INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

**ProQuest Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600**

UMI[®]

**SEISMIC ANALYSIS OF CONCRETE ARCH DAMS
WITH CONTRACTION JOINT AND NONLINEAR
MATERIAL MODELS**

by
Anton D. Tzenkov

B.Eng. (Civil Engineering)
University of Architecture, Civil Engineering and Geodesy, Bulgaria

**A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of
Master of Engineering**

April 2001

Department of Civil and Environmental Engineering*
Carleton University
Ottawa, Canada
Copyright © Anton D. Tzenkov, 2001

* The Master of Engineering program in Civil and Environmental Engineering is a joint program between Carleton University and the University of Ottawa, which is administered by the Ottawa–Carleton Institute for Civil Engineering.

**National Library
of Canada**

**Acquisitions and
Bibliographic Services**

**395 Wellington Street
Ottawa ON K1A 0N4
Canada**

**Bibliothèque nationale
du Canada**

**Acquisitions et
services bibliographiques**

**395, rue Wellington
Ottawa ON K1A 0N4
Canada**

Your file Votre référence

Our file Notre référence

The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-61026-8

Canada

Department of Civil and Environmental Engineering

The undersigned hereby recommend to the Faculty of Graduate
Studies and Research acceptance of the thesis

**SEISMIC ANALYSIS OF CONCRETE ARCH DAMS
WITH CONTRACTION JOINT AND NONLINEAR
MATERIAL MODELS**

submitted by
Anton D. Tzenkov

in partial fulfillment of the requirements for the degree of
Master of Engineering

Chair, Department of Civil
and Environmental Engineering

Thesis supervisor

Carleton University
Ottawa, Canada

April 2001

Abstract

Concrete arch dams are constructed as cantilever monoliths separated by vertical contraction joints filled with low tensile strength grouting material. Strong ground motions during earthquakes can cause high tensile and compressive stresses to develop in the body of an arch dam. The vibrational movements of the dam can result in the opening, closing, and sliding movements at the contraction joints, and in the cracking and crushing of the concrete material in the cantilever monoliths. Failure of an arch dam may lead to the sudden release of the reservoir content resulting in great loss of life and economic devastation. In order to ensure the earthquake safety of arch dams, the interaction and the coupling effects between the nonlinear inelastic behaviour of the concrete material and the movements at the contraction joints should be taken into account in the design of new structures and in the evaluation of the seismic risk of existing ones. Most previous studies on the seismic behaviour of arch dams have not accounted for these two types of nonlinearity, and only a few have considered the effect of the vertical contraction joints on the response of the analysed structure. This thesis presents a numerical model which accounts for the interaction and the coupling effects between the inelastic action of the concrete material and the movements at the contraction joints of an arch dam.

The proposed model is implemented in a new finite element computer program called CADNAP (Concrete Arch Dam Nonlinear Analysis Program). A zero-thickness joint element model is employed to simulate the opening, closing and sliding effects at the vertical contraction joints of the dam. A nonlinear concrete material

model which combines a smeared crack model with a bounding surface model is developed in the present study to investigate the effects of high tensile and compressive stresses on the concrete material of the dam. A characteristic length parameter is incorporated in the constitutive law of the smeared crack model to ensure objectivity of the obtained crack patterns. The dam-reservoir interaction is taken into account by means of the added mass concept of the impounded water. The dam-foundation interaction is considered by modelling part of the foundation with depth approximately equal to the height of the dam. The Newmark method in conjunction with the Newton-Raphson iteration scheme are employed to carry out the time-step integration of the nonlinear equations of motion of the system. The safety of the dam structure can be evaluated based on the stress and displacement response time histories and on the calculated patterns of damage.

The validity of the proposed finite element formulation is verified in the analysis of the seismic response and performance of the 142-m high Morrow Point dam. The analysis results indicate that significant stress and damage can result in the central cantilever monoliths of the dam when subjected to strong ground motions.

Dedicated to my parents, my sister, and my nieces.

Acknowledgements

My graduate studies and research at Carleton University have been funded by the Government of Japan, through the Joint Japan / World Bank Graduate Scholarship Program, and by my employer, Energoproekt plc of Sofia. The financial support from both the Government of Japan and Energoproekt plc is gratefully acknowledged.

I am thankful to my supervisor Professor David T. Lau for his guidance, encouragement, and support throughout the course of my studies and work on the present thesis.

I am also indebted to Professor John A. Goldak for his advice on formulating the nonlinear finite element procedures.

I thank Ms Christine G. Adam, Associate Coordinator of the Writing Tutorial Services, for providing advice on how to improve the readability of the text.

Last but not least, I would like to thank my fellow students for sharing with me their knowledge and experience in the fields of finite elements, earthquake engineering, and computer programming.

List of Notations

Latin Upper Case

A	co-ordinate transformation matrix
A	area
A_L, A_u	bounding surface model parameters
B	strain-displacement transformation matrix
C	damping matrix
C_T	tangent damping matrix
C_{ijkl}	compliance tensor
D	rigidity matrix
D, dD	normalised distance, its increment
D^e	elastic rigidity matrix
D^{ep}	elasto-plastic rigidity matrix
E	Young's modulus
E	strain energy
E_0	initial elastic stiffness tensor
E_s	secant elastic modulus
E_t	strain-softening modulus
F	vector of nodal point externally applied loads
F^{ef}	nodal point effective load vector
F	bounding surface
F_1	normalisation factor
G	shear modulus

G_f	fracture energy (energy per unit area dissipated during fracture)
H^e	generalised elastic shear modulus
H^p	generalised plastic shear modulus
$I_1, dI_1, I_{1,max}$	first stress invariant, its increment, its maximum value
J_2, J_3	second and third deviatoric stress invariants
J	Jacobian matrix of co-ordinate derivatives
K	stiffness matrix
K^{ef}	effective stiffness matrix
K_T	tangent stiffness matrix
K, dK, K_{max}	damage parameter, its increment, its maximum value
K_I, K_{II}, K_{III}	stress intensity factors for the I, II, and III modes of loading
K_t	tangent bulk modulus
L	loading criterion for a joint element
M	mass matrix
N	vector of nodal point restoring forces depending on U
N_i	shape function associated with node i
P	vector of nodal point restoring forces depending on U and $\dot{U}$
R	vector of nodal point resisting forces
R	matrix of direction cosines between ground acceleration and structure
R	distance of bounding surface from hydroaxis along S_{ij} direction
S_{ij}, dS_{ij}	deviatoric stress tensor, its increment
T_ϵ	strain tensor
T_σ	stress tensor
U	nodal displacement vector
$\tilde{U}$	predicted nodal displacement vector
$\dot{U}$	nodal velocity vector
$\tilde{\dot{U}}$	predicted nodal velocity vector
$\ddot{U}$	nodal acceleration vector

$\ddot{U}_g$	free-field ground motion
X, Y, Z	global co-ordinates

Latin Lower Case

a	softening parameter
b_0, b_1	Rayleigh's proportionality constants
c	cohesion
d, d^+, d^-	damage variable, that in tension, and in compression
e_{ij}, de_{ij}	deviatoric strain tensor, its increment
e_{ij}^e, de_{ij}^e	deviatoric strain tensor due to elastic response, its increment
e_{ij}^p, de_{ij}^p	deviatoric strain tensor due to plastic response, its increment
e	unit vector
f	loading function
f	friction force per unit area
f'_c	concrete strength in uniaxial compression
f'_t	concrete strength in uniaxial tension
g	acceleration of gravity
g_f	energy per unit volume dissipated during fracture
k_i	penalty parameter in direction i of a joint element
k_n, k_s	normal and tangential penalty parameters of a joint element
l_c	characteristic length
p	vector of nodal restoring forces in a joint element
r	damage threshold
r	scale factor
r	distance from a stress point to the hydroaxis on the deviatoric plane
r, θ	polar co-ordinates
r, s	natural co-ordinates

t	time
$\mathbf{u}$	vector of nodal displacements in a joint element
$\mathbf{v}$	vector of nodal relative displacements in joint element
w_p	plastic work
w_c	bandwidth of propagation of a crack
$\mathbf{x}$	vector of nodal co-ordinates
x, y, z	local co-ordinates

Greek Upper Case

Δ	finite size increment operator
Ω, Ω_r	supporting area, reduced supporting area
Ω	volume domain

Greek Lower Case

β	shear retention factor
β	shear compaction–dilatancy factor
β_1	shear compaction factor
β_2	shear dilatancy factor
β, γ	parameters of Newmark time integration
γ	specific weight of material
$\gamma_0^p, d\gamma_0^p$	plastic octahedral shear strain; its increment
δ	slip margin of a joint element
δ	crack opening displacement
ϵ_E	value of energy based convergence criterion
ϵ	uniaxial strain
ϵ	strain tensor

ϵ, ϵ_0	uniaxial strain; its value associated to f'_t
ϵ_{cr}	critical uniaxial tensile strain
$\epsilon_{ij}, d\epsilon_{ij}$	strain tensor; its increment
ϵ_p	associated to f'_c uniaxial strain
θ	Lode angle
θ	angle between projections of position vector of principal stress and that of any tensile semiaxis on deviatoric plane
λ	ratio between the residual strain upon closing of the cracks and the strain of open cracks
μ	coefficient of friction
ν	Poisson's ratio
ξ, η, ζ	natural element co-ordinates
π	trigonometric constant 'pi'
σ	uniaxial stress
σ	normal stress in a joint element
$\bar{\sigma}$	tensor of effective stress
$\sigma_{0,i}$	tensile strength in direction i of a joint element
σ'_a	apparent tensile strength of concrete
$\sigma_{ij}, d\sigma_{ij}$	stress tensor; its increment
$\sigma_I, \sigma_{II}, \sigma_{III}$	principal stresses
σ_{tu}	tensile strength of concrete in triaxial loading conditions
τ	resultant shear stress in a joint element
τ_1, τ_2	tangential stress in a joint element
$\tau_0, d\tau_0$	octahedral shear stress; its increment
ϕ	angle of friction

Contents

Abstract	iii
Acknowledgments	vi
List of notations	vii
List of tables	xvi
List of figures	xx
1 Introduction	1
1.1 General	1
1.2 Causes of nonlinear behaviour in concrete arch dams	3
1.3 Objectives and scope of the study	4
1.4 Layout of thesis	5
2 Literature survey	7
2.1 General	7
2.2 Joint models	7
2.3 Nonlinear concrete models	12
2.3.1 Fracture mechanics approach	14
2.3.2 Damage mechanics approach	17
2.3.3 Recent studies with application to concrete dams	19
2.4 Conclusions from the literature survey	22

3	Nonlinear joint model	32
3.1	General	32
3.2	Joint models	33
3.2.1	ADAP-88 joint model	34
3.2.2	Improved joint model	38
4	Nonlinear concrete model	54
4.1	General	54
4.2	Modelling of concrete behaviour in compression	54
4.2.1	Description of the implemented bounding surface model	55
4.2.2	Numerical implementation	59
4.3	Modelling of concrete behaviour in tension	65
4.3.1	Description of the implemented crack model	65
4.3.2	Numerical implementation	72
4.4	Combined model of concrete behaviour in compression and tension	75
5	Numerical implementation	90
5.1	General	90
5.2	Nonlinear solution technique	92
5.3	Solution of the equations of static equilibrium	94
5.4	Solution of the equations of motion	96
5.5	Numerical implementation of seismic analysis procedure	98
5.5.1	Damping	98
5.5.2	Effective stiffness matrix	98
5.5.3	Effective out-of-balance force vector	99
5.5.4	Algorithm for solving the nonlinear equations of motion	100
5.5.5	Convergence criteria	102
5.5.6	Finite element types	103
5.5.7	Presentation of analysis results	105

6	Seismic analysis of Morrow Point dam	113
6.1	Introduction	113
6.2	Description of Morrow Point dam	113
6.3	Finite element models	114
6.3.1	Finite element model of dam	114
6.3.2	Finite element model of foundation	115
6.3.3	Reservoir model	116
6.4	Loads and load combinations	116
6.5	Joint model	118
6.6	Concrete models	118
6.7	Static analysis	118
6.8	Seismic analysis with linear-elastic concrete model	119
6.9	Seismic analysis with smeared crack model for concrete	120
6.10	Seismic analysis with bounding surface model	122
6.11	Seismic analysis with combined smeared crack-bounding surface model	123
7	Summary and conclusions	170
7.1	Summary of the research	170
7.2	Summary of the results from the case study	170
7.3	Conclusions	171
7.4	Recommendations for future work	172
	References	173
A	Material parameters of the bounding surface model	178
A.1	Pre-failure parameters	178
A.1.1	Generalised elastic shear modulus H^e	178
A.1.2	Generalised plastic shear modulus H^p	179
A.1.3	Bulk tangent modulus K_t	180

A.1.4	Shear compaction–dilatancy factor β	180
A.2	Post–failure parameters	181
A.2.1	Shear modulus G	181
A.2.2	Generalised plastic shear modulus H^p	181
A.2.3	Bulk tangent modulus K_t	182
A.2.4	Dilatancy factor β	182

List of Tables

6.1	Material properties for the concrete of dam	126
6.2	Material properties for the rock of foundation	127
6.3	Vibration frequencies of Morrow Point dam	128

List of Figures

2.1	Equivalent joint element by Dowling and Hall	24
2.2	Mesh refinement near joints in ADAP-88	25
2.3	Hohberg's finite element model	26
2.4	Typical compressive stress-strain curve of concrete	27
2.5	Strategies for finite element fracture analysis of concrete dams	28
2.6	Modes of crack failure	29
2.7	Bilinear average stress-strain relationship for smeared crack element .	30
2.8	Simplified family tree of fracture mechanics	31
3.1	Vertical contraction joints in an arch dam	46
3.2	Kinds of contraction joints in arch dams	47
3.3	Contraction joints with shear keys	48
3.4	Geometry of the joint element	49
3.5	Constitutive relationship for the joint element of ADAP88	50
3.6	Constitutive relationships for the implemented joint element	51
3.7	Flow chart for computations at an iteration step in open state of a joint element stress point.	52
3.8	Flow chart for computations at an iteration step in closed state of a joint element stress point.	53
4.1	The bounding surface model	78
4.2	Flowchart of the bounding surface model computational algorithm . .	79
4.3	Displacement-controlled test of the bounding surface model on a single element	80
4.4	Apparent tensile strength	81

4.5	Triaxial tensile failure envelope for concrete	82
4.6	Typical stress–strain curve for mass concrete from simple tension test	83
4.7	Stress–strain relationship in tension	84
4.8	Flowchart of the smeared crack model computational algorithm . . .	85
4.9	Displacement–controlled test of the smeared crack model on a single element	86
4.10	Modelled stress–strain relationship for one–cycle uniaxial loading . . .	87
4.11	Flowchart of the combined material model computational algorithm .	88
4.12	Displacement–controlled test of the combined model on a single element	89
5.1	Illustration of the full Newton–Raphson iterative procedure for solution of a generic SDOF	107
5.2	Illustration of the initial stress Newton–Raphson iterative procedure for solution of a generic SDOF	108
5.3	Illustration of the modified Newton–Raphson iterative procedure for solution of a generic SDOF	109
5.4	Simulation of the construction sequence of a concrete arch dam . . .	110
5.5	Flowchart of the Newmark procedure applied in CADNAP for solving the nonlinear equations of motion	111
5.6	Three-dimensional solid element	112
6.1	Morrow Point dam, U.S.B.R., 1983	129
6.2	Finite element mesh of the half–model of Morrow Point dam	130
6.3	Nodes of fluid elements on the upstream face of the dam	131
6.4	Acceleration time histories for the input ground motion	132
6.5	Pseudo-acceleration response spectra for earthquake ground motion .	133
6.6	Contours of static stresses in case of empty reservoir	134
6.7	Contours of static stress in case of half–full reservoir	135
6.8	Contours of static stress in case of full reservoir	136

6.9	Linear-elastic concrete model envelopes of maximum stress in case of empty reservoir	137
6.10	Linear-elastic concrete model envelopes of minimum stress in case of empty reservoir	138
6.11	Linear-elastic concrete model envelopes of maximum stress in case of half-full reservoir	139
6.12	Linear-elastic concrete model envelopes of minimum stress in case of half-full reservoir	140
6.13	Linear-elastic concrete model envelopes of maximum stress in case of full reservoir	141
6.14	Linear-elastic concrete model envelopes of minimum stress in case of full reservoir	142
6.15	Zones with investigated response histories	143
6.16	Linear-elastic concrete model envelopes of maximum stress for monolithic dam in case of full reservoir	144
6.17	Linear-elastic concrete model envelopes of minimum stress for monolithic dam in case of full reservoir	145
6.18	Crack pattern evolution in the crown cantilever	146
6.19	Smeared crack model envelopes of maximum stress	147
6.20	Smeared crack model envelopes of minimum stress	148
6.21	Smeared crack model arch stress histories in zone B	149
6.22	Smeared crack model cantilever stress histories in zone B	150
6.23	Smeared crack model arch stress histories in zone C	151
6.24	Smeared crack model cantilever stress histories in zone C	152
6.25	Smeared crack model displacement time history at the crest of the crown cantilever	153
6.26	Bounding surface model envelopes of maximum stress	154
6.27	Bounding surface model envelopes of minimum stress	155

6.28 Bounding surface model arch stress histories in Zone B	156
6.29 Bounding surface model cantilever stress histories in Zone B	157
6.30 Bounding surface model arch stress histories in Zone C	158
6.31 Bounding surface model cantilever stress histories in Zone C	159
6.32 Bounding surface model displacement time history at the crest of the crown cantilever	160
6.33 Damage evolution in the crown cantilever	161
6.34 Combined nonlinear model envelopes of maximum stress	162
6.35 Combined nonlinear model envelopes of minimum stress	163
6.36 Combined nonlinear model arch stress histories in zone B	164
6.37 Combined nonlinear model cantilever stress histories in zone B	165
6.38 Combined nonlinear model arch stress histories in zone C	166
6.39 Combined nonlinear model cantilever stress histories in zone C	167
6.40 Comparison of the downstream arch stress time histories in zone C	168
6.41 Combined concrete model displacement time history at the crest of the crown cantilever	169

Chapter 1

Introduction

1.1 General

Dams are important structures which make possible the large scale and efficient use of one of the most important natural resources - water, for human development. Since ancient times, dams have been constructed to create reservoirs to store water for drinking water supply, agricultural irrigation, flood control, river navigation, and more recently for electric power generation and industrial use. According to a recent survey (ICOLD 1984), there are currently more than 36,000 large dams in the world, which are defined as equal to or greater than 15 metres high. Due to the ever increasing number of large dams in operation and the potential disasters that the failure of a large dam can cause, the issue of dam safety has become a matter of great public concern. To address this concern, governments have established regulatory agencies which have enacted measures and regulations to review and approve new dam projects, as well as procedures to ensure the safe operation of existing dams. Because of the many advances in structural engineering in the past few decades, especially in the areas of computer modelling and analysis techniques, it has been found that some of the previous design assumptions on material models and structural behaviour employed in large dam design may lead to inadequate structures. Consequently, the safety of many existing dams need to be re-examined in light of the new state of knowledge and developments. In order to reduce the potential needs of very costly strengthening and rehabilitation of these existing dams, numerical modelling and analysis tools which can accurately predict the behaviour and performance of dam structures under both

normal operating and extreme load conditions are required.

Many dams are located in seismically active regions. For reasons already mentioned earlier, the seismic hazards of large dam structures are major concern to dam designers, owners and operators. In typical earthquake resistant design of dams, according to the International Commission on Large Dams (ICOLD) guidelines for selecting seismic parameters, a dam must be designed to have the capacity to resist the earthquake forces induced by an Operating Basis Earthquake (OBE) and a Reservoir-Induced Earthquake (RIE) within the linear elastic response range without suffering any damage; and to resist the Maximum Design Earthquake (MDE) with only limited zones of damage due to inelastic action in the dam body but without jeopardizing the overall capacity of the structure to hold the impounded water in the reservoir (ICOLD 1989). In order to assess the performance of dam structures or design with regards to these performance requirements, comprehensive nonlinear dynamic analysis of the dam systems are required. Because large dams are very massive structures, the inelastic action in the dam body resulting from the large inertial forces of the dam, and the interaction between the dam body, the foundation and the impounded water during strong earthquakes are important structural behaviour to be considered and modelled in evaluating the earthquake responses of large dams (ICOLD 1986).

Sophisticated analysis procedures have been developed by many researchers to study the linear elastic behaviour of concrete arch dams subjected to strong ground motions during an earthquake (Clough et al. 1973, Hall and Chopra 1983, Fok and Chopra 1985, Tan and Chopra 1995). However, the nonlinear phenomena in the seismic response of large dam structures have not been addressed as extensively, partly because of lack of experimental data, and also because of the complexity of the problems and the extensive computational effort required to carry out nonlinear dynamic time history analysis. This thesis presents a new comprehensive nonlinear dynamic time history analysis procedure for evaluating the detailed responses, and behaviour and performance of concrete arch dams subjected to earthquake ground motions.

1.2 Causes of nonlinear behaviour in concrete arch dams

Nonlinear phenomena in the behaviour of an arch dam can arise in the dam body and in the foundation of the structure. The present work concerns the nonlinearities in the dam body only. The formulations proposed herein can be extended to consider similar phenomena in the rock foundation substructure of a concrete dam.

One source of nonlinearity in the responses of concrete arch dams subjected to severe earthquakes is the nonlinear stress-strain relationship of the concrete material under states of large deformations. The nonlinear behaviour of the concrete material is very complex and depends on the history and on the rate of the applied loading. In addition, concrete exhibits significantly different behaviour under tension and compression. The tensile strength of concrete is approximately 10% of its compressive strength. Seismic analyses based on the assumption of linear-elastic stress-strain relationship in both tension and compression with the same constant rigidity for concrete may lead to unrealistically high tensile and compressive stresses in the dam body. Tensile stresses exceeding the tensile strength of concrete lead to cracking of the concrete material, and thus cause the structure to respond in a nonlinear and inelastic manner to the external loads. Compressive stresses higher than approximately one third of the compressive strength of concrete cause irreversible damage to the material of crushing and permanent deformations and strains.

Another important source of nonlinearity in the seismic responses of concrete arch dams is the presence of contraction joints in the dam body. Arch dams are constructed as cantilever monoliths separated by vertical contraction joints which are grouted with cement-sand mortar before the impounding of the reservoir. The purposes of the contraction joints are to release some of the thermal stresses generated from the hydration of the concrete during the construction phase, and the thermal loads induced from the ambient temperature gradients after completion of the dam. The contraction joints also minimise the formation of tensile cracks due to shrinkage of the concrete. Because of the negligible tensile strength of the grouting material, the joints cannot transmit tensile stresses between the adjacent cantilever monoliths. Relative movements of the cantilever monoliths during an earthquake may result in

the opening, closing, and sliding of the contraction joints, resulting in significant dissipation of the seismic input energy.

The interaction between the impounded water and the movement of the dam may result in nonlinear responses of the structure. As an example, water penetrating into open cracks or open contraction joints during the seismic response of the dam can exert very high stresses on the surfaces of the cracks or contraction joints upon closing because of the high incompressibility of water. This can significantly affect the stress distribution in the body of the dam.

There have been only few cases in which nonlinear behaviour have actually been recorded in a large concrete arch dam. One of the prominent examples is the observed responses of the 111-metre high Pacoima arch dam during the 1971 San Fernando earthquake. An accelerograph installed on a ridge near the dam recorded acceleration peaks in excess of 1.0 g in both horizontal components and 0.7 g in the vertical direction. Although severely shaken, the dam body experienced only slight damage (IEC 1972). The earthquake opened a previously-grouted contraction joint on the thrust block of the left abutment and caused a small crack near the base of the thrust block. However, at the time of the earthquake, the water level was 45 meters below the crest. A comprehensive case study of the nonlinear dam responses shows that the damage suffered by the dam body would have been worse had the reservoir been full at the time of the seismic event (Dowling and Hall 1989).

1.3 Objectives and scope of the study

The objectives of the present research are to develop numerical models for non-linear dynamic time history analysis of concrete arch dams which takes into account in details the interaction and the coupling effects between the nonlinear inelastic behaviour of the concrete material and the contraction joints in the body of the dam, to verify the validity of the proposed numerical analysis models and procedures by numerical examples, and to implement a comprehensive and effective computer tool based on the developed numerical models for the seismic response and performance

evaluation of concrete arch dams. Most previous studies of arch dams have not accounted simultaneously for the two types of nonlinearity considered herein, and only a few have considered the effect of the vertical contraction joints on the response of arch dams.

The new model combines a nonlinear joint model and a nonlinear concrete model to account for the dissipation of the seismic energy through the large movements at the vertical contraction joints and through the tensile cracking and the compressive crushing of the concrete. A nonlinear zero-thickness joint element developed in an earlier phase of the study by Noruziaan (1995) and Lau et al. (1998) is employed to simulate the opening/closing and shear sliding effects at the vertical contraction joints of the dam. A concrete material model which combines a model for tensile fracture (Cervera and Hinton 1986) with a model for compressive fracture (Pagnoni et al. 1992) is developed in the present study. The joint element and the combined concrete model are implemented in a finite element computer program developed for the purposes of the current research. The new program CADNAP (Concrete Arch Dam Nonlinear Analysis Program) is based on the existing program ADAP88 (Fenves et al. 1989) as the backbone and the computer program developed by Noruziaan (1995). The validity of the proposed finite element formulation is demonstrated in the analysis of the seismic response and performance of the 142-metre high Morrow Point dam.

1.4 Layout of thesis

The thesis is organised into seven chapters. Chapter 1 introduces the problem and the motivation and significance of the current research, its objectives and scope of the study. Chapter 2 summarises the findings of the literature survey carried out to give an overview on the state of knowledge and research relevant to nonlinear analysis of concrete dams. The adopted nonlinear joint element model is described in Chapter 3. The numerical implementation of this model is discussed in details. Chapter 4 presents the basic concepts of the concrete models adopted to simulate tensile cracking and compressive fracture of concrete subjected to frequent loading reversals

typical of earthquake ground shaking. The combined model developed and its numerical implementation are discussed. Chapter 5 presents the formulation of the static equilibrium equations and the numerical algorithm for concrete arch dams seismic analysis procedures. Chapter 6 presents the case studies carried out to demonstrate the validity of the numerical models proposed for seismic analysis of large concrete arch dams. Chapter 7 presents the conclusions obtained from the present research and suggestions for future works.

Chapter 2

Literature survey

2.1 General

Previous studies on the nonlinear response of concrete arch dams subjected to static and dynamic loads generally consider separately the nonlinear kinematic behaviour of the contraction joints and the inelastic behaviour of the concrete material in the dam body. The modelling of the joint behaviour is typically carried out by means of nonlinear joint models considering only the nonlinear effects due to the kinematics of the joint movements. The material nonlinearity in the concrete cantilevers as a result of high stress states are not considered in the previous studies on the nonlinear joint behaviour of arch dams. However, in separate developments, nonlinear constitutive models developed for plain concrete have been applied to the analyses of concrete gravity dams. In the present study, results obtained from literature surveys on the current development of joint models and constitutive models for concrete, as well as their applications to analysis of concrete dams are presented.

2.2 Joint models

The utilisation of the joint elements in the modelling of the behaviour of the contraction joints is to allow discontinuity in displacements at the interface of two adjacent monolith bodies. In the formulation of the joint elements, double nodes, one on each side of the joint interface, are defined in the finite element mesh of the structure. The penalty constraint formulation is implemented to maintain compatibility in the displacements of the corresponding pairs of double nodes throughout

the response by preventing any unrealistic penetration of the interface surfaces at the joint. In previous research and earlier developments, joint elements of this nature have been referred to as “gap elements”, “joint elements”, “boundary slip elements”, and “thin elements” (Hohberg 1992). Differences in the formulations of these previous joint models are related to the finite or zero thickness of the elements, the strain or displacement constitutive relationships adopted in the formulations, the number of double nodes employed, the modelling of the deformation characteristics of the joints by a lumped or a consistent stiffness matrix, and the quadrature scheme adopted.

Dowling and Hall (1989) present a joint element formulation to model the opening and closing behaviour of contraction joints at predetermined horizontal cracking planes in concrete arch dams. Since this joint element follows a single shell element discretization scheme in the thickness direction of the dam, it is sometimes referred to as a “shell joint element” (Hohberg 1992). A translational and a rotational rigid no-tension springs are employed to model the joint, as shown in Figure 2.1. The tangent stiffness of the joint is defined based on the two-dimensional linearly elastic plane strain discretization of a slab using four-node rectangular elements. The compressive and sliding nonlinear effects of the joint are not considered in this model. In the finite element mesh of the structure, a joint is represented by a double node with translational degrees-of-freedom, two parallel and one perpendicular to the joint plane. At each of the joints, two independent sets of degrees-of-freedom are assigned: one is average, and the other is relative, each including rotations and translations. The joint elements contribute stiffness to the relative degrees-of-freedom only. Due to the no-slip assumption, relative displacement in the direction parallel to the joint is not considered. It is recognised however that the no-slip assumption could be violated in the case of an unkeyed joint or a joint with bevelled keys. Joint elements with the above mentioned characteristics have been employed in the finite element analysis of Pacoima arch dam to investigate its responses to the 1971 San Fernando earthquake. The study results reveal that the contraction joints lead to several nonlinear phenomena in the seismic behaviour of the structure. The most important of these is the opening of the joints at the dam-foundation interface and at the upper portions

of the contraction joints. The opening of the contraction joints leads to significant changes in the stress distribution pattern in the dam body. Another important observation on the nonlinear behaviour of the dam is the cracking of the cantilevers due to loss of arch stiffness. Further, in the case of a full reservoir, the computed maximum compressive stresses exceed the limit from the linear stress-strain relationship. The researchers of the study suggest that improvements to the applicability of the proposed joint model can be achieved by removing the assumptions of uniform free field input ground motions and no relative slippage between the joint surfaces. It is expected relaxation of these assumptions will lead to reduction of the intensity of the response.

Fenves et al. (1989) implement another joint finite element in the computer program ADAP-88. ADAP-88 is an extension of ADAP, a computer program originally developed by Clough et al. (1973) for linear static and seismic analysis of arch dams. The joint element adopted in ADAP-88 can model the opening and closing behaviour of vertical contraction joints. The dam's cantilevers and foundation are treated as linear elastic substructures. The only nonlinearity in the finite element model is thus associated with the joints. To further reduce the computational effort for the iterative solution of the nonlinear equations of motion, the procedure for solution of nonlinear systems with localised nonlinearity proposed by Row and Schrieker (1984) is incorporated in the computer program ADAP-88. In the thickness direction of the dam, the vertical contraction joint is modelled by three joint elements and the concrete material adjacent to the joint is modelled by multiple layers of three-dimensional solid elements in the arch direction, as shown in Figure 2.2. To reduce the computational effort, other parts of the dam body are modelled by a single layer of shell elements. The ADAP-88 arch dam model has been used in the analysis of the response of Big Tujunga dam subject to the 1971 San Fernando earthquake. The results show that the opening-closing mechanism of the contraction joints causes a reduction in the effective vibration frequencies of the structure, and a reduction in the tensile arch and cantilever stresses in the dam body.

In a subsequent parametric study performed by means of ADAP-88, Fenves et

al. (1992) investigate the optimum number of contraction joints required to adequately model the earthquake response of arch dams. The parametric study is carried out on a finite element model of Morrow Point dam. It is concluded that a model of the dam with only one contraction joint at the crown section can simulate the most important effects of joint opening on the response of the dam to a symmetric ground motion. For the case of non-symmetric response, a three-joint model of the dam structure can yield reasonably accurate results as compared to models with more than three joints. The study reports that for this particular dam and ground motion, the opening of the joints reduces the arch tensile stresses and increases the cantilever stresses. Increasing the number of modelled contraction joints leads to a larger reduction in the arch tensile stresses, but no significant change in the maximum cantilever stress. Since the opening of the contraction joints will prevent arch stress from developing, it is therefore suggested that the maximum cantilever stresses are more relevant indicators of the performance of concrete arch dams during earthquake. The results from the parametric study show that for the analysed dam a three-joint model of the whole structure provides good accuracy at reasonable computational effort.

A fairly sophisticated joint element formulation has been presented by Høhberg (1992). This joint model is developed with the objective to be applicable to concrete and rock joints. The joint model assumes small relative displacements in the joints, equivalent to the small strain assumption in continuum mechanics. Penalty parameters are adopted as elastic moduli in the formulation of the joint constitutive model. Both opening and frictional sliding of the joint are considered in the model. The former is treated as a stress-free reversible gap displacement, whereas the latter is modelled as an incremental plastic process with a non-associated flow. If the stresses exceed the joint strength, the shear capacity is computed as a function of the surface asperities. To demonstrate the applicability of the joint model, the seismic response of an idealised concrete arch dam with a symmetric geometry similar to that of the 237-metre high Mauvoisin dam is investigated. The cantilevers of the structure are modelled as linear elastic solids using 20-node serendipity brick and 15-node wedge shaped elements, as shown in Figure 2.3. In addition to the assumption of a linear

constitutive model for the cantilevers, several other simplifications are adopted: the foundation is assumed rigid, the self-weight of the structure is neglected, the reservoir is assumed empty, and only upstream/downstream excitation is considered. To investigate how the joint nonlinearity influences the response, four cases are analysed: linear monolithic structure, pure joint opening, limited shear key height and frictional sliding in unkeyed joints. It is concluded that the influence of the damping effect due to frictional sliding at the joints to the seismic response of the dam is much less than that due to the joint opening. Similar to previous studies, the analysis results confirm that joint opening leads to a lengthening of the apparent vibration period of the dam and changes to the stress distribution pattern in the dam body.

In a recent study, a comprehensive joint constitutive model is presented by Noruziaan (1995). The zero-thickness joint element developed by Noruziaan includes an enhancement to the joint model adopted in ADAP-88. It considers energy dissipation through nonlinear shear slippage. The joint slippage behaviour is formulated on the basis of a two-dimensional Mohr-Coulomb model. Parametric studies on the seismic response of the Morrow Point dam are carried out using the developed joint model. Based on the obtained results, it is concluded that contraction joints with no shear keys are susceptible to damage due to shear slippage in the upper parts of the dam.

In a study to evaluate the adequacy of a general purpose finite element computer program for the analysis of arch dams with keyed contraction joints, Mays and Roehm (1993) analyse East Canyon dam, a double curvature 80-metre high concrete arch. Both linear and nonlinear finite element models of the dam are considered in the study. In both models, the cantilevers of the dam are modelled by 20-node linear-elastic solid elements. The foundation is assumed rigid, and the dam-reservoir interaction is taken into account by means of a diagonalised added mass matrix. In the linear model, the contraction joints are modelled by rigid elements, whereas in the nonlinear model gap elements are used to connect corresponding nodes across the contraction joints. The dam models are then subjected to loading due to self-weight,

impounded water and strong ground motions. Three components of the ground motion records obtained from the 1967 Koyna earthquake event with the amplitude scaled by a factor of 1.22 to represent the maximum credible earthquake expected at the dam site are used as input excitation in the study. While the construction sequence is simulated in both models, the impounding of the reservoir is simulated by a single load step in the linear model and by multiple load increments in the nonlinear model. The results from the study indicate that the nonlinear analysis yields considerably lower horizontal tensile stress, but higher vertical tensile and compressive stresses.

2.3 Nonlinear concrete models

Nonlinear concrete models can be classified into nonlinear elastic, plastic, fracture-mechanics-based and damage-mechanics-based models. Both the elastic and the plastic concrete models utilise stress or strain based criteria to determine the failure of the material under increasing loading. This approach is more appropriate when applied to ductile materials exhibiting a prolonged yield plateau on the stress-strain relationship similar to that commonly observed in metals. As concrete is a quasi-brittle material, its failure cannot be accurately modelled by the stress criteria approach. Rather, the fracture propagation must be determined on the basis of an energy criterion, as formulated in fracture-mechanics-based and damage-mechanics-based models.

In order to completely describe the behaviour of concrete material, the constitutive model should have the capability to accurately predict the pre-fracture behaviour, the initiation and growth of the fracture process, and the post-fracture behaviour. Fracture of concrete can occur due to high tensile or high compressive stresses; it is referred to as cracking in the former case and as crushing in the latter.

Before the onset of a crack, the behaviour of concrete can be approximated as linear elastic. On the other hand, as Figure 2.4 shows, it is necessary to adopt a nonlinear constitutive law in order to properly describe the pre-crushing concrete behaviour.

As has already been stated, there are two main approaches to investigate the fracture behaviour of concrete structures, namely the damage mechanics approach and the fracture mechanics approach. While fracture mechanics is applicable only in the investigation of cracking, damage mechanics is applicable in the analysis of both cracking and crushing. It is worth mentioning that although the no-tension stress criteria design is widely believed to be on the safe side, Bazant reports (1994) that in certain cases a fracture mechanics analysis can yield a lower resistance of the structure.

Research efforts in modelling material nonlinear effects in concrete dams have focused mainly on the analysis of the two-dimensional stress and strain state in gravity dams. The material nonlinear behaviour in the three-dimensional stress and strain state of arch dams have rarely been addressed. The majority of the research work have concentrated on modelling the cracking effects due to high tensile stress. The nonlinear effects due to high compressive stress have not been considered in such details as the cracking effects. Although studies have shown that the compressive stress level in concrete gravity dams rarely exceeds the elastic limit, this may not be the case for arch dams because of their inherently more flexible characteristics.

The modelling of the behaviour of fractured concrete in a finite element formulation can be classified into two main approaches: discrete models and continuum models. Discrete models can only be applied for investigating the initiation and propagation of cracks, i.e. they cannot be used for investigating the behaviour of crushed concrete under a compressive stress state. A discrete fracture model is derived based on the principles of fracture mechanics. In the discrete fracture formulation, a crack is represented as a gap in the finite element mesh. The mesh is modified when the crack propagates. This procedure requires a significant increase in the computational effort when it is applied to seismic analysis. The discrete fracture model is more suitable for modelling the behaviour of brittle materials under slow rate of loading without too many load reversal cycles, or where the crack location is known *a priori*. The contraction joint finite element models, such as those described in the previous section, can be regarded as particular cases of the discrete crack formulation.

In finite element applications, continuum models can be used to investigate both cracking and crushing of concrete. In this approach, the finite element mesh is kept unchanged regardless of the progress of the fracturing process. The changes to the state and behaviour of the material are reflected by periodic updates of the material constitutive relationship. This approach generally requires less computational effort than the discrete model approach and has been widely adopted in the analysis of the seismic behaviour of concrete structures. Both damage-mechanics-based and fracture-mechanics-based models can be implemented in the continuum approach. When a fracture-mechanics-based model is formulated within the continuum approach, the smeared crack concept is used. The latter represents the crack tip as a blunt front spread over an entire finite element or a certain portion of it. The different strategies for finite element fracture analysis of concrete dams are summarised in Figure 2.5. In the following sections, the main concepts of the fracture mechanics and the damage mechanics approaches are presented.

2.3.1 Fracture mechanics approach

In the fracture mechanics approach, the initiation and propagation of a crack are evaluated based on the consideration of energy dissipation. Two alternative fracture mechanics formulations have been developed to determine crack propagation based on the energy criteria or stress intensity criteria.

In the energy criterion approach, the energy causing crack growth is compared to the material resistance to cracking. The resistance may include surface energy, plastic work, or any other type of energy dissipation associated with a propagating crack. At the moment of fracture, the energy release rate, G_f , defined as the rate of change in potential energy with crack area for a linear elastic material (Anderson 1991), is equal to the critical energy release rate, G_{fc} . To draw a parallel between these parameters in fracture mechanics with corresponding ones in strength of materials, the driving force for cracking, G_f , is comparable to the stress, while the material's resistance to cracking, G_{fc} , is equivalent to the material's yield strength.

In the stress intensity approach, the stress field around the tip of a crack in

an elastic material is completely characterised by stress intensity factors. The stress intensity factors are determined as follows:

$$\begin{Bmatrix} K_I \\ K_{II} \\ K_{III} \end{Bmatrix} = \lim_{r \rightarrow 0, \theta \rightarrow 0} \sqrt{2\pi r} \begin{Bmatrix} \sigma_{22} \\ \sigma_{12} \\ \sigma_{23} \end{Bmatrix} \quad (2.1)$$

where σ_{ij} denotes the near crack-tip stress, and r and θ are the polar coordinates. Each stress intensity factor corresponds to a mode of loading, as shown in Figure 2.6. In a two-dimensional analysis of a concrete gravity dam, only modes I and II of the opening and the in-plane shear modes, respectively, need to be considered. The three-dimensional stress and strain state in arch dams require that mode III, i.e. the out-of-plane shear mode, be considered as well. In the stress intensity approach, K_i is regarded as the driving force of fracture, and K_{i_c} , the critical stress intensity factor, as the measure of material resistance.

Most of the early research in the field of fracture mechanics have assumed that the material of the structure remains linear elastic except for the zone around the crack tip with dimensions negligibly small compared to the crack length or the cross section dimensions of the structure. With this assumption, linear elastic fracture mechanics models have been formulated utilising both the energy and the stress intensity approaches. The linear elastic fracture mechanics approach has been applied in both discrete and smeared crack models (Bhattacharjee and Léger 1992).

If, however, the assumption that the nonlinear material behaviour is limited at the crack tip zone cannot be justified, the linear elastic fracture mechanics models in conjunction with the stress intensity approach are not valid anymore and a nonlinear fracture mechanics model must be adopted. The nonlinear fracture mechanics models recognise that the crack front develops in a fracture process zone that is not infinitely small but has a finite size. Consequently, only the energy approach is applicable in the formulation of the cracking process. The main characteristic in the development of a nonlinear fracture mechanics model is the modelling of the strain softening behaviour of concrete in the fracture process zone. Two different models, concerned with the

fracture mode I only, have been developed to deal with strain softening: one is the fictitious crack model and the other is the crack band model.

In the fictitious crack model developed by Hillerborg (1976), the fracture process zone is represented as a fictitious crack placed ahead of the actual crack tip. The released energy per unit area during the fracture process is determined as follows:

$$G_f = \int_0^{\delta_f} \sigma(\delta) d\delta \quad (2.2)$$

where σ is the stress in the fracture process zone, δ is the crack opening displacement, and δ_f is the critical crack opening displacement when the softening stress is equal to zero. Because of the assumption that the dissipation of energy takes place over a discrete crack line, the fictitious crack model is only applicable in discrete crack models.

Alternatively, the crack band model (Bažant 1983) assumes the fracture propagates as a blunt front of micro-cracks of a width w_c . In this model, the fracture energy is expressed as the product of the area under the stress-strain curve and the width of the blunt crack as follows:

$$G_f = w_c \int_0^{\varepsilon_f} \sigma(\varepsilon) d\varepsilon \quad (2.3)$$

where ε is the strain in the fracture process zone, and ε_f is the critical strain when the softening stress is equal to zero. The width of the blunt crack w_c is assumed to be a material property in the crack band model. Based on empirical data, it is determined that w_c is approximately three times the maximum aggregate size of concrete (Shah et al. 1995).

In the formulation of the crack band model, the smeared crack model is considered. Within the bandwidth w_c , a very fine mesh of the finite elements is required to represent the crack-prone zone of the material. To avoid this limitation, numerical techniques have been developed to modify the constitutive law of the material by adjusting the area under the average stress-strain curve so that the released fracture energy G_f becomes independent of the element characteristic length l_c . A rigorous procedure to determine the characteristic length is proposed by Oliver (1989). The

characteristic length is the ratio of the fracture energy G_f (energy dissipated per unit area during the deformation process) to the specific energy g_f (energy dissipated per unit volume):

$$l_c = \frac{G_f}{g_f} \quad (2.4)$$

Considering the bi-linear stress-strain relationship shown in Figure 2.7, the strain-softening modulus E_t can be expressed in terms of the characteristic length l_c as follows:

$$E_t = \frac{(f'_t)^2 E}{(f'_t)^2 - \frac{2EG_f}{l_c}} \quad (2.5)$$

where f'_t and E are the uniaxial tensile strength and the Young's modulus of the material, respectively. Since the denominator in the above equation must be less than or equal to zero, the maximum value of the characteristic length can be obtained as follows:

$$l_{c_{max}} \leq \frac{2EG_f}{(f'_t)^2} \quad (2.6)$$

For typical concrete properties of $E = 30\,000$ MPa, $G_f = 200$ N/m, and $f'_t = 2.0$ MPa, a value of the characteristic length is determined as $l_{c_{max}} = 3.0$ m. This characteristic length presents significant difficulty when a nonlinear fracture mechanics model is used to investigate the cracking behaviour of large structures.

A schematic representation of the development of the different fracture mechanics branches is shown in Figure 2.8. The theory of elastic-plastic fracture mechanics considers plastic deformation under quasi-static conditions, whereas the theories of dynamic, visco-elastic, and visco-plastic fracture mechanics include time as a variable. Elasto-plastic, visco-elastic, and visco-plastic fracture mechanics are often referred to as Nonlinear Fracture Mechanics.

2.3.2 Damage mechanics approach

The theory of damage mechanics is based on the assumption that the material degradation is induced by distributed, arbitrarily oriented fractured zones at the

micro level. Upon increased loading, the micro fractures link up to form a macro fracture in a direction determined from the principal stress directions. The modelling methodology of damage mechanics is to express the damaged state of the material by means of an internal scalar variable called the damage variable d . The fundamentals in the formulation of a damage-mechanics-based model are presented here in this section as background materials for the present study.

In the formulation of a damage mechanics model, the strain associated with a damaged state under the applied stress σ is considered equivalent to the strain associated with its undamaged state under the effective stress $\bar{\sigma}$. In tensorial notation, the effective stress is given by:

$$\bar{\sigma} = \mathbf{E}_0 : \epsilon \quad (2.7)$$

where $\mathbf{E}_0$ is the initial elastic stiffness tensor, ϵ is the strain tensor, and $(:)$ denotes the tensorial product contracted on two indices. Assuming isotropic material, the constitutive law for a damage-mechanics-based model relates the applied stress to the effective stress as follows:

$$\bar{\sigma} = \frac{\sigma}{1 - d} \quad (2.8)$$

Cervera et al. (1996) have proposed the use of two scalar damage variables, d^+ and d^- to account for the damage resulting from tensile states and compressive states, respectively. The evolution of damage is expressed as a function of the current damage threshold r as follows:

$$d = G(r) \quad (2.9)$$

where $G(r)$ is a function obtained from experimental data. The evolution law ensure that the energy dissipated per unit volume g_f is finite, i.e. the area under the material stress-strain curve is finite. To satisfy the requirement of conservation of energy, the upper bound of the specific fracture energy g_f is related to the fracture energy of concrete as follows:

$$g_f = \frac{G_f}{l_c} \quad (2.10)$$

where the characteristic length l_c is defined similarly as in the smeared crack fracture mechanics models. Compared with the fracture mechanics theory used in the context of smeared cracks, the damage mechanics models exhibit little difference in formulation; they permit, however, easier simulation of initial damage due to thermal stress, alkali-aggregate reactions, etc.

2.3.3 Recent studies with application to concrete dams

As discussed earlier, previous studies on the fracture of dams focus mainly on the behaviour of concrete gravity dams considering only two-dimensional crack propagation. The results of these earlier studies have provided a valuable insight into the nonlinear concrete dam response to gravity, hydrostatic and seismic loads.

Bhattacharjee and Léger (1993) report a study performed by means of a smeared crack model based on nonlinear fracture mechanics. They assume a linear elastic relationship between compressive stresses and strains. The mesh-dependency of the response and the stress locking problem in the finite element model are avoided by utilising an energy conserving co-axial rotating crack model (CRCM). The energy dissipation for a unit area of crack plane propagation is conserved by adjusting the slope of the softening line in the tensile stress-strain relationship. In the co-axial rotating crack model, it is assumed that once softening initiates, a smeared band of microcracks appears in the direction perpendicular to the principal tensile strain. The material axes are rotated to align with the principal strain directions. The constitutive matrix is redefined in terms of the ratio between the softened and the initial moduli of elasticity. The rotation of the material axis system and the change to the elastic moduli due to damage to the material are reflected in the update to the global constitutive matrix. To demonstrate the applicability of the proposed model, two-dimensional seismic response analyses of Koyna dam are performed. The study points out that the predicted cracking patterns are consistent with the field observations and with the results obtained from model tests.

In a subsequent study, Bhattacharjee and Léger (1994) compare the performance of the co-axial rotating crack model against another nonlinear fracture mechanics smeared crack model, namely a fixed crack model with variable shear resistance (FCM-VSRF). The fundamental difference between CRCM and FCM-VSRF is that in the FCM-VSRF formulation the material axis system is aligned with the principal strain directions when softening initiates and is kept unchanged throughout the rest of the analysis. Both models are applied to investigate the static response of a notched shear beam, a model concrete gravity dam, and Koyna dam. The results show that the FCM-VSRF model yields a stiffer response in general, due to the significant stress locking in the finite elements. The study concludes that the CRCM model is more reliable than the FCM-VSRF model in predicting the post-peak resistance of concrete structures and that the CRCM performs consistently better in all the three cases studied.

Ghrib and Tinawi (1995, a) compare three damage mechanics based models to investigate the nonlinear responses of concrete gravity dams in two-dimensional analyses. In two of the models, the damage evolution is defined as a function of the elastic strain energy of an undamaged equivalent material. In the third model, the damage is evaluated by using a measure of the material strain field. Isotropic and anisotropic formulations of the damage are considered in the energy-based models. The mesh objectivity is achieved through the formulation of mesh dependent softening parameters in the energy-based models, and by the procedure proposed by Mazars et al. (1991) in the strain-based model. While the energy-based models require a reasonable mesh size, the strain-based model requires an extremely fine mesh, making it expensive to use for the analysis of large structures. The damage mechanics based models are implemented in a finite element program to carry out static damage analysis of a model concrete dam. The numerical results compare well with the experimental crack profile obtained from the model tests, the best accordance shown by the anisotropic energy-based model. The anisotropic energy-based model is also applied to investigate the responses and damage of the Koyna dam due to reservoir overflow. The output is compared against published results obtained from a plasticity-based model, a LEFM

discrete model, and a smeared crack model analyses. It is concluded that the damage mechanics based and the smeared crack models give very similar results. In further comparative study, the anisotropic energy-based model is applied to investigate the influence of initial damage on the seismic responses of an idealised gravity dam. The results show that initial damage due to a crack at a construction joint is more critical than a diffused damage over the entire outer skin of the dam.

In another study, Ghrib and Tinawi (1995, b) extend the capabilities of the anisotropic energy-based model for seismic response analysis of concrete gravity dams by including a criterion for crack opening and closing. The validity of the formulation is demonstrated in an analysis of the responses of the Koyna dam to the 1967 Koyna earthquake. The results show the same damage pattern reported by Bhattacharjee and Léger (1993). The impact due to initial damage on the seismic responses of the structure is demonstrated in a comparative study considering five different scenarios of initial damage of an idealised dam subjected to the 1988 Saguenay earthquake.

Cervera et al. (1996) have developed a rate-dependent isotropic damage mechanics based model. Two internal variables are introduced to consider separately the damage resulting from tension and compression. The model accounts for the effect of stiffness degradation and stiffness recovery upon load reversals. The inter-dependency between damage growth and the rate of loading in concrete is modelled by formulating rate-dependent damage threshold evolution laws. The latter are expressed as a product of a viscous damage threshold flow function and a fluidity parameter. To ensure mesh objectivity, both the softening modulus and the fluidity parameter are expressed in terms of the characteristic length of the damaged zone. The importance of considering the rate of loading in seismic analysis of concrete gravity dams is demonstrated in a study of the earthquake response of Koyna dam. It is concluded that while the displacement time history may not be greatly affected, the development of the damage changes significantly when rate-dependent material is considered. It is found that the rate-dependent property of the material leads to a significant reduction in the degradation of the material.

Lee and Fenves (1998) suggest that it is difficult to model the cyclic loading and

unloading states in concrete using purely continuum damage mechanics based models because there is no specific representation of the inelastic strain in the damaged states. They present a plastic–damage formulation in which an elastic damage model is combined with plasticity models. A cyclic rate-independent model is formulated in a manner similar to the procedure adopted by Cervera et al. (1996) with the strain decomposed into elastic and plastic terms. The rate-independent plastic damage formulation is then extended to allow for rate-dependency in order to obtain unique and mesh-objective results. The rate-dependent plastic damage constitutive model is implemented in the computer program FEAP (Taylor 1992) to analyse the seismic behaviour of Koyna dam.

Lan and Yang (1997) introduce a smeared crack model with a strength criterion for crack propagation. They investigate the response of the 250-metre high WLX arch dam to loading due to hydrostatic pressure, self-weight, and temperature change. The effect of the contraction joints is neglected and the dam body is modelled as a continuous shell. The study indicates that cracking initiates in the heel of the dam at the loading level equal to 80 % of the normal load.

2.4 Conclusions from the literature survey

The present literature survey reveals that the nonlinear effects in the behaviour of concrete arch dams have been attributed mainly to the presence of contraction joints. The phenomenon of cracking and crushing in concrete dams subjected to cyclic loading and unloading has been considered in two-dimensional analysis of concrete gravity dams but ignored in the analysis of concrete arch dams. The combined effect of joint opening/sliding and material nonlinearity in arch dams has only been partially addressed by Noruziaan (1995) where the concrete nonlinearity is restricted to the concrete material adjacent to the vertical contraction joints.

To further the development of the finite element modelling and analysis techniques, the present study extends the modelling of the concrete nonlinearity over the entire body of the arch dam model. The nonlinear concrete modelling is implemented simultaneously with the detailed modelling of the contraction joints separating the

monolith cantilevers to obtain more accurate and detailed prediction on the static and seismic behaviour of concrete arch dams.

Figure 2.1: Equivalent joint element of Dowling and Hall (1989)

Strategy of Local Mesh Refinement

Figure 2.2: Mesh refinement near joints in ADAP-88 (Fenves et al. 1988)

26

Figure 2.4: Typical compressive stress-strain curve of concrete

Figure 2.5: Strategies for finite element fracture analysis of concrete dams

Figure 2.6: Modes of crack failure

(a)

(b)

Figure 2.7: (a) Bilinear average stress-strain relationship for smeared crack element;
(b) Characteristic length

Figure 2.8: Simplified family tree of fracture mechanics

Chapter 3

Nonlinear joint model

3.1 General

Concrete arch dams are constructed in monolithic blocks separated by transverse contraction joints, as shown in Figure 3.1. The contraction joints are provided to prevent the formation of tensile cracks due to shrinkage of concrete. Usually, the contraction joints are vertical and continuous from the foundation to the top and through the thickness from the upstream to the downstream face of the dam. The design criteria of the location and spacing of the joints are based on the temperature performance of the structure and the construction requirements. The distance between two joints along the axis of the dam ranges from 15 m to 25 m.

During and after the construction of the dam blocks, the contraction joints are sealed in order to ensure the water tightness of the structure. The seals can be metal, polyvinyl chloride, or rubber. They are also used during grouting operations in order to confine the grout mixture in the joint.

The purpose of grouting the contraction joints is to bridge the adjacent blocks and thus ensure monolithness of the arch dam. The grouting fluid is a mixture of Portland cement, sand and water and is injected into the joint under pressure in lifts of 15 m to 20 m in height. Although the grouting of the joints may lead to some prestressing of the structure, the stress levels resulting from the prestressing effect are not large and the subsequent shrinkage of the concrete blocks reduces further this effect. Due to the low tensile strength of the grouting material, the contraction joints can only develop low tensile stresses.

A contraction joint may have shear keys to increase the shearing resistance between the adjacent blocks. The depth of the shear keys is usually about 200 mm which may be bevelled or unbevelled, as shown in Figure 3.2. The shear keys may be continuous or discrete within a construction lift, as shown in Figure 3.3.

During the construction of large concrete structures, construction joints are introduced whenever there is a break in the placement of concrete. A construction joint is the surface of previously placed concrete upon or against which new concrete is to be placed (USBR 1977). Construction joints represent potential crack-prone zones in an arch dam when subjected to severe earthquakes.

As has already been mentioned, arch dams are designed to behave as monolithic structures. However, both construction and contraction joints constitute planes of weakness in the body of an arch dam. In response to strong ground motions, the joints may partially open due to their low tensile strength. This results in significant change to the stiffness properties of the structure and subsequently affects its seismic response. Hence, to obtain a better insight into the seismic behaviour of an arch dam, it is necessary to model the joints. Although it is recognised that construction joints, especially these of poor construction quality, can significantly affect the seismic performance of an arch dam, the present study considers only the effect of vertical contraction joints.

3.2 Joint models

The zero-thickness joint element developed by Noruziaan (1995) and by Lau et al. (1998) is used in the present study. The formulation of the joint element by Noruziaan is based on the joint element used in ADAP-88 (Fenves et al. 1989) but with significant improvements and new modelling capabilities to simulate sliding and shear joint behaviour. In the following, the basic formulation of the joint element in ADAP-88 is presented first, followed by the formulation of the improved model.

3.2.1 ADAP-88 joint model

Element geometry

The joint element consists of two surfaces each defined by four nodes, as shown in Figure 3.4. The bottom surface is defined by nodes 1 to 4 and the top surface is defined by nodes 5 to 8. Before loading, the two surfaces of the joint are completely coincident. The global coordinate vector $\mathbf{x}$ of an arbitrary point P in the joint element is obtained as follows:

$$\mathbf{x} = \sum_{j=1}^4 N_j \mathbf{x}_j \quad (3.1)$$

where $\mathbf{x}_j$ is the global coordinate vector of node j , and N_j is the corresponding shape function evaluated at P

$$N_j = \frac{1}{4} (1 + r_j r) (1 + s_j s) \quad j = 1, 2, 3, 4 \quad (3.2)$$

where (r_j, s_j) and (r, s) are, respectively, the coordinates of node j and point P in the natural coordinate system of the element, as shown in Figure 3.4 (b).

In addition to the global and the natural coordinate systems, an orthonormal coordinate system $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ is defined at every point on the top and on the bottom surfaces of the joint element, as shown in Figure 3.4 (a). Axis 3 is perpendicular to the surface and axes 1 and 2 are tangent to the surface. The unit vectors of the orthonormal system are expressed as follows:

$$\mathbf{e}_1 = \frac{\sqrt{2}}{2} (\mathbf{e}_\alpha - \mathbf{e}_\beta); \quad \mathbf{e}_2 = \frac{\sqrt{2}}{2} (\mathbf{e}_\alpha + \mathbf{e}_\beta); \quad \mathbf{e}_3 = \frac{\mathbf{e}_r \times \mathbf{e}_s}{\|\mathbf{e}_r \times \mathbf{e}_s\|} \quad (3.3)$$

where

$$\mathbf{e}_\alpha = \frac{\mathbf{e}_r + \mathbf{e}_s}{\|\mathbf{e}_r + \mathbf{e}_s\|}; \quad \mathbf{e}_\beta = \frac{\mathbf{e}_3 \times \mathbf{e}_\alpha}{\|\mathbf{e}_3 \times \mathbf{e}_\alpha\|} \quad (3.4)$$

and

$$\mathbf{e}_r = \frac{\mathbf{x}_{,r}}{\|\mathbf{x}_{,r}\|}; \quad \mathbf{e}_s = \frac{\mathbf{x}_{,s}}{\|\mathbf{x}_{,s}\|} \quad (3.5)$$

In Eqs. 3.3 to 3.5, the symbols ' $\times$ ', ' $\|\cdot\|$ ', and ' $\cdot$ ' denote vector product, Euclidean norm, and partial derivative, respectively.

Kinematics

In the global coordinate system, the vectors of displacements of a point on the bottom surface and its corresponding point on the top surface are given by

$$\mathbf{u}_{bot} = N_1 \mathbf{u}_1 + N_2 \mathbf{u}_2 + N_3 \mathbf{u}_3 + N_4 \mathbf{u}_4 \quad (3.6)$$

$$\mathbf{u}_{top} = N_1 \mathbf{u}_5 + N_2 \mathbf{u}_6 + N_3 \mathbf{u}_7 + N_4 \mathbf{u}_8 \quad (3.7)$$

where $\mathbf{u}_j$, $j = 1$ to 8 , is the displacement vector of node j .

The constitutive relationship of the joint element is derived in the orthonormal coordinate system in terms of the relative displacement between the bottom and the top joint surfaces. Hence, the vectors of surface displacements obtained from Eqs. 3.6 and 3.7 are to be transformed from the global coordinate system to the orthonormal coordinate system. The transformation is expressed as follows:

$$\bar{\mathbf{u}}_{bot} = \mathbf{a} \mathbf{u}_{bot} \quad (3.8)$$

$$\bar{\mathbf{u}}_{top} = \mathbf{a} \mathbf{u}_{top} \quad (3.9)$$

where $\mathbf{a}$ is the transformation matrix which is a function of the natural coordinates, r and s , on the surface of the element

$$\mathbf{a} = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \end{bmatrix}^T \quad (3.10)$$

The vector of the relative displacements between the two surfaces of the joint in the orthonormal coordinate system is given as

$$\mathbf{v} = \bar{\mathbf{u}}_{top} - \bar{\mathbf{u}}_{bot} \quad (3.11)$$

To express the relative displacements $\mathbf{v}$ in terms of the nodal displacements $\mathbf{u}$, Eqs. 3.6 to 3.9 are substituted into Eq. 3.11. The following expression for the vector of the relative displacements $\mathbf{v}$ is obtained:

$$\mathbf{v} = \mathbf{B} \mathbf{u} \quad (3.12)$$

where

$$\mathbf{u} = \begin{bmatrix} \mathbf{u}_1^T & \mathbf{u}_2^T & \mathbf{u}_3^T & \mathbf{u}_4^T & \mathbf{u}_5^T & \mathbf{u}_6^T & \mathbf{u}_7^T & \mathbf{u}_8^T \end{bmatrix}^T \quad (3.13)$$

$$\mathbf{B} = \begin{bmatrix} -N_1 & -N_2 & -N_3 & -N_4 & N_1 & N_2 & N_3 & N_4 \end{bmatrix} \quad (3.14)$$

and

$$N_j = N_j \mathbf{a}, \quad j = 1, 2, 3, 4 \quad (3.15)$$

Constitutive relationship

The stresses in the joint element $\boldsymbol{\sigma}$ are developed due to the relative displacements $\mathbf{v}$ between the joint surfaces. It is assumed that the relative displacement in the direction i of the orthonormal coordinate system produces stress in that direction only. The assumed constitutive stress–relative displacement relationship is characterised by two constants of the joint: the tensile strength σ_{0i} and the stiffness k_i . The resisting stress of the joint is computed as follows:

$$\sigma_i = \begin{cases} k_i v_i & \text{for } v_i \leq \sigma_{0i}/k_i \\ 0 & \text{for } v_i > \sigma_{0i}/k_i \end{cases} \quad i = 1, 2, 3 \quad (3.16)$$

As illustrated in Figure 3.5, σ_{0i} is the maximum tensile stress the joint can resist before it opens, and k_i is the joint stiffness when the joint is closed.

The stiffness k_i is implemented by a penalty parameter. For the case of $i = 1$, it simulates the no-penetration condition of the joint faces, whereas for the cases of $i = 2, 3$, the corresponding stiffness simulates the no-slippage condition in the specified direction of the joint faces.

Restoring forces and tangent stiffness matrix

The nodal restoring forces $\mathbf{p}$ are evaluated in the global coordinate system from integration of the joint stresses over the area of the joint element surface A as follows:

$$\mathbf{p} = \int_A \mathbf{B}^T \boldsymbol{\sigma} dA \quad (3.17)$$

The tangent stiffness matrix of a joint element in the global coordinate system is given as:

$$\mathbf{k}_T = \int_A \mathbf{B}^T \bar{\mathbf{k}}_T(\mathbf{v}) \mathbf{B} dA \quad (3.18)$$

where the tangent stiffness matrix $\bar{\mathbf{k}}_T(\mathbf{v})$ in the orthonormal coordinate system is a diagonal matrix with the diagonal terms given as follows:

$$\bar{k}_{T,i} = \begin{cases} k_i, & v_i \leq \sigma_{0,i}/k_i \\ 0, & v_i > \sigma_{0,i}/k_i \end{cases} \quad i = 1, 2, 3 \quad (3.19)$$

Numerical integration

The nodal restoring force vector $\mathbf{p}$ and the tangent stiffness matrix $\bar{\mathbf{k}}_T$ of a joint element are evaluated by means of Gaussian integration as follows:

$$\mathbf{p} = \sum_l \mathbf{B}^T(r_l, s_l) \boldsymbol{\sigma}(\mathbf{v}_l) |\mathbf{J}(r_l, s_l)| w_l \quad (3.20)$$

$$\mathbf{k}_T = \sum_l \mathbf{B}^T(r_l, s_l) \bar{\mathbf{k}}_T(\mathbf{v}_l) \mathbf{B}(r_l, s_l) |\mathbf{J}(r_l, s_l)| w_l \quad (3.21)$$

where $|\mathbf{J}(r_l, s_l)|$ is the determinant of the Jacobian matrix, and w_l is the weight factor for the integration point (r_l, s_l) .

The number of Gauss integration points used to evaluate Eqs. 3.20 and 3.21 depends on the order of the integrands in Eqs. 3.17 and 3.18, respectively. If the tangential stiffness in all three directions of the element $\bar{k}_{T,i}$ is constant over the element, the integrands in both expressions are cubic functions of the natural coordinates (r, s) and a 2×2 Gauss integration with a total of four points will evaluate exactly the integrals. If $\bar{k}_{T,i}$ varies over the element, higher order integration is necessary to evaluate the integrals more accurately. However, in ADAP-88, it is assumed that a 2×2 Gauss integration gives sufficiently accurate $\mathbf{p}$ and $\mathbf{k}_T$ for the general nonlinear element. To further reduce the computational effort, it is assumed that the curvature of the joint element surface is small over a single element and the transformation matrix $\mathbf{a}$ between the global and the orthonormal coordinate systems is evaluated only at the centroid of the joint surface. Hence, the integration is carried out using a constant transformation matrix $\mathbf{a}$ at the four Gaussian points.

3.2.2 Improved joint model

The formulations of the improved joint model proposed by Noruziaan (1995) and Lau et al. (1998) are presented in this section. As in the original formulation, the open and the close states of the joint are modelled by the normal displacement greater or less than a limit value, respectively. A new parameter δ , called the “slip margin”, is introduced in the improved joint model to take into account the effect of the shear keys. A major improvement in the capability of accurately simulating the joint behaviour is that the new model can simulate nonlinear shear slippage behaviour and allow energy dissipation in the closed state of the joint. The cohesion c and the angle of friction ϕ of the joint are implemented to compute the resisting friction force during shear slippage.

Basic assumptions

The constitutive relationship of the improved joint model is derived on the basis of the following assumptions:

- Open state
 - the joint has negligible tensile strength;
 - the normal stress and the normal stiffness are zero;
 - the sliding is elastic if the shear stress is less than the shear strength or the apparent cohesion of the joint;
 - when the joint opening is less than the slip margin δ and the shear stress exceeds the shear strength of the joint, further sliding of the joint is resisted by a constant shear force resulting from the apparent cohesion c ;
 - there is no coupling between the normal and the shear displacements; and
 - the joint tangential stiffness and the tangential stresses become zero once the joint opening exceeds the slip margin δ .
- Closed state

- the friction between the two faces of the joint resists the sliding motion;
 - the joint behaves elastically or elasto-plastically and there is coupling between the shear and the normal displacements; and
 - the plastic effects in sliding are governed by associative flow rule of Mohr-Coulomb type.
- Open or closed state
 - the joint faces are isotropic; and
 - there is no mismatch in the shear keys upon closing of the opened joints in the displaced state of the dam.

Constitutive relationship in open state

The constitutive relationship employed in the new joint model is presented in Figure 3.6. As shown in Figure 3.6 (a), if the normal relative displacement at the i -th iteration of time step $(m + 1)$ v_{m+1}^i is greater than zero, the corresponding normal stress σ_{m+1}^i is set equal to zero. When the joint closes, i.e. when $v_{m+1}^i \leq 0$, the normal stiffness of the joint recovers fully.

When the normal relative displacement v_{m+1}^i exceeds the slip margin δ the shear stresses are also set equal to zero, i.e.:

$$\text{If } v_{m+1}^i > \delta \geq 0, \text{ then } \begin{cases} \sigma_{m+1}^i = 0 \\ (\tau_1)_{m+1}^i = 0 \\ (\tau_2)_{m+1}^i = 0 \end{cases} \quad (3.22)$$

In Eq.3.22, the subscripts '1' and '2' refer to the two orthogonal tangential displacements of the joint.

If the normal displacement v_{m+1}^i is greater than zero, but less than the slip margin δ , it is assumed that the shear key is still engaged. In this case, tangential displacement increments are calculated and the corresponding tangential stresses are

obtained assuming elastic behaviour in the sliding of the joint, i.e.:

$$\text{If } \delta \geq v_{m+1}^i > 0, \text{ then } \begin{cases} \sigma_{m+1}^i = 0 \\ (\tau_1)_{m+1}^i = (\tau_1)_m + (\Delta\tau_1^e)_{m+1}^i \\ (\tau_2)_{m+1}^i = (\tau_2)_m + (\Delta\tau_2^e)_{m+1}^i \end{cases} \quad (3.23)$$

where

$$(\Delta\tau_1^e)_{m+1}^i = k_s \left[(u_1)_{m+1}^i - (u_1)_m \right] \quad (3.24)$$

$$(\Delta\tau_2^e)_{m+1}^i = k_s \left[(u_2)_{m+1}^i - (u_2)_m \right] \quad (3.25)$$

Next, the apparent cohesion c is compared to the resultant tangential stress τ_{m+1}^i :

$$\tau_{m+1}^i = \sqrt{\left[(\tau_1)_{m+1}^i \right]^2 + \left[(\tau_2)_{m+1}^i \right]^2} \quad (3.26)$$

If $\tau_{m+1}^i \geq c$, then the strength of the shear key has been exceeded and the tangential stresses are reduced as follows:

$$\tau_1 = \frac{(\tau_1)_{m+1}^i}{\tau_{m+1}^i} c \quad (3.27)$$

$$\tau_2 = \frac{(\tau_2)_{m+1}^i}{\tau_{m+1}^i} c \quad (3.28)$$

If $\tau_{m+1}^i < c$, then the shear key is still in elastic state and no modification of the shear stresses is needed.

Figure 3.6 (b) illustrates the tangential constitutive relationship. Path 1-2-3-5 represents the case when the joint normal displacement v is less than the slip margin δ . Path 1-2-3-4-6-7-8 represents the case when the joint is open at point 4 and the relative normal displacement exceeds the slip margin δ . In this case, the stress is suddenly released from point 4 to point 6. The joint remains fully open till point 7. When the normal displacement becomes less than the slip margin, i.e. at point 7, the tangential stiffness is completely recovered, and the tangential stress-tangential relative displacement relationship follows path 7-8.

Constitutive relationship in closed state

The closed state is defined as the state when the normal relative displacement v is less than or equal to zero. In this case, the tangential displacements at a joint are assumed to be resisted by friction. The loading function in the contact stress space f is given by

$$f = \tau + \mu\sigma - c \quad (3.29)$$

where $\tau = \sqrt{\tau_1^2 + \tau_2^2}$ is the shear stress, $\mu = \tan \phi$ is the friction coefficient, ϕ is the angle of internal friction, σ is the normal stress, and c is the apparent cohesion of the joint.

In the above relationship, the angle of internal friction ϕ and the cohesion c are assumed constants. An elasto-plastic model is implemented to determine the normal stress σ and the tangential stress τ . For the current integration point, it is first determined whether the material is in an elastic or an elasto-plastic state. If the tangential stress τ has not exceeded the apparent cohesion c , then the joint is still in an elastic state. In this case, the incremental stress is computed as follows:

$$\{\Delta\sigma^e\}_{m+1}^i = [\mathbf{D}^e] \{\Delta\mathbf{u}\}_{m+1}^i \quad (3.30)$$

or

$$\begin{Bmatrix} \Delta\sigma^e \\ \Delta\tau_1^e \\ \Delta\tau_2^e \end{Bmatrix}_{m+1}^i = \begin{pmatrix} k_n & 0 & 0 \\ 0 & k_s & 0 \\ 0 & 0 & k_s \end{pmatrix} \begin{Bmatrix} \Delta v \\ \Delta u_1 \\ \Delta u_2 \end{Bmatrix}_{m+1}^i \quad (3.31)$$

where

$$\begin{Bmatrix} \Delta v \\ \Delta u_1 \\ \Delta u_2 \end{Bmatrix}_{m+1}^i = \begin{Bmatrix} v_{m+1}^i - v_m \\ (u_1)_{m+1}^i - (u_1)_m \\ (u_2)_{m+1}^i - (u_2)_m \end{Bmatrix} \quad (3.32)$$

In the elastic state, the current stress level is obtained from the following expression

$$\begin{Bmatrix} \sigma \\ \tau_1 \\ \tau_2 \end{Bmatrix}_{m+1}^i = \begin{Bmatrix} \sigma \\ \tau_1 \\ \tau_2 \end{Bmatrix}_m + \begin{Bmatrix} \Delta\sigma^e \\ \Delta\tau_1^e \\ \Delta\tau_2^e \end{Bmatrix}_{m+1}^i \quad (3.33)$$

Once the limiting value of the apparent cohesion c is exceeded, further increase of the loading causes plastic deformations, i.e. the relative displacement increment consists of an elastic and a plastic term

$$\{\Delta\mathbf{u}\}_{m+1}^i = \{\Delta\mathbf{u}^e\}_{m+1}^i + \{\Delta\mathbf{u}^p\}_{m+1}^i \quad (3.34)$$

In this case, the constitutive relationship is written as

$$\{\Delta\boldsymbol{\sigma}\}_{m+1}^i = [\mathbf{D}^{ep}] \{\Delta\mathbf{u}\}_{m+1}^i \quad (3.35)$$

where the elasto-plastic rigidity matrix $[\mathbf{D}^{ep}]$ is derived as follows (Noruziaan 1995):

$$[\mathbf{D}^{ep}] = \begin{pmatrix} k_n - \frac{1}{H}\mu^2 k_n^2 & -\frac{1}{H}k_n k_s \beta_1 \mu & -\frac{1}{H}k_n k_s \beta_2 \mu \\ -\frac{1}{H}k_n k_s \beta_1 \mu & k_s (1 - \frac{1}{H}k_s \beta_1^2) & -\frac{k_s^2}{H}\beta_1 \beta_2 \\ -\frac{1}{H}k_n k_s \beta_2 \mu & -\frac{k_s^2}{H}\beta_1 \beta_2 & k_s (1 - \frac{1}{H}k_s \beta_2^2) \end{pmatrix} \quad (3.36)$$

where

$$H = k_n \mu^2 + k_s \beta_1^2 + k_s \beta_2^2 = k_n \mu^2 + k_s \quad (3.37)$$

$$\beta_1 = \frac{\partial f}{\partial \tau_1} = \frac{\tau_1}{|\tau|} \quad (3.38)$$

$$\beta_2 = \frac{\partial f}{\partial \tau_2} = \frac{\tau_2}{|\tau|} \quad (3.39)$$

$$\mu = \frac{\partial f}{\partial \sigma} \quad (3.40)$$

In the derivation of Eq.3.36, it is assumed that the yield surface and the flow rule are defined by the same function, which is the basic assumption for associated plasticity.

For a joint element that is in a plastic state, it is necessary to determine whether it is in a state of loading or unloading. The loading or unloading state of a joint is determined from the plastic work of the joint element

$$\Delta w_p = \sigma \Delta v^p + \tau_1 \Delta u_1^p + \tau_2 \Delta u_2^p \quad (3.41)$$

The following loading criterion is derived from Eq.3.41 (Noruziaan 1995):

$$L = k_n \mu (\Delta v)_{m+1}^i + k_s (\beta_1)_m (\Delta u_1)_{m+1}^i + k_s (\beta_2)_m (\Delta u_2)_{m+1}^i > 0 \quad (3.42)$$

An integration point is in a loading state if $L > 0$ and, in such a case, the stress increments are obtained from Eq.3.35. If $L \leq 0$, the integration point is in an unloading state, and the elastic constitutive relationship is employed.

If, at the end of time step m , it has been determined that the point is in an elastic state, the stresses for time step $m + 1$ are calculated by assuming an elastic constitutive relationship as a first iteration. At the end of each iteration i in time step $(m + 1)$ it is determined whether the loading function f is less than, equal to, or greater than zero. If $f(\sigma_{m+1}^i) < 0$, the integration point remains in an elastic state and the calculated stress state is correct. If $f(\sigma_{m+1}^i) \geq 0$, the integration point is in an elasto-plastic state. In the latter case, it is necessary to account for overshooting the loading surface as a result of the change from an elastic to a plastic state. To do so, a scale factor r is used in order to get the stress state onto the yield surface:

$$f(\{\sigma\}_m + r \{\Delta \sigma^e\}_{m+1}^i) = 0 \quad (3.43)$$

The scale factor r is calculated as follows (Noruziaan 1995):

$$r = \frac{-[(\sigma)_m \mu + \sqrt{(\tau_1^2)_m + (\tau_2^2)_m} - c]}{[\mu (\Delta \sigma^e)_{m+1}^i + (\beta_1)_m (\Delta \tau_1^e)_{m+1}^i + (\beta_2)_m (\Delta \tau_2^e)_{m+1}^i]} \quad (3.44)$$

Hence, the adjusted stress state is computed from the following equation

$$\{\Delta \sigma\}_{m+1}^i = r \{\Delta \sigma^e\}_{m+1}^i + \int_{\{u\}_m + r \{\Delta u\}_{m+1}^i}^{\{u\}_m + \{\Delta u\}_{m+1}^i} [D^e] \{du\} \quad (3.45)$$

The expression in Eq.3.45 is evaluated numerically by subdividing the integration interval into k subintervals. It is noted that the same procedure with $r = 0$ is applied

in the case when it is determined that the integration point is in a plastic state in time step m . For each subinterval, the relative displacement sub-increment is evaluated as follows:

$$\begin{Bmatrix} \Delta \tilde{v} \\ \Delta \tilde{u}_1 \\ \Delta \tilde{u}_2 \end{Bmatrix}_{m+1}^i = \frac{(1-r)}{k} \begin{Bmatrix} \Delta v \\ \Delta u_1 \\ \Delta u_2 \end{Bmatrix}_{m+1}^i \quad (3.46)$$

With the assumption of perfect plasticity, it is necessary to determine only the elastic stress sub-increments for each subinterval in order to determine the new stress state. The sum of the sub-increments is then added to the previous stress state

$$\sigma_{m+1}^i = \sigma_m^i + \sum_{j=1}^k (\Delta \tilde{\sigma}^e)_{(m+1),j}^i \quad (3.47)$$

The elastic stress sub-increments are obtained as follows:

$$\begin{Bmatrix} \Delta \tilde{\sigma}^e \\ \Delta \tilde{\tau}_1^e \\ \Delta \tilde{\tau}_2^e \end{Bmatrix}_{(m+1),j}^i = \begin{Bmatrix} k_n (\Delta \tilde{v} - \Delta \tilde{v}^p) \\ k_s (\Delta \tilde{u}_1 - \Delta \tilde{u}_1^p) \\ k_s (\Delta \tilde{u}_2 - \Delta \tilde{u}_2^p) \end{Bmatrix}_{(m+1),j}^i \quad (3.48)$$

In Eq.3.48, the plastic sub-increments of the relative displacements are obtained as follows:

$$\{\Delta \tilde{u}^p\}_{(m+1),j}^i = \mathbf{P}_{(m+1),j}^i \{\Delta \tilde{u}\}_{(m+1),j}^i \quad (3.49)$$

where

$$\mathbf{P}_{(m+1),j}^i = \frac{1}{(k_s + k_n \mu^2)} \begin{pmatrix} k_n \mu^2 & k_s \mu \beta_1 & k_s \mu \beta_2 \\ k_s \mu \beta_1 & k_s (\beta_1)^2 & k_s \beta_1 \beta_2 \\ k_s \mu \beta_2 & k_s \beta_1 \beta_2 & k_s (\beta_2)^2 \end{pmatrix}_{(m+1),(j-1)}^i \quad (3.50)$$

Eq.3.50 shows that the determination of the plastic displacement sub-increment in the j -th subinterval uses the flow gradient of the previous $(j-1)$ -th subinterval.

In order to account for overshooting the loading surface, the stress state is corrected by adding a corrective stress increment normal to the loading surface. The

final expression for the stress state is given by

$$\begin{Bmatrix} \sigma \\ \tau_1 \\ \tau_2 \end{Bmatrix}_{m+1(cor.)}^i = \begin{Bmatrix} \sigma \\ \tau_1 \\ \tau_2 \end{Bmatrix}_{m+1}^i + a \begin{Bmatrix} \mu \\ \beta_1 \\ \beta_2 \end{Bmatrix}_{m+1}^i \quad (3.51)$$

where the scalar factor a is determined by expanding the loading function f in Taylor's series form:

$$a = \frac{-1}{\mu^2 + 1} \left(\mu\sigma + \sqrt{(\tau_1^2 + \tau_2^2)} - c \right)_{m+1}^i \quad (3.52)$$

The computational procedures employed for the open and closed states of a joint element integration point are summarised in the flow charts in Figure 3.7 and Figure 3.8, respectively.

Figure 3.1: Vertical contraction joints in an arch dam

Unkeyed contraction joint

Contraction joint with unbevelled shear key

Contraction joint with bevelled shear key

Figure 3.2: Kinds of contraction joints in arch dams

a) Continuous shear keys

b) Discrete shear keys

Figure 3.3: Contraction joints with shear keys

a) Local and global coordinate systems

b) Natural coordinate system

Figure 3.4: Geometry of the joint element

Figure 3.5: Constitutive relationship for the joint element of ADAP88

(a) Open state

(b) Closed State

- Path 1-2: Elastic loading
- Path 2-3: Shear yielding of key
- Path 3-5: Elastic unloading
- Path 4-6: Release of stress due to $v > \delta$
- Path 6-7: Unloading with no shear resistance
- Path 7-8: Recovery of shear stiffness due to $v < \delta$

Figure 3.6: Constitutive relationships for the implemented joint element

Figure 3.7: Flow chart for computations at an iteration step in open state of a joint element stress point.

Figure 3.8: Flow chart for computations at an iteration step in closed state of a joint element stress point.

Chapter 4

Nonlinear concrete model

4.1 General

The nonlinear concrete model implemented by Noruziaan (1995) is adopted for further development in the present study. A new comprehensive nonlinear material model is developed in the present study by combining a bounding surface model with a smeared crack model to investigate nonlinear inelastic actions in concrete under both compression and tension. The bounding surface model is first discussed, followed by the smeared crack model and the combined nonlinear material model.

4.2 Modelling of concrete behaviour in compression

The bounding surface theory has been developed to describe the complex multiaxial behaviour of concrete under cyclic compressive loads. A basic assumption in the bounding surface concept is that the material moduli depend on the distance of the current stress point to its projection on the bounding surface. The bounding surface is defined as the innermost locus in stress space that encloses all possible stress points for a given level of damage. The bounding surface is expressed as a function of stress invariants and a damage parameter, in which the damage parameter represents the propagation of irreversible material damage at the microscopic level. The stiffness degradation of the material caused by such damage is taken into account by shrinking the bounding surface in stress space and thus decreasing the yield stress level, as shown in Figure 4.1. To obtain a scalar measure of the damage, a mapping rule is used to associate given stress point to its image on the current bounding surface. The

bounding surface model implemented in the present work is based on the bounding surface model proposed by Pagnoni et al. (1992).

4.2.1 Description of the implemented bounding surface model

In the following discussion, compressive stresses and strains are defined as positive. The stress quantities are normalised with respect to the concrete uniaxial compressive strength f'_c . The normalised quantities are denoted by the symbol “ $\sim$ ”.

Damage parameter

Separate damage parameters are required to describe the pre-failure and post-failure states of the material. For the case of the material under deviatoric loading and unloading before failure, the damage parameter is expressed as a function of the stress state. It is taken as the normalised distance D from the stress point to the hydrostatic axis on the deviatoric plane calculated as follows:

$$D = \frac{r}{R} \quad (4.1)$$

where r is the distance from the stress point to the hydroaxis, and R is the distance from the projection of the stress point on the bounding surface to the hydroaxis, as shown in Figure 4.1. After failure has occurred, the stress point remains continuously on the bounding surface. Hence, in the post-failure phase, the normalised distance D is a constant equal to unity and cannot be used for evaluation of the current level of damage. To evaluate the damage in this phase, a strain measure based on the plastic octahedral shear strain $\hat{\gamma}_0^p$ is defined.

In the study by Pagnoni et al. (1992), the increment of the damage parameter is formulated as follows:

- For deviatoric loading and unloading in the pre-failure stage:

$$dK = \frac{RdD}{H^p F_1(\hat{I}_1, \theta)} \quad (4.2)$$

- In the post-failure stage:

$$dK = \frac{d\hat{\gamma}_0^p}{F_1(\hat{I}_1, \theta)} \quad (4.3)$$

where H^p is the generalised plastic shear modulus, F_1 is a normalisation factor, $\hat{I}_1$ is the first invariant of the normalised stress tensor, and θ is the angle between the projections of the stress vector and of the first principal stress tensile semi-axis on the deviatoric plane. In the present model, deviatoric loading is defined as the process when $dD \geq 0$ and unloading when $dD < 0$. The total damage is obtained by integrating the rate of damage along the stress path:

$$K = \int_{\text{stress_path}} dK \quad (4.4)$$

In Eqs.4.2 and 4.3, the function $F_1(\hat{I}_1, \theta)$ is defined so as to account for the sensitivity of the deviatoric damage to the hydrostatic pressure level and the stress path history. The F_1 function normalises the damage parameter K so that K is equal to unity at failure under monotonic loading.

Based on available test data from monotonic and cyclic loading, the following expressions for F_1 are proposed by Pagnoni et al. (1992):

- For deviatoric loading:

$$F_1(\hat{I}_1, \theta) = \frac{0.248\hat{I}_1^2 + 0.182\hat{I}_1}{G(\cos 3\theta)}, \quad \hat{I}_1 \leq 2 \quad (4.5)$$

$$F_1(\hat{I}_1, \theta) = \frac{1.867\hat{I}_1 - 2.379}{G(\cos 3\theta)}, \quad \hat{I}_1 > 2 \quad (4.6)$$

- For deviatoric unloading:

$$F_1(\hat{I}_1, \theta) = 1.4 \left[0.85 - \frac{\hat{I}_1 + 0.3}{\hat{I}_{1,max} + 0.3} \right] \frac{F_{1,max}}{G(\cos 3\theta)} \quad (4.7)$$

In Eqs. 4.5 to 4.7, $\hat{I}_{1,max}$ and $F_{1,max}$ are, respectively, the maximum normalised first stress invariant $\hat{I}_1$ and the maximum normalisation factor F_1 before the most recent unloading. Based also on experimental data, the term $G(\cos 3\theta)$ is expressed as follows:

$$G(\cos 3\theta) = 0.25(\cos 3\theta + 5) \quad (4.8)$$

Bounding surface

In the study by Pagnoni et al. (1992), the bounding surface F is defined as a function of the stress state $\hat{\sigma}_{ij}$ and the maximum value of damage K_{max} ever experienced by the material

$$F(\hat{\sigma}_{ij}, K_{max}) = 0 \quad (4.9)$$

By means of regression analysis of test data, the following expression is derived for the bounding surface of concrete:

$$F(\hat{\sigma}_{ij}, K_{max}) = \frac{0.25\hat{J}_2 + 3.10\sqrt{\hat{J}_2}}{4\hat{I}_1 + 3.48} (\cos 3\theta + 5) - \frac{40}{39 + K_{max}^2} \quad (4.10)$$

where $\hat{J}_2$ is the second deviatoric invariant of the normalised stress tensor.

Incremental constitutive law

The steps in the derivation of the incremental constitutive law for the bounding surface model, as presented by Chen and Buyukozturk (1985), are briefly discusses here.

The strain increment $d\epsilon_{ij}$ is first decomposed into its deviatoric, de_{ij} , and volumetric, $d\epsilon_{kk}$, components:

$$d\epsilon_{ij} = de_{ij} + \delta_{ij} \frac{d\epsilon_{kk}}{3}; \quad k=1,2,3 \quad (4.11)$$

where δ_{ij} is the Kronecker delta.

The deviatoric strain increment de_{ij} is then decomposed into its elastic, de_{ij}^e , and plastic, de_{ij}^p , components:

$$de_{ij} = de_{ij}^e + de_{ij}^p \quad (4.12)$$

The elastic deviatoric strain increment de_{ij}^e can be related to the deviatoric stress increment dS_{ij} by the Hooke's law:

$$de_{ij}^e = \frac{1}{H^e} dS_{ij} \quad (4.13)$$

where H^e is the generalised elastic shear modulus.

The plastic deviatoric strain increment de_{ij}^p is assumed proportional to the deviatoric stress S_{ij} and is given by the following equation:

$$de_{ij}^p = \frac{d\gamma_0^p}{\tau_0} S_{ij} \quad (4.14)$$

where $d\gamma_0^p$ is the increment of the plastic octahedral shear strain, and τ_0 is the octahedral shear stress. Linearising the response within the load increment, the increment of the plastic octahedral shear strain can be expressed as:

$$d\gamma_0^p = \frac{d\tau_0}{H^p} \quad (4.15)$$

where the generalised plastic shear modulus H^p is dependent on the history of the stress and the strain.

Assuming that the effects of the incremental volumetric stress dI_1 and the incremental octahedral shear stress $d\tau_0$ on the volumetric strain increment $d\epsilon_{kk}$ are decoupled, the latter can be expressed as follows:

$$d\epsilon_{kk} = \frac{dI_1}{3K_t} + \beta d\gamma_0^p \quad (4.16)$$

where K_t is the bulk tangent modulus of concrete, and β is a shear-compaction dilatancy factor. Combining Eqs.4.11 to 4.16, the following incremental constitutive law is obtained

$$d\hat{\epsilon}_{ij} = C_{ijkl} d\hat{\sigma}_{kl} \quad (4.17)$$

where the compliance tensor C_{ijkl} is expressed as follows:

$$C_{ijkl} = \frac{1}{H^e} \delta_{ij} \delta_{kl} + \frac{1}{3H^p \hat{\tau}_0} \left(\frac{\hat{S}_{ij}}{\hat{\tau}_0} + \delta_{ij} \frac{\beta}{3} \right) \hat{S}_{kl} + \left(\frac{1}{9K_t} - \frac{1}{3H^e} \right) \delta_{ij} \delta_{kl} \quad (4.18)$$

Since the described bounding surface model is implemented in a displacement-based finite element analysis, the compliance matrix $\mathbf{C}$ consisting the compliance tensor C_{ijkl} needs to be inverted in order to obtain the material rigidity matrix.

Determination of model parameters

The material parameters required to implement the bounding surface model are listed as follows:

- H^e , generalised elastic shear modulus;
- H^p , generalised plastic shear modulus;
- K_t , bulk tangent modulus; and
- β , shear compaction–dilatancy factor.

The expressions for the model parameters proposed in the study by Pagnoni et al. (1992) are used in the present study. Based on regression analyses of published experimental test data, separate expressions are specified for pre-failure and post-failure concrete behaviour, as well as for loading and unloading conditions in the pre-failure phase. The expressions for the bounding surface model parameters are presented in Appendix A.

4.2.2 Numerical implementation

In the present implementation of the bounding surface model, it is assumed that in the post-failure phase the concrete material loses its rigidity and can no longer carry any stress. This assumption is justified by observations on the behaviour of the concrete material in the post-failure phase made in the study by Noruziaan (1995). The numerical algorithm employed for the analysis is described in the following.

First, the strain field corresponding to the nodal displacements at iteration i of time step $m + 1$ is evaluated at the integration points of the finite elements

$$\boldsymbol{\epsilon}_{m+1}^i = \mathbf{B}\mathbf{U}_{m+1}^i \quad (4.19)$$

where $\mathbf{B}$ is the displacement–strain transformation matrix and $\mathbf{U}_{m+1}^i$ is the current displacement vector. The strain increment is computed as follows:

$$\Delta\boldsymbol{\epsilon}_{m+1}^i = \boldsymbol{\epsilon}_{m+1}^i - \boldsymbol{\epsilon}_m \quad (4.20)$$

In the present study, the current stress increment $\Delta\boldsymbol{\sigma}_{m+1}^i$ corresponding to the current strain increment $\Delta\boldsymbol{\epsilon}_{m+1}^i$ is computed as follows:

$$\Delta\boldsymbol{\sigma}_{m+1}^i = \frac{1}{2} (\mathbf{D}_m + \mathbf{D}_{m+1}^i) \Delta\boldsymbol{\epsilon}_{m+1}^i \quad (4.21)$$

The current estimate of the rigidity matrix $\mathbf{D}_{m+1}^i$ in Eq.4.21 depends on the current strain $\boldsymbol{\epsilon}_{m+1}^i$ and the corresponding stress $\boldsymbol{\sigma}_{m+1}^i$. Since the latter is yet to be established, an estimate of it is obtained using the rigidity matrix from the previous iteration:

$$\boldsymbol{\sigma}_{m+1}^i = \boldsymbol{\sigma}_m + \mathbf{D}_{m+1}^{i-1} \Delta \boldsymbol{\epsilon}_{m+1}^i \quad (4.22)$$

To evaluate the current rigidity matrix $\mathbf{D}_{m+1}^i$, the stress invariants of the estimated stress state $\boldsymbol{\sigma}_{m+1}^i$ are determined first. The normalised distance D from the estimated stress state to the failure surface is then computed. If $D \geq 1$, the integration point is in post-failure state and the stress tensor and the rigidity matrix associated with it are set equal to zero. If $D < 1$, the current increment of the normalised distance is determined as follows:

$$\Delta D_{m+1}^i = D_{m+1}^i - D_m \quad (4.23)$$

If $\Delta D_{m+1}^i \geq 0$, then the integration point is in a state of deviatoric loading; otherwise, it is in a state of deviatoric unloading. According to the computed first stress invariant increment ΔI_1 , the integration point is in a state of hydrostatic loading when $\Delta I_1 \geq 0$, or hydrostatic unloading when $\Delta I_1 < 0$. The material parameters associated with the integration point are then computed in accordance to the expressions presented in Appendix A. The increment of the damage parameter ΔK is computed using Eq.4.2 and the total damage is then evaluated. The current rigidity matrix $\mathbf{D}_{m+1}^i$ is computed by inverting the compliance matrix $\mathbf{C}_{m+1}^i$, and the modified stress increment $\Delta \boldsymbol{\sigma}_{m+1}^k$ is obtained using Eq. 4.21. Having computed the modified stress increment, the modified current stress state is obtained as follows:

$$\boldsymbol{\sigma}_{m+1}^i = \boldsymbol{\sigma}_m + \Delta \boldsymbol{\sigma}_{m+1}^k \quad (4.24)$$

The steps in the numerical procedure are summarized as follows:

Step 1. Based on the solution of the nodal displacements at iteration i of time step $m + 1$, evaluate the strain at all the integration point using Eq.4.19. Start the loop over the elements. For each finite element, the following steps are repeated for all the integration points.

Step 2. Check the flag for failure; if failure has occurred, go to step 16.

Step 3. Normalise the stress evaluated for the previous time step σ_m with respect to the value of concrete strength in uniaxial compression f'_c :

$$\hat{\sigma}_m = \frac{\sigma_m}{f'_c} \quad (4.25)$$

Step 4. Subtract the strain evaluated for the previous time step ϵ_m from the estimate of the current strain ϵ_{m+1}^i to obtain the current strain increment $\Delta\epsilon_{m+1}^i$, as given in Eq.4.20. Normalise the current strain increment with respect to the uniaxial strain ϵ_p corresponding to the concrete strength in uniaxial compression:

$$\Delta\hat{\epsilon}_{m+1}^i = \frac{\Delta\epsilon_{m+1}^i}{\epsilon_p} \quad (4.26)$$

Step 5. Normalise the rigidity matrix of the previous iteration D_{m+1}^{i-1} with respect to the uniaxial compressive strength f'_c and the corresponding uniaxial strain ϵ_p :

$$\hat{D}_{m+1}^{i-1} = D_{m+1}^{i-1} \frac{\epsilon_p}{f'_c} \quad (4.27)$$

Step 6. Compute an estimate of the normalised stress increment for the current iteration:

$$\Delta\hat{\sigma}_{m+1}^i = \hat{D}_{m+1}^{i-1} \Delta\hat{\epsilon}_{m+1}^i \quad (4.28)$$

Step 7. Compute the following stress invariants and invariant increments required to evaluate the normalised distance D . The tensorial notation is used here:

- Current first invariant of the normalised stress tensor:

$$\left(\hat{I}_1\right)_{m+1}^i = (\hat{\sigma}_{jj})_{m+1}^i + (\Delta\hat{\sigma}_{jj})_{m+1}^i \quad (4.29)$$

where $j = 1, 2, 3$

- Normalised deviatoric stress tensor for the current iteration and at the end of the previous time step:

$$\left(\hat{S}_{jk}\right)_{m+1}^i = (\hat{\sigma}_{jk})_{m+1}^i - \delta_{jk} \frac{\left(\hat{I}_1\right)_{m+1}^i}{3} \quad (4.30)$$

where $k = 1, 2, 3$

$$\left(\hat{S}_{jk}\right)_m = \left(\hat{\sigma}_{jk}\right)_m - \delta_{jk} \frac{\left(\hat{I}_1\right)_m}{3} \quad (4.31)$$

- Normalised second invariant of the deviatoric stress tensor for the current iteration and at the end of the previous time step:

$$\left(\hat{J}_2\right)_{m+1}^i = \frac{1}{2} \left(\hat{S}_{jk}\right)_{m+1}^i \left(\hat{S}_{jk}\right)_{m+1}^i \quad (4.32)$$

$$\left(\hat{J}_2\right)_m = \frac{1}{2} \left(\hat{S}_{jk}\right)_m \left(\hat{S}_{jk}\right)_m \quad (4.33)$$

- Current normalised third invariant of the deviatoric stress tensor:

$$\left(\hat{J}_3\right)_{m+1}^i = \frac{1}{3} \left(\hat{S}_{jk}\right)_{m+1}^i \left(\hat{S}_{kl}\right)_{m+1}^i \left(\hat{S}_{kn}\right)_{m+1}^i \quad (4.34)$$

where $l = 1, 2, 3$ and $n = 1, 2, 3$

- Current angle θ , as shown in Figure 4.1

$$\left(\theta\right)_{m+1}^i = \frac{1}{3} \arccos \left[\frac{3\sqrt{3}}{2} \frac{\left(\hat{J}_3\right)_{m+1}^i}{\left[\left(\hat{J}_2\right)^{3/2}\right]_{m+1}^i} \right] \quad (4.35)$$

- Normalised octahedral shear stress for the current iteration and at the end of the previous time step:

$$\left(\hat{\tau}_0\right)_{m+1}^i = \sqrt{\frac{2}{3} \left(\hat{J}_2\right)_{m+1}^i} \quad (4.36)$$

$$\left(\hat{\tau}_0\right)_m = \sqrt{\frac{2}{3} \left(\hat{J}_2\right)_m} \quad (4.37)$$

- Current increment of the normalised octahedral shear strain:

$$\left(\Delta\hat{\tau}_0\right)_{m+1}^i = \left(\hat{\tau}_0\right)_{m+1}^i - \left(\hat{\tau}_0\right)_m \quad (4.38)$$

- Current increment of the normalised plastic octahedral shear stress:

$$(\Delta\hat{\gamma})_{m+1}^i = \frac{(\Delta\hat{\tau}_0)_{m+1}^i}{H^p} \quad (4.39)$$

Step 8. Compute the distance of the projection of the stress point on the bounding surface from the hydrostatic axis on the deviatoric plane:

$$R = \max \left[\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right] \quad (4.40)$$

where

$$a = \frac{0.25 (\cos 3\theta + 5)}{4 (\hat{I})_{m+1}^i + 3.48},$$

$$b = 3.10 \frac{\cos 3\theta + 5}{4 (\hat{I})_{m+1}^i + 3.48}, \text{ and}$$

$$c = \frac{-40}{(39 + K_{max}^2)}$$

Step 9. Compute the distance of the stress point from the hydrostatic axis on the deviatoric plane:

$$r = \sqrt{(\hat{J}_2)_{m+1}^i} \quad (4.41)$$

Step 10. Compute the normalised distance:

$$D = \frac{r}{R} \quad (4.42)$$

Step 11. Check the failure criteria. If $D \geq 1$, failure has occurred; set flag FAILURE equal to unity. Go to step 16.

Step 12. Compute the increment of the normalised distance:

$$(\Delta D)_{m+1}^i = D_{m+1}^i - D_m \quad (4.43)$$

Step 13. Compute the current normalisation factor $[F_1(\hat{I}_1, \theta)]_{m+1}^i$ using the appropriate expression of Eqs.4.5 to 4.7 depending on the values of $(\Delta D)_{m+1}^i$ and $(\hat{I}_1)_{m+1}^i$.

Step 14. Compute the material parameters of the bounding surface model, H^e , H^p , K_t , and β , in accordance to the procedure presented in Appendix A.

Step 15. Compute the increment of the damage parameter $(\Delta K)_{m+1}^i$ using Eq.4.2, and evaluate the current damage parameter:

$$(K)_{m+1}^i = (K)_m + (\Delta K)_{m+1}^i \quad (4.44)$$

Step 16. If failure has occurred, set the components of the rigidity matrix and the resisting force vector equal to zero. Go to step 19.

Step 17. Compute the normalised components of the current compliance tensor $(\hat{C}_{jklm})_{m+1}^i$ using Eq.4.17. Transform the compliance tensor into a compliance matrix $\hat{C}_{m+1}^i$ of order 6 by 6. Impose symmetry by eliminating the nonsymmetric components. Invert $\hat{C}_{m+1}^i$ to obtain the normalised rigidity matrix $\hat{D}_{m+1}^i$.

Step 18. Compute the current approximation to the stress state using Eq.4.50.

Step 19. Repeat the procedures from step 2 for all the integration points in the current element.

Step 20. Evaluate the stress components and the rigidity matrix in the dimensional form. Compute the resisting load vector of the current element using Eq.4.45:

$$\mathbf{F}_{m+1}^i = \sum_{l=1}^8 [\mathbf{B}]^T(r, s, k)_l \{\boldsymbol{\sigma}\}_{m+1}^i | \mathbf{J}(r, s, k)_l | w(r, s, k)_l \quad (4.45)$$

Step 21. Repeat the procedures from step 2 for all elements which the bounding surface model is associated with.

Step 22. Proceed with the computational procedure for solution of the non-linear equations of equilibrium.

A flow chart of the computational algorithm for the implementation of the bounding surface model is presented in Figure 4.2.

Figure 4.3 shows the simulated by the implemented bounding surface model stress-strain behaviour of concrete under cyclic compressive loading. The test is performed on a single eight-node three-dimensional solid element subjected to specified nodal displacements.

4.3 Modelling of concrete behaviour in tension

The smeared crack model implemented in the present study follows closely the model developed by Cervera and Hinton (1986) for behaviour of concrete under tension. The model presented here includes a more detailed and accurate determination of the fracture energy dissipated during cracking. The main features and the steps of the numerical formulation of the model are outlined in the following sections. In the discussion, tensile stresses and strains are defined as positive.

4.3.1 Description of the implemented crack model

The crack model considered in this study is a three-dimensional fracture mechanics based smeared crack model. The concrete material is assumed to behave linear-elastically under compressive stress, and stress-based criteria are employed to determine the initiation of cracks. Orthotropic material characteristics are associated with the cracked state of the material. The fixed unidirectional crack approach is utilised for monitoring the status of open cracks.

Pre-fracture material behaviour

Initially, the behaviour of concrete is assumed to be linear elastic under increasing tensile stress. The stress level at which tensile failure occurs is termed the critical stress which is an important material parameter. In accordance to the assumption of linear stress-strain relationship before the peak stress in concrete, the computed tensile stress is compared with the apparent strength of concrete, as shown in Figure 4.4. To establish the apparent strength of concrete under uniaxial tension, the procedure proposed by Raphael (1984) may be used. Based on some 12 000 published individual test results, Raphael introduces the following relationship between the tensile, f_t , and the compressive, f'_c , strengths of concrete under static loading:

$$f_t = 0.324 (f'_c)^{\frac{2}{3}}, \quad \text{MPa} \quad (4.46)$$

It has been observed that the tensile strength increases with increasing rate of the applied loading, whereas the corresponding strain remains approximately the same.

Therefore, Raphael proposes that the dynamic tensile strength of concrete is obtained by increasing the static strength by 50 %:

$$f'_t = 0.486 (f'_c)^{\frac{2}{3}}, \quad \text{MPa} \quad (4.47)$$

Finally, based on experimental evidence, it is suggested that the apparent tensile strength of concrete be taken as twice of the static tensile strength.

Furthermore, based on safety assessment of concrete dams, it is reported that the initial tangent modulus does not increase at the same rate as the tensile strength under dynamic loading. The following relationship between the dynamic, E'_i , and the static, E_i , initial tangent moduli is proposed by the Canadian Electrical Association (1990):

$$E'_i = 1.25 E_i \quad (4.48)$$

Assuming that the strain corresponding to the critical stress remains the same for static and dynamic loads, the following expression for the apparent dynamic tensile strength can be derived from Eq.4.47:

$$\sigma'_a = 1.25 \left[1.3 \times 0.324 (f'_c)^{\frac{2}{3}} \right] = 0.526 (f'_c)^{\frac{2}{3}}, \quad \text{MPa} \quad (4.49)$$

The apparent dynamic tensile strength of concrete evaluated by Eq. 4.49 is about 17% lower than the corresponding one obtained by means of the procedure proposed by Raphael. The derivation of the expression in Eq. 4.49 recognises the experimental evidence that the nonlinearity in the tensile stress-strain relationship near the peak is less under dynamic conditions compared to monotonic static loading conditions.

Cracking criterion

As has been mentioned in the preceding discussion, a stress-based criterion is used in the present study to determine the initiation of a crack. For an uncracked stress point, the principal stresses are determined and arranged so that the first principal stress is the maximum stress, and the third principal stress is the minimum stress. When the first principal stress exceeds the current critical stress, a crack

is assumed to initiate in the plane orthogonal to the direction of the first principal stress. At the instance of initiation of the first crack, a second, or a second and a third, crack can form if the second, or the second and the third, principal stresses exceed the current critical stress. Records of the directions of the principal stresses at the instance of initiation of the first crack, the values of the stress and strain normal to the crack plane, and the crack opening status are maintained for each sampling point at which crack initiation has been detected.

For a sampling point with only one crack, the initiation of a second crack and a third crack is checked in all subsequent time steps and iterations. The possible second and third cracks are assumed to form in the planes orthogonal to, respectively, the directions of the second and the third principal stresses at the instance of initiation of the first crack. Only one set of three mutually perpendicular cracks is allowed to form at a sampling point.

The following assumptions are made for the stress state of a sampling point at which one or more cracks have formed:

- after crack initiation, the tensile strength of concrete decreases gradually with increased strain and eventually becomes zero;
- if a crack closes, concrete regains its strength in the direction orthogonal to the crack plane; if the crack re-opens, the elastic modulus determined for the previous cracked state is used until a new cracked state is determined;
- the Poisson's ratio effect is not applicable in the plane of a crack; and
- shear stress can be transferred across an open crack until the crack can withstand tensile stress.

The limiting value of tensile stress which the concrete material can withstand without cracking is defined for the following three cases of stress state:

- three-dimensional tension, i.e. all the three principal stresses are tensile;
- tension-tension-compression, i.e. the first and the second principal stresses are tensile and the third is compressive; and

- tension-compression-compression, i.e. only the first principal stress is tensile.

For the triaxial tension case the tensile strength σ_{tu} is determined as follows:

$$\sigma_{tu}(i) = f'_t, \quad i = 1, 2, 3 \quad (4.50)$$

For the tension-tension-compression case it is assumed that:

$$\sigma_{tu}(i) = f'_t \left(1 + 0.75 \frac{\sigma_j}{f'_c} \right); \quad \sigma_j < 0 \quad (4.51)$$

where σ_j is the principal compressive stress. Similarly, for the tension-compression-compression case, the tensile strength is given by

$$\sigma_{tu}(i) = f'_t \left(1 + 0.75 \frac{\sigma_j}{f'_c} \right) \left(1 + 0.75 \frac{\sigma_k}{f'_c} \right); \quad \sigma_j < 0, \sigma_k < 0 \quad (4.52)$$

where σ_j and σ_k are the principal compressive stresses. It is noted that in Eqs.4.51 and 4.52 the compressive stresses are negative, whereas the uniaxial compressive strength of concrete f'_c is positive. Hence, the tensile strength in the last two cases decreases with increased principal compressive stresses. Equations 4.51 and 4.52 corroborate with the observation that compression in one direction favours microcracking in the others and thus causes reduction of the tensile capacity (Cervera and Hinton 1986). The resulting triaxial failure envelope is shown in Figure 4.5.

Post-cracking material behaviour

A typical stress-strain relationship for plain concrete obtained from uniaxial tension tests is presented in Figure 4.4. This figure shows that in the post-peak phase, the concrete material can still withstand tensile stress, but its tensile strength progressively decreases with increasing tensile strain. Such material behaviour is referred to as “strain softening” and is modelled numerically by dividing the constitutive relationship into three regions:

- From zero strain to ε_0 , the strain corresponding to the uniaxial tensile strength f'_t . Linear-elastic stress-strain relationship is assumed for this region.

- From ε_0 to ε_{cr} , the strain at which concrete loses its capacity to withstand tension. In this region, a strain-softening rule is adopted to determine the stress.
- From ε_{cr} and beyond. When the strain has exceeded the critical value ε_{cr} , concrete is not capable to withstand any tensile stress.

Various models have been proposed to represent the stress-strain relationship in the interval $\varepsilon_0 < \varepsilon < \varepsilon_{cr}$. An exponential strain-softening curve proposed by Lubliner et al. (1989), is adopted in the present study:

$$\sigma(\varepsilon) = \sigma_{tu} [2e^{-a(\varepsilon-\varepsilon_0)} - e^{-2a(\varepsilon-\varepsilon_0)}] \quad (4.53)$$

where $a \geq 0$ is a softening parameter. The total area under the stress-strain curve is adjusted so that the dissipated fracture energy G_f is independent of the characteristic dimension of a finite element l_c :

$$G_f = l_c \int_0^\infty \sigma(\varepsilon) d\varepsilon \quad (4.54)$$

For the adopted strain-softening law, Eq.4.54 becomes

$$G_f = l_c \left(\frac{1}{2} \frac{\sigma_{tu}^2}{E} + \frac{3}{2} \frac{\sigma_{tu}}{a} \right) \quad (4.55)$$

From Eq.4.55, the softening parameter a is determined as follows:

$$a = \frac{3}{\varepsilon_0 \left(\frac{2EG_f}{l_c \sigma_{tu}^2} - 1 \right)} \geq 0 \quad (4.56)$$

The criterion for mesh objectivity is given by the condition that the term inside the brackets in Eq.4.56 should be positive. Therefore:

$$l_c \leq \frac{2EG_f}{\sigma_{tu}^2} \quad (4.57)$$

The value of ε_{cr} is determined by assuming a corresponding stress level and applying the adopted strain-softening law. In the present work, ε_{cr} is determined for $\sigma = \lambda \sigma_{tu}$, where λ is adopted equal to 0.005. Substituting $\sigma = \lambda \sigma_{tu}$ in Eq.4.53 and solving the resulting equation, the following expression is obtained for ε_{cr} :

$$\varepsilon_{cr} = \varepsilon_0 - \frac{\ln \left(\frac{2-\sqrt{4-4\lambda}}{2} \right)}{a} \quad (4.58)$$

For $\lambda = 0.005$,

$$\varepsilon_{cr} = \varepsilon_0 + \frac{5.990}{a} \quad (4.59)$$

Once a crack has been detected at a sampling point, an orthotropic material model is associated with this point. The material axes of the orthotropic model are fixed in the directions of the principal stresses as determined at the instant of crack initiation. At any subsequent solution step, the strain tensor is transformed from the global coordinate system into the coordinate system of the material axes at the cracked point

$$\mathbf{T}'_\epsilon = \mathbf{A}^T \mathbf{T}_\epsilon \mathbf{A} \quad (4.60)$$

where $\mathbf{T}_\epsilon$ is the strain tensor in the global coordinate system

$$\mathbf{T}_\epsilon = \begin{pmatrix} \varepsilon_x & \frac{1}{2}\gamma_{yx} & \frac{1}{2}\gamma_{zx} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & \frac{1}{2}\gamma_{zy} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_z \end{pmatrix}. \quad (4.61)$$

and $\mathbf{A}$ is the transformation matrix consisting of the cosines of the angles between the axes of the two coordinate systems

$$\mathbf{A} = \begin{bmatrix} \cos(x'x) & \cos(x'y) & \cos(x'z) \\ \cos(y'x) & \cos(y'y) & \cos(y'z) \\ \cos(z'x) & \cos(z'y) & \cos(z'z) \end{bmatrix} \quad (4.62)$$

It is assumed that the Poisson's ratio effect is not applicable at a cracked sampling point. Thereby, the stress-strain transformation matrix becomes a diagonal matrix. A secant elastic modulus can be defined in the direction along which cracking has initiated, as shown in Figure 4.7:

$$E_s = \frac{\sigma'}{\varepsilon'} \quad (4.63)$$

To account for the phenomena allowing the transfer of shear stress in cracked concrete, such as aggregate interlock, the retention of shear stress is numerically modelled by using a shear retention factor $0 \leq \beta \leq 1$. The shear modulus after cracking is

obtained by multiplying the initial shear modulus by the shear retention factor β . In some previous studies, a constant shear retention factor is used. Others have adopted a crack-strain dependent shear retention factor. A detailed discussion on the numerical effects of using a shear retention factor is presented by Rots (1989). A constant shear retention factor $\beta = 0.5$ is used in the present study.

To illustrate how the above assumptions for the Poisson's ratio effect and the retention of shear stresses at a cracked point affect the rigidity matrix in the crack coordinates, the strain-stress relationship at a sampling point with two cracks is given below

$$\begin{Bmatrix} \sigma_{x'} \\ \sigma_{y'} \\ \sigma_{z'} \\ \tau_{x'y'} \\ \tau_{y'z'} \\ \tau_{z'x'} \end{Bmatrix} = \begin{pmatrix} E_s & 0 & 0 & 0 & 0 & 0 \\ 0 & E_s & 0 & 0 & 0 & 0 \\ 0 & 0 & E & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta G & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta G & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta G \end{pmatrix} \begin{Bmatrix} \varepsilon_{x'} \\ \varepsilon_{y'} \\ \varepsilon_{z'} \\ \gamma_{x'y'} \\ \gamma_{y'z'} \\ \gamma_{z'x'} \end{Bmatrix} \quad (4.64)$$

Since seismic excitations at the base of the structure cause load reversals, a crack can repeatedly open, close and re-open. It is assumed that when a crack closes, concrete regains its elastic modulus in the direction orthogonal to the plane of the crack. If there is no open crack in all the three directions, the stresses are computed only in the global coordinate system using the stress-strain relationship for uncracked concrete

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{pmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{pmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \quad (4.65)$$

If a crack re-opens, the current secant modulus is used to establish the stress-strain relationship. If the strain in an open crack exceeds the maximum strain recorded

in the same direction, a new secant elastic modulus is computed and used for the subsequent time steps.

The stress tensor computed in the crack coordinate system is transformed back into the global coordinate system to obtain the resisting force vector

$$\mathbf{T}_\sigma = \mathbf{A}\mathbf{T}'_\sigma\mathbf{A}^T \quad (4.66)$$

where $\mathbf{T}'_\sigma$ is the stress tensor in the crack coordinate system:

$$\mathbf{T}'_\sigma = \begin{pmatrix} \sigma_{x'} & \tau_{y'x'} & \tau_{z'x'} \\ \tau_{x'y'} & \sigma_{y'} & \tau_{z'y'} \\ \tau_{x'z'} & \tau_{y'z'} & \sigma_{z'} \end{pmatrix} \quad (4.67)$$

Finally, the contribution to the resisting force vector is computed based on the stresses in the global coordinate system, using Eq.4.45.

4.3.2 Numerical implementation

The steps of the computational algorithm of the implemented crack model are presented in the following section. The procedures for the solution of the nonlinear equations of motion for iteration i of time step $m + 1$ are outlined as follows.

Step 1. Compute the components of the strain tensor in the global coordinate system

$$\boldsymbol{\epsilon}_{m+1}^i = \mathbf{B}\mathbf{U}_{m+1}^i \quad (4.68)$$

where $\boldsymbol{\epsilon}^T = [\epsilon_x, \epsilon_y, \epsilon_z, \epsilon_{xy}, \epsilon_{yz}, \epsilon_{zx}]$ is the vector representation of the strain tensor, $\mathbf{B}$ is the displacement-strain transformation matrix, and $\mathbf{U}$ is the vector of nodal displacements.

Step 2. Compute the components of the elastic stress tensor in the global coordinate system using Eq.4.65.

Step 3. Check the variable governing the status of cracking of the material. If the point has already cracked, go to Step 7. If not, continue with Step 4.

Step 4. Compute the principal stresses.

Step 5. Check the crack initiation criteria in accordance to Eqs.4.50 to 4.52. If no crack has been detected, go to Step 14; otherwise compute and store the direction cosines of the current principal axes. Set the variable governing the status of cracking to denote cracking has occurred.

Step 6. Compute and store the transformation matrix A between the global coordinate system and the crack coordinate system, which is defined by the directions of the principal strains at the instant of crack initiation, using Eq.4.62.

Step 7. Transform the strain tensor into the crack coordinate system using Eq.4.60.

Step 8. Check whether the first crack is open, i.e. whether the strain normal to the plane of the crack is positive. If this crack is closed, concrete regains its strength in the direction orthogonal to the plane of the crack. If the crack is open, compute the stress components associated with the crack, namely $\sigma_{x'}, \tau_{x'y'}, \tau_{x'z'}$, using the constitutive relationship of Eq.4.64. If the current strain $\varepsilon_{x'}$ exceeds the maximum strain recorded in this direction $\varepsilon_{x'_{max}}$, update the latter and use it in subsequent time steps.

Step 9. Check whether the second and the third cracks have formed. If both have formed, go to Step 11. If only the second has formed, go to Step 10. If neither has formed, check approximately whether a new crack forms in the second and/or the third direction using the respective stresses computed in the coordinate system associated with the first crack.

- If no new crack is detected:

and if the first crack is closed, go to Step 13.

and if the first crack is open, compute the stress components $\sigma_{y'}, \sigma_{z'}, \tau_{y'z'}$ using the constitutive relationship of Eq.4.64.

- If a new crack is detected, compute the stress components $\sigma_{y'}, \sigma_{z'}, \tau_{y'z'}$ using the constitutive relationship of Eq.4.64. If $\sigma_{z'} > \sigma_{y'}$, update the directions of the principal strains.

Step 10. The first and the second crack have formed, but not the third. Check whether the second crack is open. Check whether a new crack forms in the third direction.

- If no crack is detected in the third direction:

and if neither the first nor the second crack has formed, go to Step 13.

and if either the first or the second crack is open, compute the stress components $\sigma_{y'}, \sigma_{z'}, \tau_{y'z'}$ using the constitutive relationship of Eq.4.64.
- If a crack is detected in the third direction, compute the stress components $\sigma_{y'}, \sigma_{z'}, \tau_{y'z'}$ using the constitutive relationship of Eq.4.64.

If the strain in the second direction exceeds the maximum strain recorded in this direction, update the latter and use it in subsequent time steps.

Step 11. For the case all three cracks have formed:

- If any of them is open, compute the stress components $\sigma_{y'}, \sigma_{z'}, \tau_{y'z'}$ using the constitutive relationship of Eq.4.64.
- If none of the cracks is open, go to Step 13.
- If the current strain in any direction exceeds the maximum strain recorded in the respective direction, update the latter and use it in subsequent time steps.

Step 12. Transform the stress tensor $\mathbf{T}'_e$ back into the original coordinate system using Eq.4.66. Go to Step 14.

Step 13. No modification to the current elastic stress tensor as computed in the global coordinate system, Eq.4.65, is needed. Therefore, use the current elastic stress tensor to compute the contribution to the resisting force vector.

Step 14. Compute the resisting force vector for the current sampling point using Eq.4.45.

A flow chart of the computation algorithm of the implemented crack model is presented in Figure 4.8.

Figure 4.9 shows the simulated by the implemented smeared crack model stress-strain behaviour of concrete under cyclic loading. The test is performed on a single eight-node three-dimensional solid element subjected to specified nodal displacements.

4.4 Combined model of concrete behaviour in compression and tension

As has already been mentioned, the bounding surface model is developed to study the behaviour of concrete under cyclic compressive loading. Although the tensile modes of failure can be represented by the bounding surface, the bounding surface model cannot account for the post-cracking behaviour of concrete because it is based on the assumption of isotropic material. After cracking, the behaviour of concrete is orthotropic, if the crack is open. The behaviour of a cracked material with an open crack can be more accurately modelled by the smeared crack model.

To account for the nonlinear concrete behaviour in both compression and tension, the bounding surface and the smeared crack models are implemented in a single comprehensive algorithm. The pre-crack behaviour of concrete is simulated by the bounding surface model. The crack initiation is determined by the procedure implemented in the smeared crack model, using the initial modulus of elasticity and the apparent tensile strength of concrete. The behaviour of the concrete material in case of an open crack is described by the tension softening algorithm of the smeared crack model. For a closed crack, the bounding surface is used to evaluate the stress components in the direction orthogonal to the crack plane.

At a cracked point, if compressive unloading is followed by a tensile loading, the strain normal to the crack plane is measured from the strain where the stress is zero, as shown in Figure 4.10. When a crack re-opens, the most recent secant elastic modulus is used to compute the stress from the strain.

The steps of the computational algorithm of the combined model are outlined as follows:

Step 1. Check whether the point has failed in compression in a previous time step. If yes, set the components of the stress tensor associated with the sampling

point equal to zero and go to Step 7. Otherwise, compute the components of the elastic strain and stress tensors using Eqs.4.68 and 4.65.

Step 2. Check whether the point has cracked in a previous time step. If not, go to Step 3. If the point has cracked, transform the elastic strain and stress tensors into the crack coordinate system using Eq.4.60 and check whether there is an open crack. If there is one, re-evaluate the stresses by means of the strain softening model. Transform the stresses back into the global coordinate system using Eq.4.66.

Step 3. Apply the bounding surface model. If compressive failure is detected in the current solution step, set the components of the stress tensor equal to zero and go to Step 7. If compressive failure is not detected, re-evaluate the components of the stress tensor by means of the bounding surface model.

Step 4. If a crack has been detected in a previous time step, go to Step 5. Otherwise, compute the principal stress and check the cracking criteria. If a crack is detected in the current solution step, go to Step 6. Otherwise, no further modification of the stress is needed; go to Step 7.

Step 5. Obtain the components of the stress and strain tensors as transformed into the crack coordinate system using Eq.4.60. Check whether there is an open crack. If there is one, continue with Step 6. Otherwise, no further modification of the stress is needed; go to Step 7.

Step 6. Apply the strain softening model to re-evaluate the stress in a open crack. Transform the stress tensor back into the global coordinate system using Eq.4.66.

Step 7. Compute the resisting load vector associated with this point.

The computational algorithm of the combined bounding surface–smeared crack concrete model is presented in Figure 4.11.

Figure 4.12 shows the simulated stress–strain behaviour of concrete under cyclic load reversals. The test is performed on a single eight–node three–dimensional solid element subjected to specified nodal displacements. The figure shows that the compressive strength after tensile loading is not degraded, which means that the compressive damage is not affected by tensile cracking. On the other hand, the

tensile strength is not degraded after compressive loading.

(a)

(b)

Figure 4.1: The bounding surface model: (a) Bounding surface, (b) Measure of normalised distance D

Figure 4.2: Flowchart of the bounding surface model computational algorithm

Figure 4.3: Displacement-controlled test of the bounding surface model on a single element

Figure 4.4: Apparent tensile strength

Figure 4.5: Triaxial tensile failure envelope for concrete

Figure 4.6: Typical stress-strain curve for mass concrete from simple tension test

Figure 4.7: Stress-strain relationship in tension

Figure 4.8: Flowchart of the smeared crack model computational algorithm

Figure 4.10: Modelled stress-strain relationship for one-cycle uniaxial loading

Figure 4.11: Flowchart of the combined material model computational algorithm

Chapter 5

Numerical implementation

5.1 General

The nonlinear joint and material models described in Chapter 3 and Chapter 4 are implemented in a new finite element computer program developed for the purposes of the present work. The new program is named CADNAP (Concrete Arch Dam Nonlinear Analysis Program). The computer program ADAP88 provides the basic finite element modelling and analysis modules to form the backbone of the new computer code. Before the detailed discussion on the numerical implementation of the nonlinear joint and material models in CADNAP, a brief description of ADAP88 is presented.

ADAP88 is a finite element computer program for nonlinear static and seismic analysis of concrete arch dams developed by Fenves et al. (1989). The program has the capability to generate a detailed finite element mesh of an arch dam–foundation system with only the geometry of the dam structure provided as input parameters. A zero–thickness joint element is implemented in the program to model the nonlinear effects to the seismic response of the structure due to opening and closing of the contraction joints, as discussed in Chapter 2. The monolithic cantilevers are modelled by thick shell, three–dimensional shell, and three–dimensional solid elements. One shell element is used through the thickness of the dam. The three–dimensional solid elements are used to provide a transition between the thick shell elements and the joint elements, as shown in Figure 2.2. The foundation of the dam is modelled by three–dimensional solid elements. Linear–elastic material properties are assumed in

the formulation of the three element types. Loads due to hydrostatic forces, self-weight and temperature gradients between the two surfaces of the dam structure are considered in the static analysis module. For seismic analysis, the ground motion excitations of the structure are specified as the translational support motions at the far-field boundary of the foundation substructure. The interaction between the dam structure and the impounded water in the reservoir is simulated by the added mass technique. The substructuring technique for seismic analysis of structures with localised nonlinearities proposed by Row and Schriker (1984) is employed in ADAP88. The cantilevers of the dam and the foundation are modelled as linear-elastic substructures connected by the nonlinear contraction joints. The internal degrees of freedom of the dam cantilevers and foundation are expressed in terms of the degrees of freedom of the nonlinear substructure, i.e. the joints. After applying the static condensation procedure to eliminate the internal degrees of freedom of the linear-elastic substructures, only the degrees of freedom of the joints are included in the final assembled equations of motion. The Newmark method is employed for the direct integration in time domain of the equations of motion. The solution of the nonlinear equilibrium equations at each time step is obtained by means of the full Newton-Raphson iteration scheme.

In the new program CADNAP, the nonlinear concrete material model described in Chapter 4 is implemented in the three-dimensional solid element employed in ADAP88. A new mesh generator is formulated to model the entire dam body by the three-dimensional solid element. The number of elements through the thickness of the dam can vary from one to three. A new numerical procedure is implemented for the direct time integration of the assembled equations of motion which include all degrees of freedom of the dam-foundation structure system.

5.2 Nonlinear solution technique

The basic system of equations to be solved in nonlinear finite element analysis can be expressed as follows:

$$\mathbf{F}_{m+1} - \mathbf{R}_{m+1} = \mathbf{0} \quad (5.1)$$

where $\mathbf{F}_{m+1}$ is the vector of externally applied loads, $\mathbf{R}_{m+1}$ is the vector of nodal point resisting forces obtained by integration of the element stresses, and the subscript $(m + 1)$ denotes the current load step. Since the nodal resisting forces $\mathbf{R}_{m+1}$ depend nonlinearly on the nodal displacements, the solution of Eq.5.1 is usually obtained by means of an iterative procedure, such as the Newton-Raphson or the arc-length method. The present thesis employs the Newton-Raphson method, which is briefly discussed in the following section.

In the full Newton-Raphson iteration scheme, the increment of the solution for the nodal displacements at iteration i is obtained as follows

$$\mathbf{K}_{m+1}^{i-1} \Delta \mathbf{U}^i = \mathbf{F}_{m+1} - \mathbf{R}_{m+1} = \Delta \mathbf{F}_{m+1}^{i-1} \quad (5.2)$$

where the current tangent stiffness matrix $\mathbf{K}_{m+1}^{i-1}$ is determined by

$$\mathbf{K}_{m+1}^{i-1} = \left. \frac{\partial \mathbf{R}}{\partial \mathbf{U}} \right|_{\mathbf{U}_{m+1}^{i-1}} \quad (5.3)$$

The improved displacement solution is then calculated as follows:

$$\mathbf{U}_{m+1}^i = \mathbf{U}_{m+1}^{i-1} + \Delta \mathbf{U}^i \quad i=1,2,3 \quad (5.4)$$

The computed tangent stiffness matrix, the displacement vector, and the resisting force vector from the last iteration of load step m are used as the initial conditions in the next load step $(m + 1)$:

$$\mathbf{K}_{m+1}^0 = \mathbf{K}_m; \quad \mathbf{U}_{m+1}^0 = \mathbf{U}_m; \quad \mathbf{R}_{m+1}^0 = \mathbf{R}_m$$

Equations 5.2 to 5.4 are obtained by linearising the response of the finite element system at the previous $(i - 1)$ iteration of the current load step $(m + 1)$. The iteration is terminated when a predefined convergence criterion is satisfied. The

computational cost is high for analysing a large and complex finite element model when the tangent stiffness matrix is evaluated at each iteration, as in the case of the full Newton–Raphson method. To reduce the computational effort, other iterative schemes may be employed, such as the initial stress method and the modified Newton–Raphson method.

In the initial stress method, the initial stiffness matrix $\mathbf{K}_0$ is used throughout the analysis instead of the current tangent stiffness matrix:

$$\mathbf{K}_0 \Delta \mathbf{U}^i = \Delta \mathbf{F}_{m+1}^{i-1} \quad (5.5)$$

Therefore, the computation of the structural response is linearised about the initial configuration and the system stiffness matrix need only be evaluated and factorised once at the beginning of the analysis. A drawback of this approach is that the convergence of the solution may be slow, or the solution may even diverge.

The modified Newton–Raphson method is a compromise between the full Newton–Raphson method and the initial stress method. In this approach, the stiffness matrix is evaluated at selected equilibrium configurations

$$\mathbf{K}_r \Delta \mathbf{U}^i = \Delta \mathbf{F}_{m+1}^{i-1} \quad (5.6)$$

How often the stiffness matrix is updated depends on the degree of nonlinearity in the system. In one form of implementation, a new tangent stiffness matrix is evaluated at the end of each load step, after attaining convergence. In such case, the computation of the system response is linearised about the configuration at the end of the previous load step.

In the present study, the static equilibrium equations are solved by the full Newton–Raphson iterative scheme, whereas the dynamic equations of motion are solved by the initial stress Newton–Raphson procedure.

The iteration techniques of the Newton–Raphson family of methods are illustrated in Figures 5.1 to 5.3 where the iterations for the solution of a single-degree-of-freedom system are shown.

The preceding discussion on nonlinear problem solution techniques applies to both static and dynamic nonlinear analysis if implicit integration in time domain is

used for solving the equations of dynamic equilibrium. The only difference is that in dynamic analysis, the system's effective stiffness matrix and effective out-of-balance load vector are used in Eq.5.2 instead of, respectively, the system's stiffness matrix and load vector.

5.3 Solution of the equations of static equilibrium

The computer program CADNAP can analyse the static response of an arch dam due to the self-weight of the structure, the hydrostatic pressure of the impounded water, and the temperature changes in the dam body. For the static analysis, the dam cantilevers and the rock foundation are assumed to behave linear-elastically. Combinations of any of the above three types of static loads can be considered in the analysis.

Load due to the self-weight

The construction sequence of the cantilever monoliths has an important impact on the stress and strain state in the dam body caused by the self-weight of the concrete material. This may have a significant impact on the performance of the structure under the extreme loading conditions. In practice, the monoliths are constructed independently and each cantilever transfers its weight to the foundation. In modelling the static loads due to self-weight, the procedure originally developed in ADAP (Clough et al. 1973) which takes into account the effect of the construction sequence on the stress state of the dam is employed in CADNAP. The progress of construction is simulated by considering two sets of independent cantilever monoliths, as shown in Figure 5.4. The solution for gravity loads is therefore performed in two steps. In the first step, the analysis is performed only for the finite elements of the first set of cantilevers, by setting the elastic modulus of the second set of cantilevers equal to zero. In the second step, the stresses in the second set of cantilevers are obtained in a corresponding second analysis. Accordingly, the only interaction between the cantilever monoliths before the grouting of the contraction joints is through the foundation.

Load due to hydrostatic pressure

Since the impounding of the reservoir of an arch dam takes place after the grouting of contraction joints, the dam resists the hydrostatic pressure as a monolithic structure. Therefore, the analysis for hydrostatic load is performed by using the complete dam–foundation model. In the analysis, the dam system may be subjected to an arbitrary level of the impounded water in the reservoir. If the self-weight of the dam is considered, the displacements and the stresses due to the hydrostatic loads are added, respectively, to the corresponding displacements and stresses due to self-weight.

For the case of an adequately designed concrete arch dam, the hydrostatic pressure and the temperature gradients should not cause opening of the contraction joints of the dam. Therefore, a linear analysis procedure would be sufficient to analyse the static response of the dam. However, for the evaluation of the safety and performance of existing arch dams, the possibility of nonlinear effects due to opening and slippage at the contraction joints should be considered. Thus, a nonlinear solution procedure using the full Newton–Raphson iterative scheme is implemented in CADNAP for analysis of the static response of arch dams to hydrostatic pressure and temperature gradients.

Loads due to temperature changes

The program enables specifying temperature changes at the elevations used for generation of the finite element mesh of the dam (design elevations). Values at the mesh elevations are computed from those at the design elevations by cubic interpolation. The temperature changes are assumed constant in the arch direction. The solution for the loads due to temperature changes is carried out in the same manner as the solution for hydrostatic loads.

5.4 Solution of the equations of motion

The equations of motion of the nonlinear structure can be written as follows:

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{P}(\dot{\mathbf{U}}, \mathbf{U}_{tot}) = -\mathbf{MR}\ddot{\mathbf{U}}_g + \mathbf{F}^{st} \quad (5.7)$$

where

$$\mathbf{U}_{tot} = \mathbf{U}_{st} + \mathbf{U} \quad (5.8)$$

where $\mathbf{U}$, $\dot{\mathbf{U}}$, and $\ddot{\mathbf{U}}$ are respectively the displacement, velocity, and acceleration relative to the free-field ground support movement, $\mathbf{U}_g$, $\mathbf{U}_{st}$ is the displacement due to static loads, $\mathbf{M}$ is the mass matrix which includes the added mass from the consideration of the dam-water interaction effect, $\mathbf{P}$ is the restoring force vector which is a nonlinear function of the velocity and the total displacement, $\mathbf{R}$ is the influence matrix for $\ddot{\mathbf{U}}$, and $\mathbf{F}^{st}$ is the static load vector.

The Newmark method has been implemented in CADNAP for the direct integration of Eq.5.7 in the time domain. Accordingly, the response $\mathbf{U}$, $\dot{\mathbf{U}}$, and $\ddot{\mathbf{U}}$ of the investigated system at time step t_{n+1} are determined from the following equations:

$$\mathbf{M}\ddot{\mathbf{U}}_{n+1} + \mathbf{P}(\dot{\mathbf{U}}_{n+1}, \{\mathbf{U}_{tot}\}_{n+1}) = -\mathbf{MR}\ddot{\mathbf{U}}_g + \mathbf{F}^{st} \quad (5.9)$$

$$\mathbf{U}_{n+1} = \tilde{\mathbf{U}}_{n+1} + \Delta t^2 \beta \ddot{\mathbf{U}}_{n+1} \quad (5.10)$$

$$\dot{\mathbf{U}}_{n+1} = \tilde{\dot{\mathbf{U}}}_{n+1} + \Delta t \gamma \ddot{\mathbf{U}}_{n+1} \quad (5.11)$$

where

$$\tilde{\mathbf{U}}_{n+1} = \mathbf{U}_n + \Delta t \dot{\mathbf{U}}_n + \Delta t^2 \left(\frac{1}{2} - \beta \right) \ddot{\mathbf{U}}_n \quad (5.12)$$

$$\tilde{\dot{\mathbf{U}}}_{n+1} = \dot{\mathbf{U}}_n + \Delta t (1 - \gamma) \ddot{\mathbf{U}}_n \quad (5.13)$$

In Eqs. 5.9 to 5.13, Δt denotes the time step, $\mathbf{U}_n$, $\dot{\mathbf{U}}_n$, and $\ddot{\mathbf{U}}_n$ are the displacement, velocity and acceleration responses, respectively, at time step t_n , $\tilde{\mathbf{U}}_{n+1}$ and $\tilde{\dot{\mathbf{U}}}_{n+1}$ are

the current linearised estimates of the displacement and velocity responses in the iteration based on the solution at the end of the previous time step, $\mathbf{U}_{n+1}$ and $\dot{\mathbf{U}}_{n+1}$ are the calculated displacements and velocities at time step t_{n+1} ; and β and γ are the Newmark iteration parameters controlling the accuracy and the stability of the solution (Hughes et al. 1979). Due to the nonlinear relationship between the restoring forces $\mathbf{P}$ and the displacements and the velocities, Eqs.5.9 to 5.13 are solved iteratively by linearising the nonlinear equations of motion to an effective linear system. The solution of the linearised equations of motion at iteration $(i + 1)$ is obtained in terms of the unknown displacement increment $\Delta \mathbf{U}$ as follows:

$$\mathbf{K}^{ef} \Delta \mathbf{U} = \Delta \mathbf{F}^{ef} \quad (5.14)$$

In Eq.5.14, the effective stiffness matrix $\mathbf{K}^{ef}$ and the effective out-of-balance force vector $\Delta \mathbf{F}^{ef}$ can be obtained as follows:

$$\mathbf{K}^{ef} = \frac{1}{\Delta t^2 \beta} \mathbf{M} + \frac{\gamma}{\Delta t \beta} \mathbf{C}(\dot{\mathbf{U}}_{n+1}^i) + \mathbf{K}_T(\{\mathbf{U}_{tot}\}_{n+1}^i) \quad (5.15)$$

$$\Delta \mathbf{F}^{ef} = -\mathbf{M} \mathbf{R} \left\{ \ddot{\mathbf{U}}_g \right\}_{n+1} + \mathbf{F}^{st} - \mathbf{M} \ddot{\mathbf{U}}_{n+1}^i - \mathbf{C} \dot{\mathbf{U}}_{n+1}^i - \mathbf{N}(\{\mathbf{U}_{tot}\}_{n+1}^i) \quad (5.16)$$

where $\mathbf{K}_T(\{\mathbf{U}_{tot}\}_{n+1}^i)$ and $\mathbf{N} = \mathbf{N}(\mathbf{U}_{n+1}^i)$ are, respectively, the tangent stiffness matrix and the vector of the restoring forces. They are both nonlinear functions of the total displacements.

Having solved Eq.5.14 for $\Delta \mathbf{U}$, the improved approximations to the system responses at time step t_{n+1} are given by

$$\mathbf{U}_{n+1}^{i+1} = \mathbf{U}_{n+1}^i + \Delta \mathbf{U} \quad (5.17)$$

$$\{\mathbf{U}_{tot}\}_{n+1}^{i+1} = \mathbf{U}_{st} + \mathbf{U}_{n+1}^{i+1} \quad (5.18)$$

$$\dot{\mathbf{U}}_{n+1}^{i+1} = \dot{\mathbf{U}}_{n+1}^i + \frac{\gamma}{\beta \Delta t} \Delta \mathbf{U} \quad (5.19)$$

$$\ddot{\mathbf{U}}_{n+1}^{i+1} = \ddot{\mathbf{U}}_n^i + \frac{1}{\beta \Delta t^2} \Delta \mathbf{U} \quad (5.20)$$

The numerical procedure is repeated until the specified convergence criteria are satisfied.

5.5 Numerical implementation of seismic analysis procedure

In this section, the assumptions made in CADNAP in the formulations of the numerical procedures for seismic response analysis are discussed. The modelling of damping and the derivations of the effective stiffness and the out-of-balance force are presented. The convergence criteria adopted are also discussed.

5.5.1 Damping

Slender structures, such as concrete arch dams, have low damping ratios, varying in the range from 3% to 5%. Therefore, it is assumed in the present work that the damping of the concrete arch dam can be approximated with a reasonable accuracy by means of Rayleigh damping

$$\mathbf{C} = b_0 \mathbf{M} + b_1 \mathbf{K} \quad (5.21)$$

where b_0 and b_1 are constant parameters. To decrease the computational effort required in each time step, it is further assumed that in Eq.5.21 the changes to the structure stiffness properties $\mathbf{K}$ due to possible cracking and crushing of the concrete do not affect significantly the damping properties of the dam. Therefore, the damping matrix need only be computed once at the beginning of the time step integration procedure.

5.5.2 Effective stiffness matrix

The effective stiffness matrix $\mathbf{K}^{ef}$ is calculated using Eq.5.15. Using the assumption of Rayleigh damping, the expression for $\mathbf{K}^{ef}$ is written as follows:

$$\mathbf{K}^{ef} = \frac{1}{\Delta t^2 \beta} \mathbf{M} + \frac{\gamma}{\Delta t \beta} (b_0 \mathbf{M} + b_1 \mathbf{K}) + \mathbf{K}_T \quad (5.22)$$

Eq.5.22 can also be written in the following form:

$$\mathbf{K}^{ef} = (a_1 + a_2 b_0) \mathbf{M} + a_2 b_1 \mathbf{K} + \mathbf{K}_T \quad (5.23)$$

where $a_1 = \frac{1}{\Delta t^2 \beta}$ and $a_2 = \frac{\gamma}{\Delta t \beta}$. As discussed in section 5.2, the initial stress Newton-Raphson iterative procedure is employed in CADNAP to solve the nonlinear equations

of motion. The stiffness of the joint elements is updated at each iteration. Accordingly, the last term of Eq.5.22 is expressed as follows:

$$\mathbf{K}_T = \mathbf{K}_T^j + \mathbf{K}^c + \mathbf{K}^f \quad (5.24)$$

where $\mathbf{K}_T^j$ is the tangent stiffness matrix of the joint elements, and $\mathbf{K}^c$, and $\mathbf{K}^f$ are the initial stiffness matrices of the dam cantilevers and the foundation, respectively. The tangent stiffness of the joint elements $\mathbf{K}_T^j$ is obtained from Eq.3.21. Substituting Eq.5.24 into Eq.5.23, the expression for the effective stiffness of the dam-foundation system becomes

$$\mathbf{K}^{ef} = (a_1 + a_2 b_0) \mathbf{M} + a_2 b_1 \mathbf{K} + \mathbf{K}_T^j + \mathbf{K}^c + \mathbf{K}^f \quad (5.25)$$

5.5.3 Effective out-of-balance force vector

By substituting Eqs.5.21 into Eq.5.16, the effective out-of-balance force vector is written as follows:

$$\Delta \mathbf{F}^{ef} = -\mathbf{M} \mathbf{R} \left\{ \ddot{\mathbf{U}}_g \right\}_{n+1} + \mathbf{F}^{st} - \mathbf{M} \ddot{\mathbf{U}}_{n+1}^i - (b_0 \mathbf{M} + b_1 \mathbf{K}) \dot{\mathbf{U}}_{n+1}^i - \mathbf{N}_{n+1}^i \quad (5.26)$$

The last term of Eq.5.26, $\mathbf{N}_{n+1}^i$, is obtained by assembling the nodal restoring forces of the joint elements $\{\mathbf{N}^j\}_{n+1}^i$ and of the three-dimensional solid elements of the dam cantilevers $\{\mathbf{N}^c\}_{n+1}^i$ and of the foundation $\{\mathbf{N}^f\}_{n+1}^i$ into the global out-of-balance-force vector. It can therefore be expressed as follows:

$$\mathbf{N}_{n+1}^i = \{\mathbf{N}^j\}_{n+1}^i + \{\mathbf{N}^c\}_{n+1}^i + \{\mathbf{N}^f\}_{n+1}^i \quad (5.27)$$

The nodal restoring forces of the joint elements are obtained from Eq.3.20. Since the foundation is assumed linear-elastic, the restoring forces in the foundation elements are given by

$$\{\mathbf{N}^f\}_{n+1}^i = \mathbf{K}^f \{\mathbf{U}_{tot}\}_{n+1}^i \quad (5.28)$$

The restoring forces in the elements of the dam cantilevers can be obtained from

$$\mathbf{N}_{n+1}^i = \Sigma \int_{\Omega} \mathbf{B}^T \{\boldsymbol{\sigma}_{in}\}_{n+1}^i d\Omega \quad (5.29)$$

where the integration is carried out for each dam element associated with the nonlinear material model, and the obtained nodal forces are assembled into the global out-of-balance force vector. The final expression of the out-of-balance force thus becomes

$$\begin{aligned} \Delta \mathbf{F}^{ef} = & -\mathbf{MR} \left\{ \ddot{\mathbf{U}}_g \right\}_{n+1} + \mathbf{F}^{st} - \mathbf{M} \ddot{\mathbf{U}}_{n+1}^i - (b_0 \mathbf{M} + b_1 \mathbf{K}) \dot{\mathbf{U}}_{n+1}^i - \\ & - \{\mathbf{N}^j\}_{n+1}^i - \mathbf{K}^f \{\mathbf{U}_{tot}\}_{n+1}^i - \Sigma \int_{\Omega} \mathbf{B}^T \{\boldsymbol{\sigma}_{in}\}_{n+1}^i d\Omega \end{aligned} \quad (5.30)$$

5.5.4 Algorithm for solving the nonlinear equations of motion

The following algorithm is implemented in CADNAP to solve the nonlinear equations of motion in seismic analysis:

Step 1. Compute and assemble the constant terms of the effective stiffness matrix:

$$\mathbf{K}_{const}^{ef} = (a_1 + a_2 b_0) \mathbf{M} + a_2 b_1 \mathbf{K} + \mathbf{K}^c + \mathbf{K}^f \quad (5.31)$$

Step 2. Initialise the time integration procedure.

Step 3. If the current time step is greater or equal to the specified total number of time steps, go to Step 19.

Step 4. Compute the predictions for seismic displacements and velocities:

$$\bar{\mathbf{U}}_{n+1} = \mathbf{U}_n + \Delta t \dot{\mathbf{U}}_n + \Delta t^2 \left(\frac{1}{2} - \beta \right) \ddot{\mathbf{U}}_n \quad (5.32)$$

$$\bar{\dot{\mathbf{U}}}_{n+1} = \dot{\mathbf{U}}_n + \Delta t (1 - \gamma) \ddot{\mathbf{U}}_n \quad (5.33)$$

Step 5. Compute the external load and assemble it into the effective out-of-balance-force vector along with the static force:

$$\Delta \mathbf{F}^{ef} = -\mathbf{MR} \left\{ \ddot{\mathbf{U}}_g \right\}_{n+1} + \mathbf{F}^{st} \quad (5.34)$$

Step 6. Initialise the iterative procedure.

Step 7. If the current number of iterations is greater than a specified maximum number, go to Step 17.

Step 8. Assemble the elastic terms in the out-of-balance-force vector:

$$\Delta \mathbf{F}^{ef} = -\mathbf{MR} \left\{ \ddot{\mathbf{U}}_g \right\}_{n+1} + \mathbf{F}^{st} - \mathbf{M} \ddot{\mathbf{U}}_{n+1}^i - (b_0 \mathbf{M} + b_1 \mathbf{K}) \dot{\mathbf{U}}_{n+1}^i - \left\{ \mathbf{N}^j \right\}_{n+1}^i - \mathbf{K}^f \left\{ \mathbf{U}_{tot} \right\}_{n+1}^i \quad (5.35)$$

Step 9. Compute the tangent stiffness of the joint elements and assemble it into the global effective stiffness matrix:

$$\mathbf{K}^{ef} = \mathbf{K}_{const}^{ef} + \mathbf{K}_T^j \quad (5.36)$$

Step 10. Using the predicted nodal displacements, compute the elastic strains and stresses in the integration points of the three-dimensional solid elements of the arch dam:

$$\left\{ \epsilon \right\}_{n+1}^i = \mathbf{B} \left\{ \mathbf{U}_{tot} \right\}_{n+1}^i \quad (5.37)$$

$$\left\{ \sigma_{tot,el} \right\}_{n+1}^i = \mathbf{DB} \left\{ \mathbf{U}_{tot} \right\}_{n+1}^i \quad (5.38)$$

Step 11. Based on the current elastic strains and stresses, determine the inelastic stresses, $\left\{ \sigma_{tot} \right\}_{n+1}^i$, by means of the nonlinear model of concrete.

Step 12. Assemble the final effective out-of-balance-force vector:

$$\Delta \mathbf{F}^{ef} = -\mathbf{MR} \left\{ \ddot{\mathbf{U}}_g \right\}_{n+1} + \mathbf{F}^{st} - \mathbf{M} \ddot{\mathbf{U}}_{n+1}^i - (b_0 \mathbf{M} + b_1 \mathbf{K}) \dot{\mathbf{U}}_{n+1}^i - \left\{ \mathbf{N}^j \right\}_{n+1}^i - \mathbf{K}^f \left\{ \mathbf{U}_{tot} \right\}_{n+1}^i - \Sigma \int_{\Omega} \mathbf{B}^T \left\{ \sigma_{tot} \right\}_{n+1}^i d\Omega \quad (5.39)$$

Step 13. Solve the incremental equation of motion for the displacement increment $\Delta \mathbf{U}$:

$$\mathbf{K}^{ef} \Delta \mathbf{U} = \Delta \mathbf{F}^{ef} \quad (5.40)$$

Step 14. Compute the incremental responses:

$$\mathbf{U}_{n+1}^{i+1} = \mathbf{U}_{n+1}^i + \Delta \mathbf{U} \quad (5.41)$$

$$\left\{ \mathbf{U}_{tot} \right\}_{n+1}^{i+1} = \mathbf{U}_{st} + \Delta \mathbf{U}_{n+1}^{i+1} \quad (5.42)$$

$$\dot{\mathbf{U}}_{n+1}^{i+1} = \dot{\mathbf{U}}_{n+1}^i + \frac{\gamma}{\beta \Delta t} \Delta \mathbf{U} \quad (5.43)$$

$$\ddot{\mathbf{U}}_{n+1}^{i+1} = \ddot{\mathbf{U}}_n^i + \frac{1}{\beta \Delta t^2} \Delta \mathbf{U} \quad (5.44)$$

Step 15. If the norms of $\Delta \mathbf{U}$ and $\Delta \mathbf{F}^{ef}$ meet the specified convergence criteria, go to Step 17.

Step 16. Update the iteration counter. Go back to Step 7.

Step 17. Store the requested response quantities for output.

Step 18. Update the time step counter. Go back to Step 3.

Step 19. End the procedure.

5.5.5 Convergence criteria

To determine whether the displacements and the forces are near their equilibrium values in the iteration within a load step, the increment of internal energy at the current iteration is compared to the initial internal energy increment. Convergence is assumed when the ratio between the current and the initial increment in energy is less than a specified value:

$$\frac{|\Delta \mathbf{U}^{(i)T}| |(\mathbf{F}_{n+1} - \mathbf{R}_{n+1}^{i-1})|}{|\Delta \mathbf{U}^{(1)T}| |(\mathbf{F}_{n+1} - \mathbf{R}_n)|} \leq \epsilon_E \quad (5.45)$$

In the course of the present study it is confirmed once again that proper selection of the energy convergence tolerance ϵ_E is a prerequisite to obtaining accurate solution for reasonable computational effort. If an energy convergence tolerance is set too large, the computational effort is reduced, but so is the accuracy of the solution. On the other hand, if a very small tolerance is specified, the computational effort increases significantly without meaningful benefit to the accuracy of the solution. The experience gained from the present work shows that the optimum value for the convergence tolerance is problem-dependent and should be selected after some trial runs of the algorithm.

The convergence criteria employed in CADNAP for static and seismic analysis are discussed in the following.

In static analysis, the iterations within a load step are terminated when the ratio of the left-hand side of Eq.5.45 is less than or equal to 10^{-8} .

In seismic analysis, the procedure for determining convergence as applied in ADAP-88 is used in CADNAP. Accordingly, if the seismic analysis is preceded by a static analysis for water and/or temperature loads, ϵ_E is specified as follows:

$$\epsilon_E = 10^{-6} * E \quad (5.46)$$

where E is the strain energy of the structure under static loading. If static analysis for water and/or temperature loads is not performed, the convergence tolerance is specified as follows:

$$\epsilon_E = 10^{-8} \quad (5.47)$$

A flow chart of the numerical procedure implemented in CADNAP is shown in Figure 5.5.

5.5.6 Finite element types

The mesh generator presently used in CADNAP can generate two kinds of finite elements: a zero-thickness joint element and a three-dimensional eight-node or six-node solid element. The formulation of the joint element used in the program has been discussed in Chapter 3. The three-dimensional solid element used in the program is the eight-node hexahedron element first described by Zienkiewicz and Cheung (1967) and improved by Wilson (1970). The element is described in the rest of this section.

A typical eight-node hexahedron element defined in a global Cartesian co-ordinate system (X, Y, Z) is shown in Figure 5.6. A natural co-ordinate system (ξ, η, ζ) is defined such that ξ , η and ζ vary from -1 to +1 within the element, and the local co-ordinates of its centroid are $(0, 0, 0)$. The global co-ordinates of an arbitrary point within the element are expressed by means of the global co-ordinates of the

nodal points and a set of interpolation functions corresponding to each node:

$$\begin{aligned} X &= \sum_{i=1}^8 N_i X_i \\ Y &= \sum_{i=1}^8 N_i Y_i \\ Z &= \sum_{i=1}^8 N_i Z_i \end{aligned} \quad (5.48)$$

where $i = 1$ to 8, X_i , Y_i , and Z_i are the global co-ordinates of the nodal points, and N_i are the interpolation functions as follows:

$$\begin{aligned} N_1 &= \frac{1}{8}(1 + \xi)(1 - \eta)(1 - \zeta) \\ N_2 &= \frac{1}{8}(1 + \xi)(1 + \eta)(1 - \zeta) \\ N_3 &= \frac{1}{8}(1 - \xi)(1 + \eta)(1 - \zeta) \\ N_4 &= \frac{1}{8}(1 - \xi)(1 - \eta)(1 - \zeta) \\ N_5 &= \frac{1}{8}(1 + \xi)(1 - \eta)(1 + \zeta) \\ N_6 &= \frac{1}{8}(1 + \xi)(1 + \eta)(1 + \zeta) \\ N_7 &= \frac{1}{8}(1 - \xi)(1 + \eta)(1 + \zeta) \\ N_8 &= \frac{1}{8}(1 - \xi)(1 - \eta)(1 + \zeta) \end{aligned} \quad (5.49)$$

The displacements at a point within the element, u , v , and w , are interpolated from these at the nodal points, u_i , v_i , and w_i , by the following expressions:

$$\begin{aligned} u &= \sum_{i=1}^8 (N_i u_i + N_9 \alpha_{x1} + N_{10} \alpha_{x2} + N_{11} \alpha_{x3}) \\ v &= \sum_{i=1}^8 (N_i v_i + N_9 \alpha_{y1} + N_{10} \alpha_{y2} + N_{11} \alpha_{y3}) \\ w &= \sum_{i=1}^8 (N_i w_i + N_9 \alpha_{z1} + N_{10} \alpha_{z2} + N_{11} \alpha_{z3}) \end{aligned} \quad (5.50)$$

where:

$$\begin{aligned} N_9 &= 1 - \xi^2 \\ N_{10} &= 1 - \eta^2 \\ N_{11} &= 1 - \zeta^2 \end{aligned} \quad (5.51)$$

Interpolation functions N_9 , N_{10} , and N_{11} can be interpreted as associated with an internal nodal point. The corresponding degrees of freedom are eliminated at the element level by static condensation. Thus the internal displacements are not included in the output. However, their values are computed and used in the evaluation of the elastic strains and stresses within the element. It is noted that the inelastic stresses

are obtained on the basis of the elastic strains and stresses by means of the combined nonlinear concrete model. A comparison of the bending behaviour of the element with and without the incompatible modes is presented by Clough et al. (1973). The conclusion of the study is that the bending performance of the incompatible element is significantly better than that of the compatible element.

5.5.7 Presentation of analysis results

The response time histories of the nodal point displacements, the joint displacements, and the arch, cantilever, and shear stresses in the three dimensional solid elements are calculated. Envelops of the arch, cantilever, and shear stresses can also be determined.

The nodal displacements expressed in the global co-ordinate system (X, Y, Z) are the displacements relative to the rigid support movement. The displacements are presented relative to the static response.

The joint element displacements are the relative normal displacements between the two surfaces of the joint element evaluated at the integration points. They include components of responses due to static (hydrostatic and temperature) and seismic loads.

Stresses in the three-dimensional solid elements are calculated only for the dam elements on the upstream and the downstream layers. A local co-ordinate system (x, y, z) is defined at the free surface (i.e. the exposed surface either to the atmosphere or the water in the reservoir) of the element. The x-axis of the system passes through the midpoints of the left and the right vertical sides of the element and is directed rightward. The z-axis is normal to the free surface and is directed outward from the element. The y-axis completes the orthogonal right handed co-ordinate system. The stresses in the local co-ordinate system are evaluated at the centre of the free surface. Only σ_x , σ_y , and τ_{xy} stresses are output at the stress point, which correspond to the arch, the cantilever, and the shear stress, respectively. It is noted that the computations for the inelastic stresses are performed at the Gauss points. To obtain the inelastic components of the arch, the cantilever and the shear stresses at the

stress points, the global co-ordinate system stress components are first evaluated. If a $2 \times 2 \times 2$ Gauss integration is employed, the components of the inelastic stress in the global co-ordinate system can be evaluated as follows:

$$\begin{aligned}
\sigma_X &= \sum_{i=1}^8 N_i \sigma_{X_i} \\
\sigma_Y &= \sum_{i=1}^8 N_i \sigma_{Y_i} \\
\sigma_Z &= \sum_{i=1}^8 N_i \sigma_{Z_i} \\
\tau_{XY} &= \sum_{i=1}^8 N_i \tau_{XY_i} \\
\tau_{YZ} &= \sum_{i=1}^8 N_i \tau_{YZ_i} \\
\tau_{ZX} &= \sum_{i=1}^8 N_i \tau_{ZX_i}
\end{aligned} \tag{5.52}$$

where $i = 1$ to 8 and N_i are the interpolation functions given in Eq.5.49 but this time associated with the Gauss points, and not with the nodal points of the element. The shape functions are evaluated at the stress output point. The global co-ordinate system stress tensor is transformed into the local co-ordinate system

$$\mathbf{T}_{\sigma'} = \mathbf{A}^T \mathbf{T}_{\sigma} \mathbf{A} \tag{5.53}$$

where $\mathbf{T}_{\sigma'}$ is the stress tensor in the local co-ordinate system, $\mathbf{A}$ is the transformation matrix between the global and the local systems, and $\mathbf{T}_{\sigma}$ is the global co-ordinate system stress tensor:

$$\mathbf{T}_{\sigma'} = \begin{pmatrix} \sigma_x & \tau_{yz} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{pmatrix} \tag{5.54}$$

$$\mathbf{A} = \begin{pmatrix} \cos(xX) & \cos(xY) & \cos(xZ) \\ \cos(yX) & \cos(yY) & \cos(yZ) \\ \cos(zX) & \cos(zY) & \cos(zZ) \end{pmatrix} \tag{5.55}$$

$$\mathbf{T}_{\sigma} = \begin{pmatrix} \sigma_X & \tau_{YX} & \tau_{ZX} \\ \tau_{XY} & \sigma_Y & \tau_{ZY} \\ \tau_{XZ} & \tau_{YZ} & \sigma_Z \end{pmatrix} \tag{5.56}$$

The envelopes of the maximum and the minimum arch, cantilever, and shear stresses are obtained from the respective stress time histories at the stress points.

Figure 5.1: Illustration of the full Newton–Raphson iterative procedure for solution of a generic SDOF

Figure 5.2: Illustration of the initial stress Newton–Raphson iterative procedure for solution of a generic SDOF

Figure 5.3: Illustration of the modified Newton–Raphson iterative procedure for solution of a generic SDOF

Figure 5.4: Simulation of the construction sequence of a concrete arch dam

Figure 5.5: Flowchart of the Newmark procedure applied in CADNAP for solving the nonlinear equations of motion

Figure 5.6: Three-dimensional solid element

Chapter 6

Seismic analysis of Morrow Point dam

6.1 Introduction

To demonstrate the validity of the proposed formulations for the combined models of contraction joints and nonlinear concrete material behaviour, the seismic response of a concrete arch dam is analysed. In order to facilitate comparison of the results with those from previous studies, Morrow Point dam is selected as the example of the present study. The seismic behaviour of Morrow Point dam has been studied by many researchers (Clough et al. 1973, Fok and Chopra 1985, Fenves et al. 1992, Tan and Chopra 1995, Noruziaan 1995), which provide a good basis for comparison and validation of the present results.

6.2 Description of Morrow Point dam

Morrow Point dam is a double-curvature, single-centre concrete arch dam built on the Gunnison River in Colorado, USA. The dam was designed by the U.S. Bureau of Reclamation and was constructed from 1963 to 1968. The dam has a crest length of 221 m and a maximum height of 142 m. The thickness of the crown section varies from 3.66 m at the crest to 15.8 m at the base. The dam consists of eighteen monolith cantilevers separated by contraction joints. In the arch direction, each cantilever monolith is 12.2 m long, except for the the crown cantilever which is 9.14 m long. Figure 6.1 shows the plan view, an axis profile, and cross sections of the dam. Detailed descriptions of the geometry of Morrow Point dam can be found in the study by Hall and Chopra (1983).

6.3 Finite element models

As shown in Figure 6.1, the dam structure is almost symmetric about the central plane. To reduce the computational effort, only half of the dam–foundation–reservoir system is analysed by assuming perfect symmetry. For the excitations, the stream and the vertical components of the ground motions are considered in the present study.

6.3.1 Finite element model of dam

The mesh generator of CADNAP is utilised to generate the finite element model of the dam–foundation structure. The finite element mesh of one-half of the dam body, as shown in Figure 6.2, consists of 1503 three-dimensional eight-node and six-node solid elements and 213 joint elements. The dam body is discretized by three layers of solid and joint elements in the thickness direction, and by 28 horizontal layers in the vertical direction. Four contraction joints, uniformly spaced in the arch direction along the crest are modelled. The finite element mesh was designed in view to provide objectivity as to the conservation of the fracture energy of concrete in nonlinear analysis. The number of contraction joints modelled is chosen based on the conclusions of the study by Fenves et al. (1992), according to which including three contraction joints in the full model of an arch dam yields a reasonable accuracy of the computed response.

In the analysis, the contraction joints are modelled by the joint model described in Chapter 3. The nonlinear concrete models described in Chapter 4 are implemented with the three-dimensional solid elements to model the dam body.

Material properties of concrete

The material properties of concrete used in the present study are listed in Table 6.11.

Isotropic material properties are assumed for the uncracked concrete. The initial modulus of elasticity E , the Poisson ratio ν , and the unit weight of concrete γ , are taken the same as these used in the study by Fenves et al. (1992). The uniaxial

compressive strength f'_c , the critical uniaxial strain ε_p (the uniaxial strain corresponding to f'_c), the apparent uniaxial tensile strength f'_t , and the fracture energy G_f are selected from published experimental data (Bruhwiler 1990, Pagnoni et al. 1992). To account for the effect of high strain rate, as typically encountered in seismic excitation, E , f'_c , f'_t and G_f , are increased by 25% in the computations of the dynamic response of the structure (CEA 1990).

As has been discussed in section 4.3, the criterion for the objectivity of the finite element mesh is given by the following requirement of the characteristic length of the elements:

$$l_c \leq \frac{2EG_f}{\sigma_{tu}^2} \quad (6.1)$$

Substituting the material properties considered in the seismic analysis, the characteristic length is determined as follows:

$$l_c \leq 2.6 \text{ m} \quad (6.2)$$

For the finite element mesh used, the characteristic length associated with an integration point of the elements in regions of high tensile stresses is about 2.0 m, taken from the average linear dimension of the elements. The requirement of mesh objectivity given in Eq.6.2 is thus satisfied. The results from the present seismic analysis are then expected to be mesh independent.

6.3.2 Finite element model of foundation

An orthotropic linear-elastic material model is employed to model the material of the foundation substructure for both static and seismic response analyses. The canyon of the foundation is assumed prismatic. The foundation model consists of two layers of 270 three-dimensional solid elements extending to a depth approximately equal to the height of the dam. The mass of the foundation rock is neglected to suppress stresses caused by the inertia effect from the propagation of the seismic waves. The earthquake ground motions are specified uniformly at the far-field boundaries of the foundation rock model.

The material properties of the foundation rock used in the present analysis are listed in Table 6.2. The modulus of elasticity used in static analysis is increased by 25 % for the computations in seismic analysis.

6.3.3 Reservoir model

In the present study, the hydrodynamic pressure on the upstream face of the dam is simulated by means of the added mass concept neglecting water compressibility.

The added mass matrix representing the hydrodynamic effect is determined by the computer program RESVOR (Kuo 1982). In the analysis, the cross section of the reservoir is assumed to be constant with the far-field boundary taken at three times the height of the dam in the upstream direction. The diagonal mass coefficients are then scaled to preserve the total hydrodynamic forces due to rigid body acceleration of the dam. Although the diagonalisation of the added mass matrix is not required in the formulation of CADNAP because the structure model is a full nonlinear model involving all the degrees of freedom of the dam-foundation system, the original formulation of RESVOR is employed for convenient reason.

The fluid finite element used in RESVOR is a three-dimensional sixteen-node element with eight nodes each on its upstream and downstream face. Since eight-node three-dimensional solid elements are used in the modelling of the dam body, some of the nodes on the upstream face of the dam are not required to associate with nodes of the fluid elements at the dam-water interface. The configuration of the mesh for the fluid elements on the dam-reservoir interface of the case of full reservoir is shown in Figure 6.3.

The reservoir model consists of 1780 three-dimensional fluid elements grouped into ten layers in the stream direction.

6.4 Loads and load combinations

The responses to loads due to self-weight of the structure, hydrostatic pressure of the impounded water, and seismic excitations applied at the base of the dam are

analysed in the present study of Morrow Point dam.

In the analysis of the case of self-weight loads, the construction sequence is accounted for by considering two sets of cantilever monoliths, as discussed in Section 5.3.

The cases of empty reservoir, half-full reservoir, and full reservoir are considered in seismic response analyses carried out with a linear-elastic concrete model in order to determine the critical load case for more detailed nonlinear investigations. The full reservoir load case assumes the water level is 1.52 m below the dam crest elevation. The hydrodynamic pressure on the upstream face of the dam has been considered in the seismic analysis in the half-full and the full reservoir load cases.

The seismic analyses are carried out using the ground motion recorded at the Taft Lincoln School Tunnel during the Kern County earthquake in California on 21 July 1952. The components of ground acceleration time history applied in the stream and vertical directions are shown in Figure 6.4. The pseudo-acceleration response spectra for the ground motion components considered with 5% damping ratio are presented in Figure 6.5. Table 6.3 lists the first ten natural vibration frequencies of Morrow Point dam for the cases of empty and full reservoir considering the hydrodynamic effect by the diagonal added mass matrix. The frequencies of the first symmetric vibration mode of the monolith dam of the cases of empty and full reservoir are shown in the response spectra of the earthquake ground motion in Figure 6.5.

The response of Morrow Point dam subjected to the first 15 seconds of the selected ground motions are analysed. A time step of 0.01 seconds is used in the Newmark method for direct integration of the equations of motion of the system. The Newmark constants used in the analysis are $\beta = 0.36$ and $\gamma = 0.7$. The Rayleigh's proportionality constants b_0 and b_1 are taken as 1.37 rad/sec and 0.001732 sec/rad, respectively, to give a modal damping ratio of 5% in the first and the fifth vibration modes of the monolithic dam model with empty reservoir.

6.5 Joint model

The modelling parameters of the joint model used in this study are the normal penalty parameter k_n , the tangential penalty parameter k_s , the strength of the joint in the normal direction σ_n , the slip margin δ , the apparent cohesion c and the friction angle of the grout material ϕ . The normal and the tangential penalty parameters are set equal to 157×10^9 kN/m³. According to the parametric studies carried out by Noruziaan (1995), the apparent cohesion c and the friction angle ϕ have a negligible effect on the behaviour of the dam. The most important effect is attributed to the presence of shear keys and the slip margin model parameter δ . It is found that the shear keys are efficient barriers against shear slippage. The present study is carried out only for the case of large and strong shear keys with $\delta = 0.50$ m, with the apparent cohesion parameter taken as the tensile strength of concrete $c = f'_t$, and a friction angle $\phi = 45^\circ$ for the grout material.

6.6 Concrete models

To investigate the effects of possible cracking and crushing of concrete on the seismic response behaviour of Morrow Point dam, the combined bounding surface-smeared crack model as discussed in Chapter 4 is employed to model the concrete material of the dam. For comparison, the responses of the dam using linear-elastic concrete model, the bounding surface model and the smeared crack model are also analysed.

6.7 Static analysis

The contours of the static stresses for the cases of empty, half-full and full reservoir are presented in Figures 6.6 to 6.8. It can be seen that the arch stresses for the case of empty reservoir are very small, as expected, due to the independent construction sequence of the cantilever monoliths. The impounding of the reservoir to the middle level causes compressive arch stresses in most of the dam body and small tensile stresses at the crest around the central part of the dam. Further increase of

the water level leads to increase of the compressive arch stresses and reduction the tensile ones to zero.

6.8 Seismic analysis with linear-elastic concrete model

The envelopes for the maximum and minimum stresses for the case when the concrete behaviour in the dam is assumed to be linear-elastic are presented in Figures 6.9 to 6.14. By comparing the stress envelopes for the three investigated reservoir water levels it is concluded that the most critical case for both the maximum and the minimum stresses are obtained from the case of full reservoir. In the subsequent nonlinear analyses, the investigations are carried out for this loading case only. As can be seen from the stress envelopes of the case with full reservoir, the maximum tensile stresses occur in the crown cantilever in a region located at approximately $4/5$ the height of the dam, whereas the maximum compressive stresses occur in a region close to the crest of the dam. Based on these observations of the stress envelopes, two zones in the crown cantilever are selected for investigating the stress time histories: at the crest and at a height of $4/5$ the maximum height. The displacement time histories of the nodes at the middle of the crest are studied as well. Figure 6.15 shows the zones of the dam body for which response time histories are investigated.

For comparison, the case of full reservoir is analysed using a monolithic dam model without the vertical contraction joints. The envelopes of the maximum and the minimum stresses for this case are shown in Figures 6.16 and 6.17, respectively. It is observed that the maximum arch stresses near the dam crest on both the upstream and the downstream surfaces are much higher than the previous cases with the contraction joints. The maximum arch stress on the upstream face is about 6 MPa, and that on the downstream face is about 4 MPa. The results show that the opening of the joints causes significant decrease in the tensile arch stress on both the upstream and the downstream faces. The release of the tensile arch stress has an impact on the maximum cantilever stress and the minimum arch stress in the dam. The maximum cantilever stress on the downstream face increases from about 1.5 MPa for the monolithic dam model to 3.0 MPa for the dam model with joints. This increase

occurs when the dam displaces in the upstream direction with opened joints. The internal tensile forces in the dam are redistributed from the response mechanisms of arch action to cantilever bending. The minimum arch stress on the upstream face remains almost the same. The maximum cantilever tensile stress on the upstream face increases from 1.5 MPa to 2 MPa when the dam displaces in the downstream direction with opened joints. The smaller change in the maximum cantilever tensile stress, as compared to the increase in the corresponding stress on the downstream face, can be explained by the fact that the arch action remains important when the dam displaces in the downstream direction. It is confirmed by the observation of the decrease in the minimum arch stress on the downstream surface from -9 MPa to -10 MPa. In summary, the opening of the joints prevents the development of high tensile arch stresses and renders the downstream tensile cantilever stresses the most important tensile stress quantity.

6.9 Seismic analysis with smeared crack model for concrete

In this case, the smeared crack model for concrete described in section 4.3 is adopted in the formulation of the eight-node solid elements to model the dam. In this part of the study, the compressive behaviour of concrete is assumed to be linear-elastic. The uniaxial compressive strength of concrete is used in the computation of the tensile stress σ_{tu} from Eqs.4.50–4.52, but brittle compressive failure of the concrete material of the dam is not modelled.

Figure 6.18 shows the evolution of the crack pattern in the crown cantilever of the dam. The results show that the cracking process begins at the heel area in a limited zone. Between 5 and the 7 seconds from the beginning of the earthquake excitation, the cracking in the heel area intensifies and cracking initiates on the downstream face in the upper part of the dam. The cracks spread to the lower part of the dam between 7 and 10 seconds. Thereafter, no significant change to the crack pattern is observed.

The envelopes of the maximum and the minimum stress obtained for this case are shown in Figures 6.19 and 6.20, respectively. Compared to the linear-elastic

model, the crack model yields lower maximum cantilever stresses in the upper part of the downstream face and in the heel area of the upstream face due to the cracking that occurs in these parts of the dam. The changes to the envelopes of the maximum arch and the minimum arch and cantilever stresses are related to stress redistribution caused by the cracking process.

The arch and cantilever stress time histories for zones B and C of the dam body are compared to the corresponding stress time histories obtained from the linear-elastic model in Figures 6.21 to 6.24. Figure 6.24 shows that in zone C cracking initiates at about 7 seconds and is followed by a sudden drop of the downstream cantilever tensile stresses. After the initiation of the cracking process, when the cracks re-open, the compressive stresses increase due to the reduced load bearing area. In the same time, the tensile loads in the downstream face are redistributed from the cantilever to the arch direction, as can be seen from Figures 6.23 and 6.24. It is noted that there is no change of the stress level about which the stresses oscillate throughout the seismic excitation. This is due to the fact that the initial stress is compressive and to the assumption that upon closing of a crack concrete regains its initial compressive strength.

Figure 6.25 shows the time history of the stream and the vertical dynamic displacements at the crest of the crown cantilever (Zone A), with positive displacement in the downstream direction. Figure 6.25 shows that prior to the initiation of the cracking process on the downstream face of the dam, the crest displacements are identical to those obtained in the analysis with the linear-elastic concrete model. In the period after the crack initiation, the amplitudes of both the stream and the vertical components of the crest displacement increase. This increase is due to the proximity of the cracked zones and the reduced capacity of the concrete material at these locations to withstand tensile stresses.

6.10 Seismic analysis with bounding surface model

This analysis is aimed at evaluating the effect of high compressive stresses on the seismic behaviour of the dam. The bounding surface model for concrete described in section 4.2 is associated with the eight-node solid elements of the monolith cantilevers. The concrete material is assumed to fail in tension when the first stress invariant is greater than three quarters the tensile strength of concrete.

The envelopes of the maximum and the minimum stress obtained for this case are shown in Figures 6.26 and 6.27, respectively. The maximum compressive arch stresses are lower than the corresponding stresses obtained in the analysis with the linear-elastic concrete model. This reduction is due to the decreased compressive load bearing capacity of the concrete material caused by the damage sustained during the earthquake excitation. The compressive stress reduction is more pronounced in the crest area of the zones adjacent to the contraction joints where the compressive arch stresses attain high levels due to the impact when an open contraction joint closes. The reduction of the compressive arch stresses leads to stress redistribution in both the arch and the cantilever directions. It is noted that tensile stresses higher than the tensile strength of concrete are observed in the upstream arch and the downstream cantilever stresses. This is due to the adopted criterion for tensile failure in the bounding surface model.

The arch and cantilever stress histories for zones B and C of the dam body are shown in Figures 6.28, 6.29, 6.30, and 6.31, respectively. It is observed that the stress levels about which the stresses oscillate displace gradually throughout the seismic excitation toward tension or compression depending on the direction of stress redistribution. For example, the oscillation of the upstream arch stresses in zone C shifts by 2 MPa toward tension at the end of the investigated response, whereas the oscillation of the downstream arch stresses at the same location shifts by 1 MPa towards compression (Figure 6.30). Note that the upstream arch stresses in zone C reach 10 MPa in compression, whereas the downstream arch stresses reach only 4 MPa in compression. The shifts in the stress responses occur because of the damage to the concrete material caused by the cyclic loading during the seismic excitation and the

subsequent stress redistribution in this zone of the dam body.

The time histories of the dynamic stream and the vertical displacements at the crest of the crown cantilever as shown in Figure 6.32, are compared with the corresponding results obtained from the analysis with the linear-elastic concrete model. The accumulation of damage and the subsequent reduced capacity of the concrete material to withstand compressive loads cause gradual shift in the baseline of the stream and vertical displacements. It is therefore expected the configuration of the dam after the earthquake will differ from the configuration prior to the earthquake.

6.11 Seismic analysis with combined smeared crack-bounding surface model

The purpose of this analysis is to evaluate the combined effect of high tensile and compressive stress in the concrete monolith cantilevers on the seismic behaviour of the investigated arch dam. The combined nonlinear concrete model proposed in section 4.4 is associated with the eight-node solid elements of the monolith cantilevers.

Figure 6.33 shows the evolution of the damage pattern in the crown cantilever of the dam. It is noted that no integration point fails under compressive stress. The cracking process develops in a manner similar to the analysis with the smeared crack model. However, the analysis with the combined model shows a larger extent of the cracking in the upper one third part of the dam and in the heel area, as can be seen by comparing Figure 6.33 to Figure 6.18. This more intensive cracking is due to the compressive damage to the concrete material which causes an increase of the stress amplitudes toward tension.

The envelopes of the maximum and the minimum stress obtained with the combined smeared crack and bounding surface models are shown in Figures 6.34 and 6.35, respectively. Due to the cracking in the concrete material caused by the high tensile stresses in the cantilever direction, the maximum cantilever stresses in the upper part of the downstream face and in the heel area of the upstream face are about 1 MPa lower than the corresponding stresses obtained from the analysis with the linear-elastic concrete model. On the other hand, the high compressive

arch stresses cause damage to the concrete material and stress redistribution toward tension. If the shift of the stress levels toward tension does not exceed the tensile strength limit, then both the minimum and the maximum arch stresses increase. This is the case in dam body near the crest of the crown cantilever and near the abutments on both the upstream and the downstream faces. However, the increase in the maximum arch stress can lead to cracking of the concrete, as in the zones near the joints. As already mentioned, this additional cracking is evidenced by comparing Figure 6.33 to Figure 6.18. In such case, the minimum arch stress decrease because of the reduced load bearing area in the cracked zones.

Figures 6.36 to 6.39 show the arch and cantilever stress time histories for zones B and C of the dam body compared to the corresponding stress time histories obtained in the analysis with the linear-elastic concrete model. Before the initiation of the cracking process in zone C, the stress time histories for the combined model show the same pattern of stress redistribution as the bounding surface model. After the occurrence of the first cracks at this location, which takes place at about 7 seconds from the beginning of the seismic excitation, the stress time histories are affected by both the compressive fracture and the tensile cracking of the concrete material. These observations are clearly evident in the time history of the upstream arch stress in zone C. For convenience, the time histories of these stresses obtained from the analyses by the linear-elastic, the smeared crack, the bounding surface, and the combined smeared crack-bounding surface models are presented in Figure 6.40. As can be seen from Figure 6.40, before the crack initiation, the combined model gives higher upstream arch stress than the linear model. The same trend is observed in the stress time histories obtained by the bounding surface model. The difference between the results obtained by the bounding surface model and the combined model is due to the different formulations for tensile failure employed in the two models. After the crack initiates, similarly to the crack model, the combined model shows lower minimum and higher maximum upstream arch stress, which can be explained by the stress redistribution caused by the cracking on the downstream face of the dam. On the other hand, similarly to the bounding surface model, the combined model shifts

the base level of the stress oscillation toward tension.

The time histories of the stream and the vertical displacements at the crest of the crown cantilever are shown in Figure 6.41. It is observed that the compressive damage and the tensile cracking of the concrete material have a significant effect on the time history of the investigated displacement. This effect is more pronounced after the initiation of the cracking process on the downstream face. Figure 6.41 also demonstrates the significance of the assumption adopted in the evaluation of the tangent stiffness of the dam. Since the tangent stiffness is updated only for the changes of the stiffness of the contraction joints, the time history response of the dam shows lengthening of the apparent vibration period of the dam at the time intervals when large openings occur at the contraction joints, e.g. at 11, 12.5, and 14.5 seconds. The lengthening of the apparent vibration period will remain in effect throughout the seismic excitation, if the stiffness of the dam has been updated to account for the degradation of the concrete material caused by compressive fracture and tensile cracking. Figure 6.41 shows that at all times the dam remains stable.

Table 6.1: Material properties for the concrete of dam

Property	Static	Seismic
Modulus of elasticity, E , (MPa)	26000	32500
Poisson's ratio, ν	0.2	0.2
Unit weight, γ , (kN/m ³)	24.0	24.0
Uniaxial compressive strength, f'_c , (MPa)	25.0	31.25
Associated to f'_c axial strain, ε_p	0.002	0.002
Apparent uniaxial tensile strength, σ'_a , (MPa)	2.0	2.5
Fracture energy of concrete, G_f , (N/m)	200	250

Table 6.2: Material properties for the rock of foundation

Property	Static	Seismic
Modulus of elasticity, E , (MPa)	17240	21550
Poisson's ratio, ν	0.2	0.2
Unit weight, γ , (kN/m ³)	0.0	0.0

Table 6.3: Vibration frequencies of Morrow Point dam in (rad/sec) (After Fenves et al. 1992)

Mode	Symmetry	Empty Reservoir	Full Reservoir
1	Anti-symmetric	20.31	15.73
2	Symmetric	22.34	16.43
3	Symmetric	35.40	22.85
4	Symmetric	37.47	25.00
5	Anti-symmetric	40.40	27.50
6	Anti-symmetric	43.60	31.77
7	Symmetric	48.22	33.50
8	Symmetric	54.12	37.84
9	Anti-symmetric	60.58	42.13
10	Symmetric	60.80	44.13

Figure 6.1: Morrow Point dam, U.S.B.R., 1983

Figure 6.2: Finite element mesh of the half-model of Morrow Point dam

Figure 6.3: Nodes of fluid elements on the upstream face of the dam

Figure 6.4: Acceleration time histories for the input ground motion

Figure 6.5: Pseudo-acceleration response spectra for earthquake ground motion

Figure 6.6: Contours of static stresses (in MPa) in case of empty reservoir

Figure 6.9: Linear-elastic concrete model envelopes of maximum stress (in MPa) in case of empty reservoir

Figure 6.10: Linear-elastic concrete model envelopes of minimum stress (in MPa) in case of empty reservoir

Figure 6.11: Linear-elastic concrete model envelopes of maximum stress (in MPa) in case of half-full reservoir

Figure 6.13: Linear-elastic concrete model envelopes of maximum stress (in MPa) in case of full reservoir

Figure 6.14: Linear-elastic concrete model envelopes of minimum stress (in MPa) in case of full reservoir

Figure 6.15: Zones with investigated response histories

Figure 6.17: Linear-elastic concrete model envelopes of minimum stress (in MPa) for monolithic dam in case of full reservoir

Figure 6.18: Crack pattern evolution in the crown cantilever

Figure 6.19: Smeared crack model envelopes of maximum stress (in MPa)

Figure 6.20: Smeared crack model envelopes of minimum stress (in MPa)

Figure 6.21: Smeared crack model arch stress histories in zone B

Figure 6.22: Smeared crack model cantilever stress histories in zone B

Figure 6.23: Smeared crack model arch stress histories in zone C

Figure 6.24: Smeared crack model cantilever stress histories in zone C

Figure 6.25: Smeared crack model displacement time history at the crest of the crown cantilever

Figure 6.26: Bounding surface model envelopes of maximum stress (in MPa)

Figure 6.28: Bounding surface model arch stress histories in Zone B

Figure 6.29: Bounding surface model cantilever stress histories in Zone B

Figure 6.30: Bounding surface model arch stress histories in Zone C

Figure 6.31: Bounding surface model cantilever stress histories in Zone C

Figure 6.32: Bounding surface model displacement time history at the crest of the crown cantilever

Figure 6.33: Damage evolution in the crown cantilever

Figure 6.34: Combined nonlinear model envelopes of maximum stress (in MPa)

Figure 6.35: Combined nonlinear model envelopes of minimum stress (in MPa)

Figure 6.36: Combined nonlinear model arch stress histories in zone B

Figure 6.37: Combined nonlinear model cantilever stress histories in zone B

Figure 6.38: Combined nonlinear model arch stress histories in zone C

Figure 6.39: Combined nonlinear model cantilever stress histories in zone C

Figure 6.40: Comparison of the downstream arch stress time histories in zone C

Figure 6.41: Combined concrete model displacement time history at the crest of the crown cantilever

Chapter 7

Summary and conclusions

7.1 Summary of the research

This study presents a new numerical procedure and finite element models for the nonlinear seismic analysis of concrete arch dams. The proposed formulation combines a nonlinear joint model and a nonlinear concrete material model to consider the interaction and the coupling effects between the relative movements at the vertical contraction joints and the inelastic actions in the concrete cantilevers of the arch dam. The numerical procedures are implemented in a new computer program CADNAP.

The proposed numerical procedure is applied to the seismic analysis of Morrow Point dam, a typical concrete arch dam analysed in several previous studies. In the present study, the seismic analyses are carried out using the ground motion recorded at the Taft Lincoln School Tunnel during the Kern County earthquake in California on 21 July 1952. It is assumed that the vertical contraction joints of the dam are provided with large and strong shear keys.

7.2 Summary of the results from the case study

The main results from the seismic analysis of Morrow Point dam carried out in the present study are summarised as follows:

- The response time histories of the structure show that the relative movements of the monolith cantilevers of the dam cause opening, closing, and sliding at the vertical contraction joints.

- The opening at the contraction joints prevents the formation of high tensile stress in the arch direction and leads to stress redistribution resulting in increase of the tensile stress in the cantilever direction.
- The closing of the opened joints during dynamic response results in high compressive stress in the arch direction.
- Due to the stress redistribution caused by the relative movements at the vertical contraction joints of the dam, the most important tensile stress quantity becomes the cantilever tensile stress on the downstream face of the dam.
- High tensile cantilever stresses cause cracking in the upper part of the downstream face of the dam. High arch compressive stresses lead to stiffness degradation and stress redistribution in the affected zones of the dam body. The combined effects of compressive damage and tensile cracking result in more severe cracking in the upper parts of the downstream face and propagation of the cracks toward the lower parts of the dam body.
- The stiffness degradation due to high compressive stress leads to change of the configuration of the structure in the post-earthquake period, as compared to the configuration existing before the earthquake.
- The dam remains stable at all times.

7.3 Conclusions

Based on the case study carried out, it is concluded that relative movements at the contraction joints and tensile cracking and compressive stiffness degradation in the cantilever monoliths can significantly affect the behaviour of concrete arch dams subject to strong ground motions. Therefore, the aforementioned nonlinear phenomena should be taken into account in the design of new concrete arch dams and the safety evaluation of existing ones.

The new numerical procedure proposed in the present study enables detailed analysis of the nonlinear seismic response of concrete arch dams. The nonlinear joint

model combined with the nonlinear concrete material model allow for taking into account the relative movements at the vertical contraction joints and the inelastic effects in the concrete of the cantilever monoliths of the dam. The initiation and evolution of tensile cracking of the concrete material is accurately modelled by including a characteristic length parameter in the constitutive model for concrete subjected to tensile loading. The stiffness degradation in the zones of the arch dam with high compressive stresses is traced by the bounding surface model.

7.4 Recommendations for future work

Future work includes the implementation of a more refined concrete model, in which the strain rate effects are taken into account in a consistent manner. The material nonlinearity of the rock foundation should also be considered in the analysis of the arch dam. More research is required for improving the modelling of the interaction between the structure and the impounded water during strong ground motions. The numerical simulation of the effects of water penetration into the open cracks on the behaviour of the dam is another major field for further research.

References

1. T. Anderson. *Fracture Mechanics. Fundamentals and Applications*. CRC Press, Inc., Boca Raton, Florida, 1991.
2. Z. P. Bažant. Recent advances in fracture mechanics, size effect and rate dependence of concrete: Implications for dams. In E. Bourdarot, J. Mazars, and V. Saouma, editors, *Dam Fracture and Damage*, pages 41–54, Rotterdam, 1994. Balkema.
3. Z. P. Bažant and B. Oh. Crack band theory for fracture of concrete. *Materials and Structures*, 16(93):155–177, 1983.
4. S. S. Bhattacharjee and P. Léger. Concrete constitutive models for nonlinear seismic analysis of gravity dams — state-of-the-art. *Canadian Journal of Civil Engineering*, 19:492–509, 1992.
5. S. S. Bhattacharjee and P. Léger. Seismic cracking and energy dissipation in concrete gravity dams. *Earthquake Engineering and Structural Dynamics*, 22:991–1007, 1993.
6. S. S. Bhattacharjee and P. Léger. Application of NLFM models to predict cracking in concrete gravity dams. *Journal of Structural Engineering*, 120(4):1255–1271, April 1994.
7. E. Bruhwiler. Fracture of mass concrete under simulated seismic action. *Dam Engineering*, 1(3):153–175, 1990.
8. CEA. Safety assessment of existing dams for earthquake conditions. Seismic analysis of concrete dams. Technical Report No. 420 G 547, Canadian Electrical Association, Research and Development, 1990. Background Volume C–4.

9. M. Cervera and E. Hinton. Nonlinear analysis of reinforced concrete plates and shells using a three dimensional model. In E. Hinton and R. Owen, editors, *Computational Modelling of Reinforced Concrete Structures*, pages 327–370. Pineridge Press Limited, Swansea, U.K., 1986.
10. M. Cervera, J. Oliver, and O. Manzoli. A rate-dependent isotropic damage model for the seismic analysis of concrete dams. *Earthquake Engineering and Structural Dynamics*, 25:987–1010, 1996.
11. E.-S. Chen and O. Buyukozturk. Constitutive model for concrete in cyclic compression. *ASCE Journal of Engineering Mechanics*, 111(6):797–814, June 1985.
12. R. W. Clough, J. M. Raphael, and S. Mojtahedi. ADAP—a computer program for static and dynamic analysis of arch dams. Technical Report EERC 73-14, Earthquake Engineering Research Center, University of California at Berkeley, June 1973.
13. M. J. Dowling and J. F. Hall. Nonlinear seismic analysis of arch dams. *ASCE Journal of Engineering Mechanics*, 115(4):768–789, April 1989.
14. G. L. Fenves, S. Mojtahedi, and R. B. Reimer. ADAP-88—a computer program for nonlinear earthquake analysis of concrete arch dams. Technical Report UCB/EERC-89/12, Earthquake Engineering Research Center, University of California at Berkeley, November 1989.
15. G. L. Fenves, S. Mojtahedi, and R. B. Reimer. Parameter study of joint opening effects on earthquake response of arch dams. Technical Report UCB/EERC-92/05, Earthquake Engineering Research Center, University of California at Berkeley, April 1992.
16. K.-L. Fok and A. K. Chopra. Earthquake analysis and response of concrete arch dams. Technical Report UCB/EERC-85/07, Earthquake Engineering Research Center, University of California at Berkeley, July 1985.

17. F. Ghrib and R. Tinawi. Nonlinear behaviour of concrete dams using damage mechanics. *ASCE Journal of Engineering Mechanics*, 121(4):513–527, April 1995.
18. F. Ghrib and R. Tinawi. An application of damage mechanics for seismic analysis of concrete gravity dams. *Earthquake Engineering and Structural Dynamics*, 24:157–173, 1995.
19. J. F. Hall and A. K. Chopra. Dynamic analysis of arch dams including hydrodynamic effects. *ASCE Journal of Engineering Mechanics*, 109(1):149–167, February 1983.
20. A. Hillerborg, M. Modr, and P. Peterson. Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. *Cement and Concrete Research*, (6):773–782, 1976.
21. J. Hohberg. *A Joint Element for the Nonlinear Dynamic Analysis of Arch Dams*. PhD thesis, Institut fr Baustatic und Konstruktion ETH Zrich, July 1992.
22. T. Hughes, K. Pister, and R. Taylor. Implicit-explicit finite elements in nonlinear transient analysis. *Computer Methods in Applied Mechanics and Engineering*, 17/18(1):159–182, 1979.
23. ICOLD. World register of dams. Technical report, International Commission on Large Dams, Paris, 1973 and 1984.
24. ICOLD. Earthquake analysis procedures for dams. State of the art. Technical report, International Commission on Large Dams, Paris, 1986.
25. ICOLD. Selecting seismic parameters for large dams. Guidelines. Technical report, International Commission on Large Dams, Paris, 1989.
26. International Engineering Company, Inc. Pacoima arch dam, investigation and evaluation of effects of San Fernando earthquake. Technical report, Los Angeles County Flood Control District, 1972.

27. J. Kuo. Fluid-structure interactions: Added mass computations for incompressible fluid. Technical Report UCB/EERC-82/09, Earthquake Engineering Research Center, University of California at Berkeley, August 1982.
28. S. Lan and J. Yang. Nonlinear finite element analysis of arch dam—II. Nonlinear analysis. *Advances in Engineering Software*, (28):409–415, 1997.
29. D. T. Lau, B. Noruziaan, and G. Razaqpur. Modelling of contraction joint and shear sliding effects on earthquake response of arch dams. *Earthquake Engineering and Structural Dynamics*, 27:1013–1029, 1998.
30. J. Lee and G. L. Fenves. A plastic–damage concrete model for earthquake analysis of dams. *Earthquake Engineering and Structural Dynamics*, 27:937–956, 1998.
31. J. Lubliner, J. Oliver, S. Oller, and E. Onate. A plastic–damage model for concrete. *International Journal of Solids and Structures*, 25(3):299–326, 1989.
32. J. Mays and L. Roehm. Effect of vertical contraction joints in concrete arch dams. *Computers and Structures*, 47(4/5):615–627, 1993.
33. J. Mazars, G. Pijaudier-Cabot, and C. Saouridis. Size effect and continuous damage in cementitious materials. *International Journal of Fracture*, 51:159–171, 1991.
34. B. Noruziaan. *Nonlinear Seismic Analysis of Concrete Arch Dams*. PhD thesis, Carleton University, Ottawa, September 1995.
35. J. Oliver. A consistent characteristic length for smeared cracking models. *International Journal For Numerical Methods in Engineering*, 28:461–474, February 1989.
36. T. Pagnoni, J. Slater, R. Ameer-Moussa, and O. Buyukozturk. A nonlinear three-dimensional analysis of reinforced concrete based on a bounding surface model. *Computers and Structures*, 43(1):1–12, 1992.

37. J. Raphael. Tensile strength of concrete. *Journal of the American Concrete Institute*, 81(2):158–165, 1984.
38. J. G. Rots. *Fracture Mechanics of Concrete Structures. From Theory to Applications.*, chapter 5, pages 138–139. Chapman and Hall Ltd, 1989.
39. D. Row and V. Schriker. Seismic analysis of structures with localized nonlinearities. In *Proceedings of the Eight World Conference on Earthquake Engineering*, pages 475–482, San Francisco, California, 1984.
40. S. Shah, S. Swartz, and C. Ouyang. *Fracture Mechanics of Concrete*. John Wiley and Sons, Inc., 1995.
41. H. Tan and A. K. Chopra. Earthquake analysis and response of concrete arch dams. Technical Report UCB/EERC-95/07, Earthquake Engineering Research Center, University of California at Berkeley, July 1995.
42. R. Taylor. FEAP: A finite element analysis program for engineering workstations. Technical Report UCB/SEMM-92, Department of Civil Engineering, University of California at Berkeley, 1992.
43. United States Bureau of Reclamation, Denver, Colorado. *Design of Arch Dams*, 1977.
44. E. Wilson. SAP, a general structural analysis program. Technical Report UCSESM 70-20, Structural Engineering Laboratory, University of California at Berkeley, September 1970.
45. O. Zienkiewicz and Y. Cheung. *The Finite Element Method in Structural and Continuum Mechanics*. McGraw-Hill, 1967.

Appendix A

Material parameters of the bounding surface model

The expressions presented here for the material parameters of the bounding surface model are obtained in the studies by Chen and Buyukozturk (1985) and Pagnoni et al. (1992) through regression analysis of published experimental data.

A.1 Pre-failure parameters

A.1.1 Generalised elastic shear modulus H^e

The generalised elastic shear modulus relates the elastic component of the deviatoric strain to the deviatoric stress, as in Eq.4.13. The generalised elastic shear modulus is taken as twice as the initial shear modulus G at the beginning of the loading process, i.e. $H^e = 2G$. The initial shear modulus is evaluated as follows:

$$G = \frac{E}{2(1 + \nu)} \quad (\text{A.1})$$

where E is the initial Young's modulus and ν is the Poisson's ratio. For application with the normalised stress and strain quantities, the initial modulus is normalised by the ratio $\frac{\epsilon_p}{f'_c}$:

$$\hat{E} = E \frac{\epsilon_p}{f'_c} \quad (\text{A.2})$$

Similarly, the normalised shear modulus is obtained as follows:

$$\hat{G} = \frac{E}{2(1 + \epsilon)} \frac{\epsilon_p}{f'_c} \quad (\text{A.3})$$

The expression for $\hat{G}$ in Eq.A.3 is valid for both deviatoric loading and unloading.

A.1.2 Generalised plastic shear modulus H^p

The generalised plastic shear modulus relates the plastic octahedral shear strain to the octahedral shear stress, as in Eq.4.15. The generalised plastic shear modulus H_p reflects the accumulated damage and the distance between the current stress point and its projection on the bounding surface. The following expressions for H_p are adopted in the study by Pagnoni et al. (1992):

- For deviatoric loading:

$$H^p = \frac{R}{F_1(\hat{I}_1, \theta)} \frac{2.4(1-D)^{0.65D^2}}{(1+0.7K_{max}^2) A_L} \quad (A.4)$$

$$A_L = 1.02 - 0.81 \frac{K_R}{K_{max}}, \quad K < K_{max} \quad (A.5)$$

$$A_L = 1, \quad K = K_{max} \quad (A.6)$$

where K_R is the value of the damage parameter K at the beginning of the most recent loading process.

- For deviatoric unloading:

$$H^p = \frac{R}{F_1(\hat{I}_1, \theta)} \frac{2.4}{(1+0.7K_{max}^2) A_u} \quad (A.7)$$

$$A_u = 0, \quad K < 0.2K_{max} \quad (A.8)$$

$$A_u = \frac{K_u - 0.2K_{max}}{0.8K_{max}}, \quad K \geq K_{max} \quad (A.9)$$

where K_u is the value of the damage parameter K at the beginning of the most recent unloading process.

A.1.3 Bulk tangent modulus K_t

The bulk modulus K is a modulus of elasticity introduced by a triaxial compression test in which the same compressive loading $\sigma_{11} = \sigma_{22} = \sigma_{33} = p = \frac{\sigma_{kk}}{3}$ is applied on all surfaces of a cubic sample. The bulk modulus K is defined as the ratio between the hydrostatic pressure p and the corresponding volume change ϵ_{kk} .

The volumetric stress-strain relationship of concrete is non-linear. Therefore, it is necessary to introduce a tangent modulus K_t . Under compressive hydrostatic loading, the material exhibits softening behaviour characterised by a decrease of K_t with increasing I_1 . With further increase of I_1 , however, the material stiffens indicated by an increase of K_t with increasing I_1 . Hence, the following expressions are derived for the normalised K_t

- For hydrostatic loading:

$$\hat{K}_t = \frac{E}{3(1-2\nu)} \frac{\epsilon_p}{\left(1 + 0.358 \hat{I}_1^{1.5}\right) f'_c} \quad (\text{A.10})$$

- For hydrostatic unloading:

$$\hat{K}_t = \frac{E}{3(1-2\nu)} \frac{\epsilon_p}{f'_c} \quad (\text{A.11})$$

A.1.4 Shear compaction-dilatancy factor β

Compaction

Compaction is the result of shear crushing and void reduction. It occurs when $K = K_{max}$. The shear compaction factor is denoted by β_1 and is given by the following expressions:

$$\beta_1 = 1.1e^{-30(K_{max}-0.6)^2}, \quad K = K_{max} \quad (\text{A.12})$$

$$\beta_1 = 0, \quad K < K_{max} \quad (\text{A.13})$$

Dilatancy

The increase of the volume of material subjected to compressive loads is termed dilatancy. Experimental results show that under compressive loading, inelastic volume contraction occurs at the beginning of yielding and volume dilatation occurs at about 75% to 90% of the ultimate stress. Shear dilatancy can occur during loading, unloading, and reloading process. The shear dilatancy factor is denoted by β_2 and is given by:

$$\beta_2 = -1.97\lambda e^{-2\lambda^2} \quad (\text{A.14})$$

where:

$$\lambda = D - 0.2K_{max}^2 \quad (\text{A.15})$$

The combined shear compaction–dilatancy factor β is obtained by summing β_1 and β_2 :

$$\beta = \beta_1 + \beta_2 \quad (\text{A.16})$$

A.2 Post-failure parameters

A.2.1 Shear modulus G

To avoid numerical problems, a negligibly small value, instead of zero, is assumed for the normalised shear modulus after failure has occurred.

A.2.2 Generalised plastic shear modulus H^p

$$H^p = -0.15e^{-0.025(K_{max}-1)^2} F_2(\hat{I}_{1,max}) \quad (\text{A.17})$$

where:

$$F_2(\hat{I}_{1,max}) = \frac{0.14}{(\hat{I}_{1,max})^2 - 0.86}, \quad \hat{I}_{1,max} \leq 2.54 \quad (\text{A.18})$$

$$F_2(\hat{I}_{1,max}) = 0.025, \quad \hat{I}_{1,max} > 2.54 \quad (\text{A.19})$$

A.2.3 Bulk tangent modulus K_t

$$\hat{K}_t = \frac{E}{3(1-2\nu)} \frac{\epsilon_p}{f'_c} \quad (\text{A.20})$$

A.2.4 Dilatancy factor β

$$\beta = -1.97\lambda e^{-2\lambda^2} \quad (\text{A.21})$$

where

$$\lambda = 1 - 0.2K_{max}^2 \quad (\text{A.22})$$