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**DIFFUSION-WEIGHTED MR IMAGING OF MOVING STRUCTURES
USING A THREE ECHO NAVIGATOR IMAGING TECHNIQUE**

by

CATHY MACGILLIVRAY, B.Sc.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of

Master of Science

Department of Physics

Carleton University

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USING A THREE ECHO NAVIGATOR IMAGING TECHNIQUE"**

submitted by Cathy MacGillivray, B.Sc.
in partial fulfilment of the requirements for
the degree of Master of Science

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Abstract

The possibility of acquiring good quality diffusion-weighted MR images of moving structures using a three echo navigator imaging technique was investigated. Simulations were run to demonstrate the detrimental effects of motion in diffusion-weighted images - severe artifacts and gross error in diffusion coefficient quantification. They also showed that, in theory, the effects of such motion (rigid body, translational motion within a plane) can be corrected for in postprocessing using information derived from two specially encoded navigator echoes. Phantom experiments confirmed the theory - images of moving objects acquired using our three echo technique were markedly improved over those acquired using either a one or two echo approach. In fact, the images were often indistinguishable from those acquired while the object was stationary. Finally, an experiment carried out on a human volunteer confirms that our technique shows great promise for diffusion-weighted in vivo imaging.

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Contents

1 Introduction	1
2 Theory	6
2.1 Nuclear Spin and Resonance Theory	6
2.2 Net Magnetization and Spin Dynamics	9
2.3 Relaxation and the Spin Echo	18
2.4 Spatial Localization of the MR Signal	24
2.5 Diffusion Theory	33
2.6 The effects of motion on the MR signal	40
2.6.1 View-to-view motion in the frequency encoding direction	42
2.6.2 View-to-view motion in the phase encoding direction	43
2.6.3 Intra-view motion in all directions	44
2.7 The Navigator Echo Correction Technique	47
2.7.1 Correcting view-to-view motion in the frequency encoding direction	49
2.7.2 Correcting intra-view motion effects	52
3 Methods	54
3.1 Sequence Design	54
3.2 Three Echo Navigator Correction Technique	57
3.3 Motion Simulations	60
3.3.1 Noiseless Simulations	61
3.3.2 Simulations including noise	69

3.4 Phantom Studies	71
3.4.1 Stationary water phantom experiment . .	71
3.4.2 Moving phantom experiment	72
3.5 In Vivo Studies	75
4 Results	77
4.1 Noiseless Motion Simulations	77
4.2 Motion Simulations including Noise	86
4.3 Diffusion in a Stationary Water Phantom . . .	99
4.4 Moving Phantom Study	99
4.5 In Vivo Study	108
5 Discussion	115
5.1 Noiseless Motion Simulations	115
5.2 Simulations Including Noise	120
5.3 Phantom Studies	123
5.4 In Vivo Study	126
6 Conclusions	128
Appendix	130
References	146

List of Tables

Table 1: Motion parameters for simulation studies . . .	63
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List of Figures

Figure 1: Energy level diagram for the proton	8
Figure 2: Precession of a magnetic moment, μ , in a static magnetic field, B_0	12
Figure 3: Precession of bulk magnetization vector, M , during application of r.f. pulse B_1	17
Figure 4: Free induction decay of MR signal following a 90° r.f. pulse	21
Figure 5: Formation of a spin echo in the transverse plane	23
Figure 6: Two-dimensional, Fourier transform imaging pulse sequence diagram	31
Figure 7: Stejskal-Tanner pulsed gradient spin echo sequence for diffusion measurements	35
Figure 8: Diffusion-weighted 2DFT imaging sequence	41
Figure 9: The two echo navigator imaging sequence	48
Figure 10: The diffusion-weighted three echo navigator imaging sequence	56
Figure 11: Computer generated image of a stationary circular object	78
Figure 12: Plot of signal decay for the set of stationary, noiseless images	79
Figure 13: Computer generated image of a circular object undergoing translational rigid body motion within the field of view	80
Figure 14: Plot of signal decay for motion degraded and corrected image sets	81
Figure 15: Images of the X-NAV projection modulus values before and after correction	83
Figure 16: Images of the X-NAV projection phase values before and after correction	84

Figure 17: Computer generated image of the circular object obtained following correction of the motion degraded raw data set using the three echo navigator correction technique	85
Figure 18: Computer generated image of the circular object obtained following correction of the motion degraded raw data using the original two echo navigator correction technique	87
Figure 19: Image of a stationary circular object computer generated with a signal-to-noise level of 100:1	88
Figure 20: Plot of signal decay for stationary and corrected images having signal-to-noise levels of 100:1	89
Figure 21: Image of the circular object obtained following correction of the motion degraded raw data (signal-to-noise level of 100:1) using our three echo navigator correction technique	91
Figure 22: Image of a stationary circular object computer generated with a signal-to-noise level of 10:1	92
Figure 23: Plot of signal decay for stationary and corrected images having signal-to-noise levels of 10:1	93
Figure 24: Image of the circular object obtained following correction of the motion degraded raw data (signal-to-noise level of 10:1) using our three echo navigator correction technique	94
Figure 25: Image of a stationary circular object computer generated to have a signal-to-noise level of 3:1	96
Figure 26: Plot of signal decay for stationary and corrected images having signal-to-noise levels of 3:1	97
Figure 27: Image of the circular object obtained following correction of the motion degraded raw data (signal-to-noise level of 3:1) using our three echo navigator correction technique	98
Figure 28: Diffusion-weighted MR image of a stationary water phantom	100

Figure 29: Plot of signal decay for the stationary water phantom	101
Figure 30: Diffusion-weighted MR image of a stationary grapefruit	102
Figure 31: Plot of signal decay for the stationary and corrected grapefruit images	103
Figure 32: Diffusion map of the grapefruit generated from the set of diffusion-weighted stationary images	105
Figure 33: Diffusion-weighted MR image of the grapefruit acquired while the grapefruit was manually shifted within the imaging plane	106
Figure 34: Image of the grapefruit obtained following correction of the motion degraded raw data using our three echo navigator correction technique . . .	107
Figure 35: Diffusion map of the grapefruit generated from the set of diffusion-weighted corrected images	109
Figure 36: Image of the grapefruit obtained following correction of the motion degraded raw data using the original two echo navigator correction technique	110
Figure 37: Diffusion-weighted image of a transverse slice through the head of a human volunteer . . .	111
Figure 38: Image obtained following correction of the motion degraded image using our three echo navigator correction technique	112
Figure 39: Plot of signal decay for original and corrected images of the human brain	114
Figure 40: Flow chart summarizing the three echo navigator correction technique	131

1 Introduction

The phenomenon of nuclear magnetic resonance (NMR) was discovered in 1945 by two groups working independently, one led by Bloch [1] and the other by Purcell [2]. They demonstrated that when nuclei possessing magnetic moments are placed within an external magnetic field, the net magnetization vector can be manipulated by applying r.f. energy at a specific frequency called the Larmor or resonance frequency. The absorption of this energy causes the net magnetization vector to tip away from the static field direction into the transverse plane where it can be detected and measured by a receiver coil.

The earliest applications of NMR were in the field of spectroscopy, as a non-destructive probe of chemical structure [3] and a means of determining the self-diffusion coefficient of liquids [4]. More recently, it was discovered that magnetic field gradients could be used to spatially localize the NMR signal and produce high resolution images in a field which has come to be known as magnetic resonance imaging or MRI [5].

Contrast in MR images is determined by several factors, including nuclear spin density, magnetization relaxation times and molecular diffusion processes. The extent to which each factor affects contrast is determined by the type of imaging

sequence, magnetic field gradient pulses, r.f. pulses, and timing parameters. For example, by including large amplitude magnetic field gradient pulses in the sequence, contrast can be made to depend primarily on molecular diffusion. Such a sequence is termed a diffusion-weighted imaging (DWI) sequence.

Although still in its infancy, diffusion-weighted MRI has found many potential medical applications. It may prove useful in identifying lesions [6], determining nerve fibre orientation [7], evaluating brain ischemia caused by stroke [8], and in monitoring the progression of demyelination in patients with Multiple Sclerosis [9]. Despite its promise, DWI has yet to find widespread practical application. This is due in part to the technical difficulty of acquiring high quality diffusion-weighted images. To obtain diffusion contrast, DWI sequences are sensitized to minute molecular motions (~10 microns). However, this also sensitizes them to bulk tissue motion on this scale. The result is that, under normal imaging conditions, the diffusive motions of interest can be completely overwhelmed by the much greater bulk tissue motions associated with physiologic motions like respiration and peristalsis. Images obtained under such conditions often exhibit severe ghosting artifacts and unsharpness. In addition, signal attenuation ascribed to diffusion may, in fact, be caused by another kind of motion, resulting in an over-estimation of the self-diffusion coefficient.

Several strategies have been proposed to overcome motion related problems in diffusion-weighted MR imaging. Very fast, single-shot techniques, such as echo-planar imaging (EPI), are less sensitive to bulk tissue motion because all data are acquired in a single experiment following the application of diffusion sensitizing gradients [10]. With this technique, images may be obtained in less than 100ms, thus little motion occurs during the acquisition period. Unfortunately, single-shot imaging requires specialized hardware that is not available at most sites.

An alternate proposal is to use gating techniques - cardiac or respiratory - which trigger the imaging sequence to run at the point of least motion in the cycle. While they may be successful in reducing the amount of physiological motion that occurs during imaging, neither addresses the problem of gross patient movement during image acquisition. In addition, these techniques have not been widely adopted because they cause an increase in the image acquisition time.

A unique method for overcoming motion problems was proposed by Ehman and Felmlee in 1989 [11]. Their 'navigator' echo technique tracks motion during image acquisition and corrects its effects in postprocessing. They demonstrated that using this method, motion artifacts could be greatly reduced in images acquired with a modified, two echo, spin echo sequence. Although very promising, their technique only corrects for rigid body motion along one in-plane direction.

Also, they did not attempt to correct motion effects in diffusion-weighted images.

Several investigators [12-16] have since tested a navigator type correction scheme in diffusion-weighted images. Ordidge et al. [12] demonstrated that the effects of small scale motion (less than one pixel) could be effectively removed from diffusion-weighted images using information derived from a navigator echo. Anderson and Gore [13] went on to show that Ordidge's technique was most robust when the diffusion sensitizing gradients were applied along the phase encoding axis. First order phase errors caused by small rotations during the application of these gradients could be removed only if they were applied along the phase encoding axis. Other researchers showed that Ordidge's correction algorithm could be applied to either the motion degraded time domain data [14] or to the motion degraded image projections [15] and that multiple acquisitions further improve the technique [16]. It was also demonstrated that navigator echoes could be used to detect [17] and measure motion [18] of objects during image acquisition.

Although all of these recent studies [12-16] demonstrate that a navigator echo imaging technique dramatically reduces motion related artifacts in diffusion-weighted images, none addresses the problem of motion greater than a pixel. Large scale motion (greater than a pixel but within the field-of-view) leads not only to phase error of the measured signal but

also to frequency shifts which must be removed if an artifact free image is to be obtained. Ehman [11] did address large scale motion, but along one axis only.

In this work we expand the navigator correction technique of Ehman and Felmlee to correct for large scale motion along both in-plane directions and evaluate its effectiveness in diffusion-weighted images. We investigate the possibility of producing high quality diffusion-weighted images of moving structures which may then be used to determine self-diffusion coefficients with reasonable accuracy.

2 Theory

2.1 Nuclear Spin and Resonance Theory

Like electrons, protons and neutrons possess intrinsic angular momentum or spin. When grouped together to form a nucleus, their angular momenta add vectorially to determine the net nuclear spin. In general, nuclei with an odd number of protons and an even number of neutrons or vice versa, have non-zero nuclear spins. These are the nuclei of interest in Nuclear Magnetic Resonance (NMR) and include ^1H , ^{13}C , ^{31}P , ^{23}Na and ^{19}F . The high natural abundance of hydrogen and its relatively strong signal make it the focus of most biological NMR studies, including this work.

The angular momentum, $\mathbf{J}$, of a nucleus is given by:

$$\mathbf{J} = \hbar \mathbf{I}, \quad (1)$$

where $\mathbf{I}$ is a non-negative integer or half integer and $\hbar$ is Planck's constant. A nucleus with spin will also have a magnetic moment, $\boldsymbol{\mu}$, due to the current loop created by its spinning charge distribution. The magnetic moment is collinear with and proportional to the angular momentum vector according to the relationship:

$$\boldsymbol{\mu} = \gamma \mathbf{J}. \quad (2)$$

The constant of proportionality, γ , is called the gyromagnetic ratio and depends on the nucleus. For hydrogen, $\gamma = 42.58$ MHz/Tesla.

When the spin is placed in a static magnetic field, B_0 , there is an interaction energy of the nucleus with this field, given by the Hamiltonian, H :

$$H = -\mu \cdot B_0. \quad (3)$$

If the magnetic field is taken to be in the z -direction (i.e. $B_0 = B_0 \mathbf{k}$), then H becomes:

$$H = -\gamma \hbar B_0 I_z. \quad (4)$$

The operator I_z has eigenvalues:

$$m = I, I-1, \dots, -I+1, -I, \quad (5)$$

which correspond to $2I+1$ different values. For the proton, $I = \frac{1}{2}$, so the two possible values are $m = +\frac{1}{2}$ and $m = -\frac{1}{2}$. The eigenvalues of the Hamiltonian, H , are related to those of I_z through the constant of proportionality $-\gamma \hbar B_0$. The energy eigenvalues are then:

$$E_m = -\gamma \hbar B_0 m, \quad (6)$$

or, more specifically for the proton:

Figure 1: Energy level diagram for the proton. Zeeman splitting energy, ΔE , increases linearly with applied magnetic field, B_0 . Transitions may occur by irradiation with r.f. energy corresponding to the Zeeman splitting energy.

$$\begin{aligned}
 E_{+\frac{1}{2}} &= -\gamma \hbar B_0 \frac{1}{2} \\
 E_{-\frac{1}{2}} &= +\gamma \hbar B_0 \frac{1}{2} .
 \end{aligned}
 \tag{7}$$

These energy levels are shown in Fig. 1 and correspond to the situations in which I_z is parallel ($m = +\frac{1}{2}$) to the magnetic field B_0 , and antiparallel ($m = -\frac{1}{2}$). The difference in energy between the two states, $\Delta E = E_{-\frac{1}{2}} - E_{+\frac{1}{2}} = \gamma \hbar B_0$, is known as the Zeeman splitting. In order to excite transitions from one state to the other, one must supply an energy quantum which is equal to the Zeeman splitting energy. This may be done by irradiation with photons of a frequency such that:

$$\begin{aligned}
 \hbar \omega_0 &= \Delta E \\
 &= \gamma \hbar B_0 ,
 \end{aligned}
 \tag{8}$$

or

$$\omega_0 = \gamma B_0 .
 \tag{9}$$

This is the resonance condition and ω_0 is the resonance or Larmor frequency.

2.2 Net Magnetization and Spin Dynamics

Consider a sample containing N spins placed in a magnetic field B_0 . If the spin system is in thermal equilibrium with the lattice, the spins will be distributed between the two

energy levels according to Boltzmann statistics:

$$\frac{n_1}{n_2} = \exp \left(- \frac{\Delta E}{k T} \right) \quad (10)$$

$$= \exp \left(- \frac{\gamma \hbar B_0}{k T} \right) ,$$

where,

k = Boltzmanns Constant

T = Absolute temperature

n_1 = number of spins antiparallel to B_0

n_2 = number of spins parallel to B_0

and $n_1 + n_2 = N$ = Total number of spins.

The population difference is found to be [19] :

$$n_1 - n_2 = N \frac{1 - \exp \left(- \frac{\gamma \hbar B_0}{k T} \right)}{1 + \exp \left(- \frac{\gamma \hbar B_0}{k T} \right)} , \quad (11)$$

which, since the thermal energy (kT) for body temperature (310 K) is much greater than the Zeeman energy ($\gamma \hbar B_0$) for typical field strengths (1 Tesla), can be approximated by:

$$n_1 - n_2 = N \frac{\gamma \hbar B_0}{2 k T} . \quad (12)$$

Hence, at equilibrium, there is a slight excess in the number of spins tending to align with the field and a net magnetization will exist [19]:

$$\begin{aligned}
 M_0 &= (n_1 - n_2) \gamma \hbar \frac{1}{2} \\
 &= \frac{N \gamma^2 \hbar^2}{4 k T} B_0.
 \end{aligned}
 \tag{13}$$

This result brings to light three key points. Firstly, the bulk magnetization is proportional to N , the total number of spins. For this reason, samples containing very large concentrations of spins produce significant NMR signals. Secondly, it is proportional to the gyromagnetic ratio squared, which is why the proton is the nucleus most often studied in NMR - it has the highest gyromagnetic ratio of all naturally occurring nuclei. Finally, the bulk magnetization is proportional to the static magnetic field strength, making high fields desirable in NMR.

Applying r.f. energy at the Larmor frequency to the sample will disrupt the equilibrium magnetization by exciting transitions from the spin-up state (parallel to the field) to the spin-down state (antiparallel). In determining the behaviour of the bulk magnetization under such a perturbation, we first consider spin dynamics under equilibrium conditions.

When placed in a magnetic field B_0 , a nuclear magnetic moment experiences a couple which tends to align it with the applied field. However, because of the presence of spin angular momentum, the motion is not rotational but rather precessional, similar to that of a gyroscope. Such motion is depicted in Fig. 2. The magnetic field produces a torque, Γ ,

Figure 2: Precession of a magnetic moment, μ , in a static magnetic field, B_0 . Frequency of precession, ω_0 , depends on γ , the gyromagnetic ratio and B_0 , the strength of the applied magnetic field.

on the magnetic moment which is given by:

$$\Gamma = \mu \times B_0. \quad (14)$$

This torque produces a change in the angular momentum (Newton's second law):

$$\begin{aligned} \frac{dJ}{dt} &= \Gamma \\ &= \mu \times B_0, \end{aligned} \quad (15)$$

which, since $\mu = \gamma J$, leads to:

$$\frac{d\mu}{dt} = \gamma \mu \times B_0. \quad (16)$$

Summing over all spins, the equation of motion for the net magnetization vector is [1]:

$$\frac{dM}{dt} = \gamma M \times B_0. \quad (17)$$

In solving this, it is useful to convert to a rotating coordinate system. To make the transformation from the laboratory frame to the rotating frame of reference, the general law of relative motion is used [20]:

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + \Omega \times A, \quad (18)$$

where,

$\frac{dA}{dt}$ = time derivative of time dependent
vector $A(t)$ in laboratory frame

$\frac{\partial \mathbf{A}}{\partial t}$ = time derivative of time dependent
vector $\mathbf{A}(t)$ in rotating frame

Ω = angular velocity of rotating frame
with respect to laboratory frame.

Note that in this context, $\partial/\partial t$ does not indicate partial differentiation. Applying this to equation (17), the equation of motion of the magnetization in the rotating frame is obtained:

$$\frac{\partial \mathbf{M}}{\partial t} = \gamma \mathbf{M} \times (\mathbf{B}_0 + \frac{\Omega}{\gamma}), \quad (19)$$

which has the same form as equation (17) if the magnetic field $\mathbf{B}_0$ is replaced by an effective field, $\mathbf{B}_e = \mathbf{B}_0 + \Omega/\gamma$. One can see that the effective field vanishes for a rotating frame of $\Omega = -\gamma \mathbf{B}_0$. In this frame $\partial \mathbf{M}/\partial t = 0$ and the magnetization is a fixed vector. In the laboratory frame, it precesses about $\mathbf{B}_0$ with frequency $\omega_0 = -\gamma B_0$ which is the Larmor frequency.

As mentioned previously, exposing the spin system to r.f. energy at the resonance frequency will disrupt the equilibrium magnetization vector by exciting transitions from the lower energy state (parallel to the applied static field) to the higher energy state (antiparallel). To determine the dynamics of the net magnetization vector under such irradiation, it is necessary to determine the net magnetic field experienced by the spin system. If the r.f. pulse is generated by an alternating current in a solenoid along the x-axis, a magnetic

decay of transverse magnetization are called spin-lattice relaxation and spin-spin relaxation, respectively.

The first process, spin-lattice relaxation, governs the growth of the longitudinal magnetization back to its equilibrium value. This process involves the exchange of energy between spins and the lattice as the spins make the transition from the excited spin-down state to the lower energy spin-up state. This process has an associated time constant T_1 , and is given by:

$$\frac{dM_z}{dt} = \frac{M_0 - M_z(t)}{T_1} \quad (28)$$

The second process, spin-spin relaxation, involves the decay of the transverse magnetization due to interactions between neighbouring spins. The net magnetization vector can be thought of as being made up of a finite number of smaller magnetization vectors, or isochromats, arising from different regions of the spin ensemble. Immediately following the r.f. pulse, these vectors are all in phase and their transverse components will add to define the net transverse magnetization. However, interactions among neighbouring spins cause the vectors to precess at slightly different frequencies and lose their phase coherence over time. The result is a decay in the transverse magnetization which is given by:

$$\mathbf{B}_e = (B_0 - \frac{\omega}{\gamma}) \hat{k} + B_1 \hat{i}. \quad (24)$$

If the resonance condition applies, that is, if the r.f. pulse is applied at the Larmor frequency, $\omega = \gamma B_0$, the effective field in the rotating frame is simplified to:

$$\begin{aligned} \mathbf{B}_e &= B_1 \hat{i} \\ &= B_1 \quad , \end{aligned} \quad (25)$$

and the associated equation of motion for the magnetization vector in the rotating frame is:

$$\frac{\partial \mathbf{M}}{\partial t} = \gamma \mathbf{M} \times \mathbf{B}_1. \quad (26)$$

This has the same form as equation (17) which described a precessional motion of the magnetization about the z-axis. In this case, the magnetization will precess about $\mathbf{B}_1$, in the y' - z' plane of the rotating frame, always remaining perpendicular to $\mathbf{B}_1$. Such precessional motion is shown in Fig. 3(a), and has a frequency of $\omega_1 = \gamma B_1$. The angle through which the magnetization rotates, θ , is determined by the amplitude, B_1 , and duration, τ , of the applied r.f. pulse according to:

$$\theta = \gamma B_1 \tau. \quad (27)$$

If B_1 and τ are adjusted such that $\theta = 180^\circ$, the magnetization vector is simply inverted to lie along the $-z$ axis and the

Figure 3: Precession of bulk magnetization vector, M , during application of r.f. pulse B_1 , in (a) the rotating frame, and (b) the laboratory frame. In the rotating frame, B_1 is a static vector along the x' direction. The magnetization precesses about this vector with frequency ω_1 . In the laboratory frame, B_1 rotates about the z axis at frequency ω_0 . The magnetization will precess about this field with frequency ω_1 and simultaneously about B_0 with frequency ω_0 .

r.f. pulse is known as a π or 180° pulse. Similarly, if B_1 and τ are adjusted such that $\theta=90^\circ$, the magnetization vector is rotated from the z-direction to the y' -direction of the rotating frame. Such a pulse is termed a $\pi/2$ or 90° pulse.

In the fixed or laboratory frame of reference, the magnetization will precess about B_0 at the Larmor frequency and simultaneously about B_1 at frequency ω_1 . Such motion is depicted in Fig. 3(b), and is best described as a spiralling of the magnetization vector toward the transverse (x-y) plane.

If the r.f. pulse is not applied at the Larmor frequency (called an off-resonance pulse), the effective field in the rotating frame will have a longitudinal component and the magnetization will precess about this off-axis field.

2.3 Relaxation and the Spin Echo

It has been shown that the application of an r.f. pulse at the Larmor frequency results in a perturbation of the equilibrium magnetization. This is seen as a precession of the magnetization vector about the applied magnetic field in the rotating frame. When the perturbing field is removed, the magnetization will not remain tipped away from the z-axis indefinitely, rather it will slowly revolve or relax back to its equilibrium value. The exponential relaxation processes which govern the growth of longitudinal magnetization and

decay of transverse magnetization are called spin-lattice relaxation and spin-spin relaxation, respectively.

The first process, spin-lattice relaxation, governs the growth of the longitudinal magnetization back to its equilibrium value. This process involves the exchange of energy between spins and the lattice as the spins make the transition from the excited spin-down state to the lower energy spin-up state. This process has an associated time constant T_1 , and is given by:

$$\frac{dM_z}{dt} = \frac{M_0 - M_z(t)}{T_1} \quad (28)$$

The second process, spin-spin relaxation, involves the decay of the transverse magnetization due to interactions between neighbouring spins. The net magnetization vector can be thought of as being made up of a finite number of smaller magnetization vectors, or isochromats, arising from different regions of the spin ensemble. Immediately following the r.f. pulse, these vectors are all in phase and their transverse components will add to define the net transverse magnetization. However, interactions among neighbouring spins cause the vectors to precess at slightly different frequencies and lose their phase coherence over time. The result is a decay in the transverse magnetization which is given by:

$$\frac{dM_{x,y}}{dt} = -\frac{M_{x,y}}{T_2} \quad (29)$$

The decay time constant, T_2 , is called the spin-spin relaxation time.

When relaxation effects are taken into consideration, the equation of motion for the net magnetization vector is given by the full Bloch equation [1]:

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \gamma \mathbf{B} - \frac{M_x \hat{i} + M_y \hat{j}}{T_2} - \frac{M_z - M_0}{T_1} \hat{k} \quad (30)$$

The relaxation terms may be neglected during the application of an r.f. pulse because the pulse duration, τ , is usually much shorter than the relaxation times, T_1 and T_2 , which are sample dependent.

In NMR experiments, signal detection may be accomplished using a receiver coil positioned with its axis along the y-axis of the laboratory frame. Following an r.f. pulse, the net magnetization acquires a transverse component which precesses in the x,y plane. In doing so, it induces an e.m.f. in the receiver coil whose frequency and amplitude reflect the precessional frequency and amplitude of the transverse magnetization. Following a 90° pulse, the magnetization lies entirely in the transverse plane and the signal, called a free induction decay, resembles a damped cosine curve as shown in Fig. 4. One would expect the amplitude of this curve to decay

Figure 4: Free induction decay of MR signal following a 90° r.f. pulse. Amplitude of the transverse magnetization decays with time constant T_2^* , the effective transverse relaxation time. T_2^* decay is much faster than T_2 decay alone.

with time constant T_2 , the transverse relaxation time introduced above. However, in actuality it falls off much faster, with time constant T_2' , the effective transverse relaxation time. The cause of this increased relaxation rate is inhomogeneity of the external magnetic field. Spins at different locations experience slightly different magnetic fields and hence precess at slightly different frequencies. Consequently, spins lose their phase coherence more rapidly than by T_2 processes alone, and, in turn, the transverse magnetization decays more rapidly.

Since a perfectly homogeneous magnetic field cannot be generated, it is often advantageous to detect a spin echo rather than a free induction decay. The spin echo sequence was first proposed by Hahn in 1950 [4], and requires two r.f. pulses as shown in Fig. 5. The first pulse, a 90° pulse, tips the magnetization into the transverse plane where the isochromats begin to de-phase due to both T_2 processes and magnetic field inhomogeneity. This process continues for a time $TE/2$ at which point a 180° pulse is applied causing the distribution of magnetization isochromats to be 'reflected' about the x,z plane. Spins that had been precessing faster and thus acquiring greater phase angles will now be behind in phase compared to the slower precessing spins. The isochromats then resume precession, re-phasing until a spin echo forms at time TE - the time at which the faster spins have 'caught up' with the slower precessing spins. This echo

Figure 5: Formation of a spin echo in the transverse plane. Following a 90° r.f. pulse, the net magnetization lies in the transverse plane where it dephases due to T_2 processes and magnetic field inhomogeneity. A 180° r.f. pulse causes the spins to be reflected about the x,z plane where they rephase until a spin echo forms at time TE , the echo time. The echo is reduced in amplitude due to T_2 processes alone.

is reduced in amplitude due to T_2 processes alone and hence effects of magnetic field inhomogeneity have been removed. The time between the initial 90° pulse and the echo peak is called the echo time, TE.

2.4 Spatial Localization of the MR Signal

Since the NMR signal is generated by all spins in the sample volume, a method of spatial encoding is required to localize the signal or to produce an image. Several techniques for doing so have been developed, all of which involve the use of a magnetic field gradient superposed on the main static field. The most commonly used method, the two-dimensional Fourier transform (2DFT) technique, uses gradient pulses along each axis to achieve localization in three dimensions.

In a uniform magnetic field, all nuclei have the same Larmor frequency as given in equation (9). However, if a linear gradient, G_z , is superimposed on the main static field, the Larmor frequency will vary with position along the z-axis according to:

$$\omega = \gamma(B_0 + zG_z), \quad (31)$$

where z is the location along the z-axis. If such a gradient is activated while an r.f. pulse is applied, only those spins

in the plane having Larmor frequency equal to the r.f. pulse frequency will resonate and acquire a transverse component. Since the measured MR signal is due to the transverse magnetization, it follows that the signal will be entirely due to spins within the selected plane. In practice, we cannot limit our r.f. pulses to a single frequency; they necessarily have a spectrum of frequencies associated with them. In MRI, we normally modulate the carrier frequency, which is at the Larmor frequency with a sinc function to obtain a uniform slice profile. The thickness of the slice is determined by the r.f. pulse bandwidth and steepness of the slice select gradient, while its position is determined by the centre frequency of the r.f. pulse. The process of localizing signal to a specific plane is known as slice selection and the direction along which the localization occurs is called the slice select direction or axis. For simplicity, we have chosen the z-direction for slice selection but in practice, any direction may be selected.

Having selected a slice, spatial information must then be encoded for the two in-plane dimensions. For one direction this is done using a technique known as phase encoding. Following the slice defining r.f. pulse, the slice select gradient, G_z , is switched off and the phase encoding gradient, G_y , is activated. The role of this gradient is similar to that of the slice selecting gradient, to make precessional frequency a function of position. However, this time, the

frequency will vary with location along the y-axis. After a short period, the phase encode gradient is switched off and the spins revert to the precessional frequency determined by the main, static magnetic field. Although they are again precessing at the same rate, the spins no longer have the same phase due to the application of phase encoding. Spins that were precessing faster (i.e. those located farther from the isocenter of the magnet) have acquired larger phase angles than those that were precessing at slower frequencies (i.e. those close to the isocenter). The accumulated phase shifts, θ , are given by:

$$\theta = \gamma G_y y t , \quad (32)$$

where y is the position along the y-axis, and t is the duration of the phase encoding magnetic field gradient pulse. Because the gradient pulse was applied along the y-axis, the y-direction is also known as the phase encoding direction or axis.

Spatial localization along the other in-plane direction is accomplished using a technique known as frequency encoding. During signal measurement, or read-out, a magnetic field gradient is activated along the x-axis. This gradient, G_x , causes Larmor frequency to vary with position in the x-direction. The result is that the measured signal will contain components at each frequency produced by the field gradient. Since x-position is recorded as a function of

frequency, the x-direction is also known as the frequency encoding direction or axis.

In review, spatial localization of the MR signal involves three steps. Slice selection defines the plane to be imaged, phase encoding imparts a phase shift reflecting location along one in-plane axis and frequency encoding causes spins to precess at frequencies reflecting position along the other in-plane axis. Each position in the selected slice will therefore have a unique frequency and phase combination.

Image reconstruction would be straight-forward if the amplitude of each phase at each frequency could be deciphered from the complicated signal measured during read-out. This, unfortunately, cannot be done. Fourier transformation will give the amplitude of each frequency component only, not of each frequency-phase combination. To determine signal amplitude along the phase encoding direction, a process known as 'sampling the phase distribution' is used. This technique requires that the sequence be repeated many times, for different amplitudes of the phase encoding gradient pulse, G_y , before reconstructing the image. Each execution of the sequence is called a phase encoding step or view.

A complete raw data set will consist of N_y rows of N_x complex points. The N_y rows are due to the N_y repetitions of the sequence, where N_y is the number of phase encoding steps. The N_x points are due to digitization of the MR signal which is sampled N_x times during read-out. The points are complex

because the signal is measured in quadrature by two detectors simultaneously. Fourier transformation of each row extracts each frequency component and its amplitude from the complicated signal. Subsequent transformation of each column of data provides the amplitude at each point along the phase encoding direction. An image can then be obtained by determining the magnitude of each complex point and mapping this to a grey scale for display.

Although phase encoding and frequency encoding are done in rather different ways, they accomplish the same thing. In frequency encoding the echo is digitized as a function of time in the presence of a gradient. However, this function can also be expressed in terms of the phase, $\phi_x = \gamma G_x x t = k_x x$, where k_x has been introduced for convenience. With phase encoding, we again track the phase of a similar function, although in this case it is measured one point at a time. With phase encoding we have $\phi_y = \gamma G_y y t_0 = k_y y$, where t_0 is the duration of the phase encoding gradient pulse. Note the formal similarity between ϕ_x and ϕ_y when they are expressed in terms of k_x and k_y . In fact, for a phantom that is the same in the x and y directions, the results of phase encoding and frequency encoding will be identical.

Following a slice selective 90° r.f. pulse, a signal, S , is detected in the transverse plane whose strength is directly proportional to the number of spins lying in the selected plane:

$$S = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x,y) dx dy , \quad (33)$$

where $\rho(x,y)$ is the two-dimensional spin density in the selected plane.

If the signal is then phase encoded by applying a linear magnetic field gradient, G_y , along the y-axis for a period t_y , the signal becomes [19]:

$$S = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x,y) \exp(i\gamma G_y y t_y) dx dy , \quad (34)$$

where, for simplicity, relaxation effects, which would effectively attenuate $\rho(x,y)$, have been neglected.

If the signal is then frequency encoded by applying a linear magnetic field gradient, G_x , along the x-axis for a period t_x , the signal becomes [19]:

$$\begin{aligned} S = & \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x,y) \exp(i\gamma G_x x t_x) \exp(i\gamma G_y y t_y) dx dy \\ & = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x,y) \exp(ik_x x) \exp(ik_y y) dx dy , \end{aligned} \quad (35)$$

where $k_x = \gamma G_x t_x$ and $k_y = \gamma G_y t_y$. In examining this equation, one sees that it is a Fourier transform relationship. It follows that the spin density function can be determined from the measured signal by simply performing a two-dimensional Fourier transform operation. This computed spin density function can

then be displayed as an image after suitable scaling.

A typical 2DFT imaging pulse diagram is shown in Fig. 6. The sequence is simply a spin echo experiment with magnetic field gradient pulses applied along three orthogonal axes to achieve signal localization. Slice select gradients are activated during the 90° and 180° r.f. pulses to localize signal to a single plane. The variable amplitude phase encoding gradient pulse imparts a fixed phase twist across the imaging field of view and finally the frequency encoding gradient is activated while the signal is being measured at time TE (the echo time). In addition to these 'localizing' gradient pulses, the sequence contains 'compensator' pulses (shaded in the diagram), to ensure that any phase shift at the echo peak is due solely to the phase encoding gradient pulse. The amplitude and duration of the 'compensator' gradients is calculated such that any dephasing of the MR signal caused by the slice select and frequency encoding gradients is entirely rephased by the echo time.

The time needed to acquire a complete data set is determined by two factors: N_y , the total number of phase encoding views and TR, the repeat time, which is the time between phase encoding steps. Increasing the number of views, N_y , increases resolution along the phase encoding direction, however it also reduces the amount of signal per pixel in the final image. In choosing N_y it is important to consider both these effects and compromise between the two to obtain an

Figure 6: Two-dimensional, Fourier transform imaging pulse sequence diagram. Magnetic field gradients (trapezoids) are pulsed on in each direction to achieve signal localization in three-dimensions. Compensator gradients (shaded trapezoids) are also present to ensure the phase shift at the echo peak is due solely to the phase encoding gradient pulse.

image which has both adequate resolution and ample signal-to-noise.

In examining the sequence pulse diagram shown in Fig. 6, one might assume that following signal read-out at time TE (~15-100 ms) the sequence can be rerun immediately to acquire the next phase encoding view. This, however, is not the case. Following read-out there is a delay period during which the magnetization is allowed to relax back to its equilibrium value through T_1 processes. This ensures that the initial conditions for each phase encoding view are identical. It follows that the repeat time, TR, is dependent upon the T_1 relaxation time of the substance being imaged. Generally, a TR which is several times greater than the T_1 value is used to ensure complete relaxation. For tissue water, T_1 500ms so a TR~2s is desirable. Using 256 phase encoding steps and a TR of ~2s, the total data acquisition time would be just under 9 min. However, since different tissues relax with different characteristic T_1 values, it is often advantageous to use TR's of less than 2s to achieve tissue contrast in the reconstructed image. Tissues that relax quickly will appear much brighter than those with long T_1 's. Under such conditions, images can be obtained in just a few minutes.

2.5 Diffusion Theory

Molecular diffusion is the result of the random microscopic translational motion of molecules known as Brownian motion. Due to thermal kinetic energy, molecules are in constant motion, often colliding with other molecules. Over time, this random 'walk' process will produce a net displacement. Because of the random nature of this process, it is very difficult to predict the displacement of a given molecule after a time t . However, for large molecular populations the probability of a particular displacement is given by a Gaussian function. The variance of the distance travelled is proportional to the time the system is allowed to evolve, t , according to the Einstein equation [22]:

$$\langle (x - x_0)^2 \rangle = 2Dt. \quad (36)$$

The proportionality constant, D , is called the self diffusion coefficient, and is a measure of the mobility of molecules within the diffusing medium.

Several methods exist for the experimental determination of D in solids, liquids and gases. Many of these involve the use of radioactive or fluorescent tracers and the diffusivity is found by monitoring the concentration profiles of these tracers over time. Such techniques, however, are inherently invasive and cannot be used in vivo with humans. Fortunately, nuclear magnetic resonance techniques are completely

noninvasive and can therefore be used for in vivo human studies.

Several NMR pulse sequences have been developed for the measurement of the self diffusion coefficient. The most widely used technique is the pulsed gradient spin echo experiment proposed by Stejskal and Tanner [23]. This sequence, shown in Fig. 7, involves the addition of two magnetic field gradient pulses to the basic spin echo sequence. These gradient pulses lie on either side of the 180° refocusing r.f. pulse and are of equal amplitude, G_d , and duration, δ , and are separated by a time Δ . Their purpose is to magnetically label the spins so diffusive motions can be monitored.

The first gradient imparts a phase shift on the spin's transverse magnetization which depends on the spin position. If the spin is assumed to be stationary during the application of the gradient, this phase shift is given by:

$$\begin{aligned}\Phi_1 &= \gamma \int_0^\delta G_d y_1 dt \\ &= \gamma G_d \delta y_1 ,\end{aligned}\tag{37}$$

where y_1 is the location of the spin during this gradient pulse which is assumed to be applied along the y-axis. The 180° pulse then inverts this phase shift to $-\Phi_1$.

Figure 7: Stejskal-Tanner pulsed gradient spin echo sequence for diffusion measurements. Large amplitude gradient pulses of amplitude G_d , duration δ and spacing Δ are placed on either side of the 180° refocusing r.f. pulse.

The second gradient produces a similar phase shift:

$$\begin{aligned}\Phi_2 &= \gamma \int_{\Delta}^{\Delta+\delta} G_d y_2 dt \\ &= \gamma G_d \delta y_2,\end{aligned}\tag{38}$$

where y_2 is the spin location during this gradient pulse. The net de-phasing produced by the gradient pulse pair is:

$$\begin{aligned}\Phi_{net} &= \Phi_1 + \Phi_2 \\ &= \gamma G_d \delta (y_2 - y_1).\end{aligned}\tag{39}$$

For static spins $y_1 = y_2$ and the gradient pair produces no net de-phasing. However for moving spins a net de-phasing occurs which depends on the spin motion in the time between the two gradient pulses, Δ . Since diffusive motions are random, each spin will have a unique motion history and the result is a distribution of transverse magnetization phases. When the individual magnetization vectors are added to determine the net magnetization, the phase distribution results in an attenuation of the transverse magnetization. This damping can be evaluated by adding a diffusion term to the Bloch equation and solving it for the Stejskal-Tanner pulse sequence.

In steady-state, non-equilibrium systems, such as cells, diffusive transport occurs between regions of differing concentrations. Fick's law relates the molecular flux density, J , to the molecular concentration gradient, ∇C [22]:

$$J = -D \nabla C,\tag{40}$$

where the proportionality constant, D , is the self-diffusion coefficient. Combining this equation with the conservation of mass equation [24]:

$$\frac{\partial C}{\partial t} = -\nabla \cdot \mathbf{J}, \quad (41)$$

one obtains the diffusion equation:

$$\frac{\partial C}{\partial t} = \nabla \cdot (D \nabla C). \quad (42)$$

In NMR experiments, the diffusive transport of magnetization is accounted for by modifying the Bloch equation to include a diffusion term [25]:

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B} - \frac{(M_x \hat{i} + M_y \hat{j})}{T_2} - \frac{(M_z - M_0) \hat{k}}{T_1} + \nabla \cdot D \nabla \mathbf{M}, \quad (43)$$

which relates the applied magnetic field, $\mathbf{B}$, to the net nuclear magnetization vector, $\mathbf{M}$, in the laboratory frame.

For a pulsed-gradient spin echo experiment, the spins are subjected to the main static field, $B_0 \hat{k}$ superposed with a linear gradient in the z -direction, $\mathbf{G}$, so that:

$$\mathbf{B} = [B_0 + (\mathbf{r} \cdot \mathbf{G})] \hat{k}, \quad (44)$$

where $\mathbf{r}$ is the displacement vector. Substituting this into equation (43) and considering only the transverse magnetization, $m = M_x + iM_y$, the equation of motion becomes [24]:

$$\frac{dm}{dt} = -i\omega_0 m - \frac{m}{T_2} - i\gamma(\mathbf{r} \cdot \mathbf{G})m + \nabla \cdot (D\nabla m) . \quad (45)$$

To simplify this transport equation we transform to a coordinate system rotating about the z-axis at frequency ω_0 and remove the transverse relaxation time, T_2 , by making the following substitution:

$$m = \psi e^{-(i\omega_0 - \frac{1}{T_2})t} , \quad (46)$$

to obtain:

$$\frac{d\psi}{dt} = -i\gamma(\mathbf{r} \cdot \mathbf{G})\psi + \nabla \cdot (D\nabla \psi) . \quad (47)$$

We may simplify further by dividing ψ into a complex part which represents the solution to equation (47) without diffusion, and a real, time-dependent part, $M(t)$, that represents the attenuation due to diffusion [24]:

$$\psi = M(t) \exp(-i\mathbf{r} \cdot \mathbf{k}(t)) , \quad (48)$$

where:

$$\mathbf{k}(t) = \gamma \int_0^t \mathbf{G}(t') dt' . \quad (49)$$

Substituting this into equation (47) and simplifying, we find the equation describing $M(t)$ to be:

$$\frac{dM}{dt} = -M(t) \mathbf{k}(t) \cdot D \cdot \mathbf{k}(t) , \quad (50)$$

which has a solution:

$$\begin{aligned}
 M(TE) &= M(0) \exp(-D \int_0^{TE} \mathbf{k}(t') \cdot \mathbf{k}(t') dt') \\
 &= M(0) \exp(-D \gamma^2 \int_0^{TE} (\int_0^t G(t') dt')^2 dt) \\
 &= M(0) \exp(-bD),
 \end{aligned} \tag{51}$$

where TE, the echo time, is the time at which the net transverse magnetization is measured. Here we have introduced the 'gradient factor', b, such that:

$$b = \gamma^2 \int_0^{TE} (\int_0^t G(t') dt')^2 dt, \tag{52}$$

which must be calculated for the particular pulse sequence being used and is essentially a measure of the sequence's sensitivity to diffusion. For the pulsed-gradient spin echo sequence shown in Fig. 7, the b-factor is found to be [26]:

$$b = (\gamma G \delta)^2 \left(\Delta - \frac{\delta}{3} \right). \tag{53}$$

On examination of equation (51), one sees that the diffusion coefficient may be determined by varying b and measuring the slope of the logarithm of the signal intensity versus b.

It is possible to combine MR diffusion experiments with MR imaging sequences to produce images in which contrast is determined primarily by diffusivity. Diffusion-weighted imaging, as it has come to be known, is accomplished by adding pulsed, diffusion sensitizing gradients to a basic imaging

sequence. Fig. 8 shows the pulse diagram for a diffusion-weighted 2DFT spin echo sequence. The diffusion sensitizing gradients (DSGs) are interleaved with the imaging gradients and may be applied along any of the three axes.

In addition to diffusion-weighted images, one may produce diffusion images or maps in which the diffusion coefficient is displayed for each pixel. These diffusion maps are generated by acquiring images for several different b-values, calculating the resulting diffusion coefficient for each pixel using equation (51), then mapping these values to a grey scale for display. In determining the b-value for each image it is important that equation (52) be utilized and all imaging gradients be taken into account.

2.6 The effects of motion on the MR signal

Motion during MR image acquisition leads to many problems including image unsharpness, ghosting artifacts and loss of intensity. These effects are especially pronounced in diffusion-weighted images because of the presence of large motion-sensitizing gradients in the imaging sequence. For

Figure 8: Diffusion-weighted 2DFT imaging sequence. Diffusion sensitizing gradients are interleaved with the imaging gradients and may be applied along any of the axes.

moderately sensitized sequences, even small amplitude motion can lead to complete loss of image detail and gross error in diffusion coefficient quantification.

In analyzing the effects of motion on the MR signal we consider two basic types, view-to-view motion and intra-view motion. The first type, view-to-view, occurs when the object changes position within the field of view between phase encoding steps. We will consider the effects of such motion in both the frequency and phase encoding directions. The second type of motion, intra-view, occurs when the object changes position within the field of view during the measurement of a single phase encoding step. This leads to variable phase shifts in the MR data, which, in diffusion weighted sequences can be very large. For simplicity, we focus on a single moving object within the field of view, initially located at position x_0, y_0 .

2.6.1 View-to-view motion in the frequency encoding direction

Position along the frequency encoding direction (x -axis) determines the precessional frequency of the spin during the signal acquisition period because a magnetic field gradient, G_x , is pulsed on during the read-out period. If the x -axis location of the spin should change between phase encoding steps, its precessional frequency during read-out will change by a proportionate amount. As a result, the frequency of the

MR signal will be shifted compared with the signal obtained when the object was at x, y . In determining how this affects the complex raw data points we recall that Fourier transformation of the MR signal provides us with the corresponding frequency spectrum. For discretely sampled data, the Fourier transform relationship is given by:

$$H_f = \sum_{j=0}^{N_x-1} h_j e^{\frac{j2\pi f}{N_x}}, \quad (54)$$

where h_j are the raw data points, H_f are the frequency domain points and N_x is the total number of samples of the spin echo signal. The Fourier shift theorem states that if $f(x)$ has the Fourier transform $F(j)$, then $f(x-\Delta x)$ has the Fourier transform $\exp(-i2\pi\Delta x j)F(j)$ [27]. When discrete transforms are considered, the phase rotation in the j th ($j=0$ to N_x-1) sample caused by a displacement of Δx pixels would be:

$$\Delta\phi_x = \frac{2\pi\Delta x j}{N_x}. \quad (55)$$

Hence view-to-view displacement in the frequency encoding direction causes a unique phase rotation of each complex point in the spin echo.

2.6.2 View-to-view motion in the phase encoding direction

Effects of view-to-view motion in the phase encoding (y-

axis) direction can be appreciated by considering the role of the phase encode gradient. It is pulsed on for a brief period to generate phase shifts proportional to spin location along the y axis. For a given phase encoding view k , ranging from 0 to N_y-1 , the amplitude and duration of the gradient is adjusted such that the total phase difference across the field of view in the y direction is [11]:

$$\Delta\phi_{tot} = 2\pi \left[k - \frac{N_y-1}{2} \right]. \quad (56)$$

A displacement of Δy pixels in the y direction would therefore cause a phase shift of each complex point in the spin echo:

$$\Delta\phi_y = 2\pi \left[k - \frac{N_y-1}{2} \right] \frac{\Delta y}{N_y}, \quad (57)$$

where N_y , the number of phase encoding steps, is also the total number of pixels across the field of view in the y -direction.

2.6.3 Intra-view motion in all directions

To determine the effects of intra-view motion, we must consider the evolution of the MR signal phase, ϕ , in the presence of field gradients. If TE, the echo time, represents the time at which the MR signal is measured, the signal phase is given by:

$$\begin{aligned} \Phi(TE) &= \int_0^{TE} d\Phi \\ &= \int_0^{TE} \gamma G(t) x(t) dt, \end{aligned} \quad (58)$$

where G is the magnetic field gradient amplitude and x is the position of the object within the field of view. For stationary objects, this phase will be entirely due to the phase encoding gradient. The other gradients, slice selecting and frequency encoding, will also cause dephasing but each has an associated compensation gradient to unwind the dephasing it produces. This also explains why the diffusion-sensitizing gradients are paired. For stationary spins, the phase shift caused by the first pulse will be exactly reversed by the second pulse producing zero net phase shift at the echo peak.

Moving spins will also acquire phase shifts due to the application of phase encoding, however, they will acquire additional phase shifts due to incomplete rephasing of the shifts caused by the other gradient pulses. The magnitude of the net phase shift will depend on several factors: the direction of motion with respect to the x,y, and z axes, the gradients applied in these directions, and the time derivatives (i.e. velocity, acceleration) of the motion. To illustrate, we consider phase error caused by bulk motion (to second order only) during the application of the diffusion-

sensitizing gradients, G_x .

Let the diffusion gradient pulse sequence be that shown in Fig. 8, with the gradients being applied along the x-axis. Also, assume the object originally located at position x_0 moves with velocity v_x and acceleration a_x . At the end of the first gradient period, the signal will have acquired a phase shift:

$$\begin{aligned} \phi_\delta &= \int_0^\delta d\phi \\ &= \int_0^\delta \gamma G_d \left(x_0 + v_x t + \frac{1}{2} a_x t^2 \right) dt, \end{aligned} \quad (59)$$

where G_d is the magnetic field gradient pulse amplitude and δ is its duration. Integration of this gives:

$$\phi_\delta = \gamma G_d \left(x_0 \delta + \frac{1}{2} v_x \delta^2 + \frac{1}{6} a_x \delta^3 \right). \quad (60)$$

Treating phase evolution during the second gradient period separately, the phase shift acquired at the end of this time is:

$$\phi_{\Delta, \delta} = \int_\Delta^{\Delta+\delta} \gamma G_d \left(x_0 + v_x t + \frac{1}{2} a_x t^2 \right) dt, \quad (61)$$

or:

$$\phi_{\Delta, \delta} = \gamma G_d \left(x_0 \delta + v_x \Delta \delta + \frac{1}{2} v_x \delta^2 + \frac{1}{2} a_x \Delta \delta (\Delta + \delta) + \frac{1}{6} a_x \delta^3 \right). \quad (62)$$

The 180° r.f. pulse reverses the phase ϕ_s to $-\phi_s$, so the net phase accumulation is:

$$\Delta\phi = \phi_{\Delta\delta} - \phi_s - \gamma G_d \Delta\delta \left(v_x + \frac{1}{2} a_x (\Delta\delta) \right) . \quad (63)$$

Thus, intra-view motion causes a phase shift of each point in the spin echo which is independent of initial position and proportional to both velocity and acceleration. Intra-view motion along the other axes will cause similar phase shifts of the MR signal.

2.7 The Navigator Echo Correction Technique

In 1989, Ehman and Felmlee [11] developed a unique motion correction technique which combines pulse sequence design with image postprocessing. Their method, the 'Navigator echo' approach, uses a spin echo sequence with two spin echoes, one of which is used to track object motion during image acquisition, while the other is used to reconstruct the image. With this method, it is possible to correct for rigid body translational view-to-view motion along the frequency encoding axis and intra-view motion in all directions.

The Navigator echo imaging sequence is shown in Fig. 9. The first spin echo, termed the navigator or Nav echo, is similar to an image echo except that no phase encoding is applied. This means that any phase shift at the peak of the

Figure 9: The two echo navigator imaging sequence. Two echoes are acquired for each phase encoding step. The first echo, the Nav echo, is not phase encoded and is used to track object motion. The second echo, the image echo, is phase encoded and is used to reconstruct the image.

Nav echo is due to object motion. As the name implies, the navigator echo is used to track object motion within the imaging field of view. The second echo, the image echo, includes phase encoding in the y-direction. Data obtained from this echo is used to reconstruct the image. Slice selection for both echoes is along the z-axis and frequency encoding is in the x-direction. Phase shift information derived from the Nav echo data is used to correct the image raw data before image reconstruction takes place.

The sequence shown in Fig. 9 is for a transverse slice. The navigator echo sequence can easily be implemented for any other slice orientation, including oblique and double oblique, by changing the orientation of the field gradients. The only restriction is that the slice selection, frequency encoding and phase encoding directions be orthogonal.

2.7.1 Correcting view-to-view motion in the frequency encoding direction

As outlined in the previous section, view-to-view motion along the frequency encoding direction causes a unique phase rotation of each complex point in the spin echo. This rotation, given in equation (55), depends on the number of pixels the object moves from the reference position. The shift, Δx , can be determined by examining the modulus of the navigator data after it has been Fourier transformed.

Fourier transformation of each navigator echo produces a series of one-dimensional projections of the object along the frequency encoding axis. These projections should, in principle, all be identical in the absence of motion since no phase encoding was applied. The modulus of each complex point in a projection represents signal amplitude for a specific x axis position. Amplitude for a given pixel should be the same from one projection to the next, however, view-to-view motion in the frequency encoding direction will cause shifts of the modulus from one view to the next. The strategy for determining how much a particular projection has been shifted is to compare the projection in question with an arbitrarily chosen reference projection. The two projections are then shifted with respect to each other by a variable number of pixels until they "match". This is done computationally by calculating cross-correlation functions between the reference navigator projection and the navigator projection being considered as a function of pixel offset. The pixel offset is varied from $-N_x$ to N_x , the number of points along the projection. The shift which produces the maximum correlation is then considered to be the x-displacement (in pixels) for that projection. This procedure is repeated for each of the navigator projections. It is assumed that the displacement determined in this manner for each of the navigator projections corresponds to the displacement for the corresponding image projections as well. In other words, it

is assumed either that the navigator and image echoes are acquired close enough together that, for a particular phase encoding step, there will be no appreciable motion between the two echoes or that velocity is constant between phase encoding steps so displacement between acquisition of the NAV echoes equals displacement between acquisition of the image echoes.

To correct the image echoes, one of two approaches can be taken. The displacements may be converted to phase rotations using equation (55), which can then be used to correct the image raw data. This is done by rotating each complex point of the image echo by the reverse of the calculated phase shift. If $\Delta\phi_x$ is the phase shift due to a displacement of Δx pixels, rotation by the reverse of this, $-\Delta\phi_x$, is done using the following formulas:

$$\begin{aligned} I' &= I \cos(-\Delta\phi_x) - Q \sin(-\Delta\phi_x) \\ Q' &= I \sin(-\Delta\phi_x) + Q \cos(-\Delta\phi_x) , \end{aligned} \tag{64}$$

where I and Q are the original real and imaginary values respectively, and I' and Q' are the corrected values.

The second approach is to correct the image data after each row of the raw data has been Fourier transformed but before the column transforms have been performed. Each row of this so-called "hybrid" data set corresponds to a (phase encoded) projection of the object. If Δx is the view-to-view displacement for a given phase encoding step, the image projection for that view may be corrected by simply shifting

each complex point in the projection by $-\Delta x$ pixels. Whether this approach or the one outlined above is used, the results are the same - the final image will appear as if the object was static at the reference position.

2.7.2 Correcting intra-view motion effects

Intra-view motion is that which occurs during the process of measuring a single phase encoding view. As discussed in the previous section, view-to-view motion leads to a pixel shift of the projection data and a corresponding phase shift of each point of the time domain signal, where each of the points will have a unique phase shift. Conversely, with intra-view motion, we observe a constant phase shift in the time domain and a corresponding phase rotation for each point along the projection. The amount of rotation, or phase 'error', for each point can be determined by examining the phase of the Fourier transformed navigator echoes after they have been corrected for view-to-view motion effects.

The procedure outlined in the previous section corrects both the magnitude and phase of the image data (raw or projection data depending on the method used) for the effects of view-to-view motion. The navigator data may be corrected in exactly the same way - by either rotating the phase of the raw data points or shifting the navigator projections by an appropriate amount. Any residual phase difference between the

navigator projections following such a correction must then be due to intra-view motion.

To determine the phase rotations due to intra-view motion, all of the navigator projections are first corrected for view-to-view motion effects. Next, the phase angle, ϕ , is calculated for each complex point in every Nav projection using:

$$\phi = \arctan \left(\frac{Q}{I} \right), \quad (65)$$

where I is the real value and Q is the imaginary value. The phase of each point in the reference navigator projection is then subtracted from the phase of the corresponding point in each of the other navigator projections. The difference, $\Delta\phi_i$, for each point is the phase 'error' due to intra-view motion.

To correct the navigator data for these effects one would simply rotate each complex point by the reverse of the phase error, $-\Delta\phi_i$, using equation (64). To correct the image projection data, the phase error must be added to the phase of each point, rather than subtracted, because the phase error is reversed by the 180° r.f. pulse following the navigator echo acquisition period. Phase addition is equivalent to rotation of the real and imaginary vectors by $\Delta\phi_i$ using equation (64).

3 Methods

A diffusion-weighted imaging sequence was designed such that two-dimensional motion of an object could be tracked during the image acquisition period. A correction algorithm was then developed to remove the undesirable effects of such motion from the raw data prior to image reconstruction.

Several experiments were carried out to test the precision of our motion correction technique. First, simulations were carried out to ensure that our three-echo navigator correction technique works in theory and that all the developed routines (i.e. Fourier transform and line fitting) were functioning properly. We then used simulations to investigate the effect of signal-to-noise level on the accuracy of our correction technique. Next, phantom studies were carried out to test the three-echo navigator correction scheme in practice. Finally, an experiment was performed to investigate the possibility of using our imaging sequence to acquire in vivo diffusion-weighted images.

3.1 Sequence Design

A diffusion-weighted spin echo imaging sequence was designed such that object position could be tracked along both the x and y axes and phase shifts due to intra-view motion in

all directions could be determined. The sequence, shown in Fig. 10, is a modified three echo spin echo sequence with diffusion sensitizing gradient pulses on either side of the first 180° refocusing r.f. pulse. These variable amplitude gradient pulses have duration δ and spacing Δ and can be applied along any of the axes. Slice selection for this sequence is along the z-direction.

The first echo, the image echo, forms at time TE1 and is used to reconstruct the image. This echo is phase encoded along the y-axis and frequency encoded in the x direction. Note that following the acquisition of this echo, there is an additional gradient pulse along the y-axis which is equal in amplitude but opposite in sign to the phase encoding gradient pulse. The role of this pulse is to unwind the phase rotation caused by the phase encoding gradient pulse prior to the formation of the second echo.

The second echo, the X-NAV echo, is used to track view-to-view motion along the x-axis and phase shifts due to intra-view motion in all directions. Forming at time TE2, it is not phase-encoded but it is frequency encoded along the x-axis.

The third echo, the Y-NAV echo, forms at time TE3 and is used to track view-to-view motion along the y-axis. Similar to the X-NAV echo, it is not phase-encoded, however, frequency encoding for the Y-NAV echo occurs along the y-axis.

Figure 10: The diffusion-weighted three echo navigator imaging sequence. Three spin echoes are acquired for each phase encoding step. The first, the image echo, is used to reconstruct the image, and is the only echo to be phase encoded. The second echo, the X-NAV echo, is used to track motion along the x direction and is consequently read-out along the x-axis. The final echo, the Y-NAV echo, tracks motion along the y direction and is therefore read-out along the y-axis. Diffusion sensitizing gradients are interleaved with the imaging gradients and may be applied along any axis.

3.2 Three Echo Navigator Correction Technique

An algorithm was developed which uses displacement and phase shift information derived from the navigator echoes to correct for motion effects in the image echo raw data. The approach we use is similar to that of Ehman et al. [11], however, the addition of a Y-NAV echo to our imaging sequence makes it possible to correct not only for x-axis view-to-view displacements and intra-view displacements in all directions, but also for y-axis view-to-view displacements. The computer programs written to implement this correction technique have been included in the appendix.

Determining x-axis view-to-view displacements

To determine view-to-view displacements in the x direction, we use Ehman's [11] technique as described in section 2.7.1 .

All X-NAV echoes (i.e. one for each phase encoding step) are Fourier transformed into projections using a fast Fourier transform routine (FFT) for discretely sampled data [28]. Modulus values are then calculated for each complex point in every projection. A single one of these navigator projections is arbitrarily chosen as a reference and a set of cross-correlations (as a function of pixel offset) are calculated with respect to each of the other projections. The cross-correlation routine works as follows.

Modulus values of the reference view are shifted by an amount Δx from their original positions (i.e. the modulus originally at pixel j will now be the modulus at pixel $j+\Delta x$). Each point in this shifted projection is then multiplied by the modulus of the corresponding point in the first phase encoding view. The products for all pixels are added together and the sum is found as a measure of correlation for the two projections. The higher the sum, the greater the correlation. The same procedure is then carried out for a shift of $\Delta x+1$ pixels and so on until measures of correlation are obtained for shifts running from $-N_x$ to $+N_x$, where N_x is the total number of pixels in the x direction. The shift which produces maximum correlation is stored as the x -axis view-to-view displacement for the first phase encoding step. The entire cross-correlation procedure is repeated for every phase encoding view until displacements have been determined for each one.

Determining phase shifts due to intra-view motion

To determine phase shifts due to intra-view motion, the X-NAV projections are first corrected for the effects of view-to-view motion. This is done by shifting each projection by the negative of the displacement determined in the cross-correlation routine. For example, if the cross-correlation routine returned a shift of Δx for the first phase encoding step, each point in this projection is shifted by $-\Delta x$ pixels.

Once shifted, phase values are found (using equation (65)) for each point in every projection. The phase of each point in the reference projection is subtracted from the phase of the corresponding point in each of the other navigator projections. The phase difference, $\Delta\phi_i$, for each point is the phase shift due to intra-view motion and is stored for use in correcting the image echo data.

Determining y-axis view-to-view displacements

Y-axis view-to-view displacements are found using the same method used to determine x-axis view-to-view displacements. All Y-NAV echoes are Fourier transformed into projections using the FFT routine for discretely sampled data. Modulus values are calculated for all the complex points in every projection. The cross-correlation routine is then run for each phase encoding step until the view-to-view y-axis displacement for each projection has been determined.

Implementing the correction

Having found the view-to-view displacements and phase shifts due to intra-view motion, the image echo data may now be corrected for their effects. First, all the image echoes (i.e. the rows of the raw data set) are Fourier transformed into projections using the FFT routine. Each projection is then shifted by the negative of the x-axis displacement (in pixels) determined through the cross-correlation routine.

This removes the effects of view-to-view x-axis displacement. Next, all the complex points are rotated by the phase shift, $\Delta\phi_p$, for that point to remove the effects of intra-view motion. Finally, the y-displacements for each view are converted to phase shifts using equation (57). Each point in every projection is then rotated by the reverse of this phase shift to remove the effects of y-axis view-to-view displacement.

Following correction, the image is generated from the projection data by Fourier transforming the data set along the phase encoding direction using the FFT routine. The modulus of each complex point is then calculated and these values are mapped to a grey scale for display.

3.3 Motion Simulations

Motion simulations were performed to demonstrate the effects of object motion in diffusion-weighted spin-echo imaging sequences and to test the accuracy of a three-echo navigator correction scheme. The first study involves simulations on noiseless images while the second considers cases in which signal to noise levels decrease from 100:1 (essentially noiseless) to 3:1 (very poor signal to noise).

3.3.1 Noiseless Simulations

A series of images of a uniform circular object were computer generated with various levels of diffusion sensitization. Motion effects were then incorporated into the raw data sets and a second set of images obtained. Finally, the motion correction algorithm was run on each of the motion degraded images to produce a third set of images - the corrected images. A fitting routine was developed to determine the self-diffusion coefficient for each set of images.

Generating motionless raw data sets and images

A density function was developed to define the object of interest - a 100mm radius circle centred in a 300mm square field of view. This function generated a 256x256 matrix whose elements equalled 1 everywhere within the circle and 0 everywhere outside. Image raw data was obtained by two-dimensional inverse Fourier transformation of the density matrix using the FFT routine. Diffusion effects were then included by multiplying each point in the raw data matrix by an attenuation term, $\exp(-bD)$ (see Eq. 51), for several b values. The diffusion coefficient, D , was chosen to be $2.0E-3$ mm/sec, a value representative of the self diffusion coefficient of water at room temperature. Raw data matrices were obtained for b -values of 0, 21.37, 85.49, 192.34, 341.94

and 534.29 s/mm². These values correspond to a sequence in which the diffusion gradients have duration 40 ms, spacing 60 ms and amplitudes of 0, 2, 4, 6, 8 and 10 mT/m respectively. To obtain the actual diffusion-weighted images, two-dimensional Fourier transforms were performed on each raw data set and modulus values were found for each pixel. These values were then mapped to a grey scale for display.

Navigator echo raw data for each image was obtained by choosing one row of the image raw data (corresponding to one phase-encoding step) and making all other rows of the raw data matrix identical. For these simulations, the 128th row of image raw data was chosen, for it has the smallest amount of phase encoding. This technique was used to generate both the X-NAV and Y-NAV raw data matrices for each image.

Adding motion effects

Having obtained images corresponding to a stationary object, the raw data was then manipulated to include motion effects. Rigid body motion in two dimensions was introduced such that:

$$\begin{aligned} x &= x_0 + A \sin(2\pi\nu_x t) \\ y &= y_0 + B \sin(2\pi\nu_y t) \end{aligned} \quad (66)$$

Sinusoidal motion was chosen in an attempt to mimic physiologic motion such as respiratory motion. Different motion amplitudes and frequencies were used for each image and

Image Number and b-value (s/mm ²)	X-Axis Motion $x = x_0 + A \sin(2\pi v_x t)$		Y-Axis Motion $y = y_0 + B \sin(2\pi v_y t)$	
	A cm	v_x s ⁻¹	B cm	v_y s ⁻¹
Image 1 b = 0.0	1.0	1.5	1.8	1.6
Image 2 b = 21.37	1.8	2.0	1.2	2.2
Image 3 b = 85.49	1.4	1.8	2.0	1.8
Image 4 b = 192.34	2.0	2.5	1.0	1.0
Image 5 b = 341.94	1.2	1.0	1.6	2.5
Image 6 b = 534.29	1.6	1.3	1.4	1.2

Table 1: Motion parameters for simulation studies. Initial position, (x_0, y_0) , was the same for all images at $(0,0)$, the centre of the field of view.

are summarized in table (1). For each image it was assumed that the object started at the same position, $x_0=0$ and $y_0=0$, the centre of the field of view.

To incorporate motion effects into the raw data matrices (image, X-NAV and Y-NAV) phase shifts due to both view-to-view and intra-view motion were determined and applied to the appropriate raw data sets.

To determine x-axis view-to-view displacements, object position at the beginning of each phase encoding step was calculated:

$$x(i) = x_0 + A \sin(2\pi v_x(i-1)TR) , \quad (67)$$

where i is an index for the N_y phase encoding steps and TR is the time between phase encoding views. The displacement for a given view is then:

$$\Delta x(i) = x(i) - x_0 . \quad (68)$$

This shift, in meters, is converted to number of pixels:

$$\Delta x_{pix}(i) = \Delta x(i) \frac{N_x}{FOV} , \quad (69)$$

where N_x is the total number of pixels in the x-direction and FOV is the size of the field of view in meters. The phase rotation caused by such a displacement is calculated using equation (55) then applied to the real and imaginary parts of both the image raw data and the X-NAV raw data using equation

(64) .

Y-axis view-to-view displacements are found in a manner similar to that outlined above. Position at the beginning of each phase encoding step is found:

$$y(i) = y_0 + B \sin(2\pi v_y(i-1)TR) , \quad (70)$$

and displacements are calculated:

$$\Delta y(i) = y(i) - y_0 , \quad (71)$$

or:

$$\Delta y_{pix}(i) = \Delta y(i) \frac{N_y}{FOV} . \quad (72)$$

The phase rotation such a displacement in the phase encoding direction would cause is found using equation (57) and applied to the real and imaginary parts of the image raw data points using equation (64). To incorporate these effects into the Y-NAV echo, which is frequency encoded in the y-direction, equation (55) must be used to calculate the phase rotation caused by the displacement. Equation (64) is then used to generate the resulting Y-NAV raw data points.

In adding intra-view motion effects to the raw data, only phase shifts due to motion between the diffusion sensitizing gradient (DSG) pulses were considered. For these simulations, diffusion sensitization was chosen to be along the y-axis. Position during the first DSG pulse was found using:

$$y_1(i) = y_0 + B \sin(2\pi v_y((i-1)TR \cdot t_1)) , \quad (73)$$

where t_1 is the time from the beginning of the phase encoding step to the beginning of the gradient pulse. Similarly, position during the second DSG pulse was found:

$$y_2(i) = y_0 + B \sin(2\pi v_y((i-1)TR \cdot t_2)) , \quad (74)$$

where t_2 is the time from the beginning of the phase encoding step to the beginning of the second DSG pulse. The net dephasing produced by the gradient pulse pair is found by subtracting the phase shift caused by the first DSG from that caused by the second:

$$\begin{aligned} \Delta\phi(i) &= \phi_2(i) - \phi_1(i) \\ &= \gamma G \delta y_2(i) - \gamma G \delta y_1(i) \\ &= \gamma G \delta (y_2(i) - y_1(i)) . \end{aligned} \quad (75)$$

Here, γ is the gyromagnetic ratio, G is the amplitude of the diffusion sensitizing gradient pulses and δ is their duration. For simplicity, we assumed that displacement during each of the individual gradient pulses was negligible. To incorporate this intra-view motion phase shift into the image raw data, each point is rotated by $\Delta\phi(i)$ using equation (64). To incorporate it into the X-NAV raw data, each point is rotated by $-\Delta\phi(i)$ using equation (64).

Once motion effects were incorporated into all raw data matrices, images were obtained through 2-dimensional Fourier transformation of the image raw data sets using the FFT

routine. Modulus values were calculated for each pixel then mapped to a grey scale for display. It is important to note that these images were mapped to a grey scale using the same mapping parameters as were used in mapping the original (stationary) images to a grey scale.

Correcting the images

To obtain the third set of images - the corrected images, our motion correction algorithm was run on all the degraded image raw data sets to produce corrected image projection data sets. These projection matrices were then FFT'd along the phase encoding direction to generate corrected images. For image display, modulus values were calculated for each pixel then mapped to a grey scale for display. Again, the same mapping parameters were used as with the set of stationary images.

To demonstrate the inadequacy of a correction scheme which uses just one navigator echo (i.e. the original two echo navigator correction technique of Ehman et al. [11]), we corrected one of our motion degraded images using information derived from the X-NAV echo only. To do so, we simply set all of the y-axis view-to-view displacements determined from the Y-NAV echo to zero before correcting the image projection data. An image was then generated by Fourier transforming the resulting 'corrected' data along the phase encoding direction using the FFT routine, calculating modulus values for each

pixel and mapping the values to a grey scale (using the mapping parameters determined from the set of stationary images) for display.

Determining the self-diffusion coefficients

A fitting routine was developed so diffusion coefficients for each set of images could be determined. The routine reads in pixel intensity values and b-values for all images in a set and fits the data to an exponential of the form (see Sec 2.5):

$$S = S_0 \exp(-b \cdot D) , \quad (76)$$

where S is pixel intensity, b is b-value, and D is the self-diffusion coefficient. Diffusion coefficients may be found for individual pixels in the image or for a region of interest within the field of view. If a region is used, one simply inputs mean pixel intensity within the region into the fitting routine. Fitting is done with a general least squares fitting routine with two adjustable parameters, S_0 , the initial signal, and D , the diffusion coefficient. In addition, the routine was equipped with an error handler to return a diffusion coefficient of 0 mm/sec when a fit could not be made.

3.3.2 Simulations including noise

A study was carried out to determine the extent to which signal to noise level affects the accuracy of the three echo navigator correction technique. Raw data matrices were generated in the same manner as outlined above, however, prior to adding motion effects, noise was added to each of the raw data sets (image, X-NAV and Y-NAV).

The first step in the process of adding noise was determining the two maximum noise values - max. real noise and max. imaginary noise. This was done by determining the maximum real and imaginary values of the image raw data for the first image in the set (i.e. the one having a b-value of 0) and dividing each by the square root of two times the selected signal to noise ratio, since we define the noise level as the peak noise multiplied by the square root of two. For example, the maximum noise values for our first set of images (signal to noise level of 100:1) was found by dividing the maximum real and imaginary values of the image raw data by $100 * \sqrt{2} = 141.42$.

Once the maximum values were determined, noise was added to each raw data point in the following manner. A random number between 0 and 1 was generated then scaled and shifted so it fell in the range -1 to +1. The maximum real noise value was multiplied by this random number to produce a random noise value (in the range -maximum real noise to +maximum real

noise) which was added to the first real point in the raw data matrix. The same procedure was then carried out for all other real points in the raw data matrix. Precisely the same steps were followed to add imaginary noise to each point, however, the random numbers were multiplied by the maximum imaginary noise level to produce random imaginary noise values in the range -maximum imaginary noise to +maximum imaginary noise. Once noise had been added to all raw data sets, images were obtained by two-dimensional Fourier transformation of the image raw data matrices using the FFT routine. Modulus values were calculated for each pixel then mapped to a grey scale for display.

Having generated the 'still' images, motion effects were added to the raw data sets using the procedure outlined in the previous section. Motion parameters were again those shown in table (1). Images were produced by Fourier transformation in two dimensions using the FFT routine. Modulus values were calculated for each pixel and mapped to a grey scale (using the same mapping parameters as were used for the stationary images) for display.

The motion correction algorithm was run on each of the motion degraded images and a series of corrected images were produced by Fourier transformation of the corrected image projection data sets using the FFT routine. To display the images, modulus values were calculated for each point then mapped to a grey scale using the mapping parameters determined

from the set of stationary images.

The entire procedure of adding noise, generating still images, adding motion, generating motion degraded images, correction and generating corrected images was repeated for signal to noise levels of 100:1, 10:1 and 3:1.

3.4 Phantom Studies

A series of phantom studies were performed to demonstrate the ability of the three-echo navigator sequence to accurately determine diffusion coefficients in the absence of motion and to test the precision of the correction technique when motion has occurred. All images were acquired on a 1.5 T Siemens Magnetom whole body imaging system.

To analyze the data acquired with the imager, it was necessary to write several routines to read in the pixel information, write the corrected values back into an image file, and to display the motion corrected images and diffusion maps on the imaging console. These tasks were facilitated by incorporating Siemen's user library [30] subroutines into the software.

3.4.1 Stationary water phantom experiment

An experiment was carried out on a stationary phantom to

verify the imaging sequence's ability to accurately determine a self-diffusion coefficient in the absence of motion. A series of transverse images of a spherical water phantom were acquired using the imaging sequence described in section 3.1. Echo times for the sequence were 109 ms for the image echo, 132 ms for the X-NAV echo and 155 ms for the Y-NAV echo. Diffusion gradient pulses were of duration 40 ms and separation 49.88 ms. Imaging was performed using a 0.3 m field-of-view, 15 mm slice thickness, 256 phase-encoding views and a TR of 0.5 s. The total time to acquire one image was 2 min 12 s. In all, nine images were obtained having b-values of 16.74, 37.66, 66.95, 104.61, 150.63, 205.03, 267.79, 338.92 and 418.42 sec/mm^2 . To evaluate the diffusion coefficient for this image set, the least squares fitting routine outlined in section 3.3.1 was utilized.

3.4.2 Moving phantom experiment

This experiment was designed to evaluate the effectiveness of our three echo navigator correction scheme in experimentally acquired images.

A series of coronal images of a stationary grapefruit were acquired using the imaging sequence described in section 3.1. A grapefruit was selected for the experiment for several reasons. First, it was chosen because it has a high water

content, a feature necessary for significant signal to noise in the reconstructed images. Second, the water in the grapefruit is secured within tiny sac-like structures or membranes. This prevents a mixing or stirring effect from occurring when the grapefruit is moved about during the imaging procedure. Finally, it has an interesting internal structure, a feature which facilitates comparison between the stationary and corrected images. Echo times for the sequence were 109 ms, 132 ms and 155 ms for the image, X-NAV and Y-NAV echoes respectively. All images were obtained with a 0.2 m field of view, 15 mm slice thickness, 1.5 s repeat time and 256 phase encoding steps. The time to acquire one image was 6 min 27 s. In all, eight images were obtained, having b-values of 37.66, 66.95, 104.61, 150.63, 205.03, 267.79, 338.92 and 418.42 s/mm². The diffusion coefficient for a region of interest within the field-of-view was found using the least squares fitting routine.

In addition to finding the diffusion coefficient for a region of interest, a program was written to generate a diffusion map of the entire field of view. The algorithm uses Siemen's user library [30] routines to access pixel information from each of the image data files (one for each diffusion-weighted image). Diffusion coefficients were determined for each pixel using the least squares fitting routine and mapped to a grey scale for display.

A second series of diffusion-weighted images of the

grapefruit was obtained using the same imaging parameters as those listed above, however, this time arbitrary motion in the image plane (i.e. the xz plane) was manually imparted on the grapefruit during image acquisition. The motion was quite random, but care was taken to keep it translational rather than rotational. In order to keep the translational motion as general as possible, the speed and direction of the motion was changed arbitrarily and at random intervals so as to make any trajectory within the range of motion equally probable. Object displacement was kept to a few centimetres in each direction to ensure it did not leave the field-of-view. Rate of motion varied but did not exceed 3 cm/s. Again, eight images were obtained, having the same b-values as those listed above for the stationary grapefruit.

The correction algorithm was then run on the raw data sets and new images were obtained by Fourier transforming the corrected projection data using the FFT routine. Modulus values were calculated for each pixel then mapped to a grey scale for display. Mapping parameters used in generating these images were the same as those used in generating the sets of stationary and motion degraded images. Again, Siemen's user library routines [30] were used to facilitate reading and writing of pixel information in the image data files.

The diffusion coefficient for a region of interest was found using the fitting routine and a diffusion map was

generated from the set of corrected images using the technique outlined above.

To demonstrate the difference between our three echo correction technique and the original two echo approach, we corrected one of the motion degraded images using information derived from the X-NAV echo only. This was done by setting all of the y-axis displacements (and corresponding phase shifts) determined from the Y-NAV echoes to zero before correcting the image raw data. An image was generated from the resulting 'corrected' data by Fourier transforming the set of projections along the phase encoding direction, calculating modulus values for each pixel then mapping the values to a grey scale for display. Again, care was taken to ensure that the same mapping parameters as those used in generating the other images were used.

3.5 In Vivo Studies

An experiment was carried out to evaluate the effectiveness of our correction technique under in vivo imaging conditions. A series of transverse images of the head of a human volunteer were acquired using the imaging sequence described in section 3.1. Echo times for the sequence were 105 ms, 136 ms and 167 ms respectively for the image, X-NAV and Y-NAV echoes. Diffusion gradient pulses were of duration 40 ms and separation 49.88 ms. All images were acquired on a

Siemens Magnetom 1.5 T whole body imaging system using a .25 m field of view, 15 mm slice thickness, 256 phase encoding steps and a 1.2 s repeat time. In all, six images were obtained, having b-values of 0.0, 14.91, 59.62, 134.15, 238.48 and 372.63 s/mm². The time to acquire one image was 5 min 11 s. The diffusion coefficient for a region of interest within the field of view was calculate for this image set using the least squares fitting routine.

To correct each diffusion weighted image for motion effects, the correction algorithm was run on the raw data sets and new images generated by Fourier transforming the corrected projection data using the FFT routine. Modulus values were calculated for each pixel then mapped to a grey scale for display. The parameters used in mapping were the same as those used in displaying the set of original images. The diffusion coefficient for a region of interest was found using the least squares fitting routine. Siemen's user library [30] routines were incorporated into the software to facilitate access to the pixel data in each of the raw data files.

4 Results

4.1 Noiseless Motion Simulations

Six images of a uniform circular object were computer generated with various levels of diffusion sensitization. The diffusion coefficient used in generating the images was $2.00\text{E-}3 \text{ mm}^2/\text{s}$. One of these images, having a b-value of 192.34 s/mm^2 , is shown in Fig. 11. The other images were identical except that pixel intensity decreased as the level of diffusion sensitization increased. Plot of the signal strength of the centre pixel versus b-value for these images is given in Fig. 12. When the data was fit to an exponential function of the form $S=S_0\exp(-b*D)$, a diffusion coefficient, D, of $2.00\text{E-}3 \pm 1.56\text{E-}6 \text{ mm}^2/\text{s}$ was returned.

Motion effects were added to each of the raw data sets and a second set of images generated. The image having a b-value of 192.34 s/mm^2 is shown in Fig. 13. Ghosting artifacts and blurring make it virtually impossible to discern the circular object. The other images exhibited similar degradation due to motion effects. Signal amplitude of the centre pixel versus b-value has been plotted for the image set in Fig. 14. The diffusion coefficient returned for this data set was $4.00\text{E-}3 \pm 5.08\text{E-}4 \text{ mm}^2/\text{s}$.

Attempts were made to correct each motion degraded image

Figure 11: Computer generated image of a stationary circular object. The b-value for the image is 192.34 s/mm².

Figure 12: Plot of signal decay for the set of stationary, noiseless images. Intensity of the centre pixel of each image is plotted against b-value (set of ◇'s). Fitting the points to an exponential of the form $S=S_0\exp(-b*D)$ returned a diffusion coefficient of $D=2.00E-3 \pm 1.56E-6 \text{ mm}^2/\text{s}$, which is given by the solid line.

Figure 13: Computer generated image of a circular object undergoing translational rigid body motion within the field of view. The b-value for the image is 192.34 s/mm' . Severe ghosting artifacts and blurring cause complete loss of object detail.

Figure 14: Plot of signal decay for motion degraded and corrected image sets. When fit to an exponential the points corresponding to the motion images (the $\square$'s) returned a diffusion coefficient, D , of $4.00E-3 \pm 5.08E-4 \text{ mm}^2/\text{s}$. The points for the corrected images (the $\circ$'s) fall directly on those for the stationary images, and consequently return the same diffusion coefficient, $D=2.00E-3 \pm 1.56E-6 \text{ mm}^2/\text{s}$.

using the three-echo Navigator correction algorithm. Fig. 15(a) is an image of the modulus values of the X-NAV projection data (i.e. the lines of the raw data set have been Fourier transformed but not the columns) obtained during generation of the image shown in Fig. 13. Each horizontal line illustrates the transverse position of the object at the time of signal acquisition. Shifts from row to row are due to view-to-view motion along the x-axis. Following correction, the projections all line up as shown in Fig. 15(b).

Y-NAV projection modulus values displayed behaviour similar to that of the X-NAV modulus values. Prior to correction, there were shifts from row to row due to view-to-view motion along the y-axis. Implementation of the navigator correction scheme determined and removed these shifts so all projections lined up.

Fig. 16(a) is also an image of X-NAV projection data, however in this case phase values are displayed. Variations in phase from line to line are due to the intra-view motion of the object. Following correction, the phase along each vertical column is constant, as shown in Fig. 16(b).

Applying the three echo navigator correction technique to the raw data of the image shown in Fig. 13 yielded a projection data set which was used to reconstruct the image shown in Fig. 17. All traces of motion degradation have been removed and the image is identical to that obtained in the absence of motion. Correction of the other five images was

(A) **(B)**

Figure 15: Images of the X-NAV projection modulus values before and after correction. **(A)** Prior to correction there are shifts in the modulus from row to row due to view-to-view x-axis displacement. **(B)** Following correction, all the projections line up.

(A)

(B)

Figure 16: Images of the X-NAV projection phase values before and after correction. (A) Prior to correction, there are phase differences from row to row due to intra-view motion of the object. (B) Following correction, the phase along each vertical column is constant.

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PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
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Figure 17: Computer generated image of the circular object obtained following correction of the motion degraded raw data set using the three echo navigator correction technique. The b-value for the image is 192.34 s/mm^2 . All traces of motion have been removed and the image is identical with that generated for a stationary object, as shown in Fig. 11.

equally successful. Signal intensity of the centre pixel in each of the corrected images is plotted versus b-value in Fig. 14. The points fall directly on those for the set of static images and the fitting algorithm returns a diffusion coefficient of $2.00\text{E-}3 \pm 1.56\text{E-}6 \text{ mm}^2/\text{s}$.

One of the motion degraded images, having a b-value of 192.34 s/mm^2 , was corrected using the original two echo navigator correction technique to produce the image shown in Fig. 18. Effects of view-to-view x-axis displacements and intra-view motion have been removed but considerable artifacts due to y-axis view-to-view displacements are still visible.

4.2 Motion Simulations including Noise

A series of six diffusion-weighted images of a stationary circular object were computer generated to have a signal-to-noise level of 100:1. One of these images, having a b-value of 85.49 s/mm^2 , is shown in Fig. 19. On close examination, slight fluctuations in pixel intensity within the circular object can be observed. Mean pixel signal versus b-value for a 10 pixel by 10 pixel region at the centre of each image has been plotted in Fig. 20. Error on each point is taken as the standard deviation of the signal within the 100 pixel area. The diffusion coefficient for this set of motionless images was found to be $2.00\text{E-}3 \pm 1.57\text{E-}5 \text{ mm}^2/\text{sec}$.

Figure 18: Computer generated image of the circular object obtained following correction of the motion degraded raw data using the original two echo navigator correction technique. B-value for the image is 192.34 s/mm^2 . Although the image is improved compared with the one shown in Fig. 13, considerable artifact due to y-axis view-to-view motion remains in the corrected image.

Figure 19: Image of a stationary circular object computer generated with a signal-to-noise level of 100:1. B-value for the image is 85.49 s/mm². Slight fluctuations in pixel intensity can be seen within the object.

Signal Decay for Motion Simulations

Signal-to-Noise level of 100:1

Figure 20: Plot of signal decay for stationary and corrected images having signal-to-noise levels of 100:1. Mean pixel intensity in a 100 pixel area at the center of each image has been plotted against b-value for the set of stationary images (Δ 's) and for the set of corrected images ($\circ$'s). The points for corresponding images fall directly on top of one another. Error bars are too small to be seen in the graph. Diffusion coefficients determined for the data sets were identical at $2.00E-3 \pm 1.57E-5 \text{ mm}^2/\text{s}$.

After manipulating the raw data to include motion effects, the correction algorithm was run on each of the motion degraded images to obtain a set of corrected images. The corrected image having a b-value of 85.49 s/mm² is shown in Fig. 21. This image is identical to the one shown in Fig. 19, which is for the stationary object. Signal intensity versus b-value for the corrected images is plotted in Fig. 20. The points fall directly on those for the stationary images and fitting of the data returned a diffusion coefficient of $2.00\text{E-}3 \pm 1.57\text{E-}5 \text{ mm}^2/\text{s}$.

A set of stationary, diffusion-weighted images was generated to have a signal-to-noise level of 10:1. The image having a b-value of 192.34 s/mm² is shown in Fig. 22. Fluctuations in pixel value within the circular object are readily observable, however, the object is still obvious within the field of view. Mean pixel value within a 10 pixel by 10 pixel region at the centre of each image is plotted versus b-value for the stationary images in Fig. 23. Error on each point is taken as the standard deviation of the signal within the 100 pixel area. The diffusion coefficient determined for this set of stationary images was $2.00\text{E-}3 \pm 1.57\text{E-}4 \text{ mm}^2/\text{sec}$.

After adding motion effects to the raw data, the corresponding set of corrected images was generated, one of which is shown in Fig. 24. The image, having a b-value of 192.34 s/mm², is essentially indistinguishable from that

Figure 21: Image of the circular object obtained following correction of the motion degraded raw data (signal-to-noise level of 100:1) using our three echo navigator correction technique. B-value for the image is 85.49 s/mm². The image is identical to that for a stationary object, as shown in Fig. 19.

Figure 22: Image of a stationary circular object computer generated with a signal-to-noise level of 10:1. B-value for the image is 192.34 s/mm^2 . Fluctuations in pixel intensity are readily observable in the image, however, the object is still apparent in the field of view.

Signal Decay for Motion Simulations

Signal-to-Noise level of 10:1

Figure 23: Plot of signal decay for stationary and corrected images having signal-to-noise levels of 10:1. Mean pixel intensity in a 100 pixel area at the center of each image has been plotted against b-value for the set of stationary images (Δ 's) and for the set of corrected images ($\circ$'s). The points for corresponding images fall directly on top of each other. Diffusion coefficients calculated for the data sets were identical at $2.00E-3 \pm 1.57E-4 \text{ mm}^2/\text{s}$.

Figure 24: Image of the circular object obtained following correction of the motion degraded raw data (signal-to-noise level of 10:1) using our three echo navigator correction technique. The image is indistinguishable from that corresponding to the stationary case, as shown in Fig. 22. B-value for the image is 192.34 s/mm^2 .

obtained in the absence of motion. Mean pixel value within a 10 pixel by 10 pixel box at the centre of each image is plotted versus b-value in Fig. 23. The points fall directly on those for the corresponding stationary images and fitting returned an identical diffusion coefficient of $2.00\text{E-}3 \pm 1.57\text{E-}4 \text{ mm}^2/\text{s}$.

A third series of diffusion-weighted images was then generated to have a signal-to-noise level of 3:1. The image having b-value 341.94 s/mm^2 is displayed in Fig. 25. Although quite mottled, the circular object can still be discerned within the field of view. Mean pixel signal versus b-value for a 10 pixel by 10 pixel region at the centre of each image is plotted in Fig. 26. Error on each point is taken as the standard deviation of the signal within the region of interest. Application of the fitting algorithm to this data set returned a diffusion coefficient of $2.00\text{E-}3 \pm 5.15\text{E-}4 \text{ mm}^2/\text{s}$.

Each raw data set was then manipulated to include motion effects and our correction algorithm put to the test. The corrected image having b-value 341.94 s/mm^2 is shown in Fig. 27. Again, the image is quite mottled due to a high noise level, however there are no traces of motion artifact. Mean pixel signal within the 100 pixel region of interest is plotted against b-value in Fig. 26. The first five points fall directly on those for the stationary images and only a slight difference can be seen in the final point. The

Figure 25: Image of a stationary circular object computer generated to have a signal-to-noise level of 3:1. B-value for the image is 341.94 s/mm^2 . The image is quite mottled due to the high noise level.

Signal Decay for Motion Simulations

Signal-to-Noise level of 3:1

Figure 26: Plot of signal decay for stationary and corrected images having signal-to-noise levels of 3:1. Mean pixel intensity in a 100 pixel area at the center of each image has been plotted against b-value for the set of stationary images (Δ 's) and for the set of corrected images ($\circ$'s). The diffusion coefficient determined for the set of stationary images was $2.00E-3 \pm 5.15E-4 \text{ mm}^2/\text{s}$ while that obtained for the corrected images was $1.99E-3 \pm 4.51E-4 \text{ mm}^2/\text{s}$.

Figure 27: Image of the circular object obtained following correction of the motion degraded raw data (signal-to-noise level of 3:1) using our three echo navigator correction technique. B-value for the image is 341.94 s/mm^2 . Although quite mottled, the image contains no visible signs of motion artifact.

diffusion coefficient determined for the set of corrected images was $1.99\text{E-}3 \pm 4.51\text{E-}4 \text{ mm}^2/\text{s}$.

4.3 Diffusion in a Stationary Water Phantom

Nine diffusion-weighted images of a stationary water phantom were acquired, one of which (b-value = 205.03 s/mm^2) is shown in Fig. 28. The other images were identical except that pixel intensity decreased with increasing b-value. Mean signal in a 10 pixel by 10 pixel region of interest at the centre of each image was evaluated and is plotted versus b-value in Fig. 29. Error on each point was taken as the standard deviation of the signal within the 100 pixel area. Application of the fitting algorithm to this data returned a diffusion coefficient of $2.02\text{E-}3 \pm 4.58\text{E-}5 \text{ mm}^2/\text{s}$.

4.4 Moving Phantom Study

Eight coronal images of a stationary grapefruit were acquired, each having a different level of diffusion sensitization. One of these images, with a b-value of 66.95 s/mm^2 , is shown in Fig. 30. The other images were identical except that signal amplitude fell off as level of diffusion sensitization increased. The mean pixel value for a 14 pixel by 14 pixel box within one of the grapefruit sections is plotted versus b-value in Fig. 31. Error on each point is

Figure 28: Diffusion-weighted MR image of a stationary water phantom. B-value for the image is 205.03 s/mm^2 . The black spot at the top of the circle corresponds to an air bubble in the phantom.

Signal Decay for Stationary Water Phantom

Figure 29: Plot of signal decay for the stationary water phantom. Mean pixel intensity for a 100 pixel region at the center of each image is plotted against b -value. Error bars are too small to be seen in the graph. The diffusion coefficient determined for the data set was $2.02E-3 \pm 4.58E-5 \text{ mm}^2/\text{s}$.

Figure 30: Diffusion-weighted MR image of a stationary grapefruit. The coronal image has a b-value of 66.95 s/mm^2 . The individual sections are clearly visible and regions of low intensity corresponding to seeds can be seen near the grapefruit center.

Figure 31: Plot of signal decay for the stationary and corrected grapefruit images. Mean pixel intensity within a 196 pixel area in one of the grapefruit sections has been plotted against b-value for the set of stationary images (Δ 's) and for the set of corrected images ($\circ$'s). The diffusion coefficient found for the stationary images was $1.32E-3 \pm 6.17E-5 \text{ mm}^2/\text{s}$ while that found for the corrected images was $1.33E-3 \pm 8.19E-5 \text{ mm}^2/\text{s}$.

taken as the standard deviation of the signal in the 196 pixel region of interest. The diffusion coefficient determined for this set of images was $1.32\text{E-}3 \pm 6.17\text{E-}5 \text{ mm}^2/\text{s}$.

A diffusion map of the grapefruit was generated from the set of stationary images and is shown in Fig. 32. In this image, intensity of each pixel reflects the self diffusion coefficient for each tiny element of the grapefruit. Quite similar in appearance to the image of Fig. 30, the grapefruit outline and individual sections are still clearly visible.

Another eight diffusion weighted coronal images of the grapefruit were acquired, however, this time the phantom was manually shifted transversely in the x,y plane during the image acquisition periods (see section 3.4.2). One of these motion degraded images, having a b-value of 205.03 s/mm^2 , is shown in Fig. 33. Object detail has been completely lost due to blurring and ghosting artifacts. The other images were degraded in a similar manner.

Our correction algorithm was run on each motion image to generate a corresponding set of eight corrected images. The image that was obtained following correction of the image shown in Fig. 33 is displayed in Fig. 34. All visible traces of motion artifact have been removed and the object appears as if stationary during image acquisition. Correction of the other motion degraded images yielded similar results. The mean signal value for a 14 pixel by 14 pixel box within one of the grapefruit sections is plotted against b-value in Fig. 31.

Figure 32: Diffusion map of the grapefruit generated from the set of diffusion-weighted stationary images. Intensity of each pixel reflects the diffusion coefficient for each tiny element of the grapefruit. The individual sections and grapefruit outline are still clearly visible.

Figure 33: Diffusion-weighted MR image of the grapefruit acquired while the grapefruit was manually shifted within the imaging plane. Motion was random, although care was taken to keep it translational and within the field of view. The image has a b-value of 205.03 s/mm^2 . Object detail is completely lost due to ghosting artifacts and blurring.

Figure 34: Image of the grapefruit obtained following correction of the motion degraded raw data using our three echo navigator correction technique. The b-value for the image is 205.03 s/mm^2 . All visible signs of motion artifact have been removed and the image is indistinguishable from that obtained for the stationary grapefruit, as seen in Fig. 30.

Error on each point is the standard deviation of the signal within the 196 pixel region of interest. Application of the fitting routine to this set of images returned a diffusion coefficient of $1.33\text{E-}3 \pm 8.19\text{E-}5 \text{ mm}^2/\text{s}$.

A diffusion map was generated from the set of corrected images and is shown in Fig. 35. Although the background region is quite mottled, the grapefruit outline and individual sections are still obvious.

Attempts were made to correct one of the motion degraded images using the original two echo navigator correction scheme. The image obtained following correction of the image shown in Fig. 33 is displayed in Fig. 36. Considerable artifact is still present in the 'corrected' image because the two echo technique fails to compensate for view-to-view displacements in the y direction.

4.5 In Vivo Study

Six transverse images of a human brain were acquired, each having a different level of diffusion sensitization. One of these images, having a b-value of 134.15 s/mm^2 is shown in Fig. 37. Slight ghosting artifacts due to motion (physiologic and/or gross patient) can be seen in the image. The other images were similar, however pixel intensity decreased with increasing diffusion sensitization.

Figure 35: Diffusion map of the grapefruit generated from the set of diffusion-weighted corrected images. Although the background is quite mottled, the grapefruit is still readily discernable within the field of view. Intensity of each pixel reflects the diffusion coefficient for each tiny element of the grapefruit.

Figure 36: Image of the grapefruit obtained following correction of the motion degraded raw data using the original two echo navigator correction technique. The b-value for the image is 205.03 s/mm^2 . Motion artifacts are still evident in the image because the correction technique fails to correct for y-axis view-to-view displacements.

Figure 37: Diffusion-weighted image of a transverse slice through the head of a human volunteer. B-value for the image is 134.15 s/mm^2 . Slight ghosting artifacts due to motion are visible in the image.

Figure 38: Image obtained following correction of the motion degraded image using our three echo navigator correction technique. B-value for the image is 134.15 s/mm^2 . Signal is more uniform than in the original image (Fig. 37) and ghosting artifacts have been visibly reduced.

The three echo navigator correction algorithm was applied to each image and a second set of images generated. The image that was obtained through correction of the image shown in Fig. 37 is shown in Fig. 38. The ghosting artifacts have been visibly reduced and signal is more uniform throughout the object. Correction of the other images produced similar results.

Mean pixel value for a 10 pixel by 10 pixel region of interest within the brain tissue was found for each image (original and corrected) and is plotted against b-value in Fig. 39. Error on each point is taken as the standard deviation of the signal within the 100 pixel area. The diffusion coefficient for the set of original images was found to be $1.26\text{E-}3 \pm 2.63\text{E-}4 \text{ mm}^2/\text{s}$ while for the set of corrected images it was found to be $0.94\text{E-}3 \pm 1.33\text{E-}4 \text{ mm}^2/\text{s}$.

Figure 39: Plot of signal decay for original and corrected images of the human brain. Mean pixel intensity for a 100 pixel area of cerebral cortex in the left temporal lobe is plotted against b -value for the set of original images (Δ 's) and for the set of corrected images ($\circ$'s). The diffusion coefficient determined for the set of original images was $1.26E-3 \pm 2.63E-4 \text{ mm}^2/\text{s}$ while that found for the corrected images was $0.94E-3 \pm 1.33E-4 \text{ mm}^2/\text{s}$.

5 Discussion

5.1 Noiseless Motion Simulations

The results from the noiseless simulations demonstrate several things. First, they confirm that given a set of diffusion weighted images of a stationary object, the diffusion coefficient can be determined by examining the decline in pixel intensity with increasing level of diffusion sensitization or b-value. As shown in Fig. 12, a logarithmic plot of signal intensity versus b-value produces a straight line, the slope of which is the diffusion coefficient. In our simulations, the slope was found to be $2.00\text{E-}3 \text{ mm}^2/\text{s}$, which was to be expected since this was the value used in generating the images. It also served to confirm that the FFT and fitting routines were running properly.

The simulations also illustrate the detrimental effects of motion in diffusion weighted MR images. As seen in Fig. 13, even fairly small amplitude displacements during image acquisition lead to severe artifacts in the reconstructed image. Ghosting along the phase encoding direction and blurring in the frequency encoding direction cause complete loss of object detail. In addition, these artifacts lead to gross error in diffusion coefficient quantification. The value determined from the motion degraded images was $4.00\text{E-}3$

mm^2/s , an overestimation of the true value by a factor of two. Precision on the fit for the motion images was also poor. As can be seen in Fig. 14, signal versus b-value data points for the set of motion images do not fall in a straight line on the logarithmic plot. Such deviations from linearity lead to greater uncertainty in the fit. In this case, the uncertainty was $5.1\text{E-}4 \text{ mm}^2/\text{s}$ or 25% of the fitted parameter.

Lastly, and most importantly, the noiseless simulations demonstrate that it is indeed possible to remove the undesirable effects of motion from MR images acquired using our specialized three echo navigator imaging sequence. As seen in Fig. 17, the images generated following correction of the motion degraded raw data matrices are identical with those obtained for a stationary object - all traces of motion artifact are gone. Pixel intensities in the corrected images are the same as those for the stationary images and hence, the diffusion coefficient found from them is as accurate and precise as that found from the set of stationary images, $2.00\text{E-}3 \pm 1.56\text{E-}6 \text{ mm}^2/\text{s}$.

Correction of the motion degraded images using just one navigator echo (i.e. using the original technique of Ehman [11]) was not nearly as successful. As seen in Fig. 18, artifacts persist in the corrected image because the two echo technique fails to remove the effects of y-axis view-to-view motion from the image echo data. It is obvious then, that the acquisition of a third echo (i.e. a second navigator echo)

represents a significant enhancement of the correction technique.

It must be noted that in adding motion effects to the raw data sets, we used a simple approach - one designed to test the theory of our correction technique without trying its limits. The correction technique is designed to remove the effects of rigid body translational motion that is either of constant velocity or of slow enough speed that no appreciable motion occurs between the image and navigator echoes. For example, our correction algorithm assumes x-axis view to view displacements are the same for the image and X-NAV echoes (i.e. pixel shifts determined from X-NAV projection data are used to correct the image projection data). Only if object velocity is constant during image acquisition is this strictly true - displacement along the x-axis from the time of acquisition of one X-NAV echo to the next will equal x-axis displacement from the time of acquisition of one image echo to the next. Under more general conditions, where speed and/or direction of motion change during the imaging sequence, shifts may differ for the two echoes. However, if the two echoes are acquired close enough together, shifts will be the same regardless of a change in speed or direction of motion. This is because no appreciable motion can occur in the very short time between the acquisition of the two echoes. In state of the art machines, motion between the two echoes is less of a problem because they are capable of producing very large

gradients, a feature that allows the echo times to be very close together.

In our simulations, we chose to include effects due to a sinusoidal type motion along both the x and y axes. To ensure that x-axis view-to-view displacements were equal for both the image and X-NAV echoes, object position at the beginning of each phase encoding step was calculated, the shift (in pixels) from the reference view was determined for each phase encoding step then motion effects were added to both the image and X-NAV echoes according to these shifts. A more rigorous approach would determine x-axis position at the time of acquisition of each image echo, calculate the shift from the reference view for each phase encoding step, then add motion effects to the image raw data according to these shifts. Next, object x-axis position at the time of acquisition of each X-NAV echo would be determined, shifts from the reference view calculated, and motion effects added to the X-NAV raw data according to these shifts. If motion is slow, little displacement can occur in the short time between the image and X-NAV echoes (~20 ms) so shifts for the image and X-NAV echoes may be the same. However, if motion is rapid and direction and/or speed of motion changes during the imaging sequence, shifts may differ for the two echoes and the correction would not entirely remove the effects of view-to-view x-axis displacements from the image data.

In a similar manner, our correction algorithm assumes

view-to-view displacements in the y direction to be the same for the image and Y-NAV echoes. To ensure they were, motion effects were added to the raw data sets based on shifts in position at the beginning of each phase encoding step. Again, a more rigorous approach would consider position at the time of acquisition of each echo, determine shifts from these positions and add motion effects independently to the image and Y-NAV raw data sets.

The final feature of our correction technique that must be discussed is that in correcting the effects of intra view motion, it compensates for all motion that occurs from the beginning of each phase encoding step until the acquisition of the X-NAV echo. Since the X-NAV echo is acquired after the image echo, it may have greater intra-view motion effects than the image echo. In such cases, our correction algorithm over compensates for intra-view motion effects in the image data and residual artifact will be present in the corrected image.

In our simulations, we ensured that intra-view motion effects were the same in the image and X-NAV echoes by only including effects due to the diffusion sensitizing gradient pulses. Since these pulses precede both echoes, phase shifts they produce will effect both echoes equally and the correction technique will work flawlessly in removing their effects from the image data. Gradient pulses that follow the image echo, but precede the X-NAV echo produce phase shifts in the X-NAV echoes only. Had the effects of such gradients been

included in our simulations, the correction algorithm would have assumed they affected both echoes equally and hence attempted to remove their effects from the image data. The result is that residual phase shifts would have been introduced in the image data. Fortunately, phase shifts caused by gradient pulses which follow the image echo are very small compared with those produced by the diffusion sensitizing gradient (DSG) pulses. This is because the DSG's are of much higher amplitude and much longer duration than the imaging gradient pulses. The residual error that would have been introduced in the corrected image had the effects of the gradients following the image echo been included would have been very small. The bulk of the phase error due to intra-view motion is caused by the diffusion sensitizing gradient pulses, and fortunately, our correction technique corrects for this phase error flawlessly.

5.2 Simulations Including Noise

To investigate the effects of noise on the competency of our correction technique, we evaluated its accuracy in three sets of images, each having a different signal to noise level.

The first set of images generated had a very high signal to noise level of 100 to 1. The diffusion coefficient determined from the set of stationary images was $2.00\text{E-}3 \pm 1.57\text{E-}5 \text{ mm}^2/\text{s}$. By examining a region containing many pixels

(100), the noise averaged out and an accurate measure of the diffusion coefficient was obtained. Precision on the fit was also very good, with uncertainty being less than one percent of the fitted parameter. The diffusion coefficient for the same region was then found from the set of corrected images. Again, D was determined to be $2.00\text{E-}3 \pm 1.57\text{E-}5 \text{ mm}^2/\text{s}$. Since accuracy and precision on the fit are identical with those for the stationary images, we conclude that at this low level, noise has no effect on the ability of our correction algorithm to remove the effects of motion from MR images.

The second set of images we investigated had a moderate signal to noise level of 10 to 1. As with the case of the signal to noise level of 100 to 1, diffusion coefficients found from the stationary and corrected images were identical at $2.00\text{E-}3 \pm 1.57\text{E-}4 \text{ mm}^2/\text{s}$. Again, noise is averaged out by considering average pixel intensity within a region containing many pixels (100), and an accurate determination of the diffusion coefficient is obtained. Even at this moderate level, noise has no effect on the ability of our correction algorithm to remove motion effects from MR images. The only impact noise has is in reducing the precision with which the diffusion coefficient can be determined. Although uncertainty is still reasonably low, at $\sim 8\%$ of the fitted parameter, it is much higher than in the first case (signal to noise level of 100 to 1), where uncertainty was just $\sim 0.8\%$ of the fitted parameter.

The final set of images we investigated had a very low signal to noise level of 3 to 1. The diffusion coefficient found from the set of stationary images was $2.00\text{E-}3 \pm 5.15\text{E-}4$ mm^2/s while that found from the corrected images was $1.99\text{E-}3 \pm 4.51\text{E-}4$ mm^2/s . The two values are in close agreement with just a slight difference being due to variation in average pixel intensity in the image having the highest diffusion sensitivity or b-value. As can be seen in Fig. 26, all points except the last are identical for the stationary and corrected images. The deviation in the final point is due to the very high noise level in the images of that b-value. It should be noted that, although the signal to noise level was set to 3:1 in this case, this applies only to the b=0 data. For higher b-values the signal intensity is significantly reduced and the signal to noise ratios are correspondingly lower. Had the experiment been rerun, the point for the final corrected image may have fallen beneath the point for the stationary image (rather than above as it is now), with the average corrected point falling directly on the average stationary point for that b-value. It appears then, that even at very high noise levels, our correction technique removes the motion effects from MR images acquired with our special navigator echo sequence.

5.3 Phantom Studies

Phantom studies were carried out to test the performance of our three echo navigator correction technique in practice. The results show that the acquisition of high quality diffusion weighted images of moving structures is possible using our specialized imaging sequence in combination with the postprocessing correction algorithm we have developed.

The first experiment, carried out on a stationary water phantom, confirmed that our three echo navigator imaging sequence, when implemented on the Siemens Magnetom imaging system, generates high quality diffusion weighted images of a stationary object, and that from these images, we can experimentally determine the diffusion coefficient with good accuracy and precision. A water phantom was chosen for the experiment because free diffusion of water has been widely studied and values of the diffusion coefficient at temperatures near room temperature are well documented [29,31,32]. The value we determined for a 100 pixel area at the centre of the water phantom was $2.02\text{E-}3 \pm 4.58\text{E-}5 \text{ mm}^2/\text{s}$, which is in good agreement with the literature values which range from $2.0 - 2.2 \text{ E-}3 \text{ mm}^2/\text{s}$. Precision on the fit is also good with uncertainty $\sim 2.3\%$ of the fitted parameter.

The results of our second phantom experiment, that carried out on a grapefruit, demonstrate that it is possible to remove motion effects from experimentally acquired images

of an object undergoing rigid body, translational motion within the field of view during the image acquisition period. The diffusion coefficient for a 14 by 14 pixel region within one of the grapefruit sections was found from the set of stationary images to be $1.32\text{E-}3 \pm 6.17\text{E-}5 \text{ mm}^2/\text{s}$. We assume this to be accurate since the value we determined for the water phantom was reasonable. The diffusion coefficient obtained from the set of corrected images was $1.33\text{E-}3 \pm 8.19\text{E-}5 \text{ mm}^2/\text{s}$, which is in good agreement with the value obtained from the stationary images.

In examining the plot of signal intensity versus b-value for the grapefruit experiment, shown in Fig. 31, we see that all points but those corresponding to one b-value are essentially the same for the stationary and corrected images. A discrepancy exists for the $b-b_0$ point of 230.13 s/mm^2 : the S/S_0 value for the corrected image is noticeably less than that for the stationary image. It appears that in this one case, our correction algorithm was unable to remove all motion effects from the raw data matrix and residual artifact was present in the corrected image. A few factors may have contributed to this deficiency. First, because motion was manually imparted on the grapefruit during image acquisition, it may not have been strictly translational - slight rotations may have occurred, the effects of which would not be removed by our correction algorithm. Secondly, intra-view phase errors may have been different for the image and X-NAV echoes.

As discussed previously, our correction technique assumes they are the same, and if they are not, residual error remains in the corrected image. Finally, view-to-view displacements may have differed for the image and navigator echoes. Although care was taken to keep motion relatively slow ($<3\text{cm/s}$), displacements determined from the X and Y-NAV echoes may have been incorrect for some of the image echoes, the result being an incomplete correction of view-to-view motion effects in the final image.

Results of our grapefruit study also show that given a set of good quality diffusion weighted images, it is possible to generate a diffusion map of the object of interest. Both the map obtained from the set of stationary images (Fig. 32) and the one obtained from the corrected images (Fig. 35) display quite clearly variations in diffusion coefficient within the grapefruit. The background of the map obtained from the corrected images is much more mottled than that of the map obtained from the set of stationary images. This is because the images used to generate the corrected map (i.e. the corrected images) were mapped to a grey scale having a wider range of values than the grey scale used in generating the original images. The result is that the background pixels (regions of noise) of the corrected images have greater variations of intensity than those of the original images. When the diffusion map was generated from the set of stationary images, all background pixels were found to have

the same diffusion coefficient, $D=0.0$, because the fitting routine was unable to make a fit to the data. However, when the map was generated from the set of corrected images, a variety of diffusion coefficients were found for the background pixels - the greater variation in intensities allowed a fit to be made using our least squares routine.

Correction of the motion degraded grapefruit images using information derived from just one navigator echo (the X-NAV echo) was not nearly as successful as when information from both navigator echoes was used. As can be seen in Fig. 36, the image obtained following correction using the original two echo navigator technique of Ehman [11] is still badly degraded with motion artifacts. None of the effects of view-to-view y-axis displacement were removed from the image. Our technique, the three echo navigator approach, is clearly a great improvement over the original technique.

5.4 In Vivo Study

Acquiring good quality diffusion weighted in vivo images has been quite difficult in the past. Motion artifacts seriously degrade the images, even in cases where bulk patient motion has been minimized by comfortably securing the subject within the magnet during image acquisition. The main problem is internal motion, due to physiologic processes like

respiration and cardiac function. Fig. 37 shows a typical diffusion weighted image of the brain of a healthy human volunteer. Although she remained quite still during the imaging procedure, bulk motion of the brain, due to pulsatile CSF flow, has caused ghosting artifacts in the reconstructed image. The diffusion coefficient found from the set of uncorrected images for a 100 pixel region of cerebral cortex in the left temporal lobe of the brain was $1.26\text{E-}3 \pm 2.63\text{E-}4 \text{ mm}^2/\text{s}$. This value is higher than values quoted in the literature which range from $0.66 - 1.04 \text{ mm}^2/\text{s}$ [31,33,34].

Our correction technique was successful in removing visible signs of motion from the image, as can be seen in Fig. 38. In addition, the diffusion coefficient found from our set of corrected images, $0.94\text{E-}3 \pm 1.33\text{E-}4 \text{ mm}^2/\text{s}$ was more reasonable than that found from the corrected images. From this, we conclude that our correction technique shows great promise in producing high quality in vivo diffusion images of moving structures.

6 Conclusions

Although the navigator echo scheme was first proposed seven years ago, it has not been used a great deal in practice. One of the main reasons for this is that it is restricted to rigid body motion in one direction only. In this thesis we have shown that by acquiring two navigator echoes, rigid body motion in a plane can be tracked and corrected for in postprocessing. This represents a significant enhancement of the technique.

Using our three echo navigator routine, we were able to acquire diffusion-weighted images of moving structures that were essentially artifact free and that could be used to calculate diffusion coefficients with good accuracy. The technique shows great promise for in vivo diffusion studies - especially with state of the art MR imagers. These machines allow echo times for multi-echo sequences to be much smaller, resulting in higher signal to noise levels as well as reduced effects due to motion between the image and navigator echoes.

MR imaging using navigator echoes is a field with great potential for growth. Continued work might include investigating the effects of more complex types of motion such as rotations and non-rigid body motions. If an adaptive correction technique could be developed to remove the effects of such motions, in vivo diffusion imaging of any and all

internal structures may become clinically feasible.

Appendix

The following pages display the computer programs written to execute the three echo navigator correction technique. In addition, a flow chart has been developed which summarizes the correction process and is shown in Figure 40.

The x-navigator, y-navigator and image raw data sets are Fourier transformed into projections using our FFT routine [28]. The x-navigator projection data is then analyzed to determine the x-axis view-to-view shifts and phase errors due to intraview motion. The y-navigator data is next analyzed to determine the y-axis view-to-view displacements which are subsequently converted into phase shifts. (Note the distinction between these phase shifts and those determined from the x-navigator projections for intraview motion). Finally, the image projection data is corrected using the information derived from the navigator echoes.

The program which carries out the correction process is entitled Navcorr and is given below. Input to the program are the files containing the motion degraded projection data for each echo. Returned is the corrected image projection data. The program calls several subroutines as it carries out the correction. These subroutine programs have also been included.

Following correction, the image projection data is

Figure 40: Flow chart summarizing the three echo navigator correction technique.

Fourier transformed along the phase encoding direction (using our FFT routine [28]) and modulus values are calculated for each point. Finally, the modulus values are mapped to a grey scale and the resulting image is displayed.

Below are the various computer programs used to correct the images. The programs were written in FORTRAN and implemented on a VAX 4300 computer.

```

!-----
!                               NAVCORR
! THIS PROGRAM CORRECTS FOR MOTION USING NAVIGATOR ECHO DATA!
!
! READ IN ARE THE X-NAV HYBRID DATA, Y-NAV HYBRID DATA AND !
! THE IMAGE HYBRID DATA.
!
! THE FOLLOWING SUBROUTINES ARE CALLED: MODULUS, CCORR
! SHIFT, PHASE, PHADIF, PHASHIFT, ROTAT.
!-----

```

```

PROGRAM NAVCORR

```

```

IMPLICIT NONE

```

```

! THE FOLLOWING 'INCLUDE' STATEMENTS ARE NECESSARY IF SIEMEN'S
! USER LIBRARY ROUTINES ARE TO BE USED.
! NOTE THAT ALL ROUTINES WHICH BEGIN WITH U2 ARE SIEMEN'S USER
! LIBRARY ROUTINES.

```

```

INCLUDE '($SSDEF)'
INCLUDE 'INCLULB_(U2$ERRDEF)/LIST'
INCLUDE 'INCLULB_(U2$MODULES)'
INCLUDE 'INCLULB_(HEADEF)'

```

```

! DECLARING THE VARIABLES.

```

```

INTEGER*2 STATUS, NCHAR, STBLK, NBLK, EXBLK
INTEGER*2 A(131072), NROW, NCOL, VREF, DELX(256)
INTEGER*2 IOCHAN, SIZE, DELY(256)

```

```

INTEGER*4 NUM, I, J

```

```

REAL*4 C(1024), VAL(256,512)
REAL*4 MOD(256,256), DELPHIY(256)
REAL*4 PHI(256,256), DELPHI(256,256), NVAL(256,512)

```

```
CHARACTER*80  FILENAME1,FILENAME2,FILENAME3, FILEOUT
RECORD/HEADER/H
```

```
! READ IN THE X-NAVIGATOR HYBRID DATA FILENAME, THE
! Y-NAVIGATOR HYBRID DATA FILE NAME, THE IMAGE HYBRID DATA
! FILENAME, THE NUMBER OF ROWS AND COLUMNS IN THE IMAGE, AND
! THE NAME OF THE OUTPUT FILE.  SPECIFY WHICH VIEW YOU WOULD
! LIKE AS THE REFERENCE VIEW.
```

```
WRITE(6,80)
READ(5,7) NCHAR, FILENAME1
WRITE(6,40)
READ(5,9) NCHAR, FILENAME2
WRITE(6,60)
READ(5,11) NCHAR, FILENAME3
WRITE(6,*) 'ENTER THE NUMBER OF FOURIER LINES:'
READ(6,*) NROW
WRITE(6,*) 'ENTER THE NUMBER OF SAMPLES IN EACH
LINE:'
READ(6,*) NCOL
WRITE(6,*) 'WHICH VIEW WOULD YOU LIKE AS THE
REFERENCE VIEW:'
READ(6,*) VREF
WRITE(6,110)
READ(5,13) NCHAR,FILEOUT
```

```
! *****
! ANALYSING THE X-NAV DATA
! *****
! THE X-NAV HYBRID DATA IS USED TO DETERMINE THE X-AXIS
! VIEW-TO-VIEW SHIFTS AND THE PHASE ERROR DUE TO INTRAVIEW
! MOTION.
! *****
```

```
! READ IN THE FREQUENCY-NAVIGATOR HYBRID DATA FILE HEADERS.
```

```
STATUS = U2GETHEAD(FILENAME2(1:NCHAR),H,IOCHAN)
IF (.NOT.STATUS) THEN
WRITE(6,120)
GOTO 5
ENDIF
```

```
! CERTAIN HEADERS OF INTEREST ARE WRITTEN TO THE SCREEN.
```

```
WRITE(6,*) 'PATIENT NAME: ',H.HEA_T_PATIENT_NAME
WRITE(6,*) 'DATA TYPE: ',H.HEA_W_IMAGE_DATA_TYPE
WRITE(6,*) 'START BLOCK: ',H.HEA_W_ENTRY_4_START
WRITE(6,*) 'NUMBER OF BLOCKS:
',H.HEA_W_ENTRY_4_LENGTH
WRITE(6,*) 'NUMBER OF EXTRA
```

```

BLOCKS: ',H.HEA_W_ENTRY_13_LENGTH+1
      WRITE(6,*) 'FIRST FREE
BLOCK: ',H.HEA_W_FIRST_FREE_BLOCK
      WRITE(6,*) 'NO. RAW FOURIER
LINES: ',H.HEA_L_RAW_LIN_NO
      WRITE(6,*) 'NO. RAW COLUMNS:',H.HEA_L_RAW_COL_NO
      WRITE(6,*) 'NO. IMAGE LINES:',H.HEA_W_IMAGE_SIZE_Y
      WRITE(6,*) 'NO. IMAGE COLUMNS:',H.HEA_W_IMAGE_SIZE_X

      STBLK = H.HEA_W_ENTRY_4_START
      NBLK = H.HEA_W_ENTRY_4_LENGTH
      EXBLK = H.HEA_W_ENTRY_13_LENGTH+1

```

```

! READ IN THE X-NAVIGATOR HYBRID DATA VALUES AND EXTRA HEADERS
! THEN CLOSE THE I/O CHANNEL.

```

```

      STATUS = U2READBLK(STBLK,NBLK,IOCHAN,A)
      IF (.NOT.STATUS) CALL LIB$SIGNAL (%VAL(STATUS))
      STATUS = U2READBLK(STBLK+NBLK,EXBLK,IOCHAN,C)
      IF (.NOT.STATUS) CALL LIB$SIGNAL (%VAL(STATUS))
      STATUS = U2CLOSE(IOCHAN)
      IF (.NOT.STATUS) CALL LIB$SIGNAL (%VAL(STATUS))

```

```

! PIXEL VALUES ARE STORED IN THE 2-D (256*512) ARRAY VAL.

```

```

      NUM = 0
      DO I = 1,NROW
        DO J = 1,2*NCOL
          NUM = NUM + 1
          VAL(I,J) = FLOAT(A(NUM))
        ENDDO
      ENDDO

```

```

! THE MODULUS IS CALCULATED FOR EACH POINT IN HYBRID SPACE.

```

```

      WRITE(6,*) 'CALCULATING THE MODULUS'
      CALL MODULUS(NROW,NCOL,VAL,MOD)

```

```

! CROSSCORRELATIONS FOR EACH ROW WITH THE REFERENCE VIEW ARE
! CALCULATED.

```

```

! THE X-AXIS VIEW-TO-VIEW DISPLACEMENT IS DETERMINED FOR EACH
! VIEW - THIS WILL BE USED TO CORRECT THE IMAGE HYBRID DATA.

```

```

      WRITE(6,*) 'CALCULATING CROSSCORRELATIONS'
      CALL CCORR(NROW,NCOL,MOD,VREF,DELX)

```

```

! ROWS ARE SHIFTED SO MAXIMUM CORRELATION WITH REFERENCE VIEW
! IS ACHIEVED.

```

```

! THIS REMOVES X-AXIS VIEW-TO-VIEW MOTION EFFECTS.

```

```
WRITE(6,*) 'SHIFTING ROWS TO MAX. CORRELATION POSITION'
CALL SHIFT(NROW,NCOL,DELX,VAL,NVAL)
```

```
! THE PHASE ANGLE IS CALCULATED FOR EACH POINT IN HYBRID
! SPACE.
```

```
WRITE(6,*) 'CALCULATING PHASE ANGLE'
CALL PHASE(NROW,NCOL,NVAL,PHI)
```

```
! THE PHASE DIFFERENCE IS CALCULATED FOR EACH POINT.
! THIS WILL BE USED TO CORRECT THE IMAGE HYBRID DATA FOR
! INTRA-VIEW MOTION EFFECTS.
```

```
WRITE(6,*) 'CALCULATING PHASE DIFFERENCE'
CALL PHADIF(NROW,NCOL,PHI,VREF,DELPHI)
```

```
! *****
! ANALYSING THE Y-NAV DATA
! *****
! THE Y-NAV HYBRID DATA IS USED TO DETERMINE THE Y-AXIS
! VIEW-TO-VIEW SHIFTS.
! *****
```

```
! READ IN THE PHASE-NAVIGATOR HYBRID DATA FILE HEADERS.
```

```
STATUS = U2GETHEAD(FILENAME3(1:NCHAR),H,IOCHAN)
IF (.NOT.STATUS) THEN
  WRITE(6,120)
  GOTO 5
ENDIF
```

```
! CERTAIN HEADERS OF INTEREST ARE WRITTEN TO THE SCREEN.
```

```
WRITE(6,*) 'PATIENT NAME: ',H.HEA_T_PATIENT_NAME
WRITE(6,*) 'DATA TYPE: ',H.HEA_W_IMAGE_DATA_TYPE
WRITE(6,*) 'START BLOCK: ',H.HEA_W_ENTRY_4_START
WRITE(6,*) 'NUMBER OF BLOCKS:
',H.HEA_W_ENTRY_4_LENGTH
WRITE(6,*) 'NUMBER OF EXTRA
BLOCKS: ',H.HEA_W_ENTRY_13_LENGTH+1
WRITE(6,*) 'FIRST FREE
BLOCK: ',H.HEA_W_FIRST_FREE_BLOCK
WRITE(6,*) 'NO. RAW FOURIER
LINES: ',H.HEA_L_RAW_LIN_NO
WRITE(6,*) 'NO. RAW COLUMNS: ',H.HEA_L_RAW_COL_NO
WRITE(6,*) 'NO. IMAGE LINES: ',H.HEA_W_IMAGE_SIZE_Y
WRITE(6,*) 'NO. IMAGE COLUMNS: ',H.HEA_W_IMAGE_SIZE_X

STBLK = H.HEA_W_ENTRY_4_START
NBLK = H.HEA_W_ENTRY_4_LENGTH
```

```
EXBLK = H.HEA_W_ENTRY_13_LENGTH+1
```

```
! READ IN THE Y-NAVIGATOR HYBRID DATA VALUES AND EXTRA HEADERS
! THEN CLOSE THE I/O CHANNEL.
```

```
STATUS = U2READBLK(STBLK,NBLK,IOCHAN,A)
IF (.NOT.STATUS) CALL LIB$SIGNAL (%VAL(STATUS))
STATUS = U2READBLK(STBLK+NBLK,EXBLK,IOCHAN,C)
IF (.NOT.STATUS) CALL LIB$SIGNAL (%VAL(STATUS))
STATUS = U2CLOSE(IOCHAN)
IF (.NOT.STATUS) CALL LIB$SIGNAL (%VAL(STATUS))
```

```
! PIXEL VALUES ARE STORED IN THE 2-D (256*512) ARRAY VAL.
```

```
NUM = 0
DO I = 1,NROW
  DO J = 1,2*NCOL
    NUM = NUM + 1
    VAL(I,J) = FLOAT(A(NUM))
  ENDDO
ENDDO
```

```
! THE MODULUS IS CALCULATED FOR EACH POINT IN HYBRID SPACE.
```

```
WRITE(6,*) 'CALCULATING THE MODULUS'
CALL MODULUS(NROW,NCOL,VAL,MOD)
```

```
! CROSSCORRELATIONS FOR EACH ROW WITH THE REFERENCE VIEW ARE
! CALCULATED.
```

```
! THE Y-AXIS VIEW-TO-VIEW DISPLACEMENT IS DETERMINED FOR EACH
! VIEW.
```

```
WRITE(6,*) 'CALCULATING CROSSCORRELATIONS'
CALL CCORR(NROW,NCOL,MOD,VREF,DELY)
```

```
! THE PHASE CORRECTION NEEDED FOR THE IMAGE ECHO IS CALCULATED
! I.E. THE SHIFT IN PIXELS IS CONVERTED TO A PHASE ROTATION.
```

```
WRITE(6,*) 'CALCULATING PHASE CORRECTION FACTOR'
CALL PHASHIFT(NROW,NCOL,DELY,DELPHIY)
```

```
! *****!
! CORRECTING THE IMAGE DATA!
! *****!
! SHIFTS AND PHASE ROTATIONS DETERMINED FROM THE NAVIGATOR!
! ECHO DATA SETS ARE USED TO CORRECT THE IMAGE HYBRID DATA!
! *****!
```

```
! READ IN THE HEADERS OF THE IMAGE HYBRID DATA FILE.
```

```

STATUS = U2GETHEAD(FILENAME1(1:NCHAR),H,IOCHAN)
IF (.NOT.STATUS) THEN
  WRITE(6,120)
  GOTO 5
ENDIF

```

```

WRITE(6,*) 'PATIENT NAME: ',H.HEA_T_PATIENT_NAME
WRITE(6,*) 'DATA TYPE: ',H.HEA_W_IMAGE_DATA_TYPE
WRITE(6,*) 'START BLOCK: ',H.HEA_W_ENTRY_4_START
WRITE(6,*) 'NUMBER OF BLOCKS:
           ',H.HEA_W_ENTRY_4_LENGTH
WRITE(6,*) 'NUMBER OF EXTRA
BLOCKS:',H.HEA_W_ENTRY_13_LENGTH+1
WRITE(6,*) 'FIRST FREE
BLOCK:',H.HEA_W_FIRST_FREE_BLOCK
WRITE(6,*) 'NO. FOURIER LINES:',H.HEA_L_RAW_LIN_NO
WRITE(6,*) 'NO. COLUMNS RAW
DATA:',H.HEA_L_RAW_COL_NO

```

```

STBLK = H.HEA_W_ENTRY_4_START
NBLK = H.HEA_W_ENTRY_4_LENGTH
EXBLK = H.HEA_W_ENTRY_13_LENGTH+1

```

```

! READ IN THE IMAGE HYBRID DATA VALUES AND EXTRA HEADERS
! THEN CLOSE THE I/O CHANNEL.

```

```

STATUS = U2READBLK(STBLK,NBLK,IOCHAN,A)
IF (.NOT.STATUS) CALL LIB$SIGNAL (%VAL(STATUS))
STATUS = U2READBLK(STBLK+NBLK,EXBLK,IOCHAN,C)
IF (.NOT.STATUS) CALL LIB$SIGNAL (%VAL(STATUS))
STATUS = U2CLOSE(IOCHAN)
IF (.NOT.STATUS) CALL LIB$SIGNAL (%VAL(STATUS))

```

```

! STORE THE IMAGE HYBRID DATA VALUES IN THE 256*512 ARRAY VAL.

```

```

NUM = 0
DO I = 1,NROW
  DO J = 1,2*NCOL
    NUM = NUM + 1
    VAL(I,J) = FLOAT(A(NUM))
  ENDDO
ENDDO

```

```

! ROWS ARE SHIFTED TO MAXIMUM CORRELATION POSITION.
! THIS CORRECTS FOR X-AXIS VIEW-TO-VIEW MOTION.

```

```

WRITE(6,*) 'SHIFTING ROWS TO MAX. CORRELATION POSITION'
CALL SHIFT(NROW,NCOL,DELX,VAL,NVAL)

```

```

! IMPLEMENT THE PHASE CORRECTION AT EACH POINT.

```



```
! THIS CORRECTS FOR INTRAVIEW MOTION AND Y-AXIS VIEW-TO-VIEW
! MOTION.
```

```
      WRITE(6,*) 'CORRECTING THE IMAGE HYBRID DATA'
      CALL ROTAT(NROW,NCOL,DELPHI,DELPHIY,NVAL)
```

```
! VALUES ARE ROUNDED TO THE NEAREST INTEGER AND STORED IN THE
! ARRAY A.
```

```
      NUM = 0
      DO I = 1,NROW
        DO J = 1,2*NCOL
          NUM = NUM + 1
          A(NUM) = NINT(NVAL(I,J))
        ENDDO
      ENDDO
```

```
! CREATE A NUMARIS HYBRID DATA FILE AS THE OUTPUT FILE.
```

```
      SIZE = NBLK + EXBLK + 16
      STATUS = U2CREATE(FILEOUT(1:NCHAR),SIZE,IOCHAN)
      IF (.NOT.STATUS) THEN
        WRITE(6,120)
        GOTO 5
      ENDIF
```

```
! WRITE IN THE PIXEL VALUES, HEADERS AND EXTRA HEADERS THEN
! CLOSE THE FILE.
```

```
      STATUS = U2WRITBLK(STBLK,NBLK,IOCHAN,A)
      IF (.NOT.STATUS) CALL LIB$SIGNAL(%VAL(STATUS))
      STATUS = U2WRITBLK(STBLK+NBLK,EXBLK,IOCHAN,C)
      IF (.NOT.STATUS) CALL LIB$SIGNAL(%VAL(STATUS))
      STATUS = U2PUTHEAD(H,IOCHAN)
      IF (.NOT.STATUS) CALL LIB$SIGNAL(%VAL(STATUS))
```

```
5      CONTINUE
7      FORMAT (Q,A80)
9      FORMAT (Q,A80)
11     FORMAT (Q,A80)
13     FORMAT (Q,A80)
40     FORMAT (/1X'ENTER THE FREQ-NAVIGATOR HYBRID DATA
FILENAME:'$)
60     FORMAT (/1X'ENTER THE PHASE-NAVIGATOR HYBRID DATA
FILENAME:'$)
80     FORMAT (/1X'ENTER THE IMAGE HYBRID DATA FILENAME:'$)
110    FORMAT (/1X'ENTER THE NAME OF THE OUTPUT FILE TO BE
CREATED:'$)
120    FORMAT (/1X'ERROR OPENING FILE ...'$)
END
```

```

!-----!
! THIS SUBROUTINE CALCULATES THE MODULUS OF EACH COMPLEX !
! NUMBER IN AN ARRAY. !
! THE ARRAY DATA IS AN NR*NC SET OF COMPLEX NOS. THIS IS !
! EQUIVALENT TO A REAL ARRAY OF DIMENSION NR*2NC. AN ARRAY !
! CALLED MOD IS RETURNED. IT HAS DIMENSION NR NC AND IS !
! REAL. !
!-----!

```

```

SUBROUTINE MODULUS (NR, NC, DATA, MOD)

```

```

  INTEGER*2 I, NR, NC, J
  INTEGER*4 NUM
  DIMENSION DATA (NR, 2*NC), MOD (NR, NC)
  REAL*4 DATA, MOD, A, B

```

```

  DO I = 1, NR
    NUM = 0
    DO J = 1, NC*2-1, 2
      NUM = NUM + 1
      A = (DATA(I, J))**2.0
      B = (DATA(I, J+1))**2.0
      MOD(I, NUM) = ((A + B)**0.5)
    ENDDO
  ENDDO

```

```

  RETURN
  END

```

```

!-----!
! This subroutine evaluates the crosscorrelation of each !
! row with a reference view. The array val is of dimension !
! nr*nc and is real. The reference view is read in as vref. !
! The crosscorrelation is calculated for a range of shifts !
! to the reference view. The shift that leads to maximum !
! correlation for each row with the reference view is !
! stored and returned in the 1-D array delx, which has !
! dimension nr. !
!-----!

```

```

subroutine ccorr(nr,nc,val,vref,delx)

```

```

integer*2 i,nr,nc,x,j,maxx,vref

```

```

dimension val(nr,nc),delx(nr)

```

```

integer*2 delx

```

```

real*4 val,max,a,b,prod

```

```

real*8 sum

```

```

do i = 1,nr

```

```

  max = 0.0

```

```

  do x = -nc,nc

```

```

    sum = 0.0

```

```

    do j = 1,NC

```

```

      if (((j+x).lt.1).or.((j+x).gt.nc)) then

```

```

        a = 0.0

```

```

      else

```

```

        a = val(vref,j+x)

```

```

      endif

```

```

      b = val(i,j)

```

```

      prod = a*b

```

```

      sum = sum + prod

```

```

    enddo

```

```

      if (sum.gt.max) then

```

```

        max = sum

```

```

        maxx = x

```

```

      endif

```

```

    enddo

```

```

    delx(i) = maxx

```

```

  enddo

```

```

return

```

```

end

```

```

!-----!
! This subroutine shifts each row of an nr*2nc array by a !
! specified number of pixels. The array is read in as !
! val(nr,2*nc) and the output is returned as nval(nr,2*nc)..!
! The shift for each row is read in through the array !
! delx(nr). !
!-----!

```

```

SUBROUTINE SHIFT(NR,NC,DELX,VAL,NVAL)

```

```

  INTEGER*2 i,J,NR,NC,NUM
  DIMENSION VAL(NR,2*NC), NVAL(NR,2*NC), delx(nr)
  integer*2 delx
  REAL*4 VAL,NVAL

```

```

  DO i = 1,NR
    DO J = 1,2*NC
      NUM = J - 2*delx(i)
      IF ((NUM.LT.1.0).OR.(NUM.GT.2*NC)) THEN
        NVAL(I,J) = 0.0
      ELSE
        NVAL(I,J) = VAL(I,NUM)
      ENDIF
    ENDDO
  ENDDO

  RETURN
END

```

```

!-----!
! THIS SUBROUTINE CALCULATES THE PHASE ANGLE FOR EACH
! COMPLEX NUMBER IN AN ARRAY. THE ARRAY PASSED IN, DATA,
! CONSISTS OF NR*NC COMPLEX NUMBERS. THIS IS EQUIVALENT TO
! A REAL ARRAY OF DIMENSION NR*2NC. THE PHASE ANGLES ARE
! RETURNED IN THE ARRAY PHI WHICH IS A REAL ARRAY OF
! DIMENSION NR*NC.
!-----!

```

```

SUBROUTINE PHASE(NR,NC,DATA,PHI)

```

```

  INTEGER*2 I,NR,NC,J

```

```

  INTEGER*4 NUM

```

```

  DIMENSION DATA(NR,2*NC),PHI(NR,NC)

```

```

  REAL*4 A,B,DATA,PHI,PI

```

```

  PI = 3.14159

```

```

  DO I = 1,NR

```

```

    NUM = 0

```

```

    DO J = 1,2*NC-1,2

```

```

      NUM = NUM + 1

```

```

      A = DATA(I,J)

```

```

      B = DATA(I,J+1)

```

```

      IF ((A.EQ.0.0).AND.(B.GE.0.0)) THEN

```

```

        PHI(I,NUM) = PI/2.0

```

```

      ELSEIF ((A.EQ.0.0).AND.(B.LT.0.0)) THEN

```

```

        PHI(I,NUM) = 3.0*PI/2.0

```

```

      ELSEIF ((A.GT.0.0).AND.(B.GE.0.0)) THEN

```

```

        PHI(I,NUM) = ATAN(B/A)

```

```

      ELSEIF ((A.GT.0.0).AND.(B.LT.0.0)) THEN

```

```

        PHI(I,NUM) = ATAN(B/A) + (2.0*PI)

```

```

      ELSE

```

```

        PHI(I,NUM) = ATAN(B/A) + PI

```

```

      ENDIF

```

```

    ENDDO

```

```

  ENDDO

```

```

  RETURN

```

```

END

```

```

!-----!
! This subroutine calculates the phase difference !
! between each point and the corresponding point in the !
! reference view. The phase angle of each point in the !
! reference view is subtracted from the phase angle of !
! the corresponding point in each other view. The phase !
! differences are returned in the nr*nc array delphi. The !
! array of phase angles is read in as the array val. !
! Vref is the reference view. !
!-----!

```

```

subroutine PHADIF(nr,nc,val,vref,delphi)

```

```

integer*2 i,nr,nc,vref,j
dimension val(nr,nc),delphi(nr,nc)
real*4 val,delphi,pi

```

```

pi = 3.14159

```

```

do i = 1,nr
  do j = 1,nc
    delphi(i,j) = val(i,j) - val(vref,j)
  enddo
enddo

return
end

```

```
!-----!  
! THIS SUBROUTINE CONVERTS THE PIXEL SHIFTS, FOUND FROM !  
! THE Y-NAV DATA TO PHASE ROTATIONS FOR CORRECTION OF THE !  
! IMAGE ECHO. PHA IS THE PHASE TWIST ACROSS THE FOV FOR !  
! THE IMAGE ECHO. !  
!-----!
```

```
subroutine phashift(nr,nc,dely,delphiy)
```

```
integer*2 i,j,nr,nc  
dimension dely(nr),delphiy(nr)  
INTEGER*2 DELY  
real*4 delphiy,pi,pha
```

```
pi = 3.14159
```

```
do i = 1,nr  
    pha = 2.0*pi*((float(i-1))-((float(nr-1))/2.0))  
    delphiy(i) = pha*FLOAT(dely(i))/FLOAT(nr)  
enddo
```

```
return  
end
```

```

!-----!!
! THIS ROUTINE DETERMINES THE TOTAL PHASE CORRECTION !
! NEEDED FOR EACH POINT IN THE IMAGE ECHO THEN APPLIES THE !
! APPROPRIATE PHASE ROTATION TO THE COMPLEX IMAGE HYBRID !
! DATA ACCORDING TO: !
!      I' = I*cos(delta) - Q*sin(delta) !
!      Q' = I*sin(delta) + Q*cos(delta) !
! !
! FOR SIMPLICITY, WE COMBINE THE PHASE SHIFTS DUE TO !
! INTRAVIEW MOTION AND Y-AXIS VIEW-TO-VIEW MOTION BEFORE !
! CORRECTING THE DATA SO ONLY ONE ROTATION MUST BE DONE !
! FOR EACH POINT. !
!-----!

```

```

SUBROUTINE ROTAT(NR,NC,DELPHI,DELPHIY,DATA)

```

```

  INTEGER*2 I,NR,NC,J
  INTEGER*4 NUM
  DIMENSION DATA(NR,2*NC),DELPHI(NR,NC),DELPHIY(NR)
  REAL*4 A,B,C,D,DATA,DELPHI,DELPHIY,DELTA

```

```

  DO I = 1,NR
    NUM = 0
    DO J = 1,2*NC-1,2
      NUM = NUM + 1
      delta = delphi(i,num) - delphiy(i)
      A = DATA(I,J)*COS(delta)
      B = DATA(I,J)*SIN(DELTA)
      C = DATA(I,J+1)*COS(DELTA)
      D = DATA(I,J+1)*SIN(DELTA)
      DATA(I,J) = A-D
      DATA(I,J+1) = B+C
    ENDDO
  ENDDO

  RETURN
END

```


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END

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