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THE BEHAVIOUR OF FLEXIBLE CONDUITS BURIED

AT

SHALLOW DEPTHS

by

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A thesis submitted to

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in partial fulfillment of the requirements for

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ABSTRACT

An extensive review of available literature on the behaviour of flexible buried conduits is presented in order to trace the development of the present state-of-the-art and to highlight the areas in which more research is needed. This research is centered on the utilization of two 2-D dimensional physical models in studying the behaviour of buried pipes at shallow depths under concentric and eccentric loads. A steel rod model and a sand box apparatus were used to observe the failure patterns of the pipe - soil system and to obtain qualitatively the influence of eccentricity on the failure loads. In the steel rod model, experiments were performed with pipes founded on rigid and flexible beddings.

The eccentric loadings caused asymmetrical buckling. When the pipes are buried at a cover depth greater than the pipe diameter, the influence of eccentric loading was found to be negligible. For shallow covers (less than one diameter), the eccentric critical loadings were smaller than those for the concentric loadings. This significant influence seemed to increase with the decrease of cover height. Providing reinforcement over the pipe drastically increased the carrying capacity of the flexible pipe even with eccentric loading, particularly for pipe buried at shallow depth.

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To my parents

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LIST OF SYMBOLS

A	Cross Sectional Area of Conduit Wall
B	Footing Width, Empirical Soil Support Coefficient
C	Soil Arching Coefficient
c	Shear Strength Parameter
C	Coefficient of Uniformity
C	Eccentricity Load Factor
D, d	Diameter of Pipe
D _r	Relative Density
D _l	Deflection Lag Factor
E	Young's Modulus (conduit)
E _s	Young's Modulus (soil), Secant Modulus of Soil
E	Tangent Modulus of Soil
e	Eccentricity, Void Ratio
F _C	Critical Buckling Thrust Force
F _k	Burial Depth Factor
F _p	Conduit Flexibility Ratio (Peck)
F	Conduit-Soil Flexibility Factor
f	Conduit Thrust Stress
f _b , f _c	Critical Buckling Thrust Stress
f _y	Yield Stress
G	Specific Gravity
g	Gravity
H, h	Cover Depth
I	Moment of Inertia of Conduit Wall Cross-Section, Impact Factor

K	Bedding Constant
K_s	Modulus of Soil Support (FL^{-2})
K_r	Relative Stiffness
k	Soil Stiffness Factor
k_T, k_R	Tangential and Rotational Elastic Stiffness at Each Supports
k_x	Load Relief Spring Constant
k_s	Subgrade Modulus (FL^{-3}) or Modulus of Soil Reaction
L	Axial Length of Conduit
l	Half-Wave Length of Local Buckle
l_t	Length of Dispersed Load
M_s	Constrained Soil Modulus
m_f	Modification Factor for Multi-lane Loading
n	Buckling Mode
P_e	Eccentric Critical Load
P_o	Concentric Critical Load
P_{air}	Critical Load Without Soil Support
p_b, p_c	Critical Radial Buckling Pressure
p_l	Live Load pressure
p_o	Overburden Pressure at Conduit Crown
R	Conduit Radius
R_i	Inside Radius of Supporting Soil Ring
R_o	Outside Radius of Supporting Soil Ring
$\bar{R}$	Equivalent Radius
r	Radius of Gyration
T	Circumferential Thrust

T_D	Dead Load
T_L	Live Load
t	Conduit Wall Thickness
U	Unevenness Factor for Imperfections
W_C	Vertical Load
α, ϕ_o	Angles Defining Arch Sections of Conduit
α_D	Dead Load Factor for Soil
α_L	Live Load Factor for Soil
γ	Unit Weight of Soil
ν	Poisson's Ratio (Conduit)
ν_s	Poisson's Ratio (Soil)
ν_e	Elastic Poisson's Ratio
ν_p	Plastic Poisson's Ratio
ϕ	Angle of Internal Friction
ψ_B	Angle Defining Distribution of Active Soil Pressure Around Conduit
λ	Active Earth Pressure Coefficient
λ_a	Angle of Friction of Arborite Sheet
λ_p	Angle of Friction of Plexiglass Sheet
λ_r	Angle of Friction of Rough Surface
λ_s	Angle of Friction of Stainless Steel Sheet
σ'	Effective Soil Pressure
μ_1	Conduit Shape Factor
ϕ	Performance Factor

CONVERSION OF UNITS

Symbol and Conversion Factors for SI Units

m = metre = 39.37 inches

mm = millimeter = 0.03937 inches

N = Newton = 0.22481 pound force

kN = KiloNewton = 224.81 pound force

Conversion from Imperial to SI Units

1 inch = 25.4 mm

1 foot = 304.8 mm = 0.3048 m

1 lb. force = 4.4482 N

1 kip = 4448.2 N = 4.4482 kN

Conversion of Units for Pressure and Stress

1 psi = 1 lb. force per square inch = 6.8947 kPa

or

1 kPa = 1 kiloNewton per square metre = 0.14504 psi

(xx)

CHAPTER 1

INTRODUCTION

1.1 General

The rapid expansion of modern transportation facilities requires the construction of many culverts and other underground structures like tunnels and passways. Large size corrugated steel culverts with spans of up to 20m are becoming increasingly popular as economical and efficient substitutes for traditional bridges. Since these culverts are larger in size and are subjected to heavier loads than those built previously, their behaviour should be studied very accurately, in order to provide safe and economical structures.

The study of the interaction between a buried pipe and its supporting soil media is of fundamental importance to a geotechnical and/or structural designer because of the composite performance of both the pipe and the soil in which it is buried. A buried flexible pipe shows far greater resistance to loading than when it is not surrounded by soil. The soil performs two conflicting roles. On one hand, it constitutes a considerable portion of the load that the pipe has to sustain, and on the other hand it provides the support needed by the pipe to increase its resistance to loading, due to the interaction between the soil and the buried structure.

A flexible pipe may fail due to the following reasons: 1. Wall

crushing, which occurs when compressive stress - due only to circumferential thrust - exceeds the yield stress, 2. Separation of seam when the thrust exceeds the seam strength, 3. Initiation of buckling from an elastic state of stress, 4. Inelastic buckling, and 5. Excessive deflection, or flattening, due to plastic yielding under combined compressive and bending stresses. These failure modes are shown in Figure 1.1 (Leonards and Stetkar (1978)).

It is relatively easier to guard against the first two modes of failure, since the magnitude of the circumferential thrust is relatively less sensitive to the properties of soil or to the stiffness of the pipe. However, bending stresses, which play an important role in failure modes (3), (4) and (5), are very sensitive to the above two parameters.

Several empirical, theoretical and rational methods have been derived to analyse the above stated failure modes of pipes (see literature review). All of them, with the exception of Kloppel and Glock (1970), deal only with uniformly applied boundary pressures on circular cross sections. But in real situations where a pipe is buried at shallow depth either under railways, airfields or highways, it experiences non uniform pressure due to the concentrated axle load, which acts eccentrically to the pipe. Therefore, the present work was undertaken to study the effects of eccentric loading on flexible pipes buried at shallow depths.

Wall Crushing

Seam Separation

Elastic Local Buckling

Inelastic (Snap-through)-Buckling

Plastic Yielding
(Excessive Deflection)

FIGURE 1.1 FAILURE MODES OF FLEXIBLE CONDUITS

(Ref. Leonards and Stetkar, 1978)

1.2 Statement of Problem

In analysing the failure properties of a buried pipe the following variables are to be considered.

1. Properties of the buried structure

- flexibility/rigidity
- smoothness/roughness
- geometry - D/t ratio, roundness etc.
- shape - circular, elliptical, oviform etc.

A buried structure can be considered as flexible, if the relative stiffness of the soil-structure system, $K_r = EI(1-\nu_s^2) / E_s(1-\nu^2)R^3$, is less than 0.001, where E, E_s and ν , ν_s are the moduli of elasticity and Poisson's ratios of the structure and soil respectively, R is the radius of curvature, and I is the moment of inertia of the structure wall cross-section. For semi-flexible structures K_r ranges from 0.001 to 0.01 (Meyernhof, 1979).

2. Properties of supporting media

- elastic/elasto-plastic
- type - sand/clay
- relative density
- condition - drained/undrained
- rigid/flexible bedding or surrounded by same media

- cover depth/pipe diameter ratio

3. Interaction

- E of structure/E of soil ratio
- slippage between structure and supporting media
- composite action - soil ring
- arch action
- modes of failure

Since it is difficult to cover all the above parameters, the present study is confined only to flexiole ($K_T = 0.0001$), smooth and circular pipes with a fixed diameter (150 mm) and thickness (0.39 mm). Sand is used as the supporting media. Loading is applied through a rigid and rough footing. The experiments were performed in plane strain condition.

1.3 Scope of the Study

Three types of experiments were conducted. In the first two types, steel rods of three different diameter were used to simulate sand particles. Flexible (rubber) and rigid (wood) beddings were used in these two sets. In the third set of experiments, the pipe was buried in sand.

The first set of tests were used to visualize the failure pattern of soil by chalk markings and they were video-taped. The second and

third set were used to obtain critical loads.

Figure 1.2 is a flow - chart which outlines the scope of this study. It explains how the two-dimensional physical models with varied cover depths, different loading positions and with different beddings were used to determine the failure pattern of soil and pipe and to estimate the critical load. The test results were then compared with theoretical analysis by using Finite Element - numerical technique to aid in the formulation of the conclusions and recommendations.

1.4 Brief Outline of the Thesis

The need for the study of the behaviour of buried structures, the soil-pipe interaction, the nature of the problem and the scope of the study are presented in Chapter 1. Chapter 2 contains the summary of the literature survey. The existing theories and the formulation of the Finite Element techniques which are used to compare the test results are outlined in Chapter 3. The descriptions of the two dimensional physical models, i.e, testing apparatus are presented in Chapter 4. In Chapter 5 the sequence of the testing program is given. The analysis and the discussion of results appears in Chapter 6. Chapter 7 gives the summary, conclusions and recommendations for further work.

FIGURE 1.2 SCOPE OF THE STUDY

CHAPTER 2

REVIEW OF LITERATURE

2.1 Laboratory Experiments on Buried Conduits

Many researchers have conducted a number of controlled laboratory experiments in prefabricated conduits supported with soil to compare theory with observation because of the complexity of the interaction between the flexible conduits and the surrounding soil, and the vagaries of field construction practices. Some of the works that have been done in laboratories with controlled conditions to simulate the field situations are reviewed in this section.

Leonards and Stetkar (1978) have recently presented the works that have been performed by Bulson (1963), Luscher (1966), Allgood and Ciani (1968), Dorris (1965), Albritton (1968), Watkins and Moser (1969), Meyerhof and Baikie (1963), and Howard (1971), and reviewed critically the apparatus and procedures used in the experiments and hence they will not be recounted here. A brief description for each of the rest of the controlled experiments is given below.

-- Donnellan (1964)

Donnellan conducted experiments with 100 mm diameter, and with thicknesses of 0.89 mm and 0.41 mm aluminium cylinders using 6 mm epoxy lap joint, to investigate their response to quasi-static overpressures.

The cylinders were buried horizontally in dense, dry Ottawa Sand, contained in a 750 mm diameter, 400 mm high bin and loaded by air pressure through a membrane which was in contact with the surface of sand. During the loading and unloading cycle the pipes were monitored with six electric strain gauges and five electric displacement gauges.

From the experimental results, the following conclusions were drawn:

1. The existence of very large strains did not imply the imminence of collapse.
2. The overpressure required to cause collapse increased with the depth of the burial. Collapse occurred by snap-through at the crown.
3. The ratio of the overpressure required to cause collapse at any depth to the theoretical in-air primary buckling pressure varied inversely with stiffness.
4. Two zones of burial were identified. In the first zone (shallow burial), the displacements and strains were highly sensitive to small changes in the depth of the burial. In the second zone (deep burial), the displacements and strains were insensitive to changes in the depth of the burial. The minimum depth required for deep burial increased with the overpressure.
5. The deflected shape assumed by a buried cylinder depends on its stiffness and the zone of burial. Deeply buried, instrumented cylinders

of both stiffnesses deformed into low-order modes. At shallow burial, the more rigid, instrumented cylinder assumed a high-order mode in the vicinity of the crown. Again, at shallow burial, the more flexible, instrumental cylinder experienced inward radial displacements at the springline.

6. The direct compressive strains were uniform around the more rigid, instrumented cylinders at deep burial.

7. Local buckling of the more flexible cylinder resulted in spurious strain data.

-- Marino and Riley (1964)

Studies were conducted on cylinders of 125 mm diameter and 375 mm long with three different thickness ratios (D/t of 40, 80 and 160) and hemispherical end closures. The other model ($D/t=80$) had an end closure system which could not transmit load to the shell. The soil tank of 1.22 m diameter and 1.83 m depth, contained Ottawa sand which was compacted by two small portable vibrators. Nitrogen gas was used to apply the load at the soil surface with a rubber diaphragm. The shells were monitored by electrical resistance gauges and linear variable differential transformers (LVDT).

After the preliminary cycling operation, static pressure was applied in 690 KPa increments over the range from 0 to 2760 KPa. The data used for analysis was recorded during the fifth and sixth load

2.3 Theoretical Concepts of the Failure of Buried Flexible Conduits

2.3.0 General

As mentioned in section 1.1, it is relatively simple to design for the failure criteria of crushing of wall and separation of seam. However, the other modes of failure (buckling etc.) are very complicated because of the complication of soil-pipe interaction. There are several expressions available which are suggested by various mathematicians and researchers with different assumptions, to estimate the critical buckling loads.

Leonards and Stetkar (1978) have made a literature review to trace the development of the present state-of-art for predicting buckling of flexible conduits. In their report, the assumptions and models used in the theoretical analyses were discussed and evaluated, and their applicability to the problem at hand was assessed. Because most of the buckling theories were elaborately presented in that report, only a very short description of the theories is given in this chapter.

In the following theories, buckling of flexible thin-walled cylindrical shells or rings were considered. The buckled shape was taken to be constant along the axis of the cylinders and thus no longitudinal buckling modes resulting from axial forces were considered. Although some solutions for longitudinal buckling are

model to simulate gravity loading. Tubes with a diameter of 101.6 mm and a length of 254 mm were fabricated from a steel plate of 0.152 mm thickness by soft soldering 15 mm wide lap joint. The circumferential and axial strains were measured by strain gauges attached to both inner and outer surfaces of the pipes. A uniform pressure was applied to the soil and the pipe by the centrifugal action.

In all the cases the maximum hoop stress seemed to occur at the spring line. The line of thrust was seen to lie within the middle third of the pipe, thus indicating that the pipe was resisting the loads coming from the backfill almost entirely by compressive action, as predicted by ring compression theory. The ring compression theory was shown to be valid in the case of pipes placed in wide trenches or under embankments. In contrast, for pipes placed in narrow or battered trenches the results showed that the line of thrust may be outside the wall thickness, and that relatively large bending moments were induced.

-- Valsangkar and Britto (1979)

Valsangkar and Britto extended their study with 0.354 mm thick melinex plastic pipes formed with lap joint using an adhesive, to analyse the influence of effective stiffness of the pipes on the reduction in loads due to arching.

For tests with low cover/diameter ratios (< 0.25), where the applied surface load was closest to the crown and shoulders of the

pipe, the magnitude of the hoop stresses increased with surface loads. For greater ratios (> 0.5), the effect of increasing surface load was small and resulted in a uniform increase in thrust around the pipe. In general, it was observed that the induced bending moments are of small magnitude. The distribution of hoop stresses was seen to be increasingly nonuniform as the pipe approached failure. It was observed that the induced bending moment was of small magnitude up to the eventual failure of the pipe.

2.2 Field Experiments on Buried Conduits

Since instrumentation is seldom provided to measure all soil properties and conduit loads in field tests, it is not possible to compare those field experiments with any mathematical model or laboratory physical model. Moreover, field installations are expensive and hence are seldom designed to reach failure. Thus, the failure mechanisms are unclear and the factors leading to failure are uncertain.

Leonards and Stetkar (1978) commented about the Prairie Farm Rehabilitation Administration's (1957), Demmin's (1966) and Wu's (1977) works in detail and therefore they are not described here again.

-- Trott and Gaunt (1972)

A steel pipe 1.83 m diameter, 9 m long and comprised two welded semi-circular sections of 12.7 mm thick rolled steel was buried at 1.2 m average depth in a trench 2.75 m wide, on a shaped sand bed, bedding angle of 60 degrees. Sand was used as side-fill for 150 mm above the horizontal diameter and backfill was then continued with excavated material. A small vibrating tamper was used to compact the sand around the sides of the pipe, and a man operated vibrating roller compacted the backfill above the level of crown. Material was compacted in nominal 250 mm layers.

Resistance foil strain gauges were cemented at 45 degrees spacing along the inside and outside the pipe. Pipe deflections were studied by means of internal diameter measurements. Small stainless steel studs were cemented to the pipe wall at 45 degrees spacing adjacent to the strain gauge positions and diameters were checked with a gauge. Dynamic deflections (with moving loads) were monitored with an inductive displacement transducer. Soil pressures were measured with a pressure cell.

The maximum vertical deflection was only about 0.2% of the pipe diameter. Static deflections were very small, reaching to a maximum of only 0.14 mm vertically and 0.10 mm horizontally. Dynamic deflections with the vehicle running at an average speed of 9 km/h, were of the same order as the static ones. At an average speed of 30 km/h, it

showed a maximum vertical deflection of 0.27 mm, corresponding to an impact factor of approximately 2.10.

Strains measured during backfilling and following surcharge loading were very low for a steel pipe, and at no time exceeded 100 microstrain. The radial pressures were highest around the top half of the pipe initially, but tended to increase over the lower half with increasing load, and to approach a state of nearly uniform compression under sustained surcharge.

The recorded dynamic pressures across the pipe crown are much lower than the value predicted by Boussinesq's analysis. This is to be expected, as the calculated pressure at the pipe crown does not take into account load-spreading effect of road construction. From the measured results, a relationship between vertical and horizontal pressures (P_V & P_H) was found to be as follows:

$$P_V = 0.79 P_H + 1.22 \quad 2.1$$

The difference between this relationship and the predicted using Spangler's equation may be explained by the fact that the experimental pipe was buried at a shallow depth in relation to its diameter, and Spangler's assumption of a uniform vertical pressure acting across the diameter of the pipe was not in accordance with the observed pressure distribution.

2.3 Theoretical Concepts of the Failure of Buried Flexible Conduits

2.3.0 General

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available (Almroth, 1962; Reynolds, 1962; Weingarten, 1962), the buckling theories mentioned here were concerned with the two-dimensional problem in which sections transverse to the axis of the pipe were assumed to behave identically, i.e., assuming plane strain conditions.

2.3.1 Elastic Buckling in High Modes

Donnell (1956) considered imperfection effects on shell buckling by modifying the extended von Mises theory considering finite shell lengths to include an unevenness factor in the load-deformation equations. The normalized critical buckling pressure was given by equation 2.2.

$$p_b = \frac{E t^{5/2}}{R^{3/2} L} \quad 2.2$$

Bodner (1958) extended von Mises theory of shell buckling which calculated the critical buckling pressure of a long cylinder subjected to uniform normal pressure, to consider uniform constant direction boundary pressures and obtained the following equations 2.3a and 2.3b. He concluded that in general, a uniform constant direction pressure is less critical than a uniform normal pressure.

$$p_b = \frac{3 E I}{R^3 (1-\nu^2)} \quad (\text{Von Mises}) \quad 2.3a$$

$$p_b = \frac{4 E I}{R^3 (1-\nu^2)} \quad (\text{Bodner}) \quad 2.3b$$

Timoshenko and Gere (1961) derived an expression (equation 2.4a) for the critical external normal pressure, by approximating buckling of flexible conduits buried in soil to a circular ring subjected to uniform hydrostatic external pressure. The critical compressive stress within the ring was given by equation 2.4b.

$$p_b = E I (n^2 - 1) / R^3 \quad 2.4a$$

$$f_b = 3 E I / R^2 t \quad 2.4b$$

Armenkas and Herrmann (1963) derived a more general solution that considered buckling of cylinders of finite length under uniform constant direction and uniform normal pressures (equation 2.5). The effect of buckling mode on the critical pressure was also accounted for.

$$p_b = K^* E I / (1 - \nu^2) R^3 \quad 2.5$$

$$\begin{aligned} \text{where } K^* &= n^2 / (1 + \frac{t}{R}) && \text{for constant direction pressure} \\ &= (n^2 - 1) && \text{for normal pressure} \end{aligned}$$

Cheney (1963) analyzed the stability of circular ring under plane stress conditions, subjected to circumferential support by elastic springs and acted upon by a uniform external pressure on the ring wall of constant magnitude and direction. Initial strain prior to buckling was neglected. The closed form solution is given by the following equation.

$$p_b = \frac{E I}{R^3} (n^2 - 1) + \frac{K_s}{n^2 - 1} \quad 2.6$$

Meyerhof and Baikie (1963) extended the plate buckling theory developed by Timoshenko and Gere (1961) to approximate the buckling behaviour of elastically supported cylindrical shells, and obtained the following equation 2.7.

$$p_b = \frac{f_y}{1 + f_y A \sqrt{\frac{(1-\nu^2)(1-\nu_s^2) R}{2 E_s E I}}} \quad 2.7$$

Valentine (1964) suggested a method to estimate the critical buckling load on shallow (less than one diameter of soil cover at the crown) flexible conduits subjected to live loads, assuming that a portion of the top of the conduit acted as a two-hinged curved bar (equation 2.8).

$$p_b = \frac{E I}{R^3} \left[\frac{\pi^2}{\alpha^2} - 1 \right] \quad 2.8$$

where 2α = central angle forming the arch, in radians, $0 < \alpha < \frac{\pi}{2}$

Forrestal and Herrmann (1965) extended Cheney's analysis to consider cylindrical shells subjected to a uniform, constant directional external pressure on the shell wall and supported by a continuous elastic medium rather than discrete springs. They obtained the following expressions (eq. 2.9a and 2.9b) for critical buckling pressure for fully bonded and for no interface friction cases. This theory indicates that slip at the conduit-soil interface decreases the

critical buckling pressure significantly.

$$\frac{p_b}{p_0} = \frac{\frac{n^2(n^2-1)(3-4\nu_s)}{3} + \frac{E_s/(1+\nu_s)}{3EI/R^3(1-\nu^2)} \left[2n(1-\nu_s) + (1-2\nu_s) \left\{ \frac{n^2 t}{R} - 1 \right\} \right]}{(n^2-1)(3-4\nu_s) + 2n(1-2\nu_s) + 1} \quad 2.9a$$

$$\frac{p_b}{p_0} = \frac{n^2-1}{3} + \frac{E_s/(1+\nu_s)}{3EI/R^3(1-\nu^2)} \left[\frac{1}{(1-2\nu_s)(n+1) + n} \right] \quad 2.9b$$

$$\text{where } p_0 = 3EI/R^3(1-\nu^2) \quad 2.9c$$

Luscher (1966) formulated a semi-empirical solution (equation 2.10a) for the critical, uniform, radially applied buckling pressure acting on the wall of a buried conduit, from his soil-surrounded tubes experiment. Luscher gave a relationship between the modulus of soil support, obtained from buckling tests, and the constrained modulus, obtained from oedometer tests (equations 2.10b and 2.10c).

$$p_b = 2\sqrt{K_s EI/R^3} \quad 2.10a$$

$$\text{where } K_s = B E_s = \frac{B(1+\nu_s)(1-2\nu_s)}{(1-\nu_s)} M_s \quad 2.10b$$

M_s = Constrained modulus from Oedometer test.

$$\text{and } B = \frac{1 - \left(\frac{R_i}{R_0}\right)^2}{(1+\nu_s) \left\{ 1 + \left(\frac{R_i}{R_0}\right)^2 (1-2\nu_s) \right\}} \quad 2.10c$$

Duns and Butterfield (1971) employed many of the same assumptions used by Forrestal and Herrmann (1965) and extended Bodner's (1958) solution to include the effects of elastic support. The critical thrust was given by equation 2.11a when the failure would occur by elastic buckling in high modes. Duns and Butterfield also obtained an empirical expression (equation 2.11b) relating the critical thrust to the uniform applied pressure over the soil surface by Finite Element Analysis. A depth of burial influence factor, similar to the B factor developed by Luscher (1966) (equation 2.10c), was also incorporated into the analysis (equation 2.11c). By minimizing with respect to n, the lower-bound critical soil over-pressure is given by the equation 2.11d, where the critical buckling mode is given by the equation 2.11e.

$$F_c = \frac{EI}{1 - \nu^2} \left(\frac{n}{R} \right)^2 + k_s \left(\frac{R}{n} \right)^2 \quad 2.11a$$

$$\text{where } k_s = \frac{E_s (n^2 - 1)}{R (1 + \nu_s) \{2n + 1 - 2\nu_s (n + 1)\}}$$

$$\text{and } n^2 (n + 1)^2 = \frac{k_s (1 - \nu^2) R^4}{EI}$$

$$F_c = \frac{3 (1 - \nu_s) R p_b}{2 - \nu_s} \quad 2.11b$$

$$F_k = \frac{1 - \left(\frac{R_i}{R_o} \right)^2}{1 + \left(\frac{R_i}{R_o} \right)^2 (1 - 2\nu_s)} \quad 2.11c$$

$$\text{where } p_b = \frac{2 - \nu_s}{1 - \nu_s} \cdot \frac{F_k E_s}{4 n (1 - \nu_s^2)} \quad 2.11d$$

$$n = \left[\frac{F_k E_s R^3 (1 - \nu^2)}{4 (1 - \nu_s) E I} \right]^{1/3} \quad 2.11e$$

In order to represent more closely the boundary conditions at the soil-pipe interface, Chelapati and Allgood (1972) derived a more rigorous theoretical solution, using a bonded interface and minimum total potential energy from large deflection theory and two cases of soil support which were continuous and only on outward deflecting lobes of circumferential buckling waves. For the continuous circumferential support, the critical uniform external pressure is given by equation 2.12a. In the other case the soil support is halved. Minimizing the equation 2.12a with respect to n and assuming Poisson's ratio of 0.3, the critical pressures for both soil support conditions are given by equations 2.12b and 2.12c.

$$p_b = \frac{(n^2 - 1) E I}{(1 - \nu^2) R^3} + \frac{K_s}{n^2 - 1} \quad 2.12a$$

$$p_b = 2.1 \sqrt{K_s E I / R^3} \quad 2.12b$$

$$p_b = 1.5 \sqrt{K_s E I / R^3} \quad 2.12c$$

Cheney (1976) revised his equation using Duns and Butterfield's (1971) and Luscher's (1966) expressions for modulus of soil support (equation 2.13a). Hence, Cheney's new expression is given by the equation 2.13b and minimized with respect to n gives equation 2.13c. Even though the critical pressure was lower bound, it made conservative predictions. Therefore, Cheney believed that tangent soil modulus would be more appropriate rather than the secant modulus. By reducing the secant modulus in equation 2.13c to $1/8$, the equation 2.13d resulted,

which fitted the experimental results of Allgood and Ciani quite well.

$$k_s = \frac{E_s (n^2 - 1)}{R (1 - \nu_s^2) (2n + 1 - \nu_s)} \quad 2.13a$$

where E_s is secant modulus of soil

$$p_b = \frac{E I (n^2 - 1)}{R^3 (1 - \nu^2)} + \frac{E_s}{(2n + 1 - \nu_s) (1 - \nu_s^2)} \quad 2.13b$$

$$p_b = 1.2 \left[\frac{E I}{(1 - \nu^2) R^3} \right]^{1/3} \left[\frac{E_s}{1 - \nu_s^2} \right]^{2/3} \quad 2.13c$$

$$p_b = 0.3 \left[\frac{E I}{(1 - \nu^2) R^3} \right]^{1/3} \left[\frac{E_s}{1 - \nu_s^2} \right]^{2/3} \quad 2.13d$$

Meyerhof (1979) defined a relative stiffness, K_r

$$K_r = \frac{E I}{R^3 (1 - \nu^2)} \cdot \frac{(1 - \nu_s^2)}{E_s} \quad 2.14a$$

and derived an expression for buckling pressure

$$\frac{p_b}{p_0} = 0.47 K_r^{-1/2} \quad 2.14b$$

where p_0 is defined by equation 2.9(c)

2.3.2 Elastic Local Buckling of Buried Conduits

Amstutz (1953) derived an expression for the critical pressure producing a local inward buckle in a cylinder rigidly restrained from outward buckling, assuming the buckling deformations as a cosine function. The critical normal stress was determined by equation 2.15 in the lower bound ($n=1$) case.

$$\begin{aligned} & \frac{f_c (1 - \nu^2)}{E} \left[1 + 12 \left(\frac{R}{t} \right)^2 \frac{f_c (1 - \nu^2)}{E} \right]^{3/2} \\ = & \frac{2 R}{t} \frac{(f_y - f_c) (1 - \nu^2)}{E} \left[1 - \frac{R}{t} \frac{(f_y - f_c) (1 - \nu^2)}{E} \right] \quad 2.15 \end{aligned}$$

Anderson and Boresi (1962) derived an expression to estimate the critical buckling pressure of a ring in air subjected to a non-uniform external pressure. The external pressure was assumed to be of the form of $p = p_0 \sin^2 \theta$ where p_0 is the pressure at the crown of the pipe. The critical buckling pressure is,

$$(p_0)_{cr} = 9.0 E I / R^3 \quad 2.16a$$

and the average critical pressure $\bar{p}_{cr}$ which corresponds to $(p_0)_{cr}$ is

$$\bar{p}_{cr} = (p_0)_{cr} / 2 = 4.5 E I / R^3 \quad 2.16b$$

which is 50 percent greater than that is predicted by Timoshenko and

Gere (1961) for $n=2$. (equation 2.4a).

Kloppel and Glock (1970) developed a rational method to predict the instability of initially deflected, buried conduits. They assumed two zones of interaction : one upper portion where the conduit deflects away from the soil (active) and the other lower portion pressures the soil (passive). Because of the reduced soil support of the upper section, it was reasoned that an instability is most likely to occur near the crown of the conduit. They modelled the upper portion by a hinged arch with radial elastic (soil) support along its circumference, tangential and rotational elastic support at the hinges, thereby representing soil support and restraint provided by the lower conduit section. The distribution of active radial pressure about the conduit wall was expressed as a cosine function (equation 2.17a and Figure 2.1a). The boundary pressure distribution on the arch section was decomposed into a uniform component and a non uniform component (equation 2.17b). In order to solve the instability problem an arch section (Figure 2.1b) was considered.

$$p = p_s \cos \left[\frac{\pi}{2} \cdot \frac{\psi}{\psi_B} \right] \quad 2.17a$$

$$\text{where } \psi_B = \frac{\pi^2}{4 \cos^{-1} \lambda}$$

and $\lambda =$ coefficient of active earth pressure.

$$p = p_0 + p_\ell \sin \left[\frac{\pi}{2} \cdot \frac{\phi}{\phi_0} \right] \quad 2.17b$$

$$\text{where } p_0 = p_s \cos \frac{\pi \phi_0}{2 \psi_B}$$

and $p_s = p_0 + p_\ell =$ vertical overburden plus live load pressures

Figure 2.1a

Active Pressure Distribution Around Circular Conduit

Figure 2.1b

Pressure Distribution on Two-Hinged Arch
(Kloppel and Glock, 1970)

The snap-through instability problem was solved by assuming an initial value and deformation into a single mode buckle, defined by a cosine function. By minimizing potential energy and using variational calculus Kloppel and Glock solved the problem and concluded the following:

1. For small values of the conduit-soil flexibility factor, variation of the conduit or soil stiffness has little effect on the snap-through buckling.
2. The buckling load of an elliptical conduit (with the major axis horizontal) is approximately one-half that of a circular conduit.
3. The amount of soil-conduit interface friction has negligible influence on the magnitude of the buckling load.
4. Increasing nonuniformity of the pressure distribution decreases the buckling load.
5. Very flexible conduits with good soil support will tend to buckle in high modes.

Cheney (1971) reviewed Luscher's (1966) analysis and discussed some unsatisfactory aspects of Luscher's results. Cheney attempted to model theoretically the local buckling behaviour of a buried conduit by energy consideration and subsequently developed an empirical equation for the modulus of soil support (equation 2.18). Cheney's concept for the distribution of critical pressure during local buckling of a

soil-surrounded tube is shown in Figure 2.2.

$$K_s = 720 \sigma' \left[\frac{R_o - R_i}{R_i} \right]^{0.153} \quad 2.18$$

El-Bayoumy (1972) analysed snap through-buckling in a rigidly confined, flexible circular ring with similar boundary conditions in Cheney's (1971) analysis. He also used energy methods to obtain equilibrium equations but assumed no boundary pressure reduction at the local buckling. Rather, a snap-through buckle was assumed to occur when the critical pressure was reached.

Snap-through buckling will not occur in perfect rings free from external disturbance (El-Bayoumy, 1972). Consequently, flexible conduits without imperfection subjected to uniform external pressure should fail by some other mechanism such as elastic buckling in high modes. The fact that snap-through buckling occurs more frequently in practice indicates that geometric and material imperfections (and possibly residual stresses) play an important role in conduit performance.

Figure 2.2

Distribution of Critical Pressure During Local
Buckling of Soil Surrounded Tubes (Cheney, 1971)

2.3.3 Inelastic Buckling of Buried Conduits

Inelastic buckling of pipes is a complex problem that has been analysed only for simple boundary conditions and pressure distributions. Many researchers (Bilaard, 1949; Gerard, 1957; Reynolds, 1960; Lee, 1962) have analysed inelastic buckling of cylindrical shells of finite length subjected to uniform external pressure but without soil. Gerard (1957) derived the following expression for an "effective" modulus to be substituted into expressions obtained for elastic buckling to estimate the inelastic buckling load.

$$E_{\text{eff}} = \sqrt{E E_t \left[\frac{1 - \nu_e^2}{1 - \nu_p^2} \right]} \quad 2.19$$

Gjelsvik and Bodner (1962) suggested a simple, yet reasonable, approach to predict the critical loads in the inelastic range. Assuming elastic-purely plastic behaviour, the use of elastic instability equations on the unyielded section was proposed to predict the onset of buckling. Thus an equivalent section was suggested rather than an effective modulus. No theories have, as yet, been advanced to predict the inelastic buckling of radially soil-supported conduits, according to the author.

2.4 Finite Element Approach

A vast amount of literature (Ashwell and Gallagher, 1976; Crisfield, 1981) has been produced on the application of Finite Elements to the analysis of thin shells subjected to various types of loading. Until now, most of this work has related to linear analysis and a few dealt with non-linear Finite Element analysis. However, none of them considered the buckling of conduits buried in soil.

Assuming linear properties of soil and pipe, numerous works have been done (Nath, 1977; Nayak, 1977; Ahmad, 1981) on buried conduit to find out the stresses and deflections in conduit as well as in soil.

-- Nath (1977)

Nath carried out a finite element analysis to predict the structural performance of a buried pipe and compared the calculated stresses and deformations with the results of a full scale experiment conducted at Kirtling in Cambridgeshire (Trott and Gaunt, 1972). Triangular finite elements with two degrees of freedom at each node were used to represent the surrounding soil. The pipe was represented by a combination of two concentric rings. The primary ring consisted of line elements and the secondary ring was formed by curved elements. The optimum location of the external boundaries of the mesh was determined by carrying out an iterative process until a stage had reached when the differences between both the stresses and displacements from successive

calculations were acceptably small. The optimum location was found to be at a distance 3 to 4 times the diameter from the centre of the pipe.

The soil parameters (secant modulus etc.) were obtained from the standard experiments, performed on a remoulded soil specimen. The stresses and displacements determined by the Finite Element analysis related only to those produced by the self-weight of the pipe and backfill, and a surcharge loading applied over the completed backfill. No account was taken of the stress condition in the system due to other causes such as excavation of the trench and the compaction of the backfill. Three types of limiting cases were analysed: (a) theoretical lower limit bending and compressive strength of pipe were neglected, by omitting both the primary and secondary rings from the mesh, (b) practical lower limit, the bending strength of the pipe was included by including the secondary ring, the primary ring was neglected to simulate smooth (frictionless) interface between pipe and soil, and (c) upper limit, the pipe was represented by both rings, where the bending and compressive strength of the pipe was mobilised and with non-slip frictional contact existing along the soil-pipe interface.

The experimental results were very close to the upper limit analysis (case(c)). Overall, the results obtained from the full-scale experiment were in fair agreement with those obtained from a Finite Element analysis. The deformations of the pipe calculated by using Spanglers (1971), empirical design formula were about 10-20 times

larger than the measured values, and those predicted by the Finite element method are considered very reasonable.

-- Valliappan et. al (1977)

The soil medium was considered to be elasto-plastic. The continuum was discretized using isoparametric 'quadratic' quadrilateral elements whereas the flexible pipe itself was modelled using one dimensional bar element. In order to investigate the effects of slippage and no slippage between the pipe and soil medium, joint elements were used at the interface. The free slippage condition was introduced by assigning a very small value to both cohesion and friction. Two different yield criteria - von Mises and Drucker's modified von Mises - were adopted for the elasto-plastic analysis, to idealize the undrained condition and the drained condition respectively. Two different shapes of pipe - circular and oviform - were used for the analysis. The applied loads were due to surface pressure only and the width over which they acted were taken to be one diameter of the buried pipe. The depth of cover was twice the diameter of the pipe.

From the results, it was found that the interface conditions did not affect the displacements very much whereas the shape of the pipe did. For the circular pipe, the plastic yielding was the cause of failure whereas for the oviform pipe, the tensile cracking in the medium was the cause of failure. The compressive stresses for the smooth cases were uniform while they were not symmetrical for rough

cases. The peak values of the stresses were reduced considerably (nearly half). Moreover, it was concluded that the drained condition allowed the pipe to be subjected to higher surface pressure than the undrained condition, due to the effect of the internal friction on the soil strength.

As far as the maximum surface pressure which can be applied to the pipe was concerned, there was very little difference between rough and smooth cases irrespective of whichever yield criterion (von Mises or modified von Mises) was adopted. However, Drucker's modified Von Mises criterion which incorporated the cohesion and friction properties of the soil was preferred to the other one.

--Hafez and Abdel (1983)

It comes as a surprise to the author, that a similar piece of work has been done and published very recently.

This paper introduces a few improvements to the existing finite element analysis (Duncan, 1979; Katona et al., 1976) of soil steel structures which takes into account the state of stresses in the soil also. The domain consists of four types of elements. The structure wall is represented by beam elements and soil - below the spring line and away from it - is simulated by constant strain triangular elements. Linear strain quadrilateral elements are used above the spring line. Two noded spring type interface elements are used between the structure

nodes and corresponding soil nodes.

The analyses are performed in increments using hyperbolic stress-strain relationships for the soil (Duncan and Chang, 1970; Wong and Duncan, 1974). In the case of a shear or tension failure in a soil element, the stress is transformed into equivalent nodal forces. These forces are then dissipated to neighbouring elements through stress transfer technique. Thereafter, soil element is assigned new properties based on an arbitrary confining pressure of 1/10 atmospheric pressure in case of tension failure, and maximum principal stress of 90 percent of the principal stress at failure in case of shear failure (Duncan et al., 1980). Similarly the shear stress - tangential displacement relationship for the interface element is assumed to follow the hyperbolic models (Clough and Duncan, 1971).

Maximum thrust and negative moment occur under symmetrical live loads at haunches of the upper half of the steel structure and maximum positive moment occurs at the crown zone. Both thrust and moment in the bottom third of the structure are negligible under surface live loads.

The analytical results show satisfactory agreement with those obtained experimentally at White Ash Creek and Adelaide Creek by Ontario Ministry of Transportation and Communications. They also show that the American Association of State Highway and Transportation Officials (AASHTO) provisions overestimate the thrust due to live load

and under estimate the thrust due to dead load in the case of circular pipes and overestimate the ~~in~~ in the case of horizontal elliptic shapes when compared with Ontario Highway Bridge Design Code (OHBDC) and the finite element results.

CHAPTER 3

THEORETICAL CONCEPTS

3.1 Introduction

This chapter presents the empirical equations which are currently being used to design the flexible buried conduits and the description of the linear finite element analysis adopted to find out the deflections and the stresses in the pipe and the surrounding soil. It is worthy to note that the empirical equations take into account only the uniform pressure distribution on the pipe, and therefore they are not capable to handle the problems with eccentric loading. Therefore the empirical equations will be restricted to symmetric cases only and will be compared to the corresponding experimental and Finite Element Analysis results. For asymmetrical buckling due to eccentric loading conditions, the experimental results will be compared with the Finite Element Analysis only.

3.2 Closed Form Solutions

3.2.1 General

The load carrying capacity of a flexible conduit is governed by compression failure, buckling failure or by a combination of both. The compression - crushing - failure occurs in the wall when the stresses approach the yield point of the material. This type of failure was

observed in the conduits with low wall flexibility buried in soil of high passive resistance (Watkins, 1966). The snap-through buckling due to localized reversal of curvature occurs in soil with low passive earth resistance with conduits of high ring flexibility.

Earlier researchers did not pay much attention to buckling as a criterion for metal conduits. Assuming a very good backfill was provided around the conduit, either deformation (Spangler, 1971) or ring stress (White and Layer, 1960) criteria were considered sufficient to assure the safety of the conduit.

Because larger culverts with heavier loads were recently introduced, the buckling nature was studied more carefully. A thorough literature survey of the buckling of pipes is given in Chapter 2. In this chapter, some of the theories commonly used in practice and the provision made in the codes such as American Association of State Highway and Transportation Officials Specifications (AASHTO) (1977), and Ontario Highway Bridge Design Code (OHBDC) (1983), along with Spangler's, and White and Layer's solutions are presented.

3.2.2 Spangler's Theory (1971)

To avoid buckling by controlling deflection, Spangler suggested from his experimental investigation that, if the vertical diameter of a circular flexible pipe decreases by about 20 percent from the initial diameter, the pipe is in a state of incipient collapse. Spangler

derived an equation for estimating the horizontal deflection x in mm,
(also known as the Iowa Deflection Formula)

$$\Delta x = D_1 \frac{K W_c R^3 (35.26)}{E I + .061 K_s R^3} \quad 3.1$$

where,

D_1 - deflection lag factor

K - bedding constant (depends on bedding angle)

W_c - vertical load per unit length of pipe (N/m)

R - mean radius of pipe (m)

E - modulus of elasticity of pipe (kN/m^2)

I - moment of inertia per unit length (m^4/m)

K_s - modulus of soil support (kN/m^2)

3.2.3 Ring Compression Theory

White and Layer (1960) suggested that the conduit wall can be analysed as a ring in uniform compression. This assumption is believed to be valid only if the depth of cover exceeds $1/8$ of the conduit diameter (Abdel and Bakht, 1981). The uniform pressure is taken as the overburden pressure plus any live load, p_e , including impact, which is

considered uniformly distributed at the top of the conduit.

$$p = \gamma_s h + p_l \quad 3.2$$

The circumferential thrust,

$$T = p R \quad 3.3a$$

The compressive stress in the wall,

$$f = p R / t \quad 3.3b$$

3.2.4 Kloeppel and Glock (1970)

Kloeppel and Glock suggested the pressure at the crown p_c , as,

$$p_c = \gamma_s h + p_l \quad \text{for } p_l < \gamma_s h \quad 3.4 a$$

$$p_c = 1.1(\gamma_s h + p_l) \quad \text{for } p_l > \gamma_s h \quad 3.4 b$$

The 10 percent increase of pressure with shallow depths of cover accounts for the effect of concentrated live loads.

3.2.5 AASHTO Specifications (1977)

(a) Seam strength

Seam strength must be sufficient to withstand the thrust developing from the total load supported by the conduit.

$$\text{Seam strength} > F.S. (LL + DL) (1/2) \text{Span} \quad 3.5 a$$

(b) Failure of the conduit wall

For diameters less than D, where,

$$D = \frac{r}{k} \sqrt{\frac{24 E}{f_y}} \quad 3.5 b$$

the ring compression stress f_c , at which buckling becomes critical, is given by,

$$p_c = f_y - \frac{f_y^2 (k \cdot D)^2}{48 E r} \quad (\text{MPa}) \quad 3.5 c$$

For diameters greater than D,

$$p_c = \frac{12 E}{(kD/r)^2} \quad (\text{MPa}) \quad 3.5 d$$

where,

f_y = minimum tensile strength (MPa)

p_c = critical stress (MPa), not to exceed yield stress

k = soil stiffness factor

D - pipe diameter (m)

r - radius of gyration (corrugation)

E - modulus of elasticity (MPa)

(c) Deflection

The deflection is calculated from the Iowa Deflection Formula given above (Eq. 3.1).

3.2.6 OHBDC Specifications (1983)

(a) Thrust

The thrust, T, in the conduit wall at the ultimate limit is calculated as,

$$T = \alpha_D T_D + \alpha_L T_L \quad 3.6 a$$

where,

α_D - dead load factor for soil, specified in section 2 = 1.25

α_L - live load factor for soil, specified in section 2 = 1.40

T_D - dead load, calculated as follows

T_L - live load, calculated as follows

$$T_D = \frac{\mu_1 \gamma h R}{1000}$$

3.6 b .

where,

μ_1 - conduit shape factor

h - height above the crown (m)

R - radius of the pipe (mm)

γ - density (kN/m³)

$$T_L = 0.5 \text{ (lesser of } D_h \text{ and } l_t) \sigma_L m_f (1+I) \quad 3.6 c$$

and,

σ_L - equivalent uniformly distributed load at the crown (Bakht, 1981)

l_t - length of the dispersed load, (1.45 + 2H) m.

I - impact factor, obtained in clause 2.4.3.2

is 0.4 for $H = 1$ and 0.1 for $H > 2$ m.

m_f - modification factor for multi-lane loading,

from clause 2.4.3.1.4 is 1.0, 0.95 and 0.85

for one, two and three loaded cases respectively.

(b) Strength Requirements

At the ultimate limit state the stress, $= T/A$ shall not exceed the factored buckling stress (MPa) given either by,

$$p_b = \phi \left[f_y - \frac{f_y^2}{12E} \left\{ \frac{K R}{r} \right\}^2 \frac{1}{\rho} \right] \quad \text{for } R \leq \bar{R} \quad 3.6 d$$

or by,

$$p_b = \phi \frac{3 E \rho}{(K R / r)^2} \quad \text{for } R > \bar{R} \quad 3.6 e$$

where,

ϕ - performance factor

$= 0.80$ for $R \leq 4 \text{ m}$

$= 0.75$ for $R > 4 \text{ m}$

$\bar{R}$ - an equivalent radius given by,

$$\bar{R} = \left\{ \frac{6 E \rho}{f_y} \right\}^{1/2}, \text{ mm}$$

K - flexibility factor given by,

$$\lambda \left[\frac{E I}{\bar{K}_S R^3} \right]^{1/4} \leq 0.5$$

λ - a factor governing K and is given by,

$$1.22 \left[1.0 + 2 \left\{ \frac{E I_S}{\bar{K}_S R^3} \right\} \right]^{1/4}$$

$\bar{K}_S$ - modified modulus of soil reaction, given by,

$$K_S \left[1 - \left\{ \frac{R}{R + 1000H} \right\}^2 \right]$$

ρ - reduction factor for buckling stress in conduit walls and is given by,

$$1.0, \text{ for } (1000H/2R) \geq 1.0$$

$$(1000H / 2R)^{1/2} \text{ for } 0.10 < (1000H/2R) < 1.0$$

r - radius of gyration of corrugation profile $(I/A)^{1/2}$, mm

f_y - yield stress (MPa)

K - a factor representing the relative stiffness of the conduit w.r.t. the adjacent soil

The above buckling theories were derived by Abdel (1978) by combining Kloeppel and Glock (1970) and Deflection control (Spangler, 1941) requirements. (To determine the influence of conduit shape on arching, plane strain finite element programs (CANDE) were carried out. (Katona et al, 1976)).

3.3 Finite Element Solution

The finite element method (F E M) gaining impetus after the advent of the digital computers, is today probably the most effective and widely used numerical technique for many engineering problems in several fields. Of the many finite element approaches (viz. displacement, stress, mixed, and hybrid) the displacement method is the most popular and this is the method used in this research work. The theory of the finite element method, in which load and displacement relation of a structure is expressed as a set of linear simultaneous equations, can be found elsewhere (Zienkiewicz, 1971; Gallagher, 1975).

The finite element method has been successfully employed to determine the stresses and deformations of buried conduits (Katona et al. (1976), Nath (1977), Valliappan et al. (1977), Duncan (1979), and Hafez and Abdel (1983)). The early applications of the method treated the soil media as an isotropic material (Abel et al, 1973). Thereafter, the nonlinear and stress-dependent characteristics of the soil have been taken into account (Duncan and Chang 1970).

Abdel and Bakht (1981) used the finite element program CANDE (Katona et al 1976), assuming the soil as linear elastic and nonlinear elastic, and found that the wall thrust in the conduit remained almost unchanged by the latter analyses.

Even though this research is mainly concerned with the experimental investigation, some attempt was made here to analyze the problem using a linear elastic finite element program. In fact, a non-linear analysis which takes into account both material and geometric non-linearities or Eigen value problem to solve buckling - instability - problems would be a more appropriate treatment. But due to the limitations on time, such non-linear or other complicated analyses are well beyond the scope of the present work and are left for future research.

The computer code basically used here for the formulation of the element stiffness matrix and load vectors was taken from Hinton and Owen (1977). However, the profile equation solver by Bathe (1982) was used for the assembling the element matrices and solving the resulting simultaneous equations, since this technique considerably saves the computer core memory among all the available in-core solution techniques. The automatic mesh generation program (Hinton and Owen, 1979) was also incorporated in order to alleviate much of the effort involved in data preparation. The complete program listing together with the comments indicating the function of each subroutine is appended in Appendix A. In addition,

instructions for preparing the input data are also included in the appendix.

The program basically uses the 8-node serendipity isoparametric, quadrilateral element (Figure 3.1) with curved sides and a quadratic variation of displacement field. This element has been found to be extremely versatile and a good performer for the two dimensional linear elastic analysis. Each node of the element can take displacements parallel to the x and y axes and has two degrees of freedom.

Figure 3.1 8-node Quadrilateral Element

3.3.1 Size of the Domain and Boundary Conditions

The accuracy of the finite element analysis can be improved by increasing the number and/or the complexity of the elements and by locating the boundaries well beyond the region influenced by the culvert. By experimentally (Valsangkar and Britto, 1978) and analytically - by FEM - (Katona and Smith, 1976; Nath, 1977), it is found that the vertical walls be located 3 - 4 diameter (or span) of the culvert from the centre of the culvert, to get accurate results. The bottom boundary should be located 1.5 - 2 diameter (or rise) of the culvert below the culvert. It is worth mentioning here that the size of the sand box, pipe and its location in the experiment were selected to satisfy the above criterion in selecting the size of the domain.

The F E program can accept any desired displacement boundary conditions. Thus, any nodal point can be constrained in the x, y or both directions. In the problem concerned, the vertical boundaries are considered as smooth walls, that is, roller joints which have movement only in the y direction and no movement in the x direction. The horizontal (bottom) boundary is considered as rough and therefore pin joints are simulated (no movements in both direction).

A typical finite element mesh representing the domain under consideration with the prescribed boundary conditions is shown in

Figure 3.2.

3.3.2 Application of Load and Footing

The program accepts both point as well as distributed loads. Gravity loads are also included. The footing was taken as rigid and rough; thus all the points directly under the footing should have equal vertical displacements and no horizontal displacements. The nodal points beneath the footing are constrained in x the direction to simulate the roughness of the footing. To achieve equal vertical displacements beneath the footing, to satisfy the rigidity of the footing, the load applied to the footing was distributed to 5 nodal points in the ratio of 0.2457, 0.1849, 0.1388, 0.1849 and 0.2457 which are obtained from the analysis of normal stress distribution under a rigid foundation (Antoniadis, 1978).

The selection of soil parameters and the results are presented in Chapter 6. The deflections obtained in the FEM analysis are plotted against the load - deformation curves obtained from experiments, for the comparison purposes. The stress obtained from the analysis are presented in the same chapter, compared with experimental and closed form solutions are then discussed.

Figure 3.2 Finite Element Mesh Discretization with Boundary Conditions

CHAPTER 4

TESTING APPARATUS

4.1 Steel Rod Model

4.1.1. General

In the steel rod apparatus three dimensional cohesionless soil particles are simulated by two dimensional steel rods of different diameters. This system has been proved to be an excellent mechanical analog to granular, cohesionless idealised soil and found to be highly suitable for investigating plane strain problems such as long pipes buried in soil where there is no strain in the longitudinal direction (Holtz and Broms, 1978).

The advantages in using this model are: 1. There is no wall friction, 2. Each particle can be identified from others and therefore, it is possible to trace the movement of the soil particles and, 3. It is easy to make horizontal and vertical colour lines on the cross section in order to observe the deformation of the soil mass. However, the soil-pipe interaction is not exactly similar to the actual one because of use of different materials which have different modulus of elasticity ratio, which is a major draw back in this system. Therefore, this model is used merely to observe the failure patterns and to get a rough idea about the influence of eccentric loading. Since the simulation of steel-steel to sand-steel is not perfect, the results

obtained are not qualitative and they will be just reported but not compared with other results.

4.1.2 Description of the Testing Frame

The metal rods which were used to simulate the two dimensional, granular, cohesionless and ideal soil medium required a frame to contain the rods stacked in a vertical plane and to act as a support for the loading mechanisms. The frame consisted of three aluminum U-channels and a steel I-beam which were fastened together at the corners. The base beam of the apparatus consisted of a 150x90x15 mm aluminum channel section and 1.15m in length which was sturdier and wider than the rest of the beams so as to provide the necessary stability against overturning. The two upright (vertical) columns were made of 130x60x10 mm aluminum channels which were 1.59 m in height. The reaction beam was made of 60x75x0.6 mm steel I-section with a span of 0.9 m. The base beam and the uprights were fastened together using aluminum angle brackets of 150x100x95 mm and 15 mm thick, along with eight 12 mm bolts per connection and the top beam and the uprights were fastened together with a 130x100x10 mm thick steel plate welded to the top beam along with four 12 mm bolts per connection to ensure absolute rigidity of the frame. Furthermore, the top beam was designed in such a way that it could be moved to different heights. The details of the steel rod apparatus is shown in Figures 4.1.

Figure 4.1 Detail of Steel Rod Testing Apparatus

The entire frame was clamped to a rigid wooden table in order to provide a convenient working height. Further, the frame was anchored to the wall at the top to avoid any possibility of overturning or tilting.

4.1.3 Load Application

U

a) Footing

A model footing, consisting of two plates of 13 mm and 10 mm thickness welded perpendicularly, was used to apply the load at the surface of the soil. The base plate of the footing was 100 mm wide and 150 mm in length, the same as the length of the steel rods. The surface of the base was roughened by welding thirty one 5 mm diameter rods side by side along the breadth of the footing. The shaft of the footing was 50 mm in length at the top and 86 mm at the bottom. A stud was welded to the shaft to fix the load cell firmly to the footing. All weldings were continuous and the footing was rigid and not allowed to rotate.

b) Loading mechanism

The footing load was applied by a 20kN capacity hydraulic jack which was mounted on the reaction beam. For the first set of experiments, to observe the failure pattern of soil and pipe a hydraulic jack was used to apply loads. But for the next series this loading system was not satisfactory because it could not retain the load. Therefore, the hydraulic jack was replaced by a mechanical jack

for the subsequent experiments.

The footing load was applied through a mechanical gear jack assembly mounted to the reaction beam. The gear system was driven by a variable speed electric drill. The drill used was equipped with a reverse mechanism so that the loading shaft could be withdrawn after each testing.

4.1.4 Steel Rod Soil

a) Description of the Steel Rods

The two dimensional cohesionless soil was modelled using steel rods of three different diameters of 3.2, 4.8 and 6.4 mm and using quantities in the ratio of 2:2:1 respectively, a combination which performed very well and yielded void ratios similar to realistic field values for granular fills (Houle, 1982). The reason for using different diameter bars was to enable a dense packing of the steel rods to provide a higher density of simulated soil which is similar to the actual backfill material for pipe trenches.

b) Stacking Procedure

The steel rods were stacked up horizontally in a parallel array in between the loading frame. This arrangement eliminates the necessity of side walls normal to the direction of movement of steel rods, and

hence eliminates wall friction, which is difficult to account for in other models. In order to maintain the same density of the soil in each experiment, the rods were removed from the frame and re-stacked after each test. It is suggested that the rods be vibrated with a mechanical vibrator after stacking has been done. However, this procedure might interfere with the flexible pipe. To guard against this, each 50 mm layer of soil was gently compacted by hand which in turn forced the rods into a denser arrangement and left a uniform and same density soil mass in each experiment. Furthermore, the flexible pipe was also filled simultaneously with the same steel rods to prevent any distortion that might occur to the pipe by the lateral pressure of the soil mass. After filling to the required height, the steel rods inside the pipe were removed.

Because of the large number of tests and steel rods involved in this series of tests, it was essential to keep the rods in place quickly and conveniently. This was performed by keeping the rods in a special bar container in such a fashion that they all lie parallel to each other, saving a lot of time in placement.

c) Soil Parameters

The average unit weight of this soil was determined to be 61.8 kN/m³ and the void ratio was 0.21. Since the simulated soil is a granular media, the cohesion C is zero. The value of the angle of internal friction was determined in a conventional Wykenham Farrance

Direct Shear Box apparatus and was found to be 33 degrees according to Houle (1982), whereas Scalzitti (1979) obtained 33.7 degrees for the same material.

4.1.5 Conduits

A cold drawn or cast pipe is very suitable for this study because it is free from residual stresses. But the available model pipes in Canada are very thick and they do not fail before the soil fails in bearing. For example, the minimum thickness available in 100 mm diameter pipes are 2.11 mm and 3.81 mm aluminum and steel respectively and the buckling load required is about twenty times the bearing capacity of the soil. Because of this problem fabricated pipes made from very thin steel sheets were used.

The conduits used in these model tests were made from 0.39 mm thick cold rolled steel sheets. They were cut into 100 mm wide and 324 mm (100 mm + lap = 314 + 10 mm) length sheets from which pipes were manufactured by bending in a roller machine and welded by a spot welding device, which would eliminate the chance of any residual stresses being present. Due to the cold work (bending and welding) the residual bending stress could be believed as minimal. Also, thermal stress is reduced greatly because the spot welding device makes use of instantaneous heat to melt and weld by making a short circuit instead of a welding rod or gas.

A few trial experiments were conducted to find out the effect of lap joints at different locations such as Invert, Haunch, Springing, Shoulder and Crown. These tests did not show any variation with different locations and they proved that the lap joints did not change the symmetrical properties significantly.

From the strain gauge measurements, the stresses and strains were found to be very small in the region between Haunch and Invert or more generally in the lower half section. Therefore, the lap joint was kept at the invert location to eliminate any critical or major effects and to ensure the symmetry of the pipe.

The mechanical and material properties of the 100 mm diameter pipes are given in Appendix B.

4.1.6 Bedding

To support the pipes two types of bedding were used in the steel rod model tests. Rubber was used as compressible bedding and wood as rigid bedding. The shape of the footing was adopted from Spangler's (1971) model which has the bedding angle of 40 degrees.

4.1.7 Instrumentation

a) Load Cell

The first series of tests in the steel rod model was performed to obtain the failure loads and to visualize the deformed patterns of soil and pipe. Therefore, not much instrumentation was done except for measuring the load by a load cell and a digital strain indicator. As mentioned in section 4.1.3, the load cell was connected to the model footing mounted between the hydraulic jack and the footing.

The hydraulic jack used for this set of experiments had a load capacity of 20 kN and was operated manually. A precision universal load cell with a capacity of 44.5 kN and a digital strain indicator with an accuracy of 0.01 kN were used to measure the applied vertical loads. A view of digital strain indicator, load cell (with the footing used in sand box test) is shown in Figure 4.2. Complete specifications of the instruments and limitations are contained in Appendix C.

b) Deformation of Soil

The deformation of the soil was observed by marking chalk lines on the front face of the steel rods. For the purpose of clarity, the entire face was painted white and 50 mm square blue colored grids were drawn on it. The load was applied continuously until the pipe failed completely, and the whole process was video-taped.

Figure 4.2
A View of Digital Strain Indicator and Load Cell

C) Deformation of Conduits

The second series of steel rod tests were performed to obtain Load-Deformation characteristics of the conduits and the failure loads were determined from the graphs.

From the preliminary experiments, load-deformation and load-strain measurements showed that the deflection and the strain were very small at Haunch and at Invert. Therefore, no attempts were made to take deflection measurements at Haunch.

Since the length of the pipe and the steel rods were the same, the pipes were completely covered by steel rods. Therefore, a difficulty arose in taking deflection measurements at the outside of the pipes. At the same time, because the pipe diameter was too small to accomodate ordinary dial gauges, the internal measurements were also impossible. These problems were overcome by fixing protruding metal pieces of sufficient lengths to hold the dial gauge rods. The 30 mm long and 7 mm wide metal pieces were cut from a 0.5 mm thick stainless steel sheet and glued internally at the Crown, Shoulders, Springings and Invert. The metal pieces had wide tops to accomodate lateral movements of the pipe.

The radial deflections at the crown, both shoulders and at both springings of the pipe were measured using dial gauges with an accuracy of 0.01 mm. In the case of compressible bedding the deformation at

Invert was also measured. The arrangements of dial gauges to measure deflections are shown in Figures 4.3 and 4.4 for rigid and flexible beddings respectively. The model footing through which load was applied, was not allowed to rotate and vertical displacements at all four corners were taken with dial gauges, to ensure plane strain condition and uniform loading.

The entire readings were taken in load controlled condition.

4.2 Sand Tank

4.2.1 General

In this model study "Ottawa" Quartz sand was used as the supporting media which was contained in a sand box (will be described in the next section) having a transparent - plexiglass sheet as one side and a plywood wall on the other. The same loading frame and loading mechanism was adopted here as in the steel rod model but a different size footing was used.

Figure 4.3

Dial Gauge Arrangements to Measure Radial
Deflection of pipe on Rigid Bedding

Figure 4.4

Dial Gauge Arrangements to Measure Radial
Deflection of Pipe on Flexible Bedding

This experimental work, as in the steel rod experiments, was restricted to analysing only a limited number of variables. For example, the dimensions and the material of the pipe (t , d , E and therefore I) were taken as constant. Only one type of soil - i.e., quartz sand, was used. The flexibility of the footing was fixed and can be considered rigid. The width of the footing was fixed. A strip footing was modelled and was always placed at the surface. The bottom of the footing was roughened by glueing sand particles to it. Therefore, the interface of the footing and soil can be considered rough. The load application can be considered as static and was applied vertically through the central axis of the footing. The walls of the sand tank were assumed to be reasonably smooth and the problem being modelled was that of a rough rigid strip footing and a flexible pipe tested under plane strain condition.

The cover depths and the locations of the load application at the surface were varied to study the influence of overburden pressure and the eccentricity of load application. The pipe was completely buried under soil, that is, no bedding was used as in the steel rod model. The width and the length of the tank were kept constant. The entire experiment was performed under embankment condition.

By limiting the number of factors to be analysed, the effect of the main parameters under consideration can be studied more closely. Ditch conditions and the effect of friction of the ditch were not analysed.

4.2.2 Description of the Sand Tank

The entire third series of experiments were performed in a rigid tank with inside dimensions of 1.10 m x 0.90 m x 0.15 m. The base and the ends of the tank were made of 150x90x10 mm aluminum channels. The side walls were made of 1.22 m x 0.915 m x 13 mm plexiglass to visualize the failure pattern and plywood respectively. They were fastened to the channels at 155 mm intervals with 7 mm hexagonal bolts and nuts. Since plane strain condition imply no outward movement of the side walls, additional stiffness was incorporated by four 63x63x6 mm aluminum angles fixed horizontally throughout the length of the plywood wall at equal intervals with five 7 mm countersunk bolts. The plexiglass wall was reinforced by a frame which was formed by four 850x63x6 mm aluminum vertical stiffeners welded to four 63x63x6 mm aluminum angles. To reduce the side wall friction, the inner side of plywood sheet was lined with a 1 mm thick arborite sheet. Measurements taken for the lateral movements of the side walls at the worst points (central point on the top side of the wall) indicated that the provided stiffening system was satisfactory. A line diagram and a side view of the tank are shown in Figures 4.5 and 4.6.

- 1 - 63 x 63 x 6 mm ANGLE (Al) 2 - 850 x 63 x 6 mm PLATE STIFFNER (Al)
 3 - 150 x 90 x 10 mm CHANNEL (Al) 4 - 1220 x 915 x 13 mm PLEXIGLASS
 5 - 1220 x 915 x 13 mm PLYWOOD 6 - ROTATING DEVICE
- All dimensions in mm

Figure 4.5 Detail of Sand Box Testing Apparatus

Figure 4.6

A Side view of Sand Box, Loading frame Loading mechanism
and Hopper

4.2.3 Study of the Effect of the Wall Friction

As mentioned previously, wall friction on the plywood sheets was minimized by affixing an arborite sheet to the plywood wall. The interior surface of the plexiglass wall was polished to reduce the side friction in order to ensure plane strain condition. The coefficients of friction of plexiglass and arborite were determined by the following pull out test to make sure that the frictional effects were small compared to the working load level.

Because there are some limitations in the direct shear apparatus to determine coefficient of friction such as scale effects, sand raining process, disturbance to the top surface of sand due to levelling and the possibility of locking of grains (Rabbaa, 1981) the Pull-out test was used.

The purpose of this series of tests was to determine the ratio between the angle of friction of plexiglass plate (λ_p), Arborite sheet (λ_a) and the angle of friction of another plate roughened with sand (λ_r). The ratio will be an estimation of the smoothness of plexiglass and arborite.

Two arborite sheets of 1 mm thick were glued together, keeping the smooth surfaces outside, to both sides of a piece of 250x250x8 mm plywood to get a rigid plate. The plate was vertically placed in a sand tank and the sand was rained with the same hopper until the plate was

completely immersed. Pull-out loads were applied and the corresponding vertical displacements were taken at both corners of the plate. The set-up is shown in Figure 4.7. Then the smooth surfaces were roughened by glueing sand and the same procedure was repeated. The above experiment was performed for a 250x250x6 mm plexiglass plate too. The pull-out load-displacement curves for the both plates and both cases are shown in Figure 4.8. The theory was given by Rabbaa (1981) and from the experiments the following relationships are found.

$$\tan \lambda_a / \tan \lambda_r = 0.21 \quad \text{and} \quad \tan \lambda_p / \tan \lambda_r = 0.24$$

$$\text{and therefore, } \lambda_a = 10.54^\circ \quad \text{and} \quad \lambda_p = 11.74^\circ$$

From these results, it can be assumed that the arborite and the plexiglass are fairly smooth and the small frictional effects will not change the results appreciably within the working load levels.

Figure 4.7

Pull-out Test

2

Figure 4.8

4.2.4 Load Application

a) Footing

To simulate strip loading, a model footing, consisting of two plates of 25 mm thick comprising the shaft and base welded perpendicular to each other, was used to apply the load at the surface of the soil. The base plate of the footing was 150 mm in length, the same as the diameter of the pipe and 150 mm in width same as the length of the pipe and width of the tank. The bottom surface of the base was roughened by glueing sand using white wood glue. The shaft of the footing was 50 mm wide at the top and 195 mm at the bottom. The footing was rigid and was allowed to neither rotate nor tilt by using wide (spread) plates to transfer load from the mechanical jack to the footing. As in the steel model test, the same load cell was placed in between the jack and the footing, and connected rigidly to the footing as shown in Figure 4.2.

b) Loading Frame

The same testing frame used in the steel model test was used for the load application in the sand test with a few modifications. The reaction beam was shifted to the top and the whole frame was stepped down from the table to the floor, to create a convenient working height. The sand box described previously was kept inside the frame and rested on the base beam of the frame. The weight of the tank and soil

gave an additional stability to the loading frame. The frame was anchored at the top to the wall for further safety.

The sand box could be moved with respect to the loading frame to apply loadings at several locations.

c) Loading Device

The same mechanical screw jack used in the steel rod model test, was mounted to the reaction beam and the ram applying the load was advanced by driving gears with an electrical varying speed drill.

4.2.5 Cohesionless Soil

a) General Characteristics of Ottawa Sand

The granular material used in this study was commercially available Quartz sand supplied by the Ottawa Silica Company, Ottawa, Illinois. The Quartz sand whose commercial designation was grade No. 24, was a dry, medium to fine white quartz sand composed of rounded to subrounded particles. The sand was poorly graded with a coefficient of uniformity of 1.7, and had the grain size distribution shown in Figure 4.9. Using the methods outlined by Bowles (1978), the maximum and minimum densities were found to be 18.0 kN/m^3 and 15.3 kN/m^3 respectively (Rabbaa, 1981). The specific gravity and the maximum and the minimum void ratios were found to be 2.66, 0.74 and 0.50 respectively

(Kempthorne, 1978).

b) Method of Deposition

The soil-pipe interaction is highly dependent on the compactness of the surrounding soil mass. Since investigation of the effect of some other variables was the main concern of this study, it is imperative to control the compactness and thus to keep the density of the soil constant throughout the entire experiments. Therefore, the soil placement plays an important role in these tests to perform all the tests in similar soil conditions. The technique that was employed had been used successfully by several researchers (Kempthorne (1978), Moore (1979) and Rabbaa (1981)).

To obtain a uniform density sand bed, the rate of deposition of the sand was controlled. In general, the greater the rate of deposition, the greater the porosity of the sand; and the greater the velocity of fall of the sand, the greater the energy and hence the greater the density of the sand.

The placement of the sand in the sand box was performed by allowing the sand particles to "rain" onto the bed at a controlled rate of deposition. This was accomplished by depositing the sand in a wooden hopper with a wire mesh at the bottom. The hopper traversed at the top of the test tank at a constant velocity (Figure 4.10). The intensity of sand deposition is controlled by the hole diameter and the hole spacing

Figure 4.10
Deposition of Sand

in the wire mesh. Drupsteen (1977) showed that increasing the hole diameter or increasing the number of holes per unit mesh area increased the rate of deposition and resulted in a less dense sand bed. However, the tests performed by Drupsteen (1977), Kempthorne (1978), Moore (1979) indicated that the density was not increased significantly by placing the sand at a height greater than 0.2 m. Hence, the height of deposition was maintained at 0.2 m above the sand bed. Table 4.1, obtained from Kempthorne (1978) shows the variation of density with hole size and with fall. The increase in layer thickness per sweep was maintained at 5-6 mm. Using a 1.6 mm hole diameter, and a spacing of 2.8 mm for the mesh, a density of $1760 \pm 10 \text{ kg/m}^3$ was obtained (Rabbaa, 1981). This converted into a relative density (D_r) of $90\% \pm 2\%$ and a 'dense' identification, based on the criteria outlined by Holtz (1973).

When the sand reached about 5 mm above the required level, the top surface was levelled with a smooth scraper, supported against the straight edges of the tank, which removed the 5 mm extra sand and left a perfectly levelled plane surface.

Table 4.1

Quartz Sand Density Calibration Tests (kg/m^3)

(from Kempthorne, 1978)

(A)

EFFECT OF HEIGHT OF FALL WITH 1.6 MM HOLE DIAMETER

Trial Number	Height of Fall (m)	Density Tin Number				Average
		1	2	3	4	
1	0.69	1747	1753	1751	1753	1749
2		1742	1753	1744	1756	
3		1753	1743	1738	1753	
Average		1747	1750	1744	1754	
4	0.46	1778	1749	1746	1766	1753
5		1738	1743	1744	1766	
6		1748	1756	1749	1763	
Average		1753	1749	1746	1765	
7	0.23	1742	1750	1740	1746	1750
8		1754	1740	1744	1755	
9		1748	1745	1741	1763	
Average		1756	1745	1742	1755	
10	0.13	1729	1733	1735	1752	1738
11		1726	1742	1737	1762	
12		1718	1739	1742	1763	
Average		1724	1729	1738	1759	
In-Situ Test	0.53	1767	1752	1750	1775	

(B)

EFFECT OF HOLE DIAMETER ON DENSITY

Hole Size (mm)	Height of Fall (m)	Average Density (kg/m^3)	Source
1.60	0.43	1753	(from Kempthorne, 1978)
2.38	0.44	1737	from Drupsteen (1977)
2.78	0.44	1731	" " "
3.18	0.44	1716	" " "
3.57	0.44	1712	" " "

c) Shear Properties of Ottawa-Sand

Kemphorne (1978) performed experiments with the same sand in the direct shear box deposited in the identical method used in the plane strain apparatus. He obtained the angle of internal friction, as 40 degrees for the Quartz sand.

From a series of triaxial compression tests, Rabbaa (1981) determined the value of the angle of internal friction, as 41 degrees for the Quartz sand with a relative density (D_r) of 90%. The value of ϕ was corrected by increasing 4 degrees, to suit the plane strain condition with assuming smooth walls. But, when the angle of friction of stainless steel ($\lambda_s = 8^\circ$) was taken into account, the same value 41 degrees was obtained.

4.2.6 Conduits

The conduits used in this series of tests were fabricated from 0.39 mm thick cold rolled steel sheets, the same as the conduits used in the steel rod model tests. But instead of 100 mm diameter and 100 mm long circular pipes, 150 mm diameter and 150 mm long circular pipes were chosen. The circular pipes were manufactured by bending 150 mm wide and 481 mm long ($\pi d + \text{lap} = 471 + 10 \text{ mm}$) sheets in a rolling machine and then welded with a spot welding device.

Similar to the steel rod trial tests, some preliminary tests were

performed to study the effect of lapping by keeping the joints at various positions (Invert, Haunch, Springing, Shoulder and Crown). These tests yielded the same results as in the rod model test, i.e., the pipes behaved similarly irrespective of the location of joint. To avoid any anomaly due to the lap joint, it was always kept at the invert to ensure the symmetry about the vertical axis and because the strains in the lower half of the pipe were much smaller compared to the upper half. The mechanical and material properties of the 150 mm diameter pipes are given in Appendix B. Strips of soft foam rubber were glued to pipe at both ends as shown in Figure 4.11, to avoid any leakage of sand at the interface between the pipe and walls of the box.

4.2.7 Instrumentation

a) Load measurement

The load measurements in the sand box experiments were exactly similar to the measurements in the steel rod model tests which were described in section 4.1.7 (a).

Figure 4.11
Pipes Used in Sand Box Tests

b) Deformation of Soil

The deformations of the soil and the pipe and the displacement of the footing were observed during the loading through the transparent-plexiglass wall. Layers of about 3 mm thick colored sand which was obtained by dyeing the same Quartz sand with black ink was deposited horizontally at 50 mm intervals to visualize clearly the deformation of soil and the pipe and to check whether the soil below (or around) the footing failed in punching. A front view of the tank and sand with colour lines is shown in Figure 4.12. The failure patterns were photographed after the collapse of the conduits and some typical results are given in Chapter 6.

c) Deformation of conduits

This is the most difficult measurement to accomplish because the pipe is completely buried under sand and access to the interior of the pipe was difficult due to the ends of the pipes being blocked by the front and rear walls of the testing tank. Any type of insertion for Linear Variational Differential Transformer (LVDT) or dial gauge (like extension rod or shaft) seemed to be impossible due to the sand raining technique. Furthermore, the deflection measurement through the transparent wall was not very accurate. Therefore, the deflection measurements from outside the pipe were given up and attempts were made to measure the deflection inside the pipe. If LVDT's were to be used, they can not be rotated and in turn there was not enough room to

Figure 4.12 The Front View of Sand Box and Colour Lines in Sand

accommodate all eight LVDT's. An ordinary dial gauge (125 mm) was used to take all eight deflection measurements by rotating the gauge inside the pipe.

A device was designed to firmly hold the dial gauge at the centre of the pipe and to rotate it through 360 degrees without any wobble. Further the device was designed in such a fashion that the dial gauge could be withdrawn from the tank either to place or to remove the pipe. The rotation device was checked for its wobbling accuracy and the maximum wobbling effect during the rotation was found to be only 0.02 mm which is negligible compared to the deflection of pipe which was in the order of 5-10 mm or about 0.5%. A view of the rotating device is shown in Figure 4.13 after fastened to the stiffeners of the tank. A 125 mm diameter hole was bored into the plywood wall to insert and to remove the dial gauge after placing the pipe. The rotation of dial gauge was eased with substituting the regular tip of the gauge by a roller contact point. The accuracy of the dial gauge used was 0.01 mm.

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Figure 4.13 The View of Rotating Device

d) Strain gauging

Two 150 mm diameter pipes were thoroughly cleaned by sand blasting to remove all oil dirt. The eight cardinal points (at 45 degrees spacing) were marked at the inside of the pipe and eight 120 ohms resistance strain gauges were circumferentially attached (Figure 4.14). The strain gauge configuration was a full bridge setup with dummy gauges, that completely neglected the effects of bending. Therefore, only hoop tensile strains were measured in the circular pipes. The complete specifications of the strain gauges used here are given in Appendix C.

2.8 Soil reinforcing

To study the effect of soil reinforcement on the shallow buried flexible pipes, a few experiments were performed for the critical cases of the two very shallow depth cases with one and two steel strips. The 40 mm wide and 550 mm long reinforcing strips were cut from 0.04 mm thick cold rolled steel sheets. These reinforcing strips had an ultimate tensile strength of 6.4 kN, which would not fail during the test.

The reinforcing strips were placed when the height of soil bed reached to half the cover depth, then soil raining was continued till the required final level.

Figure 4.14 Strain Gauging of Pipe

CHAPTER 5

TESTING PROGRAM

5.1 Introduction

The survey of previously published literature revealed that there was a dearth of analysis of soil-pipe interaction with asymmetrical buckling of flexible buried conduits due to eccentrically applied concentrated load, in both model tests and full scale tests. Most of the theories derived and the laboratory experiments conducted with the deeply buried pipes and therefore uniformly distributed loads on the pipes. But in reality, the conduits are buried at shallower depths or with smaller cover depth/diameter ratio under highways, railways and airfields where they undergo highly concentrated (moving) eccentric loading which can cause asymmetrical buckling. For this reason a program of investigation was started to observe the manner in which the moving-eccentric loads respond to a flexible pipe buried at shallow depth. The loading was modelled into a strip footing and the whole problem was tested in plane strain condition.

As mentioned in Section 1.2, it was not possible to vary all parameters in this study, and therefore only the following conditions were varied while others were kept constant.

- (i) Loading was applied at $e/B = 0, 0.5, 1.0$ and 1.5 , where e is the distance between the centre lines of the pipe and the

footing; and B is the width of the footing.

(11) The the ratio of cover depth to diameter of the pipe ratio (H/D) was varied as 0.25, 0.5, 1.0 and 2.0.

All the experiments were performed in the embankment conditions, i.e., the walls of the ditch are far apart and therefore the wall friction is negligible. If the walls are farther than 6 D, then that can be considered as embankment condition (Valsanger and Britto, (1978)). The footing was rough and rigid and not allowed to rotate or tilt.

5.2 Steel Rod Model

The two dimensional granular soil medium was simulated using different diameter steel rods and they were stacked in a parallel array in between a frame. The loading mechanism was mounted on a loading frame and the load was applied at the surface of soil through a footing.

5.2.1 Visualisation of Failure Patterns of Soil and Pipe

In this series of tests, the front face of the steel rods was painted with white paint and 50 mm vertical and horizontal grid lines were drawn with a line marker. The load was applied by a 20 kN capacity hydraulic jack mounted to the loading frame at the surface of the soil through the rough footing.

Two types of bedding- flexible (rubber) and rigid (wood)- were used and the cover depths were varied as $H/D=1$ and 2. The loading position was changed from $e/B=0$ to 1.5 in steps of 0.5 increments. The whole experiment was video-taped and a few still shots were taken from the replay.

5.2.2 Estimation of Critical Load

The same steel rod model was used to obtain the critical load with a few modifications. Firstly, the hydraulic loading mechanism was substituted by a mechanical assembly in order to load at a slower rate and to maintain the load for a while, so that all dial gauge readings could be taken. The deflections of the pipes at the crown, both shoulders and springings and in the case of flexible bedding, at the invert were measured with dial gauges. The set up was already shown in Figures 4.3 and 4.4.

In this series of experiments too, the cover depth was changed to 1D and 2D and flexible and rigid beddings were used. Loadings were applied at centre and eccentrically at 0.5 B, 1.0 B and 1.5 B.

These experiments were performed as preliminary tests to find out the feasibility of extending it to real soil. Further, the soil-pipe interaction will not be the same as in real situation. Therefore no attempt was made to compare these results with theory.

5.3 Sand Tank

A detailed description about the sand tank, loading mechanism, deflection measurement by a rotation mechanism, and the materials involved in these experiments (sand and pipe) has already been presented in Chapter 4.

In this series of experiments, pipes were buried in sand and tests were performed with four depths, and at four loading locations. H/D was varied as 0.25, 0.5, 1.0 and 2.0. As in the steel rod experiments e/B was changed from 0 to 1.5 in steps of 0.5.

All of these experiments were repeated twice, to make sure the consistency of experimental results and to find out the variation in the results. The critical cases ($H/D=0.25$, $e/B=0$, $H/D=0.25$, $e/B=0.5$, $H/D=0.5$, $e/B=0.5$; & $H/D=1$, $e/B=0.5$) were repeated again (i.e., 3 tests).

5.3.1 Strain Gauging

Two 150 mm circular pipes were strain gauged at the eight cardinal points (at 45 degrees spacings) and strains were measured with full bridge setup with dummy gauges for cases of $H/D=1$, $e/B=0$ and $e/B=0.5$.

5.3.2 Soil Reinforcing

When the conduits are buried at shallower depths, they failed in asymmetric buckling mode first, rather than other modes of failure, due to the eccentric concentrated loads. This type of failure could be improved by distributing the concentrated loads all over the conduit using reinforcement in soil.

The 40 mm wide and 550 mm long reinforcing strips were cut from 0.04 mm thick cold rolled steel sheets and were placed at the middle of the cover depth, that is, half distance between the surface and the top of the culvert.

Experiments were performed for the critical cases such as $H/D=0.25$, $e/B=0.5$ and $H/D=0.25$, $e/B=0$ with one reinforcing strip and two reinforcing strips for the both cases. An additional test was performed for $H/D=1$, $e/B=0.5$ with one reinforcing strip.

A summary of the all experiments performed is given in Table 5.1.

Table 5.1 Schedule of Experiments

Test Series	Objective	Surrounding media	Bedding	H/D	e/B	R/f strips	No. of Repeated tests	Total No. of tests
1*	To observe failure patterns of soil and pipe	Steel rods	Flexible and Rigid	1 & 2	0, 0.5, 1.0, 1.5	--	--	16
2*	To determine eccentric critical loads	Steel rods	Flexible and Rigid	1 & 2	0, 0.5, 1.0, 1.5	--	--	16
3**	To determine eccentric critical loads	Sand	Sand	0.25, 0.5, 1 & 2	0, 0.5, 1.0, 1.5	--	2 and 3 (critical)	36
4**	To measure strains	Sand	Sand	1	0.0, 0.5	--	--	2
5**	To study the effect of reinforcing soil on the failure of pipes	Sand	Sand	0.25 & 1	0.0, 0.5	1 and 2	5	4
				1	0.5	1	--	1

* 100 mm dia. 100 mm long pipes, 150 x 100 mm footing

** 150 mm dia. 150 mm long pipe, 150 x 150 mm footing

CHAPTER 6

PRESENTATION AND DISCUSSION OF RESULTS

6.1 Introduction

In this chapter, the experimental results, and the analytical and the finite element solutions are presented. The experimental results are analysed and compared with the theoretical predictions. The experimental observations from the steel rod model tests are presented. The failure patterns of the soil - pipe system are shown by a series of photographs. The load - deformation curves for the steel rod model are given in the following section. Next, the same type of curves for the sand box tests are presented. The deflection and the stresses for the pipe used in the sand box test, calculated according to various theories, are also given in this chapter. The results of the finite element analysis are compared to the corresponding experimental values. At last the results from the load - deformation curves are discussed for both test series and they are compared to theoretical predictions.

6.2 Experimental Results

6.2.1 Failure Pattern of Soil - Pipe System

In the first series of tests, that is, the steel rod model, the pipes were supported on both rigid and flexible beddings and the soil

was simulated by steel rods. The loads were applied at the surface of the soil until the pipes collapsed completely. The maximum loads were noted for all cases, that is, $H/D = 1$ and 2 , and $e/B = 0, 0.5, 1.0$, and 1.5 , where H is the soil cover above the crown and D the diameter of the pipe. No deflection measurements were taken in this series but the movements of soil particles and the pipe were recorded by video-tape from the initial loading stage to complete failure. A few photographs were also taken after the failure. Further, from the video-tape a number of still photographs were reproduced at different stages of failure. Some of these are shown in Figures 6.1 to 6.5. The failure zone of the soil can be clearly distinguished by the blurred region. As shown by the video - tape replay the failure occurred within a region between the footing and the pipe. In the case of eccentric loadings, the failure of the pipes was caused by collapsing the shoulder of the pipe, the failure zone of the soil extends over a large region. This can be seen in Figure 6.5 very distinctly.

Figure 6.1 Failure Pattern of Soil-Pipe System $h/D = 2$, $e/B = 0$

Figure 6.2 Failure Pattern of Soil-Pipe System $H/D = 2$, $e/B = 0.5$

Figure 6.3 Failure Pattern of Soil-Pipe System $H/D = 2$, $e/B = 1.0$

LOAD VS. DEFLECTION FOR $H/D=2, E/B=0$
[RIGID BEDDING]

Figure 6.8

6.2.2 Load - Deformation Curves for Steel Rod Tests

The second series of tests were carried out to estimate critical loads under various conditions, as described in chapter 5, that is with rigid and flexible beddings, $H/D = 1$ and 2, and $e/B = 0, 0.5, 1.0$, and 1.5, where e is the eccentricity of load application and B is the footing width. The radial deformations of the pipe were measured at five points for pipes supported on rigid bedding and at six locations in the case of flexible bedding. The displacements at all four corners of the footing were taken and the average value was obtained. The weight of the footing and the load cell, which was connected to the footing, were added to the load cell reading as the applied load. The applied load versus the radial deflection and the vertical displacement of footing curves were plotted and are shown in Figures 6.6 to 6.21.

The failure loads in this series of tests and the maximum loads found in the first set of tests were more or less the same and these results are analysed in the section 6.5.1.

LOAD VS. DEFLECTION FOR $H/D=1, E/B=0$

[RIGID BEDDING]

RADIAL DEFLECTION (mm)

"+" inward radial deflection "-" outward radial deflection

Figure 6.6

LOAD .VS. DEFLECTION FOR $H/D=1$, $E/B=0$

Figure 6.7

LOAD VS. DEFLECTION FOR $H/D=2, E/B=0$
[RIGID BEDDING]

Figure 6.8

LOAD VS. DEFLECTION FOR $H/D=2, E/B=0$
[FLEXIBLE BEDDING]

LOAD .VS. DEFLECTION FOR $H/D=1, E/B=0.5$
[RIGID BEDDING]

"+" inward radial deflection "-" outward radial deflection

Figure 6.10

LOAD VS. DEFLECTION FOR $H/D=1$, $E/B=0.5$

[FLEXIBLE BEDDING]

Figure 6.11

LOAD .VS. DEFLECTION FOR $H/D = 2, E/B = 0.5$

[RIGID BEDDING]

"+" inward radial deflection "-" outward radial deflection

Figure 6.12

LOAD VS. DEFLECTION FOR $H/D=2, E/B=0.5$

[FLEXIBLE BEDDING]

Figure 6.13

LOAD VS. DEFLECTION FOR $H/D=1, E/B=1$

[RIGID BEDDING]

"+" inward radial deflection "-" outward radial deflection

Figure 6.14

LOAD VS. DEFLECTION FOR $H/D=1, E/B=1$

[FLEXIBLE BEDDING]

Figure 6.15

LOAD VS. DEFLECTION FOR $H/D=2, E/B=1$
[RIGID BEDDING]

Figure 6.16

LOAD VS. DEFLECTION FOR $H/D=2, E/B=1$

[FLEXIBLE BEDDING]

"+" inward radial deflection "-" outward radial deflection

Figure 6.17

[RIGID BEDDING]

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LOAD VS. DEFLECTION FOR $H/D=1$, $E/B=1.5$
[FLEXIBLE BEDDING]

Figure 6.19

[RIGID BEDDING]

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LOAD VS. DEFLECTION FOR $H/D=2, E/B=1.5$
[FLEXIBLE BEDDING]

Figure 6.21

6.2.3 Load - Deformation Curves for Sand Box Tests

Since the soil-pipe interaction in these tests* is similar to an actual case, more attention was given to this series of tests. All experiments were repeated at least once and third tests were performed for the critical cases as indicated in the schedule of tests (Table 5.1). Since in this series of tests, numerous dial gauge readings were taken, a computer program (given in Appendix D) was written to calculate the radial deformations and the average vertical displacement of the footing. The average deformations were calculated and the corresponding applied load versus radial deflection curves were drawn using the computer program called "APL GRAF", available through the Carleton University computer program library. The Load - Deformation curves obtained in the sand box tests are shown in Figures 6.22 to 6.36. The differences of the deflection values in the repeated experiments ranged within +5% from the corresponding average values.

6.2.4 Measurement of Strain

The maximum strain measured for $H/D = 1$, $e/B = 0$ experiment was 8.8 microinch/inch which corresponds to a hoop stress of 1.76 MPa. For the eccentric loading $H/D = 1$, $e/B = 0.5$ the maximum strain at failure occurred at the shoulder of the pipe and was 15.72 microinch/inch which corresponds to a hoop stress of 3.14 MPa.

LØAD. VS. DEFLECTION FØR $H/D = .25, E/B = 0$

Figure 6.22

LØAD. VS. DEFLECTION FØR $H/D = .25, E/B = .5$

Figure 6.23

Figure 6.24

Figure 6.25

Figure 6.26

Figure 6.27

Figure 6.28

Figure 6.29

Figure 6.30

Figure 6.31

Figure 6.32

Figure 6.33

Figure 6.34

LØAD. VS. DEFLECTIONØN FØR $H/D=2, E/B=.5$

Figure 6.35

Figure 6.36

6.3 Calculation of Pipe Deflections and Stresses by Closed Form Solutions

6.3.1 Introduction

The Iowa deflection formula (Eq. 3.1) is the only empirical expression available to calculate the deflection of a flexible conduit.

$$\Delta x = D_1 \frac{K W_c R^3 (35.26)}{E I + .061 K_s R^3} \quad 3.1$$

The values of the bedding constant (K) and the deflection lag factor (D_1) in the formula are suggested by Spanzler (1971) and the rest of the values must be the respective values of the particular case on hand. Again the values of the radius (R) and the moment of inertia (I) of the pipe are given by the geometry of the conduit. The modulus of elasticity of the pipe material (E) is also generally known. But the value of the modulus of the soil reaction (K_s) is not known and it is difficult to determine a suitable value for the particular case. This problem arises because the value of K_s depends on several factors, such as the unit weight of the soil, the type of the soil, compaction, Poisson's ratio etc. Further the K_s value will change while the conduit is deformed. The determination of the modulus of soil reaction by means of laboratory testing, such as triaxial compression tests, can in general not duplicate the soil-pipe load-deformation response. Therefore, it is difficult to obtain the

true value for K_s and a reasonable approximation should be made.

The compressive stresses in the wall of the conduit due to circumferential thrusts can be calculated by the ring compression theory (White and Layer, 1960) without any problem because this does not depend on the soil properties and depends only on the yield stress of the pipe material and the pipe geometry. However, determination of buckling stresses of a flexible conduit buried in soil, is not that simple because of its complexity and dependence on several factors. Most of the buckling theories (described in Chapter 2) are basically of the same nature, as shown below, because they were derived from the same principles except the theory proposed by Meyerhof and Baikie (1963), which was obtained from plate buckling phenomenon.

The buckling pressure p_b is generally given by:

$$p_b = \frac{E I}{R^3} (n^2 - 1) + \frac{K_s}{n^2 - 1} \quad 2.6$$

$$p_b = 2 \sqrt{K_s E I / R^3} \quad 2.10a$$

The differences in the expressions for the buckling pressure are due to the manner in which the modulus of soil reaction K_s , is related to the modulus of deformation of the soil, E_s . The relation between K_s and E_s depends on the Poisson's ratio of the soil and the buckling mode of the pipe. The different values for K_s suggested by various

researchers are given in Table 6.1. To compare the buckling pressures predicted by various theories, an average value for K_s equal to $0.71E_s$ is taken in all cases analysed analytically.

6.3.2 Calculation of Deflection

The horizontal deflection of the pipe (it can be considered the same as the vertical deflection) is calculated from the Iowa formula (Eq. 3.1). Because a uniform pressure was assumed in the derivation of this formula, it cannot be used for eccentric loading. Therefore, deflections are found only for cases of symmetric loading.

From Raabaa (1981), who used the same sand and the same compaction, the modulus of deformation of the soil, E_s , is taken as 17,650 kPa (180 kg/cm^2). The value of the modulus of soil support K_s is obtained from $K_s = 0.71E_s$ (Table 6.1), as 12,530 kPa, which is similar to the value given by OHBDC (1983) for a dense, poorly graded sand.

From Spangler (1971), for an excellent fill material (95 percent Standard Proctor Density), the value of deflection lag factor D_1 is taken as 1.25 and the bedding constant for a bedding angle $\alpha = 90^\circ$, $K = 0.083$.

Table 6.1

Comparison of Theoretical Formulations of the Soil Support Modulus

 $(\nu_s = 0.33)$

Author	Proposed Expression	Value of K_s	
		$n = 2$	$n = 7$
Meyerhof and Baikie (1963)	$\frac{E_s}{2(1 - \nu_s^2)}$	$0.60 E_s$	$0.60 E_s$
Luscher (1966)	$\frac{\left\{ 1 - \left(\frac{R_f}{R_0} \right)^2 \right\} E_s}{(1 + \nu_s) \left\{ 1 + \left(\frac{R_f}{R_0} \right)^2 (1 - 2\nu_s) \right\}}$	$0.71 E_s$	$0.71 E_s$
Kloppel and Glock (1970)	$\frac{E_s}{(1 + \nu_s)}$	$0.71 E_s$	$0.71 E_s$
Dun and Butterfield(1971)	$\frac{E_s (n^2 - 1)}{R (1 + \nu_s) \{ 2n + 1 - 2\nu_s (n + 1) \}}$	$0.82 E_s$	$4.0 E_s$
Cheney (1976)	$\frac{E_s (n^2 - 1)}{R (1 - \nu_s^2) (2n + 1 - \nu_s)}$	$0.776 E_s$	$3.9 E_s$

The vertical load per unit length of the pipe W_c is calculated by assuming a 2:1 load distribution. For the pipe used in the sand box test, $R = 75 \text{ mm}$, $t = 0.39 \text{ mm}$ and $I = 0.0049 \text{ mm}^4/\text{mm}$. The calculated maximum deflections for the maximum concentric loading obtained from the experiments for the four different H/D ratios are tabulated in Table 6-2.

6.3.3 Calculation of Compressive Stress due to Circumferential Thrust

(a) Ring Compression Theory (From Chapter 3),

The compressive stress in the wall,

$$f = p R / t \quad 3.3b$$

$$p_c = 1.1(\gamma_s h + p_L) \quad \text{for } p_L > \gamma_s h \quad 3.4 b$$

$$\begin{aligned} \text{The unit weight of sand} &= 1760 \text{ kg/m}^3 \times 9.81 \times 10^{-3} \text{ kN} \\ &= 17.26 \text{ kN/m}^3 \end{aligned}$$

For H/D = 1 case,

$$p_L = 34.67 \text{ kPa/m}$$

$$\gamma h = 2.59 \text{ kPa/m}$$

Since, $p_L > \gamma h$, according to Kloppel and Glock (1970) (Eq. 3.4b), the total pressure should be increased by 10 percent (to cover impact loads). Therefore, the total pressure, p , is 40.99 kPa/m. From the ring compression theory, the compressive stress, (Eq. 3.3b) on the wall is 7.89 MPa. But the yield stress of the pipe material (steel), f_y is 230 MPa. Therefore the pipe did not fail due to wall crushing.

Table 6.2 Maximum deflection calculated by Iowa formula.

H/D	Critical Load (N)	Max. Deflection (mm)
0.25	790	20.08
0.50	1030	21.85
1.00	1560	24.81
2.00	2250	23.86

(b) OHBDC (1983)

According to the Code, (Eq 3.6a)

$$\begin{aligned}\text{Thrust } T &= \alpha_D T_D + \alpha_L T_L \\ &= (1.25 \times 2.59 + 1.4 \times 34.67 \times 1.25) \times 0.150/2 \\ &\quad \text{kN/m} \\ &= 4.79 \text{ kN/m.}\end{aligned}$$

Compressive stress = 12.29 MPa $\ll$ f_y (230 MPa)

Therefore the failure of the pipe is not due to wall crushing.

6.3.4 Calculation of Buckling Stress

As described in section 6.3.1, the determination of buckling stress of a flexible buried culvert is very difficult because the buckling pressure depends on various factors mainly the soil reaction and others such as buckling mode number (n), the flexibility of the pipe, Poisson's ratio of the soil and pipe, friction at the interface etc. For example, by compacting the soil, the modulus of soil reaction can be increased several times and the buckling pressure will increase roughly by the same order.

In the following calculation, for the purpose of comparison of all buckling theories, the value for K_3 as $0.71E_s$ will be used (Table 6.1). Usually very flexible conduits with good soil support will tend to buckle in high modes, whereas those with weak soil support will buckle in low modes. However, due to geometric imperfections and the

presence of residual stresses, even in the case of buckling in high modes, crimping of the conduit wall may occur at one or two locations. From the experimental results it was found that all pipes failed in the primary buckling mode ($n = 2$). So the buckling stresses were calculated for the primary mode ($n = 2$).

Table 6.3 presents the buckling stress for the pipe used in the sand box experiment, calculated by several buckling theories discussed in Chapter 2.

The following values were used in estimating the buckling stresses.

$$E = 2 \times 10^8 \text{ kPa}$$

$$\nu = 0.3$$

$$E_s = 17,650 \text{ kPa}$$

$$\nu_s = 0.33$$

$$K_s = 0.71E_s = 12,530 \text{ kPa} \quad f_y = 2.3 \times 10^5 \text{ kPa}$$

$$n = 2$$

The dimensions of the pipe were already given in section 6.3.2.

Table 6.3 Buckling pressures predicted by various theories

Authors	Buckling Pressure (kPa)	Remarks
Cheney (1963) (eq 2.6)	4184	Circular ring under plane stress condition, supported by elastic springs, constant direction and magnitude external pressure
Meyerhof and Baikie (1963) (eq 2.7)	64411	Elastically supported cylinders, using plate buckling theory
Forrestal and Herrmann (1965) (eq 2.9a)	4216	Constant directional pressure, elastic support
(eq 2.9b)	4402	Fully bonded No friction
Luscher (1966) (eq 2.9b)	4184 402	Radial elastic support Lower bound solution
Chelapati and Allgood (1972) (eq 2.12a)	4184	Continuous radial elastic support around the entire circumference
(eq 2.12b)	357	Soil support only on outward deflecting lobes of circumferential buckling waves
(eq 2.12c)	255	-do- Lower bound solution
Cheney (1976) (eq 2.13a)	4249	Supported by elastic continuum
(eq 2.13b)	1197	Minimized w.r.t. n
(eq 2.13c)	300	To match exp. results
Meyerhof (1979) (eq 2.14)	320	Lower limiting bound sol.
AASHTO (1977) (eq 3.6e)	56450	Buckling pressure of the lower zone of corrugated steel pipes (passive state)
OHBDC (1983) (eq 3.5d)	70570	-do-

6.4 Calculation of Deflection and Stress in the Pipe by the Finite Element Method

6.4.1 Sensitivity Analysis of Finite Element Mesh

The finite element method was used to estimate the deflections and the stresses in and around the pipe, produced by eccentric surface loadings. The accuracy of the results of the finite element analysis depends on several factors such as suitable values for the material properties, boundary conditions and mainly discretization of the elements. For this particular study, the boundary conditions were not changed as described in Chapter 3. Selection of suitable values for the soil parameters is very important because the density (loose or dense compaction) influences severely the deformation and the stress. The soil parameters obtained from the experiments ($E_s = 17.65 \text{ MPa}$, $\nu_s = 0.33$, $\rho_s = 1760 \text{ kg/m}^3$) were reported earlier. The parameters for steel are taken from OHBDC (1979) which are $E = 2 \times 10^5 \text{ MPa}$, $\nu = 0.3$, $\rho = 7850 \text{ kg/m}^3$. The influence of the discretization of elements on the results will be discussed in this section.

Generally, the more elements one uses for a given bounded region, the greater the sensitivity of the load - deformation response. This is true up to a certain degree. Also, a larger number of elements increases the computing time and expenditure. Therefore, one usually performs a sensitivity study in order to find a suitable mesh configuration. For the sensitivity analysis, the loading position, the

cover depth and the magnitude of load were kept constant ($H/D = 1$ and $e/B = 0$) and the number of divisions and the ratio of divisions were changed. Several mesh configurations were considered consisting of 66, 100, 144, 152, 160, and 176 elements. The 160 element mesh as shown in Figure 6.37 was selected, because the values of stresses and the deflections were converging in this mesh. The ratio of division did not show much variation in the results and therefore a equal division in the first direction and a 1:2:4 ratio in the second direction were selected.

6.4.2 Method of Analysis

After selecting the 160 element mesh with 560 nodes from the sensitivity analysis, different cover depths were simulated by changing the coordinates. The number and ratio of divisions were selected in such a fashion that the load application nodal points were at multiples of 75 mm (because e/B is in steps of 0.5 and $B = 150$ mm).

To simulate a rough footing, the nodal points below the footing were constrained and could move only vertically. The applied load was distributed to the 5 nodes at the footing - soil interface in the ratios of 0.2457, 0.1849, 0.1388, 0.1849, and 0.2457 respectively as mentioned in Chapter 3, in order to have equal displacements as required by the rigidity of the footing.

Figure 6.37 Finite Element Mesh Discretization

Several finite element analyses were performed to verify the experimental results and to check the failure loads obtained by closed form solutions. For the analysis, the cover depths were changed from say 37.5, 75, 150, and 225 mm and the load was applied eccentrically at 0, 75, 150, and 225 mm ($H/D = 0.25, 0.5, 1.0, \text{ and } 2$; $e/B = 0, 0.5, 1.0, \text{ and } 1.5$). Further the critical failure loads obtained from the experiments were used as the applied loads in the finite element analysis.

6.5 Analysis of Experimental Results

6.5.1 Steel Rod Model

(a) Failure Patterns of Soil - Pipe System

An examination of the failure sequence in the steel rod tests revealed that the pipes started to deform at the point of intersection of the perimeter of the pipe and the line connecting the centre of pipe and footing. This implies that in the case of eccentric loading, the maximum deflection will not occur at the crown but at a point closer to the footing. After the initial pipe failure, further loading caused the soil mass located between the footing and the pipe to move as a block. In the case of the eccentric loading, the failure zone was similar to a wedge failure as shown in Figure 6.5. As expected, for central loading conditions symmetric buckling occurred and eccentric loading produced asymmetric buckling. In all experiments, hardly any

deflection below the springing line could be observed and all the deflection, either pushing inward or kicking outward, occurred above the upper half of the pipe. This may be due that the upper half of the pipe-soil system is more flexible (soil dilatancy) whereas the soil surrounding the lower half is being compressed and therefore more rigid.

(b) Instrumented Tests in Steel Rod Model

The results of the instrumented tests, performed in the steel rod model are shown in Figures 6.6 to 6.21 in the form of load - deflection curves. From the graphs, it can be observed that, for the symmetric loading cases ($H/D = 1$ and 2 , $e/B = 0$), there is not much difference in the results (same failure loads and equal deflections), between rigid and flexible beddings. When the cover depth was doubled, the failure load was also more or less doubled for both bedding conditions. In all the cases the deflection at the crown was the highest, then at the springing and the least radial deflection occurred at the shoulder. For $e/B = 0.5$ cases, the deflections at the shoulder were the highest in all four cases. The bedding condition influenced the failure loads and deflections. In the rigid bedding cases the pipes failed with less deflection and larger deflections were noticed in the flexible bedding tests. This difference can be attributed to the larger deformability of the flexible bedding. The failure loads for flexible were higher than that for rigid bedding cases. The larger deformation for pipes on flexible bedding allowed the formation of positive arching and thus a

decrease in load exerted on the pipe. The same phenomena were observed in the next four tests with $e/B = 1$. The failure loads were approximately doubled when the cover depth was doubled for both bedding conditions. However, for $H/D = 2$ cases, the footing had to undergo large movements, compared to the deflections of the pipe. This implies that the soil must have failed first before the pipe failed, because the soil between footing and pipe moved as a block. This was also borne out by the larger deflections observed at the springing and at the shoulders. A reversal in the failure loads was noticed when the eccentricity was increased to $1.5B$, that is, the pipes on the flexible bedding failed before the pipes on the rigid bedding. This may be due to lateral compression of the flexible bedding which in turn would have disturbed the whole soil-pipe system. In the case of $H/D = 2$, $e/B = 1.5$, as in the previous case ($H/D = 2$, $e/B = 1$), the soil must have failed first. Therefore the critical loads for the pipe must be higher than that found for the $H/D = 2$, $e/B = 1$ and $e/B = 1.5$ cases.

The summary of the above results are depicted in the Figures 6.38 and 6.39 in a non - dimensional form (P_e/P_o and P_e/P_{air} versus e/B where P_e is the eccentric critical load, P_o is the concentric critical load for the same cover depth and P_{air} is the the failure load of pipe without soil support). The figures show that for shallow cover depth eccentric critical loadings are smaller than concentric critical loadings. But for the deeper burial depths, the eccentricity does not influence the critical load and as the eccentricity increases, the critical load also increases. The pipes on flexible bedding show a

NORMALIZED CRITICAL LOAD P_e/P_o VS. e/B

(STEEL ROD MODEL)

Figure 6.38

NORMALIZED CRITICAL LOAD P_e/P_{air} VS. e/B

(STEEL ROD MODEL)

Figure 6.39

higher load carrying because of arching action of the soil.

6.5.2 Sand Box

(a) Failure Patterns of Soil and Pipe

Figures 6.40 to 6.42 show the deformation of soil and pipe after the failure of the pipes. The horizontal deformation of the color lines infers the smooth transference of load from the footing to the pipe. The soil at the edges of the footing did not undergo any punching failure and no slip lines were noticed between the pipe and soil. This was observed also in the steel rod model tests where greater pipe deflections occurred at points which are closer to the footing,

(b) Load - Deformation Behaviour

The loads and deformations obtained in the sand box tests were plotted by computer and shown in Figures 6.22 to 6.36. Basically all curves have the same appearance, initially at low applied loads they are linear and eventually become horizontal.

Figure 6.40 Failure Pattern of Soil-Pipe System, $H/D = 1$, $e/B = 0.5$

Figure 6.41 Failure Pattern of Soil-Pipe System $H/D = 1$, $e/B = 0.5$

Figure 6.42 Failure Pattern of Soil-Pipe System $H/D = 1$, $e/B = 1.5$

In general, a concentric loading produced symmetric deflections and the largest deflection was measured at the crown, as expected. For the shallower cover depths ($H/D = 0.25$ and 0.5) the deflections at the crown were higher than the vertical movement of the footing. This shows that the pipe is more flexible when it is buried at shallow depths, because of less soil support. In decreasing order of magnitude the maximum deflection occurred at the crown then the shoulders followed by the springing. The pipes did not experience much deflections below the springing line. This sequence of deflection was similar as in the steel rod model tests.

For the eccentric loading cases ($e/B = 0.5$ and 1.0), the radial deflections at the shoulders were the highest and generally larger than the footing movement for shallow depth tests ($H/D = 0.25$ and 0.5). The behaviour of the other tests were similar to the steel rod tests and therefore are not discussed again.

The failure loads obtained from the experiments for various cover depths and eccentricity conditions are normalised using ratios of eccentric loads P_e to concentric loads P_o and the failure load without soil support P_{air} . The P_e/P_o versus e/B plots are shown in Figures 6.43 and 6.44 and reveal some interesting results. When the cover depth decreases, the critical load for an eccentric position $e/B = 0.5$ decreases. Beyond this eccentricity the normalized load increases with decrease in cover depth. For a cover depth of $H/D=1$ and less the failure load at $e/B = .5$ is lower than that for the concentric load.

Figure 6.43

NORMALIZED CRITICAL LOAD P_e/P_{air} VS. e/B

(SAND BOX MODEL)

Figure 6.44

But the failure load is steadily increasing with the eccentricity for cover depths of $H/D = 2$. From these results it can be deduced that the effect of eccentricity is negligible for deep buried culverts (say $H > D$) and the eccentric loading should be taken into account for the analysis of pipes buried at shallow depths. Further the influence of eccentric loading becomes more pronounced as the cover depth is decreased. The P_e/P_{air} versus e/B graph also depicts the same phenomenon, in addition it shows the increment of failure with the increase of cover depth.

Therefore, the effect of eccentricity should be included in the design of large span culverts where the relative cover depth is small or for ordinary (drainage) culverts buried at shallow depths. This effect can be associated with the usual symmetric loading design by increasing the applied load by an appropriate factor. To get the exact factor more experiments would have to be done with very small H/D and e/B ratios. However, from the experiments on hand, at $e/B = 0.5$, a tentative load factor C_e is proposed and given (in graphic form) in Figure 6.45.

Figure 6.45

(c) Effect of Reinforcing the Soil over the Culvert.

The experiments with reinforcing steel strips above the pipe showed a clear increase in load carrying capacity of the pipe. In fact the reinforcement distributes the load over a larger area and therefore the effect of eccentric loading has been diminished. However, for the case of $H/D = 0.25$, $e/B = 0.5$ the recorded radial deflections at the shoulder were the largest and were larger than the vertical movement of the footing. This was true for the both cases of reinforcing, that is, with one and two reinforcing strips. In all other cases, including $H/D = 1$, $e/B = 0.5$, the crown deflection was the highest. The Load - Deformation curves for the experiments performed with reinforcement are shown in Figures 6.46 to 6.50.

As discussed earlier, the most critical situation occurred when $H/D = 0.25$ and $e/B = 0.5$. For this case 40 mm wide reinforcing strips were introduced at half-depth above the pipe. The load carrying capacity of the pipe was increased by 21 percent. For the two reinforcing strips placed side by side at half-depth, the load carrying capacity was increased by 54 percent compared to the unreinforced case. This shows the effectiveness of the reinforcement at shallow depths. However, for $H/D = 0.25$, $e/B = 0$ case one reinforcement strip increased the load by 14 percent and two reinforcement strips by 30 percent. When the cover depth increased the effect of reinforcement was less pronounced. For example in the experiment for $H/D = 1$, and $e/B = 0.5$, one reinforcement strip increased the failure load by 10 percent only.

Figure 6.46

Figure 6.47

LOAD. VS. DEFLECTION $H/D = .25$, $E/B = .5$, $1-R/R$

↑ "+" inward radial deflection "-" outward radial deflection

Figure 6.48

Figure 6.49

LOAD. VS. DEFLECTION $H/D=1, E/B=.5, 1-R/F$

Figure 6.50

Therefore, placing some reinforcement, such as wire mesh, metal strips, geotextile or casting concrete slabs above the crown of the culvert buried at shallow depth will increase the load carrying capacity considerably. This technique is permitted in the OHBDC(1983).

6.6 Comparison and Discussion of Results

6.6.1 Deflection

The maximum deflections calculated by the Iowa formula, as shown in Table 6.2, are in the order of 20 to 25 mm for the failure loads found from the experiments for various cover depths. According to Spangler (1941) if the deflection exceeds 20 percent of the pipe diameter, that is, 30 mm, the pipe is in a state of incipient collapse. But in all the experiments performed in the sand tank (43 tests) during pipe collapse the maximum deflections were in the order of 10mm or about 7 percent of the pipe diameter.

Therefore, Spangler's deflection control method to avoid buckling failure is not satisfactory for very flexible pipes buried at shallow depths.

The deflections obtained from the linear elastic finite element analysis are shown against the experimental results in the same plots (Figure 6.22 to 6.36). As discussed earlier the amount of deflection is highly dependent on the input parameters, mainly the modulus of

elasticity of the soil. Since the finite element analysis was carried out by assuming the soil as a linear elastic material, the load - deformation curves are linear and closer to the initial linear portion of the experimental curves. However, despite the large difference in the failure region, the general trend (large deflection at crown, shoulder etc) of both analytical and experimental curves are similar.

6.6.2 Stress

From the strain gauge measurements the hoop stress at failure was found as 1.72 MPa and 3.14 MPa for symmetric loading and $e/B = 0.5$ with one diameter cover depth. According to the ring compression theory the compressive stress in the wall of the conduit was found to be 7.89 MPa which is much smaller than the yield stress of 230 MPa. The OHBDC predicts a higher compressive stress in the wall of the conduit (12.29 MPa).

A very wide range of values were obtained from the various buckling theories with different assumptions. The values varied from a small value of 0.25 MPa to a large value of 70.57 MPa. In fact, both the codes (AASHTO and OHBDC) predict the highest buckling stress values, because the code expressions were derived from the envelope of all theoretical predictions. The largest predictions comes from Spangler's deflection control theory and Meyerhof and Baikie's (1963) theory. Reasonable average - buckling stresses are in the range of 1.2 MPa to 4.4 MPa which are very close to the experimental observations.

The maximum stresses estimated by the finite element analysis for various cases are tabulated in Table 6.4. They range from 11 MPa to 25.5 MPa. As mentioned previously the results of the FEM analysis depends on the input parameters and on the assumptions made. Obviously, the soil is not a linear elastic material and the linear analysis is not valid at large deflections. Generally, deflections of the order of the thickness of the pipe can be analysed using a linear - elastic model. In the case on the hand the maximum deflection was in the order of 20 times the pipe thickness. Furthermore, it is interesting to note that the influence of eccentricity at maximum stress levels. From Table 6.4, the values are more or less the same and equal to 11.5 MPa for $e/B = 0$, regardless of the cover depths. These values increase with increasing eccentric loading to about 16.5 and 24 MPa for $e/B = 0.5$ and $e/B = 1$, respectively for all cover depths and decrease with further eccentricity ($e/B = 1.5$) to about 15 MPa. These results show that the pipe attains the largest stress when the eccentricity is equal to the footing width which is similar to the results obtained from the steel rod model tests.

3 3
OF / DE

Table 6.4 Maximum stress estimated by FEM

$\begin{array}{c} e/B \\ \backslash \\ H/D \end{array}$	0.0	0.5	1.0	1.5
0.25	11.47	18.30	25.5	12.6
0.50	11.54	13.77	25.5	13.2
1.00	11.92	16.58	23.1	17.9
2.00	11.00	18.40	22.8	-

As a whole, the theories derived by Cheney (1963), Forrestal and Hermann (1965), Luscher (1966), Chalapati and Allgood (1972) with the assumption of elastically supported pipes, were predicting the buckling stress for symmetrical loading condition close to the experimental values and the finite element analysis. The predictions by the design codes (AASHTO and OHBDC) and Meyernof and Baikie (1963) were much higher - about 20 to 30 times of the observed values. On the other hand, the lower bound solutions given by Luscher (1966), Chalapati and Allgood (1972), Cheney (1976), and Meyerhof (1979) were underestimating the buckling stresses by as much as a factor of 10.

CHAPTER 7

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

7.1 Summary

An extensive literature review pertaining to the buckling behaviour of buried culverts showed that the asymmetrical buckling of conduits have not been treated so far. Asymmetrical buckling is a major consideration in the design of large culverts with shallow soil covers because heavy concentrated surface loads might induce asymmetric buckling. Two 2-dimensional physical models were utilized, with different bedding conditions and varying cover depths to determine the asymmetrical failure pattern of the pipes and to find out the influence of eccentric loading on the flexible pipes. In addition, the investigation also included the linear elastic finite element analysis which can handle asymmetrical loading. In both analyses, the footing was kept at four different locations at the surface of the soil and the deflections and loads (or stresses) at the pipe were monitored.

An analysis of the results and comparisons with existing theories and code recommendations and finite element analysis are presented and discussed.

7.2 Conclusions

Based on the experimental and the finite element analysis results obtained from this study, the following conclusions can be drawn:

1. The two - dimensional physical models used in this research were capable of depicting the failure patterns of pipe - soil system.

2. The eccentric critical loadings were smaller than the corresponding concentric critical loadings in the cases where the cover depth is smaller than the pipe diameter. This must be because of the asymmetrical buckling of the pipe. Therefore, the effect of eccentric loading should be taken into account in the design of flexible pipes buried at relatively shallow depths.

3. The pipes founded on flexible bedding experienced larger deflections (at all points) compared to pipes on rigid bedding. The difference was attributed to the deformability of the flexible bedding. Further, the pipes founded on flexible bedding carried more load than those on rigid bedding. This is because of larger movement and hence larger positive arching over the pipe. Therefore, the load carrying capacity of pipes could be increased by providing compressible bedding and therewith making use of the arching effect of the soil.

4. In the sand box tests, most of the pipes reached a maximum deflection of 10 mm which is only 7 percent of the initial pipe diameter. Therefore, Spangler's 20 percent deflection control criteria to avoid buckling failure is not suitable for flexible pipes buried at

shallow depths. They would fail in buckling before they reach instability due to excessive deflection.

5. The theories proposed by Cheney, Forrestal and Herrmann, Luscher, Chelapati and Allgood with the assumption of elastically supported pipes were predicting the buckling stress (in symmetrical loading condition) closer to the experimental values and the finite element analysis. The predictions by the design codes (AASHTO and OHBDC) and Meyerhof and Baikia were much higher - about 20-30 times of the above values. On the other hand, the lower bound solutions given by Luscher, Chelapati and Allgood, Cheney, and Meyerhof underestimating the buckling stresses by as much as a factor of 10. Therefore, from this study, the theories proposed with the elastically supported assumption by Cheney, Forrestal and Herrmann, Luscher, and Chelapati and Allgood are recommended to estimate the buckling stresses of flexible buried culverts.

6. Placing a reinforcing element in the soil over the pipe, distributes the concentrated load over the pipe and reduces the deflection. Using reinforcing, the detrimental effect of eccentric loading is also reduced. Therefore, to minimize asymmetric buckling, and to increase the load carrying capacity of the pipe, reinforcing the soil is a beneficial method. The reinforcing element can be a wire mesh, metal strip, geotextile or even a concrete slab to relieve and distribute the concentrated loads.

7. The results obtained from the finite element analysis also show higher stresses for eccentric loading compared to corresponding

concentric loading. For the case where $e/B = 1$ the stresses were 100 percent higher. This was similar to the results obtained in the steel rod model. All these studies show that asymmetrical buckling failure occurs before symmetrical buckling failure. Therefore, an additional load factor should be included in the theoretical predictions for asymmetrical failure behaviour as suggested in Figure 6.45.

7.3 Recommendations for Further Work

This work has identified the importance of asymmetrical buckling failure due to eccentric loadings. To date, except for a few full scale tests performed by the Ontario Ministry of Transportation and Communications with eccentric loading no controlled experiments have been performed to study this effect. Therefore, further research is necessary (analytical and experimental) in the following areas:

1. The effect of interface condition with eccentric loading.
2. The influence of the shape of conduit (elliptical, arch etc.) on the asymmetrical buckling behaviour.
3. The effect of eccentric loading was studied for single static load only. It can be extended to multiple axle loads and moving loads with dynamic (impact) effects.
4. The finite element investigation should be extended to analyse the symmetrical and asymmetrical buckling (by solving eigen value -

instability problem) with non linear properties of materials as well as geometries.

5. The effects of special features associated with the construction of buried structures such as earth reinforcing, thrust beams and tie rods should be studied.

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APPENDIX A

```

C*****
C
C   FINITE ELEMENT PROGRAM TO CALCULATE DEFLECTION AND STRESS
C*****
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/TWQ/NDFPN,NDFPE,NNPE,NSTRE,NDIME
      COMMON/TP/NEL,NNODE,NBNODE,NODCL,NEQ,NEQ1,NTYPE,IGLOD
      COMMON/GAS/POSGP(3),WEIGP(3),NGAUS
      COMMON/BLK1/NODE(300,8),ID(2,750),NBN(100),NLOAD(30),
1COORD(750,2),PCON(30,2),MATNO(300),PROPS(10,5)
      COMMON/BK9/ASM(90000),V(1500)
      COMMON/TX/LM(16)
      COMMON/BLK2/STIF(136),ELOAD(300,16),DEL(16)
      COMMON/BLK4/MHT(1500),MAXA(1500)
      COMMON/XX1/SHAPE(8),DERIV(2,8),XJACM(2,2),XJACI(2,2),
1CARTD(2,8),ELCOD(2,8),BMTX(3,16),DMATX(3,3),
2 DBMAT(3,16),SMATX(3,16,4),GPCOD(2,4)
      READ(5,10) MSHON,NODSTS
      NGAUS=2
      NDIME=2
      READ(5,15) ICORE,IEQ
15  FORMAT(8I10)
      CALL GAUSSQ
      READ(5,10)NPROB
      DO 5678 IPROB=1,NPROB
      CALL INBLOC
      CALL GENER
      IF(MSHON.EQ.0) GO TO 5678
      READ (5,10)NBNODE,NDFPN,NODCL,NMATS,NPROP,NTYPE,IGLOD
      WRITE(6,10)NBNODE,NDFPN,NODCL,NMATS,NPROP,NTYPE,IGLOD
      IF(NODCL.EQ.0)GO TO 25
      READ(5,10)(NLOAD(I),I=1,NODCL)
      WRITE(6,10)(NLOAD(I),I=1,NODCL)
      DO 1050 I=1,NODCL
      READ(5,5)(PCON(I,K),K=1,NDFPN)
1050 WRITE(6,6)(PCON(I,K),K=1,NDFPN)
25  CONTINUE
      WRITE(6,10)(MATNO(I),I=1,NEL)
      DO 991 IMAT=1,NMATS
      READ(5,5)(PROPS(IMAT,J),J=1,5)
991 WRITE(6,6)(PROPS(IMAT,J),J=1,5)
5   FORMAT(8F10.0)
6   FORMAT(2X,5E12.5)
10  FORMAT(20I4)
      IF(IGLOD.EQ.0) GO TO 3912
      CALL GRLOD

```

```

3912 CONTINUE
      DO 700 I=1,NDFPN
      DO 700 J=1,NNODE
700   ID(I,J)=0
      READ(5,10)(NBN(I),I=1,NBNODE)
      WRITE(6,10)(NBN(I),I=1,NBNODE)
      DO 651 I=1,NBNODE
      N=NBN(I)
      READ(5,10)(ID(J,N),J=1,NDFPN)
      WRITE(6,10)(ID(J,N),J=1,NDFPN)
651   CONTINUE
      NEQ=0.
      DO 703 N=1,NNODE
      DO 703 I=1,NDFPN
      IF(ID(I,N))704,705,704
705   NEQ=NEQ+1
      ID(I,N)=NEQ
      GO TO 703
704   ID(I,N)=0
703   CONTINUE
      IF(NEQ.LE.IEQ) GO TO 1121
      WRITE(6,1122)NEQ
1122  FORMAT(2X,22HEQUATION EXCEEDED,NEQ=,I10)
      STOP
1121  CONTINUE
      NEQ1=NEQ+1
      DO 897 I=1,NEQ
897   MHT(I)=0
      NDFPE=NNPE*NDFPN
      NSTRE=3
      DO 711 IEL=1,NEL
      CALL NCODE(IEI)
      CALL COLHT
711   CONTINUE
      WRITE(6,11)NEQ,NEQ1
      CALL ADDRES(MK,NWK)
      WRITE(6,11)NWK
      IF(NWK.LE.ICORE) GO TO 1125
      WRITE(6,1126)NWK
1126  FORMAT(2X,25HCORE MEMORY EXCEEDED,NWK=,I10)
      STOP
1125  CONTINUE
11   FORMAT(8I10)
      DO 742 I=1,NWK
742   ASM(I)=0.
      DO 945 I=1,NEQ
945   V(I)=0.
      DO 1000 IEL=1,NEL
      CALL NCODE(IEI)
      CALL ELSTIF(IEI)
      CALL ADDBAN
1000  CONTINUE
      IF(NODCL.EQ.0)GO TO 1001

```

```

DO 1070 I=1,NODCL
N=NLOAD(I)
DO 1070 J=1,NDFPN
NA=ID(J,N)
IF(NA.EQ.0)GO TO 1070
V(NA)=V(NA)+PCON(I,J)
1070 CONTINUE
1001 IF(IGLOD.EQ.0) GO TO 3913
DO 3914 IEL=1,NEL
KK=1
DO 3915 INOD=1,NNPE
N=NODE(IEI,INOD)
DO 3915 J=1,NDFPN
NA=ID(J,N)
IF (NA.EQ.0) GO TO 3915
V(NA)=V(NA)+ELOAD(IEI,KK)
3915 KK=KK+1
3914 CONTINUE
3913 CONTINUE
CALL COLSOL(NWK)
C
C*** OUTPUT THE DEFORMATIONS
C
WRITE (6,900)
WRITE (6,905)
900 FORMAT(//,17X,25HDEFORMATIONS AT THE NODES,/)
905 FORMAT(12X,8HNODE NO.,7X,14HX-DISPLACEMENT,7X,14HY-DISPLACEMENT,
1//)
DO 2700 I=1,NNODE
DO 2710 J=1,NDFPN
N=ID(J,I)
DEL(J)=0.
IF(N.NE.0)DEL(J)=V(N)
2710 CONTINUE
WRITE(6,2097)I,(DEL(K),K=1,2)
2700 CONTINUE
2097 FORMAT(15X,I5,2(6X,F15.2))
REWIND 3
IF(NODSTS.EQ.0) GO TO 5678
CALL STSNOD
5678 CONTINUE
STOP
END
FUNCTION MINO(I,J)
IMPLICIT REAL*8(A-H,O-Z)
IF(I.GT.J)MINO=J
IF(J.GE.I)MINO=I
RETURN
END
SUBROUTINE COLSOL(NWK)
IMPLICIT REAL*8(A-H,O-Z)
COMMON/BLK4/MHT(1500),MAXA(1500)
COMMON/TP/NEL,NNODE,NBNOOE,NODCL,NEQ,NEQ1,NTYPE,IGLOD

```

```

COMMON/BK9/ASM(90000),V(1500)
NN=NEQ
DO 140 N=1,NN
KN=MAXA(N)
KL=KN+1
KU=MAXA(N+1)-1
KH=KU-KL
IF(KH)110,90,50
50 K=N-KH
IC=0
KLT=KU
DO 80 J=1,KH
IC=IC+1
KLT=KLT-1
KI=MAXA(K)
ND=MAXA(K+1)-KI-1
IF(ND)80,80,60
60 KK=MINO(IC,ND)
C=0.
DO 70 L=1,KK
KIL=KI+L
KLTL=KLT+L
70 C=C+ASM(KIL)*ASM(KLTL)
ASM(KLT)=ASM(KLT)-C
80 K=K+1
90 K=N
B=0.
DO 100 KK=KL,KU
K=K-1
KI=MAXA(K)
C=ASM(KK)/ASM(KI)
B=B+C*ASM(KK)
100 ASM(KK)=C
ASM(KN)=ASM(KN)-B
110 IF(ASM(KN))120,120,140
120 WRITE(6,2000)N,ASM(KN)
STOP
140 CONTINUE
DO 180 N=1,NN
KL=MAXA(N)+1
KU=MAXA(N+1)-1
IF(KU-KL)180,160,160
160 K=N
C=0.
DO 170 KK=KL,KU
K=K-1
170 C=C+ASM(KK)*V(K)
V(N)=V(N)-C
180 CONTINUE
DO 200 N=1,NN
K=MAXA(N)
200 W=V(N)/ASM(K)
IF(NN.EQ.1)RETURN

```

```

      N=NN
      DO 230 L=2,NN
      KL=MAXA(N)+1
      KU=MAXA(N+1)-1
      IF(KU-KL)230,210,210
210    K=N
      DO 220 KK=KL,KU
      K=K-1
220    V(K)=V(K)-ASM(KK)*V(N)
2000  FORMAT(//,22H STIF NOT POS.DEFINITE,/5H EQU=,I4,/7H PIVOT=,E15.7)
230    N=N-1
      RETURN
      END
      SUBROUTINE NCODE(IEL)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/TP/NEL,NNODE,NBNODE,NODCL,NEQ,NEQ1,NTYPE,IGLOD
      COMMON/TWO/NDFPN,NDFPE,NNPE,NSTRE,NDIME
      COMMON/BLK1/NODE(300,8),ID(2,750),NBN(100),NLOAD(30),
1COORD(750,2),PCON(30,2),MATNO(300),PROPS(10,5)
      COMMON/TX/LM(16)
      II=1
      DO 10 I=1,NNPE
      J=NODE(IEL,I)
      DO 10 L=1,NDFPN
      LM(II)=ID(L,J)
10    II=II+1
      RETURN
      END
      SUBROUTINE COLHT
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/TX/LM(16)
      COMMON/BLK4/MHT(1500),MAXA(1500)
      COMMON/TP/NEL,NNODE,NBNODE,NODCL,NEQ,NEQ1,NTYPE,IGLOD
      COMMON/BLK1/NODE(300,8),ID(2,750),NBN(100),NLOAD(30),
1COORD(750,2),PCON(30,2),MATNO(300),PROPS(10,5)
      COMMON/TWO/NDFPN,NDFPE,NNPE,NSTRE,NDIME
      ND=NDFPE
      LS=10000
      DO 100 I=1,ND
      IF(LM(I))110,100,110
110    IF(LM(I)-LS)120,100,100
120    LS=LM(I)
100    CONTINUE
      DO 200 I=1,ND
      II=LM(I)
      IF(II.EQ.0) GO TO 200
      ME=II-LS
      IF(ME.GT.MHT(II))MHT(II)=ME
200    CONTINUE
      RETURN
      END
      SUBROUTINE ADDRES(MK,NWK)
      IMPLICIT REAL*8(A-H,O-Z)

```

```

COMMON/TX/LM(16)
COMMON/BLK4/MHT(1500),MAXA(1500)
COMMON/TP/NEL,NNODE,NBNODE,NODCL,NEQ,NEQ1,NTYPE,IGLOD
COMMON/BLK1/NODE(300,8),ID(2,750),NBN(100),NLOAD(30),
1COORD(750,2),PCON(30,2),MATNO(300),PROPS(10,5)
COMMON/TWO/NDFPN,NDFPE,NNPE,NSTRE,NDIME
NN=NEQ+1
DO 20 I=1,NN
20  MAXA(I)=0
    MAXA(1)=1
    MAXA(2)=2
    MK=0
    IF(NEQ.EQ.1)GO TO100
    DO 10 I=2,NEQ
    IF(MHT(I).GT.MK)MK=MHT(I)
    10  MAXA(I+1)=MAXA(I)+MHT(I)+1
    100  MK=MK+1
        NWK=MAXA(NEQ+1)-MAXA(1)
        RETURN
    END
SUBROUTINE ADDBAN.
IMPLICIT REAL*8(A-H,O-Z)
COMMON/BLK4/MHT(1500),MAXA(1500)
COMMON/TX/LM(16)
COMMON/BK9/ASM(30000),V(1500)
COMMON/TP/NEL,NNODE,NBNODE,NODCL,NEQ,NEQ1,NTYPE,IGLOD
COMMON/BLK1/NODE(300,8),ID(2,750),NBN(100),NLOAD(30),
1COORD(750,2),PCON(30,2),MATNO(300),PROPS(10,5)
COMMON/TWO/NDFPN,NDFPE,NNPE,NSTRE,NDIME
COMMON/BLK2/S(136),ELOAD(300,16),DEL(16)
ND=NDFPE
NDI=0
DO 200 I=1,ND
II=LM(I)
IF(II)200,200,100
100  MI=MAXA(II)
    KS=I
    DO 220 J=1,ND
    JJ=LM(J)
    IF(JJ)220,220,110
    110  IJ=II-JJ
        IF(IJ)220,210,210
    210  KK=MI+IJ
        KSS=KS
        IF(J,GE.I)KSS=J+NDI
        ASM(KK)=ASM(KK)+S(KSS)
    220  KS=KS+ND-J
    200  NDI=NDI+ND-I
        RETURN
    END
SUBROUTINE SFR2(S,T)
IMPLICIT REAL*8(A-H,O-Z)

```

C

```

C**** CALCULATES SHAPE FUNCTIONS AND THEIR
C DERIVATIVES FOR 2D ELEMENTS
C
COMMON/GAS/POSGP(3),WEIGP(3),NGAUS
COMMON/TWO/NDOPN,NEVAB,NNODE,NSTRE,NDIME
COMMON/TP/NELEM,NPOIN,NBNODE,NODCL,NEQ,NEQ1,NTYPE,IGLOD
COMMON/BLK1/LNODS(300,8),ID(2,750),NBN(100),NLOAD(30),
1COORD(750,2),PCON(30,2),MATNO(300),PROPS(10,5)
COMMON/BLK2/STIF(136),ELOAD(300,16),DEL(16)
COMMON/XX1/SHAPE(8),DERIV(2,8),XJACM(2,2),XJACI(2,2),
1CARD(2,8),ELCOD(2,8),BMATX(3,16),DMATX(3,3),
2 DBMAT(3,16),SMATX(3,16,4),GPCOD(2,4)
S2=S*2.0
T2=T*2.0
SS=S*S
TT=T*T
ST=S*T
SST=S*S*T
STT=S*T*T
ST2=S*T*2.0

C
C**** SHAPE FUNCTIONS
C
SHAPE(1)=(-1.0+ST+SS+TT-SST-STT)/4.0
SHAPE(2)=(1.0-T-SS+SST)/2.0
SHAPE(3)=(-1.0-ST+SS+TT-SST+STT)/4.0
SHAPE(4)=(1.0+S-TT-STT)/2.0
SHAPE(5)=(-1.0+ST+SS+TT+SST+STT)/4.0
SHAPE(6)=(1.0+T-SS-SST)/2.0
SHAPE(7)=(-1.0-ST+SS+TT+SST-STT)/4.0
SHAPE(8)=(1.0-S-TT+STT)/2.0

C
C**** SHAPE FUNCTIONS DERIVATIVES
C
DERIV(1,1)=(T+S2-ST2-TT)/4.0
DERIV(1,2)=-S+ST
DERIV(1,3)=(-T+S2-ST2+TT)/4.0
DERIV(1,4)=(1.0-TT)/2.0
DERIV(1,5)=(T+S2+ST2+TT)/4.0
DERIV(1,6)=-S-ST
DERIV(1,7)=(-T+S2+ST2-TT)/4.0
DERIV(1,8)=(-1.0+TT)/2.0
DERIV(2,1)=(S+T2-SS-ST2)/4.0
DERIV(2,2)=(-1.0+SS)/2.0
DERIV(2,3)=(-S+T2-SS+ST2)/4.0
DERIV(2,4)=-T-ST
DERIV(2,5)=(S+T2+SS+ST2)/4.0
DERIV(2,6)=(1.0-SS)/2.0
DERIV(2,7)=(-S+T2+SS-ST2)/4.0
DERIV(2,8)=-T+ST
RETURN
END
SUBROUTINE JACOB2(IELEM,DJACB,KGASP)

```

```

      IMPLICIT REAL*8(A-H,O-Z)
C
C**** CALCULATES COORDINATES OF GAUSS POINTS
C      AND THE JACOBIAN MATRIX AND ITS DETERMINANT
C      AND THE INVERSE FOR 2D ELEMENTS
C
      COMMON/GAS/POSGP(3),WEIGP(3),NGAUS
      COMMON/TWO/NDOFN,NEVAB,NNODE,NSTRE,NDIME
      COMMON/TP/NELEM,NPOIN,NBNODE,NODCL,NEQ,NEQ1,NTYPE,IGLOD
      COMMON/BLK1/LNODS(300,8),ID(2,750),NBN(100),NLOAD(30),
1COORD(750,2),PCON(30,2),MATNO(300),PROPS(10,5)
      COMMON/BLK2/S(136),ELOAD(300,16),DEL(16)
      COMMON/XX1/SHAPE(8),DERIV(2,8),XJACM(2,2),XJACI(2,2),
1CARTD(2,8),ELCOD(2,8),BMATX(3,16),DMATX(3,3),
2 DBMAT(3,16),SMATX(3,16,4),GPCOD(2,4)
C
C**** CALCULATE COORDINATES OF SAMPLING POINT
C
      DO 10 IDIME=1,NDIME
      GPCOD(IDIME,KGASP)=0.0
      DO 10 INODE=1,NNODE
      GPCOD(IDIME,KGASP)=GPCOD(IDIME,KGASP)+
      , ELCOD(IDIME,INODE)*SHAPE(INODE)
10 CONTINUE
C
C**** CREATE JACOBIAN MATRIX XJACM
C
      DO 20 IDIME=1,NDIME
      DO 20 JDIME=1,NDIME
      XJACM(IDIME,JDIME)=0.0
      DO 20 INODE=1,NNODE
      XJACM(IDIME,JDIME)=XJACM(IDIME,JDIME)+
      , DERIV(IDIME,INODE)*ELCOD(JDIME,INODE)
20 CONTINUE
C
C**** CALCULATE DETERMINANT AND INVERSE OF
C      JACOBIAN MATRIX
C
      DJACB=XJACM(1,1)*XJACM(2,2)-XJACM(1,2)*
      , XJACM(2,1)
      IF(DJACB.GT.0.0) GO TO 30
      WRITE (6,900) IELEM
      STOP
30 XJACI(1,1)=XJACM(2,2)/DJACB
      XJACI(2,2)=XJACM(1,1)/DJACB
      XJACI(1,2)=-XJACM(1,2)/DJACB
      XJACI(2,1)=-XJACM(2,1)/DJACB
C
C**** CALCULATE CARTESIAN DERIVATIVES
C
      DO 40 IDIME=1,NDIME
      DO 40 INODE=1,NNODE
      CARTD(IDIME,INODE)=0.0

```



```

DO 40 JDIME=1,NDIME
  CARTD(IDIME,INODE)=CARTD(IDIME,INODE)+
    . XJACI(IDIME,JDIME)*DERIV(JDIME,INODE)
40 CONTINUE
900 FORMAT(/,24HPROGRAM HALTED IN JACOB2,
  . /,11X,22H ZERO OR NEGATIVE AREA,/,
  , 10X,16H ELEMENT NUMBER ,I5)
  RETURN
END
SUBROUTINE BMATPS
  IMPLICIT REAL*8(A-H,O-Z)
  COMMON/GAS/POSGP(3),WEIGP(3),NGAUS
  COMMON/TWO/NDOFN,NEVAB,NNODE,NSTRE,NDIME
  COMMON/TP/NELEM,NPOIN,NBNODE,NODCL,NEQ,NEQ1,NTYPE,IGLOD
  COMMON/BLK1/LNODS(300,8),ID(2,750),NBN(100),NLOAD(30),
  1COORD(750,2),PCON(30,2),MATNO(300),PROPS(10,5)
  COMMON/BLK2/S(136),ELOAD(300,16),DEL(16)
  COMMON/XX1/SHAPE(8),DERIV(2,8),XJACM(2,2),XJACI(2,2),
  1CARTD(2,8),ELCOD(2,8),BMATX(3,16),DMATX(3,3),DBMAT(3,16),
  2SMATX(3,16,4),GPCOD(2,4)
  NGASH=0
  DO 10 INODE=1,NNODE
    MGASH=NGASH+1
    NGASH=MGASH+1
    BMATX(1,MGASH)=CARTD(1,INODE)
    BMATX(1,NGASH)=0.0
    BMATX(2,MGASH)=0.0
    BMATX(2,NGASH)=CARTD(2,INODE)
    BMATX(3,MGASH)=CARTD(2,INODE)
    BMATX(3,NGASH)=CARTD(1,INODE)
10 CONTINUE
  RETURN
END
SUBROUTINE MODPS(LPROP)
  IMPLICIT REAL*8(A-H,O-Z)
  COMMON/GAS/POSGP(3),WEIGP(3),NGAUS
  COMMON/TWO/NDOFN,NEVAB,NNODE,NSTRE,NDIME
  COMMON/TP/NELEM,NPOIN,NBNODE,NODCL,NEQ,NEQ1,NTYPE,IGLOD
  COMMON/BLK1/LNODS(300,8),ID(2,750),NBN(100),NLOAD(30),
  1COORD(750,2),PCON(30,2),MATNO(300),PROPS(10,5)
  COMMON/BLK2/S(136),ELOAD(300,16),DEL(16)
  COMMON/XX1/SHAPE(8),DERIV(2,8),XJACM(2,2),XJACI(2,2),
  1CARTD(2,8),ELCOD(2,8),BMATX(3,16),DMATX(3,3),
  2 DBMAT(3,16),SMATX(3,16,4),GPCOD(2,4)
  YOUNG=PROPS(LPROP,1)
  POISS=PROPS(LPROP,2)
  DO 10 ISTRE=1,NSTRE
    DO 10 JSTRE=1,NSTRE
      DMATX(ISTRE,JSTRE)=0.0
10 CONTINUE
  IF (NTYPE.NE.1) GO TO 20

```

C
C***** D MATRIX FOR PLANE STRESS CASE

```

C
CONST=YOUNG/(1.0-POISS*POISS)
DMATX(1,1)=CONST
DMATX(2,2)=CONST
DMATX(1,2)=CONST*POISS
DMATX(2,1)=CONST*POISS
DMATX(3,3)=(1.0-POISS)*CONST/2.0
GO TO 30
20 IF(NTYPE.NE.2) GO TO 30
C
C***** D MATRIX FOR PLANE STRAIN CASE
C
CONST=YOUNG*(1.0-POISS)/((1.0+POISS)*
. (1.0-2.0*POISS))
DMATX(1,1)=CONST
DMATX(2,2)=CONST
DMATX(1,2)=CONST*POISS/(1.0-POISS)
DMATX(2,1)=CONST*POISS/(1.0-POISS)
DMATX(3,3)=CONST*(1.0-2.0*POISS)/
. (2.0*(1.0-POISS))
30 CONTINUE
RETURN
END
SUBROUTINE ELSTIF(IELEM)
IMPLICIT REAL*8(A-H,O-Z)
COMMON/GAS/POSGP(3),WEIGP(3),NGAUS
COMMON/TWO/NDOFN,NEVAB,NNODE,NSTRE,NDIME
COMMON/TP/NELEM,NPOIN,NBNODE,NODCL,NEQ,NEQ1,NTYPE,IGLOD
COMMON/BLK1/LNODS(300,8),ID(2,750),NBN(100),NLOAD(30),
1COORD(750,2),PCON(30,2),MATNO(300),PROPS(10,5)
COMMON/BLK2/S(136),ELOAD(300,16),DEL(16)
COMMON/XX1/SHAPE(8),DERIV(2,8),XJACM(2,2),XJACI(2,2),
1CARTD(2,8),ELCOD(2,8),BMATX(3,16),DMATX(3,3),
2 DBMAT(3,16),SMATX(3,16,4),GPCOD(2,4)
DIMENSION SS(136)
C
C***** LOOP OVER EACH ELEMENT
C
LPROP=MATNO(IELEM)
C
C***** EVALUATE THE COORDINATES OF THE ELEMENT
C NODAL POINTS
C
DO 400 I=1,136
400 S(I)=0.0
DO 10 INODE=1,NNODE
LNODE=LNODS(IELEM,INODE)
DO 10 IDIME=1,NDIME
10 ELCOD(IDIME,INODE)=COORD(LNODE,IDIME)
C
C***** EVALUATE THE D-MATRIX
C
CALL MODPS(LPROP)

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```

      THICK=PROPS(LPROP,3)
C
C**** INITIALIZE THE ELEMENT STIFFNESS MATRIX
C
      DO 20 I =1,136
      20 S(I)=0.
      KGASP=0
C
C**** ENTER LOOPS FOR AREA NUMERICAL INTEGRATION
C
      DO 50 IGAUS=1,NGAUS
      DO 50 JGAUS=1,NGAUS
      KGASP=KGASP+1
      EXISP=POSGP(IGAUS)
      ETASP=POSGP(JGAUS)
C
C**** EVALUATE THE SHAPE FUNCTIONS, ELEMENTAL
C      VOLUME, ETC.
C
      CALL SFR2(EXISP,ETASP)
      CALL JACOB2(IELEM,DJACB,KGASP)
      DVOLU=DJACB*WEIGP(IGAUS)*WEIGP(JGAUS)
      IF(THICK.NE.0.0) DVOLU=DVOLU*THICK
C
C**** EVALUATE THE B AND DB MATRICES
C
      CALL BMATPS
      CALL DBE
C
C**** CALCULATE THE ELEMENT STIFFNESSES
C
      KK=1
      DO 30 IEVAB=1,NEVAB
      DO 98 JEVAB=IEVAB,NEVAB
      SS(KK)=0.
      DO 99 ISTRE=1,NSTRE
      99 SS(KK)=SS(KK)+
      . BMATX(ISTRE,IEVAB)*DBMAT(ISTRE,
      . JEVAB)*DVOLU
      98 KK=KK+1
      30 CONTINUE
C
C**** STORE THE COMPONENTS OF THE DB MATRIX FOR
C      THE ELEMENT
C
      DO 40 ISTRE=1,NSTRE
      DO 40 IEVAB=1,NEVAB
      40 SMATX(ISTRE,IEVAB,KGASP)=DBMAT(ISTRE,IEVAB)
      DO 500 I=1,136
      500 S(I)=S(I)+SS(I)
      50 CONTINUE
C
C**** CONSTRUCT THE LOWER TRIANGLE OF THE

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C      STIFFNESS MATRIX
      WRITE(3) SMATX,GPCOD
C
C
C**** STORE THE STIFFNESS MATRIX, STRESS MATRIX
C      AND SAMPLING POINT COORDINATES FOR EACH
C      ELEMENT ON DISC FILE
C
      RETURN
      END
      SUBROUTINE GAUSSQ
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/GAS/POSGP(3),WEIGP(3),NGAUS
      IF(NGAUS.GT.2) GO TO 10
      POSGP(1)=-.577350269189626
      WEIGP(1)=1.
      GO TO 20
10     POSGP(1)=-0.774596669241483
      POSGP(2)=0.
      WEIGP(1)=0.555555555555556
      WEIGP(2)=.888888888888889
20     KGAUS=NGAUS/2
      DO 30 IGASH=1,KGAUS
      JGASH=NGAUS+1-IGASH
      POSGP(JGASH)=-POSGP(IGASH)
      WEIGP(JGASH)=WEIGP(IGASH)
30     CONTINUE
      RETURN
      END
      SUBROUTINE DBE
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/TWO/NDOFN,NEVAB,NNODE,NSTRE,NDIME
      COMMON/TP/NELEM,NPOIN,NBNODE,NODCL,NEQ,NEQ1,NTYPE,IGLOD
      COMMON/BLK1/LNODS(300,8),ID(2,750),NBN(100),NLOAD(30),
1COORD(750,2),PCON(30,2),MATNO(300),PROPS(10,5)
      COMMON/BLK2/S(136),ELOAD(300,16),DEL(16)
      COMMON/XX1/SHAPE(8),DERIV(2,8),XJACM(2,2),XJACI(2,2),
1CARTD(2,8),ELCOD(2,8),BMATX(3,16),DMATX(3,3),
2DBMAT(3,16),SMATX(3,16,4),GPCOD(2,4)
C
C**** CALCULATES D X B
C
      DO 10 ISTRE=1,NSTRE
      DO 10 IEVAB=1,NEVAB
      DBMAT(ISTRE,IEVAB)=0.
      DO 10 JSTRE=1,NSTRE
      DBMAT(ISTRE,IEVAB)=DBMAT(ISTRE,IEVAB)+
1DMATX(ISTRE,JSTRE)*BMATX(JSTRE,IEVAB)
10     CONTINUE
      RETURN
      END
      SUBROUTINE INBLOC
      IMPLICIT REAL*8(A-H,O-Z)

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        DIMENSION TITLE(20)
        COMMON/TWO/NDOFN,NEVAB,NNODE,NSTRE,NDIME
        COMMON/TP/NELEM,NPOIN,NBNODE,NODCL,NEQ,NEQ1,NTYPE,IGLOD
        COMMON/BLK1/LNODS(300,8),ID(2,750),NBN(100),NLOAD(30),
1COORD(750,2),PCON(30,2),MATNO(300),PROPS(10,5)
        COMMON/XX1/SHAPE(8),DERIV(2,8),XJACH(2,2),XJACI(2,2),
1CARD(2,8),ELCOD(2,8),BMTX(3,16),DMATX(3,3),
2 DBMAT(3,16),SMATX(3,16,4),GPCOD(2,4)
        COMMON/MESH1/MNODE
        DATA LNODE/8/
C
C**** THIS SUBROUTINE ACCEPTS THE INPUT DATA DEFINING
C**** THE STRUCTURAL OUTLINE
C
        READ(5,900) TITLE
        WRITE(6,900) TITLE
900  FORMAT(20A4)
        READ(5,905) NPOIN,NELEM,NNODE,NDIME
        WRITE(6,905) NPOIN,NELEM
905  FORMAT(10I5)
        WRITE(6,910) NPOIN,NELEM
910  FORMAT(5X,30HAUTOMATIC DATA GENERATION FROM,I4,
121H LOCATION POINTS AND,I3,13H DATA BLOCKS/)
        WRITE(6,915) NNODE
915  FORMAT(5X,29HNUMBER OF NODES PER ELEMENT =,I3/)
        WRITE(6,920)
920  FORMAT(5X,11HDATA BLOCKS/)
        WRITE(6,925)
925  FORMAT(1X, 9HBLOCK NO.,8X,17HDEFINITION POINTS,10X,
18HMATERIAL/)
        DO 10 IELEM=1,NELEM
            READ(5,905) NUMEL,(LNODS(NUMEL,INODE),INODE=1,LNODE),
1MATNO(NUMEL)
10  CONTINUE
        WRITE(6,905) NUMEL,(LNODS(NUMEL,INODE),INODE=1,LNODE),
1MATNO(NUMEL)
        WRITE(6,930)
        WRITE(6,935)
930  FORMAT(5X,15HLOCATION POINTS)
935  FORMAT(5X,9HPOINT NO.,2X,10H X-COORD.,4X,10H Y-COORD./)
        DO 20 IPOIN=1,NPOIN
            READ(5,940) JPOIN,(COORD(JPOIN,IDIME),IDIME=1,NDIME)
20  CONTINUE
        WRITE(6,950) JPOIN,(COORD(JPOIN,IDIME),IDIME=1,NDIME)
940  FORMAT(I10,2F10.0)
950  FORMAT(I10,2F15.5)
        RETURN
        END
        SUBROUTINE GENER
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION WEITX(40),WEITY(40),TCORD(500,2),TNODS(50,8),TMATO(50)
1,LREP(350),LASOC(350),LFINN(350),LFASC(350)
        COMMON/TWO/NDOFN,NEVAB,NNODE,NSTRE,NDIME

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COMMON/TP/NELEM,NPOIN,NBNODE,NODCL,NEQ,NEQ1,NTYPE,IGLOD
COMMON/BLK1/LNODS(300,8),ID(2,750),NBN(100),NLOAD(30),
1COORD(750,2),PCON(30,2),MATNO(300),PROPS(10,5)
COMMON/XX1/SHAPE(8),DERIV(2,8),XJACM(2,2),XJACI(2,2),
1CARD(2,8),ELCOD(2,8),BMATX(3,16),DMATX(3,3),
2 DBMAT(3,16),SMATX(3,16,4),GPCOD(2,4)
COMMON/MESH1/MNODE
DATA MREPN/350/,MPOIN/750/,LNODE/8/
C
C**** THIS SUBROUTINE PERFORMS THE MESH SUBDIVISION
C
C**** INITIALISATION SECTION
C
DO 10 IREP=1,MREPN
10 LREP(IREP)=0
NPONT=NPOIN
NBLOC=NELEM
NPOIN=0
NELEM=0
MNODE=4
IF(MNODE.EQ.8) MNODE=8
KNODE=MNODE/4
FNODE=KNODE
DO 20 IPONT=1,NPONT
DO 20 IDIME=1,NDIME
20 TCORD(IPONT,IDIME)=COORD(IPONT,IDIME)
DO 30 IPOIN=1,MPOIN
DO 30 IDIME=1,NDIME
30 COORD(IPOIN,IDIME)=0.0
DO 40 IBLOC=1,NBLOC
TMATO(IBLOC)=MATNO(IBLOC)
DO 40 INODE=1,LNODE
40 TNODS(IBLOC,INODE)=LNODS(IBLOC,INODE)
C
C**** READ AND WRITE BLOCK SUBDIVISION DATA
C
DO 170 IBLOC=1,NBLOC
READ(5,900) KBLOC,NDIVX,NDIVY
READ(5,905) (WEITX(IDIVX),IDIVX=1,NDIVX)
READ(5,905) (WEITY(IDIVY),IDIVY=1,NDIVY)
WRITE(6,910) KBLOC
WRITE(6,915)
WRITE(6,920) NDIVX,NDIVY
WRITE(6,925)
WRITE(6,906) (WEITX(IDIVX),IDIVX=1,NDIVX)
WRITE(6,930)
WRITE(6,906) (WEITY(IDIVY),IDIVY=1,NDIVY)
900 FORMAT(3I5)
905 FORMAT(8F10.0)
906 FORMAT(8F10.3)
910 FORMAT(5X,14HDATA BLOCK NO.,I3)
915 FORMAT(6X,17H-----)
920 FORMAT(5X,31HNO. OF DIV. IN FIRST DIRECTION=,I3,',',2X,

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132HNO. OF DIV. IN SECOND DIRECTION=,13/)
925 FORMAT(5X,44HLIST OF WEIGHTING FACTORS IN FIRST DIRECTION)
930 FORMAT(5X,45HLIST OF WEIGHTING FACTORS IN SECOND DIRECTION)

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C
C**** DIVIDE EACH BLOCK INTO ELEMENTS
C

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```

TOTAL=0.0
DO 50 IDIVX=1,NDIVX
IF(WEITX(IDIVX).EQ.0.0) WEITX(IDIVX)=1.0
50 TOTAL=TOTAL+WEITX(IDIVX)
XNORM=2.0/TOTAL
TOTAL=0.0
DO 60 IDIVY=1,NDIVY
IF(WEITY(IDIVY).EQ.0.0) WEITY(IDIVY)=1.0
60 TOTAL=TOTAL+WEITY(IDIVY)
YNORM=2.0/TOTAL
NXTWO=NDIVX*KNODE+1
NYTWO=NDIVY*KNODE+1
IASY=0
ETASP=-1.0
KWETY=0
KONTY=-1
DO 160 IYTWO=1,NYTWO
IASY=IASY+1
IF(NNODE.NE.8.AND.IASY.EQ.3) IASY=2
IF(NNODE.EQ.8.AND.IASY.EQ.4) IASY=2
IASX=0
EXISP=-1.0
KWETX=0
KONTX=-1
DO 130 IXTWO=1,NXTWO
IASX=IASX+1
IF(NNODE.NE.8.AND.IASX.EQ.3) IASX=2
IF(NNODE.EQ.8.AND.IASX.EQ.4) IASX=2
IF(IASX.EQ.2.AND.IASY.EQ.2.AND.NNODE.EQ.8) GO TO 100
NPOIN=NPOIN+1
CALL SFR2(EXISP,ETASP)
DO 70 INODE=1,LNODE
JTEMP=TNODS(IBLOC,INODE)
DO 70 IDIME=1,NDIME
70 COORD(NPOIN,IDIME)=COORD(NPOIN,IDIME)+SHAPE(INODE)*TCORD(JTEMP,
1IDIME)
GO TO(80,90),KNODE
80 IF(IASX.NE.2.OR.IASY.NE.2) GO TO 100
NELEM=NELEM+1
JPOIN=NPOIN-NXTWO
LNODS(NELEM,1)=JPOIN-1
LNODS(NELEM,2)=JPOIN
LNODS(NELEM,3)=NPOIN
LNODS(NELEM,4)=NPOIN-1
MATNO(NELEM)=TMATO(IBLOC)
GO TO 100
90 IF(IASX.NE.3.OR.IASY.NE.3) GO TO 100

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NELEM=NELEM+1
IPOIN=NPOIN-IXTWO-NDIVX+(IXTWO-1)/2
JPOIN=NPOIN-NXTWO-NDIVX-1
LNODS(NELEM,1)=JPOIN-2
LNODS(NELEM,2)=JPOIN-1
LNODS(NELEM,3)=JPOIN
LNODS(NELEM,4)=IPOIN
LNODS(NELEM,5)=NPOIN
LNODS(NELEM,6)=NPOIN-1
LNODS(NELEM,7)=NPOIN-2
LNODS(NELEM,8)=IPOIN-1
MATNO(NELEM)=TMATO(IBLOC)
100 CONTINUE
GO TO (110,120),KNODE
110 KWETX=KWETX+1
GO TO 130
120 IF(KONTX.LT.0)KWETX=KWETX+1
KONTX=KONTX*(-1)
130 EXISP=EXISP+XNORM*WEITX(KWETX)/FNODE
GO TO (140,150),KNODE
140 KWETY=KWETY+1
GO TO 160
150 IF(KONTY.LT.0) KWETY=KWETY+1
KONTY=KONTY*(-1)
160 ETASP=ETASP+YNORM*WEITY(KWETY)/FNODE
170 CONTINUE
C
C**** ELIMINATE REPEATED NODES AT BLOCK INTERFACES
C
NREP=0
DO 210 IPOIN=1,NPOIN
IF(NREP.EQ.0) GO TO 190
DO 180 IREP=1,NREP
IF(IPOIN.EQ.LREP(IREP)) GO TO 210
180 CONTINUE
190 CONTINUE
LPOIN=IPOIN+1
DO 200 JPONT=LPOIN,NPOIN
TOTAL=ABS(COORD(IPOIN,1)-COORD(JPONT,1))+ABS(COORD(IPOIN,2)-
COORD(JPONT,2))
IF(TOTAL.GT.0.0001) GO TO 200
NREP=NREP+1
LREP(NREP)=JPONT
LASOC(NREP)=IPOIN
200 CONTINUE
210 CONTINUE
IF(NREP.EQ.0) GO TO 360
INDEX=0
DO 240 IPOIN=1,NPOIN
DO 220 IREP=1,NREP
IF(LREP(IREP).EQ.IPOIN) GO TO 230
220 CONTINUE
GO TO 240

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```

230 INDEX=INDEX+1
    LFINN(INDEX)=LREPN(IREP)
    LFASC(INDEX)=LASOC(IREP)
240 CONTINUE
    DO 250 IREP=1,NREP
        LREPN(IREP)=LFINN(IREP)
250 LASOC(IREP)=LFASC(IREP)
    DO 260 IREP=1,NREP
        DO 260 IELEM=1,NELEM
            DO 260 INODE=1,MNODE
260 IF(LNODS(IELEM,INODE).EQ.LREPN(IREP)) LNODS(IELEM,INODE)=
    1LASOC(IREP)
    DO 310 IPOIN=1,NPOIN
        DO 270 IREP=1,NREP
            IF(IPOIN.EQ.LREPN(IREP)) GO TO 310
270 CONTINUE
            IF(IPOIN.LT.LREPN(1)) GO TO 310
            IDIFF=IPOIN-NREP
            IF(IPOIN.GT.LREPN(NREP)) GO TO 290
            DO 280 IREP=1,NREP
                KREP=NREP-IREP+1
280 IF(IPOIN.LT.LREPN(KREP)) IDIFF=IPOIN-KREP+1
290 DO 300 IDIME=1,NDIME
300 COORD(IDIFF,IDIME)=COORD(IPOIN,IDIME)
310 CONTINUE
    DO 350 IELEM=1,NELEM
        DO 350 INODE=1,MNODE
            NPOSI=LNODS(IELEM,INODE)
            DO 320 IREP=1,NREP
                IF(NPOSI.EQ.LREPN(IREP)) GO TO 350
320 CONTINUE
                IF(NPOSI.LT.LREPN(1)) GO TO 350
                IDIFF=NPOSI-NREP
                IF(NPOSI.GT.LREPN(NREP)) GO TO 340
                DO 330 IREP=1,NREP
                    KREP=NREP-IREP+1
330 IF(NPOSI.LT.LREPN(KREP)) IDIFF=NPOSI-KREP+1
340 LNODS(IELEM,INODE)=IDIFF
350 CONTINUE
360 CONTINUE
    NPOIN=NPOIN-NREP
    RETURN
    END

```

```

C   CALCULATE AVERAGE STRESSES AT THE NODES
    SUBROUTINE STSNOD
    IMPLICIT REAL*8(A-H,O-Z)
    COMMON/GAS/POSGP(3),WEIGP(3),NGAUS
    COMMON/TWO/NDOFN,NEVAB,NNODE,NSTRE,NDIME
    COMMON/TP/NELEM,NPOIN,NBNODE,NODCL,NEQ,NEQ1,NTYPE,IGLOD
    COMMON/BLK1/LNODS(300,8),ID(2,750),NBN(100),NLOAD(30),
1COORD(750,2),PCON(30,2),MATNO(300),PROPS(10,5)
    COMMON/BLK2/STIF(136),ELOAD(300,16),DEL(16)
    COMMON/XX1/SHAPE(8),DERIV(2,8),XJACM(2,2),XJACI(2,2),

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```

1CARTD(2,8),ELCOD(2,8),BMATX(3,16),DMATX(3,3),
2 DBMAT(3,16),SMATX(3,16,4),GPCOD(2,4)
COMMON/BK9/ASM(90000),ASDIS(1500)
DIMENSION XINOD(8),ETANOD(8),STRS(300,8,4),STSA(4),STRSG(4),
1STRSP(3),ELDIS(2,8)
DATA XINOD/-1.,0.,1.,1.,1.,0.,-1.,-1./
DATA ETANOD/-1.,-1.,-1.,0.,1.,1.,1.,0./
NSTRE=3
KGASP=1
NSTR1=NSTRE+1
DO 1000 IELEM=1,NELEM
LPROP=MATNO(IELEM)
POISS=PROPS(LPROP,2)
DO 10 INODE=1,NNODE
LNODE=LNODS(IELEM,INODE)
DO 10 IDIME=1,NDIME
10 ELCOD(IDIME,INODE)=COORD(LNODE,IDIME)
CALL MODPS(LPROP)
DO 200 INODE=1,NNODE
LNODE=LNODS(IELEM,INODE)
DO 102 IDOFN=1,NDOFN
NNP=ID(IDOFN,LNODE)
ELDIS(IDOFN,INODE)=0.
IF(NNP.NE.0)ELDIS(IDOFN,INODE)=ASDIS(NNP)
102 CONTINUE
200 CONTINUE
DO 50 INODE=1,NNODE
EXISP=XINOD(INODE)
ETASP=ETANOD(INODE)
CALL SFR2(EXISP,ETASP)
CALL JACOB2(IELEM,DJACB,KGASP)
CALL BMATPS
CALL DBE
DO 20 ISTRE=1,NSTRE
STRSG(ISTRE)=0.
KGASH=0
DO 20 INOD=1,NNODE
DO 20 IDOFN=1,NDOFN
KGASH=KGASH+1
STRSG(ISTRE)=STRSG(ISTRE)+DBMAT(ISTRE,KGASH)*ELDIS(IDOFN,
1INOD)
20 CONTINUE
IF(NTYPE.EQ.2) STRSG(4)=POISS*(STRSG(1)+STRSG(2))
IF(NTYPE.EQ.1) STRSG(4)=0.0
DO 30 ISG=1,4
30 STRS(IELEM,INODE,ISG)=STRSG(ISG)
50 CONTINUE
1000 CONTINUE
C
C*** OUTPUT THE NODAL STRESSES
C
WRITE (6,900)
WRITE (6,905)

```

```

900 FORMAT(/,45X,21HSTRESSES AT THE NODES,/)
905 FORMAT(3X,8HNODE NO.,11X,8HX-STRESS,7X,8HY-STRESS,6X,
19HXY-STRESS,7X,8HZ-STRESS,7X,8HMAX P.S.,7X,8HMIN P.S.,7X,
25HANGLES,/)
DO 2000 IPOIN=1,NPOIN
KK=0
DO 25 ISA=1,4
25 STSA(ISA)=0.
DO 600 IELEM=1,NELEM
DO 700 INODE=1,NNODE
NOD=LNODS(IELEM,INODE)
IF(NOD.NE.IPOIN) GO TO 700
KK=KK+1
DO 70 ISA=1,4
70 STSA(ISA)=STSA(ISA)+STRS(IELEM,INODE,ISA)
GO TO 600
700 CONTINUE
600 CONTINUE
DO 800 ISA=1,4
800 STSA(ISA)=STSA(ISA)/KK
XGASH=(STSA(1)+STSA(2))/2.
XGISH=(STSA(1)-STSA(2))/2.
XGESH=STSA(3)
XGOSH=SQRT(XGISH**2+XGESH**2)
STRSP(1)=XGASH+XGOSH
STRSP(2)=XGASH-XGOSH
IF(XGISH.EQ.0.)XGISH=0.1E-20
STRSP(3)=ATAN(XGESH/XGISH)*28.647889757
WRITE(6,915) IPOIN,(STSA(ISTR1),ISTR1=1,NSTR1),(STRSP(ISTRE),
1ISTR1=1,NSTRE)
915 FORMAT(5X,I5,5X,6F15.2,F12.2)
2000 CONTINUE
RETURN
END
SUBROUTINE GRLOD
IMPLICIT REAL*8(A-H,O-Z)
COMMON/TWO/NDOFN,NEVAB,NNODE,NSTRE,NDIME
COMMON/TP/NELEM,NPOIN,NBNODE,NODCL,NEQ,NEQ1,NTYPE,IGLD
COMMON/GAS/POSGP(3),WEIGP(3),NGAUS
COMMON/BLK1/LNODS(300,8),ID(2,750),NBN(100),NLOAD(30),
1COORD(750,2),PCON(30,2),MATNO(300),PROPS(10,5)
COMMON/BK9/ASM(90000),V(1500)
COMMON/TX/LM(16)
COMMON/BLK2/STIF(136),ELOAD(300,16),DEL(16)
COMMON/BLK4/MHT(1500),MAXA(1500)
COMMON/XX1/SHAPE(8),DERIV(2,8),XJACM(2,2),XJACI(2,2),
1CARTD(2,8),ELCOD(2,8),BMATX(3,16),DMATX(3,3),
2 DBMAT(3,16),SMATX(3,16,4),GPCOD(2,4)
C
C*** GRAVITY LOADING SECTION
C
C
C*** READ GRAVITY ANGLE AND GRAVITATIONAL CONSTANT

```

```

C
  READ(5,920) THETA,GRAVY
920  FORMAT(2F10.0)
  WRITE(6,925) THETA,GRAVY
925  FORMAT(1X,16H GRAVITY ANGLE = ,F6.2,19H GRAVITY CONSTANT =,
      1F6.2)
  THETA=THETA/57.295779514
C
C*** LOOP OVER EACH ELEMENT
C
  DO 1567 IEL=1,NELEM
    DO 1567 J=1,16
1567  ELOAD(IEEL,J)=0.
    DO 90 IELEM=1,NELEM
C
C*** SET UP PRELIMINARY CONSTANTS
C
  LPROP=MATNO(IELEM)
  THICK=PROPS(LPROP,3)
  DENSE=PROPS(LPROP,4)
  IF(DENSE.EQ.0.0) GO TO 90
  GXCOM=DENSE*GRAVY*SIN(THETA)
  GYCOM=-DENSE*GRAVY*COS(THETA)
C
C*** COMPUTE COORDINATES OF THE ELEMENT NODAL POINTS
C
  DO 60 INODE=1,NNODE
    LNODE=LNODS(IELEM,INODE)
    DO 60 IDIME=1,NDIME
60    ELCD(IDIME,INODE)=COORD(LNODE,IDIME)
C
C*** ENTER LOOPS FOR AREA NUMERICAL INTEGRATION
C
  DO 80 IGAUS=1,NGAUS
    DO 80 JGAUS=1,NGAUS
    EXISP=POSGP(IGAUS)
    ETASP=POSGP(JGAUS)
C
C*** COMPUTE THE SHAPE FUNCTIONS AT THE SAMPLING POINTS AND
C    ELEMENTAL VOLUME
C
  CALL SFR2(EXISP,ETASP)
  KGASP=1
  CALL JACOB2(IELEM,DJACB,KGASP)
  DVOLU=DJACB*WEIGP(IGAUS)*WEIGP(JGAUS)
  IF(THICK.NE.0.0)DVOLU=DVOLU*THICK
C
C*** CALCULATE LOADS AND ASSOCIATE WITH ELEMENT NODAL POINTS
C
  DO 70 INODE=1,NNODE
    NGASH=(INODE-1)*NDOFN+1
    MGASH=(INODE-1)*NDOFN+2
    ELOAD(IELEM,NGASH)=ELOAD(IELEM,NGASH)+GXCOM*SHAPE(INODE)*DVOLU

```

70 ELOAD(IELEM, MGASH) = ELOAD(IELEM, MGASH) + GYCOM * SHAPE(INODE) * DVOLU
80 CONTINUE
90 CONTINUE
RETURN
END

INSTRUCTIONS FOR PREPARING INPUT DATA

The user's instructions for preparing the input data are provided here.

CARD SET 1 (20I4) - One card

columns 1-4 MSHON

Flag for generating Mesh only

= 1 if complete analysis is to be done

= 0 if only mesh is to be generated for
checking

5-8 NODSTS

Flag for calculation of average
nodal stresses

= 1 if average nodal stresses are to
be calculated

= 0 if nodal stresses are not required

CARD SET2 (8I10) - One card

columns 1-10 ICORE

Dimension given to the assembled
stiffness matrix (=90000 in the
present listing)

11-20 IEQ

Dimension given to the assembled
load vectors (=1500 in the present
listing)

CARD SET3 (20I4) - One card

columns 1-4 NPROB

Total number of problems to be
solved in one run

CARD SET4 (20A4) - One card

columns 1-80 TITLE

Title of the problem - limited to
80 alpha numeric characters

INPUT DATA DEFINING THE STRUCTURAL OUTLINE

CARD SET5 (10I5)

columns 1-5 NPOIN

Total number of nodal points in the
data blocks

6-10 NELEM

Total number of elements in the
data blocks

11-15 NNODE

Number of nodes per element (=8)

16-20 NDIME

Number of coordinate dimensions (=2)

CARDS SET6 (10I5) - one card for each element

columns 1-5 NUMEL

Block numbers

6-10 LNODS(NUMEL,1) 1st Nodal connection number

11-15 LNODS(NUMEL,2) 2nd nodal connection number

nn nn

nn nn

41-45 LNODS(NUMEL,8) 8th nodal connection number

46-50 MATNO (NUMEL) Material property number

Note : The nodal connection numbers must be listed in an anticlockwise
sequence, starting from any corner node.

CARD SET7 (I10,2F10.0) one card for each node whose coordinates
are to be input

columns 1-10 JPOIN

Nodal point numbers

11-20 COORD (JPOIN,1) x- coordinate of node

21-30 COORD (JPOIN,2) y- coordinate of node

BLOCK SUBDIVISION DATA - FOR EACH BLOCK

CARD SET8 (3I5)

columns 1-5 KBLOC	Block number
6-10 NDIVX	Number of element subdivisions in the local .. direction
11-15 NDIVY	Number of element subdivisions in the local .. direction

CARD SET9 (8F10.0) as many cards as necessary

columns 1-10 WEITX(1)	List of weighting factors for
11-20 WEITX(2)	element subdivision in the ..
21-30 WEITX(3)	direction

"

"

WEITX(NDIVX)

CARD SET10 (8F10.0) as many cards as necessary

columns 1-10 WEITY(1)	List of weighting factors for
11-20 WEITY(2)	element subdivision in the ..
21-30 WEITY(3)	direction

"

"

WEITY(NDIVY)

The following datas are for the automatically generated finer mesh

CARD SET 11 (20I4)

columns 1-4 NBNODE	Number of boundary nodes
5-8 NDFPN	Number of degrees of freedom per node(=2)
9-12 NODCL	Number of nodes on which point loads are acting
13-16 NMATS	Number of different types of material
17-20 NPROP	Number of independent properties per material (=5)
21-24 NTYPE	Problem type parameter = 1 plane stress = 2 plane strain
25-28 IGLOD	FLAG for gravity load = 0 No gravity load =1 gravity load is present

CARD SET12(20I4) -Read only if NODCL is not equal to 0

columns 1-4 NLOAD(1)	node numbers on which point load is
5-8 NLOAD(2)	acting
9-12 NLOAD(3)	
"	
"	

CARD SET13 (8F10.0) Read only if NODCL is not equal to zero

	one card for each node with point load
columns 1-10 PCON(I,1)	Point load in the x - directon
11-20 PCON(I,2)	Point load in the y - directon

CARD SET14 (8F10.0) one card for each different material

columns 1-10 PROPS (IMAT,1) Elastic modulus, E
 11-20 PROPS (IMAT,2) Poisson's ratio, ..
 21-30 PROPS (IMAT,3) Material thickness, t
 31-40 PROPS (IMAT,4) Mass density, ..
 41-50 PROPS (IMAT,5) Coefficient of thermal expansion,..

CARD SET15 (2F10.0) READ ONLY IF IGLOD IS NOT EQUAL TO ZERO

columns 1-10 THETA Gravity angle
 11-20 GRAVY Gravity constant

CARD SET16 (20I4) Read the nodes where boundary conditions are
 prescribed

Columns 1-4 NBN(1) 1st Bouondary node numbers
 5-8 NBN(2) 2nd Bouondary node numbers
 9-12 NBN(3) 3rd Bouondary node numbers

"

"

CARD SET17 (20I4) One card for each boundary node

columns 1-4 ID(1,N) Code for constraint in the x direction
 = 1 if it is constrained
 = 0 if it is free

5-8 ID(2,N) Code for constraint in the y-
 direction (same as above)

APPENDIX B

Properties of 100 mm Diameter Pipe Used in Steel Rod Model

Length of the pipe = 100 mm

Diameter of the pipe = 100 mm

Thickness of the pipe = 0.39 mm

Second Moment of Inertia I = $1/12 \text{ bt}$

= $1/12 \times 1 \times (0.39) \text{ mm}^3/\text{mm}$

= 0.004943 mm³/mm

Modulus of Elasticity of Steel = 200000 MPa

Poisson's Ratio = 0.3

EI/R = 0.0079 MPa

Properties of 100 mm Diameter Pipe Used in Sand Box Test

Length of the pipe = 150 mm

Diameter of the pipe = 150 mm

Thickness of the pipe = 0.39 mm

Second Moment of Inertia I = $1/12 bt^3$

= $1/12 \times 1 \times (0.39)^3 \text{ mm}^4$

= 0.004943 mm⁴

Modulus of Elasticity of Steel = 200000 MPa

Poisson's Ratio = 0.3

EI/R = 0.0023 MPa

APPENDIX C

INSTRUMENTATION SPECIFICATIONS

(A) Load Cell Specifications

Interface, Inc.

7401 East Butherus Drive,

Scottsdale, Arizona.

Model 1210-AF Precision Universal Load Cell

Load range	+/- 44.5KN
Input resistance	350.2 Ohms
Output resistance	350.2 Ohms
Recommended excitation	10 VDC or VAC
Maximum excitation	20 VDC or VAC
Maximum error band, computed on a best straight line through zero basis including non-linearity hysteresis and repeatability	+/- 0.015% Full Scale (FS)
Compensated Temperature range	- 10 C to + 46 C
Thermal Zero Shift	0.0002 % FS/ C
Zero Balance	0.18% FS
Compression/Tension Output	4.085/4.196 MV/V

(B) Strain Gauge Specifications

Bean Strain Gauges

William T. Bean, Inc.,

Detroit, Michigan

Gauge Type BAE - 06 - 250BB - 120p

Serial No 1 - CX - 3 - A - SJ LOT No. 305

Resistance	120.0 +/- 0.2 ohms
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Gauge factor	2.03 +/- 1%
--------------	-------------

k	0%
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Temperature Range	- 200.C to + 200.C
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(C) Digital Strain Indicator

Vishay / Ellis - 20.

APPENDIX D

C *****

C

C TO CALCULATE DEFLECTIONS FROM THE DIAL GAUGE READINGS

C

C *****

DIMENSION RIN(9),DEF(100,13),S(100),TITLE(20)

READ (5,1) TITLE

WRITE (6,1) TITLE

* 1 FORMAT (20A4)

READ (5,*) N

READ (5,*)(RIN(I),I=1,9)

WRITE (6,2)

2 FORMAT(///2X,4HLOAD,3X,'1',5X,'2',5X,'3',5X,'4',5X,'5',5X,'6',
15X,'7',5X,'8',5X,'9',4X,'10',4X,'11',4X,'12'/)

WRITE(6,10) (RIN(I),I=1,9)

DO 100 I=1,N

READ (5,*) (DEF(I,J),J=1,13)

WRITE(6,10)(DEF(I,J),J=1,13)

10 FORMAT(/2X,I4,12F6.2)

100 CONTINUE

WRITE(6,11)

11 FORMAT(////)

WRITE (6,3)

3 FORMAT(2X,4HLOAD,5X,'1',7X,'2',7X,'3',7X,'4',7X,'5',7X,'6',

```

17X,'7',7X,'8',7X,'9'//)
DO 200 I=1,N
DO 250 J=1,9
250 DEF(I,J)=DEF(I,J)-RIN(J)
S(I)=0.0
DO 260 J=1,4
260 S(I)=S(I)+DEF(I,J+9)
S(I)=S(I)/4.0
200 WRITE (6,20) (DEF(I,J),J=1,9),S(I)
20 FORMAT (2X,I4,9F8.2/)
STOP
END

```


END

3 1 0 8 8 4

FIN