$$\sigma_\rho = \sqrt{k_1^2 - k_{x;\rho}^2} \quad (A-23b)$$

$$\rho = \sqrt{k_3^2 - k_{x;\rho}^2} \quad (A-23c)$$

Radiation modes of the other type are bound at only one cladding interface. Modes bound at the interface of the core with cladding region 1, but unbound at the interface with cladding region 3, correspond to solutions of the form:

$$E_{y;\gamma} = D_1 e^{K_{1;\gamma} z} + B_1 e^{-K_{1;\gamma} z}, \quad d < z \quad (A-24a)$$

$$E_{y;\gamma} = A_1 e^{jk_{z;\gamma} z} + A_2 e^{-jk_{z;\gamma} z}, \quad -d < z < d \quad (A-24b)$$

$$E_{y;\gamma} = B_2 e^{j\gamma z} + D_2 e^{-j\gamma z}, \quad z < -d \quad (\text{A-24c})$$

where

$$k_{z;\gamma} = \sqrt{k_2^2 - k_{x;\gamma}^2} \quad (\text{A-25a})$$

$$K_{1;\gamma} = \sqrt{k_{x;\gamma}^2 - k_1^2} \quad (\text{A-25b})$$

$$\gamma = \sqrt{k_3^2 - k_{x;\gamma}^2} \quad (\text{A-25c})$$

A similar solution can be written for modes unbound at the region 3 interface but bound at that of region 1.

The radiation modes each carry infinite power since their amplitudes remain constant to infinite distances in one or both cladding regions. It is therefore necessary to normalize these modes to the delta function rather than to unity. The power carried by the modes of a portion of the radiation continuum is then given by the integral of the modal radiation intensities over that portion of the continuum.

We examine first the radiation modes that are unbound on both sides of the waveguiding core. Applying the boundary conditions requiring continuity of the tangential fields at the core boundaries to Eq. A-22 yields:

$$D_1 = \frac{1}{2} \left[\sin(k_{z;\rho} d + \phi_\rho) - j \frac{k_{z;\rho}}{\sigma_\rho} \cos(k_{z;\rho} d + \phi_\rho) \right] e^{-j\sigma_\rho d} \quad (\text{A-26})$$

$$D_2 = \frac{1}{2} \left[\sin(-k_{z;\rho} d + \phi_\rho) - j \frac{k_{z;\rho}}{\sigma_\rho} \cos(-k_{z;\rho} d + \phi_\rho) \right] e^{j\sigma_\rho d} \quad (A-27)$$

where we have written Eq. A-22b in the form

$$E_{y;\rho} = A_\rho \sin(k_{z;\rho} z + \phi_\rho)$$

which is the same form as Eq. A-6b. The boundary conditions also require that $B_1 = D_1^*$ and $B_2 = D_2^*$ where the asterisk represents the complex conjugate.

Modal quantities for radiation modes of this type are labelled with a subscript ρ , the transverse component of the wavevector of the unbound radiation in region 3, rather than with a mode index. This notation is intended to indicate that the radiation modes form a continuum rather than a discrete set. No eigenvalue equation similar to Eq. A-17 can be obtained for the radiation modes because the two equations, A-26 and A-27, cannot be solved for the three unknown amplitude coefficients A_ρ , D_1 , and D_2 . The transverse component of the wavevector in the core, $k_{z;\rho}$, therefore remains arbitrary, which means that the wave equation is satisfied in the waveguide for a continuum of radiation modes that are unbound at both sides of the core. Because of the normalization used for the radiation modes, the dimension of the field amplitude variable $E_{y;\rho}$ is volts/(metre)^{1/2}. The square of its absolute value has the dimensions of intensity per unit of transverse wavevector ρ .

With the time-harmonic factor rendered implicit, the normalized radiation modes of this type have field distributions in the waveguiding structure given to within a constant phase factor by

$$\mathcal{E}_{y;\rho} = E_{y;\rho} e^{-jk_{x;\rho}x} \quad (\text{A-28})$$

where

$$E_{y;\rho} = A_{\rho} \left\{ B_{\rho} e^{j\sigma_{\rho}z} + B_{\rho}^* e^{-j\sigma_{\rho}z} \right\}, \quad d < z \quad (\text{A-29a})$$

$$E_{y;\rho} = A_{\rho} \sin(k_{z;\rho}d + \phi_{\rho}), \quad -d < z < d \quad (\text{A-29b})$$

$$E_{y;\rho} = A_{\rho} \left\{ D_{\rho} e^{j\rho z} + D_{\rho}^* e^{-j\rho z} \right\}, \quad z < -d \quad (\text{A-29c})$$

and where

$$B_{\rho} = \frac{1}{2} \left[\sin(k_{z;\rho}d + \phi_{\rho}) - j \frac{k_{z;\rho}}{\sigma_{\rho}} \cos(k_{z;\rho}d + \phi_{\rho}) \right] e^{-j\sigma_{\rho}d} \quad (\text{A-30})$$

$$D_{\rho} = \frac{1}{2} \left[\sin(-k_{z;\rho}d + \phi_{\rho}) - j \frac{k_{z;\rho}}{\rho} \cos(-k_{z;\rho}d + \phi_{\rho}) \right] e^{j\rho d} \quad (\text{A-31})$$

The normalization factor A_{ρ} is obtained by imposing the orthonormality condition

$$\int_{-\infty}^{\infty} E_{y;\rho}(z) E_{y;\rho'}^*(z) dz = \delta(\rho - \rho') \quad (\text{A-32})$$

Marcuse⁽⁵³⁾ has given a normalization of radiation modes that are unbound at both interfaces that results in even and odd field distributions as the refractive index distribution forming the waveguide approaches symmetry. By transforming the co-ordinate system used in (53) to the symmetric system used here, and normalizing the modes as in Eq. A-32 rather than to unit modal power as in (53), we obtain:

$$A_p = \sqrt{\frac{2}{\pi} (1+F_i)^2 \left\{ \frac{\rho}{\sigma_p} \left[\cos(2k_{z;p}d) - F_i \sin(2k_{z;p}d) \right]^2 + \frac{k_{z;p}^2}{\sigma_p^2} \left[\sin(2k_{z;p}d) + F_i \cos(2k_{z;p}d) \right]^2 + 1 + \frac{k_{z;p}^2}{\sigma_p^2} F_i^2 \right\}^{-1}} \quad (A-33)$$

where

$$F_i = \frac{1}{(k_{z;p}^2 - \rho^2) \sin(4k_{z;p}d)} \left\{ (k_{z;p}^2 - \rho^2) \cos(4k_{z;p}d) + \frac{\rho}{\sigma_p} (k_{z;p}^2 - \sigma_p^2) \right. \\ \left. \pm \sqrt{(k_{z;p}^2 - \rho^2) + \frac{2\rho}{\sigma_p} (k_{z;p}^2 - \rho^2) (k_{z;p}^2 - \sigma_p^2) \cos(4k_{z;p}d) + \frac{\rho^2}{\sigma_p^2} (k_{z;p}^2 - \sigma_p^2)^2} \right\} \quad (A-34)$$

The + sign in Eq. A-33 is associated with odd modes in symmetric structures, the - sign with even modes. The index i equals 1

or 2 for odd and even modes respectively, in the symmetrical structure. The co-ordinate transformation also yields the relation

$$F_i = \cot(k_{z;\rho} d + \phi_{\rho;i}) \quad (A-35)$$

where $\phi_{\rho;i}$ is the phase of the field distribution of the radiation mode, corresponding to the transverse propagation constant ρ , that degenerates to an even or an odd mode according to the value of i as discussed above.

The second type of radiation mode is bound only at one interface between core and cladding which we choose to be the interface with region 1. Such modes can be treated by methods similar to those used for radiation modes unbound at both interfaces, to produce field distributions in the form

$$E_{y;\gamma} = E_{y;\gamma} e^{-jk_{x;\gamma} x} \quad (A-36)$$

where

$$E_{y;\gamma} = A_{\gamma} \sin(k_{z;\gamma} d + \phi_{\gamma}) e^{-K_{1;\gamma} z}, \quad z > d \quad (A-37a)$$

$$E_{y;\gamma} = A_{\gamma} \sin(k_{z;\gamma} z + \phi_{\gamma}), \quad -d < z < d \quad (A-37b)$$

$$E_{y;\gamma} = A_{\gamma} C_{\gamma} \sin(\gamma z + \theta_{\gamma}) \quad z < -d \quad (A-37c)$$

and

$$\theta_{\gamma} = \arctan \left\{ \frac{\gamma}{k_{z;\gamma}} \tan(k_{z;\gamma} d + \phi_{\gamma}) \right\} \quad (A-38)$$

$$C_{\gamma} = \frac{\sin(k_{z;\gamma} d + \phi_{\gamma})}{\sin(\gamma d + \theta_{\gamma})} \quad (A-39)$$

By making use as before of the results of reference (80), the normalization can be carried out to yield:

$$A_Y = \sqrt{\frac{\gamma^2 \left[k_{z,Y} \cos(2k_{z,Y} d) + \frac{K_{1,Y}}{j} \sin(2k_{z,Y} d) \right]^2}{\gamma^2 \left[k_{z,Y} \cos(2k_{z,Y} d) + \frac{K_{1,Y}}{j} \sin(2k_{z,Y} d) \right]^2 + k_{z,Y}^2 \left[k_{z,Y} \sin(2k_{z,Y} d) - \frac{K_{1,Y}}{j} \cos(2k_{z,Y} d) \right]^2}} \quad (A-40)$$

The normalized field distributions of the bound modes and the two types of radiation modes of a slab waveguide have been determined in this Appendix. These field distributions play a part in determining the modal gain, loss and coupling coefficients of the waveguiding structure. To obtain each of these parameters requires performing an inner product integral of two modal field distributions. This inner product is a weighted overlap integral that describes the interactions of the modal fields with each other and with the waveguide.

APPENDIX B

COUPLED-MODE EQUATIONS FOR HYBRID OSCILLATIONS

The coupled mode equations for hybrid oscillations, in which modes of different order are coupled, are developed in this Appendix. We consider the waveguiding structure defined by the dielectric constant distribution $\epsilon(x,z)$, where

$$\epsilon(x,z) = \epsilon_0(z) + (\Delta\epsilon - j\epsilon''')S(x,z) + j\epsilon''b(z) \quad (B-1)$$

and the expressions on the right-hand side of Eq. B-1 are defined in Section 4.2.1. The electric field distribution $\mathcal{E}_{y;m}(x,z)$ for the m th propagation mode of the passive waveguiding structure, in the absence of any corrugation, has the form:

$$\mathcal{E}_{y;m}(x,z) = E_{y;m}(z) \left\{ e^{j(\omega t - k_{x;m}x)} + e^{-j(\omega t - k_{x;m}x)} \right\} \quad (B-2)$$

where $E_{y;m}(z)$ is the modal transverse field distribution.
 $k_{x;m}$ is the propagation constant of the mode.

Because of the convention that a positive mode-order is associated with a wave travelling in the direction of the positive x -axis, and a negative mode-order with a wave travelling in the reverse direction, so that $k_{x;m} = -k_{x,-m}$, the normalized modal transverse field distributions obey the orthogonality relations

$$\int_{-\infty}^{\infty} E_{y;m}(z) E_{y;n}(z) dz = \begin{cases} 1 & m=n \\ 0 & \text{otherwise} \end{cases} \quad (\text{B-3})$$

and also have the property

$$E_{y;m}(z) = E_{y;-m}(z) \quad (\text{B-4})$$

We may consider the structure defined by the dielectric constant distribution $\epsilon(x,z)$ to be a perturbed case of the passive, corrugationless waveguiding structure that is defined by $\epsilon_0(z)$, and accordingly express the electric field distribution $\mathcal{E}_y(x,z)$ in the perturbed waveguide by an expansion in terms of the modal field distributions of the unperturbed waveguide in the form

$$\mathcal{E}_y(x,z) = \sum_p E_{y;p}(z) \left\{ C_p(x) e^{j(\omega t - k_x x; p^x)} + C_p^*(x) e^{-j(\omega t - k_x x; p^x)} \right\} \quad (\text{B-5})$$

where $C_p(x)$ is a complex mode-amplitude coefficient and, for simplicity, the modes of the continuum are not included, so that the summation is over the finite number of bound modes. To ignore the radiation modes is a valid procedure for lasers that operate by feedback on the first order of the corrugation. When feedback is on a higher order, there is coupling to the radiation modes by the lower orders of the corrugation. This coupling causes a loss which can be included in the coupled-mode equations by techniques that are discussed in Appendix D.

The wave equation and appropriate boundary conditions must be satisfied in the perturbed waveguide by the field distributions $\mathcal{E}_y(x, z)$ that occur in the perturbed structure, so that we can write:

$$\frac{\partial^2 \mathcal{E}_y(x, z)}{\partial z^2} + \frac{\partial^2 \mathcal{E}_y(x, z)}{\partial x^2} = \epsilon(x, z) \frac{\partial^2 \mathcal{E}_y(x, z)}{\partial t^2} \quad (\text{B-6})$$

By using the expression for $\mathcal{E}_y(x, z)$ given in Eq. B-5 and making use of the fact that the modal field distributions $\mathcal{E}_{y;p}(x, z)$ are solutions of the wave equation in the unperturbed structure, we obtain from Eq. B-6 the result:

$$\begin{aligned} & \sum_p E_{y;m}(z) \left\{ \frac{\partial^2 C_p(x)}{\partial x^2} - 2jk_{x;p} \frac{\partial C_p(x)}{\partial x} \right\} e^{j(\omega t - k_{x;p}x)} \\ & + \text{complex conjugate} \\ & = -\mu_0 \omega^2 \left\{ (\Delta\epsilon - j\epsilon'') S(x, z) + j\epsilon'' L(z) \right\} \left[\sum_p E_{y;p}(z) C_p(x) e^{j(\omega t - k_{x;p}x)} \right. \\ & \quad \left. + \text{complex conjugate} \right] \quad (\text{B-7}) \end{aligned}$$

The complex mode-amplitude coefficients $C_p(x)$ can be determined from Eq. B-7 by using the orthogonality relationships for the modal transverse field distributions given in Eq. B-3. Multiplying Eq. B-7 by $E_{y;m}(z) dz$ and integrating, we obtain from the first terms in the left- and right-hand sides, the equation:

$$\begin{aligned}
& \left(\frac{\partial^2 C_m(x)}{\partial x^2} - 2jk_{x;p} \frac{\partial C_m(x)}{\partial x} \right) e^{j(\omega t - k_{x;m}x)} + \left(\frac{\partial^2 C_{-m}(x)}{\partial x^2} - 2jk_{x;-m} \frac{\partial C_{-m}(x)}{\partial x} \right) e^{j(\omega t - k_{x;-m}x)} \\
& = -\mu_0 \omega^2 \sum_p \int_{-\infty}^{\infty} \left\{ (\Delta\epsilon - j\epsilon'') S(x,z) + j\epsilon'' L(z) \right\} E_{y;p}(z) C_p(x) e^{j(\omega t - k_{x;p}x)} E_{y;m}(z) dz
\end{aligned}
\tag{B-8}$$

The second terms on the left- and right-hand sides of Eq. B-7, i.e. the complex conjugates of the first terms, yield an equation similar to Eq. B-8 that gives an identical set of coupled-mode equations. By rendering the time dependence $e^{j\omega t}$ implicit in Eq. B-8 and by using the approximation that the mode-amplitudes change slowly along the propagation axis of the waveguide so that

$$\frac{\partial^2 C_p(x)}{\partial x^2} \ll \frac{\partial C_p(x)}{\partial x} k_{x;p}$$

we obtain

$$\begin{aligned}
& 2jk_{x;m} \left(\frac{\partial C_m(x)}{\partial x} - \frac{\partial C_{-m}(x)}{\partial x} e^{2jk_{x;m}x} \right) \\
& = +\mu_0 \omega^2 \sum_p C_p(x) e^{j(k_{x;m} - k_{x;p})x} \int_{-\infty}^{\infty} E_{y;p}(z) \left\{ (\Delta\epsilon - j\epsilon'') S(x,z) + j\epsilon'' L(z) \right\} E_{y;m}(z) dz
\end{aligned}
\tag{B-9}$$

To obtain an expression for the complex amplitude coefficients $C_m(x)$, the differential equation given by Eq. B-9 must be integrated with respect to x . The term on the left-hand side containing the factor $e^{2jk_{x;m}x}$ is an oscillating function of x , so that it does not contribute significantly to the integral on the left-hand side, and it is therefore neglected. For conciseness of notation, it is advantageous to define a modal gain coefficient and a coupling coefficient that arise from the integrations with respect to z on the right-hand side of Eq. B-9. The modal gain coefficient α_m is:

$$\alpha_m = \frac{\epsilon'' \mu_0 \omega^2}{2 |k_{x;m}|} \int_{-\infty}^{\infty} E_{y;m}(z) L(z) E_{y;m}(z) dz \quad (B-10)$$

This gain coefficient is exactly half the intensity-gain coefficient $\alpha_{G;m}$ of the externally-pumped evanescent-wave amplifier, as can be seen by comparing Eq. 2-11 with Eq. B-10. It is proportional to the perturbation of the eigenvalue $k_{z;m}$ of the time-independent wave equation caused by the presence of a cladding region with gain.

The coupling constant also is proportional to a perturbation in $k_{z;m}$, in this case by the presence of some component of the corrugation $S(x,z)$ in the waveguide wall. In order to apply perturbation theory, it was assumed that the periodic perturbation that causes contradirectional

mode-coupling is weak. When the coupling perturbation is periodic and weak, mode-coupling only occurs when pseudomomentum⁽³¹⁾ is conserved. Pseudomomentum is conserved in the axial direction when the axial components of the wavevectors corresponding to the bound modes, and the wavevector corresponding to the order of the corrugation that provides the coupling, all sum to zero. Coupling thus occurs only between pairs of bound modes or between single bound modes and narrow regions of the radiation-mode continuum. As we have neglected the modes of the continuum in the derivation, the latter coupling will be ignored here. It is treated in Appendix D. Since coupling only occurs between pairs of modes, the summation in Eq. B-9 can be dropped, and we have:

$$\frac{\partial C_m(x)}{\partial x} = \frac{\mu_0 \omega^2}{2jk_{x;m}} C_n(x) e^{j(k_{x;m} - k_{x;n})x} \int_{-\infty}^{\infty} E_{y;n}(z) (\Delta\epsilon - j\epsilon'') S(x,z) E_{y;m}(z) dx + \alpha_m C_m(x) \quad (B-11)$$

The coupling coefficient arises from the integral in Eq. B-11. We consider coupling by the r th component of the corrugation. An expression of the r th component of the function $S(x,z)$ that represents the corrugation is given by Eq. 4-5. It is:

$$G_r(z) \left\{ e^{j[r\Lambda x - \theta_r(z)]} + e^{-j[r\Lambda x - \theta_r(z)]} \right\} \quad (B-12)$$

where $G_r(z)$ and $\theta_r(z)$ are real functions.

Since the expression of Eq. B-12 for the r th component of the corrugation contains two terms that are conjugate to each other, we need only define a single complex coupling coefficient. This will be given by $K_{mn}(\Delta\epsilon - j\epsilon'')$ where

$$K_{mn} = \frac{\mu_0 \omega^2}{2\sqrt{|k_{x;m} k_{x;n}|}} \int_{-\infty}^{\infty} E_{y;m}(z) G_r(z) e^{j\theta_r(z)} E_{y;n}(z) dz \quad (B-13)$$

Using Eqs. B-12 and B-13, we can write Eq. B-11 in the form:

$$\begin{aligned} \sqrt{|k_{x;m}|} \frac{\partial C_m(x)}{\partial x} = & C_n(x) \sqrt{|k_{x;n}|} \left[K_{mn} e^{j(k_{x;m} - k_{x;n} - r)x} \right. \\ & \left. + K_{mn}^* e^{j(k_{x;m} - k_{x;n} + r)x} \right] \frac{(\Delta\epsilon - j\epsilon'')}{j} \\ & + \alpha_m C_m(x) \sqrt{|k_{x;m}|} \end{aligned} \quad (B-14)$$

Since coupling occurs between modes travelling in opposite directions, one of the two mode-orders m and n in Eq. B-14 must be a negative number representing the mode travelling in the direction of the negative x -axis. We choose mode order m to be positive and mode order n to be negative. The magnitude r of the component of the corrugation that provides mode-coupling is intrinsically a positive number. The term containing K_{mn}^* in Eq. B-14 is therefore an oscillatory function of x ,

and does not contribute significantly to the integration of the equation. The term containing K_{mn} on the other hand oscillates only very slowly because the argument of the exponential is very small when pseudomomentum is conserved. We therefore ignore the term with K_{mn}^* and obtain the first of the two coupled-mode equations:

$$\sqrt{|k_{x;m}|} \frac{\partial C_m(x)}{\partial x} = -j K_{mn} (\Delta\epsilon - j\epsilon'') e^{j(k_{x;m} - k_{x;n} - r\Lambda)x} \sqrt{|k_{x;n}|} C_n(x) + \alpha_m \sqrt{|k_{x;m}|} C_m(x) \quad (B-15)$$

To obtain the coupled-mode equation describing the growth of the complex amplitude coefficient of the mode that propagates in the negative x-direction, we multiply both sides of Eq. B-9 by $e^{-2jk_{x;m}x}$, drop the term on the left that is an oscillating function of x, and set $m=-n$ and $n=-m$ to obtain:

$$2jk_{x;n} \frac{\partial C_n(x)}{\partial x} = \mu_0 \omega^2 \sum_p C_p(x) e^{j(k_{x;n} - k_{x;p})x} \int_{-\infty}^{\infty} E_{y;p}(z) \{ (\Delta\epsilon - j\epsilon'') S(x,z) + j\epsilon'' L(z) \} E_{y;n}(z) dz \quad (B-16)$$

Making use of the gain and coupling coefficients defined in Eqs. B-10 and B-13 in Eq. B-16 gives:

$$\begin{aligned}
\sqrt{|k_{x;n}|} \frac{\partial C_n(x)}{\partial x} = & - C_m \sqrt{|k_{x;m}|} \left[K_{mn} e^{j(k_{x;n} - k_{x;m} - r\Lambda)x} \right. \\
& \left. + K_{mn}^* e^{j(k_{x;n} - k_{x;m} + r\Lambda)x} \right] \frac{(\Delta\epsilon - j\epsilon'')}{j} \\
& - \alpha_n C_n(x) \sqrt{|k_{x;n}|}
\end{aligned} \tag{B-17}$$

The negative sign in front of the gain coefficient, in Eq. B-17 arises because the gain coefficient is defined in such a way that it is independent of the propagation direction. The term containing K_{mn} in Eq. B-17 is an oscillating function of x and can therefore be dropped. We are left with the second of the coupled mode equations:

$$\begin{aligned}
\sqrt{|k_{x;n}|} \frac{\partial C_n(x)}{\partial x} = & j K_{mn}^* (\Delta\epsilon - j\epsilon'') e^{j(k_{x;n} - k_{x;m} + r\Lambda)x} \sqrt{|k_{x;m}|} C_m(x) \\
& - \alpha_n C_n(x) \sqrt{|k_{x;n}|}
\end{aligned} \tag{B-18}$$

We define the detuning parameter Δ_{mn} to be

$$\Delta_{mn} = k_{x;m} - k_{x;n} - r\Lambda \tag{B-19}$$

The detuning parameter represents the degree to which pseudo-momentum is not conserved in the axial direction. Periodic

perturbations of waveguiding structures may have the effect of preventing oscillations in which pseudomomentum is exactly conserved because there can exist a stop-band at the Bragg condition. See for example (34). We have finally the two coupled-mode equations:

$$\sqrt{|k_{x;m}|} \frac{\partial C_m(x)}{\partial x} = -j K_{mn} (\Delta\epsilon - j\epsilon'') e^{j\Delta_{mn}x} \sqrt{|k_{x;n}|} C_n(x) + \alpha_m \sqrt{|k_{x;m}|} C_m(x)$$

$$\sqrt{|k_{x;n}|} \frac{\partial C_n(x)}{\partial x} = j K_{mn}^* (\Delta\epsilon - j\epsilon'') e^{-j\Delta_{mn}x} \sqrt{|k_{x;m}|} C_m(x) - \alpha_n \sqrt{|k_{x;n}|} C_n(x)$$

(B-20)

APPENDIX C

SOLUTION OF THE COUPLED-MODE EQUATIONS

The coupled-mode equations, Eqs. 4-2, can be solved with the appropriate boundary conditions to obtain the complex mode-amplitude distributions $C_m(x)$ and $C_n(x)$ for the hybrid oscillations of the corrugated waveguide laser. These are the steady-state axial field-distributions of the two propagation modes that are established when oscillation occurs on the hybrid mode that includes them both. Such a solution does not take into account any non-linear effects such as gain saturation, that in reality would limit the growth of the oscillation amplitude. The field distributions that result, therefore, are determined relative to an unspecified absolute field-amplitude. They describe the threshold condition, in which gain and feedback are just sufficient to initiate oscillation. The threshold solutions of the coupled-mode equations yield eigenvalues that specify the minimum gain that is required to establish oscillation on each hybrid mode. These solutions are therefore useful in determining which modes would and which would not be expected to oscillate in a particular laser.

The simultaneous solution of the coupled-mode equations, Eqs. 4-2, yields a dual-eigenvalue equation. The two eigenvalues of this equation are the hybrid-mode gain-coefficient

α'_{mn} and the detuning parameter Δ_{mn} for the hybrid, at threshold. The equation can be solved for these two eigenvalues by a numerical method. The complex field-amplitude distributions can then be determined for each pair of eigenvalues. In this Appendix, we obtain the eigenvalue equation appropriate to the structure of the PCL and indicate the numerical method used to solve it for the eigenvalues. Since the PCL was pumped only in a central region of the waveguide structure, the solution given here assumes a structure in which gain is available only in a central region of the corrugated length. The gain in this region is assumed to be uniform. The boundaries between the gain region and the passive end regions occur at the planes $x=\pm l/2$. Losses that may occur in the passive end regions are considered in the solution.

To begin, we consider the fields in the central gain region and the passive end regions separately, and introduce the notation $D_m(x)$ and $D_n(x)$ for the complex field-amplitude coefficients of the modes in the end regions, to distinguish them from $C_m(x)$ and $C_n(x)$, the corresponding coefficients in the gain region. The coefficients $C_m(x)$ and $C_n(x)$ in the gain region satisfy the coupled-mode equations in the form:

$$\sqrt{|k_{x;m}|} \frac{\partial C_m(x)}{\partial x} = -j K_{mn} (\Delta\epsilon - j\epsilon'') e^{j\Delta_{mn}x} \sqrt{|k_{x;n}|} C_n(x) + \alpha'_m \sqrt{|k_{x;m}|} C_m(x)$$

$$\sqrt{|k_{x;n}|} \frac{\partial C_n(x)}{\partial x} = j K_{mn}^* (\Delta\epsilon - j\epsilon'') e^{-j\Delta_{mn}x} \sqrt{|k_{x;m}|} C_m(x) - \alpha_n' \sqrt{|k_{x;n}|} C_n(x) \quad (C-1)$$

where α_m' and α_n' are the net modal gain-coefficients, in which the modal loss-coefficients have been subtracted from the modal gain coefficients. The coefficients $D_m(x)$ and $D_n(x)$ in the end regions satisfy the coupled-mode equations in the form:

$$\begin{aligned} \sqrt{|k_{x;m}|} \frac{\partial D_m(x)}{\partial x} &= -j K_{mn} (\Delta\epsilon) e^{j\Delta_{mn}x} \sqrt{|k_{x;n}|} D_n(x) - \alpha_m^L \sqrt{|k_{x;m}|} D_m(x) \\ \sqrt{|k_{x;n}|} \frac{\partial D_n(x)}{\partial x} &= j K_{mn}^* (\Delta\epsilon) e^{-j\Delta_{mn}x} \sqrt{|k_{x;m}|} D_m(x) + \alpha_n^L \sqrt{|k_{x;n}|} D_n(x) \end{aligned} \quad (C-2)$$

where α_m^L and α_n^L are the magnitudes of the modal loss coefficients.

The general solution of Eqs. C-1 can be obtained by differentiating the first and making substitutions from the second to form a single second-degree linear differential equation in the single unknown $C_m(x)$:

$$\begin{aligned} \frac{\partial^2 C_m(x)}{\partial x^2} - \frac{\partial C_m(x)}{\partial x} \left[\alpha_m' - \alpha_n' + j\Delta_{mn} \right] - C_m(x) \left[|K_{mn}|^2 (\Delta\epsilon - j\epsilon'')^2 + \alpha_m' \alpha_n' - j\Delta_{mn} \alpha_m' \right] &= 0 \end{aligned} \quad (C-3)$$

This equation has the general solution

$$C_m(x) = A_1 e^{(a+b)x} + A_2 e^{(a-b)x} \quad (C-4)$$

where

$$a = (\alpha_m' - \alpha_n' + j\Delta_{mn})/2 \quad (C-5)$$

$$b = \sqrt{r^2 + |K_{mn}|^2 (\Delta\epsilon - j\epsilon'')^2} \quad (C-6)$$

and

$$r = -(\alpha_m' + \alpha_n' - j\Delta_{mn})/2. \quad (C-7)$$

By a similar procedure, we also obtain a general solution for $C_n(x)$:

$$C_n(x) = B_1 e^{(a^*+b)x} + B_2 e^{(a^*-b)x} \quad (C-8)$$

where the asterisk indicates the complex conjugate.

The same method, when applied to Eqs. C-2, gives general solutions for the complex field amplitude distributions in the end regions. These are:

$$D_m(x) = f_1 e^{(c+d)x} + f_2 e^{(c-d)x} \quad (C-9)$$

and

$$D_n(x) = g_1 e^{(c^*+d)x} + g_2 e^{(c^*-d)x} \quad (C-10)$$

where

$$c = (\alpha_n^L - \alpha_m^L + j\Delta_{mn})/2 \quad (C-11)$$

$$d = \sqrt{s^2 + |K_{mn}|^2 (\Delta\epsilon - j\epsilon'')^2} \quad (C-12)$$

and

$$s = (\alpha_m^L + \alpha_n^L + j\Delta_{mn})/2 \quad (C-13)$$

The coefficients A_1 , A_2 , B_1 , B_2 , f_1 , f_2 , g_1 , and g_2 are all constants determined by the boundary conditions. The boundary conditions are established by the fact that no radiation at the signal frequency is incident on the device. The amplitude of each mode must therefore be zero at the end of the device away from which it propagates. By convention, mode m travels in the positive x -direction, and mode n in the opposite direction. The boundary conditions are thus:

$$D_m(-\frac{L}{2}) = 0 = D_n(\frac{L}{2}) \quad (C-14)$$

We employ these boundary conditions directly to eliminate the unknown constant f_2 at one end of the device, and the corresponding constant g_2 at the other. It is convenient to relate all the remaining unknown constants in the solution to f_1 , in the case of mode m and g_1 in the case of mode n . Accordingly, we write the solutions of the coupled-mode equations in the three regions of the device as follows:

$$\begin{aligned} D_m(x) &= f_1 \left[e^{(c+d)x} - e^{(c-d)x-dL} \right] \\ D_n(x) &= g_1 \left[G_1 e^{(c^*+d)x} + G_2 e^{(c^*-d)x} \right] \end{aligned} \quad -\frac{L}{2} < x < \frac{L}{2}$$

$$\begin{aligned}
 C_m(x) &= f_1 \left[A_1 e^{(a+b)x} + A_2 e^{(a-b)x} \right] & \frac{l}{2} < x < \frac{l}{2} \\
 C_n(x) &= g_1 \left[B_1 e^{(a^*+b)x} + B_2 e^{(a^*-b)x} \right] \\
 D_m(x) &= f_1 \left[F_1 e^{(c+d)x} + F_2 e^{(c-d)x} \right] & \frac{l}{2} < x < \frac{l}{2} \\
 D_n(x) &= g_1 \left[e^{(c^*+d)x} - e^{(c^*-d)x+dL} \right]
 \end{aligned}
 \tag{C-15}$$

By substituting the expressions given by Eqs. C-15 for the field distributions into one of the equations of the appropriate coupled pair, Eqs. C-1 or C-2, and then collecting terms in like exponents e^{-dx} and e^{dx} or e^{-bx} and e^{bx} , as appropriate, two independent equations that relate the remaining undetermined constants to each other can be obtained for each of the three longitudinal regions of the device. By using the first equation of each of the coupled pair, we obtain in this way the relations:

$$\begin{aligned}
 \frac{\sqrt{k_{x;m}} f_1}{\sqrt{k_{x;n}} g_1} (s+d) &= -j K_{mn} \Delta \epsilon G_1 \\
 \frac{\sqrt{k_{x;m}} f_1}{\sqrt{k_{x;n}} g_1} (s-d) &= +j K_{mn} \Delta \epsilon G_2 e^{dL}
 \end{aligned}
 \tag{C-16}$$

$$\frac{\sqrt{|k_{x;m}|} f_1}{\sqrt{|k_{x;n}|} g_1} A_1(r+b) = -j K_{mn} (\Delta\epsilon - j\epsilon'') B_1 \quad (C-17)$$

$$\frac{\sqrt{|k_{x;m}|} f_1}{\sqrt{|k_{x;n}|} g_1} A_2(r-b) = -j K_{mn} (\Delta\epsilon - j\epsilon'') B_2$$

$$\frac{\sqrt{|k_{x;m}|} f_1}{\sqrt{|k_{x;n}|} g_1} F_1(s+d) = -j K_{mn} \Delta\epsilon \quad (C-18)$$

$$\frac{\sqrt{|k_{x;m}|} f_1}{\sqrt{|k_{x;n}|} g_1} F_2(s-d) = +j K_{mn} \Delta\epsilon e^{dL}$$

The ratio f_1/g_1 can be obtained from the symmetry of the device. The phase of the corrugation coupling coefficient is arbitrary in the sense that it only relates the phase of the component of the corrugation responsible for the feedback to the phase of the fundamental. We can therefore choose the phase of the coupling coefficient to be zero, since the relative phase of the fundamental of the corrugation is of no consequence to the oscillation thresholds. The device is thus assumed to be symmetrical for the purposes of the analysis and the field distributions C_m and C_n will have either odd or even symmetry when both modes of the hybrid are of the same order, i.e. $m=n$. The lowest-threshold solutions, corresponding to the lowest-order longitudinal modes, have odd symmetry in the

formulation used. (The phase of the solution goes through $-\pi$ at $x=0$.) By setting $C_m(-x) = -C_m(x)$, we have from Eqs. C-16 and C-18 an expression for f_1/g_1 :

$$\frac{f_1}{g_1} = -e^{dL} \quad (C-19)$$

Expressions that give the constants F_1, F_2 and G_1, G_2 in terms of the device properties and the constant f_1/g_1 can be obtained directly from Eqs. C-16 and C-18.

To obtain similar expressions for the constants A_1, A_2 and B_1, B_2 we note that the modal field-amplitudes in the end regions and the central region must be equal at the planes $x = \pm l/2$ where the regions meet. The following conditions must therefore hold:

$$\begin{aligned} D_m(\pm l/2) &= e_m(\pm l/2) \\ D_n(\pm l/2) &= C_n(\pm l/2) \end{aligned} \quad (C-20)$$

By substituting the expression given by Eq. C-15 for the field-amplitude coefficients into the four equations set by the requirement for field continuity across the interface between the active and passive regions, two sets of simultaneous, linear equations are formed. These can be solved to obtain the coefficients A_1, A_2 and B_1, B_2 . The resulting expressions are:

$$A_1 = \frac{1}{(e^{-bL} - e^{bL})} \left[e^{(a-b-c-d)L/2} - e^{(a-b-c+d)L/2-dL} \right. \\ \left. - F_1 e^{(-a+b+c+d)L/2} - F_2 e^{(-a+b+c-d)L/2} \right]$$

$$A_2 = \frac{1}{(e^{bL} - e^{-bL})} \left[e^{(a+b-c-d)L/2} - e^{(a+b-c+d)L/2-dL} \right. \\ \left. - F_1 e^{(-a-b+c+d)L/2} - F_2 e^{(-a-b+c-d)L/2} \right]$$

$$B_1 = \frac{1}{(e^{-bL} - e^{bL})} \left[G_1 e^{(a^*-b-c^*-d)L/2} + G_2 e^{(a^*-b-c^*+d)L/2} \right. \\ \left. - e^{(-a^*+b+c^*+d)L/2} + e^{(-a^*+b+c^*-d)L/2+dL} \right]$$

$$B_2 = \frac{1}{(e^{bL} - e^{-bL})} \left[G_1 e^{(a^*+b-c^*-d)L/2} + G_2 e^{(a^*+b-c^*d)L/2} \right. \\ \left. - e^{(-a^*-b+c^*+d)L/2} + e^{(-a^*-b+c^*-d)L/2+dL} \right] \quad (C-21)$$

In addition to the properties of the device structure and the constant f_1/g_1 , the expressions given in Eqs. C-21 also depend on the modal gain coefficients and the hybrid detuning parameter through the expressions a and b , which are given by Eqs. C-5 and C-6. The two equations C-17 thus form the basis from which the dual eigenvalue equation for gain and detuning parameter can be obtained. By dividing the first of

Eqs. C-17 by the second, we construct the eigenvalue equation in the form

$$\frac{r+b}{r-b} = \frac{B_1 A_2}{B_2 A_1} \quad (C-22)$$

The eigenvalues of this equation are the hybrid-mode gain coefficient and the detuning parameter, which are the real and imaginary parts of r as given by Eq. C-7.

To show the numerical method used to solve Eq. C-22, we first rewrite the equation in a simplified notation in which left- and right-hand sides are expressed simply as functions of the eigenvalues

$$E(\alpha_{03}, \Delta) = F(\alpha_{03}, \Delta) \quad (C-23)$$

where the relations

$$\begin{aligned} \alpha'_m &= \alpha_{03} \alpha_{0;m} - \alpha_m^L \\ \alpha'_n &= \alpha_{03} \alpha_{0;n} - \alpha_n^L \end{aligned} \quad (C-24)$$

have been used to express the modal net gain coefficients, α'_m and α'_n terms of the bulk gain coefficient α_{03} of the active region and two coefficients $\alpha_{0;m}$ and $\alpha_{0;n}$ that represent the efficiency with which each mode obtains gain from evanescent-wave interactions. The coefficients $\alpha_{0;m}$ and $\alpha_{0;n}$ are deter-

mined by the parameters of the passive waveguiding structure and by the frequency of the hybrid oscillation, which is set by the conservation of pseudomomentum condition (except for the detuning represented by Δ_{mn} , which is small). They are, in fact, the gain coefficients of the two modes of the hybrid when the bulk gain coefficient of the active region has unit value. The two eigenvalues to be determined numerically are thus α_{03} , the bulk gain coefficient of the active region, and Δ_{mn} , the detuning parameter.

We expand the left- and right-hand sides of Eq. C-22 in Taylor's series about the two assumed eigenvalues α_0 and Δ_0 to obtain

$$\begin{aligned} E(\alpha_0, \Delta_0) + \frac{\partial E(\alpha_0, \Delta_0)}{\partial \alpha_0} (\alpha_{03} - \alpha_0) + \frac{\partial E(\alpha_0, \Delta_0)}{\partial \Delta_0} (\Delta - \Delta_0) \\ = F(\alpha_0, \Delta_0) + \frac{\partial F(\alpha_0, \Delta_0)}{\partial \alpha_0} (\alpha_{03} - \alpha_0) + \frac{\partial F(\alpha_0, \Delta_0)}{\partial \Delta_0} (\Delta - \Delta_0) \end{aligned} \quad (C-25)$$

The real and imaginary parts of Eq. C-25 form two simultaneous equations that can be solved to obtain the corrections $(\alpha_{03} - \alpha_0)$ and $(\Delta - \Delta_0)$ to the assumed eigenvalues. The procedure is then repeated with the corrected eigenvalues until the left- and right-hand sides are equal to within the required accuracy.

APPENDIX D

THE EFFECTIVE CORRUGATION

Two coupling coefficients were defined in the development that led to calculations of the thresholds of the corrugated waveguide laser. One of these represented contra-directional coupling between two bound modes caused by the r th order Fourier component of the corrugation. The other represented coupling of bound modes to radiation modes by Fourier components of the corrugation of order less than r , denoted s . The two coupling coefficients contain factors defined by Eqs. 4-4 and 4-14 that contain integrals representing the inner product of the field distributions of the coupled modes as weighted by a factor of the form $G_r(z)e^{-j\beta_r(z)}$. This factor represents the strength of the corrugation component causing the coupling, and has been called the effective perturbation caused by the corrugation. In this Appendix, the expressions given in Eqs. 4-9 and 4-15 for the effective perturbation caused by the different orders of a sinusoidal corrugation and of the blazed grating corrugation are developed from the Fourier expansion given in Eq. 4-8.

When modes are coupled, the pseudomomentum that is represented by the axial and transverse modal wavevectors and the axial and transverse components of the corrugation wave-vector must be nearly conserved. The corrugation is represented by a two-

dimensional function $S(x,z)$ that takes on the values $\pm 1, 0$ over the x,z plane. It can be resolved into spatial Fourier components in the x and z directions. To determine the effective corrugation for the r th order component, the inverse Fourier transform in two dimensions is performed on the r th axial component of the function $S(x,z)$. This procedure leads to Eq. 4-8.

We can write Eq. 4-8 in the form:

$$G_r(z') \cos[rAx' - \phi_r(z')] = \frac{\Lambda}{\pi} \left\{ \left[\int_0^{2\pi/\Lambda} g(x,z') \cos(rAx) dx \right] \cos rAx' + \left[\int_0^{2\pi/\Lambda} g(x,z') \sin(rAx) dx \right] \sin rAx' \right\} \quad (D-1)$$

where

$$g(x,z') = \int_d^{d+R(x)} \delta(z-z') dz \quad (D-2)$$

It can be recognized that Eq. D-1 is a Fourier series representation of a periodic function $g(x,z')$ whose period in the x -direction is the same as that of the corrugation itself. We now examine this function.

From the definition of the functions $S(x,z)$ and $R(x)$ given in Section 4.2.1, the relation follows:

$$\int_{-\infty}^{\infty} S(x,z) dz = \int_d^{d+R(x)} (1) dz \quad (D-3)$$

From the definition of $g(x)$ given by Eq. D-2, it is clear that

$$g(x, z') = \int_{-\infty}^{\infty} \delta(z - z') S(x, z) dz \quad (D-4)$$

Thus, $g(x, z')$ is the function of x defined by evaluating $S(x, z)$ at $z = z'$:

$$g(x, z') = S(x, z') \quad (D-5)$$

The integral over z in Eq. 4-8 has only the effect of producing a function of x that represents a cross-section of the function $S(x, z)$ taken at $z = z'$, as sketched in Fig. D-1.

In Eq. 4-8, the effective corrugation is thus expressed simply as a component of a Fourier series expansion of this function. For the sinusoidal corrugation and the corrugation of the blazed diffraction grating with which we are concerned, the simplest procedure for determining the effective corrugation is to obtain the function $g(x, z')$ by inspection from the function $R(x)$. The r th component of the Fourier series representation of this function is then the effective corrugation, and can be expressed in the form:

$$G_r(z') = \left\{ e^{j[r\Lambda x' - \theta_r(x')]} + e^{-j[r\Lambda x' - \theta_r(x')]} \right\} \\ = M_r(z') \sin(r\Lambda x') + N_r(z') \cos(r\Lambda x') \quad (D-6)$$

FIG. D-1. The function $g(x, z')$ is a cross-section of $S(x, z)$ taken at $z = z'$.

where

$$M_r(z') = \frac{\Lambda}{\pi} \int_0^{2\pi/\Lambda} g(x, z') \sin(r\Lambda x) dx$$

$$N_r(z') = \frac{\Lambda}{\pi} \int_0^{2\pi/\Lambda} g(x, z') \cos(r\Lambda x) dx \quad (D-7)$$

By manipulating both sides of Eq. D-6, we obtain:

$$2G_r(z') \cos[r\Lambda x' - \theta_r(z')] = \left\{ \cos \left[r\Lambda x' - \arctan \left(\frac{M_r(z')}{N_r(z')} \right) \right] \right\} \sqrt{M_r^2(z') + N_r^2(z')} \quad (D-8)$$

There result the relations:

$$G_r(z') = \frac{\sqrt{M_r^2(z') + N_r^2(z')}}{2}$$

$$\theta_r(z') = \arctan \left(\frac{M_r(z')}{N_r(z')} \right) \quad (D-9)$$

The formulas given by Eq. D-9 for $G_r(z')$ and $\theta_r(z')$ can be applied directly to obtain the weighting functions for the sinusoidal and blazed-grating corrugations that are given by Eqs. 4-9 and 4-15. The primes denoting co-ordinates of

retransformed functions are dropped from this point.

For the sinusoidal corrugation, the function $g(x, z)$ is as sketched in Fig. D-2. The Fourier coefficient $M_r(z)$ is given by:

$$M_r(z) = \frac{2}{T} \int_{-\frac{T}{2\pi} \arccos(z/a)}^{\frac{T}{2\pi} \arccos(z/a)} (-1) \sin(r\pi x/T) dx \quad (D-10)$$

where

$$T = \frac{2a}{\Lambda}$$

and a is the amplitude of the corrugation function $R(x)$, in this case a sinusoid. The factor (-1) in the integrand is the value of $g(x, z)$ over the range of integration. By evaluating the integral of Eq. D-10, we obtain:

$$M_r(z) = 0.$$

Similarly, the function $N_r(z)$ is given by:

$$N_r(z) = \frac{2}{T} \int_{-\frac{T}{2\pi} \arccos(z/a)}^{\frac{T}{2\pi} \arccos(z/a)} (-1) \cos(r\pi x/T) dx \quad (D-11)$$

and evaluating the integral gives:

$$N_r(z) = -\frac{2}{r\pi} \sin[r \arccos(z/a)] \quad (D-12)$$

The function on the right-hand side of Eq. D-12 can be expressed as a polynomial:

$$N_r(z) = -\frac{2}{r\pi} U_{n-1}(z/a) \sqrt{1 - (z/a)^2} \quad (D-13)$$

where U_{n-1} is the Chebyshev polynomial of the second kind of order $n-1$.

By substituting Eq. D-13 in the second of Eqs. D-19, we obtain:

$$\theta_r(z) = 0 \quad (D-14)$$

and we have thus developed Eqs. 4-9.

$$G_r(z) = -\frac{1}{r\pi} U_{n-1}(z/a) \sqrt{1 - (z/a)^2} \quad (D-15)$$

$$\theta_r(z) = 0$$

The expressions given in Eqs. 4-16 for the Fourier sine and cosine coefficients corresponding to the corrugation of the blazed grating are found by similar procedures. They are obtained from the function $g(x, z)$ sketched in Fig. D-2b.

The integrals of Eq. D-7 take the forms:

$$M_r(z) = \frac{1}{T} \int_{z+d/Q}^{T - [(z+d)/s]} (-1) \sin(r\pi x/T) dx \quad (D-16)$$

$$N_r(z) = \frac{1}{T} \int_{z+d/Q}^{T - [(z+d)/s]} (-1) \cos(r\pi x/T) dx$$

where S and Q are the slopes of the two faces of the corrugation as shown in Fig. 4-2. By evaluating the integrals of Eq. D-16, we obtain:

$$M_r(z) = \frac{1}{r\pi} \left\{ (-1)^r \cos \left[\frac{r\pi(z+d)}{ST} \right] - \cos \left[\frac{r\pi(z+d)}{QT} \right] \right\} \quad (D-17)$$

$$N_r(z) = \frac{1}{r\pi} \left\{ (-1)^r \sin \left[\frac{r\pi(z+d)}{ST} \right] - \sin \left[\frac{r\pi(z+d)}{QT} \right] \right\}$$

The effective strength of the corrugation components of any order have been found as a function of z for the two corrugation shapes studied. Since these two functions can be made antisymmetric about $z=d$, by a correct choice of the axis $x=0$, the results of the procedure used are independent of where the section is taken through $S(x,z)$ to obtain $g(x,z')$. This is not the case in general. The integrals of Eqs. D-7 will have to be performed separately for positive and negative values of $(z-d)$ for corrugation functions that lack this symmetry.

APPENDIX E

THE RADIATION-LOSS COEFFICIENTS

The modes of the radiation continuum were not included in the expansion of the corrugated-waveguide field-distributions used in Appendix B to obtain the coupled-mode equations. The coupled-mode equations, as given in Appendix B therefore only apply to coupling between bound modes, and do not take into account coupling that may occur between bound modes and modes of the radiation continuum. Feedback in corrugated-waveguide lasers may result from a corrugation component of order higher than the first, in which case the lower-order modes can cause strong coupling between bound modes and mode-groups of the radiation continuum. When such coupling occurs, each bound mode suffers a loss of power to the radiation modes. This loss of power can be represented by a modal loss coefficient $\alpha_{m,s}^L$ where m is the bound-mode order and s is the order of the corrugation component that causes coupling to the radiation continuum. An expression for $\alpha_{m,s}^L$ in terms of the characteristics of the bound mode and the waveguide structure is given by Eq. 4-13. In this Appendix, we derive Eq. 4-13 by considering the power transfer between bound modes and modes of the continuum.

When the radiation modes are included, the expansion of the electric field distributions $\vec{E}_y(x, z)$ in the corrugated-waveguide, evanescent-wave amplifier structure is given by a modified form of Eq. B-5:

$$\begin{aligned} \vec{E}_y(x, z) = & \sum_p E_{y;p}(z) \left\{ C_p(x) e^{j(\omega t - k_x; p x)} + C_p^*(x) e^{-j(\omega t - k_x; p x)} \right\} \\ & + \int_{-\infty}^{\infty} E_{y;p}(z) \left\{ g(p, x) e^{j(\omega t - k_x; p x)} \right. \\ & \left. + g^*(p, x) e^{-j(\omega t - k_x; p x)} \right\} \end{aligned} \quad (E-1)$$

where

$g(p, x)$ is a complex mode-amplitude coefficient for the radiation mode labelled p ,

p is the transverse component of the propagation constant of a particular radiation mode, in region 3 of the waveguiding structure. The use of the transverse propagation constant to label the radiation modes is discussed in Appendix A.

For simplicity, only radiation modes that are totally unbound have been included in the expansion given by Eq. E-1. In the corrugated waveguide laser that is specifically considered in Chapter 4, all the leaky modes were of this type. The extension

to include radiation modes that are bound at one side of the waveguiding core, which may be required in the consideration of lasers operating by feedback on high orders of the corrugation, follows by a straight-forward application of the methods that are presented in this Appendix.

With the radiation modes included in the expansion, the methods that led from Eq. B-5 to Eq. B-9 can be applied to obtain the equation:

$$\begin{aligned}
 & 2j k_{x;m} \left(\frac{\partial C_m(x)}{\partial x} - \frac{\partial C_{-m}(x)}{\partial x} e^{j2k_{x;m}x} \right) \\
 & = \mu \omega^2 \sum_p C_p(x) e^{j(k_{m;x} - k_{p;x})x} \int_{-\infty}^{\infty} E_{y;p}(z) [(\Delta\epsilon - j\epsilon'') S(x, z) \\
 & \quad + j\epsilon'' L(z)] E_{y;m}(z) dz + \mu^2 \int_{-\infty}^{\infty} d\rho g(\rho, x) e^{j(k_{m;x} - k_{\rho;x})x} \\
 & \quad \int_{-\infty}^{\infty} dz E_{y;\rho}(z) [(\Delta\epsilon - j\epsilon'') S(x, z) + j\epsilon'' L(z)] E_{y;m}(z) \quad (E-2)
 \end{aligned}$$

The transverse field distributions of the radiation modes, $E_{y;\rho}(z)$, are normalized to the delta function, so that they obey the orthogonality relation:

$$\int_{-\infty}^{\infty} E_{y;\rho}(z) E_{y;\rho'}(z) dz = \delta(|\rho| - |\rho'|) \quad (E-3)$$

By employing this orthogonality relationship and the methods used to obtain Eq. E-2 from Eq. E-1, we obtain a differential equation for the complex amplitudes $g(\rho, x)$ that is similar to Eq. E-2:

$$\begin{aligned}
 & 2j k_{x;\rho} \left(\frac{\partial g(\rho, x)}{\partial x} - \frac{\partial g(-\rho, x)}{\partial x} \right) e^{jk_{x;\rho} x} \\
 &= \mu \omega^2 \int_{-\infty}^{\infty} d\rho \sum_p C_p(x) e^{j(k_{x;p} - k_{x;\rho})x} E_{y;p}(z) [(\Delta\epsilon - j\epsilon'')S(x, z) \\
 &+ j\epsilon''L(z)] E_{y;\rho}(z) + \mu \omega^2 \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} dz g(\eta, z) e^{j(k_{x;\eta} - k_{x;\rho})x} \\
 &E_{y;\eta}(z) [(\Delta\epsilon - j\epsilon'')S(x, z) + j\epsilon''L(z)] E_{y;\rho}(z) \quad (E-4)
 \end{aligned}$$

The oscillating terms on the left-hand sides of Eqs. E-2 and E-4 can be ignored, as explained in Appendix B, and we are left with expressions describing the rate of change along the x -direction of the complex amplitudes of, respectively, the bound mode m and the radiation mode ρ .

It is important to establish the meaning of the two terms on the right-hand sides of Eqs. E-2 and E-4. The first term on the right-hand side of Eq. E-2 represents the contribution to the growth of bound mode m , as it propagates in the positive x -direction, of energy that is coupled from the

other bound modes. Because of the requirement for conservation of pseudomomentum discussed in Appendix B, each bound mode is coupled to only one other bound mode by any single component of the corrugation. The first term therefore represents the contribution of the contradiirectional mode n to the growth of bound mode m where (m,n) are the two modes of the hybrid pair. The second term on the right-hand side of Eq. E-2 represents the contribution to bound mode m of power coupled from the radiation modes. We assume that energy in the radiation modes is carried away from the waveguiding core sufficiently rapidly that this term is insignificant. In the case of the laser studied in Chapter 4, the leaky radiation is on a group of radiation modes that propagate away from the core nearly at right angles to the propagation axis, so this assumption is justified.

The first term on the right-hand side of Eq. E-4 represents the contribution to the radiation mode ρ of coupling from all the bound modes in the waveguide. This term is the one that represents the loss with which we are concerned. Single terms can be extracted from the sum over the bound modes p to study the contribution of a single bound mode to the radiation continuum. Because of the requirement for the conservation of pseudomomentum for strong coupling, we are concerned with the coupling of single modes to the radiation continuum. The second term on the right-hand side of Eq. E-4 represents

other bound modes. Because of the requirement for conservation of pseudomomentum discussed in Appendix B, each bound mode is coupled to only one other bound mode by any single component of the corrugation. The first term therefore represents the contribution of the contradiirectional mode n to the growth of bound mode m where (m,n) are the two modes of the hybrid pair. The second term on the right-hand side of Eq. E-2 represents the contribution to bound mode m of power coupled from the radiation modes. We assume that energy in the radiation modes is carried away from the waveguiding core sufficiently rapidly that this term is insignificant. In the case of the laser studied in Chapter 4, the leaky radiation is on a group of radiation modes that propagate away from the core nearly at right angles to the propagation axis, so this assumption is justified.

The first term on the right-hand side of Eq. E-4 represents the contribution to the radiation mode p of coupling from all the bound modes in the waveguide. This term is the one that represents the loss with which we are concerned. Single terms can be extracted from the sum over the bound modes p to study the contribution of a single bound mode to the radiation continuum. Because of the requirement for the conservation of pseudomomentum for strong coupling, we are concerned with the coupling of single modes to the radiation continuum. The second term on the right-hand side of Eq. E-4 represents

coupling between radiation modes, and is not significant because the leaky radiation occurs on only a narrow range of radiation modes - other radiation modes carry no significant power. We can write the dependence of the growth in the complex amplitude of radiation mode ρ , as it propagates in the negative x -direction on the complex amplitude of bound mode m by extracting the appropriate term from Eq. E-4:

$$\frac{\partial g(\rho, x)}{\partial x} = - \frac{\mu \omega^2}{2j k_{x;\rho}} C_m(x) \int_{-\infty}^{\infty} \Delta \epsilon G_s(z) E_{y;m}(z) E_{y;\rho}(z) e^{-j[(k_{x;\rho} - k_{x;m} - s\Lambda)x - \theta_s(z)]} dz \quad (E-5)$$

where we have dropped the perturbation terms containing the factor ϵ'' , since this is insignificant compared with $\Delta \epsilon$ while the magnitudes of $S(x, z)$ and $L(z)$ are roughly equal over the range of z where the integrand has significant magnitude. The s th component of the perturbation function $S(x, z)$ which is the component causing the coupling to radiation modes has been written in the form given by Eq. 4-5 and a term that results which is an oscillating function of x has been dropped from Eq. E-5.

By defining the factor $K_{m\rho}$ to be

$$K_{m\rho} = \frac{\mu \omega^2}{2\sqrt{|\rho k_{x;m}|}} \int_{-\infty}^{\infty} G_s(z) e^{-j\theta_s(z)} E_{y;m}(z) E_{y;\rho}(z) dz \quad (E-6)$$

We can write Eq. E-5 in the form:

$$\sqrt{|k_{x;\rho}|} \frac{\partial g(\rho, x)}{\partial x} = -\frac{1}{j} \sqrt{|k_{x;m}|} C_m(x) K_{m\rho}^* \Delta \rho \sqrt{\frac{|\rho|}{|k_{x;\rho}|}} e^{-j[k_{x;\rho} - k_{x;m}]x} \quad (E-7)$$

Because the modes of the radiation continuum must be normalized to the delta function, power is not carried by individual radiation modes, but only by segments of the radiation continuum, over which the infinitesimal contribution from each, normalized radiation mode is integrated. For pseudomomentum to be nearly conserved, such segments of the continuum must be narrow, in the sense that they span modes that propagate nearly in the same direction. The power carried by the radiation mode group to which the bound mode is coupled is proportional to

$$\int_{-\infty}^{\infty} k_{x;\rho} |g(\rho, x)|^2 d\rho$$

Because all the radiation modes propagate nearly in the same direction, we can make the approximation:

$$\int_{-\infty}^{\infty} k_{x;\rho} |g(\rho, x)|^2 d\rho \approx k_{x;\rho'} \int_{-\infty}^{\infty} |g(\rho, x)|^2 d\rho \quad (E-8)$$

where ρ' is the central radiation mode of the group.

Into the relation

$$\frac{\partial}{\partial x} \left\{ k_{x;p} \int_{-\infty}^{\infty} |g(\rho, x)|^2 d\rho \right\} = k_{x;p} \int_{-\infty}^{\infty} \left[g(\rho, x) \frac{\partial g(\rho, x)}{\partial x} + g^*(\rho, x) \frac{\partial g(\rho, x)}{\partial x} \right] d\rho \quad (E-9)$$

we substitute the expression for $\partial g(\rho, x)/\partial x$ given by Eq. E-7 to obtain:

$$\begin{aligned} \frac{\partial}{\partial x} \left[k_{x;p} \int_{-\infty}^{\infty} |g(\rho, x)|^2 d\rho \right] &= \int_{-\infty}^{\infty} d\rho |C_m(x)|^2 K_m^2(\rho, \epsilon)^2 \frac{k_{x;m} \rho}{k_{x;p}} \\ &\quad \left\{ e^{-j[k_{x;m} - k_{x;p} - s]x} \int_0^x e^{j[k_{x;m} - k_{x;p} - s]x} dx + \text{complex conjugate} \right\} \quad (E-10) \end{aligned}$$

Since the power transferred to the group of radiation modes all comes from bound mode m , the irradiances of the bound mode and the radiation modes are related by:

$$\frac{\partial}{\partial x} \left[\int_{-\infty}^{\infty} |k_{x;p}| |g(\rho, x)|^2 d\rho \right] = + |k_{x;m}| \frac{\partial}{\partial x} |C_m(x)|^2 \quad (E-11)$$

The positive term on the right arises because the radiation modes have been assumed to be contradiirectional to the bound mode m .

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Using Eqs. E-8 and E-11 in Eq. E-10, and performing the integration gives:

$$+ \frac{\partial |C_m(x)|^2}{\partial x} = \int_{-\infty}^{\infty} \frac{|\rho|}{k_{x;p}} |C_m(x)|^2 k_{m0}^2 (\Delta\epsilon)^2 \frac{2 \sin(k_{x;m} - k_{x;p} - s\Lambda)x}{(k_{x;m} - k_{x;p} - s\Lambda)} d\rho \quad (E-12)$$

The function $\sin(k_{x;m} - k_{x;p} - s\Lambda)x / (k_{x;m} - k_{x;p} - s\Lambda)$ contained in the integrand of Eq. E-12 is a good approximation to the delta function $\delta(k_{x;m} - k_{x;p} - s\Lambda)$ if $x \gg (k_{x;m} - k_{x;p} - s\Lambda)$, which is true over most of the range of x because conservation of pseudomomentum requires $(k_{x;m} - k_{x;p} - s\Lambda)$ to be nearly zero. The transverse and longitudinal propagation constants of the radiated mode are related by:

$$k_{03}^2 = k_{x;p}^2 + \rho^2 \quad (E-13)$$

and therefore

$$d\rho = - \frac{k_{x;p}}{\rho} dk_{x;p} \quad (E-14)$$

By changing the variable of integration, we write Eq. E-12 in the form:

$$\frac{\partial |C_m(x)|^2}{\partial x} = - \int_{-\infty}^{\infty} |C_m(x)|^2 k_{m0}^2 (\Delta\epsilon)^2 \delta(k_{x;m} - k_{x;p} - s\Lambda) dk_{x;p} \quad (E-15)$$

The integral over the delta function gives:

$$\frac{\partial |C_m(x)|^2}{\partial x} = -|C_m(x)|^2 |K_{mp}|^2 (\Delta\epsilon)^2 \cdot 2\pi \quad (E-16)$$

By integrating Eq. E-16 over x , we obtain the complex amplitude $C_m(x)$ as a function of distance x along the waveguide

$$C_m(x) = e^{-\pi |K_{mp}|^2 (\Delta\epsilon)^2 x} \quad (E-17)$$

The amplitude loss-coefficient $\alpha_{m;s}^L$ is thus

$$\alpha_{m;s}^L = -|K_{mp}|^2 (\Delta\epsilon)^2 \quad (E-18)$$

This loss coefficient describes the effect of the s th corrugation component, which is independent of the effects of any other component. The loss coefficient $\alpha_{m;s}^L$ therefore combines linearly with the gain coefficient to result in a net, or effective gain coefficient, as indicated by Eq. 4-12. Other loss coefficients corresponding to other corrugation components combine similarly, as indicated by the summation over the loss coefficients in Eq. 4-12.

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