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"Downflow-Induced Flow Distortion and Its Influence on the
Performance of a Vertical Axis Lifting Fan", submitted by
Dw. Walter Schaub, in partial fulfilment of the requirements for
the degree of Doctor of Philosophy in Mechanical Engineering.

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CROSSFLOW-INDUCED FLOW DISTORTION
AND ITS INFLUENCE ON THE PERFORMANCE
OF A VERTICAL AXIS LIFTING FAN

by

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B.Sc., Queen's University, 1959

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A thesis submitted to the Faculty of Graduate
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ABSTRACT

VTOL lifting fans experience extremely large crossflow distortions during transition manoeuvres from fan-supported to wing-supported flight, yet are expected to perform both stably and efficiently. It is the purpose of this study to ascertain the character of the flow distortions involved and their influence on fan performance, and to devise a complete analytical model for simulating the flow distortions and the fan through-flow.

It is evident from the results of wind tunnel studies involving simple suction wing and powered fan-in-wing models that common flow mechanisms are responsible for considerable similarities in the inlet distortions. However, there are also fundamental differences which can be associated with the fan's reaction to inflow and back-pressure distortion effects.

The crossflow induced inflow distortion, being mainly a distortion of the potential flow field, appears to be well represented analytically by means of a polar lattice vortex potential flow model. The crossflow induced distortion at the fan's exit plane is shown to be primarily a circumferential non-uniformity in back-pressure. It is simulated by a three-dimensional network of vortex lines attached to the surface of the jet plume, whose centreline trajectory is described by an experimentally determined functional relationship.

The fan is simulated by a collection of constant flow area fan segments, each occupying a fraction of the total annulus

area. Mean line average through-flows are calculated for the fan segments by use of standard cascade loss and deflection data. The fan model provides a solution for imposed inlet distortion swirl and radial velocities and back-pressure conditions.

The segment through-flows are matched with the external flow by adjustments of the back-pressure distribution to suit the interaction between the crossflow and the whole efflux jet. The solution to the whole fan-in-crossflow problem is obtained for a specified fan geometry, speed and crossflow condition on coupling together the above three analytical models and converging iteratively on an operating point.

The analysis demonstrates that the fan segments operate with constant speed characteristics that are different from one another and from that of the complete fan by virtue of the fact that they depend mainly on the local inlet velocity distortion. The analysis predicts the circumferential distribution of performance variables and indicates, for example, that the advancing fan blades (relative to the crossflow) achieve larger axial velocities, total pressures and temperatures than the retreating blades, and that the advancing fan blade Mach numbers are greatest.

The fan model can be improved on relaxing the segment constant axial area constraints, thereby facilitating the inclusion of the previously omitted direct crossflow distortion of the axial velocities. A valid balance of areas in any axial plane can be assured on the basis of geometric compatibility, force equilibrium and continuity requirements; but, even a highly

relaxed form of equilibrium requires a knowledge of radial velocities within the machine. These may be estimated as being induced by the blading and wall bound vortex lines and the generated vorticity. In order to satisfy the equilibrium requirements considerable modifications have to be made to the calculation method; but, incorporating area change on the basis of geometric compatibility and continuity alone, while being less accurate, is not difficult to do.

In spite of the complexity of the aerodynamic problems involved, the simple analytical model represents the more important crossflow distortion effects quite well and requires no more input information than is normally available. It is amenable to improvements and, above all, it is simple to use.

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LIST OF SYMBOLS

A	Area (ft^2)
A_1, A_2	Constant coefficients incorporating surface normals
$\vec{a}$	Acceleration (ft/sec^2)
a_r, a_θ, a_z	Component acceleration magnitudes in the r, θ, z directions, resp. (ft/sec^2)
a_1, a_2	Constant coefficients
B	Coefficient determinant
B_1, B_2	Constant coefficients incorporating surface normals
b	Length dimension along fan axis (ft)
$\vec{b}$	Length vector
b_{ji}	Influence functions
b_1, b_2	Constant coefficients
C	Magnitude of velocity in absolute co-ordinates (ft/sec)
$\vec{C}$	Velocity in absolute co-ordinates (ft/sec)
C_1	Integration constant
C_D	Drag coefficient
C_{DP}	Blade profile drag coefficient
C_{DA}	Annulus boundary layer loss coefficient
C_{DS}	Secondary flow loss coefficient
C_L	Lift coefficient
C_r, C_θ, C_z	Component velocity magnitudes in the r, θ, z directions, resp. in the absolute reference frame (ft/sec)
c	Blade chord length (ft)
c_p	Specific heat at constant pressure ($\text{BTU}/\text{lbm-R}^\circ$)
D	Fan diameter (ft), also the substantial derivative and drag force (lbf)

D_r	Hub-tip radius ratio
d	Infinitesimal change in a quantity
e	Specific internal energy (BTU/lbm- R^0), also used for exponent
F	Force (lbf)
g_o	Constant of proportionality: $g_o = 32.17 \left[\frac{\text{lbm-ft}}{\text{lbf-sec}^2} \right]$
h	Blade height (ft)
h	Specific enthalpy (BTU/lbm- R^0)
i	Cartesian base vector, symbol also used to indicate imaginary number, fan blading incidence angle (deg.), and radial counter, also an alternate counter for j
J	Joule's constant: $J = 778.17$ (ft-lb/BTU)
j	Cartesian base vector, symbol is also used to denote radial index counter for $\Delta\phi$ elements
k	Column Matrix, also reduced frequency: $k = \frac{V_\infty}{W}$
k	Cartesian base vector
L	Lift force (lbf)
λ	Length of vortex filament (ft), also primary distortion wave length (radians)
$\ln$	Naperian logarithm
M	Mach number, also lattice coefficient
$\dot{m}$	Mass rate of flow (lbm/sec)
N	Surface normal ratio: $N = n_o/n_r$, also Yan R.P.M.
$\hat{n}$	Unit surface normal
n_r, n_θ, n_z	Components of the unit surface normal
P	Pressure (lbf/ft 2), also point in space
Q	Amount of heat changed (BTU)
q	Dynamic pressure (lbf/ft 2), also any quantity on a Wu surface

R	Radius of curvature (ft); radius of fan, as specified in text (ft); gas constant, as specified in text $\left(\frac{\text{lb-ft}}{\text{lbm-R}^\circ}\right)$
$\vec{R}$	Position vector (ft)
R_L	Bellmouth lip radius divided by fan tip radius
r	Radial co-ordinate distance (ft)
r_o	Outer radius of jet (ft)
r, θ, z	Cylindrical co-ordinates
σ, σ_2	Stream surfaces
$S(K)$	Sears' function
s	Specific entropy (BTU/lbm-R ^o), also distance along a streamline (ft) and blade pitch-spacing (ft)
T	Temperature (°F, °R, °K, as defined)
$T(K)$	Horlock's function
t	Time (sec), also blade thickness (ft)
U	Wheel speed of fan (ft/sec)
u	Unsteady velocity component parallel to the blade chord (ft/sec)
u_o	Modulus of u (ft/sec)
V	Velocity (ft/sec)
V_{x1}	Average axial segment velocity (ft/sec)
V_{∞}	Crossflow velocity (ft/sec)
v	Unsteady velocity component parallel to the blade chord (ft/sec)
v_o	Modulus of v (ft/sec)
W	Velocity relative to blade (ft/sec)
w_m	Mean velocity relative to blade (ft/sec)

x, y Cartesian co-ordinates relative to the blade chord
 x, z Cartesian co-ordinates in the crossflow and fan axial flow directions
 L Axial length of blade row (ft)
 ∇ Gradient

Greek Alphabet

α Angle of attack or air angle, as indicated in text (Deg.)

β Blade metal angle (Deg.)

γ Ratio of specific heats

Γ Circulation (ft^2/sec)

Δ Change in a quantity

δ Incremental change in a quantity

δ^* Displacement thickness of the boundary layer

η Quantity related to entropy change

θ Circumferential co-ordinate distance (Deg. or Radians)

$\dot{\theta}$ Blade stagger angle (Deg.), or time rate of position change (ft/sec)

N Number of sides in vortex polygon and number of fan segments

ω Circular frequency of the disturbance (radians/sec)

ρ Density (lbm/ft^3)

s Blade surface distance (ft)

ϕ Inlet swirl angle, or vortex filament angle (Deg.)

ψ Sears' and Horlock's transient function data

ψ Stream function

ψ, ψ_T	Sears' and Horlock's transient function data
Ω	Rotor angular speed (sec^{-1})
ω	Vorticity magnitude (sec^{-1})
$\omega_r, \omega_\theta, \omega_z$	Vorticity components in the r, θ, z directions, resp. (sec^{-1})

Subscripts

AV	Averaged quantity
a	Averaged axial component
D	Based on drag
i	Alternate index for influence functions, also a radial index
j	Circumferential index functions
L	Based on Lift
m, mean	Mean average
a	Denotes the amplitude of a fluctuating quantity
P	At constant pressure
r, θ, z	Denotes the radial, circumferential, and axial directions
s	At the blade surface
k	Relative to moving reference frame
s	Denotes a static condition
t	Designates a total or stagnation condition
u	Due to velocity fluctuation parallel to the mean velocity direction
v	Due to velocity fluctuation perpendicular to the blade chord. Also denotes association with blade or vane

Denotes a static condition in the undisturbed approach stream

Station upstream of a blade row

Station downstream of a blade row

At the inlet of the rotor blade row, also denotes a stream surface

At the exit of the rotor blade row, also denotes a stream surface

At the inlet of the stator blade row

At the exit of the stator blade row

Superscripts

A vector quantity

An unsteady condition, also used to denote a relative quantity

A rate of change

An instantaneous quantity

An average quantity

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CHAPTER 1

INTRODUCTION

1.1. Definition of Problem

The general attractiveness of lifting fans to serve as thrusters for V.T.O.L. aircraft has been well recognized⁽¹⁾. Such axial fans, with the fan axis nominally perpendicular to the flight direction, pose novel and important aerodynamic problems that must be understood. While lifting fans are expected to perform efficiently, they must also provide adequate distortion tolerance in order to assure stable operation for all transition manoeuvres from vertical to horizontal flight, and vice versa, especially those responsible for extremely high distortion levels.

A large amount of experimental test data already exists on the effect of approach velocity (crossflow) on flow distortion and fan performance;* but most of this experience involves model tests because full scale experiments tend to be expensive and difficult to carry out. The character of the fan-in-crossflow distortion problem is not well understood and analytical simulations of this difficult problem have not yet been achieved.

Inflow distortion problems associated with conventional compressor intakes, usually expressed in terms of total pressure, are not new. A survey of published information in this important area was made by Dunham⁽²⁾. An examination of the serious compressor inflow distortion problems reveals that there are basically two types: unsteady distortions, normally associated with inlet turbulence, and spatial or steady state distortion.

* One such relevant work by the author is given in Appendix E.

It has been shown⁽³⁾ that the principal destabilizing influence associated with inlet turbulence is a substantial increase in total pressure distortion for a period of time equal to compressor response time, i.e. of the order of one revolution. Inlet turbulence, therefore, produces a time variant spatial distortion in total pressure. The second type of distortion is a steady state distortion involving total pressure defects (intake loss) that are large in scale and area, relative to flow unsteadiness. The reported^(3,4) inherent tendency of compressors to induce relatively uniform velocity regardless of local circumferential total pressure defects prevents transfer of existing compressor experience to lifting fan applications where the character of the flow distortions is fundamentally different.

Most lifting fan configurations involve "axially thin" installations with typical axial lengths of the order of half a fan diameter. While with the short inlet bellmouths involved it is apparently possible now to design bellmouth lip contours for non-separating boundary layers under crossflow conditions⁽⁵⁾, the walls generally are incapable of providing adequate guidance for the entering flow and a very large residual momentum remains at the fan inlet face⁽⁶⁾. The primary inflow distortion problem takes on a totally new dimension in lifting fans: Instead of involving a total pressure loss (as in the compressor inlet) it now involves a severe maldistribution of velocity at essentially constant total pressure⁽⁷⁾. The distortion is, in fact, a "potential flow distortion". As in the case of compressor inflow distortions, lifting fans are probably apt to experience time

variant spatial distortions in addition to spatial distortions, depending upon the presence and severity of inlet turbulence. The spatial distortions, being of the order of the crossflow ratio $\left[\frac{V_{\infty}}{V_a} \right]$, can be very large and, therefore, insight into their mechanisms and interactions with the fan is considered to be fundamental.

The significance of velocity distortion is most easily visualized by considering its polar components. In this form distortion comprises axial, swirl and radial distortion velocities. Of these only the first two feature direct influence on fan performance because they affect rotor blade incidence. The third relates to spanwise flow over the fan blading and hence is involved with radial flow adjustments inside the fan.

In addition to causing inlet distortion problems the crossflow interacts directly with the efflux jet, which emerges from the fan's exit plane, and causes another very important distortion in the exit plane. This distortion takes the form of a circumferential nonuniformity in back-pressure.

Unlike the multi-stage axial compressor, the lifting fan typically features only a single stage and often little separation between the rotor and stator. It can, therefore, not be expected to achieve the same amount of circumferential internal flow redistribution. As a result, the fan is subjected to both distortions simultaneously and is not capable of attenuating the circumferential inlet distortions to the same degree.

It is the combined influence of inflow and outlet distortions that is responsible for the cyclically fluctuating

load on the rotor blading. Interactive effects cause fan performance to deviate substantially from the axisymmetric flow case.

1.2. Outline of Related Work

1.2.1. External and Internal Flow Models

There is now a considerable background in potential flow theory involving external flows and entrance duct flows. Only an extensive literature survey could come close to listing the more valuable contributions. One excellent method was devised by A.M.O. Smith and colleagues at Douglas Aircraft⁽⁸⁾. This method involved distributed sources and sinks by means of which body shapes and surface pressures were generated. Rubbert and Saaris of Boeing Aircraft pioneered a similar method using sources and doublets⁽⁹⁾. A whole wing with two fans and their jet wakes was modelled by representing the surfaces as arrays of panels, each containing a source. The wing wake, fan-in-wing face and wake were simulated by doublet distributions. The jet efflux trajectory was assumed from experimental measurements. Independently, Monical, also of Boeing, simulated the fan-in-wing configuration by means of a three-dimensional vortex lattice network⁽¹⁰⁾. The non-planar wing was represented by a network of horseshoe vortices with bound elements attached to constant percent chord and span lines. The trails followed the surface, then became parallel to the local flow. The fan efflux wake, whose trajectory also was shaped according to measurements, was a vortex tube comprising segments of square cross-section. Efflux

flow was induced by flaring the tube some distance below the wing. These latter methods appeared to predict successfully the aerodynamic forces and moments on the model due to external flow, but the fans were badly represented and consequently the results were not as satisfactory as hoped for.

There are other works relating to entrance and internal flow involving vortex rings, but these studies related only to aligned flow conditions. Schultz-Piszachich determined the inflow profile and surface pressure distribution on an engine cowl by means of a distribution of vortex rings⁽¹¹⁾. Dyne published a method for the design of ducted propellers in a uniform approach flow using a semi-infinite sheet of vortex rings⁽¹²⁾. The outer shroud and jet stream were simulated by a semi-infinite distribution of ring vortices, and the propeller and its wake were represented by a distribution of vortex lines.

There have been a multitude of studies involving the interactive flow field of a jet issuing from a surface parallel with an oncoming crossflow. Many of the recent works are to be found in the publications of NASA and AIAA. The entrainment or mixing flow theories appear a little tentative at this time and, consequently, there is considerable emphasis on potential interaction of the two fluid streams with entrainment neglected. The calculation of base pressures on the basis of a two-dimensional curved impervious jet appear to give reasonably good results, as reported by Bechert⁽¹³⁾.

1.2.2. Fan and Compressor Through-Flow Models

A large amount of useful work has been accomplished with

various compressor through-flow models. The many independent approaches feature a great variation in the simplifying assumptions and mathematical procedures employed in obtaining solutions. The published work has been initially classified into two fundamentally different solution groups. One group represents those methods that characterize the inlet flow distortion in terms of a small perturbation of the main flow. The reduced problem is then solved using the known steady state performance of the compressor and approximating the compressor in terms of actuator disks. The second group represents all methods which consider the general equations directly and seeks a solution for the through-flow on stream surfaces. These latter methods generally take into account streamline curvature terms and are especially well suited for representing detailed internal flows. They are also more accurate in dealing with large nonuniformities.

It is to be noted that at present there is no method in the published literature that makes possible the calculation of the highly three-dimensional flow in a lifting fan.

1.2.2.1. Perturbation Methods.

Small disturbance theory has been applied to the problem of flow distortion in axial compressors with considerable success. The merit of this method lies in the fact that the mathematical method is relatively straightforward, as it bears resemblance to thin airfoil theory, and that the predictions compare quite well with measurements for an important and large class of problems. The application that this method has been associated with in the published literature generally involves axial compressors that

feature a constant static pressure field downstream of the last blade row and that are subjected to small amounts of total pressure distortion in the inlet.

It seems feasible that the small perturbation method could be extended to admit velocity distortion in the inlet and a nonuniform but known static pressure field at the compressor exit.

Small perturbation methods are expected, however, to give incorrect results when the inlet distortion is large, i.e. when the fluctuating component is of the same order as the mean value, as is frequently the case for velocity distortions in the inlets of lifting fan configurations.

Since the small perturbation method is relevant to the general problem of inlet distortion it is of interest to indicate a sampling of some appropriate existing theories.

Seidel's work in this field is typical of an early application⁽¹⁴⁾. He dealt with a single stage large hub-tip ratio machine and assumed incompressible two-dimensional flow. By rearranging the linearized forms of the equations of motion and continuity he was able to devise three novel equivalent forms that demonstrated constancy of total pressure along a streamline and the satisfaction of the Cauchy-Riemann conditions. He made a modified actuator disk analysis in terms of these new "potential theory functions" using a mean blade force and two unsteady force contributions, the Sears' and the Isaac's forces.

A more recent and extensive effort has been expended by Pratt and Whitney Aircraft and Lehigh University. Plourde and

Stenning⁽¹⁵⁾ made a small perturbation analysis of a two-dimensional incompressible flow field upstream, within, and downstream of a high hub-tip ratio multi-stage compressor which is subjected to a small circumferential total pressure distortion in its inlet. They demonstrated that for this high hub-tip ratio "long" compressor (one for which it is assumed that the pressure downstream is constant) there was a considerable amount of flow redistribution upstream of the compressor inlet plane. The overall attenuation of both total pressure and axial velocity distortions was primarily dependent upon the slope of the pressure rise versus flow rate characteristic, and that it would increase if the slope were more negative. This analysis was extended by Callahan⁽¹⁶⁾ for a low hub-tip ratio compressor where the flow was expected to be three-dimensional. Callahan compared the previous solution by "unwrapping" the annulus (i.e. approximating the field with a cartesian reference frame) with an analysis using a cylindrical coordinate system and found that serious errors in the rates of attenuation of distortion result if such a simplification is made. In comparing the attenuation rates of compressors featuring constant pressure just downstream and far downstream of the stage exit plane he showed that only 50% of the attenuation of distortion and hence smaller radial and crossflow velocities resulted upstream of the compressor for short compressors as opposed to long compressors. The generality of Callahan's theory is limited, however, because it admitted only small distortions and small pressure rise compressors; it required constancy of static pressure far downstream of the exit plane; and also limiting was

the fact that the distortion wave was centered along the forward and leeward sides of the intake - a limitation that disallows any amount of circumferential mass transfer.

1.2.2.2: Stream Surface Solution Methods

The general procedure followed by methods in this group is to seek a solution of the equations of motion in their most general forms on defined stream surfaces. This approach has met with considerable success under axisymmetric flow conditions in simple duct flows, compressors, and in turbines.

Because of the great difficulties encountered in finding a truly three-dimensional mathematical solution the usual expedient resorted to was to reduce the problem to mathematical two-dimensionality. In practice such a reduction was achieved by solving the relevant equations on defined stream surfaces. In certain cases the stream surfaces are naturally axisymmetric and hence this simplification was automatically in force. Even so, the mathematical solutions were not simple and generally led to the stage where one or more non-homogeneous first or second order non-linear differential equations had to be solved. It is mainly in the handling and in the solution of these equations that differences arise between various authors^(17,18,19). Wu published a highly detailed general physical formulation⁽¹⁷⁾, later Marsh adapted this method for calculations through use of matrix solution techniques⁽²⁰⁾.

1.2.3. Segmented Fan and Compressor Concept

Reid⁽⁴⁾ appears to have been first to propose the concept

of the segmented compressor in which the complete unit is imagined to act as a collection of individual elemental compressors, all working in parallel and each accepting a local inflow distortion. Beeler and Przedpelaki, concerned with inlet distortion problems in lifting fans proposed an analogy between inlet distortions and the velocity field generated by an imaginary set of perfect inlet guide vanes⁽²¹⁾. Goethert and Kimzey⁽²²⁾, as part of a study of high frequency fluctuations in the inlet flow of multi-stage axial compressors, used the concept of parallel compressors together, with shared inlet, interstage and outlet reservoir volumes.

1.3. Object and Scope of this Study

An appropriate analytical model representing the fan and the basic crossflow distortion mechanisms can achieve three things. It can aid greatly in promoting an improved understanding of the interrelationship between the fan and its crossflow environment. By characterizing the distortions and representing the fan through-flow the model can aid the extrapolation of full scale effects from test data. Finally, the model can act as part of a vehicle for an analytical simulation of a complete V.T.O.L. aircraft in computer studies of flight performance optimizations.

The highly complicated flow distortions and internal fan flow adjustments make it unrealistic to seek a proper three-dimensional through-flow solution. The present approach offers a convenient and simple analytical model comprising a segmented fan with simulated inflow and outlet distortions. This model, while not representing the character of all known flow phenomena,

does provide a reasonable approximation of the problem. The calculated flow distortions and fan performance are compared with corresponding experimental measurements involving a 12-inch diameter fan-in-wing model and reasonable agreement has been achieved.

Possible logical extensions of the basic method leading to an improved simulation are indicated, but it is realized that their inclusion would necessarily introduce much greater complexity in the calculation method.

1.4 Organization of the Report

The work presented is organized into five parts, as follows: inlet plane flow distortions; outlet plane flow distortions; segmented fan model; complete simulation of inflow distortion, segmented fan, and outlet plane distortions; extension theories; and additional experimental work.

The fundamental flow mechanisms of crossflow induced inflow distortion in simple annular inlets with inlet axis perpendicular to the crossflow and lifting fan inlets are examined in Chapter 2. An analytical model for predicting inlet distortion velocities under crossflow conditions is developed and compared with experimental measurements.

The character of crossflow induced outlet plane flow distortion is studied in Chapter 3, and an analytic jet plume model representing the primary distortion effects is developed. The generated back-pressure distribution is compared with standard potential flow model and measured static pressures in the outlet

plane of a lifting fan mounted in a wing.

Chapter 4 deals with the development of a simple "segmented fan" analytical model for calculating performance and mean radius flow distribution for given inlet and outlet plane flow distortions. The limitations and implications of assuming each fan segment to occupy equal fractions of the annulus area at each calculation plane are also considered. The calculated fan performance is compared with experimental data at zero cross-flow. Methods are developed to assure the matching of externally imposed flow distortion and the fan through-flow. The flow distributions generated by the analytical segmented fan model subjected to crossflow are compared with experimentally measured ones for the same experimental distortion inputs.

The calculation procedure of the complete fan model with self-generating inlet and outlet distortions is outlined in Chapter 5, and various aspects of solution convergence are discussed. Comparisons are then made under typical crossflow conditions between the inlet and outlet flow distributions and performance as predicted by the complete model and the measured data obtained from a lifting fan under the same crossflow conditions.

Chapter 6 deals with the development of appropriate extension theories for incorporating fan segment area change from one calculation plane to the next. The proposed methods differ from one another only in the techniques for calculating compressor segment area in each calculation plane.

Simple improvements in the inlet and outlet plane

distortion generating models are suggested in Appendix A.

Some experimental work, published earlier by the author⁽⁷⁾ and having direct relevance to the present work, is given in Appendix E. This work deals with some wind tunnel tests involving a powered fan-in-wing model at various transition conditions. The model, experimental method, and tunnel interference corrections are discussed.

The computer programs of the analytical models are not included in this report, but can be obtained from the National Research Council of Canada, Division of Mechanical Engineering.

CHAPTER 2

INFLOW DISTORTION

2.1. Causes of Inflow Distortion

Inlet flow distortion, in the absence of fan feedback effects, comprises three basic phenomena. First there is simple sink flow from still ambient air.

It features contracting streamlines in the inlet and well down in a long intake duct the streamlines become parallel. Correspondingly, the velocity after increasing from zero tends to a constant final inflow value. Secondly, since the inlet has a definite geometric shape, far different from the mathematical sink concept of a singularity, the inflow is subjected to wall caused flow curvature and results in near surface velocities being larger than velocities far away. Both simple and annular inlet flows feature these two effects. In the case of the annular inlet both the outer and hub walls cause flow curvature and this

FIGURE 1. General flow curvature effect

FIGURE 2. Flow curvature effects in an annular inlet

results in the "cusped" velocity profile sketched in Figure 2. Thirdly, when crossflow is imposed on the normal inlet configuration the fluid originates in the crossflow and bends round the leading lip corner into the inlet.

This gives rise to an additional overall front-to-back corner flow problem: the streamlines are more closely spaced near the corner around which the flow bends than away from it, see Figure 3.

FIGURE 3. Crossflow/inlet flow streamline pattern

The passage velocity profile is evidently dependent on crossflow ratio, inlet surface geometry and inflow distance. While a lifting fan in close proximity to the inlet reacts to the collective effect of the velocity profile, in the absence of flow separations only the front-to-back flow bending phenomenon may properly be construed as crossflow distortion.

2.2. Appearance of Inflow Distortion

2.2.1. Swirl Velocity

The tangential velocity component in the plane of the fan is called swirl, and is defined as being positive when it is counter to the direction of fan rotation, see Figure 4.

The basic character of swirl distribution may be explored by means of a simple artifice. The simplest possible representation of this character would reflect only the sense of

the velocity perturbation on the mean flow. However, in order to show that this perturbation is as large as the crossflow itself, it is represented by a double plus and a double minus, see

Figure 5.

FIGURE 4. Inlet swirl due to crossflow

The casing surface,

being an "internal" surface with respect to the wetting flow, provides more effective flow deflection than the hub surface, which by the same definition is an "external" surface and is, therefore, capable of providing greater three-dimensional relief.

FIGURE 5. Unit representations of distortion swirl effects.

The crossflow residual is therefore reduced more at the outer wall. This variable effectiveness in providing flow guidance can be imagined to induce a cellular motion flow perturbation, as indicated in Figure 5. Simple superposition of the two individual unit form contributions yields a combined unit form representation, again as seen in Figure 5. The character of the measured distribution appears to be well represented, as can be seen on comparison with the measured swirl distribution in a simple suction inlet (through which air was drawn by plant suction machinery far away

FIGURE 6. Swirl velocity C_θ/V_0
in a simple suction
inlet, $s=D/6$, $V_0/V_0 = 0.27$, $V_0 = 212$ ft/sec

from the inlet) under crossflow conditions, Figure 6. The undulating distribution with respect to circumference angle of the measured distortion swirl velocity is clearly noticeable. Swirl magnitude was at a maximum at the sides of the inlet, particularly near the hub, where the guidance was least and the residual was nearly 100% of the crossflow velocity.

FIGURE 7. Swirl velocity C_θ/V_0
in a fan inlet, $s=D/7$,
 $V_0/V_0 = 0.24$,
 $V_0 = 515.0$ ft/sec

This cyclic variation of inlet swirl velocity was also observed in the inlet of a lifting fan, as Figures 7 and 8 show. There were discernible differences, but these were clearly of a minor nature: 1) a small phase shift of the distribution in the direction

of fan rotation (probably a rotor blading upwash effect), and
 2) second order differences in magnitude at the hub and outer wall sides. Swirl distortion is expected to exert a powerful influence on the fan because it is directly involved in causing cyclic swings in rotor blade incidence.

FIGURE 8 Circumferential distribution of measured inlet distortion swirl velocity of mean radius. Fan speed = 12,000

The influence of crossflow ratio on the circumferential swirl profile at the mean radius is shown in Figure 8: swirl velocity increased with crossflow ratio. The general likeness of the two swirl distributions (suction inlet and fan inlet) and the obvious dependence of swirl on crossflow ratio are taken as sufficient evidence to suggest that swirl velocity, and therefore

also crossflow ratio, is one prime "independent" distortion variable that should be considered in off-design fan performance.

The effects of inlet swirl on the rotor inlet velocity triangle are suggested in Figure 9 for a hypothetical case of

FIGURE 9. Hypothetical velocity diagram showing the swing in incidence at constant axial velocity.

constant axial velocity. The advancing blades face counter-whirl (positive swirl) and consequently experience incidence excursions towards the positive. The retreating blades are subjected to the opposite, moreover because of triangle

geometry these swings towards the negative are likely to be larger

for the same swirl velocity magnitude. Since outlet angle deviation in unstalled flow is much less than incidence variation (but always in the same sense) one can estimate the influence of distortion swirl on axial velocity, as is shown in Appendix B. Equation B-50, derived in Appendix B, indicates that positive distortion swirl causes axial velocity to increase. At the same time, rotor incidence becomes less positive than it would have been had axial velocity remain constant. Negative distortion swirl has, of course, the opposite effect. Axial velocity increases also, and rotor incidence becomes less positive, if back-pressure is reduced.

The zero crossflow ideal flow angles can, therefore, not be maintained at crossflow conditions, and at the same time,

incidence angle swings are less than would be expected if axial velocity were to remain unchanged, see Figure 10.

FIGURE 10. Effective velocity triangles incorporating adjustment of axial velocity due to change in incidence

The lifting fan backpressure is highly non-uniform due to mutual interaction between the crossflow and efflux jet. The short axial length of the lifting fan is insufficient to isolate the rotor from this exit plane condition. Consequently, actual axial velocity adjustments

and rotor incidence deviations cannot be predicted on the basis of simple suction inflow distortion data alone.

The fan feedback effect suggested in Figure 10, which tends to reduce axial velocity on the retreating blade side and to increase it on the advancing blade side, is the type velocity perturbation necessary for correlating the simple suction

FIGURE 11. Unit representation of fan feedback on axial velocity due to distortion swirl

axial velocity data with the measured lifting fan data. The proposed swirl-induced fan feedback effect may be characterized in unit form by indicating the sense of the correction, as shown in Figure 11.

2.2.2. Radial Velocity

Radial velocity in the inlet plane is generated by the combined effects of flow contraction and crossflow residual.

The zero crossflow and crossflow effects on radial velocity may be represented in unit form.

Denoting radial inflow as plus and radial outflow by minus, the contraction effect of zero crossflow is indicated in Figure 12.

The crossflow residual features

radial inflow at the front, zero

at the sides and radial outflow at the rear of the annulus, as

FIGURE 13. Unit representation of radial velocity residual due to crossflow in a simple suction inlet

FIGURE 12. Unit representation of radial velocity in a simple suction inlet at zero crossflow

is suggested in Figure 13. The

combined result is the superposition of these two, and is

shown in Figure 14. In actual

fact, the radial velocity must be

zero at the walls and at the

middle of the annulus. Appropriate

representation of this basic

requirement in Figures 12 and

13 would have led to a greater

likeness between the result shown in Figure 14 and the measured distributions.

A measured distribution in the simple suction inlet under crossflow conditions is shown in Figure 15. The difference

FIGURE 14. Unit representations of radial velocity effects

in the measured magnitude and distribution of radial velocity between the crossflow and zero crossflow cases was considerable, as can be seen on comparing Figure-15 with Figure 16. Radial

FIGURE 15. Radial velocity C_r/V_0 in a simple section inlet, $S=D/8$, $V_0/V_0 = 0.27$, $V_0 = 212$ ft/sec

FIGURE 16. Radial velocity C_r/V_0 in a simple section inlet, $S=D/8$, $V_0/V_0 = 0.00$, $V_0 = 212$ ft/sec

velocities were a minimum in the middle of the annulus, small but positive (flow towards the centre) near the outer wall, and small and negative (radial outflow) near the hub.

The zero crossflow data depicted the character of the flow contraction effect; clearly the inlet velocity profile was still developing at a depth of $D/8$.

Some measured radial velocities in the inlet of a lifting fan are shown in

FIGURE 17. Radial velocity C_r/V_a
in a fan inlet, $s=D/7$
 $V_a/V_a = 0.24$, $V_a =$
518.9 ft/sec

Figure 17. Since radial velocities in a fan imply spanwise flow on the blades they have no first order effect on performance.

Consequently, the radial velocity distribution ahead of the fan was not fundamentally different from that measured in the simple suction inlet, Figure 15.

Although the swirl and radial unit form distortions were devised separately, there is in fact a simple coupling between the swirl and radial distortion velocity distributions. As shown in Appendix D, although highly distorted the inflow is actually irrotational, and because the z -vorticity (as well as the r and θ -vorticities) is zero the swirl velocity distribution is directly related to the radial velocity distribution. This interdependency is demonstrated by both the combined unit form results and the experimental data, and does, in part, explain why the swirl and radial distortions in the fan inlet are not basically different from those in the simple suction inlet.

2.2.3. Axial Velocity

The flow curvature effects, associated with stream tube contraction discussed earlier, cause near surface velocities to be larger than velocities at the middle of the passage. The salient features of this distribution, i.e. the sense of the

perturbation on the mean axial velocity, may be depicted in unit form, as Figure 18 indicates.

FIGURE 18. Unit representation of axial velocity, simple suction inlet at zero crossflow

As crossflow increases the axial velocity distribution features more and more the effect of the flow curvature associated with flow bending into the inlet from the flight direction.

Excluding the flow phenomena near the wall, the axial velocity in simple suction flow was quasi-two

dimensionally disposed with magnitudes decreasing from the front

to the rear of the inlet, as shown in Figure 22. The effect of crossflow bending towards the axial direction alone is depicted in unit form in Figure 19. The mean radius circumferential profile of velocity perturbation due to bending into the inlet is also indicated.

FIGURE 19. Unit representation of pure flow bending due to crossflow on axial velocity

The complete axial velocity distribution (including both flow contraction and bending)

is obtained by superpositioning the individual representations given in Figures 18 and 19, as well as the uniform distribution of average axial flow. The individual perturbations may be

assigned with unit magnitudes of ± 0.1 and the mean flow with a magnitude of 1.0. Figure 20 shows the superpositioning operation and the result. This assembly of basic flow phenomena does, in

FIGURE 20. Superposition of unit representations: total distortion of axial velocity in a simple inlet due to crossflow

effect, explain the character of the measured axial velocity distributions.

FIGURE 21. Axial velocity C_a/V_a in a simple suction inlet, $s=D/8$, $V_a/V_a = 0.00$, $V_a = 210$ ft/sec

The flow contraction effect was clearly shown experimentally with crossflow velocity being zero or near zero, see Figure 21. The distribution of axial velocity was axisymmetric and featured a 'cusped' profile (with radius). The crossflow case, depicted in Figure 22,

demonstrates (as the unit representations suggested) the prominent front-to-back variation in velocity.

The lifting fan inflow was observed to feature a similar distribution, but in addition the retreating blades

FIGURE 22. Axial velocity C_a/V_a in a simple suction inlet, $s=D/8$, $V_a/V_a = 0.27$, $V_a = 212$ ft/sec

FIGURE 23. Axial velocity C_a/V_a in a fan inlet, $s=D/7$, $V_a/V_a = 0.24$, $V_a = 518.8$ ft/sec

experienced lower axial velocities than the advancing blades, as shown in Figure 23. This side to side difference, as previously discussed, is attributed to the fan feedback effect due to distortion swirl (see Figure 11). The skewing of the axial

FIGURE 24. Superposition of unit representations; total distortion of axial velocity in the inlet of a lifting fan

velocity distribution in the counter rotational direction is depicted schematically in Figure 24 and is evidently a result of the superposition of individual unit forms. The quality of comparison with the measured distortion data (Figure 23) is evident.

The degree in which axial velocity distortion varied with crossflow ratio is illustrated by the mean radius circumferential profiles of measured axial velocity in Figure 25. Crossflow tended to increase the axial velocity perturbation, but the influence was not as great as it was on swirl velocity, see Figure 8.

FIGURE 25. Circumferential distribution of measured inlet axial velocity at mean radius, fan speed = 11,000 R.P.M.

2.3. Proposed Analytical Model for Inflow Distortion

The apparent ease of superpositioning elementary distortion effects to create inflow distributions that are consistent with measured distortion data suggests that perturbation swirl, radial, and axial velocities should be considered in formulating an analytical distortion model. A separate analytical fan model, capable of accepting the generated inflow distortion, can then be employed to produce the mean axial flow as well as any feedback effects resulting from performance under the imposed inlet and exit conditions.

The generation of inflow distortion by means of a potential vortex model is a logical extension of the work done by Dyne⁽¹²⁾, who modelled successfully the entrance effects of an inlet aligned with the approach flow, and Monical⁽¹⁰⁾ who modelled the external flow over a three-dimensional lifting surface not aligned with the flight vector. A simple vortex model was devised for generating specifically the distortion velocities subject to entrance effects of nonaligned approach flows. The particular vortex model choice can be rationalized by examining several simple potential flow examples.

A two-dimensional vortex pair inducing flow from quiescent surroundings is illustrated in Figure 26. As can be seen, all fundamental aspects of flow curvature are represented. Clearly, the irrotational nature of pure inflow is not violated by the vortex model, provided the vortex cores are excluded from the region of interest. A case of plane approach flow, normal to an inducing two-dimensional vortex pair, is shown in Appendix A.

The region between the vortex cores exhibits contracting streamlines, as is expected for accelerating inlet flows approaching on axis. However, when the two-dimensional vortex pair lies in the plane of flow the induced flow

FIGURE 26. Two-dimensional vortex pair in quiescent surroundings

exhibits the basic features of crossflow distortions. Appropriate illustrations are presented in Appendix A. These elementary models demonstrate the presence of all necessary ingredients for constructing a meaningful, albeit more complicated, three-dimensional inflow distortion model.

A vortex ring lying in the plane of approach flow, the three-dimensional equivalent of the two-dimensional vortex pair, is more appropriately related to inflows involving bellmouths. As will be shown shortly, there are advantages in using a regular polygon ring instead of a circular ring. The Biot-Savart law for induced velocity was applied to the ring and to each line segment of the inscribed polygon, and a comparison was made of the centreline induced velocity in the plane of the vortex models, both of identical strength. As can be seen from Figure 28, the centreline

Biot Savart law:

$$\mathbf{v} = -\frac{\Gamma}{4\pi} \int \frac{\mathbf{r} \times d\mathbf{l}}{r^3}$$

FIGURE 27. Induced velocity due to an element dl of a ring vortex

Induced velocity of the polygon approaches the ring induced velocity asymptotically as the number of polygon sides increase, e.g. the inscribed eleven sided polygon features a centreline velocity only 1% in excess of that generated by the ring.

The induced velocity field in the neighbourhood of the eleven sided vortex lattice is compared with Dyne's exact solution⁽¹²⁾ for a uniform vortex ring in Appendix A.

FIGURE 28. Induced velocity along the axis of symmetry in the plane of the ring

Annular inflows can be modelled by means of concentric co-planar vortex rings. As sketched in Figure 29, two rings are capable of generating collectively the wall contraction effects and provide a continuous velocity profile of the required cusped shape. The fields external to the outer ring and internal to the inner ring are of no interest since the relevant flow field is restricted to the annular gap.

FIGURE 29. Simulation of outer and hub wall-flow curvature effects.

The effect of inflow distortion due to crossflow can be generated by assigning varying vortex strengths to the polygon sides: increased vortex strength at the front and a reversed vortex strength at the rear relative to the approach flow direction. In order to satisfy the Helmholtz vortex law, shed vorticity is introduced, and the resultant vortex configuration becomes a polar

lattice of horseshoe vortex elements. The bound elements produce both radial and axial induced velocities. Apart from satisfying the Helmholtz condition, the trails serve a vital second function: they induce radial and swirl velocities in the inlet, reduced radial velocities and increased swirl velocities at the sides of the bellmouth lip and hub, and increased radial velocities and reduced swirl velocities at the front and the rear of the inlet.

FIGURE 30. Polar lattice of horseshoes representing crossflow distortion

The induced radial velocities are shown schematically in Figure 31.

FIGURE 31. Variable strength vortex polar lattice necessary to satisfy the boundary condition at the walls in the inlet of a lifting fan.

In order to assure that the surface radii of curvature are approximately correct and the surface velocities remain bounded, the core lines of the bounded vortex elements are placed to be

congruent with the locus of the flare radii of the bellmouth and centrebody lips, as shown in Figure 32.

The calculating procedure is to divide the outer lip and hub surfaces into μ panels and to assign to each a horseshoe vortex of unknown strength. A suitable trail length, typically

FIGURE 32. Location of vortex horseshoe and control point relative to the inlet

several fan diameters long, is then specified. 2μ control points are chosen on the surfaces of the inlet where a boundary condition has to be met. These control points may conveniently be located in the fan inlet plane.

The radial component of the induced velocity due to all vortex

horseshoes is then calculated at each control point in terms of influence functions and unknown vortex strengths. This gives the left hand side of a set of 2μ equations involving 2μ influence function coefficients and 2μ unknown vortex strengths. The boundary condition requires that there be no flow through the wall. This condition has to be met when there is crossflow. Expressed in vector notation the boundary condition is:

$$\hat{n} \cdot (\vec{V}_\infty + \vec{C}) = 0 \quad (1)$$

where $\hat{n}$ is the unit normal of the wetted surface and is always directed towards the flow. In polar coordinates the condition requires that:

$$V_\infty \cos \theta_j + C_{rj} = 0 \quad (2)$$

The set of equations is completed with the boundary condition making up the right hand side.

$$\begin{aligned}
 b_{11}\Gamma_1 + b_{12}\Gamma_2 + \dots b_{1i}\Gamma_i + \dots b_{12\mu}\Gamma_{2\mu} &= k_1 \\
 \vdots & \\
 b_{j1}\Gamma_1 + b_{j2}\Gamma_2 + \dots b_{ji}\Gamma_i + \dots b_{j2\mu}\Gamma_{2\mu} &= k_j \\
 \vdots & \\
 b_{2\mu 1}\Gamma_1 + b_{2\mu 2}\Gamma_2 + \dots b_{2\mu i}\Gamma_i + \dots b_{2\mu 2\mu}\Gamma_{2\mu} &= k_{2\mu}
 \end{aligned} \tag{3}$$

where $k_j = -V_\infty \cos \theta_j$

b_{ji} = the influence function at control point "j" due to horseshoe "i" and is one element of many making up C_{rj} .

The above set of equations $B\Gamma = K$ has a unique solution because the coefficient determinant B is non-zero and the matrix is square. The solution is the inverted matrix: $\Gamma = B^{-1}K$.

With the vortex strengths (Γ_i) now known it is possible to solve for the induced velocity at any field point by calculating the appropriate influence functions and using the found vortex strengths.

The calculated axial velocity field has no net (axial) inflow because the distortion field generated gives only velocity perturbations. To obtain the complete axial velocity distribution, it is necessary only to add the required bulk velocity, or a more realistic axial velocity profile reflecting flow contraction and fan feedback effects.

2.4. Comparison of Analytical Results with the Experiment

For comparison purposes an inflow distortion was generated using an eleven sided vortex polar lattice for a cross-flow ratio (V_∞/V_θ) equal to 0.33. The lip radius was set at 0.09D, and the vortex trail length at 5D. A greatly increased

trail length did not change the results significantly since the vortex strengths would adjust to suit. The boundary condition was met and the distortion velocities were calculated in the inlet plane, i.e. at an inflow depth of $D/8$. Axial location of the boundary condition was not critical within a depth of $D/3$ as far as velocities were concerned, since the strengths of the horseshoe vortices would adjust to suit; however, with a single plane polar lattice model, radial velocities at the wall can only be zero in the plane where the boundary condition is satisfied.

The calculated radial velocity distribution, shown in Figure 31, at first glance does not resemble those measured in the simple suction model (Figure 15) or the fan inlet (Figure 17). The apparent discrepancy cannot be accounted for in terms of the small differences in crossflow ratio. The distribution is, however, what one expects from a simple unit form representation if the requirement for zero velocity at the walls had been included. The calculated vortex-induced distribution (Figure 33) represents, in fact, only the crossflow distortion radial velocity perturbation since no mechanism was introduced in the distortion model to simulate normal inflow contraction effects, such as depicted in Figure 12.

It would, of course, be a relatively simple matter to include such effects, and one possible technique showing how to proceed is

FIGURE 31. Radial velocity C_r/V_a induced by the inlet vortex lattice, $z=D/8$, $V_a/V_a=0.11$, $V_a=208$ ft/sec

indicated in Appendix A. The distributions of generated swirl and axial velocities for the same vortex lattice model are shown in Figure 24. Although bulk axial velocity is not a requirement for the solution of the inflow model, the example offered here features a bulk velocity corresponding to a crossflow ratio of 0.33, and was the most appropriate available at the time of writing. The calculated data do, in fact, represent, as a first approximation, the characteristics of the measured distortions in the simple suction inlet, see Figures 6 and 22. At the cost of a little more complexity in the vortex lattice model, further refinements in the inlet distributions and their axial distance

Swirl velocity C_s/V_a

Axial velocity C_a/V_a

FIGURE 24. Distributions of velocity induced by the inlet vortex lattice, $s=D/8$, crossflow ratio $(V_\infty/V_a)=0.33$, $V_a=206$ ft/sec

developments are possible by pursuing one of the extensions to the theory, as suggested in the Appendix A.

The inflow model described meets all the requirements imposed by the general distortion through-flow calculation method. In fact, it is only necessary to use the generated swirl and radial velocity perturbations; the axial velocity perturbations are intermixed with the fan feedback effects and are not used as an input to the overall through-flow calculation program.

CHAPTER 3

OUTLET PLANE DISTORTION

3.1. Causes and Appearance of Outlet Plane Distortion

The dominant variable in controlling the constant speed performance of axial flow stages with axisymmetric inflow is the relative level and distribution of back-pressure. Even with ideal inflow conditions the back-pressure determines to a great degree the flow through the machine, and hence its axial velocities and blading incidence. Lifting fan configurations, which feature typically only a single stage, have a relatively short axial length in order to minimize frontal area under cruise conditions. Such configurations virtually assure that the pod or wing surface pressure distribution and the pressure field resulting from the mutual interaction between the crossflow and the fan efflux impose themselves directly on the exit plane of the fan. The variations of back-pressure around the periphery of the efflux jet under crossflow conditions are expected to influence fan performance to a variable degree in the circumferential direction because of the short axial length available for internal flow redistribution. The need for considering the distribution of back-pressure is, therefore, essential.

A measured static pressure distribution in a plane just downstream of the stator exit plane for a crossflow velocity of 127 ft/sec and a crossflow ratio of 0.24 is shown in Figure 34. The data are strongly influenced by blade to blade variations and

were not corrected for wind-tunnel interference and compressibility effects.

The pressure recovery on the leeward side of the efflux jet implies that the wake left by the jet in the crossflow tends to be much smaller than would be expected if the jet acted like a

bluff body. Swirl velocity measurements made in the same plane indicate that the whole efflux jet rotated reasonably uniformly, see Figure 36. A qualitative resemblance is evident between the measured pressure distribution around the circumference of the jet and that calculated for a potential flow involving a rotating cylinder in crossflow meeting the same crossflow and tangential velocity conditions.

The observed flow distribution in the exit plane suggests that the static pressure distribution is the combined result of several interactive effects, the more prominent ones being:

- 1) crossflow over a long cylindrical "body" essentially perpendicular to the approach flow,
- 2) The Magnus effect associated with rotation of the jet boundary, that results in a reduction in wake width,

FIGURE 35. Back-pressure ($\Delta P_0/q_\infty$) in the fan exit plane, $V_\infty/V_{A0} = 0.24$, $q_\infty = 19.1 \text{ PSF}$

FIGURE 36. Swirl velocity (C_θ/V_{A0}) in the fan exit plane, $V_\infty/V_{A0} = 0.24$, $V_{A0} = 166.7 \text{ ft/sec}$

- 3) the trajectory of efflux flow due to the convection of the jet in the crossflow,
- 4) the presence of an end wall parallel to the crossflow and from which the jet issues,
- 5) end wall surface curvature,
- 6) the entrainment and mixing flow processes involved in developing the cross-section of the jet and the scavenging of the wing boundary layer,
- 7) the capability of the jet's fluid boundary to support pressure gradients and the internal structure of the jet,
- 8) the base flow phenomenon downstream of the fan hub,
- 9) the induced effects due to the presence of shed vorticity from the fan.

The interactions between these effects are highly complex, not well understood, and no realistic analytical models exist. The fluid boundary of the jet, while effecting a pressure recovery in the lee of the jet, cannot support the full magnitude of pressure gradients associated with the potential flow Magnus effect. The efflux jet is far from being homogeneous in the exit plane, as shown in Appendix E; the total pressure and velocity distributions are highly nonuniform, not only because the work done by the fan is not constant around the circumference and along the radius, but also because strong wakes are being shed by the fan. While the stator wakes assume, at least initially, well ordered helical trajectories, the rotor wakes are badly chopped up and generate considerable turbulence. The jet is presumed to experience a substantial amount of internal mixing as

both the well ordered and the decimated wakes decay, particularly early on, when the shear stresses are high. Some distance downstream of the exit plane the wakes become no longer identifiable and the effects of the mixing processes at the jet boundary due to entrainment begin to influence the internal structure of the jet. The interaction between the crossflow and the jet results in a deformation of the jet's cross-section, and the jet spreads more easily when it has an initial swirl. This complicated flow field is not amenable to easy analytical modelling, and warrants considerable study on its own.

The present purpose for seeking a model of the jet plume is to obtain a representative back-pressure distribution for the fan under crossflow conditions. As will be demonstrated in Chapter 4, a detailed two-dimensional distribution of static pressure in the exit plane of the fan is actually not required because the proposed fan through-flow model makes use of locally constant properties.

The primary requirement is, therefore, to devise an approximate static pressure distribution around the circumference of the jet, and this, as will be shown, obviates an involvement with the detailed structure of the jet, unless second order improvements are sought. The compromise made is to model only what are thought to be the first order effects. This is consistent with the practice of other investigators, who were faced with the same problem^(9,10). The result of such a simplified approach will, of course, be compared with experimental measurements.

Following Monical's example, use was made of experimentally measured no swirl jet core centreline trajectories. This expedient,

along with the assumption that end wall surface curvature is apt to be a second order effect if the jet orifice is small relative to the surface radius of curvature, makes possible an analytical model of the efflux jet that meets the first four of the nine effects. The remaining five are, of course, not insignificant, but no method suggests itself for their inclusion at the present time.

3.2. Proposed Analytical Model for the Outlet Plane Pressure Distortion

The proposed model, like the inlet polar lattice, comprises a network of horseshoe vortex elements. The boundary condition that there be no flow through the jet "wall" is satisfied at each vortex panel under crossflow conditions. Whereas Monical⁽¹⁰⁾ actually simulated a fan wing by means of a horseshoe network, the model proposed here is a simpler one, which neglects the wing lift and thickness problems and treats the wing as an image plane. Unlike the inlet model, however, the jet model is piece-wise curved or swept, its centreline lying on the measured trajectory used by Monical. The trajectory was expressed in non-dimensional form:

$$z/D = \left[(V_a/V_m)(x/D)^{0.4/1.2} \right]^{1/1.11} \quad (4)$$

The horseshoe network used, sketched in Figure 37, features "bound" elements forming a polygon in each cross-sectional plane with trails lying in the surface of the jet plume and extending fully to an empirically defined extremity. The surface of the plume is therefore generated by a polar family of nested horseshoes. The whole vortex network in real space is reflected in the image plane in order to overcome unrealistic end effects at the exit plane. The vortex strengths depend on the

FIGURE 37. Efflux jet vortex lattice

jet dynamic pressure error in the original measurement would lead to an uncertainty of unity (100% of free stream dynamic pressure) in the plotted pressure coefficients. Moreover, the shown distribution is influenced by the effects of a thick vane at $\theta = 90^\circ$, which houses the drive shaft, and has not been corrected for local tunnel constraints. For these reasons this distribution is considered only approximately representative of free air conditions, i.e. the minimum pressure coefficient is somewhat too negative.

If the distributions A and B are compared, it can be seen that the pressure maxima and minimum are approximately similar.

FIGURE 50. Circumferential distribution of measured exit swirl velocity at mean radius, $V_\infty = 108.0$ ft/sec., fan speed = 1,335 R.P.M.

crossflow and average efflux velocities, efflux circulation, number of sides of the plume's polygon cross-section, radius at which the pressures are to be generated (in the $z = 0$ symmetry plane of the vortex model) and the number and spacing of the cross-sectional planes bounding the plume segments.

7.3. Shortcomings of the Vortex Model

When one simulates the surface of a plume with a distribution of finite vortex elements, one important problem arises with the shape of the "surface" generated. The surface has waves or undulations near the vortex elements. The effect of these undulations in the far field are minimal; but near the surface there are local diffusion effects due to flow past depressions in the surface and accelerations over the waves. The reduced velocities in a depression that are implied by the expanded streamline spacing are shown in Figure A.9. This undulating flow problem can be overcome in two ways. If the number of surface singularities is increased, then in the limit the problem is eliminated; however, this leads to very large vortex arrays and greatly increased computer time. The second possibility is really more of an expedient than a solution. In evaluating pressure coefficients at some distance from the surface the local irregularities are no longer felt. For the present eleven sided vortex polygon it is necessary to use a radius of 1.1 of the polygon circumscribing radius (i.e. the jet plume's outer radius) before a reasonable amount of attenuation has taken place, see Figure 38.

A second problem with the vortex model results at zero crossflow. This problem is a mathematical one involving the matrix solution. It can be overcome if needed by means of the simple geometric expedient, as proposed by Monical⁽¹⁰⁾, to change the diameter of the plume far downstream. In fact, this problem

will not occur because there is no point in calculating the vortex plume model under zero crossflow conditions.

A third problem is associated with all cases involving zero efflux swirl. While there is efflux circulation the actual wake in the lee of the fan's jet plume is narrow, and consequently the potential flow model can be counted on to give a reasonable simulation of the inviscid flow field effects. Without swirl the actual wake is large and highly disorganized, and a good simulation is far more difficult. The vortex model can, however, be adapted with minor geometric modifications to represent gross blockage effects.

3.4. Comparison of the Jet Plume Vortex Model Results with the Experiment

The quality of the vortex model simulation of the jet plume may be assessed by means of comparing circumferential pressure distributions in Figure 39. Several distributions are shown: two experimental distributions A and B, calculated from efflux measurements in the exit plane of the lifting fan; a pressure distribution generated by a doublet with circulation (curve C); and two vortex plume generated pressure distributions for the same circulation, curves D and E.

The experimental distributions of pressure coefficient shown are for a crossflow velocity of 106.9 ft/sec and a tip Mach number of 0.468, this corresponds to a fan speed of 9995 R.P.M. and a crossflow ratio of 0.26. Each point on curve A represents the average of five pressure samples across the annulus

and measurements were made by means of five 5-tube pressure probes inside the efflux stream. Curve B represents the pressure distribution at the outer radius of the efflux jet. Although care had been taken to avoid stator wake flows, the field was

FIGURE 10. Circumferential pressure distributions at the fan exit plane, $V_0 = 10.5$ ft/sec., $V_0 / V_{tip} = 0.75$

still highly nonuniform and quite unsteady. The difficulties of measuring static pressures generally, and particularly in this manner, are well known, as is evidenced here by the scatter of the data. While independent measurement of a few surface pressures on the underside of the fan wing near the efflux verified that the shown distribution was reasonable, the fact remains that a 6%

jet dynamic pressure error in the original measurement would lead to an uncertainty of unity (100% of free stream dynamic pressure) in the plotted pressure coefficients. Moreover, the shown distribution is influenced by the effects of a thick vane at $\theta = 90^\circ$, which houses the drive shaft, and has not been corrected for local tunnel constraints. For these reasons this distribution is considered only approximately representative of free air conditions, i.e. the minimum pressure coefficient is somewhat too negative.

If the distributions A and B are compared, it can be seen that the pressure maxima and minimum are approximately similar.

FIGURE 10. Circumferential distribution of measured exit swirl velocity at mean radius, $V_\infty = 100.0$ ft/sec., fan speed = 1,101 R.P.M.

The outer distribution (curve B) is, however, not representative of the mean pressure and it appears that pressure decreases gradually with decreasing radius. The difference between the two curves is not large, in fact, it is within the uncertainty of measuring errors. While it cannot be explained easily in terms of measured swirl velocities, as Figure 40 shows, the difference may be attributed in part to flow curvature effects in the meridional plane, i.e. the flow may be closing in downstream of the centrebody. An assumption can be made that static pressure is constant radially within an error band of two free stream dynamic heads.

The assumption that static pressure inside the efflux jet in the exit plane is uniform radially and the calculation of the circulation at the mean radius instead of the outer radius are necessary in the general fan through flow method developed here. As will be shown, fan performance is, in fact, calculated only at the mean radius and without uncertain further assumptions about radial distributions (in addition to the implied free vortex flow in the free jet) only mean line data are available.

The doublet in crossflow distribution (curve C) has been calculated for a relative uniform circulation strength $\left(\frac{\Gamma}{2\pi r_o V_\infty}\right)$ of 1.06, which relates to the test crossflow condition and the estimated circulation based on fan mean radius and measured mean radius efflux swirl velocities. While the basic form looks very similar to the measured pressure distribution, the magnitudes of the pressure coefficient over the interval $0 \leq \theta \leq \pi$ are exaggerated.

The vortex model, curve D, is seen to produce at a radius ratio of 1.1, the same general pressure distribution form as the measured one, but the magnitudes are significantly underestimated. One may, therefore, ask two questions: 1) since both the doublet and the vortex plume are potential flow models, why is there such a large difference between them, and 2) how can the difference between the vortex plume result and the experimental distribution be explained and resolved?

On the first question, the fact that the vortex plume underestimates is attributed to the wall undulations. This difference can be minimized as explained earlier. While the quality of the comparison with the doublet model is only of academic interest, that with the measured back-pressure distribution is highly important.

As curve E demonstrates, much of the discrepancy between the vortex plume model and experiment can be accounted for in terms of the value of circulation used, for example, a uniform increase in circulation reduces the error considerably. But how valid is the assumption of uniform circulation in the first place? An examination of the circumferential distribution of exit swirl velocity provides some insight.

Figure 40 is a typical circumferential distribution of relative exit swirl at mean radius. The shown nonuniformity, excluding the atypical effect of the thick stator blade at $\theta = 90^\circ$, is about 15% of the average, with maxima at the sides of the fan, and is evidence of the degree of isolation of individual stator passage flows. It is not apt to persist very far downstream of the

exit plane because of strong mixing action, and may conceivably introduce a second order modification (with the same phase relationship as the swirl distribution) in the assumed uniform jet efflux circulation.

The nonuniformity in swirl velocity can be allowed for, and a considerable improvement in the vortex plume pressure distribution can be made possible simultaneously, if the uniform circulation is augmented with a second order correction proportional to $\cos 2\theta$. How this may be done is suggested in Appendix A.6.

Additional improvements in the pressure distribution may be realized if the previously neglected interaction effects could be modelled analytically in terms of trigonometric or other functional relationships involving n_0 , with $n \geq 1$. The vortex model is amenable to incorporating some or all of these effects if they can be represented analytically. The requirement for flow not to penetrate the plume surface can, for instance, be relaxed in order to admit a specified entrainment flow.

The present form and use of the vortex model jet plume does not provide highly visible advantages over the doublet model as far as generating back-pressure is concerned; but the vortex modelling technique does

FIGURE 41. Degeneration of jet wakes

offer considerably greater potential for extension. It has, for example, inherently the necessary mechanism to represent the rolling up of the jet wake far downstream of the model, as depicted in Figure 41. The jet plume model, together with the analytic fan model would also be more suitable for simulating the external aerodynamics of whole lifting fan aircraft configurations.

CHAPTER 4

ANALYTICAL MODEL OF THE SEGMENTED FAN

4.1. Choice of Performance Model - The Segmented Fan Concept

With the analytic models of the crossflow distortion available to serve as boundary conditions for the lifting fan, the analytical modelling of matched fan performance may be considered next. This model must meet the following requirements: 1) it should respond to the flow distortions imposed upon it, 2) it should feature the inherent freedom to find its own operating point, and 3) it must produce valid feedback on the external flow.

Any accurate solution for the complete fan-in-crossflow problem should include also the mutual induction effects of the efflux, inlet, and fan flows. The relative importance of mutual inductions increases as the number of compressor stages decrease and becomes most significant for free rotors where the advance ratio $\left(\frac{V_a}{U}\right)$ is less than unity. Yet this nonlinear problem has never been solved for even the most simple cases involving axisymmetric ducted propeller flows.

The inlet and outlet distortion patterns are remarkable for their complexity. These, together with the mutual induction effects and the intricate three-dimensional internal flows, make it most difficult to devise a proper and tractable through-flow solution.

The approach chosen is relatively simple, and while many of the above requirements have been relaxed, the method does admit

the two basic building blocks: the character of steady-state cross-flow distortions and a variable fan performance. Both of these have already been introduced in the form of unit representations in Chapter 2.

One finds from compressor experience that little attenuation of circumferential distortion is possible if the available interstage volume (here proportional to the separation between rotor and stator) is small and the machine has a low hub tip ratio with considerable spanwise variation in blade camber⁽⁴⁾. These conditions appear to be well met in the "axially thin" lifting fan. In fact, in the fan-in-wing configuration, where the rotor stator spacing is a minimum, internal flow redistribution in the circumferential direction is so highly constrained that each blade passage flow is effectively isolated circumferentially from its neighbouring through-flows.

Consequently, Reid's parallel compressor concept⁽⁴⁾ appears highly appropriate here, and the whole fan may be viewed as a segmented fan comprising a collection of elements, all working in parallel. This theory does not upset the presumed greater ability of the fan blading to redistribute the flow radially, as dictated by the radial pressure gradients that probably exist within the fan.

This concept enables the calculation of mean performance of each fan segment in an assumed isolation of its neighbouring elements. Each fan segment accepts an externally imposed local inflow distortion and exhausts to a uniform back-pressure that is determined by the interaction of the total efflux with the crossflow.

The shortcomings of deducing fan segment performance on the basis of mean line calculations are very evident; however, in view of the many unsolved distortion flow phenomena involved, it would be unwise to seek a more complicated method initially.

4.2. The Mean Line Design Method

4.2.1. General Overview

The general analytic through-flow model is basically a simple mean line constant flow area direct design method. The fan model is completely defined if the following input data are specified: stage geometry (outer fan diameter, hub/tip radius ratio and blading details) and performance figures (in the form of listings of experimentally determined cascade deflection and loss data). Specification of fan segment axial flow areas, inlet total temperature and pressure, inflow distortion velocities, back-pressure and fan speed defines the aero-thermodynamic through-flow problem. Provided that the problem does not fall outside the range of known cascade data, a solution can be obtained either for a specified rotor incidence angle (in which case back-pressure becomes a dependent variable) or for a specified back-pressure with rotor incidence finding its own value. The former type of solution is useful in generating compressor characteristics which are unique for the particular inflow distortion velocities imposed. The latter type of solution yields operating points and may be double valued, depending upon the level of back-pressure and the nearness of the operating point to the maximum of the pressure characteristics.

The actual procedure used in the analytical program is to calculate through-flows for a set of rotor blade incidence angles over a preselected range. A new set of incidence angles are calculated for a reduced range, which brackets the prescribed back-pressure condition, and new through-flows are computed. The process of reducing incidence steps and thereby converging on the prescribed back-pressure is continued until a specified iteration tolerance is met.

4.2.2. Implications of Constancy of Axial Flow Area with Axial Distance

When each fan segment is assumed to have a constant axial area, and only the solutions on the negative slope sides of the constant speed fan characteristics are admitted, each segment mass flow is uniquely determined for any given set of fan speed, distortion swirl and radial velocities, and either back-pressure

or rotor incidence angle (but not both) conditions. This fact can be deduced by viewing Figure 42, and reasoning as follows.

Assume for the moment, that the radial distortion velocity C_r is negligible. If the rotor incidence is a free parameter, then it can be seen that $\dot{C}_{\theta 1}$ is also "free to slide vertically", i.e. perpendicular to the axial direction.

However, as soon as "1" is

FIGURE 42. Implication of constant axial flow area on inlet velocity triangle

specified the axial velocity, and hence the mass flow, is also completely determined. With the inlet velocity triangle now completely defined, the fan will realize a unique back-pressure value according to its appropriate cascade deflection and loss characteristics. Alternately, if a given back-pressure is imposed, the incidence angle, and hence axial velocity, will be determined exactly. This is true for any constant area fan segment with any number of blade rows. The presence of radial distortion velocities does not alter the above relationship because these velocities are instrumental only in modifying the static conditions, and take no part in the blading performance.

It is evident, therefore, if inflow distortion swirl and radial velocities and backpressure are imposed on a constant area fan segment that associated axial distortion velocities (see Figure 19) are redundant since the problem is implicitly stated without them. In order to make use of the axial velocity perturbations generated by the inflow distortion model, it is necessary to have another independent variable, such as a flow area that changes from one axial plane to the next. The presence of the additional independent parameter would make it possible to adjust axial velocity (and with it also rotor incidence angle), while meeting the same back-pressure condition. This point is demonstrated alternately by continuity considerations. If mass flow changes from the above constant area requirement, while meeting the same back-pressure conditions, there must be associated changes in outlet area.

To accommodate area change increases the complexity of

the through-flow fan segment model considerably, as will be explained in Chapter 5. The present analytical method is limited, therefore, to developing a fan segment model based on constant area, and therefore, the axial inflow distortion velocities, illustrated symbolically in Figure 19, cannot be included. Indications are given in Chapter 5 and Appendix B on how a variable area segmented fan model may be formulated.

4.2.3. Blade Row Performance

The aerodynamic performance of the fan blading can be calculated by means of listed sets of available subsonic speed cascade deflection and loss data. These data sets are given in Appendix B. Flow deflections can be calculated either by Howell's nominal deflection method⁽²¹⁾ or directly by using NGTE data. Really appropriate data are not available and a compromise must be made between the Howell method, which accounts for solidity and camber effects, but not Mach number; and a direct method using NGTE data, which accounts for Mach number, but not solidity and camber (camber is approximately correct for the test fan).

4.2.4. Segment Performance Calculations

The aero-thermodynamic calculations are standard, embodying the well known concepts of cascade mechanics: conservation of mass, momentum, moment of momentum and energy across each blade row. Indications are given in Appendix B where and how changes in flow area may be incorporated. Heat losses are assumed to be negligibly small. Temperatures are moderate, and constant values for specific heat are assumed throughout. All calculations

are made on the specified mean line on the basis of one-dimensional constant area flow upstream and downstream of each blade row.

The following parameters have to be calculated at the inlet face:

$$D_m = \frac{D}{2} (D_r + 1) \quad (6)$$

$$s/H = \frac{\pi}{n_{rot}} \left(\frac{1 + D_r}{1 - D_r} \right) \quad (7)$$

$$U = \frac{\pi}{60} D_m N \quad (8)$$

where α , β , i , D_m , D , D_r , s , H , n_{rot} , N are the air, blade metal and incidence angles, the mean and fan diameters, the hub/tip radius ratio, the blade pitch and height, the number of rotor blades and the fan R.P.M., respectively.

A rotor blade incidence i is chosen, and together with the blade inlet metal angle and the initial value of inlet swirl C_{θ} , the inlet velocity triangle can be evaluated.

$$\alpha_1 = \beta_1 + i \quad (9)$$

$$C_{z1} = \frac{U + C_{\theta 1}}{\tan \alpha_1} \quad (10)$$

The inlet swirl angle ϕ is:

$$\phi = \tan^{-1} \frac{C_{\theta 1}}{C_{z1}} \quad (11)$$

The presence of radial velocity in the inlet plane to the rotor blading has no first order importance on blading performance

since it contributes only to spanwise flow. Consequently, velocity triangle relationships can be derived in the absence of radial velocities. The trial absolute inlet velocity (disregarding radial velocities) is:

$$C_1 = \frac{C_{z1}}{\cos \phi} \quad (12)$$

The corresponding relative blade velocity affecting blade row performance and its swirl component is:

$$W_1 = \frac{C_{z1}}{\cos \alpha_1} \quad (13)$$

$$W_{\theta 1} = W_1 \sin \alpha_1 \quad (14)$$

The radial velocities make, however a real, albeit small, contribution in the calculation of resultant velocities and the properties of state. The inlet static and relative total temperatures follow from application of the steady flow energy equation:

$$T_1 = T_{t1} + \frac{C_1^2 + C_{r1}^2}{2g_o J c_p} \quad (15)$$

$$T_{t1} = T_{t1} - \frac{\left[C_1^2 + C_{r1}^2 \right] - W_1^2}{2g_o J c_p} \quad (16)$$

The static pressure and density follow from the equation of state and the pressure-density relationship

$$P_1 = P_{t1} \left(\frac{T_1}{T_{t1}} \right)^{\frac{\gamma}{\gamma-1}} \quad (17)$$

$$p_1 = \frac{P_1}{RT_1} \quad (18)$$

Where R is the gas constant. In the relative reference frame the following relationships hold: the dynamic pressure (referenced to the blade and exclusive of radial flow);

$$q_1' = \frac{\rho_1 w_1^2}{2g_o} \quad (19)$$

the Mach number (again, referenced to the blade and exclusive of radial flow):

$$M_1' = \frac{w_1}{\sqrt{g_o \gamma R T_1}} \quad (20)$$

the total pressure:

$$P_{t1}' = P_1 \left(\frac{T_{t1}'}{T_1} \right)^{\frac{\gamma}{\gamma-1}} \quad (21)$$

These and subsequent quantities are shown in the temperature-entropy diagram (Figure 43), and the velocity diagram (Figure 44). The axial stations in the fan are defined in Figure 45.

Depending upon the blading geometry and stream Mach number, one of the two indicated cascade methods may be used to determine flow deflection. If the Howell method is used it is necessary to determine flow parameters associated with 0.8 of low speed stalling deflections, since that is the basis of the Howell design method⁽²³⁾. Once the flow deflection has been calculated the outlet angle follows.

Although the variation of flow area with axial distance

FIGURE 43. Temperature-entropy diagram for the fan

is assumed to be zero the density downstream of the blade row and pressure loss are unknown. Therefore, the outlet axial velocity is calculated on an iterative basis by assuming, on a trial basis, a value that is the mean between two previously calculated values. Choosing a low initial trial value, such as $C_{2z} = 0.4 C_{2r}$, assures convergence and the iterative process is sufficiently rapid.

For small radial velocities the radial displacements of streamlines inside the rotor blading and hence the centrifugal effect due to the associated change in tangential speed may be neglected in the first order analysis. The new temporary

FIGURE 44. Mean radius velocity diagram of the fan

FIGURE 45. Schematic section through the fan

outlet triangle may now be calculated.

$$W_2 = C_{z2} / \cos \alpha_2 \quad (22)$$

$$W_{\theta 2} = W_2 \sin \alpha_2 \quad (23)$$

$$C_2 = \sqrt{C_{\theta 2}^2 + C_{z2}^2} \quad (24)$$

The swirl change is:

$$\Delta C_{\theta} = [W_{\theta 1} - W_{\theta 2}] \quad (25)$$

Since UAC_0 is the temporary specific work input, the outlet total temperature follows if it is assumed that there is no heat loss within the blade row $(T_{t2} = T_{t1})$

$$T_{t2} = T_{t1} = \frac{UAC_0}{h_0 J c_p} \quad (26)$$

If it is assumed that the average radial velocity at the blade row outlet is zero, the static temperature may be obtained directly from the relative flow:

$$T_2 = T_{t2} - \frac{W_2^2}{2h_0 J c_p} \quad (27)$$

In order to calculate the total and static pressures downstream of the blade row, losses must be estimated first. Following Howell's example, the total loss is regarded as the result of three individual sources: secondary flow loss, profile loss, and annulus boundary layer loss:

$$C_D = C_{DS} + C_{DP} + C_{DA} \quad (28)$$

A satisfactory expression for secondary flow loss has not yet been achieved⁽²⁴⁾; but in the absence of more recent reliable information it is considered to be proportional to flow deflection, and hence to lift coefficient⁽²³⁾. Howell proposed the following relationship⁽²³⁾:

$$C_{DS} = 0.018 C_L^2 \quad (29)$$

The lift coefficient for incompressible flow with associated area change is:

The coefficient has been rederived in Appendix B for constant area flow:

$$C_{DP} = \frac{a}{c} \left(\frac{2C_{z1}}{C_{z1} + C_{z2}} \right) \cdot \frac{\Delta P_{t_{wake}}}{\frac{\rho_1 W_1^2}{2g_0}} \cdot \frac{\cos^3 \alpha_m}{\cos^2 \alpha_1} \quad (34)$$

The annulus wall boundary layer loss was allowed for in Howell's manner:

$$C_{DA} = 0.02 \frac{a}{\Pi} \quad (35)$$

The three loss coefficients C_{DS} , C_{DP} , C_{DA} are summed and the total pressure loss can be obtained directly.

The calculated pressure loss now makes it possible to compute the relative and absolute total pressures and static pressure:

$$P_{t2} = P_{t1} - C_{DQ_1} \quad (36)$$

$$P_2 = P_{t2} \left(\frac{T_2}{T_{t2}} \right)^{\frac{\gamma}{\gamma-1}} \quad (37)$$

$$P_{t2} = P_2 \left(\frac{T_{t2}}{T_2} \right)^{\frac{\gamma}{\gamma-1}} \quad (38)$$

The outlet density can be calculated using the equation of state and the previous values for static temperature and pressure.

The accuracy of the assumed estimate of outlet axial

$$C_L = 2 \frac{s}{c} (\tan \alpha_1 - \tan \alpha_2) \cos \alpha_m \quad (30)$$

where c is the blading chord length.

Under constant area flow conditions compressibility causes axial velocity to change across the blade row. The following relationship was derived in Appendix B for constant area flow:

$$C_L = 4 \frac{s}{c} \frac{\left[\tan \alpha_1 - \frac{C_{z2}}{C_{z1}} \tan \alpha_2 \right]}{\left[1 + \frac{C_{z2}}{C_{z1}} \right]} \cos \alpha_m \quad (31)$$

The direction at which the blade force acts relative to the plane of the blade row α_m is given for incompressible flow as:

$$\tan \alpha_m = \frac{1}{2} (\tan \alpha_1 + \tan \alpha_2) \quad (32)$$

The extended formula for constant area flow, derived in Appendix B is:

$$\tan \alpha_m = \frac{\frac{C_{z1}}{C_{z2}} \tan \alpha_1 + \tan \alpha_2}{\left[\left(1 - \tan^2 \alpha_2 \right) - \frac{C_{z1}}{C_{z2}} \left(1 - \tan^2 \alpha_1 \right) \right]}$$

The profile loss coefficient C_{DP} for incompressible flow was given by Howell in terms of the wake defect loss for incompressible flow:

$$C_{DP} = \frac{s}{c} \frac{\Delta P_{t_{wake}}}{\frac{\rho_1 W_1^2}{2g_0}} \frac{\cos^3 \alpha_m}{\cos^2 \alpha_1} \quad (33)$$

constant speed characteristic.

When all fan segment operating points have been calculated the performance calculation is complete for the imposed inlet and outlet conditions, but unless the imposed exit static pressures are correct for the crossflow condition a matching solution has not yet been achieved.

4.3. Comparison of Analytical Results with the Experiment at Zero Crossflow Conditions

The mean line design method was tested against measured lifting fan performance data presented in Reference (25). The test fan was of the constant flow area type with an outer diameter of 12 inches and a hub-tip ratio of 0.5. The axial length of the fan, including the intake, was $D/2$. The fan was submerged to a depth of $D/4$ at 35% of the 40 inch chord of a two-dimensional wing with a NACA 0015 section and had an intake with a circular arc lip shape ($R/D = 4$) that was faired with the wing surface. The specifications of the blading are given in Table 1 below.

The inlet and discharge conditions of the analytical model were assumed to be uniform, flow being drawn from and discharged into quiescent surroundings - as was the case during the experiment. With inlet and outlet conditions being uniform, the fan was treated as one complete unit. The fan flow was assumed to be axisymmetric and of the constant area type.

Analytical calculations were made to establish the fan characteristic as well as the running performance.

The non-dimensional parameters used to define the fan

velocity may now be checked by means of continuity:

$$C_{z2} = \frac{\dot{m}}{\rho_2 A_2} \quad (39)$$

The calculated value must agree with the assumed one within the assigned tolerance before the stator calculations can begin.

Possible calculation inaccuracies arising in the C_L and $\tan \alpha_m$ formulae, when blade row flow deflections are small and large trial axial velocity ratios are used, are averted by temporarily setting the axial velocity ratio to unity in these expressions.

The stator calculations are similar to those of the rotor in the relative reference frame. In the absence of heat loss and no work input, the total temperature remains constant. Basically, the same formulations may be used again if the relative conditions are regarded as absolute ones.

It can be seen that the downstream static pressure is completely determined by the choice of rotor incidence, inlet swirl and flow area. Consequently, if inlet swirl and radial velocity are given and flow area is defined, the back-pressure condition can be satisfied merely by variations in rotor incidence angle. Moreover, as incidence angle changes so does axial velocity and mass flow.

Repeating the calculations for a range of rotor incidence angles results in varying exit pressures and mass flow and, therefore, creates a unique compressor constant speed characteristic for the imposed inflow distortions. Satisfying a particular back-pressure results in an operating point on the

constant speed characteristic.

When all fan segment operating points have been calculated the performance calculation is complete for the imposed inlet and outlet conditions, but unless the imposed exit static pressures are correct for the crossflow condition a matching solution has not yet been achieved.

4.1. Comparison of Analytical Results with the Experiment at Zero Crossflow Conditions

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Analytical calculations were made to establish the fan characteristic as well as the running performance.

The non-dimensional parameters used to define the fan

	Rotor	Stator
Number of blades	18	19
n/c	1.046	1.372
Profile	C7	Double Circular Arc
Blade Inlet Angle (mid)	51.5°	45°
Blade Outlet Angle (mid)	6.5°	0.0°
(t/c) _{max} (mid)	0.090	0.159
Stagger Angle (mid)	29°	22.5°

Table 1. Geometry of Test Fan for
Zero Crossflow Comparison

FIGURE 96. Calculated performance of the experimental lifting fan of Reference 24 at zero crossflow

performance were: work coefficient and flow coefficient. These are presented in Figure 96 for a range of fan speeds from 1,000 to 15,000 R.P.M. Running line performance data (air flow, power and thrust) are shown in Figure 97. The design point speed for the experimental fan was assumed to be at 13,000 R.P.M., i.e., at the point where the thrust curve changes slope. The computed

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operating point at this speed gives an adiabatic efficiency of 74.4%. The measured fan data, also indicated in the same figure, demonstrate that a reasonable agreement was achieved with the

FIGURE 47. Calculated running line performance of a lifting fan at zero crossflow

analytical model based on constant area flow. In fact, the basic character of the experimental fan performance appeared to be well simulated to within a few percentage points over the entire operating speed range.

As indicated previously, the computation method furnished the inputs to the compressor map directly since rotor

incidence was varied until the imposed back-pressure condition was met. Figure 48 shows the calculated compressor map for the speed range 9,000 to 15,000 R.P.M. The computed running line

FIGURE 48. Calculated characteristics of a lifting fan at zero crossflow

and measured operating points are also shown. The results indicate again good local agreement, but unfortunately test data were not available to check additional points on the compressor map.

The calculated map indicates a considerable surge margin for the fan if one assumes the points of maximum pressure ratio to define the surge line. While no

quantitative experimental data are available at this time to verify this observation, a multitude of crossflow tests involving a similar model and producing severe inflow distortions appeared to be totally unsuccessful in bringing about a surge condition [7].

4.4. Matching Considerations

4.4.1. Efflux Circulation

One important feature of the efflux jet, explained previously in Chapter 3, was swirl velocity. Whenever the fan stator does not remove the rotor swirl fully the rotation of the efflux has a major influence on the static pressure distribution around the jet. This effect should, and can be, simulated by assigning to the centreline of the jet plume model an effective vortex strength that imparts an equivalent average swirl velocity on and inside the jet boundary. To achieve this it is necessary to calculate the circulation around the fan tip radius circle. Unfortunately, the swirl velocities can be determined only at mean radius, and so the circulation is calculated at mean radius as well. The implication of this approximation is that the efflux has a swirl distribution of the type $rC_{\theta 4} = \text{constant}$. This is, of course, true for free vortex fan designs and not an unreasonable approximation for fans with untwisted stator blades, such as in the present test fan.

Based on mean radius, the circulation is

$$\Gamma = \oint C_{\theta 4} \frac{D_m}{2} d\theta \quad (40)$$

$$= \frac{\pi D_m}{\mu} \sum_{i=1}^{\mu} C_{\theta 4_i} \quad (41)$$

where μ is the total number of fan segments. This circulation becomes an input for the analytical jet plume model.

4.4.2. Effective Circumferential Transport Distance

The match between each fan segment and its associated back-pressure is an important aspect of the problem. Because the fan segments are defined as volumes through which a constant rate of flow passes, the fluid is transported circumferentially with respect to the fan inlet because the mean flow path through the machine is helical. These transport distances can be estimated on the basis of average "stay time" inside the rotor while the rotor moves tangentially, and lateral shift distances relative to the blades that exist because the blades have stagger.

Let each fan segment be considered as a stream tube through which steady one-dimensional flow passes. The motion of an "average" fluid particle is governed by the nonuniform spatial acceleration associated with stream curvature and blockage effects caused by flow deflection and effective blade thickness (blade thickness plus boundary layer thickness), respectively*.

The rules of vector mechanics permit determining stay time on the basis of the axial components of velocity and acceleration. In fact, it is not necessary even to become involved with the complicated mechanics of motion if an integral approach is taken.

A knowledge of initial and final axial velocities and areas is not always sufficient for a computation of stay time.

*This can be shown formally for a simplified case of zero blockage by equating particle acceleration to convective acceleration terms in the case of incompressible flow for various assumed area variations $A(z)$.

Figure 49 illustrates a possible axial velocity distribution.

FIGURE 49. Possible variation of axial velocity inside a blade passage

Fluid particles moving along streamlines are subjected to varying accelerations, such as sketched in Figure 50.

While moving a distance δz in a small interval of time with an average axial velocity component of C_z , a fluid particle has an instantaneous acceleration of a_z .

FIGURE 50. Possible axial acceleration of fluid inside a blade passage

Hence an increment stay time is:

$$\delta t = \frac{\delta z}{C_z}$$

The total stay time is, therefore,

$$t = \int_{t_1}^{t_2} dt = \int_{z_1}^{z_2} \frac{dz}{C_z} \quad (42)$$

The continuity requirement is

$$\dot{m} = \rho(z) A_z(z) C_z(z)$$

which is a constant for each stream tube. The stay time, therefore, is simply:

$$t = \frac{1}{\dot{m}} \int_{z_1}^{z_2} \rho(z) A_z(z) dz \quad (43)$$

The integral is the mass contained in the stream tube and can be expressed more meaningfully as a difference between the mass if there were zero blockage and the displaced mass due to blockage, i.e.

$$\Delta t = \frac{1}{\dot{m}} \left[\rho_{\text{effective}} \text{ volume} \left(1 - \frac{\text{Blocked volume}}{\text{volume}} \right) \right] \quad (44)$$

In the special case of constant density and constant axial flow area

$$\Delta t = \frac{z}{C_z} \left(1 - \frac{t_c c}{z} \right) \quad (45)$$

where $t_c = \frac{c^2}{Z} \int_0^1 \left(\frac{t}{c} + \frac{2\delta^*}{c} \right) d\left(\frac{z}{c}\right)$, it is the effective blockage

thickness where t = blade thickness

δ^* = displacement thickness

If axial flow area is not constant, but is a linear function of

axial distance, or is almost linear:

$$\Delta t = \frac{A_{\text{mean}}}{A_1} \frac{Z}{C_{z_1}} \left(1 - \frac{t_c c}{sz} \right) \quad (46)$$

$$= \frac{Z}{C_{z_{\text{mean}}}} \left(1 - \frac{t_c c}{sz} \right)$$

It is interesting to note that only in the case of zero thickness blades and inviscid flow is stay time equal to $\frac{Z}{C_{z_{\text{mean}}}}$. If the area change is far from being linearly dependent on axial velocity, e.g. if the radius of curvature of the mean streamline is considerable in the meridional plane, the above formula can be suitably corrected by means of the prismoidal formula for volumes. For most cases it should be sufficiently accurate (in view of the assumption of constant density) to use the relationship:

$$\Delta t = \frac{Z}{C_{z_{\text{mean}}}} \left(\frac{2}{3} + \frac{1}{3} \frac{A_{Z/2}}{A_{\text{avg.}}} \right) \left(1 - \frac{t_c c}{sz} \right) \quad (47)$$

where $A_{Z/2}$ = axial area at $Z/2$, and

$$A_{\text{avg.}} = \frac{A_{z_1} + A_{z_2}}{2} \quad (48)$$

An inexact density correction can be made in Equations (45) to (47) by multiplying the expressions by $\rho_{\text{mean}}/\rho_{z_1}$.

The total shift of each stream tube in the circumferential direction including that due to flow deflection is:

$$\Delta \theta = \frac{U}{r_{\text{mean}}} \Delta t - \frac{Z}{r_{\text{mean}}} \tan \left(\frac{\alpha_1 + \alpha_2}{2} \right) \quad (49)$$

for the rotor. For closely spaced rotor and stator sets the gap

distance is negligible. Inside the stator only a circumferential shift due to stagger is involved. The total shift across the fan segment stage is therefore:

$$\Delta\theta = \frac{U}{r_{\text{mean}}} \cdot \Delta t - \frac{Z_{\text{ROT}}}{r_{\text{mean}}} \cdot \tan \left(\frac{\alpha_1 + \alpha_2}{2} \right) + \frac{Z_{\text{STAT}}}{r_{\text{mean}}} \cdot \tan \left(\frac{\alpha_3 + \alpha_4}{2} \right) \quad (50)$$

In the constant area through-flow analysis the effective circumferential shift is obtained through use of Equation (50) together with Equation (45), adjusted simply for density and by mass averaging all individually calculated fan segment cell shifts. In the extended analysis, featuring area change, this shift becomes the datum shift, additional shifts arising out of axial flow area changes.

4.5. Fan Feedback Effects

The feedback of the analytic fan model on the inlet flow distributions is entirely in the form of adjustments in axial velocity and rotor incidence angle, as depicted by the superpositioning of unit forms in Chapter 1. The chosen model is not capable of changing the upwash on the blades and therefore cannot adjust distortion swirl. Adjustments in radial velocities in the inlet can similarly not be achieved by the chosen fan model, because it is based on mean line calculations. Feedback distortions on mean line axial velocity can be obtained by determining the difference when the calculated axial velocities of the fan segments are subtracted from the corresponding zero crossflow velocities.

The fan's response in the efflux plane to crossflow distortion manifests itself in changes in the distribution of exit static and total pressures, static and total temperature, and efflux velocity. Again, the importance of these distortions may best be assessed by comparing the zero crossflow and crossflow cases.

The primary reaction of the fan to crossflow distortion lies in the deviation of fan segment constant speed characteristic lines from the basic constant speed characteristic of the whole fan at zero crossflow. These lines are unique for a certain (externally imposed) inflow distortion. The operating points of the fan segments on their respective characteristic lines are determined by the back-pressure conditions.

4.6. Comparison of the Segmented Fan Model with the Experiment at Crossflow Conditions

In order to make the best possible comparison between the performance predicted by the analytical segmented fan model and the measured performance of a lifting fan, measured inlet swirl and radial distortion velocities and back-pressures were imposed on the analytical fan.

The experimental fan was a fan-in-wing model comprising a constant 40-inch chord wing with a NACA 0015 section. The wing spanned completely the 10 x 20-ft. N.R.C. Propulsion Tunnel working section at the 10-ft. elevation. The lifting fan had a 12-inch diameter and was mounted mid-span in the wing at 35% of the chord. Six inch high fences were fitted to the wing ends

10 inches away from the tunnel walls. The fan rotor and stator blades were similar to those described in Chapter 4, with the following differences: there were eleven stator blades with a mean radius solidity of 1.58. The rotor was driven by a turbine powered shaft which passed through a thick stator blade at $\theta = 90^\circ$. The inlet and centrebody surfaces had elliptic (2:1) profiles with the major axis parallel to the fan axis.

The test condition selected involved a fan speed of 9995 R.P.M. (tip Mach number of 0.468) and a crossflow velocity of 106.9 ft/sec. The crossflow static conditions were assumed as $P_\infty = 2110.4$ C.B.S.F. and $T_\infty = 518.7^\circ\text{R}$. This test condition resulted in a measured crossflow ratio of 0.26. Figures 51 and 52 give the swirl and radial inlet distortion velocities measured at mid-annulus. These distributions are also reasonably representative of segment averages. Figure 53 is a plot of the (radially averaged) measured back-pressure.

The fan annulus was divided into eleven equal area fan segments, starting at the front of the annulus ($\theta = 0^\circ$) and the input variables were determined at each segment centre. The N.G.T.E. deflection and loss data were chosen for blading performance. These data were known to result in less flow deflection than measured in the test fan, and much less than predicted by means of the Howell data.

Figures 54 to 61 illustrate the result of the analytical segmented fan calculations and enable comparisons to be made with measured mid-annulus distributions. The measured inlet distributions were unfortunately a distance of $0.0625D$ upstream of the

FIGURE 51. Circumferential distributions of measured inlet distortion swirl velocity at mean radius, $V_{a1} = 106.6$ ft/sec., fan speed = 9,995 R.P.M., tip Mach number = 0.466

FIGURE 52. Circumferential distribution of measured inlet distortion radial velocity at mean radius, $V_{a1} = 106.6$ ft/sec., fan speed = 9,995 R.P.M., tip Mach number = 0.466

FIGURE 53. Radially averaged measured circumferential pressure distribution at the fan exit plane, $V_\infty = 106.4$ ft/sec., fan speed = 9,995 R.P.M., $V_\infty/V_0 = 0.26$

inlet calculation plane for measuring probe hardware considerations.

The calculated axial velocity distribution in Figure 54 is seen to have a functional relationship with θ that is predominantly proportional to $(1 - b_1 \sin \theta)$. This relationship is attributed to the combined effect of feedback due to distortion swirl velocity, which tends to increase axial velocities on the advancing blade side and to reduce them on the retreating blade side (as illustrated schematically in Figures 10 and 11, and as is explained in Appendix B), and the imposed back-pressure

FIGURE 54. Comparison between the measured and calculated inlet axial velocity distributions at mean radius, $V_{\infty} = 100.9$ ft/sec, fan speed = 9,995 R.P.M., tip Mach number = 0.466

distribution, shown in Figure 53. As is known from the constant speed stable portion of the pressure-mass flow characteristics of conventional axial compressor stages, and as was shown in Figure 48, reductions in back-pressure level are met with increases in axial velocity. Such increases diminish, of course, with increasing Mach number. The distortion swirl velocity and back-pressure distributions have, therefore, counteracting feedback effects on axial velocity, as depicted in Figure 55 for the fan speed of 9995 R.P.M. over a range of back-pressure conditions.

FIGURE 55. Influence of inlet distortion swirl velocity, back-pressure and Mach number on axial velocity and rotor incidence angle. Analytical segmented fan model, $V_0 = 106.9$ ft/sec, fan speed = 9,995 R.P.M.

As can be seen from the indicated operating regime of the different fan segments in Figure 55, the effect of distortion swirl on axial velocity is somewhat more significant than the back-pressure effect. This order of importance can be expected to be maintained over the range of distortion swirl and back-pressure shown here, as well as any other low Mach number condition.

The measured distortion wave, unlike the calculated one, has its velocity maximum at $\theta = 320^\circ$. This gives the whole axial velocity distribution a counterrotational orientation, or bias, with respect to the crossflow direction, as shown in Figure 32. To force the calculated distortion towards the measured one a function of the type $(a_n \cos n\theta + b_n \sin n\theta)$ with a very strong a_1 term is necessary. The axial distortion velocity perturbation due to bending was shown in Chapter 1 to be of the $a_1 \cos \theta$ type; but, as explained in Section 4.2.2., the restriction on axial area change present in the segmented fan model precludes inclusion of this independent crossflow distortion effect. The higher order effects ($a_n \cos n\theta$ and $b_n \sin n\theta$ with $n > 1$) are expected to arise from additional, but much smaller, fan feedback due to the $a_1 \cos \theta$ distortion.

All of the distortion effects discussed could be described by one or more trigonometric functions on the interval 0 to 2π . The back-pressure is the only variable not featuring a symmetrical waveform distribution. These types of circumferential distributions suggest that, regardless of how badly the circumferential distributions are predicted by theory, such average quantities as axial velocity (mass flow, thrust, etc.) should not

be greatly different from measured results. This, in fact, was found to be the case: the predicted average axial velocity was only 4% larger than the measured one. These types of circumferential distributions are also indicative of the probable trends with increasing crossflow. The variables causing axial velocity, and hence mass flow, to increase with crossflow are the total conditions and the back-pressure distribution. Providing no new inflow distortion phenomena (such as flow separation) appear and there is no performance fall-off with increasing Mach number, at constant fan speed the average axial velocity and mass flow are apt to increase gradually with crossflow.

As Figure 56 shows, all rotor blades operated with negative incidence, but the most negative swings took place on the retreating side of the fan. The analytical fan segments exhibit similar values of incidence. Reference to the interactive effects on incidence of the various crossflow distortions, Figure 55, reveals the large negative incidence angle swings on the retreating sides to be the result of the reinforced effects of reduced back-pressure and negative distortion swirl velocity. The reinforcing effect on the advancing side is not as strong because the back-pressure rise is limited by the crossflow stagnation pressure and, consequently, the incidence is somewhat less negative and considerably more uniform.

The differences between the predicted and the measured rotor incidence distributions in Figure 56 are not great. The distortion perturbation in axial velocity, which was omitted from the segmented fan model, would, according to the relationships

FIGURE 56. Comparison between measured and calculated rotor incidence angle circumferential distributions at mean radius, $V_{a1} = 186.9$ ft/sec, fan speed = 8,995 R.P.M., tip Mach number = 0.468.

shown in Figure 55, decrease the magnitude of incidence at $\theta = 180^\circ$ and increase it at $\theta = 0^\circ$. The differences between the predicted and measured incidence data are therefore attributed mostly to the lack of including the axial velocity distortions associated with bending in the segmented fan model. Inadequately predicted blading performance and the axial separation between the planes of observation and calculation are to a lesser extent also involved in making up the differences.

How effectively the fan manages to attenuate crossflow

distortion may be assessed on the basis of exit plane distributions. The reduction in the absolute value of the axial velocity distortion wave is nearly 1/3, as can be ascertained from a comparison of Figures 57 and 54. It is solely a density effect in the analytical segmented fan model. The correlation between

Comparison between measured and calculated efflux axial velocity circumferential distributions at mean radius, $V_m = 106.9$ ft/sec, fan speed = 9,745 R.P.M., ρ_1 Mach number = 0.468

the nondimensional predicted and measured axial velocity data is excellent, but such cannot be claimed for the absolute magnitudes. The apparent large differences are this time associated with the measurements. Probe measurements, which were local values and hence subject to the blade to blade profile shape, indicated

larger than profile averaged values. In addition, the predicted segment efficiencies are probably different from those which were realized in the test fan. The presence of the thickened stator vane at $\theta = 90^\circ$ is evident in the predicted data as well as the measured data because the measured distortion data were used as an input for the calculations.

FIGURE 10. Comparison between measured and calculated efflux swirl velocity distributions at mean radius, $V_0 = 106.9$ ft/sec, fan speed = 9,995 R.P.M., tip Mach number = 0.466

The predicted nondimensional exit swirl velocity distribution appears to be a fair simulation of the measured one, except near the thickened stator vane ($t/c = 0.25$), where the flow deflection was considerably less than elsewhere. No allowance was made in the analytical model for this local thickness problem. The magnitudes of the predicted swirl velocities depend on the

stator outlet angles used. The N.G.T.E. deflection and loss data used in this calculation gave less flow deflection and hence larger outlet angles than were actually the case. However, this result is considerably more realistic than the Howell deflection method result, which predicted greater flow deflection than were realized in the experimental fan. The predicted average outlet swirl velocity (202.3 ft/sec.) is therefore some 34% higher than the measured one.

The velocity triangle analysis that produced Figure 55, and is given in Appendix B, also shows that positive distortion swirl (advancing blades) increases the swirl change across the

FIGURE 55 Comparison between the measured and calculated segment total pressure ratio at mean radius, $V_m = 106.9$ ft/sec, fan speed = 9,935 R.P.M., tip Mach number = 0.468

rotor and that negative distortion swirl (retreating blades) decreases the swirl change. The circumferential distributions of total pressure ratio and total temperature rise (divided by average total temperature rise) that this variation in swirl change produces are given in Figures 59 and 60, respectively. In fact, the visible nonuniformity in these variables is direct evidence of the fan's response to crossflow distortion.

The segmented fan model predictions resemble the measured results reasonably well, in spite of the fact that the distortion perturbation in axial velocity was not included in the

FIGURE 59. Comparison between the measured and calculated segment relative total temperature rise at mean radius, $V_m = 106.9$ ft/sec, fan speed = 9,995 R.P.M., tip Mach number = 0.466

model. At $\theta = 0^\circ$ and 180° , where one would expect the greatest error due to exclusion of the axial velocity distortion perturbation, the predicted results are actually very close to measured results. The evidence of swirl velocity distortion being the more significant distortion velocity effect here is supported by the fact that the absolute value of the measured swirl velocity distortion was nearly twice as large as the absolute value of the measured perturbation in inlet axial velocity, see Figures 51 and 54.

The segmented analytical fan model dealt with the segments as separate fans with fan speed and shared inflow and outlet flow field being the only common links between them. As was shown, the distortions that each fan segment was exposed to varied from segment to segment, and were, in fact, totally responsible for making the operation of each fan segment effectively different from the others. This feature of the whole problem was used to construct Figure 55 and makes it possible to determine effective constant speed characteristic lines for each segment.

Figure 61 shows a family of constant speed lines for the inlet distortions experienced by each segment. It can be seen that these lines deviate substantially from the standard (zero crossflow) characteristic line for the complete fan. With crossflow the fan operates with the inflow distortion as if there existed a loss-free set of inlet guide vanes providing the fan with different degrees of pre-whirl. This analogy, which was first proposed by Beeler and Przedpelski⁽²¹⁾, while being very convenient for visualizing the main inflow distortion effect,

FIGURE 61. Fan segment effective operating characteristics for a fan speed of 9,901 R.P.M. and a crossflow velocity of 106.9 ft/sec.

turns out to be only a rough approximation because it does not allow for either the radial or axial inlet distortion velocities. Moreover, it also does not lend itself to predicting the fan segment and complete fan operating points because the back-pressure (distribution and level), which is the second most important cross-flow distortion effect, is not being taken account of.

How well the analytical segmented fan model predicts the individual segment through-flows at higher fan speeds is indicated

in Figure 82. This figure shows the effect of blade inlet Mach number for constant back-pressure over a range of fan speeds and is fashioned after Figure 55. In order to indicate general trends simple artificial data were used as inputs to the calculations: a crossflow velocity of 127.0 ft/sec., zero radial velocity, and a uniform back-pressure of 2110.4 P.S.F. These data were contrived

FIGURE 82. Influence of inlet distortion swirl velocity and Mach number on axial velocity for a range of fan speeds and blade inlet Mach numbers. Analytical segmented fan model, $V_c = 127$ ft/sec, $P_b = 2110.4$ P.S.F.

as a time saving expedient, but they actually cover the range of conditions between 9995 and 13,450 R.P.M. reasonably well, as can be seen on comparing Figure 62 with Figures 55 and 63. Figure 63 represents a range of uniform back-pressure conditions at the fan speed of 13,450 R.P.M.

It is evident from Figures 62 and 63 that Mach number

FIGURE 63. Influence of inlet distortion swirl velocity, back-pressure and Mach number on axial velocity and rotor incidence angle. Analytical segmented fan model, $V_a = 127.8$ ft/sec, fan speed = 13,450 R.P.M.

has a profound effect on segment performance predictions. The effect becomes first noticeable at a Mach number of 0.62 and becomes unacceptably high for Mach numbers greater than 0.70. Although the blading incidence angles are far more negative for the segments on the retreating side of the fan, the segments on the advancing side are first afflicted with high Mach numbers. This detrimental effect of positive distortion velocities could not have been anticipated had the fan been treated as a complete unit instead of as a collection of fan segments.

The deterioration of segment performance is, of course, associated with the blading loss and deflection data used. The N.G.T.E. data employed in the model are actually not suitable for high Mach number performance predictions. However, while the present data can no longer yield acceptable prediction results, it must be noted that the experimental fan also experienced a significant deterioration of performance at speeds in excess of 12,000 R.P.M., as shown in Figure 47, presumably also because of high blading Mach numbers⁽²⁵⁾. In this respect at least, the present data continue to be useful because the results are indicative of a real operational problem. More appropriate cascade data would really not obviate this difficulty; at best they would help in predicting more accurately the deterioration of fan performance.

It is interesting to observe, while the predictions of fan segment performance becomes less accurate with increasing Mach number, that averaged performance quantities remain to be reasonable estimates, as evidenced in Table 2 below.

Fan Speed	R.P.M.	9995			13450		
Data Source	Units	Experiment		Theory	Experiment		Theory
Tip Mach No.	1		0.468	0.468		0.631	0.631
V_m	ft/sec	106.9	106.9	106.9	127.0	127.0	127.0
V_{al}	ft/sec	421.5*	414.6**	430.0	526.7*	518.9**	555.5
$\dot{m}$	lbm/sec		17.5	17.9		21.0	22.1
HP	HP		104.2	111.8		236.8	228.9
P_{t4}/P_{t1}	1		1.11	1.10		1.16	1.15
$\frac{\dot{m}\sqrt{T_1}}{A P_1}$	$\frac{\text{lbm} \cdot \text{OK}^4}{\text{ft}^2 \cdot \text{sec}}$		0.238	0.244		0.285	0.299
Thrust	lbf	230.2*	183.3 [†]	208.5 ^{††}	343.9*	270.2 [†]	321.8 ^{††}
AT_t	or			18.3			30.4
$\eta_{adiab.}$	1			0.79			0.72
$\dot{V}$	ft ³ /sec		-356.3	-476.2			-564.72
cell-shift	Deg.			18.1			17.6

* Based on mass averaged traverse data, i.e. a momentum thrust.

** Based on one-dimensional flow estimate for average mass flow and total area.

[†] Based on force balance measurements.

^{††} Based on calculated efflux momentum, average back-pressure and centrebody base drag.

TABLE 2. COMPARISONS OF PERFORMANCE AVERAGES

The reason for this seemingly inconsistent trend involves the circumferential distribution form of the distortion variables, and was explained earlier in this chapter.

Because the fan segments appear to operate well away from positive incidence, crossflow conditions are not expected to make fan operation unstable. A gradual deterioration of fan performance at low fan speeds and large crossflow velocities (increasing crossflow ratios) appears likely, however, because the retreating blades experience increasingly large negative incidence angles. At high fan speeds a gradual deterioration of fan performance with increasing crossflow again seems certain because the advancing blades operate at higher and higher Mach numbers. These aspects of performance deterioration have, in fact, been observed experimentally under various crossflow conditions (7,25).

CHAPTER 5

COMPLETE ANALYTICAL MODEL OF THE SEGMENTED FAN WITH
MATCHED INLET AND OUTLET FLOW DISTORTIONS

5.1. Outline of Calculation Procedure

5.1.1. Inflow Distortion

A once-only calculation is made of the inflow distortion velocities for a given crossflow velocity by means of the inflow polar lattice vortex model. This calculation yields the distortion axial, swirl, and radial velocities at the inlet plane of the fan (or any other plane near the inlet plane). In addition to crossflow velocity certain other geometric inputs are necessary, as explained in Chapter 2. One of these is the number of sides of the polar lattice. The number of sides should be the same as the number of fan segments to be used. Another important geometric input variable is the field point location. Obviously, the distortion velocities need to be known at the centres of the fan segments (or at several points if an average distortion is to be used).

5.1.2. Exit Plane Pressures

Fan segment back-pressure values are necessary for calculating performance, but since the average axial through-flow and exit swirl velocities are not initially known, the efflux jet plume model cannot be used for calculating exit pressures. It suffices to use a constant back-pressure, possibly at

ambient level, to start the calculation procedure. This incorrect initial choice does not affect the segment characteristics and, as will be shown presently, does not obstruct the convergence of the calculation process to a unique solution.

5.1.3. Performance Calculation for Each Fan Segment

The inputs necessary for this phase of calculation were explained in Section 4.2.1. The number of fan segments is chosen to be the same as the number of sides in the inlet polar lattice (eleven segments were used in the test case because the stator had as many blades). The constant area mean line direct design method is used in conjunction with specified fan speed, inflow distortion swirl and radial (but not axial) velocities, and other geometric (hardware) specifications to calculate the thermodynamic state of the air for each fan segment at each axial station over a range of rotor incidence angles. This calculation establishes uniquely the compressor constant speed characteristics of each fan segment, each being distinctively different from the others because of the imposed inflow distortion velocities, for the chosen cascade loss and deflection data and the assumed annulus boundary layer and secondary flow losses.

The back-pressures, also specified as input data, are used in the calculating program to create temporary "solution points" for the fan segments until finally the desired fan-in-crossflow operating point is arrived at. In having the calculation model satisfy these pressures each fan segment merely "slides along" its previously established constant speed distortion characteristic line.

FIGURE 84. General flow chart for the complete analytical model

When such solutions are achieved (i.e. the back-pressure condition is met) the corresponding total mass flow and the efflux average axial and swirl velocities are available; but these results must be considered tentative, unless a match between the assumed back-pressures and those resulting from subsequently derived jet plume model static pressures using the most recent available efflux velocities is obtained as well.

5.1.4. Efflux Circulation

The efflux swirl velocities derived from the fan segment performance calculations are integrated around the circumference of the mean radius circle to yield the strength of the jet plume's centreline vortex. This vortex line is responsible

for simulating the Magnus effect of the jet plume in crossflow.

5.1.5. Circumferential Transport Distance

The calculated static pressures around the efflux plume's circumference can only be properly associated with the fan segments if the circumferential positions of the outlet areas relative to the inlet areas are known. This shift, which is due to the helical path taken by the fluid through the stage, can now be calculated using available performance data.

5.1.6. Revision of Exit Plane Static Pressure Distortion

The average axial velocity, the crossflow velocity, the plume centreline vortex strength, the circumferential transport distance, and the jet efflux plume model are used to calculate new exit pressures at the centres of the fan segments at the exit.

These revised static pressure data become the new input data for a new through-flow performance calculation of all the fan segments.

5.1.7. Convergence and Meaning of Solution

As explained previously, except for the first iteration pass, when the true character of the back-pressure distribution emerges, no large changes in back-pressure level are encountered. Consequently, one may expect, and actually does find, that such total quantities as fan mass flow and pressure ratio do not change greatly between iterative calculation passes. As is known from general compressor constant speed characteristics, back-pressure changes lead to adjustments of axial velocity, and these

result in variation in blade incidence angle. However, away from stalling flow conditions the blade row outlet angles do not vary as much as the inlet angles. In fact, both the analytical model and experiment⁽⁷⁾ verify that stator outlet angle was highly uniform circumferentially (and radially). For ease of discussion one may, therefore, assume that the stator outlet angle was constant.

The constant speed characteristics for individual fan segments are different from each other only because the inflow

FIGURE 31. Typical sensitivity of fan segments

distortions of each segment are different. Depending on the segment sensitivity $\left(\frac{\Delta P}{\Delta U_x}\right)$ for a given drop in exit pressure the segment responds with a proportional increase in axial velocity, an increase in outlet swirl velocity (because $\bar{U}_u$ is constant), and an increase in the circulation

of the efflux jet. Since this increase in circulation is, however, distributed over the whole jet, as far as the jet plume model is concerned, there is an attenuation factor of $(1/\mu)$. Within the broad limits, changes in circulation do not introduce significant static pressure changes on the jet circumference inside the interval $\pi \pm \theta \pm 2\pi$, see Figure 39. Consequently, even if the segment pressure-circulation sensitivity $\left(\frac{\Delta P}{\Delta \Gamma}\right)$ were equal to the jet plume pressure-circulation sensitivity the iterations on Γ would be convergent. In actual fact, the linear portion of the

5.2. Comparison of the Complete Analytical Model with the Experiment at Crossflow Conditions

Up to this point separate assessments have been made of the generated inflow distortions (Chapter 2), the generated back-pressure distortions (Chapter 3), and the through-flow, as predicted by the analytical segmented fan model (Chapter 4).

The previous comparison of the analytical segmented fan model (hereafter called the partial analytical model), with the experiment led to a comprehensive analysis of the qualities and shortcomings of the fan modelling technique. This critical appreciation is also applicable to the complete model and so does not require repetition. The final stage of comparison involves the results of a complete simulation of a particular experimental operating point using all three analytical models in closed loop manner. As before, the comparison is based on the crossflow test condition of 9995 fan R.P.M. and the crossflow velocity of 106.9 ft/sec. The comparison is made in order to, a) show and account for the differences between the complete and partial model results, which arise solely out of imperfect simulation of inflow and outlet distortions, and b) to demonstrate how well the actual test condition can be simulated by theory alone, i.e. without resorting to any aspect of the experiment.

Figures 67 and 68 illustrate the generated distortion swirl and radial velocities. Both the values actually used in the partial analytical model portion of the complete program and the values at an axial distance of $0.0625D$ above the fan inlet plane, which corresponds to the observation plane, are shown. It

can be seen that the generated inlet swirl distortions are aligned with the fore and aft axis of the inlet, but the measured

FIGURE 67. Comparison between the complete and partial analytical models and experimental results. Circumferential distributions of inlet distortion swirl velocity at each radius. $V_0 = 106.9$ ft/sec, fan speed = 9,995 R.P.M., tip Mach Number = 0.468

distortions are not. The circumferential bias of the measured swirl distribution is presumed to be a nonlinear feedback effect that cannot be modelled easily. The small phase difference (nearly 10 degrees) between the generated and the measured distributions gives rise to a considerable error in the end result of the calculations, as will be shown presently. The calculated distribution of radial velocity, while being in phase with the measured

distribution, overestimates the absolute magnitude by nearly 100%.

FIGURE 68. Comparison between the complete and partial analytical models and experimental results. Circumferential distributions of inlet distortion radial velocity at mean radius, $V_{a1} = 410.6 \text{ ft/sec}$, fan speed = 0.055 R.P.M., tip Mach number = 0.650

As can be seen, the values actually used in the calculation are not nearly as much in error. The large difference in radial velocities stems from the imperfect simulation of the inflow velocity field, and could be reduced if the inflow distortion model were modified in the manner suggested in Appendix A. The error in radial velocities is, however, not excessively detrimental in the overall calculations because it affects only the static inlet conditions.

The back-pressure distribution shown in Figure 69 is the result of the fourth and final calculation pass during which

FIGURE 59. Comparison between the complete analytical model and experimental results. Circumferential distribution of back-pressure coefficient at mean radius, $V_\infty = 100.0$ ft/sec, fan speed = 0.001 R.P.M., tip Mach number = 0.458

the efflux circulation converged to within 0.05% of its previous value. Although the predicted pressure distribution compares reasonably well with the measured one, it would feature an inferior degree of likeness if the calculated exit swirl had been the same as the measured one. A critique of the back-pressure model was presented in Chapter 3.

The results presented to this point serve as "inputs" to the partial analytical model. The following results are predicted by the analytical model of the segmented fan and hence are dependent variables. Because the previous distortion variables

act either directly or indirectly on the axial velocity in each fan segment the most severe test the whole method can be subjected to is to compare the predicted axial velocities with the measured ones. This comparison may be made in Figure 70.

FIGURE 70. Comparison between the complete and partial analytical models and experimental results. Circumferential distributions of inlet axial velocity at mean radius, $V_0 = 195.0$ ft/sec, fan speed = 9,995 R.P.M., tip Mach number = 0.468

The predicted axial velocities are, in fact, a considerable underestimation of the measured axial velocity distribution, the most obvious difference being proportional to $\cos\theta$. The difference between the measured and partial model data is attributed to the lack of including axial distortion velocities associated with flow bending effects, as explained in Chapter 4. The difference

between the partial and complete model results must, therefore, arise out of the inadequacies of predicting input distortion variables, particularly inlet distortion velocities, since the back-pressure distribution simulation was reasonably acceptable.

The error in phase angle of the distortion swirl distribution is suspected of being responsible for most of the deviation. As Figure 67 showed, this phase difference effectively caused the generated distortion swirl velocities to be too large at the rear of the inlet on the advancing blade side and not negative enough on the retreating blade side. An estimate of the effect of these differences in distortion swirl velocities on axial velocities may be made by means of Figure 71. Following a constant pressure line, for example, shows that a reduction in positive swirl velocity decreases axial velocity.

The second, albeit minor, cause of difference in axial velocity between the partial and the complete model results is attributed to the exaggerated radial velocities generated in the latter model. While radial velocities do not affect blading performance directly, they do increase resultant velocities.

Since it is the resultant velocities that determine static conditions in the inlet, the fan segments have to move air against a larger pressure difference. The segments can do this only by reducing mass flow, and hence axial velocity. Assuming that the segment pressure-mass flow characteristic changes are negligible, this adjustment in axial velocity is proportional to density changes, and hence is of a minor nature. The indicated adjustment due to the radial velocity errors, shown in Figure 68, are

FIGURE 71. Influence of inlet distortion swirl velocity, back-pressure and Mach number on axial velocity and rotor incidence angle. Complete analytical segmented fan model, $V_m = 106.9$ ft/sec, fan speed = 9,895 R.P.M.

FIGURE 77. Fan segment and complete fan operating lines for a range of crossflow and fan speed conditions. Calculated performance is indicated by the solid symbols.

FIGURE 71 Comparison between the complete and partial analytical models and experimental results. Circumferential distributions of off-flow axial velocity at mean radius, $V_0 = 114.9 \text{ ft/sec}$, fan speed = 2,105 R.P.M., tip Mach number = 0.668.

Figures 73 to 76 illustrate the predicted complete and partial model and experimental axial and swirl velocity, total pressure ratio and relative total temperature rise results at the exit plane of the fan. The comparisons made in Chapter 4 are highly appropriate here as well, because the complete and partial model results are very similar. The magnitude of the exit swirl velocities, as in the partial model, tends to be too high because the blading performance data used produced less than real flow deflections.

The predicted fan results are generally a reasonable simulation of the measured data, but it must be conceded that the

FIGURE 74. Comparison between the complete and partial analytical models and experimental results. Circumferential distributions of efflux swirl velocity at mean radius, $V_m = 126.9 \text{ ft/sec}$, fan speed = 9,895 R.P.M., tip Mach number = 0.468

FIGURE 75. Comparison between the complete and partial analytical models and experimental results. Circumferential distributions of total pressure ratio at mean radius, $V_m = 126.9 \text{ ft/sec}$, fan speed = 9,895 R.P.M., tip Mach number = 0.468

predicted segment through-flow distortions are not quite as severe as measured. On the one hand, it would appear that inclusion of the axial inflow distortion velocities is essential in order to improve the match considerably. On the other hand, the degree of success achieved with the present analytical model suggests that

FIGURE 76. Comparison between the complete and partial analytical models and experimental results. Circumferential distributions of relative total temperature rise at mean radius, $V_a = 105.9$ ft/sec, fan speed = 9,995 R.P.M., tip Mach number = 0.444

the major distortion characteristics have already been emulated, and only lesser improvements are necessary.

Figure 77 is a summary plot of the experimental operating lines in terms of total pressure ratio and mass flow function of the eleven fan segments, as shown in the insets. The predicted operating points for two crossflow conditions (9995 R.P.M. and

FIGURE 77. Fan segment and complete fan operating lines for a range of crossflow and fan speed conditions. Calculated performance is indicated by the solid symbols.

106.9 ft/sec., and 13,540 R.P.M. and 127.0 ft/sec) have been superposed on the experimental data and indicate where in the range of experimental crossflow conditions the comparisons have been made.

It can be seen that while the predicted mass flows are somewhat larger than measured, the calculated segment pressure ratios are not unreasonable.

The operation of each fan segment within the crossflow regime shown is distinctively different from all others and the whole fan. The retreating blade side segments show the greatest variation in performance with crossflow. These segments feature not only the largest negative incidence angles, but also suffer the greatest degradation in performance (considerable loss in total pressure and only little gain in mass flow with increasing crossflow velocity). The fan segments on the advancing side of the fan experience considerable augmentation in mass flow at low fan speeds and an appreciable increase in total pressure at low crossflow velocities. At high fan speeds and crossflow velocities both the segment mass flow and delivery total pressure ratio decline.

There is, unfortunately, not enough calculated data to see if the same crossflow distortion trends can be predicted with theory; but, the two calculated points show in a quantitative way where and how fan performance deteriorates with crossflow.

The calculated fan averaged performance variables (mass flow, thrust and power) have been added to the experimental performance plots. These are shown in Figures 78 to 80. On the whole, for reasons discussed in Chapter 4, the predicted results are reasonably accurate.

FIGURE 76 Measured and calculated total fan thrust under crossflow conditions

FIGURE 77 Measured and calculated average inlet axial velocity under crossflow conditions

FIGURE 88. Measured and calculated fan power under crossflow conditions

CHAPTER 6

EXTENSIONS OF THE ANALYTICAL SEGMENTED FAN MODEL

6.1. First Extension

6.1.1. General Considerations

The logical first improvement in the analytical model would be to relax the constant area requirement and thereby to allow inclusion of the generated axial velocity perturbation of the inflow distortion model.

The actual crossflow induced inlet distortion and the method used to simulate them analytically were seen to involve potential flow only. Since only perturbations in velocity were generated by the distortion model, the axial distortion velocities sum up to zero net flow. Consequently, any total flow adjustments that arise as a result of including axial distortion velocities may be regarded as fan feedback effects. The previous constant area method was seen to be implicitly defined by the distortion variables used, and if inlet axial velocity distortions are to be admitted an additional degree of freedom must be provided. The additional degree of freedom enables fan segment mass flow to be different from the constant axial flow area case while meeting the same, or nearly the same, back-pressure condition. It is suggested that these distortions can be accommodated if the axial flow area of each fan segment is permitted to vary with axial distance.

6.1.2. Outline of the Proposed Analytical Method

6.1.2.1. Inflow Distortion

The inflow distortion velocities are generated in the previously established method at the centres of the fan segments and remain constant for the whole of the calculation.

6.1.2.2. Inclusion of Swirl and Radial Distortion Velocities

The previous constant area mean line design method is applied to each fan segment using inlet swirl and radial distortion velocities only. Repeated calculations are made for all segments with revision of back-pressure using the jet plume model, as outlined in the previous method, until a constant area solution is obtained.

6.1.2.3. Inclusion of Axial Distortion Velocities

The resulting inlet axial velocities, the swirl, radial and (previously unused) axial distortion velocities are used in the mean line design method to achieve the same back-pressures by allowing the area downstream of the rotor to change. The fan segment area is assumed to remain constant through the stator. Modifications to the various blade row relationships are indicated in Appendix B.

"Temporary" solutions can be obtained for each fan segment for appropriate area ratios across the rotor blade row; but, the sum of the downstream "solution" areas may not generally be equal to the total area, nor need the back-pressure necessarily

be correct since the efflux circulation, the net circumferential transport distance, and the total mass flow may all be slightly different.

Without applying circumferential equilibrium, as is done in the second extension method, geometric compatibility can be re-established by pro-rating all fan segment areas (by mass, momentum, or some other averaging technique). In order to maintain continuity for these revised areas the performance of each fan segment is then recalculated for the same back-pressures, this time allowing axial velocity to vary. This adjustment is interpreted as a fan feedback effect. The result of this calculation satisfies continuity, geometric compatibility, and blading performance for the constant area solution back-pressure values.

6.1.2.4. Revision of Back-Pressures

The latest estimates of efflux axial velocity, efflux circulation and circumferential transport distance now make it possible to correct the back-pressures by means of the jet plume model. Although the exit areas of the fan segments are no longer equal fractions of the total annulus area, the jet plume's cross-section may continue to be a regular polygon. New pressures are calculated at the circumferential positions corresponding to the fan segment centres and are not expected to be greatly different from the previous set.

6.1.2.5. Revision of Fan Segment Performance

With these corrected pressures, the same distortion velocities and the previous solution values of inlet axial velocity

(which include the axial distortion velocity) the fan segment performance can now be recalculated, as before.

6.1.2.6. Uniqueness of Solution

One might expect the process to have similar convergence properties as the constant area calculations and one arrives at a final solution when no further changes in back-pressure result. Double valued solutions for individual fan segments associated with the constant speed characteristic lines may arise as before but the unwanted solution may be rejected on the basis of pressure/mass flow curve slopes, as done in the constant area method.

The rotor incidence angle, which becomes a "free" parameter in this first extension method, will in general not be the same as for the constant area model; but, this again appears to be a valid fan feedback effect.

6.2. Second Extension

6.2.1. General Considerations

In developing the simple constant area segmented fan model several basic physical requirements were relaxed in order to accomplish an approximate solution. This model made use of generated inlet swirl and radial distortion velocities and back-pressure; but the generated axial distortion velocities were redundant. Consequently, the crossflow bending effect, which was seen in Section 1 to be the smaller of the two important crossflow induced inflow distortion effects, was not provided for.

The proposed first extension method includes this second inflow distortion effect by relaxing the constant area requirements; but, in pro-rating the unequal segment areas so that the sum of the segment areas equals the annulus area the first extension method neglects the presence of an important principle: the equilibrium of forces. In failing to satisfy the equilibrium requirements the theory remains limited to "axially thin" fans.

Many lifting fan configurations do not have a severe axial constraint and so varying amounts of circumferential flow redistribution between the blade rows and at the exit plane do exist. The fan-in-pod arrangement is a typical example where axial distance may actually be large and interblade row volumes cannot be neglected. Indications are that the character of the cross-flow induced efflux and inlet distortions here is very similar to the "axially thin" fan, and that the demonstrated principles of simulating these distortions analytically are valid; however, the flow through the segmented fan and just outside the exit plane cannot be treated as superficially.

Basically, the second extension is only an improvement on the first extension of the through-flow model. The general features of the segmented fan mean line design method do still apply. Two major modifications of the basic method are proposed: the incorporation of area change (as in the first extension), and the interblade row and exit plane pro-ration of areas consistent with equilibrium requirements. Possible provisions for area change in various aerodynamic and thermodynamic formulae are

indicated in Appendix B.

Equilibrium requirements, being a vital part of this new method, will now be examined in some detail.

6.2.2. Equilibrium in the Axial Planes

6.2.2.1. The Equation of Motion in the Absolute Frame of Reference

Consider a subsonic, inviscid, three-dimensional flow field which is determined by a given inlet distortion, an exit plane static pressure field, and the known off-design performance of blade rows of a given fan. Let it be assumed that there is no boundary layer separation from the end walls and that normal boundary layer growth can realistically be accounted for in loss coefficients and area change terms.

The equation of motion of a compressible inviscid and adiabatic fluid expressed in vector form is:

$$\frac{\partial \vec{C}}{\partial t} + (\vec{C} \cdot \nabla) \vec{C} = - \frac{g_0}{\rho} \nabla P \quad (51)$$

This equation together with its scalar forms in terms of cylindrical coordinates is derived in Appendix C. The forms for the radial, circumferential, and axial components are as follows: the r-component is:

$$\frac{\partial C_r}{\partial t} + C_r \frac{\partial C_r}{\partial r} + \frac{C_\theta}{r} \frac{\partial C_r}{\partial \theta} + C_z \frac{\partial C_r}{\partial z} - \frac{C_\theta^2}{r} = - \frac{g_0}{\rho} \frac{\partial P}{\partial r} \quad (51a)$$

the θ -component is:

$$\frac{\partial C_\theta}{\partial t} + C_r \frac{\partial C_\theta}{\partial r} + \frac{C_\theta}{r} \frac{\partial C_\theta}{\partial \theta} + C_z \frac{\partial C_\theta}{\partial z} + \frac{C_\theta C_r}{r} = - \frac{g_0}{r\rho} \frac{\partial P}{\partial \theta} \quad (51b)$$

the z-component is:

$$\frac{\partial C_z}{\partial t} + C_r \frac{\partial C_z}{\partial r} + \frac{C_\theta}{r} \frac{\partial C_z}{\partial \theta} + C_z \frac{\partial C_z}{\partial z} = - \frac{g_0}{\rho} \frac{\partial P}{\partial z} \quad (51c)$$

Equation (51) and its scalar forms (51a, 51b, and 51c) can be expressed in a slightly different form which shows the stream vorticity and its components in cylindrical coordinates. The derivation of these expressions is given in Appendix C.

$$\frac{\partial \zeta}{\partial t} - \zeta = (\nabla \times \zeta) + \nabla^2 C^2 = - \frac{g_0}{\rho} \nabla P \quad (52)$$

the r-component becomes:

$$\frac{\partial C_r}{\partial t} - \frac{C_\theta}{r} \left(\frac{\partial r C_\theta}{\partial r} - \frac{\partial C_r}{\partial \theta} \right) + C_z \left(\frac{\partial C_r}{\partial z} - \frac{\partial C_z}{\partial r} \right) + \frac{\partial}{\partial r} \zeta C^2 = - \frac{g_0}{\rho} \frac{\partial P}{\partial r} \quad (52a)$$

the θ -component becomes:

$$\frac{\partial C_\theta}{\partial t} - \frac{C_z}{r} \left(\frac{\partial r C_z}{\partial \theta} - \frac{\partial r C_\theta}{\partial z} \right) + \frac{C_r}{r} \left(\frac{\partial r C_\theta}{\partial r} - \frac{\partial C_r}{\partial \theta} \right) + \frac{\partial}{\partial \theta} \zeta C^2 = - \frac{g_0}{\rho} \frac{\partial P}{\partial \theta} \quad (52b)$$

the z-component becomes:

$$\frac{\partial C_z}{\partial t} - C_r \left(\frac{\partial C_r}{\partial z} - \frac{\partial C_z}{\partial r} \right) + \frac{C_\theta}{r} \left(\frac{\partial C_z}{\partial \theta} - \frac{\partial r C_\theta}{\partial z} \right) + \frac{\partial}{\partial z} \zeta C^2 = - \frac{g_0}{\rho} \frac{\partial P}{\partial z} \quad (52c)$$

where, $C^2 = C_r^2 + C_\theta^2 + C_z^2$.

The continuity equation in cylindrical coordinate form

is:

$$\frac{\partial P}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho C_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho C_\theta) + \frac{\partial}{\partial z} (\rho C_z) = 0 \quad (53)$$

6.2.2.2. Consideration of Quasi-Steady Flow

Even under normal steady running conditions, the flow

locally is usually quite unsteady as the rotor carries with it a network of bound and sheds trailing vortices and viscous wakes that subsequently interact with the stationary blade row. For aerodynamic design purposes involving undistorted flow and conventional machinery it usually suffices to ignore the detail of unsteady flow and to use time averages of the flow variables*.

In the case of a once-per-revolution cyclically recurring velocity nonuniformity, the validity of this simplification cannot be assumed on the basis of apriori arguments. The relative importance of flow unsteadiness was examined on the basis of a calculation involving the aerodynamically most sensitive elements of a compressor - the blades. It is not difficult to estimate the transient lift developed by the blades in response to a fluctuating velocity field, if it is assumed that the boundary layers on the blade surfaces adjust instantaneously to the potential velocity field of the main flow. Actually, the reaction of boundary layers to unsteady motion of the main stream is not well understood, and therefore the assumption of negligibly short time lag cannot be made easily. It is suggested, however, that for low frequency fluctuations (once-per-revolution) the boundary layer development will not be greatly out of phase with the main flow⁽³⁾, and that this assumption, one which other authors had to make as well^(15,16), can at least be assessed in retrospect of the solution.

In order to deal with the unsteady velocity field, it was

In the study of aerodynamic noise this expedient is, of course, not possible since the basic mechanism of noise generation originates from the local flow unsteadiness and is augmented by various interaction effects.

convenient to resolve it into two mutually perpendicular component velocity profiles. The transient lift developed by an airfoil in a transverse gust (a velocity perturbation normal to the chord) has been analyzed by Von Kármán and Sears⁽²⁶⁾ and Sears⁽²⁷⁾. Kemp and Sears⁽²⁸⁾ have subsequently applied the original analysis for a calculation of the transient lift of rotor and stator blades in axial flow turbomachines.

The fluctuating lift developed by an airfoil subjected to a streamwise velocity perturbation has been studied by Horlock⁽²⁹⁾. Morfey⁽³⁰⁾ dealt with orthogonality aspects of Horlock's work.

The results of these studies were used to estimate the unsteady lift increments associated with a typical inflow distortion for a typical reduced frequency $K = \frac{VC}{W}$ of 0.30, and this showed that the time dependent terms may be neglected, provided that instantaneous velocity and angle of attack data are used. In other words, the inflow distortion data may be used directly as input information for the segmented fan model.

The through-flow may, therefore, be represented reasonably also by quasi-steady flow if the time dependent terms are omitted and instantaneous values of the flow variables are used

as if they were locally "permanently" maintained. This amounts to a considerable simplification. Now the streamlines have become well defined space curves and can be used to define stream tubes containing part of the total mass flow.

The appropriate reduced form of the momentum equation is:

$$-\vec{C} \times (\nabla \times \vec{C}) + \nabla h_c^2 = -\frac{g_0}{\rho} \nabla P \quad (54)$$

and its scalar forms are:

$$-\frac{C_\theta}{r} \left(\frac{\partial r C_\theta}{\partial r} - \frac{\partial C_r}{\partial \theta} \right) + C_z \left(\frac{\partial C_r}{\partial z} - \frac{\partial C_z}{\partial r} \right) + \frac{\partial}{\partial r} h_c^2 = -\frac{g_0}{\rho} \frac{\partial P}{\partial r} \quad (54a)$$

$$-\frac{C_z}{r} \left(\frac{\partial C_z}{\partial \theta} - \frac{\partial r C_\theta}{\partial z} \right) + \frac{C_r}{r} \left(\frac{\partial r C_\theta}{\partial r} - \frac{\partial C_r}{\partial \theta} \right) + \frac{\partial}{\partial \theta} h_c^2 = -\frac{g_0}{\rho} \frac{\partial P}{\partial \theta} \quad (54b)$$

$$-C_r \left(\frac{\partial C_r}{\partial z} - \frac{\partial C_z}{\partial r} \right) + \frac{C_\theta}{r} \left(\frac{\partial C_z}{\partial \theta} - \frac{\partial r C_\theta}{\partial z} \right) + \frac{\partial}{\partial z} h_c^2 = -\frac{g_0}{\rho} \frac{\partial P}{\partial z} \quad (54c)$$

Or, in terms of its equivalent Crocco form (as derived in Appendix C):

$$\frac{1}{g_0 J} \vec{C} \times (\nabla \times \vec{C}) = \nabla h_t - T \nabla s \quad (55)$$

with the following scalar component equations:

$$\frac{1}{g_0 J} \left[\frac{C_\theta}{r} \left(\frac{\partial r C_\theta}{\partial r} - \frac{\partial C_r}{\partial \theta} \right) - C_z \left(\frac{\partial C_r}{\partial z} - \frac{\partial C_z}{\partial r} \right) \right] = \frac{\partial h_t}{\partial r} - T \frac{\partial s}{\partial r} \quad (55a)$$

$$\frac{1}{g_0 J} \left[\frac{C_z}{r} \left(\frac{\partial C_z}{\partial \theta} - \frac{\partial r C_\theta}{\partial z} \right) - \frac{C_r}{r} \left(\frac{\partial r C_\theta}{\partial r} - \frac{\partial C_r}{\partial \theta} \right) \right] = \frac{\partial h_t}{\partial \theta} - T \frac{\partial s}{\partial \theta} \quad (55b)$$

$$\frac{1}{g_0 J} \left[C_r \left(\frac{\partial C_r}{\partial z} - \frac{\partial C_z}{\partial r} \right) - \frac{C_\theta}{r} \left(\frac{\partial C_z}{\partial \theta} - \frac{\partial r C_\theta}{\partial z} \right) \right] = \frac{\partial h_t}{\partial z} - T \frac{\partial s}{\partial z} \quad (55c)$$

The equation of continuity is simply

$$\frac{1}{r} \frac{\partial}{\partial r} (r C_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (C_\theta) + \frac{\partial}{\partial z} (C_z) = 0 \quad (56)$$

where the velocity components C_r , C_θ , C_z in the equation above are all understood to be instantaneous values.

6.2.2.3. Special Features of the Assumed Flow Field and Implications on the Solution Method

The through-flow was shown to be quasi-steady in the sense that instantaneous values of the flow variables are used. Since the distortion is stationary with respect to the absolute reference frame, each stream tube can be assumed to carry steady flow which is part of the total mass flow. The flow within each stream tube is assumed to be inviscid and adiabatic*. This may be expressed mathematically as:

$$\vec{C} \cdot \nabla n_t = 0$$

$$\vec{C} \cdot \nabla s = 0$$

This means that the gradients above are normal to the local velocity vector. The local velocity vector is, of course, tangent to a streamline and streamlines and particle paths coincide under steady flow conditions. Therefore, the above gradients are normal to the stream tubes. Normality to the streamlines does not

*Strictly speaking, this assumption excludes consideration of all types of shear flows. To admit boundary layer development more realistic allowances would have to be made for the stream tubes adjacent to the end walls. Viscous wakes with large velocity defects also are not represented adequately; it is assumed that rotor wakes in a stationary reference frame are sufficiently well represented by time averages of the flow variables. The assumptions are deemed reasonable if the distortion wavelength is large relative to the stream tube width, as is the case for the type of inlet distortion considered here.

necessarily imply normality to the stream surface, as is illustrated in the sketch below; one case where it does imply this is axisymmetric flow, e.g. Novak's problem.

If a natural coordinate system (a streamline coordinate system) had been chosen instead of a cylindrical one, it would have been found that the normal derivative of static pressure (i.e. $\frac{\partial p}{\partial n} = \frac{\rho}{g_0} \frac{C^2}{R}$) is also normal to the stream surface containing the streamline. This particular feature of the normal derivative is valid for general three-dimensional flows as well as for axisymmetric flows, and clearly distinguishes itself from the pressure gradient which need not act perpendicularly to either

FIGURE 81. Relationship of gradient of enthalpy relative to a stream surface

streamline or surface. In fact, whenever area change is involved (in the streamline direction) the gradient cannot act perpendicularly to the streamline.

The operation indicated in the previous equations can be applied to Equation (55),

$$\frac{1}{g_0} \dot{\vec{C}} \cdot \dot{\vec{C}} \times (\nabla \times \dot{\vec{C}}) = \dot{\vec{C}} \cdot \nabla h_t - \tau \dot{\vec{C}} \cdot \nabla s$$

The left hand side of the above is

$$C_r [\dot{\vec{C}} \times (\nabla \times \dot{\vec{C}})] + C_\theta [\dot{\vec{C}} \times (\nabla \times \dot{\vec{C}})] + C_z [\dot{\vec{C}} \times (\nabla \times \dot{\vec{C}})] = 0$$

As C_r , C_θ , C_z are non-zero "constants" which make the identity true, and the acceleration components are not all zero, a linear dependence is implied. That is, each term must be zero on its own. Hence, from the rule of linear dependence we know that any two acceleration components are linearly independent and the third is redundant.

Therefore, if there exists any solution to the problem, it can be determined by considering any two of the scalar forms of the Equation (55), and the r and θ components are chosen for this purpose since a solution will be attempted with $z = \text{constant}$, i.e. in a chosen z -plane.

Novak⁽¹⁸⁾, in solving the axisymmetric case without circumferential distortion, was able to do this by changing to a new variable $C_{\text{meridional}}$. This velocity is the intermediate resultant of the velocity components C_r and C_z , i.e. the projection of the velocity in a meridional plane. This change of variable has special meaning in the axisymmetric case because $C_{\text{meridional}}$ is not a function of circumference and because the stream tubes are co-axial cylinders. In the three-dimensional problem $C_{\text{meridional}}$ has lost this special relevance and one has to seek a solution in more general terms.

The relationships (C-27a) and (C-27b) from Appendix C

are substituted in Equations (55a) and (b). After some rearrangement the following two linearly independent equations result:

In the r-direction:

$$\frac{\partial}{\partial r} C_z^2 - a_1 C_z^2 = b_1 \quad (57)$$

$$\text{where } a_1 = \frac{2}{C_z} \frac{\partial}{\partial z} C_r - \frac{1}{R} \frac{\partial}{\partial r} 0 \quad (57a)$$

$$b_1 = 2g_o J \frac{1}{0} \frac{\partial}{\partial r} (h_t 0) - (C_r^2 + C_\theta^2) \frac{1}{0} \frac{\partial}{\partial r} 0 - 4C_\theta \omega_z \quad (57b)$$

and the quantity 0, as derived in the Appendix C is

$$0 = e^{-\left(\frac{u_- - u_+}{c_p}\right)}$$

In the θ -direction:

$$\frac{\partial}{\partial \theta} C_z^2 - a_2 C_z^2 = b_2 \quad (58)$$

$$\text{where } a_2 = \frac{2}{C_z} \frac{\partial}{\partial z} (r C_\theta) - \frac{1}{0} \frac{\partial}{\partial \theta} 0 \quad (58a)$$

$$\text{and, } b_2 = 2g_o J \frac{1}{0} \frac{\partial}{\partial \theta} (h_t 0) - (C_r^2 + C_\theta^2) \frac{1}{0} \frac{\partial}{\partial \theta} 0 + 4C_r \omega_z r \quad (58b)$$

where ω_z is the z-vorticity term (Equation (C-7c))

$$2\omega_z = \frac{1}{r} \left(\frac{\partial}{\partial r} (r C_\theta) - \frac{\partial}{\partial \theta} C_r \right)$$

6.2.2.4. Choice of Solution Surfaces

The above Equations (57) and (58) are nonlinear partial differential equations that are difficult to solve in this form.

One possible approach employs Wu's S_1 or S_2 surfaces⁽¹⁷⁾.

These surfaces are mutually orthogonal and are stream surfaces.

A solution found on either, or both, surfaces automatically satisfies continuity as well. Wu's derivation is shown in Appendix C.

Since a stream surface is defined as one which is made up of streamlines, any quantity "q" on it can be considered to be a function of θ and z only because on the surface r can be expressed in terms of θ and z . This leads to Wu's special derivatives on the S_1 and S_2 stream surfaces at fixed axial stations that are the means for converting the partial differential Equations (57) and (58) into ordinary differential equations

$$\left. \frac{dq}{d\theta} \right|_{\substack{z = \text{const.} \\ S_1 = \text{const.}}} = \frac{\partial q}{\partial \theta} - rN \frac{\partial q}{\partial r} \quad (59a)$$

$$\left. \frac{dq}{dr} \right|_{\substack{z = \text{const.} \\ S_2 = \text{const.}}} = \frac{\partial q}{\partial r} - \frac{1}{rN} \frac{\partial q}{\partial \theta} \quad (59b)$$

where

$$N = - \frac{\frac{dq}{dr}}{\frac{dq}{d\theta}}$$

Thus, the flow along S_1 is aptly described by:

$$\frac{dC_z^2}{dr} - A_1(r, \theta, z) C_z^2 = B_1(r, \theta, z) \quad (60)$$

while the flow along S_2 is represented by

$$\frac{dC_z^2}{d\theta} - A_2(r, \theta, z) C_z^2 = B_2(r, \theta, z) \quad (61)$$

The coefficients are linear combinations of the previous set, i.e.

$$A_1 = a_1 - \frac{1}{rN} a_2 \quad (60a)$$

$$B_1 = b_1 - \frac{1}{rN} b_2 \quad (60b)$$

$$A_2 = -rNA_1 \quad (61a)$$

$$B_2 = -rNB_1 \quad (61b)$$

or, written out in detail:

$$\left. \begin{aligned} A_1 &= \left[\frac{2}{C_z} \frac{\partial}{\partial z} C_r - \frac{1}{\theta} \frac{\partial \theta}{\partial r} \right] + \frac{d\theta}{dr} \left[\frac{2}{C_z} \frac{\partial}{\partial z} (rC_\theta) - \frac{1}{\theta} \frac{\partial \theta}{\partial \theta} \right] \\ A_2 &= \left[\frac{2}{C_z} \frac{\partial}{\partial z} (rC_\theta) - \frac{1}{\theta} \frac{\partial \theta}{\partial \theta} \right] + \frac{dr}{d\theta} \left[\frac{2}{C_z} \frac{\partial}{\partial z} C_r - \frac{1}{\theta} \frac{\partial \theta}{\partial r} \right] \\ B_1 &= \left[2g_o J \frac{1}{\theta} \frac{\partial}{\partial r} (h_t \theta) - (C_r^2 + C_\theta^2) \frac{1}{\theta} \frac{\partial \theta}{\partial r} - 4C_\theta \omega_z \right] \\ &\quad + \frac{d\theta}{dr} \left[2g_o J \frac{1}{\theta} \frac{\partial}{\partial \theta} (h_t \theta) - (C_r^2 + C_\theta^2) \frac{1}{\theta} \frac{\partial \theta}{\partial \theta} + 4rC_r \omega_z \right] \\ B_2 &= \left[2g_o J \frac{1}{\theta} \frac{\partial}{\partial \theta} (h_t \theta) - (C_r^2 + C_\theta^2) \frac{1}{\theta} \frac{\partial \theta}{\partial \theta} + 4rC_r \omega_z \right] \\ &\quad + \frac{dr}{d\theta} \left[2g_o J \frac{1}{\theta} \frac{\partial}{\partial r} (h_t \theta) - (C_r^2 + C_\theta^2) \frac{1}{\theta} \frac{\partial \theta}{\partial r} - 4C_\theta \omega_z \right] \end{aligned} \right\} \quad (62)$$

Since the coefficients are not functions of the independent variable alone, and there are products of the unknown function C_z , these equations are nonlinear and hence difficult to solve. Moreover, either one together with the equation of continuity make a simultaneous set.

6.2.2.5. Possible Solution Methods

Two possible solution methods of the equilibrium equations suggest themselves.

A) Either or both equations can be solved by a method of successive approximations while satisfying simultaneously continuity if A_1 , A_2 , B_1 , and B_2 are treated as known functions of the independent variable^{*}. The "linearized" forms of the equations are therefore:

$$\frac{dC_z^2}{dr} - A_1(r)C_z^2 = B_1(r) \quad (63)$$

$$\frac{dC_z^2}{d\theta} - A_2(\theta)C_z^2 = B_2(\theta) \quad (64)$$

with constant coefficients defined as previously indicated. The solution of a first order non-homogeneous equation with constant coefficients is straightforward. It is simply the general solution formed by the linear combination of a complementary function u and a particular solution v , i.e. $q = u + v$.

$$\frac{dq}{dx} - Aq = B$$

The complementary function is $C_1 e^{Ax}$ and the particular solution is C_2 , so that the general solution is $q = C_1 e^{Ax} + C_2$. The value of C_2 is $-\frac{B}{A}$ and comes directly from back substitution. The constant C_1 can be evaluated from boundary conditions. As the coefficients A and B are known not to be constants, the solution is cast in finite difference form and the constant C_1 is evaluated using the boundary condition of the backward step,

^{*}This technique of dealing with some nonlinear ordinary differential equations is a standard one and was the one employed by Jansen and Moffett in the axisymmetric problem⁽¹⁹⁾.

$$C_1 = \left(q_j + \frac{B_j}{A_j} \right) e^{-A_j x}$$

Therefore,

$$q_{j+1} = \left[\left(q_j + \frac{B_j}{A_j} \right) e^{-A_j x} \right] e^{A_{j+1}(x+\Delta x)} - \frac{B_{j+1}}{A_{j+1}}$$

While the coefficients should be evaluated at the boundaries it is assumed that they are constant throughout the interval Δx and, consequently, the solution of q is expressed in the following form:

$$q_{j+1} = q_j e^{A \Delta x_j} - \frac{B}{A} (1 - e^{A \Delta x_j}).$$

By analogy, the solutions of C_z^2 of the equilibrium equations on the S_1 and S_2 surfaces are:

$$C_{z_{i+1}}^2 = C_{z_i}^2 e^{A_1 \Delta r_i} - \frac{B_1}{A_1} (1 - e^{A_1 \Delta r_i}) \quad (65)$$

$$C_{z_{j+1}}^2 = C_z^2 e^{A_2 \Delta \theta_j} - \frac{B_2}{A_2} (1 - e^{A_2 \Delta \theta_j}) \quad (66)$$

In order that C_z^2 be the general solution it is necessary that C_z should simultaneously satisfy both equilibrium and continuity.

The intervals Δr_i and $\Delta \theta_j$ may consequently be regarded as the local stream surface spacing (in any z -plane) and the corresponding pairs of stream surfaces S_1 and S_2 as the surface of the stream tube ji . The continuity requirement for constant rate of flow gives the additional relationship between cross-sectional area

and velocity, i.e.

$$\dot{m}_{ij} = \rho_{ij} C_{z_{ij}} A(z)_{ij} \quad (67)$$

where $A(z)_{ij}$ is the axial projection of the stream tube area at the particular z-station and is suitably related to (Δr_i) and $(r\Delta\theta_j)$.

It is to be noted that the method of finite differences is consistent with thinking of the whole flow field as a collection of stream tubes and leads naturally to considering the whole flow field as an array of individual fan segments, working in parallel with another and sharing common inlet, interstage, and outlet fields.

B) Instead of solving Equations (60) and (61) in finite difference form, as indicated above, an alternate approach is possible. The coefficients can be simplified in terms of known variables and an algebraic equation results.

Away from torque producing elements and for isentropic steady flow the following variables are constant along streamlines: (rC_θ) , (θ) , (h_t) . The substantial and local derivatives of any of these variables are, therefore, zero. For example:

$$C_r \frac{\partial}{\partial r} (rC_\theta) + C_\theta \frac{\partial}{\partial \theta} (rC_\theta) + C_z \frac{\partial}{\partial z} (rC_\theta) = 0 \quad (68)$$

Dividing by C_z and using the stream surface tangency condition

$$dr:r d\theta:dz = C_r:C_\theta:C_z \quad (69)$$

this relationship can be re-arranged as follows:

$$\frac{\partial}{\partial z} (rC_\theta) = - \frac{C_\theta}{rC_z} \left[\frac{\partial}{\partial \theta} (rC_\theta) + \frac{dr}{d\theta} \frac{\partial}{\partial r} (rC_\theta) \right]$$

The bracketed expression has the same form as Equation (59.a):

$$\therefore \frac{\partial}{\partial z} (rC_\theta) = - \frac{C_\theta}{rC_z} \frac{d}{d\theta} (rC_\theta) \bigg|_{\substack{z = \text{constant} \\ S_1 = \text{constant}}} \quad (70)$$

where the total differential is to be evaluated in a z -constant plane along the stream surface S_1 , i.e. $r = f(\theta)$. Similarly,

$$\frac{1}{\theta} \left(\frac{\partial \theta}{\partial \theta} + \frac{dr}{d\theta} \frac{\partial \theta}{\partial r} \right) = \frac{1}{\theta} \frac{d\theta}{d\theta} \bigg|_{\substack{z = \text{const.} \\ S_1 = \text{const.}}} \quad (71)$$

$$2g_o J \frac{1}{\theta} \left[\frac{\partial}{\partial \theta} (h_t \theta) + \frac{dr}{d\theta} \frac{\partial}{\partial r} (h_t \theta) \right] = 2g_o J \frac{1}{\theta} \frac{d}{d\theta} (h_t \theta) \bigg|_{\substack{z = \text{const.} \\ S_1 = \text{const.}}} \quad (72)$$

$$(C_r^2 + C_\theta^2) \frac{1}{\theta} \left[\frac{\partial \theta}{\partial \theta} + \frac{dr}{d\theta} \frac{\partial \theta}{\partial r} \right] = (C_r^2 + C_\theta^2) \frac{1}{\theta} \frac{d\theta}{d\theta} \bigg|_{\substack{z = \text{const.} \\ S_1 = \text{const.}}} \quad (73)$$

Consider the term $\frac{\partial}{\partial z} C_r$:

The θ -vorticity is available from considerations of secondary flow

$$2\omega_\theta = \frac{\partial}{\partial z} (C_r) - \frac{\partial}{\partial r} (C_z)$$

$$\therefore \frac{\partial}{\partial z} C_r = 2\omega_\theta + \frac{\partial}{\partial r} C_z \quad (74)$$

The partial derivative with respect to r may be difficult to evaluate if the stream surfaces are badly twisted, but it can be expressed in alternate form:

$$\frac{\partial^2}{\partial r^2} C_z = \frac{C_0}{r^2} \left(\frac{\partial}{\partial r} C_z \right) \Big|_z = \frac{\partial^2}{\partial z^2} C_z \quad (75)$$

From the v-vorticity relationship one has

$$\frac{\partial}{\partial z} C_z = 2r\omega_r + \frac{\partial}{\partial z} (rC_\theta)$$

$$= 2r\omega_r + \frac{C_0}{r^2} \frac{\partial}{\partial r} (rC_\theta) \Big|_z$$

$$\frac{\partial^2}{\partial z^2} C_z = 2\omega_r + \frac{C_0}{r^2} \frac{\partial}{\partial r} C_z \Big|_z = \frac{C_0}{r^2} \omega_r + \frac{C_0^2}{r^2} \frac{\partial}{\partial r} (rC_\theta) \Big|_z \quad (76)$$

The through-flow model chosen has no means of generating radial vorticity because radial displacement of streamlines is assumed as zero and in the inlet, where radial displacements are apt to be large, the radial vorticity is zero.

$$\therefore \omega_r = 0$$

The terms are substituted in the coefficients A and B

$$\left. \begin{aligned} A_2 &= -\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{C_0}{r^2} \frac{\partial}{\partial r} C_z \right) + \frac{C_0}{r^2} \frac{\partial}{\partial z} C_z \\ B_2 &= 2r\omega_r + \frac{1}{r} \frac{\partial}{\partial r} (rC_\theta) = (C_r^2 + C_\theta^2) \frac{1}{r} \frac{\partial}{\partial r} \end{aligned} \right\} \quad (77)$$

The coefficients A_1 , B_1 may be simplified in similar manner. The θ -differential equation takes the following form:

$$\frac{d}{d\theta} C_z^2 - \left(-\frac{1}{\theta} \frac{d\theta}{d\theta} + \frac{4rC_r}{C_\theta C_z} \omega_\theta + \frac{2}{C_z} \frac{d}{d\theta} C_z \right) C_z^2$$

$$= 2g_o J \frac{1}{\theta} \frac{d}{d\theta} (h_t \theta) - (C_r^2 + C_\theta^2) \frac{1}{\theta} \frac{d\theta}{d\theta}$$

The $\frac{d}{d\theta} C_z^2$ is seen to cancel out and the reduced form is

$$\frac{C_z^2}{\theta} \frac{d\theta}{d\theta} - \frac{4rC_r C_z}{C_\theta} \omega_\theta = 2g_o J \frac{1}{\theta} \frac{d}{d\theta} (h_t \theta) - (C_r^2 + C_\theta^2) \frac{1}{\theta} \frac{d\theta}{d\theta}$$

or,

$$C_z^2 - \frac{4rC_r \omega_\theta}{C_\theta \frac{1}{\theta} \frac{d\theta}{d\theta}} C_z - \frac{2g_o J \frac{d}{d\theta} (h_t \theta)}{\frac{d\theta}{d\theta}} + (C_r^2 + C_\theta^2) = 0 \quad (78)$$

The variables except for C_z , ω_θ , and C_r are obtained from the fan performance mean line design method. The latter two variables are calculated from the secondary flow model. Normally there would be a contribution to C_r from the primary flow as calculated from radial equilibrium considerations, i.e. the

$\frac{d}{dr} \left| \begin{matrix} z \\ \psi_1 \end{matrix} \right| = \text{const.}$ equation; however, the solution of two simultaneous

equations in the z -plane is difficult for convergence reasons. On

the strength of the observation that the major crossflow distortion effect is associated with the circumferential direction the expedient is used to relax the radial equilibrium requirement.

As a result of this approach only the secondary flow (including wall vorticities) remains as the agent causing radial velocities.

It is possible now to solve Equation (78) as a quadratic for C_z . This can be achieved by resorting again to the segmented

fan concept and dividing the inlet annulus plane up into a number of cells of equal width $r\Delta\theta$ and the $z = \text{constant}$ planes downstream of the rotor and stator into areas defined by continuity requirements for stream tubes originating in the inlet. The stream variables are evaluated at the cell centres and treated as temporary constant averages for each cell in a z -plane.

Of all the variables appearing in the coefficients of Equations (90) or (19), only the radial velocity C_r and the θ and z vorticities cannot be deduced from the mean line design method for fan performance. As previous discussions indicated, these quantities are by no means negligible and now need to be considered in the light of the velocity field induced by the fan rotor and stator.

6.2.3. Blade Row Generated Vorticity

In the current state of the art of treating the vortex motions inside turbomachines analytically, it seems necessary to have some previous knowledge of the primary fluid motions. This condition is consistent with the Helmholtz law (vortex lines are associated with fluid lines) and Kelvin's circulation theorem (the circulation around a closed fluid line remains constant), both of which are founded on the principle that vorticity is carried by the primary flow.

Smith⁽³¹⁾ suggested that the primary flow be defined arbitrarily as the idealized axisymmetric main flow calculated on the basis of standard cascade deflection and loss data as an approximation of the actual blade to blade flow. The primary flow

may or may not be rotational. The difference between the actual and the so-defined primary flow is called secondary flow and therefore embraces all aspects of vortex motion generated within the blade row, including both viscous and potential shear flow effects.

In actual fact, however, these simple definitions are not entirely satisfactory because some internal flows are so highly three-dimensional that no clear distinction between the types of motion is possible. Moreover, the generation and convection of identifiable vorticity in alternate rotating and stationary stages is greatly complicated by the fact that the main flow can never be steady itself due to potential interaction and wake chopping action. Von Kármán and Sears⁽²⁶⁾ investigated the aspects of instantaneous lift (steady lift plus unsteady lift) of isolated airfoils in unsteady flow, and showed that discrete vortex sheets are always created. Normally in turbomachine through-flow calculations the complexity of the actual problem is relaxed by neglecting this unsteady production of vorticity⁽³¹⁾ and treating the flow as if the flow variables were time averages.

A large amount of work has been done on various analytical treatments of the secondary flow problem in the axial compressor. Recent excellent surveys of this work by Woods⁽²⁴⁾ and Lakshminarayana and Horlock⁽³²⁾ demonstrate that while there are a variety of calculation approaches, most^(31,33,34) relate to plane and annular cascade flow. In most cases, in order to reduce the complexity of the problem, restrictive assumptions had to be made that ultimately compromised the generality of the

results. There is, for example, considerable uncertainty about applying these results to rotating blade rows. There are approaches especially for rotating blade rows^(35,36), but their application is difficult unless the detailed blade to blade primary flow field is known beforehand and, what is far worse, such methods have neither been applied to a general case nor compared with experimental data. Finally, the success of any of these theories cannot be considered complete because, firstly, viscous and turbulent stresses are ignored and, secondly, a full explanation of losses of total pressure that are associated with secondary flow has not been achieved.

Woods' review⁽²⁴⁾ of the origins of vorticity in the axial compressor showed that vorticity is generated by surface or near-surface flows involving boundary layer action and in the inviscid flow region when the primary flow carries vorticity and is deflected.

6.2.3.1. Origin in the Viscous Flow Region

Corner vortices will be formed at the junctions of the blade suction surfaces and end walls. They are fed low energy material by two boundary layers, as illustrated in Figure 82. Tip clearance will generally allow some boundary layer fluid to escape experiencing flow deflection and result in a reduction of the bound circulation at the tip and a tip (shed) vortex on the suction side of the blade. At the same time, the pressure surfaces of the compressor blading have been observed to roll up some of the casing wall boundary layer by scraping action into a stream-wise vortex that tends to offset the tip clearance vortex. Some

of the material accumulating in the corner vortices has been flowing along the blade span as a result of radial pressure gradients (and centrifugal force in the case of the rotor). This radial flow, which makes only a very small direct contribution to vorticity, feeds the hub corner vortex in the case of the stator and either hub or tip corner vortex depending on the blading design in the case of the rotor.

FIGURE 82. Mechanisms of vortex generation in an axial flow compressor rotor (adapted from Ref. 24)

The scraping and the radial-flow effects have been observed experimentally only. A limited amount of success has been achieved in modelling tip leakage and corner vortex flows, but these models usually involve the knowledge of detailed initial and primary flow conditions. The tip clearance dimensions are generally not known beforehand and are, for example, not applicable in the

case of a shrouded lifting fan with a tip turbine. Consequently, the generation of vorticity due to boundary layer development will not be accounted for in the present model. Progress in the analytical treatment of secondary flows has been possible on the basis of various inviscid flow models and these contributions are now to be examined.

6.2.3.2. Origin in the Inviscid Flow

Hawthorne and Novak⁽³³⁾ have shown when a nonuniform steady flow with small shear or vorticity, but with a large perturbation in velocity, enters a stationary cascade that there are three components of vorticity downstream of the cascade outlet plane. The first is a distributed passage vorticity that entered the cascade as an "axisymmetric" vorticity, and regardless of how it came to exist originally, is convected by the primary (irrotational) flow. The other two are trailing shed and trailing filament vorticities, and are concentrated in the vortex wakes of the blades. The strength of the vortex wake leaving the blades of turbomachines apparently cannot simply be determined by the well known Prandtl wing theorem (which states that the strength of the vortex wake shed is equal to the spanwise gradient in circulation) because this theorem assumes the approach flow to be irrotational. Instead, as Smith⁽³¹⁾ demonstrated, the strength is approximately proportional to $\frac{1}{W_m \rho} \frac{d}{dy} (\Gamma_v W_m \rho)$ i.e.:

$$\frac{1}{W_m \rho} \frac{d}{dy} (\Gamma_v W_m \rho) = \frac{d \Gamma_v}{dy} + \frac{\Gamma_v}{\rho} \cdot \frac{d \rho}{dy} + \frac{\Gamma_v}{W_m} \cdot \frac{d W_m}{dy} \quad (79)$$

where Γ_v is the vane circulation involving the uniform flow only. The second term is normally negligible even for compressible flows in comparison with the first and last terms⁽³¹⁾. The first term is the familiar Prandtl wing term, and the last called trailing filament circulation in Reference (33), arises because the incoming flow is rotational relative to the blade row. Vortex filaments originally transverse in the approach flow become stretched in the blade wakes because fluid particles travelling with different velocities over the suction and pressure surfaces of the blades have different arrival times.

6.2.3.3. Initial Value of Vorticity

As demonstrated in Appendix D, the absolute flow entering the rotor, although highly distorted, is nevertheless irrotational. The relationship for change of reference frames from absolute to relative for vorticity vectors is

$$\mathbf{v}_R \times \hat{\mathbf{w}} = \mathbf{v} \times \hat{\mathbf{c}} - 2\hat{\mathbf{n}} \quad (80)$$

with the absolute vorticity entering the rotor being zero, the relative vorticity is equal to a solid body rotation of $+2\hat{\mathbf{n}}$, because the direction of wheel rotation $\left(\frac{\hat{\mathbf{n}}}{n}\right)$ is assumed to be counter to increasing θ . Comparing components of the curls of $\hat{\mathbf{w}}$ and $\hat{\mathbf{c}}$ of the above equation we find that

$$\frac{1}{r} \frac{\partial(rw_\theta)}{\partial r} - \frac{\partial c_r}{r \partial \theta} = 2n \quad (81)$$

and, as was shown in Appendix D, $\frac{\partial c_r}{\partial \theta} \neq 0$. Let the initial value of vorticity be designated w_1 , which in this particular application happens to be a z-vorticity as well.

The initial value of vorticity for the stator ($\dot{\omega}_3$) depends on the absolute vorticity generated by the rotor and is therefore not exclusively a z-vorticity.

6.2.3.4. Derivation of Blade Row Generated Vorticity

The limited amount of success shown by the various available analytical inviscid secondary flow models demonstrates that none can claim a clear advantage and, consequently, further work is required, particularly with respect to rotor flows. The approach chosen here follows Dixon's⁽³⁶⁾ work while maintaining the fundamental concepts featured by Smith⁽³¹⁾ and Hawthorne and Novak⁽³³⁾.

Smith devised a secondary flow channel theory for steady compressible flow with axisymmetric inlet vorticity where the imagined channel boundaries are formed by the axisymmetric flow solution stagnation streamlines and blade surfaces. He made three assumptions about the flow model: there was to be no rotation of the Bernoulli surfaces (i.e. surfaces of constant total pressure), no bending of vortex filaments (these were assumed to have an average strength), and the blade row was stationary (no centrifugal effects). The theory was formulated to yield secondary vorticity as the difference between the actual and the axisymmetric vorticities at the blade row outlet. By definition, therefore, the calculated secondary flow was not referenced to the blade surfaces, but instead to the outlet primary flow.

Hawthorne and Novak pointed out, however, that in order to calculate radial velocities the total vorticity and not just the secondary vorticity is required. Dixon's method for calculating

passage vorticity satisfies this requirement directly. It affords a more convenient approach, yet being consistent with Smith's method and embodying all assumptions made about the flow. The salient features of Dixon's derivation, modified to include effects of compressibility, are now presented.

Consider a vortex filament entering a blade row passage. The Helmholtz theorem requires that the vorticity per unit length of a vortex line divided by its density must remain constant as the filament is convected through the passage. This requirement is met by the normal to stream and meridional vorticities for example, as can be seen from Figure 83.

$$\frac{\omega_{n1}}{\rho_1 s_1 \cos \alpha_1} = \frac{\omega_{n2}}{\rho_2 s_2 \cos \alpha_2} \quad (82)$$

and

$$\frac{\omega'_{t1}}{\rho_1 AF \cos \alpha_1} = \frac{\omega'_{t2}}{\rho_2 GD' \cos \alpha_2} \quad (83)$$

where the prime indicates associations with convected properties. Since GD' is a fluid line that was AF before convection on a

streamline away from the blade surfaces,

$$\frac{AF}{GD'} = \frac{W_1}{W_2} \quad (84)$$

in order to satisfy the conservation of mass and mass rate of flow requirements.

$$\therefore \frac{\omega'_{t1}}{\rho_1 W_1 \cos \alpha_1} = \frac{\omega'_{t2}}{\rho_2 W_2 \cos \alpha_2} \quad (85)$$

FIGURE 83. Geometry of vorticity convected through a blade row

The following identity may be expressed:

$$\frac{\frac{\omega_{n1}}{\rho_1 s_1 \cos \alpha_1}}{\frac{\omega_{n2}}{\rho_2 s_2 \cos \alpha_2}} = \frac{\frac{\omega_{t1}}{\rho_1 W_1 \cos \alpha_1}}{\frac{\omega_{t2}}{\rho_2 W_2 \cos \alpha_2}} \quad (86)$$

The convected downstream meridional vorticity is therefore:

$$\omega_{t2} = \frac{W_2}{W_1} \frac{s_1}{s_2} \frac{\omega_{n2}}{\omega_{n1}} \omega_{t1} \quad (87)$$

This vorticity may be expressed in terms of its streamwise and normal-to-stream component and is:

$$\omega_t = \omega_n \sin \alpha + \omega_s \cos \alpha \quad (88)$$

Combining the continuity relationship in the relative reference frame with Equation (84) one obtains:

$$\omega_{n2} W_2 \Delta y_2 = \omega_{n1} W_1 \Delta y_1 \quad (89)$$

Equations (88) and (89) are substituted in Equation (87).

Continuity based on axial area is used and after some algebraic operations the downstream convected streamwise vorticity is:

$$\omega_{s2} = \frac{A_1}{A_2} \left[\frac{\sin \alpha_1}{\sin \alpha_2} \omega_{n1} + \frac{\cos \alpha_1}{\cos \alpha_2} \omega_{s1} \right] - \frac{\rho_2 s_2 \sin \alpha_2}{\rho_1 s_1 \cos \alpha_2} \omega_{n1} \quad (90)$$

or,

$$\omega_{s2} = \frac{A_1}{A_2} \frac{1}{\cos \alpha_2} \omega_{t1} - \frac{\rho_2 s_2 \sin \alpha_2}{\rho_1 s_1 \sin \alpha_1} \omega_{n1} \quad (91)$$

The actual streamwise vorticity includes another important contribution - the trailing filament vorticity. This quantity was dealt with in detail by Smith⁽³¹⁾ and Hawthorne and Novak⁽³²⁾, but the Dixon method used did not demonstrate its direction. The

salient features of Smith's work are therefore incorporated here in a rederivation.

It is assumed that the relative flow approaching the blade row has a vorticity normal to the stream direction such that $\vec{W}_1 \times \vec{\omega}_{n1} \parallel \vec{y}$, where y is pointed radially outward from the fan centreline, in the direction of increasing blade span, i.e. $\vec{\omega}_{n1}$, $\vec{W}_1$, $\vec{y}$ form a right handed co-ordinate system, as shown in Figure 84.

Consider a circuit ABCDEFA enclosing a blade but a small distance away from its surface so that BAF and CDE are separated

FIGURE 84. Placement of circuit ABCDEFA enclosing the blade and fluid particles of the same circuit a moment before upstream (after Smith(11))

in the span direction and lie on axisymmetric surfaces, with BC||FE at the trailing edge. Before the fluid particles are in this position around the blade they occupied the shown position upstream of the blade in the primary flow A'B'C'D'E'F'A'. Fluid particles F and B, side by side at the trailing edge, are displaced radially by δy upstream because: (a) the induced cross velocity C_r has not yet acted on them, and (b) there may be an upstream streamwise vorticity ω_{01} that would cause displacement in the y and n directions. The same fluid particles (F and B) when upstream were also separated by a streamwise distance f_1 because owing to larger velocities over the suction side of the blade than at the pressure side F' took less time to arrive at F than B' took to reach B. This time difference or drift time, as Hawthorne and Novak⁽³³⁾ called it, is:

$$\Delta t = \int_A^B \frac{ds}{W_0} - \int_A^{F'} \frac{ds}{W_0} \quad (92)$$

where s is the blade surface distance and W_0 the local velocity. This time difference is obviously related to the effective blade lift and, according to Smith, may be expressed for a cascade approximately as:

$$\Delta t = \frac{\Gamma VA}{W_m^2} \quad (93)$$

The streamwise separation f_1 is, therefore, approximately equal to:

$$f_1 = W_1 \Delta t = W_1 \frac{\Gamma VA}{W_m^2} \quad (94)$$

Consider the circuit C'D'E'F'A'B'C'. The circulation around this path is equal to the sum of circulations around three paths:

$$\Gamma_{C'D'E'F'A'B'C'} = \Gamma_{C'D'E'C'} + \Gamma_{E'F'B'C'E'} + \Gamma_{D'A'B'F'} \quad (95)$$

Smith⁽³¹⁾ assumed that

$$\Gamma_{C'D'E'C'} = - \Gamma_{F'A'B'F'}$$

which is justified if area A'F'B' = area C'E'D' and $\dot{\omega}_1 = f(y)$.

This means that

$$\Gamma_{C'D'E'F'A'B'C'} = \Gamma_{E'F'B'C'E'} \quad (96)$$

where:

$$\Gamma_{E'F'B'C'E'} = \dot{\omega}_1 \cdot \text{Area}_{E'F'B'C'E'} \quad (96)$$

$$= \omega_{n1} f_1 \Delta y_1 \quad (97)$$

$$= \omega_{n1} \frac{W_1 \Gamma_{VA}}{W_m} \Delta y_1 \quad (98)$$

FIGURE 85. Summation of closed fluid line circulations

Consider circuit CDEFABC. As previously,

$$\Gamma_{CDEFABC} = \Gamma_{CDEC} + \Gamma_{EFBCE} + \Gamma_{FABF} \quad (99)$$

$$= \Gamma_{CDEC} + \Gamma_{EFBCE} - \Gamma_{BAFB}$$

$$= \Gamma_{CDEC} + \Gamma_{EFBCE} - \left(\Gamma_{CDEC} + \frac{\partial \Gamma_{CDEC}}{\partial y} \Delta y_2 \right)$$

$$= - \frac{\partial \Gamma_{CDEC}}{\partial y} \Delta y_2 + \Gamma_{EFBCE} \quad (100)$$

The Kelvin circulation theorem requires that for steady flow the circulation of a fluid circuit remains constant in time and the Helmholtz theorem associates vorticity with the same fluid particles for all time. Consequently,

$$\Gamma_{C'D'E'F'A'B'C'} = \Gamma_{CDEFABC} \quad (101)$$

and,

$$\omega_{n1} \frac{W_1}{W_m} \frac{\Gamma_{VA}}{2} \Delta y_1 = - \frac{\partial \Gamma_{CDEC}}{\partial y} \Delta y_2 + \Gamma_{EFBC} \quad (102)$$

The above relationship is the trailing filament circulation and was derived by Smith for an isolated aerofoil. It shows the relationship between upstream normal to stream vorticity, the Prandtl wing spanwise variable circulation (first R.H.S. term) which is independent of stream vorticity and the generated wake vorticity (second R.H.S. term). The sense indicated in this equation is important. The Prandtl wing term results in a shed vorticity with the same direction as $\vec{W}_2$ if lift decreases with increasing y ($\frac{\partial \Gamma}{\partial y}$ is a negative quantity), the direction of the trailing filament circulation evidently depends on the blade lift ($\vec{L}$) and the normal to stream vorticity upstream of the blade. If $\vec{L} \times (\vec{W} \times \vec{\omega}_{n1})$ is in the direction of $-\vec{W}_m$, then the trailing filament circulation has the same direction as $-\vec{W}_2$.

For the rotor, therefore, which has an initial relative vorticity of 2Ω , the trailing filament circulation is directed parallel to $\vec{W}_2$ and upstream, as indicated in Figure 84.

Using Equation (89) and after dividing the circulation by the outlet flow area, one obtains the equivalent vorticity

distributed over the appropriate passage element ($\Delta y_2 s_2 \cos \alpha_2$) due to trailing filament circulation:

$$\omega_f = \frac{\omega_{n2} W_2 \frac{\Gamma_{VA}}{W_m^2} \Delta y_2}{\Delta y_2 s_2 \cos \alpha_2} \quad (103)$$

or,

$$\omega_f = \frac{\omega_{n1} W_1}{s_2 \cos \alpha_2} \frac{\Gamma_{VA}}{W_m^2} \frac{\Delta y_1}{\Delta y_2} \quad (104)$$

where ω_f is a negative quantity for the rotor here. It assumes the same sign as $\vec{L} \times (\vec{W}_3 \times \vec{\omega}_{n3})$ in case of the stator.

Assuming that the circumferential velocity distortion does not introduce serious errors in ω_f , the actual streamwise vorticity is therefore equal to the sum of the previous vorticities:

$$\omega_{s2} = \omega_f + \omega_{s2} \quad (105)$$

$$= \underbrace{\left[W_1 \frac{\Gamma_{VA}}{W_m^2} \right] \frac{\Delta y_1}{\Delta y_2} \frac{\omega_{n1}}{s_2 \cos \alpha_2}}_{\text{negative quantity}} + \frac{A_1}{A_2} \left[\frac{\sin \alpha_1}{\sin \alpha_2} \omega_{n1} + \frac{\cos \alpha_1}{\cos \alpha_2} \omega_{s1} \right] - \frac{\rho_2 s_2}{\rho_1 s_1} \frac{\sin \alpha_2}{\cos \alpha_1} \omega_{n1} \quad (106a)$$

$$= \underbrace{\left[W_1 \frac{\Gamma_{VA}}{W_m^2} \right] \frac{\Delta y_1}{\Delta y_2} \frac{\omega_{n1}}{s_2 \cos \alpha_2}}_{\text{negative quantity}} + \frac{A_1}{A_2} \frac{\omega_{t1}}{\cos \alpha_2} - \frac{\rho_2 s_2}{\rho_1 s_1} \frac{\sin \alpha_2}{\cos \alpha_1} \omega_{n1} \quad (106b)$$

Smith⁽³¹⁾ concluded that the magnitude of mainstream secondary flow is inversely proportional to the number of blades, and for the large number of blades associated with turbomachinery little error is incurred if it is assumed that the actual blade

circulation (Γ_{VA}) is equal to the circulation in uniform flow (Γ_V).

The total streamwise vorticity at the blade row outlet comprises several contributions. On the R.H.S. of Equation (106b), the first term is the trailing filament circulation and is present only if the blade row has lift, the second term is the streamwise component of the convected relative axisymmetric initial vorticity, the last term accounts for the directional change from inlet. Equation (106b) is not equal to Smith's so-called secondary flow vorticity for the reasons already explained; but it is a simple task to cast it again completely in terms of meridional vorticities and thereby achieve agreement. In Smith's form the secondary vorticity was shown to be equal to the sum of trailing filament and trailing shed vorticity. Equation (106b) includes both these as well, however, the latter vorticity does not show up directly. When the inflow is distorted, as in the present case, there are additional terms to the Prandtl variable circulation terms.

As pointed out by Hawthorne⁽³⁷⁾, the trailing filament circulation appears always of opposite sign to the convected passage vorticity. Equation (106b), when written for the present rotor (i.e. with the trailing filament circulation being a negative quantity), shows that the passage vorticity is actually being reduced.

The rotor generated vortex wake has been used successfully in classical propeller theory involving axisymmetric inlet flows to deduce the slipstream velocity field. Dyne⁽¹²⁾ has shown, for

example, that the helical wakes of a ducted propeller in irrotational flow may be decomposed into a network of ring vortices spaced axially at a pitch distance of $\frac{2\pi R}{N} \tan \alpha_2$ and axial trails leading away from the propeller plane. The unsteady effect of the finite-bladed propeller and its rotating helical wake was averaged out by distributing the actual vortex lines as if the propeller had a great number of blades with the same total circulation, and by making the vortex lines stationary with respect to the duct.

This artifice has, however, not been used on rotor stator combinations, and its advantages are lost if the flow is not axisymmetric.

The circumferential inflow distortion results in a circumferentially nonuniform generated vorticity distribution, as can be seen from the input variables (relative velocity and air angle ϕ) in Equation (106). Such a distribution, even if approximated by a piecewise uniform helical patchwork of vortex lines, as Figure 86

FIGURE 86. Convection of generated vorticity in relative and absolute space

lines, as Figure 86

suggests, is really not amenable to decomposition.

The advantage of distributing the vortex network to achieve a change in reference frame is still valid and can be used here.

Because velocity calculations are to be made

in absolute space the rotor generated vorticity must be re-referenced using Equation (80):

$$\omega_{z2} = \omega_{s2} \cos \alpha_2 - \omega_{n2} \sin \alpha_2 - 2\Omega \quad (107a)$$

$$\omega_{\theta 2} = \omega_{s2} \sin \alpha_2 - \omega_{n2} \cos \alpha_2 \quad (107b)$$

The minus sign in front of the 2Ω stems from a negative direction of wheel speed. These are the absolute vorticity components associated with a fluid line being transported in the relative reference frame, as shown by

the vector diagram in Figure 87. The vector addition has the result of developing the rotating reference frame into a "two-dimensional" one, but the vorticity is still being convected by the relative motion, i.e. in the direction of $\hat{W}_2$. It is more

FIGURE 87. Removal of effect of rotation of rotor

convenient to calculate the

vorticity in the absolute reference frame, where it is convected in the direction of $\hat{C}_3$, see Figure 88. While the axial transport rate in the two reference frames is identical the streamline spacing is not: the fluid line $a'b'$ in relative motion has a different length ab in absolute motion. In the relative reference frame the vorticity has the direction and magnitude of

$$\phi_2 = \cos^{-1} \left(\frac{\omega_{\theta 2}}{\omega_2} \right) \quad (108a)$$

$$\omega_2 = \sqrt{\omega_{z2}^2 + \omega_{\theta 2}^2} \quad (108b)$$

Since vortex elements are actually transported in absolute space by the same fluid lines, they are "stretched". Let us denote this state by asterisks: ω_2^* , l_2^* . In order to fit between the wakes in absolute space the vorticity ω_2 has to be scaled according to the Helmholtz relationship (Equation (82))

$$\frac{\omega_2^*}{l_2^*} = \frac{\omega_2}{l_2}$$

$$\omega_2^* = \frac{\omega_{1,2}}{\cos(\alpha_2 - \phi_2)}$$

The scaled vorticity is shown in Figure 88. The

corresponding magnitude of vortex filaments in the absolute reference frame again has to satisfy the Helmholtz relationship:

$$\frac{\omega_3}{\rho_2 l_3} = \frac{\omega_2^*}{\rho_2 l_2^*}$$

$$\therefore \omega_3 = \omega_2^* \frac{l_3}{l_2^*} \quad (109)$$

FIGURE 88. The Helmholtz relationship $\frac{\omega}{\rho l}$ in relative and absolute space

From geometry the ratio λ_3/λ_2^* is:

$$\frac{\lambda_3}{\lambda_2} = \frac{\cos \phi_2 - \sin \phi_2 (\cos \alpha_2 \cos \phi_2 + \sin \alpha_2 \sin \phi_2)}{\sin \phi_3 (\cot \phi_2 + \tan \alpha_2)} \quad (110)$$

where:

$$\phi_3 = \tan^{-1} \left(\frac{1}{\cot \phi_2 + \tan \alpha_2 + \tan \alpha_3} \right) \quad (111)$$

The circulation for the area $s_2 \Delta y_2 \sin \phi_3$ is

$$\Gamma_3 = \omega_3 s_2 \Delta y_2 \sin \phi_3 \quad (112)$$

and it has an axial pitch distance of

$$\Delta z_{23} = \frac{s_2 \sin \phi_2 \cos \alpha_2}{\cos (\alpha_2 - \phi_2)} \quad (113)$$

The exact location of the core of an equivalent vortex filament of strength Γ_3 with the area $s_2 \Delta y_2 \sin \phi_3$ cannot be ascertained from the previous formulations. Since the vorticity $\bar{\omega}_3$ is a distributed vorticity, however, it would be mathematically correct to calculate $\bar{\omega}_3$ for many small stream tubes if the streamlines could be defined in the annular space. The equivalent vortex filaments could then be located at the centre of the small areas and the position error would be minimized.

The distributed vorticity and vortex wakes apparently do not persist in the actual fluid field⁽²⁴⁾; it splits up into discrete vortices and these, being unstable in turn, roll up into a single large vortex pair, as depicted in Figure 82. The fluid particles carrying the vorticity generated due to viscous action join this vortex pair and the end result appears to be a single

large vortex pair of indistinguishable combined strength somewhere in the corner regions formed by the blade suction surface and end walls. The concentration of vorticity in these two major vortices is obviously dependent on the axial distance available and cannot be considered complete for closely spaced rotor and stator sets. The exact location of secondary flow vortex cores is at best uncertain, but Lakshminarayana⁽³⁸⁾ appears to have been successful in locating theoretical vortex cores in the passage corners formed by the end walls and the blade suction surface. In the absence of better information it is advantageous here to split the annular passage height up into two or three equal spacings of Δy .

The Helmholtz restriction on the existence of finite vortex line segments in space requires that there be additional vortex lines leading away from the boundaries of the helical patches of vorticity depicted in Figure 86. It seems unlikely that these filaments would lie parallel to the streamlines under steady flow conditions because this would entail ever-increasing radial velocities in the axial direction. It is assumed, therefore, that the additional vortex lines must be parallel to $\vec{\omega}_\theta = \vec{e}_\theta$, and since this orientation is radial, or mostly radial, these additional lines experience considerable cancellations with the result that their net effect is not expected to be large. From the outset these additional vortex lines can be neglected.

Following Dyne's example⁽¹²⁾, the rotor blade bound vorticity may be distributed over each passage as a collection of bound vortex lines of uniform local strength located in the

mid-chord plane of the rotor. The stator blades, being already in an absolute frame of reference, may be directly represented by bound vortex lines again in a mid-chord plane. The helical patches of rotor generated vorticity can be assumed to span the axial distance between the rotor and stator mid-chord stations.

The stator accepts the rotor vorticity as a distributed vorticity in the absolute flow and acts on it in the prescribed manner. Here the reference frame changes are omitted. The total vorticity downstream of the stator includes both stator and rotor generated contributions and may again be represented in helical patches of discrete vortex filaments starting at the stator mid-chord plane and extending downstream several fan diameters.

The boundary condition of zero radial velocity at the walls must be satisfied when calculating the induced velocities. It is automatically satisfied under normal conditions of axisymmetric free-vortex flow because the blades have uniform spanwise loading and the relative flow is irrotational. This is the annular blade row equivalent of cascade flow - the bound vorticity is constant, pierces the end walls and continues to plus and minus infinity. The boundary condition is also satisfied if the blade ends do not support a pressure difference, as in the case of isolated free three-dimensional aerofoils. Vavra's suggestion⁽³⁹⁾ of a discrete trailing vortex at the tip and a "fan" of distributed trailing vortices at the root is the turbomachine equivalent of the free aerofoil case; but it does not admit the observed tendency of annular blade rows to support a finite blade loading at both hub and tip ends. A more promising approach was

presented by Labreyer, Loeve and Slooff⁽⁴⁰⁾ on three-dimensional circulatory flow involving lifting wing-body combinations. It admits finite wing-root loading through the use of a buried network of horseshoe vortices, as shown in Figure 89.

FIGURE 89. Schematic view of selected system of singularity distributions (adapted from Ref. 40)

This type of approach is reported⁽²⁴⁾ to have met with considerable success in turbomachinery flows where the local velocity (cross and downwash) is induced by the vortex elements everywhere. It appears to be attractive for the present model; but the induced downwash velocities, which have an important effect on the deflection of the primary flow, may be considered in the solution only if another iteration loop in the through-flow can be

justified.

The strengths of the buried network of vortices that satisfy the conditions of zero radial velocities at the walls and finite blade loading at the blade ends are not known beforehand. These will have to be solved for indirectly as indicated in the outline of the proposed calculation procedure.

6.2.4. Outline of Second Proposed Analytical Method

The prerequisites for this method are the same as for the first extension model: inflow distortion velocities and a solution of the constant area through flow of each fan segment.

6.2.4.1. Inclusion of Axial Distortion Velocities

The axial distortion velocities, which were redundant in the formulation of the constant area segmented fan model, can now be imposed on the through-flow because the constant area requirement has been relaxed. For "axially thin" fans the total area change per fan segment can be taken across the rotor entirely since the stator exit plane is directly exposed to the crossflow. This option parallels the approach of the first extension method. For longer axial fan installations, where there is additional duct length downstream of the stator, some additional area change may be involved downstream of the stator; but this cannot be accounted for unless yet another governing physical relationship is applied (such as spline fits for the streamlines through the machine).

"Temporary" solutions can now be calculated for each fan segment by iterating on area ratio across the rotor while

forcing the calculation to meet the previous back-pressure values and the previous mass flows adjusted by the segment's axial distortion velocities. The sum of the segment areas calculated in this manner may not be equal to the total annulus area. The method used to adjust these areas represents the main difference between this and the first extension method.

6.2.9.7. Evaluation of Variables Appearing in the Equilibrium Equation

In order to serve the needs for later calculations, each segment may be divided along an axial plane into three equal sized sub-segments. The thrust and performance of each sub-segment is calculated using the established area ratio, back-pressure (which is assumed to be constant for each segment) and the appropriate distortion velocities. For the purpose for which the information is to be used, it is not critically important to maintain constant inlet conditions.

The blade row generated vortex model may now be used to determine the radial velocities induced at the walls by all the trailing filament and (spanwise) variable circulation shed vortex elements. Additional horseshoe vortices placed outside of the annulus space, as explained earlier, can be made to assume sufficient strength to yield zero net radial velocities at the walls. To solve for their strengths, influence functions are first calculated at as many control points as there are unknown strengths. The solution is achieved by matrix inversion techniques similar to the process used in the jet plume model. The complete vortex system

is then used to estimate the induced radial velocities and the desired vorticity at the fan segment centres in the axial plane between rotor and stator and, if appropriate, downstream of the stator.

The obvious limitations of calculating average radial velocities from discrete vortex lines in the near field are acknowledged; however, there appears to be no other simple alternative available.

The remaining variables appearing in the equilibrium equation are evaluated using mean line performance data calculated for the fan segments.

6.2.4.3. Application of Equilibrium Conditions

The tangential equilibrium equation may now be solved in either of the indicated ways for either C_z or $\Delta\theta$. Relaxing of the radial equilibrium requirements implies that the warping of the S_1 and S_2 stream surfaces is ignored.

In order to satisfy continuity in each fan segment from axial plane to axial plane, new areas (or axial velocities) are obtained. Since the sum of all areas must equal the annulus area, the previous schedule of areas has to be revised to satisfy total area and to be some mean between the original and latest derived sets.

6.2.4.4. Revision of Through-Flow Calculations

These revised areas are now used with the inlet swirl and radial distortion velocities and still the same back-pressure distribution to recalculate the meanline performance of all fan

segments. Although the back-pressure distribution is unchanged the back-pressure values should be adjusted by the new circumferential transport distances.

The axial velocity changes that arise are interpreted as valid fan feedback effects.

The calculation procedure is considered to lead to a fixed schedule of fan segment areas downstream of the rotor. This schedule of areas is, however, only correct for the initial distribution of back-pressure.

6.2.4.5. Revision of Back-Pressures

The efflux circulation may be calculated in the established manner. It, together with average efflux velocity and net circumferential transport distance, enables a revision of the jet plume model.

6.2.4.6. Continued Calculations and Convergence

The new static pressure distribution may now form the basis of a new round of calculations starting with the constant area flow solution. However, because the integrated results of the previous fan segment calculations are highly convergent, the back-pressure distribution will converge quickly to a final result. In spite of the implied size of the computing program to do all this, the convergence process of the whole procedure is expected to require only a few rounds of calculation.

The whole second extension method program is really generating only new back-pressures for revised segment areas. Consequently, the final solution is attained when some tolerance on segment area distribution has been met.

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CONCLUSIONS

1. Under normal (zero crossflow) inflow conditions flow curvature effects associated with contracting entrance flow have been observed to lead to minor axisymmetric nonuniformities in radial and axial velocity.
2. Under crossflow conditions, flow deflection from the approach direction towards the perpendicular inlet axis may not be complete, depending upon such variables as inlet geometry and the presence and proximity of the fan inlet face. As a consequence, overall (deflection) flow curvature effects and crossflow residuals remain.
3. Such flow curvature effects manifest themselves as a front-to-back nonuniformity in axial velocity, and the residuals generate variations in swirl and radial velocity wherever flow guidance is minimal. The crossflow-induced distortions are evidently nonuniformities of velocity at essentially constant total pressure.
4. Because the above basic potential flow effects are common to both pure suction and fan inlets it is possible to identify a first order correspondence between their inflow distortions. Some differences exist however, they are evident primarily in the distribution of axial velocities, and are regarded as fan feedback effects due to crossflow distortion.
5. Because the mainstream entering an inlet is irrotational the character of the inlet distortion can be modelled in terms of simple potential flow effects.

6. The zero crossflow radial distribution of velocity may be simulated by constant strength concentric vortex rings. The independent crossflow-induced inlet distortion, i.e. without fan feedback, may be simulated by a polar lattice of vortex horseshoes of nonuniform strengths.

7. Crossflow interaction with the fan efflux leads to a circumferentially nonuniform back-pressure distribution. The observed circumferential variation of static pressure in the exit plane of the fan suggests a relatively strong Magnus effect between the swirling efflux and the crossflow, and a considerably reduced wake width in the lee of the efflux jet.

8. It therefore appears possible to simulate the efflux jet plume by means of a network of nested vortex horseshoes and a jet circulation that corresponds to the swirl velocities present in the fan efflux.

9. The quality of the plume simulation depends on the number of circumferential vortex elements chosen and the manner of distributing the vortex lines representing the jet circulation. The static pressure distribution around the base of the jet can be made to resemble closely the back-pressure distributions at the exit plane of the fan.

10. The limited amount of circumferential flow redistribution inside a lifting fan enables the whole fan to be simulated by a collection of fan segments, all working in parallel.

11. For crossflow distortions of the spatial type the fan segment through-flows may be calculated on the basis of steady flow provided the instantaneous distortions are treated as if they

permanently maintained within the segments.

12. The assumption that the fan segments have constant axial flow area admits the use of distortion swirl and radial velocities in the analytical model; but, at the same time, it bars the use of direct crossflow distortions in axial velocities.

13. Of the four identified spatial crossflow-induced distortion variables, i.e. swirl, radial and axial distortion velocities and back-pressure, the first and the last constitute the prime distortion effects influencing fan performance. The radial distortion velocities are least important.

14. The through-flow and performance calculations of the fan segments are severely compromised by the general lack of suitable cascade loss and deflection data and the inadequacies of the available secondary flow loss relationships.

15. The integration of the segment efflux swirls for furnishing the jet plume with a core circulation, for matching the calculated fan through-flow with the external field, leads to a highly convergent solution involving the three separate analytical models (inlet distortion, back-pressure, and segmented fan).

16. The complete analytical model demonstrates that under crossflow conditions the fan segments operate on constant speed characteristic lines that are different from one another and from the complete fan. With increasing crossflow the characteristic lines tend away from a universal characteristic line because they depend on the local inlet distortions and Mach number effects. A previously proposed analogy between the

crossflow distortions and the velocity distribution created by a perfect set of inlet guide vanes is, in fact, substantiated.

17. The variations in the operating characteristic lines of the segments result in feedback on the inlet plane distributions of axial velocity and rotor incidence angle.

18. The adjustments in axial velocity result in a large perturbation, which accounts for most of the observed skewness in the fan inlet axial velocity distribution.

19. The prime distortion variables cause the advancing fan blades (relative to the crossflow) to experience incidence swings towards the positive and the retreating ones to suffer even larger swings towards negative incidence. Fan segment operation appears, therefore, to be well away from surge conditions.

20. The axial velocity feedback adjustments due to distortion swirl reduce the magnitude of rotor incidence angles.

21. Since the retreating blades do not achieve a large swirl change and operate so far away from minimum loss incidence, they cannot produce the zero crossflow total pressure and temperature rises. The advancing blades, on the other hand, achieve larger swirl changes and operate at better incidence angles, but because they experience higher velocities, the relative Mach numbers are higher also. Consequently, at high fan speed the advancing blades tend to suffer from increased losses, partly because of the reduced range of minimum loss incidence. Thus, while total pressure rise across the retreating blades always decreases with increasing crossflow velocity, across the advancing blades it decreases only at very high fan speeds and

actually increases at low fan speeds.

22. The advancing fan blades see higher axial velocities, relative Mach numbers, total temperature rise and total pressure rise, except at very high Mach numbers. As a result, the exit plane distributions of total temperature and pressure are also nonuniform circumferentially and radially.

23. The calculated inlet distortion swirl is a reasonable approximation of the measured distortion swirl, but there is small phase angle between the distributions. A fan reaction effect (which cannot be predicted by this model) results in skewing the measured distortion swirl distribution by about 10 to 15 degrees.

24. The measured radial velocity tends to be more positive (more flow towards the fan axis) than the calculated one. This defect could be corrected by including flow contraction effect in the form of co-axial vortex rings.

25. The calculated back-pressure distribution underestimates the static pressure depressions at the sides of the exit plane.

26. This shortcoming could be remedied by splitting up the efflux jet circulation core into a vortex pair separated by a small distance.

27. The calculated circumferential distortions are generally not as severe as measured. The deviations appear to involve both the absolute magnitude of and a phase angle between the distortion waves of such variables as axial velocity, rotor incidence, total pressure, temperature, etc. The calculated

distributions are basically antisymmetrical about the front to back fan axis, while the measured distributions seem to be skewed at an approximate angle of $\theta = 45^\circ$.

28. There is acceptable agreement between the calculated and measured annulus averaged performance variables.

29. Relaxing the constant axial flow area appears to be the most promising possible improvement to the analytical segmented fan model. This modification to the model would make it possible to include the crossflow distortion velocities calculated by means of the inlet distortion model. As was indicated by the superposition of unit representations, inclusion of this crossflow distortion effect would reduce significantly both the phasing and magnitude errors between the calculated and measured distortion distributions.

30. The simplest way of controlling area change in and between fan segments is on the basis of continuity and geometric compatibility; but, this approach does not necessarily ensure satisfaction of circumferential equilibrium.

31. Application of an approximate circumferential equilibrium condition seems possible, but even if it is highly simplified considerable additional complexity is introduced because of the requirement for calculating radial velocities inside the machine.

32. The theories describing vorticity generated by blade rows in three-dimensional primary flow are not well developed at present, particularly as far as rotating blade rows are concerned. One must, therefore, attach considerable uncertainty

to vorticities calculated on the basis of existing theories.

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APPENDIX A

A.1. Examples of Simple Vortex Models

$$v = v_2 + v_1 = \frac{\Gamma}{4\pi} \ln \left[\frac{x^2 + (y - b)^2}{x^2 + (y + b)^2} \right] - v_\infty y$$

FIGURE A.1 Two-dimensional vortex pair perpendicular to and advancing in the crossflow

The fundamental aspects of potential streaming flow past a two-dimensional vortex pair in the plane of flow and perpendicular to it are illustrated in Figures A.1 and A.2, respectively. These simple models exhibit the flow curvature effects that are responsible for generating the inflow distortion velocity distributions in the more complicated three-

dimensional vortex lattice described in Section 1.

A.2. Comparison of Vortex Polygon with Ring Vortex Solution

As the number of sides of the vortex polygon is increased

the approximate solution of induced velocity tends in the limit to the exact solution, or ring vortex solution. Dyne⁽¹²⁾ presented the velocity distribution of the exact solution, Figure A.3, and this may be compared with the approximate solution of an

FIGURE A.2. Two-dimensional vortex pair in the plane of crossflow

eleven-sided vortex polygon of constant vortex strength, Figure A.4. The distribution of the approximated axial velocity is very nearly the same as generated by the exact solution. As one might expect, the greatest differences occurred at the vortex ring itself; away from the ring and the ring plane the approximate solution approaches the exact solution asymptotically.

The inlet vortex model used here does not use the velocity field near the vortex elements because the induced velocities are too high, consequently, in the region of interest, the agreement is very good.

FIGURE A.3. Axial component of the velocity at a point that is induced by a ring vortex of radius " R ".

FIGURE A.4. Axial component of the velocity at a point that is induced by a uniform strength vortex polygon of eleven sides circumscribed by a circle of radius " R ".

Use cannot be made of the exact solution in the present application because the mathematics involved in accommodating circumferentially variable vortex strengths are unreasonably complicated for the purpose for which the data are to be used.

A.3. Possible Extension of the Inflow Distortion Model to Include Flow Contraction Effects

The inflow distortion model was devised to specifically characterize crossflow distortion effects. The means of including normal inflow contraction effects are suggested in Figure 29: one needs to consider pairs, or stacks of pairs of uniform strength concentric vortex rings at or buried inside the annulus walls. If a single pair were used it would have to be coplanar with the location at which the boundary condition is met so that the $\tau_r = 0$ condition at the wall would not be violated. A stack of equal strength rings, such as Dyne⁽¹²⁾ used, would be much better since, (a) the boundary condition would be maintained automatically, and (b) it would provide for axial as well as radial variation of C_r , with C_r eventually attenuating at large inflow depths. The same model would, however, provide uniform induced axial velocities as well, a feature that could not be adapted easily in the fan through-flow method.

A.4. Further Improvements to the Inflow Distortion Model

The distortion model can be refined by sweeping the vortex trails rearwards, or curving them along a prescribed efflux jet path. A logical further extension is possible by

replacing the simple polar lattice with a stacked polar lattice, as sketched in Figure A.5.

FIGURE A.5. Improved polar vortex lattice for generating inlet distortion

In this case, the boundary condition of no flow through the surface has to be satisfied at the centroids of all additional vortex panels so formed. The method is, of course, not different in principle, only in complexity, and hence requires much more computer storage space.

It is relevant to note also that in the case of the stacked polar lattice, the vortex elements may be placed on the wetted surface as long as precautions are taken when evaluating surface velocities on top of vortex lines. The vortex elements under the field points should be disregarded during velocity calculations.

The ultimate extension of the polar lattice method is to represent the complete lifting fan body or wing; however, due care must be taken to look after the wake. In the fan-in-wing case this amounts to meeting the Joukowski condition at the trailing edge of the wing and representing the downwash velocities properly.

A.5. Comparisons of Simple External Flow Models

A comparison is made in two-dimensional space between the disturbance in the external flow field due to a vortex line generated cylinder and a doublet in a crossflow. The vortex model assessed comprises eleven single vortices located at the corners of an eleven-sided polygon. For symmetrical flow over the "cylinder" this is equivalent to a linear superposition of five vortex pairs and one vortex of zero net strength. The stream function for a single pair in a uniform cross flow is

$$\psi_i = \frac{\Gamma_i}{2\pi} \ln \left(\frac{B_{i1}}{B_{i2}} \right) - v_\infty y \quad (A-1)$$

where

$$B_{i1}^2 = (x - x_i)^2 + (y - y_i)^2$$

$$B_{i2}^2 = (x - x_i)^2 + (y + y_i)^2$$

FIGURE A.6. Co-ordinates used to calculate induced / velocities of vortex pairs

For an eleven-sided vortex polygon one vortex is placed on the symmetry axis, hence it has zero strength. The stream function for the remaining five vortex pairs is

FIGURE A.7. Streamline plot for an eleven-sided vortex polygon pole in crossflow

FIGURE A.8. Streamline plot for the comparison model: doublet in crossflow

$$\psi = \sum \psi_i$$

$$= -\frac{1}{2\pi} \ln \left| \left(\frac{B_1}{B_2} \right)_1^{\Gamma_1} \left(\frac{B_1}{B_2} \right)_2^{\Gamma_2} \left(\frac{B_1}{B_2} \right)_3^{\Gamma_3} \left(\frac{B_1}{B_2} \right)_4^{\Gamma_4} \left(\frac{B_1}{B_2} \right)_5^{\Gamma_5} \right| - V_\infty y \quad (A-2)$$

The streamline pattern for the crossflow ratio of 0.325 and the respective vortex strengths of -40.16, -73.49, 56.2, and -20.9 is shown in Figure A.7. This figure may be compared with the potential flow model of a doublet in uniform streaming flow, Figure A.8. The vortex model evidently does not generate the high surface and near-surface velocities associated with doublet flow since the streamlines are spaced more widely. Both models

FIGURE A.9. Streamline plot for Munk's doublet and square vortex model comparison. (Adapted from Ref. 10)

induce similar results in the far field, but since continuity has to be satisfied in both the near and far fields, at distances of the order of a diameter the streamlines are spaced more closely, i.e. the velocities are higher than for the doublet model.

Monical's result for a square and circle comparison⁽¹⁰⁾, see Figure A.9, illustrates these differences even more dramatically. A comparison of the stream velocities at the sides of the simple vortex and doublet models in streaming flow

is afforded in Figure A.10. The major difference between the two models involves the surface and near surface velocities. While the doublet has a circular cylindrical surface with velocity being a maximum at the wall, the vortex model has an undulating cylindrical surface with streamlines being closely spaced at the crests and more widely spaced in the troughs. The surface velocities

FIGURE A.10. Velocity profile comparison for two-dimensional flow

at the crest are consequently larger and those in the troughs smaller than for the doublet model. Figure A.18 makes a velocity comparison in a trough. It can be seen that the velocity profiles are really quite dissimilar. The direct influence of the wall undulations has attenuated at a radius ratio of 1.5, but even the far field shows some small differences for continuity reasons.

The peculiar near surface flow associated with the vortex model is believed to give a more effective representation of the mixing region around the highly nonuniform efflux jet.

A.6. A Further Improvement to the Vortex Plume Model

The inadequacies of using a uniform efflux circulation in the lifting fan's exit plane were discussed in Chapter 3. One simple means of creating a distributed effect while maintaining the same circulation is to split the vortex strength assigned to the plume's centreline into two equal parts. Two vortex lines separated by a small distance of $2s$ will have the same total circulation but will not generate the same swirl velocities on the surface of the plume. The (total) circulation may be calculated as before, i.e. by summing $(\frac{2\pi}{N} r_m C_{\theta 4})$ for all fan segments. The distance s is found by matching the distribution of swirl velocities around the circumference of the jet's mean radius.

This match should not be difficult to achieve; however, the extra degree of freedom obtained in having two vortex lines may create convergence problems in the complete through-flow solution because the "Cos 2θ " effect could well be self-generating between subsequent calculation passes through the segmented fan

and jet plume models. If this should occur it may be necessary to introduce some damping in the solution by limiting the rate at which the distance's grows from one calculation pass to the next.

Part of the problem of replacing the single centreline vortex line with two or more vortices is maintaining the boundary condition at the jet plume surface. The boundary condition must remain satisfied at all times.

APPENDIX B

B.1. Cascade Performance Data

The aerodynamic performance of the fan blading can be calculated by means of listed sets of available low speed cascade deflection and loss data. A choice between two different sets of data is offered. For low speed operation and "conventional" low speed fan blades, Howell's deflection data and NGTE loss data are available. The Howell cascade data covers a range of solidities ($0.75 \leq \frac{S}{C} \leq 1.0$) and a range of blade camber angles, but Mach number effect is not covered. For high subsonic speed operation ($M \leq .8$) with 10C7/40C50 sections for the rotor and 10C3/45C50 sections for the stator (as in the present test fan) NGTE loss and deflection are available. The solidity is, however, restricted to unity, and the 10C3/45C50 data lack inlet angle range. The general lack of cascade data for the 10C3/45C50 section over sufficiently wide operating and solidity ranges is very evident. The 10C3/45C50 data listing has, consequently, been augmented with some extracted data wherever necessary in order to extend the operating range within reasonable limits.

Table B-1. Rowell's Actual Deflection Data⁽²³⁾

$(i-i^*)/\epsilon^*$	-0.600	-0.550	-0.500	-0.450	-0.400	-0.350
ϵ/ϵ^*	+0.370	+0.426	+0.485	+0.543	+0.603	+0.660

$(i-i^*)/\epsilon^*$	-0.300	-0.250	-0.200	-0.150	-0.100	-0.050
ϵ/ϵ^*	+0.713	+0.768	+0.823	+0.872	+0.920	+0.962

$(i-i^*)/\epsilon^*$	-0.000	+0.050	+0.100	+0.150	+0.200	+0.250
ϵ/ϵ^*	+1.005	+1.043	+1.084	+1.122	+1.160	+1.192

$(i-i^*)/\epsilon^*$	+0.300	+0.350	+0.400	+0.450	+0.500	+0.550
ϵ/ϵ^*	+1.222	+1.238	+1.250	+1.254	+1.258	+1.260

Table B-2. Rowell's Nominal Deflection Data⁽²³⁾

a_2^*	-10.0	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0
$s/c = 0.0$	67.5	56.3	47.6	40.0	32.9	26.5	20.45	14.75	9.2
$s/c = 0.5$	51.30	42.90	36.50	30.70	25.40	20.60	16.10	11.70	7.50
$s/c = 1.0$	39.5	32.20	28.00	22.50	19.60	15.80	12.50	9.25	6.10
$s/c = 1.5$	31.3	26.8	22.55	18.6	15.4	12.9	10.35	7.6	4.95
$s/c = 2.0$	26.5	22.3	19.0	15.9	12.3	10.8	7.55	6.2	3.85

Table B-3. NGTE Deflection Data for 10C7/40C50, $s/c = 1.0$ ⁽⁴¹⁾

$\alpha_1 \backslash M_1$	0.8	0.70	0.60	0.50	0.40	0.30
29.20	4.00	5.00	7.50	10.70	8.80	7.50
32.40	7.50	9.50	13.70	16.80	16.40	10.60
35.60	9.50	12.90	20.00	19.50	17.40	14.10
38.90	12.50	20.00	23.60	23.70	22.00	17.50
42.20	15.00	25.50	27.00	26.70	25.80	23.50
45.60	18.00	29.92	29.98	29.96	29.50	29.00
49.10	23.00	27.50	31.80	32.50	32.50	32.50
52.50	29.00	32.60	32.00	30.50	31.50	32.50
54.50	29.00	32.00	31.60	30.00	30.00	31.50
56.40	27.70	30.20	29.80	29.60	29.50	30.00
58.2	28.20	25.80	27.70	28.60	28.50	25.40

Table B-4. NGTE Loss Data $\left(\frac{\Delta P_t}{P_t - P} \right)$ for 10C7/40C50, $s/c = 1.0$ ⁽⁴¹⁾

$\alpha_1 \backslash M_1$	0.80	0.70	0.60	0.50	0.40	0.30
29.20	0.5250	0.3500	0.1800	0.0830	0.0850	0.0975
32.40	0.4000	0.2450	0.0850	0.0280	0.0375	0.0850
35.60	0.3000	0.1400	0.0370	0.0370	0.0530	0.0825
38.90	0.2160	0.0460	0.0280	0.0285	0.0410	0.0770
42.20	0.2160	0.0800	0.0200	0.0250	0.0350	0.0570
45.60	0.2360	0.0280	0.0200	0.0200	0.0240	0.0400
49.10	0.2380	0.0950	0.0290	0.0230	0.0270	0.0330
52.50	0.2475	0.1200	0.0650	0.0450	0.0350	0.0350
54.50	0.2825	0.2140	0.1230	0.0970	0.1190	0.2150
56.40	0.3575	0.3180	0.2910	0.2730	0.2650	0.2680
58.20	0.4330	0.4350	0.4240	0.3950	0.3760	0.3620

Table B-5. NGTE Deflection Data for 10C3/45C50 $s/c = 1.0$ ⁽⁴²⁾

$\alpha_1 \backslash M_1$	0.8	0.70	0.60	0.50	0.40	0.30
20.00	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*
25.00	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*
30.00	0.00*	1.30*	2.75*	2.20*	0.00*	0.00*
35.00	5.80	8.50	10.00	9.70	8.00	5.80
40.00	12.50	17.20	18.20	17.50	16.00	14.10
43.30	17.50	22.50	24.00	22.80	22.50	20.00
45.60	21.50	28.60	30.00	28.80	27.50	25.60
46.70	24.40	26.00	28.25	28.00	27.00	26.00
47.90	24.60	28.10	26.70	28.00	27.00	24.60
49.10	34.20	30.50	27.00	27.00	26.50	25.00
51.90	30.00	28.10	26.50	25.50	25.50	26.20

Table B-6. NGTE Loss Data $\left(\frac{\Delta P_t}{P_t - P} \right)$ for 10C3/45C50, $s/c = 1.0$ ⁽⁴²⁾

$\alpha_1 \backslash M_1$	0.80	0.70	0.60	0.50	0.40	0.30
20.00	0.0920*	0.0840*	0.1380*	0.1680*	0.2130*	0.2600*
25.00	0.1100*	0.0950*	0.1220*	0.1480*	0.1840*	0.2200*
30.00	0.1300*	0.0980*	0.1070*	0.1250*	0.1500*	0.1800*
35.00	0.1530	0.0960	0.0880	0.0980	0.1200	0.1420
40.00	0.1650	0.0780	0.0650	0.0700	0.0850	0.1040
43.30	0.1750	0.0520	0.0460	0.0420	0.0500	0.0720
45.60	0.1850	0.0340	0.0180	0.0230	0.0320	0.0500
46.70	0.1730	0.0200	0.0250	0.0250	0.0250	0.0270
47.90	0.1950	0.0310	0.0400	0.0310	0.0280	0.0270
49.10	0.2500	0.1000	0.0600	0.0470	0.0400	0.0370
51.90	0.3000	0.2500	0.1570	0.1400	0.1350	0.1320

* Extrapolated data.

B.2. Development of Formulae for Mean Line Design Method

B.2.1. Control Volume, Continuity, Force and Energy Relationships

The fan annulus is unwrapped at the mean radius and control volumes are formed around the blade row sets that are imagined to be contained by each fan segment. Consider steady, compressible } low-speed flow through the two-dimensional control volume.

FIGURE B.1. Control volume containing blade row

Continuity:

$$\rho_1 W_1 A_1 = \rho_2 W_2 A_2 \quad (B-1)$$

$$\rho_1 C_{z1} A_1 = \rho_2 C_{z2} A_2 \quad (B-2)$$

Force Balance:

$$z\text{-direction: } F_z = (P_2 - P_1)A_2 + \frac{\rho_1 A_1 C_{z1}}{g_0} (C_{z2} - C_{z1}) \quad (B-3)$$

$$\theta\text{-direction: } F_\theta = \frac{\rho_1 A_1 C_{z1}}{g_0} (W_{\theta 1} - W_{\theta 2}) \quad (B-4)$$

Energy:

$$h'_t = h + \frac{W^2}{2g_0 J} \quad (B-5)$$

$$T'_t = T + \frac{W^2}{2g_0 J c_p} \quad (B-6)$$

Compressible flow relationships:

$$T'_t = T \left(1 + \frac{\gamma - 1}{2} M'^2 \right) \quad (B-7)$$

$$P'_t = P \left(1 + \frac{\gamma - 1}{2} M'^2 \right)^{\frac{\gamma}{\gamma - 1}} \quad (B-8)$$

The primes designate quantities referred to the relative reference frame.

B.2.2. Static Pressure Rise Coefficient

$$P_2 - P_1 = \frac{P'_{t2}}{\left(1 + \frac{\gamma - 1}{2} M'^2_2 \right)^{\frac{\gamma}{\gamma - 1}}} - \frac{P'_{t1}}{\left(1 + \frac{\gamma - 1}{2} M'^2_1 \right)^{\frac{\gamma}{\gamma - 1}}} \quad (B-9)$$

For the ideal static pressure rise it is generally assumed that

$$P'_{t2} = P'_{t1} \quad (B-10)$$

Consider the quantity:

$$\frac{P_1}{\frac{\rho_1 W_1^2}{2g_0}} \quad (B-11)$$

from the equation of state:

$$\frac{P_1}{\rho_1} = RT_1 \quad (B-12)$$

and,

$$M_1^2 = \frac{W_1^2}{g_0 \gamma R T_1} \quad (B-13)$$

$$\therefore \frac{P_1}{\frac{\rho_1 W_1^2}{2g_0}} = \frac{2g_0 RT_1}{W_1^2} = \frac{2}{\gamma M_1^2} \quad (B-14)$$

Combining all these:

$$\frac{P_2 - P_1}{q_1} = \frac{2}{\gamma M_1^2} \left[\left(\frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2} \right)^{\frac{\gamma}{\gamma - 1}} - 1 \right] \quad (B-15)$$

This is the ideal pressure rise coefficient for compressible, constant or variable area, flow.

An approximate blade row calculation can be made in which compressibility effects and total pressure loss are crudely accounted for, and that is more amenable for hand calculations than the accurate compressible flow calculation.

The compressible flow stagnation pressure relationship

$$\frac{P_t}{P} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}}$$

expanded in a binomial series, the first three terms are:

$$\frac{P_t'}{P} \approx 1 + \frac{\gamma}{2} M'^2 + \frac{\gamma}{8} M'^4 \quad (B-16)$$

$$\therefore P_t' \approx P + \frac{\rho W^2}{2g_0} \left(1 + \frac{1}{4} M'^2 \right) \quad (B-17)$$

The static pressure difference between the blade inlet and outlet stations is

$$P_2 - P_1 \approx P_{t2}' - P_{t1}' + \frac{\rho_1 W_1^2}{2g_0} \left(1 + \frac{1}{4} M_1'^2 \right) - \frac{\rho_2 W_2^2}{2g_0} \left(1 + \frac{1}{4} M_2'^2 \right)$$

The (first order) compressibility correction terms are:

$$\left(\frac{1}{4} M_1'^2 \right) \frac{\rho_1 W_1^2}{2g_0} \quad \text{and} \quad \left(\frac{1}{4} M_2'^2 \right) \frac{\rho_2 W_2^2}{2g_0}$$

When these are negligible the flow can be considered to be incompressible ($\rho_1 = \rho_2$) and the familiar incompressible pressure difference is obtained. For compressible flow the error incurred in neglecting the compressibility correction terms is approximately:

$$\frac{1}{4} M_1'^2 \left[1 - \frac{A_1' W_2}{A_2' W_1} \left(\frac{M_2'}{M_1'} \right)^2 \right] \frac{\rho_1 W_1^2}{2g_0} \quad (B-18)$$

For diffusing flows this error is at most equal to:

$$\left(\frac{1}{4} M_1'^2 \right) \frac{\rho_1 W_1^2}{2g_0} \quad (B-19)$$

Based on inflow dynamic pressure the static pressure coefficient is approximately:

$$\frac{P_2 - P_1}{\frac{\rho_1 W_1^2}{2g_0}} = \frac{P_{t2} - P_{t1}}{\frac{\rho_1 W_1^2}{2g_0}} + 1 - \frac{A_1 W_2}{A_2 W_1} + \frac{1}{4} M_1'^2 \quad (B-20)$$

$$C_p = -C_{DP} + 1 - \frac{A_1 W_2}{A_2 W_1} + \frac{1}{4} M_1'^2 \quad (B-21)$$

In deriving the lift coefficient and angle at which the lift force acts on the blades, the loss term is normally neglected, and for incompressible flow the correction term is zero. The measured cascade profile loss data shows that $C_{DP} = f(\alpha_1, M_1')$. In the minimum loss region $C_{DP} \sim M_1'$, whereas at very large (negative and positive) angles of incidence C_{DP} is proportional to some higher power of M_1' . If the arbitrary but convenient guess is made that $C_{DP} = \frac{1}{4} M_1'^2$ both the pressure loss and compressibility effects are in part, albeit empirically and imperfectly, accounted for over a range of incidence angles.

The approximate relationship for $(P_2 - P_1)$ may, therefore, be substituted in the z-force equation (B-3), which (after noting that axial area is considered constant) becomes:

$$\frac{F_z}{A} = \frac{\rho_1 W_1^2}{2g_0} - \frac{\rho_2 W_2^2}{2g_0} + \frac{\rho_1 C_{z1}}{g_0} (C_{z2} - C_{z1}) \quad (B-22)$$

Dividing by $q_1 = \frac{\rho_1 C_{z1}^2}{2g_0}$ and substituting for $\frac{\rho_2}{\rho_1} = \frac{C_{z1}}{C_{z2}}$ one obtains:

$$\frac{F_z}{q_1 A} = \left(\frac{W_1}{C_{z1}} \right)^2 - \frac{C_{z2}}{C_{z1}} \left(\frac{W_2}{C_{z2}} \right)^2 + 2 \left(\frac{C_{z2}}{C_{z1}} - 1 \right) \quad (B-23)$$

After some re-arranging and substituting $\frac{C_z}{W} = \tan \alpha$ the following relationship results:

$$\frac{F_z}{q_1 A} = \frac{C_{z2}}{C_{z1}} \left[\left(1 - \tan^2 \alpha_2 \right) - \frac{C_{z1}}{C_{z2}} \left(1 - \tan^2 \alpha_1 \right) \right] \quad (B-24)$$

Similarly, the tangential force may be expressed as:

$$\frac{F_\theta}{q_1 A} = \frac{2}{C_{z1}} (W_{\theta 1} - W_{\theta 2}) \quad (B-25)$$

This is expressed more conveniently in terms of air angles.

$$\frac{F_\theta}{q_1 A} = 2 \left(\tan \alpha_1 - \frac{C_{z2}}{C_{z1}} \tan \alpha_2 \right) \quad (B-26)$$

The ratio of the two forces is

$$\frac{F_z}{F_\theta} = \frac{1}{\frac{C_{z1}}{C_{z2}} \tan \alpha_1 - \tan \alpha_2} \left[\left(1 - \tan^2 \alpha_2 \right) - \frac{C_{z1}}{C_{z2}} \left(1 - \tan^2 \alpha_1 \right) \right] \quad (B-27)$$

Finally, the tangent of the angle of the force triangle relative to the axial plane is $\tan \alpha_m$. This is the angle at which the lift force acts

$$\tan \alpha_m = \frac{1}{\frac{C_{z1}}{C_{z2}} \tan \alpha_1 - \tan \alpha_2} \left[\left(1 - \tan^2 \alpha_2 \right) - \frac{C_{z1}}{C_{z2}} \left(1 - \tan^2 \alpha_1 \right) \right] \quad (B-28)$$

Note that the incompressible form is obtained by setting

$$C_{z1} = C_{z2}$$

B.2.3. Direction of Blade Force for Compressible Flow with Area Change

The axial force equation expressed in the previous terms is:

$$\begin{aligned} \frac{F_z}{q_1} &= \frac{2A_2}{\gamma M_1^2} \left[\frac{\left(1 + \frac{\gamma-1}{2} M_1'^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(1 + \frac{\gamma-1}{2} M_2'^2\right)^{\frac{\gamma}{\gamma-1}}} - 1 \right] + \frac{\frac{\rho_1 A_1 C_{z1}}{g_0}}{\frac{\rho_1 W_1^2}{2g_0}} (C_{z2} - C_{z1}) \\ &= \frac{2A_2}{\gamma M_1^2} \left[\frac{\left(1 + \frac{\gamma-1}{2} M_1'^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(1 + \frac{\gamma-1}{2} M_2'^2\right)^{\frac{\gamma}{\gamma-1}}} - 1 \right] + 2A_1 \cos^2 \alpha_1 \left(\frac{C_{z2}}{C_{z1}} - 1 \right) \quad (B-29) \end{aligned}$$

The tangential force is:

$$\begin{aligned} F_\theta &= \frac{\rho_1 A_1 C_{z1}}{g_0} (W_{\theta 1} - W_{\theta 2}) \\ \therefore \frac{F_\theta}{q_1} &= 2A_1 \left(\frac{C_{z1}}{W_1} \right)^2 \left(\frac{W_1}{C_{z1}} - \frac{C_{z2}}{C_{z1}} \frac{W_{\theta 2}}{C_{z2}} \right) \\ &= 2A \cos^2 \alpha_1 \left\{ \tan \alpha_1 - \frac{C_{z2}}{C_{z1}} \tan \alpha_2 \right\} \quad (B-30) \end{aligned}$$

The ratio of the axial and tangential forces is F_z/F_θ .

The direction of the blade force is α_m from the axial:

$$\tan \alpha_m = \frac{\frac{A_2}{A_1} \left[\frac{\left(1 + \frac{\gamma-1}{2} M_1'^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(1 + \frac{\gamma-1}{2} M_2'^2\right)^{\frac{\gamma}{\gamma-1}}} - 1 \right]}{\gamma M_1'^2 \cos^2 \alpha_1 \left(\tan \alpha_1 - \frac{C_{z2}}{C_{z1}} \tan \alpha_2 \right)} + \frac{\frac{C_{z2}}{C_{z1}} - 1}{\tan \alpha_1 - \frac{C_{z2}}{C_{z1}} \tan \alpha_2}$$

or,

$$\tan \alpha_m = \frac{1}{\frac{C_{z1}}{C_{z2}} \tan \alpha_1 - \tan \alpha_2} \left\{ \frac{C_{z1}}{C_{z2}} \cdot \frac{A_2}{A_1} \left[\frac{1 + \frac{\gamma-1}{2} M_1'^2}{1 + \frac{\gamma-1}{2} M_2'^2} \right]^{\frac{\gamma}{\gamma-1}} - 1 \right\} + \left(1 - \frac{C_{z1}}{C_{z2}} \right)$$

(B-31)

B.2.4. Lift Coefficient

B.2.4.1. Constant Area Flow Case

$$L = \frac{F_\theta}{\cos \alpha_m} - F_D \tan \alpha_m \quad (B-32)$$

Assuming an ideal lift coefficient, i.e. $F_D \approx 0$:

$$C_L = \frac{L}{\frac{\rho_m W_m^2}{2g_0} cH} \quad (B-33)$$

$$= \frac{F_\theta}{\frac{\rho_m W_m^2}{2g_0} cH \cos \alpha_m} \quad (B-34)$$

But, using sH as axial area,

$$F = \frac{\rho_1 sH C_{z1}}{g_0} (W_{\theta 1} - W_{\theta 2}).$$

Substitute this in C_L and obtain:

FIGURE B.2. Orientation of lift vector

$$\frac{\frac{\rho_1 s H C_{z1}}{g_0} (W_{\theta 1} - W_{\theta 2})}{\frac{\rho_m W_m^2}{2g_0} cH \cos \alpha_m} = \frac{2 \frac{s}{c} \rho_1 C_{z1} (W_{\theta 1} - W_{\theta 2})}{\rho_m W_m^2 \cos \alpha_m}$$

Now from continuity:

$$\rho_1 A_1 C_{z1} = \rho_m A_m C_{zm}$$

In the case of constant area flow:

$$\frac{A_m}{A_1} = 1$$

Let $C_{zm} = \frac{C_{z1} + C_{z2}}{2}$

After making substitutions in the lift coefficient formula using the above relationships one obtains

FIGURE B.3. Rotor blade velocities

$$C_L = 4 \frac{s}{c} \left(\frac{W_{\theta 1} - W_{\theta 2}}{C_{z1} - C_{z2}} \right) \cos \alpha_m \quad (B-35)$$

$$= 4 \frac{s}{c} \left(\frac{1}{1 + \frac{C_{z2}}{C_{z1}}} \right) \left(\tan \alpha_1 - \frac{C_{z2}}{C_{z1}} \tan \alpha_2 \right) \cos \alpha_m \quad (B-36)$$

B.2.4.2. Compressible Flow with Area Change Case

The basic formula for a flow area of sH is

$$C_L = \frac{F_\theta}{q_m c H \cos \alpha_m}$$

$$= \frac{2 \frac{\gamma}{\gamma-1} \rho_1 C_{z1} (W_{\theta 1} - W_{\theta 2})}{\rho_m W_m^2 \cos \alpha_m}$$

The magnitude of W_m is defined as

$$W_m = \frac{W_1 + W_2}{2}$$

W_m is assumed to act normal to the lift vector, i.e. α_m to the fan axis. Consider the dynamic pressure $\frac{1}{2} \rho_m W_m^2$. In the relative reference frame losses are assumed to be zero for ideal lift coefficient calculations. As before, let the relative total conditions be T_t' and P_t'

$$\therefore T_m = T_t' - \frac{W_m^2}{2 g_o J c_p}$$

$$P_m = P_t' \left(\frac{T_m}{T_t'} \right)^{\frac{\gamma}{\gamma-1}}$$

and

$$\rho_m = \frac{P_m}{R T_m}$$

now:

$$c_p = \frac{\gamma}{\gamma-1} \frac{R}{J}$$

so,

$$\frac{T_m}{T_t'} = 1 + \frac{\gamma-1}{2} \left(\frac{W_m}{\sqrt{g_o R T_t'}} \right)^2$$

$$\therefore \rho_m = \frac{P_t'}{RT_1} \frac{\left[1 + \frac{\gamma-1}{2} \left(\frac{W_m}{\sqrt{\gamma g_0 RT_t'}} \right)^2 \right]^{\frac{\gamma}{\gamma-1}}}{\left[1 + \frac{\gamma-1}{2} \left(\frac{W_m}{\sqrt{\gamma g_0 RT_t'}} \right)^2 \right]} \quad (B-37)$$

consequently we obtain for $\rho_m W_m^2$

$$\rho_m W_m^2 = \frac{W_m^2 P_t'}{RT_t'} \left[1 + \frac{\gamma-1}{2} \left(\frac{W_m}{\sqrt{\gamma g_0 RT_t'}} \right)^2 \right]^{\frac{1}{\gamma-1}} \quad (B-38)$$

the variable group $\frac{P_t'}{RT_t'} = \rho_t'$.

In terms of the relative inlet Mach number and inlet and outlet relative velocities:

$$\rho_t' = \rho_1 \left(1 + \frac{\gamma-1}{2} M_1'^2 \right)^{\frac{1}{\gamma-1}}$$

this is substituted above:

$$\frac{\rho_1}{\frac{P_t'}{RT_t'}} = \frac{1}{\left(1 + \frac{\gamma-1}{2} M_1'^2 \right)^{\frac{1}{\gamma-1}}}$$

The group of variables in the C_L numerator $C_{z1} (W_{\theta 1} - W_{\theta 2})$ is

$$C_{z1}^2 \left(\tan \alpha_1 - \frac{C_{z2}}{C_{z1}} \tan \alpha_2 \right)$$

The appropriate substitutions in C_L lead to

$$C_L = \frac{\frac{1}{2} \frac{s}{c} \left(\frac{C_{z1}}{W_1 + W_2} \right)^2 \left[\tan \alpha_1 - \frac{C_{z2}}{C_{z1}} \tan \alpha_2 \right]}{\left(1 + \frac{\gamma - 1}{2} M_1'^2 \right)^{\frac{1}{\gamma - 1}} \left[1 - \frac{\gamma - 1}{8} \left(\frac{W_1 + W_2}{\sqrt{\gamma R g_o T_t}} \right)^2 \right]^{\frac{1}{\gamma - 1}} \cos \alpha_m} \quad (B-39)$$

W_m, M_1', T_t' are relative quantities.

B.2.5. Blade Profile Pressure Loss Considerations

The pressure loss due to blade drag alone will be considered here.

B.2.5.1. Constant Area Flow Case

$$F_{DP} = F_\theta \sin \alpha_m - F_{z0} \cos \alpha_m \quad (B-40)$$

$$= \frac{\rho_1 A_1 C_{z1}}{g_o} (W_{\theta 1} - W_{\theta 2}) \sin \alpha_m - \frac{\rho_1 W_1^2 A}{2 g_o} + \frac{\rho_2 W_2^2 A}{2 g_o} - \frac{\rho C_{z1} A}{g_o} (C_{z2} - C_{z1})$$

$$= - (P_{t2}' - P_{t1}') A \cos \alpha_m \quad (B-41)$$

The mean relative dynamic pressure is $\frac{\rho_m W_m^2}{2 g_o}$. From previous work,

$$\frac{\rho_m W_m^2}{2 g_o} = \frac{\rho_1 C_{z1} (C_{z1} + C_{z2})}{4 g_o \cos^2 \alpha_m}$$

The drag coefficient is:

$$\frac{F_D}{\frac{\rho_m W_m^2}{2 g_o}} = c_H \quad (B-42)$$

$$C_{DP} = \frac{(P'_{t1} - P'_{t2}) \frac{s}{c} \cos \alpha_m}{\frac{\rho_m W_m^2}{2g_0}}$$

(B-43)

$$= \frac{s}{c} \frac{\Delta P_t \cos^3 \alpha_m}{\frac{\rho_1 C_{z1}}{2g_0} \left(\frac{C_{z1} + C_{z2}}{2} \right)}$$

$$\therefore \frac{\Delta P_t}{\frac{\rho_1 C_{z1}}{2g_0}} = C_{DP} \frac{c}{s} \left(\frac{C_{z1} + C_{z2}}{2C_{z1}} \right) \frac{1}{\cos^3 \alpha_m}$$

Converting to relative inlet dynamic pressure

$$C_{z1} = W_1 \cos \alpha_1$$

$$\frac{\Delta P_t}{\frac{\rho_1 W_1^2}{2g_0}} = C_{DP} \frac{c}{s} \left(\frac{C_{z1} + C_{z2}}{2C_{z1}} \right) \frac{\cos^2 \alpha_1}{\cos^3 \alpha_m}$$

(B-44)

B.2.5.2. Compressible Flow with Area Change

$$C_{DP} = \frac{(P'_{t1} - P'_{t2}) \frac{s}{c} \cos \alpha_m}{\frac{\rho_m W_m^2}{2g_0}}$$

As was seen, $\rho_m = f(P'_t, \text{etc.})$. P'_t itself will be changed by a fraction (of the order of one percent) of the yet unknown pressure loss. Hence there does not appear to be any direct way of finding a correct value of ρ_m , the only possibility of obtaining a correct solution is to resort to an iterative solution where P'_{tm} is recursively improved. But in view of the fact that an accuracy

of the order of 1% does neither exist in the experimental data of C_D , nor the interpolation methods used, this iterative procedure cannot be justified. Consequently, the previously calculated value for $\rho_m W_m^2$ will be used again:

$$\rho_m W_m^2 = \frac{W_m^2 P'_{t1}}{RT_t} \left[1 + \frac{\gamma - 1}{2} \left(\frac{W_m}{\sqrt{\gamma g_o RT_t}} \right)^2 \right]^{\frac{1}{\gamma - 1}}$$

The pressure loss, made dimensionless by relative inlet dynamic pressure, is

$$\frac{\Delta P'_t}{\frac{\rho_1 W_1^2}{2g_o}} = \frac{C_{DP} \frac{c}{s} \sec \alpha_m \frac{W_m^2 P'_{t1}}{2g_o RT_t} \left[1 + \frac{\gamma - 1}{2} \left(\frac{W_m}{\sqrt{\gamma g_o RT_t}} \right)^2 \right]^{\frac{1}{\gamma - 1}}}{\frac{\rho_1 W_1^2}{2g_o}}$$

$$= C_{DP} \frac{c}{s} \sec \alpha_m \frac{P'_{t1}}{\rho_1 RT_t} \left(\frac{W_m}{W_1} \right)^2 \left[1 + \frac{\gamma - 1}{2} \left(\frac{W_m}{\sqrt{\gamma g_o RT_t}} \right)^2 \right]^{\frac{1}{\gamma - 1}}$$

In terms of relative inlet and outlet quantities the above is:

$$\frac{\Delta P'_t}{\frac{\rho_1 W_1^2}{2g_o}} = C_{DP} \frac{c}{s} \sec \alpha_m \left(1 + \frac{W_2}{W_1} \right)^2 \left(1 + \frac{\gamma - 1}{2} M_1'^2 \right)^{\frac{1}{\gamma - 1}} \left[1 - \frac{\gamma - 1}{8} \left(\frac{W_1 + W_2}{\sqrt{\gamma g_o RT_t}} \right)^2 \right]^{\frac{1}{\gamma - 1}}$$

(B-45)

B.3. Influence of Distortion Swirl Velocity on Axial Velocity

As was shown in Chapter 1, crossflow induced inlet flow distortion provided the advancing blades with positive swirl and

the retreating blades with negative swirl. Since outlet angle deviation in unstalled flow is normally much less than imposed incidence angle variation, but always in the same sense, positive swirl causes axial velocities to increase while negative swirl causes them to decrease. This effect of inlet swirl on axial velocity may be demonstrated for incompressible flow with constant

flow area. Let it be assumed that the back-pressure, wheel speed, the total pressure upstream of the fan, and the rotor blade outlet air angle are constant, and that the rotor exhausts directly to the atmosphere. The rotor pressure rise is equated to the pressure drop between the approach stream and the fan inlet face,

FIGURE B.4. Effect of inlet swirl on the rotor inlet velocity triangle at constant axial velocity

and when one considers the effects of positive inlet swirl, as depicted in Figure B.4, the relative velocities are:

$$W_1^2 = (U + C_{\theta 1})^2 + C_z^2 \quad (B-46)$$

$$W_2 = C_z / \cos \alpha_2 \quad (B-47)$$

The ideal blade row pressure rise coefficient is:

$$\frac{\Delta P}{\rho W_1^2} = 1 - \left(\frac{W_2}{W_1} \right)^2 \quad (B-48)$$

The ideal pressure drop coefficient, in the inlet to the fan is:

$$\frac{\Delta P}{\rho W_1^2 / 2g_o} = \frac{P_t - P_1}{\rho W_1^2 / 2g_o} = \frac{C_{\theta 1}^2 + C_z^2}{W_1^2} \quad (\text{B-49})$$

Since the pressure rise through the fan is assumed to equal the inlet pressure drop, one can equate (B-48) to (B-49) in terms of (B-46) and (B-47):

$$1 - \left(\frac{W_2}{W_1} \right)^2 = \frac{C_{\theta 1}^2 + C_z^2}{W_1^2}$$

or,

$$U^2 + 2UC_{\theta 1} + C_{\theta 1}^2 + C_z^2 - C_z^2 / \cos^2 \alpha_2 = C_{\theta 1}^2 + C_z^2$$

After simplification and division by U^2 , one obtains:

$$\frac{2C_{\theta 1}}{U} = \left(\frac{C_z}{U} \right)^2 \frac{1}{\cos^2 \alpha_2} - 1 \quad (\text{B-50})$$

Since α_2 and U are assumed to be constant, Equation

(B-50) shows that C_z must increase if $C_{\theta 1}$ increases, and vice versa. Figure B.5

illustrates a typical inlet

velocity diagram with this feedback effect included. In actual fact, α_2 does not remain constant, but changes with rotor incidence

FIGURE B.5. Effect of inlet swirl on the rotor inlet velocity triangle with axial velocity adjustments

as mentioned above. Equation (B-50) shows that deviations in outlet angle cause additional increases in axial velocity due to positive distortion swirl, etc.

If the outlet angle is constant, or nearly constant, positive distortion swirl causes an increase in swirl change across the rotor on the advancing side and a decrease in swirl change on the retreating side of the rotor. This leads to similar variations in total conditions at the fan exit.

The above reasoning remains valid even when back-pressure is nonuniform circumferentially and when a stator is present, but the degree of feedback produced obviously becomes dependent on back-pressure distribution and other effects which are less than ideal.

An increased back-pressure results in a larger static pressure rise across the fan. Let the associated values of axial velocity and outlet angle be $\hat{C}_z$, $\hat{\alpha}_2$. With the increase in back-pressure being denoted as ΔP , Equation (B-50) becomes:

$$\frac{2C_{\theta 1}}{U} = \left(\frac{\hat{C}_z}{U} \right)^2 \frac{1}{\cos^2 \hat{\alpha}_2} - 1 + \frac{\Delta P}{\frac{\rho U^2}{2g_0}} \quad (\text{B-51})$$

If $\Delta P > 0$, then either $\hat{C}_z < C_z$, or $\hat{\alpha}_2 < \alpha_2$, or both. Since the deviation in outlet angle is not apt to be great, an increase in back-pressure results in a decrease in axial velocity, and thereby drives the rotor incidence towards the positive.

The effect that the above variables have on rotor incidence is indicated in Figure B.6 below. The exact position and shape of the zero rotor incidence line is, however, not

universal, and depends on the blade geometry used and the operating performance of the blades.

FIGURE B.6. Influence of inlet distortion on axial velocity and rotor incidence

APPENDIX C

C.1. Equilibrium Equations in the Absolute Reference Frame

C.1.1. Equation of Motion

The equation of motion for an inviscid and compressible fluid is

$$\frac{D\vec{C}}{Dt} = -g_0 \frac{1}{\rho} \nabla P \quad (C-1)$$

where C is the velocity and $\frac{D\vec{C}}{Dt}$ is the acceleration vector. The equation of motion split into scalar forms can be written in the familiar form:

r-direction:

$$\frac{\partial C_r}{\partial t} + C_r \frac{\partial C_r}{\partial r} + \frac{C_\theta}{r} \frac{\partial C_r}{\partial \theta} + C_z \frac{\partial C_r}{\partial z} - \frac{C_\theta^2}{r} = -\frac{g_0}{\rho} \frac{\partial P}{\partial r} \quad (C-2a)$$

θ -direction:

$$\frac{\partial C_\theta}{\partial t} + C_r \frac{\partial C_\theta}{\partial r} + \frac{C_\theta}{r} \frac{\partial C_\theta}{\partial \theta} + C_z \frac{\partial C_\theta}{\partial z} + \frac{C_\theta C_r}{r} = -\frac{g_0}{\rho} \frac{\partial P}{r \partial \theta} \quad (C-2b)$$

z-direction:

$$\frac{\partial C_z}{\partial t} + C_r \frac{\partial C_z}{\partial r} + \frac{C_\theta}{r} \frac{\partial C_z}{\partial \theta} + C_z \frac{\partial C_z}{\partial z} = -\frac{g_0}{\rho} \frac{\partial P}{\partial z} \quad (C-2c)$$

The substantial derivative $\frac{D}{Dt}$ in terms of its local and convective derivatives is:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + C_r \frac{\partial}{\partial r} + C_\theta \frac{\partial}{r \partial \theta} + C_z \frac{\partial}{\partial z} \quad (C-3)$$

so that Equation (C-1) may also be written as:

$$\frac{\partial \vec{C}}{\partial t} + (\vec{C} \cdot \nabla) \vec{C} = - \frac{g_0}{\rho} \nabla P \quad (C-4)$$

The component forms may also be expressed in terms of the vorticity relationships. This is achieved by adding and subtracting

$C_\theta \frac{\partial C_\theta}{\partial r} + C_z \frac{\partial C_z}{\partial r} + \frac{C_r}{r} \frac{\partial C_r}{\partial \theta} + \frac{C_z}{r} \frac{\partial C_z}{\partial \theta} + C_r \frac{\partial C_r}{\partial z} + C_\theta \frac{\partial C_\theta}{\partial z}$ in the Equations (C-2a,b,c), respectively. The familiar result is:

r-direction:

$$\frac{\partial C_r}{\partial t} - \frac{C_\theta}{r} \left(\frac{\partial r C_\theta}{\partial r} - \frac{\partial C_r}{\partial \theta} \right) + C_z \left(\frac{\partial C_r}{\partial z} - \frac{\partial C_z}{\partial r} \right) + \frac{\partial}{\partial r} \frac{1}{2} C^2 = - \frac{g_0}{\rho} \frac{\partial P}{\partial r} \quad (C-5a)$$

θ -direction:

$$\frac{\partial C_\theta}{\partial t} - \frac{C_z}{r} \left(\frac{\partial C_z}{\partial \theta} - \frac{\partial r C_\theta}{\partial z} \right) + \frac{C_r}{r} \left(\frac{\partial r C_\theta}{\partial r} - \frac{\partial C_r}{\partial \theta} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \frac{1}{2} C^2 = - \frac{g_0}{\rho r} \frac{\partial P}{\partial \theta} \quad (C-5b)$$

z-direction:

$$\frac{\partial C_z}{\partial t} - C_r \left(\frac{\partial C_r}{\partial z} - \frac{\partial C_z}{\partial r} \right) + \frac{C_\theta}{r} \left(\frac{\partial C_z}{\partial \theta} - \frac{\partial r C_\theta}{\partial z} \right) + \frac{\partial}{\partial z} \frac{1}{2} C^2 = - \frac{g_0}{\rho} \frac{\partial P}{\partial z} \quad (C-5c)$$

These scalar equations are recognized as components of a general vector equation

$$\frac{\partial \vec{C}}{\partial t} - \vec{C} \times (\nabla \times \vec{C}) + \nabla \frac{1}{2} C^2 = - \frac{g_0}{\rho} \nabla P \quad (C-6)$$

where $C^2 = C_r^2 + C_\theta^2 + C_z^2$ and where the vorticity terms are

$$2\omega_r = \frac{1}{r} \left(\frac{\partial C_z}{\partial \theta} - \frac{\partial r C_\theta}{\partial z} \right) \quad (C-7a)$$

$$2\omega_\theta = \left(\frac{\partial C_r}{\partial z} - \frac{\partial C_z}{\partial r} \right) \quad (C-7b)$$

$$2\omega_z = \frac{1}{r} \left(\frac{\partial r C_\theta}{\partial r} - \frac{\partial C_r}{\partial \theta} \right) \quad (C-7c)$$

C.1.2. Equation of Continuity

The equation of continuity for a control volume is mass rate of flow out + rate of storage - mass rate of flow in = 0. For a cylindrical coordinate system and a compressible flow the continuity equation in differential form is

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho C_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho C_\theta) + \frac{\partial}{\partial z} (\rho C_z) = 0 \quad (C-8)$$

C.1.3. Energy Considerations

The through-flow is assumed to be quasi-steady (as will be demonstrated), hence it is well-ordered along streamlines with no convection taking place normal to streamlines. The flow within stream tubes of small cross-section away from blade rows is assumed to be inviscid and isentropic so that there exists no mechanism internal to the stream tubes to change either the total enthalpy or entropy of the contained fluid. Different values of total enthalpy and entropy are possible for neighbouring stream tubes, however, in order to allow for variations in blade row performance and inflow distortion. Blade row performance is introduced as bounded discontinuities, i.e. step changes in h_t and s , imposed locally on the flow inside each stream tube between control stations.

Consider the fluid inside a typical stream tube. The Steady Flow Energy equation in differential form per unit mass is:

$$dq = de + \frac{1}{J} d\left(\frac{P}{\rho}\right) + \frac{1}{2g_o J} dC^2 + dW \quad (C-9)$$

and applies for a reversible steady flow process in the stream tube in the absence of body forces. On using the entropy relationship $Tds = dq$ for the same process, and the definition of enthalpy $h = e + \frac{P}{\rho}$ we obtain the following useful relationship:

$$\frac{1}{\rho J} dP = dh - Tds. \quad (C-10)$$

For the fluid considered the thermodynamic quantities P, h, s , are functions of space alone. Hence the total derivative forms may be expressed in terms of their partial derivatives.

$$\begin{aligned} \frac{1}{\rho J} \left(\frac{\partial P}{\partial r} dr + \frac{\partial P}{\partial \theta} d\theta + \frac{\partial P}{\partial z} dz \right) &= \\ &= \left(\frac{\partial h}{\partial r} dr + \frac{\partial h}{\partial \theta} d\theta + \frac{\partial h}{\partial z} dz \right) - T \left(\frac{\partial s}{\partial r} dr + \frac{\partial s}{\partial \theta} d\theta + \frac{\partial s}{\partial z} dz \right) \end{aligned}$$

Each term in parenthesis is recognized as the dot product $\nabla \cdot d\vec{s}$, i.e. the scalar product of the gradients and the signed elemental length of the stream tube. On dividing out the vector $d\vec{s}$ we obtain the useful gradient relationship

$$\frac{1}{\rho J} \nabla P = \nabla h - T \nabla s \quad (C-11)$$

On substituting $h = h_t - \frac{C^2}{2g_0 J}$ in Equation (C-11) one obtains:

$$\frac{1}{\rho J} \nabla P = \nabla h_t - \frac{1}{g_0 J} \nabla \frac{C^2}{2} - T \nabla s \quad (C-12)$$

This relationship is most meaningful in a mathematical sense for flow along streamlines, as Novak⁽¹⁸⁾ pointed out and as will be demonstrated later.

The pressure gradient term of Equation (C-6) is replaced by its equivalents as given in Equation (C-11) above. After

re-arranging and dropping the time dependent term the following equation results:

$$\left(\frac{1}{g_{0J}}\right) \tilde{C} \times (\nabla \times \tilde{C}) = \nabla h_t - T \nabla s \quad (C-13)$$

This equation is known as the Crocco equation in absolute coordinates. The corresponding scalar forms of the above equation are:

$$\frac{1}{g_{0J}} \left[\frac{C_{\theta}}{r} \left(\frac{\partial r C_{\theta}}{\partial r} - \frac{\partial C_r}{\partial \theta} \right) - C_z \left(\frac{\partial C_r}{\partial z} - \frac{\partial C_z}{\partial r} \right) \right] = \frac{\partial h_t}{\partial r} - T \frac{\partial s}{\partial r} \quad (C-13a)$$

$$\frac{1}{g_{0J}} \left[\frac{C_z}{r} \left(\frac{\partial C_z}{\partial \theta} - \frac{\partial r C_{\theta}}{\partial z} \right) - \frac{C_r}{r} \left(\frac{\partial r C_{\theta}}{\partial r} - \frac{\partial C_r}{\partial \theta} \right) \right] = \frac{\partial h_t}{r \partial \theta} - T \frac{\partial s}{r \partial \theta} \quad (C-13b)$$

$$\frac{1}{g_{0J}} \left[C_r \left(\frac{\partial C_r}{\partial z} - \frac{\partial C_z}{\partial r} \right) - \frac{C_{\theta}}{r} \left(\frac{\partial C_z}{\partial \theta} - \frac{\partial r C_{\theta}}{\partial z} \right) \right] = \frac{\partial h_t}{\partial z} - T \frac{\partial s}{\partial z} \quad (C-13c)$$

C.2. Derivation of a Property Related to Entropy

The partial derivatives of entropy appearing in the scalar forms of the Crocco equation are difficult to evaluate directly. Instead, it is more convenient to consider an associated property θ comprising both temperatures and pressures.

Equation (C-10) contained only static quantities and is known to be valid for both an internally reversible non-flow process and an internally reversible steady flow process. One relevant example for the former is the process involved in bringing the gas to rest isentropically (stagnation condition). In this case the left hand side of the equation is zero. A relevant example for the latter is inviscid adiabatic flow along a stream

tube. If there is zero energy input, the left hand side is again zero. It is admissible, however, to impose a change in both s and h from the outside (provided this condition involves a series of equilibrium states) without violating this relationship. This is assumed to happen when the blades do work on the air and change both its internal and kinetic energy, and hence it is permissible to apply Equation (C-10) across a blade row (regardless of whether it is moving or is stationary).

Using the following relationships:

$$P = \rho RT \quad (C-14)$$

$$dh = c_p dT \quad (C-15)$$

$$c_p = \frac{R}{J} \left(\frac{\gamma}{\gamma-1} \right) \quad (C-16)$$

where c_p and γ are considered to be invariant with temperature

$$\frac{ds}{c_p} = \frac{dT}{T} - \frac{\gamma-1}{\gamma} \frac{dP}{P} \quad (C-17)$$

As all functions herein are point functions an integration may be carried out between the states existing in the inlet (constant entropy and total enthalpy) and at a control station on the same streamline downstream of the rotor.

$$\frac{s_- - s_+}{c_p} = \ln \frac{T_-}{T_+} - \frac{\gamma-1}{\gamma} \ln \frac{P_-}{P_+} \quad (C-18)$$

$$= - \ln \frac{(P_-/P_+)^{\frac{\gamma-1}{\gamma}}}{(T_-/T_+)} \quad (C-19)$$

FIGURE C.1. Temperature entropy diagram for blade row

On using the former example of a stagnation process the static temperatures and pressures may be converted into their respective total values.

$$\frac{s_- - s_+}{c_p} = - \ln \left[\frac{\left(\frac{P_{t+}}{P_+} \right)^{\frac{\gamma-1}{\gamma}} \left(\frac{P_{t-}}{P_-} \right)^{\frac{\gamma-1}{\gamma}}}{\left(\frac{T_{t+}}{T_+} \right)^{\frac{\gamma-1}{\gamma}} \left(\frac{T_{t-}}{T_-} \right)^{\frac{\gamma-1}{\gamma}}} \right] \quad (C-20)$$

The ratio $\frac{\left(\frac{P_t}{P} \right)^{\frac{\gamma-1}{\gamma}}}{\frac{T_t}{T}}$ is, of course, equal to unity. Hence, if

$\frac{\left(\frac{P_{t-}}{P_{t+}} \right)^{\frac{\gamma-1}{\gamma}}}{\frac{T_{t-}}{T_{t+}}}$ is called θ then we obtain exactly the same expression as indicated by Novak⁽¹⁸⁾:

$$\frac{s_- - s_+}{c_p} = - \ln \theta \quad (C-21)$$

where $\theta = e^{-\left(\frac{s_- - s_+}{c_p} \right)}$ is called the entropy associated function.

This function is, however, not a definition as indicated in Novak's paper because the relationship holds only under special conditions,

as was shown above.

The quantity s_+ may be regarded as a reference value as it is considered constant in this problem. Because of this feature it will drop out when derivatives are taken. Hence,

$$\frac{ds_+}{c_p} = - d(\ln \theta) \quad (C-22)$$

Since s and θ are functions of space only, the total derivatives may be expressed in terms of their partial derivatives:

$$\frac{1}{c_p} \frac{\partial s}{\partial r} = - \frac{\partial}{\partial r} (\ln \theta) \quad (C-23)$$

and

$$\frac{1}{c_p} \frac{\partial s}{r \partial \theta} = - \frac{1}{r} \frac{\partial}{\partial \theta} (\ln \theta) \quad (C-24)$$

where s is understood to mean s_+ , i.e. downstream of the rotor,

or,

$$T \frac{\partial s}{\partial r} = - c_p T \frac{\partial}{\partial r} (\ln \theta) \quad (C-25)$$

$$T \frac{\partial s}{r \partial \theta} = - c_p T \frac{\partial}{r \partial \theta} (\ln \theta) \quad (C-26)$$

On using the definitions for static and total enthalpy the above expressions may be cast directly into a form containing only total temperatures, total pressures and velocity components.

$$T \frac{\partial s}{\partial r} = \left(-h_t + \frac{C_r^2 + C_\theta^2 + C_z^2}{2g_o J} \right) \frac{\partial}{\partial r} \ln \theta \quad (C-27a)$$

and,

$$T \frac{\partial s}{r \partial \theta} = \left(-h_t + \frac{C_r^2 + C_\theta^2 + C_z^2}{2g_o J} \right) \frac{\partial}{r \partial \theta} \ln \theta \quad (C-27b)$$

C.3. Wu's Surfaces in an Absolute Reference Frame

Consider an S_1 surface which is initially in the e_θ and e_z direction (e_θ and e_z are the base vectors in the cylindrical coordinate system), i.e. at the first axial station all streamlines lie on a circular arc co-axial with the machine axis of rotation. The equation of the surface is

$$S_1(r, \theta, z) = 0 \quad (C-28)$$

Hence,

$$\Delta S_1 = \frac{\partial S_1}{\partial r} \Delta r + \frac{\partial S_1}{\partial \theta} \Delta \theta + \frac{\partial S_1}{\partial z} \Delta z = 0 \quad (C-29)$$

Since each r, θ, z are functions of time we divide each term by a small interval of time Δt and take the limit as $\Delta t \rightarrow 0$

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta S_1}{\Delta t} = \frac{dS_1}{dt} = \frac{\partial S_1}{\partial r} \frac{dr}{dt} + \frac{\partial S_1}{\partial \theta} \frac{d\theta}{dt} + \frac{\partial S_1}{\partial z} \frac{dz}{dt} = 0 \quad (C-30)$$

On interpreting the total derivatives of r, θ , and z with respect to time as the component velocities, one finds that

$$\frac{\partial S_1}{\partial r} C_r + \frac{\partial S_1}{\partial \theta} C_\theta + \frac{\partial S_1}{\partial z} C_z = 0$$

This is recognized as the dot product of two vectors

$$\vec{n} \cdot \vec{C}$$

where $\vec{n}$ is the surface normal and is by definition normal to the velocity vector $\vec{C}$, as is evidenced by the dot product being zero.

$$\vec{n} = n_r \mathbf{e}_r + n_\theta \mathbf{e}_\theta + n_z \mathbf{e}_z \quad (C-31)$$

$$= \frac{\partial S_1}{\partial r} \mathbf{e}_r + \frac{\partial S_1}{\partial \theta} \mathbf{e}_\theta + \frac{\partial S_1}{\partial z} \mathbf{e}_z$$

The relationship between the scalar components is conveniently put as:

$$\frac{n_r}{\frac{\partial S_1}{\partial r}} = \frac{n_\theta}{r \frac{\partial S_1}{\partial \theta}} = \frac{n}{\frac{\partial S_1}{\partial z}} \quad (C-32)$$

C.3.1. Flow Along the S_1 and S_2 Surfaces

Since the stream surface S_1 was defined as one which is made up of streamlines, any quantity "q" on it can be considered a function of θ and z only, since on the surface r can be considered as a function of θ and z .

$$q = q [\theta, z, r(\theta, z)] \quad (C-33)$$

The total change (along a streamline) would be

$$dq = \frac{\partial q}{\partial r} dr + \frac{\partial q}{\partial \theta} d\theta + \frac{\partial q}{\partial z} dz$$

but,

$$dr = \frac{\partial r}{\partial \theta} d\theta + \frac{\partial r}{\partial z} dz$$

so that:

$$dq = \frac{\partial q}{\partial r} \left(\frac{\partial r}{\partial \theta} d\theta + \frac{\partial r}{\partial z} dz \right) + \frac{\partial q}{\partial \theta} d\theta + \frac{\partial q}{\partial z} dz$$

The total change is a superposition of partial effects, and we are free to consider changes in the r - θ directions only, i.e. with z held fast.

$$\left. \begin{aligned} dq \\ z = \text{const.} \\ S_1 = \text{const.} \end{aligned} \right\} = \frac{\partial q}{\partial \theta} d\theta + \frac{\partial q}{\partial r} \cdot \frac{dr}{d\theta} d\theta \quad (C-34)$$

Now consider Equation (C-29) again as applied to a differential distance, dS_1 and in the special case of $z = \text{constant}$.

$$dS_1 = \frac{\partial S_1}{\partial r} dr + \frac{\partial S_1}{\partial \theta} d\theta$$

But, since $r = r(\theta, z)$ and $z = \text{constant}$

$$\frac{\partial S_1}{\partial r} \frac{dr}{d\theta} + \frac{\partial S_1}{\partial \theta} = 0$$

This relationship is an expression for the slope of the S_1 surface at an axial station ($z = \text{constant}$)

$$\frac{dr}{d\theta} = - \frac{\frac{\partial S_1}{\partial \theta}}{\frac{\partial S_1}{\partial r}} \quad (C-35)$$

Or, using the surface normal relationship it is more concisely put as:

$$\frac{dr}{d\theta} = - \frac{r n_\theta}{n_r} \quad (C-36)$$

The surface normal ratio $\frac{n_\theta}{n_r}$ will be designated "N" from now on.

Equation (C-34) may be expressed in terms of Equation (C-36):

$$\left. \begin{aligned} dq \\ S_1 = \text{const.} \end{aligned} \right|_{z = \text{const.}} = \frac{\partial q}{\partial \theta} d\theta - rN \frac{\partial q}{\partial r} d\theta$$

or,

$$\left. \frac{dq}{d\theta} \right|_{\substack{z = \text{const.} \\ S_1 = \text{const.}}} = \frac{\partial q}{\partial \theta} - rN \frac{\partial q}{\partial r} \quad (37a)$$

For flow along the S_2 surface analogous formulations lead to

$$\left. \frac{dq}{dr} \right|_{\substack{z = \text{const.} \\ S_2 = \text{const.}}} = \frac{\partial q}{\partial r} - \frac{1}{rN} \frac{\partial q}{\partial \theta} \quad (C-37b)$$

The surfaces S_1 and S_2 are normal to one another as can be seen from taking the ratio of the slopes (Equation (C-35) and its equivalent for S_2).

On taking the ratio of the q-derivatives (Equations (C-37a) and (C-37b)) a second useful relationship can be found for N.

$$\begin{aligned} \frac{\frac{dq}{d\theta}}{\frac{dq}{dr}} &= \frac{\frac{\partial q}{\partial \theta} - rN \frac{\partial q}{\partial r}}{\frac{\partial q}{\partial r} - \frac{1}{rN} \frac{\partial q}{\partial \theta}} \\ &= -rN \left(\frac{-\frac{1}{rN} \frac{\partial q}{\partial \theta} + \frac{\partial q}{\partial r}}{\frac{\partial q}{\partial r} - \frac{1}{rN} \frac{\partial q}{\partial \theta}} \right) \\ \therefore \frac{dq}{rd\theta} / \frac{dq}{dr} &= -N. \end{aligned}$$

The q-derivative expressions (Equations (C-37a) and (C-37b)) are the means for converting the partial differential equations (Equations 55a, b, c) into ordinary differential equations for the special case of $z = \text{constant}$. By making this change to the differential equations an important simplification will be achieved toward a solution as the problem will have been made mathematically two-dimensional. The problem will no longer be solved on a cylindrical reference frame, instead a solution shall be sought on stream surfaces.

APPENDIX D

D.1. Derivation of the Crocco Equation in the Relative Reference Frame

The vector relationship connecting stationary and moving frames of reference is:

$$\vec{C} = \vec{\Omega} + \vec{\Omega} \times \vec{r} \quad (D-1)$$

The quantity $\vec{\Omega} \times \vec{r}$ has the magnitude $U = \Omega r$ and a tangential direction, as dictated by the r.h. screw rule. The formulation of the Crocco equation in relative co-ordinates involves time changes from t to $t + dt$ as the rotor turns through an angle Ωdt . The rotor motion has the direction of $\frac{\vec{\Omega}}{\Omega}$. While the rotor moves Ωdt an observer on the rotor sees a fluid particle position change from $\vec{r}_R$ to $\vec{r}_R + \vec{\Omega} dt$. The arc length vector of this position change (in the plane of rotation) is $\vec{\lambda}$.

$$\begin{aligned} \vec{\lambda} &= \frac{\vec{\Omega}}{\Omega} \times (\vec{r}_R + \vec{\Omega} dt) \Omega dt \\ &= \vec{\Omega} \times \vec{r}_R dt + \vec{\Omega} \times \vec{\Omega} dt^2 \end{aligned} \quad (D-2)$$

Neglecting terms of order higher than dt :

$$\vec{\lambda} \approx \vec{\Omega} \times \vec{r}_R dt \quad (D-3)$$

The subscript R indicates reference to the relative reference frame. The vector $\vec{r}_R$ is on the moving rotor while $\vec{r}$ is in the absolute reference frame, as shown in Figure D.1., i.e.

$$\vec{b} + \vec{r}_R = \vec{r} \quad (D-4)$$

$$\therefore \dot{\vec{n}} \times \vec{r} = \dot{\vec{n}} \times (\vec{b} + \vec{r}_R)$$

$$= \dot{\vec{n}} \times \vec{b} + \dot{\vec{n}} \times \vec{r}_R$$

but $\dot{\vec{n}} \parallel \vec{b}$, so that $\dot{\vec{n}} \times \vec{b} = 0$

$$\therefore \dot{\vec{n}} \times \vec{r} = \dot{\vec{n}} \times \vec{r}_R \quad (D-5)$$

The time rate of position change $\dot{\vec{x}}$ is equal to the change from relative to absolute

$$\therefore \dot{\vec{c}} = \dot{\vec{w}} + \frac{d\vec{x}}{dt}$$

$$= \dot{\vec{w}} + \dot{\vec{n}} \times \vec{r},$$

which is the basic equation stated at the outset.

FIGURE D.1. Motion in the relative reference frame

The relation-

ship between absolute and relative accelerations are obtained in much the same way. Consider a change in velocities $D_R \vec{w}$ and $D\vec{c}$, with D denoting the substantial derivative.

$$D_R \vec{w} = \frac{\partial \vec{w}}{\partial t} dt + d\vec{r}_R \cdot \nabla \vec{w} \quad (D-6)$$

The new relative velocity is, therefore,

$$\vec{w}_{t+dt} = \vec{w} + D_R \vec{w}$$

The velocity triangle may be completed as before (see Equation (D-1)):

$$[\vec{c} + D\vec{c}] = [\vec{w} + D_R \vec{w}] + [\dot{\vec{n}} \times (\vec{r}_R + \vec{w}dt)].$$

This is, however, a "stationary" relationship since in time dt the rotor has actually moved through an angle Ωdt and the change in direction must be accounted for. As previously (see Equation (D-2)), the change in the arc direction is obtained by the cross product of unit rotation and signed arc length, i.e.

$$\frac{\hat{n}}{\Omega} \times [\hat{C} + D\hat{C}] \Omega dt$$

$$\therefore \hat{C} + D\hat{C} = [\hat{\omega} + D_R \hat{\omega}] + [\hat{\Omega} \times (\hat{r}_R + \hat{\omega} dt)] + \hat{\Omega} \times ([\hat{\omega} + D_R \hat{\omega}] + [\hat{\Omega} \times (\hat{r}_R + \hat{\omega} dt)]) dt \quad (D-7)$$

The change in absolute velocity is $D\hat{C}$, to obtain this

$\hat{C} = \hat{\omega} + \hat{\Omega} \times \hat{r}_R$ can be subtracted from the right hand side. The terms are multiplied out and the differentials of higher order than unity (i.e. $\hat{\Omega} \times D_R \hat{\omega} dt$ and $\hat{\Omega} \times (\hat{\Omega} \times \hat{\omega} dt^2)$) are neglected.

$$D\hat{C} = D_R \hat{\omega} + 2\hat{\Omega} \times \hat{\omega} dt + \hat{\Omega} \times (\hat{\Omega} \times \hat{r}_R dt) \quad (D-8)$$

The time rate of change of absolute velocity is obtained by dividing by the small interval of time

$$\begin{aligned} \dot{\hat{a}} &= \frac{D\hat{C}}{dt} \\ &= \frac{D_R}{dt} \hat{\omega} + 2\hat{\Omega} \times \hat{\omega} + \hat{\Omega} \times (\hat{\Omega} \times \hat{r}_R) \end{aligned} \quad (D-9)$$

or since $\frac{D_R}{dt} \hat{\omega}$ is the total derivative of relative velocity, $\hat{\omega}(\hat{r}_R, t)$

$$\dot{\hat{a}} = \frac{\partial}{\partial t} \hat{\omega} + \hat{\omega} \cdot \nabla_R \hat{\omega} + 2\hat{\Omega} \times \hat{\omega} + \hat{\Omega} \times (\hat{\Omega} \times \hat{r}) \quad (D-10)$$

(since $\hat{\Omega} \times \hat{r}_R = \hat{\Omega} \times \hat{r}$).

The component contributions to acceleration have the following meanings:

$\frac{\partial_R}{\partial t} \vec{w}$: unsteady relative acceleration

$\vec{w} \cdot \nabla_R \vec{w}$: spatial linear acceleration

$2\vec{\Omega} \times \vec{w}$: Coriolis acceleration for helical flow

$\vec{\Omega} \times (\vec{\Omega} \times \vec{r})$: centripetal acceleration, whose magnitude is $\Omega^2 r$ and a direction towards the axis of rotation. At constant rotational speed the acceleration increases with radius, the sign being indicated by the vector product. ∇_R is a unit vector, positive in the increasing r -direction.

$$\begin{aligned} \therefore \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) &= -\Omega^2 R \nabla_R \\ &= -\nabla \frac{\Omega^2 R^2}{2} \end{aligned}$$

The relative acceleration can be expressed in terms of known quantities:

$$\frac{D_R}{Dt} \vec{w} = \frac{D\vec{C}}{Dt} - 2\vec{\Omega} \times \vec{w} - \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) \quad (D-11)$$

Using the pressure and energy relationships derived for the absolute frame of reference (Equation (C-27)) and Equation of motion (C-11), the Crocco equation in relative co-ordinates may now be obtained:

$$\frac{D_R}{Dt} \vec{w} = -g_0 J V h + g_0 J T V s - 2\vec{\Omega} \times \vec{w} - \vec{\Omega} \times (\vec{\Omega} \times \vec{r})$$

The substantial derivative of $\vec{w}$ may be expressed as follows:

$$\begin{aligned} \frac{D_R}{Dt} \vec{w} &= \frac{\partial_R}{\partial t} \vec{w} + \vec{w} \cdot \nabla_R \vec{w} \\ &= \frac{\partial_R}{\partial t} \vec{w} - \vec{w} \times (\nabla_R \times \vec{w}) + \nabla_R \frac{w^2}{2} \end{aligned}$$

$$\frac{\partial R}{\partial t} \vec{\omega} - \vec{\omega} \times (\nabla_R \times \vec{\omega}) = -\nabla I + g_0 J \nabla s - 2\vec{\Omega} \times \vec{\omega} \quad (D-14)$$

This is the desired Grocco equation with:

$$h_t' = h + \frac{w^2}{2g_0 J}$$

and the group of terms $(g_0 J h_t' - \frac{w^2 R^2}{2})$ was called "relative total enthalpy" by Vavra⁽³⁹⁾; but since this terminology is inconsistent with the notation here, Wu's definition⁽¹⁷⁾ of the quantity, "I" seems more appropriate.

D.2. Discussion of Special Features of Inflow

It was shown in Reference 7 that total pressure loss due to fluid friction was small and not measurable because of the encountered experimental errors. Consequently, the only significant losses are, for all practical purposes, confined to the boundary layers on the outer wall and centrebody wall. Heat transfer effects along streamlines are nearly zero; hence each integral is assumed to be zero on its own in the primary flow region and the time rate of change of circulation is zero. The mainstream inflow is therefore irrotational, no matter how distorted it appears after completing its turn from the approach direction.

The special character of inflow can be studied by means of the quantity I. One important feature of I is its constancy along a relative streamline, as can be demonstrated:

Consider steady $\left(\frac{\partial}{\partial t} = 0\right)$, isentropic flow ($\nabla s = 0$),

without friction forces between streamlines. Equation (D-12) reduces to:

$$\vec{\omega} \times (\nabla_R \times \vec{\omega}) = \nabla I + 2\vec{\Omega} \times \vec{\omega} \quad (D-13)$$

The relationship between absolute and relative vorticity

$$\nabla_R \times \vec{\omega} = \nabla \times \vec{\zeta} - 2\vec{\Omega} \quad (D-14)$$

is substituted above and the result is

$$\nabla I = \vec{\omega} \times (\nabla \times \vec{\zeta}) \quad (D-15)$$

The total change of I along a streamline is expressed by the substantial derivative:

$$\frac{D_R I}{Dt} = \frac{\partial_R I}{\partial t} + \vec{\omega} \cdot \nabla I \quad (D-16)$$

Substituting for ∇I and noting that $\frac{\partial I}{\partial t} = 0$

$$\frac{D_R I}{Dt} = \vec{\omega} \cdot \vec{\omega} \times (\nabla \times \vec{\zeta}) \quad (D-17)$$

But $\vec{\omega} \cdot \vec{\zeta} = 0$, consequently along a streamline:

$$\frac{D_R I}{Dt} = 0 \quad (D-18)$$

Consider the inlet triangle relationships:

$$\vec{\omega} + \vec{\Omega} R = \vec{\zeta}$$

$$W^2 = \Omega^2 R^2 + 2\Omega R C_\theta + C_\theta^2 + C_z^2$$

The terms $(C^2 = C_\theta^2 + C_z^2)$ are subtracted from above to obtain:

$$W^2 - C^2 = \Omega^2 R^2 + 2\Omega R C_\theta$$

After re-arrangement and division by two:

$$\frac{W^2}{2} - \frac{\Omega^2 R^2}{2} = \Omega R C_\theta + \frac{C^2}{2}$$

This is substituted in the definition of $I = g_o Jh + \frac{W^2}{2} - \frac{\Omega^2 R^2}{2}$

$$\therefore I = g_o Jh + \frac{C^2}{2} + \Omega R C_\theta \quad (D-19)$$

The association between the quantity I and total enthalpy may now be shown.

$$\text{Absolute co-ordinates: } g_o Jh_t = g_o Jh + \frac{C^2}{2} \quad (D-20)$$

$$\text{Rotating co-ordinates: } I = g_o Jh_t + \Omega R C_\theta \quad (D-21)$$

Therefore, whereas in absolute co-ordinates the constancy of h_t along a streamline implies a reversible exchange between the specific thermal energy h and the specific kinetic energy $\frac{C^2}{2}$, within the rotor the constancy of I along a streamline implies a reversible exchange between (absolute) total energy h_t and the work term $\Omega R C_\theta$. Stated in a slightly different way, the constancy of I implies that the rate of change of (absolute) total enthalpy along a streamline is equal to the angular speed of the blade times the rate of change of angular momentum about the fan axis of a fluid particle along its streamline. This, as Wu⁽¹⁷⁾ pointed out, is an equivalent statement of the well known Euler pump-turbine equation.

It has been demonstrated that the absolute flow entering the rotor has zero absolute vorticity ($\nabla \times \tilde{C} = 0$), uniform s and h_t . Consequently, $\nabla I = 0$ because

$$\nabla I = \tilde{\Omega} \times (\nabla \times \tilde{C}) = 0$$

The condition of $VI = 0$ can be satisfied in one of three ways:

1) $I = 0$ is clearly not the desired solution, 2) $I = \text{constant}$, or 3) $\frac{\partial I}{\partial r} i + \frac{\partial I}{r \partial \theta} j + \frac{\partial I}{\partial z} k = 0$ with the terms summing to zero.

Since $I = g_0 J h_t + \Omega R C_\theta$, and with h_t being a constant, how does the free vortex term $\Omega R C_\theta$ vary? The experimental evidence (Figures 6 and 7) suggest that C_θ varies from zero at the front of the inlet to some finite value approaching V_∞ at the sides of the fan. Clearly, therefore, the second condition, i.e. $I = \text{constant}$, is not possible and the only remaining possibility is that $\frac{\partial I}{\partial r}$, $\frac{\partial I}{r \partial \theta}$, $\frac{\partial I}{\partial z}$ cannot all be zero at the same time. Further useful information can be obtained from the absolute vorticity/ Equations (C-7):

$$r\text{-vorticity} = (\nabla \times \vec{C})_r = 2\omega_r = \frac{1}{r} \left(\frac{\partial C_z}{\partial \theta} - \frac{\partial r C_\theta}{\partial z} \right) = 0$$

$$\theta\text{-vorticity} = (\nabla \times \vec{C})_\theta = 2\omega_\theta = \left(\frac{\partial C_r}{\partial z} - \frac{\partial C_z}{\partial r} \right) = 0$$

$$z\text{-vorticity} = (\nabla \times \vec{C})_z = 2\omega_z = \frac{1}{r} \left(\frac{\partial r C_\theta}{\partial r} - \frac{\partial C_r}{\partial \theta} \right) = 0$$

Consider, for example, the z-vorticity since its implications can be most easily interpreted by means of experimental measurements (Figures 6 and 15 for the simple suction inlet or 7 and 17 for the fan inlet). We have $\frac{\partial}{\partial r} (r C_\theta) = \frac{\partial}{\partial \theta} C_r$. At the sides of the inlet $\frac{\partial}{\partial \theta} C_r$ appears to be a maximum (falling off with θ) and $\frac{\partial}{\partial r} (r C_\theta)$ also appears to be a maximum (falling off with increasing r), both slopes being of the same order. At the front and rear of the inlet $\frac{\partial}{\partial r} (r C_\theta) = \frac{\partial}{\partial \theta} C_r = 0$. Furthermore,

$$C_{\theta \text{ hub}} = C_{\theta \text{ outer wall}}$$

The experimental data evidently agree with the vorticity condition and the field distribution of I . In order that I be uniform the entrance flow would have had to be of the free vortex type, i.e. $rC_{\theta} = \text{constant}$. The entrance flow here $\left(\frac{\partial}{\partial r} rC_{\theta} = \frac{\partial}{\partial \theta} C_r \right)$ is of a general type with free vortex flow ($rC_{\theta} = \text{constant}$, $C_r \neq f(\theta)$) as a much more restrictive special case. In classifying entrance flow fields Wu⁽¹⁷⁾ did not single out this type of flow:

constant absolute total enthalpy	}
zero absolute vorticity	
constant entropy	
variable quantity I	
steady absolute and relative flow	

By virtue of Kelvin's circulation theorem the absolute vorticity in the primary flow will remain zero through the rotor and, as was shown, along streamlines the quantity I and the relative total enthalpy ($g_0 J h_t' = g_0 J h + \frac{W^2}{2}$) will remain constant. The non-uniformity of the quantity I and h_t' normal to the streamlines will cause the generation of secondary flow.

D.3. Feature of Rotor Flow

Vavra⁽³⁹⁾ showed that the rate of change of circulation along a streamline in the rotor can only be zero if the relative velocity is parallel to the axis of rotation. This vector relationship could be obtained if special inlet guide vanes were

employed in the inlet; but the resultant distribution of velocity would lead to a negative reaction fan design (a net increase of relative velocity through the rotor blades), and high relative Mach numbers. The fan cannot be expected to do useful work on the air if $\vec{W}$ is to remain parallel to $\vec{n}$. Consequently, under all conceivable conditions the rotor flow is bound to be rotational.

Swirl changes within the rotor produced by work done on the air changes the vector quantity $\vec{n} \times \vec{W}$, and hence also, the stream vorticity.

APPENDIX E

Flow Distortion and Performance Measurements on a 12 inch
Fan-in-Wing Model for a Range of Forward Speeds and Angle of Attack
Settings.

A reprint of a paper by U.W. Schaub and R.W. Bassett, presented
to the 38th Meeting of the AGARD Propulsion and Energetics Panel.

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PAPER No 17

**FLOW DISTORTION AND PERFORMANCE MEASUREMENTS ON A
12" FAN-IN-WING MODEL FOR A RANGE OF FORWARD SPEEDS
AND ANGLE OF ATTACK SETTINGS**

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38TH MEETING OF THE AGARD PROPULSION AND ENERGETICS PANEL

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