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Estimation of Cell Area Coverage and Cell-Site Diversity Gain in 30 GHz Fixed Cellular Systems under Rainy Conditions

by

Gamantyo Hendrantoro, B.Eng., M.Eng.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of

Doctor of Philosophy

**Ottawa-Carleton Institute for Electrical and Computer Engineering
Faculty of Engineering
Department of Systems and Computer Engineering**

**Carleton University
Ottawa, Ontario, Canada**

May, 2001

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submitted by Gamantyo Hendrantoro, B.Eng., M.Eng., in partial fulfillment of the
requirements for the degree of Doctor of Philosophy.

Thesis Supervisor

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Abstract

Since a number of new broadband wireless access systems considered the use of millimetre-wave frequencies, especially those around 30 GHz, for their operating frequency, rain attenuation has been one of the main concerns in the system design. Beside the well-studied high attenuation on radio links operating at these high frequencies, the spatio-temporal variation thereof also requires in-depth research, especially with regards to its impact on area coverage in a cellular radio network. The research reported in this thesis was aimed at examining the effects of rain attenuation on cell area coverage and the feasibility of applying cell-site diversity in such cellular systems. The study was conducted by simulating circular radio cells on images of radar-measured reflectivity factors, from which statistics of rain attenuation at 30 GHz and area coverage were derived.

Initial stages of this research were dedicated to derivation of an appropriate set of parameters of the power-law relationship between specific attenuation and radar reflectivity factor to be used later in radar data processing. Examination of a large number of raindrop size measurements made in Montreal and Toronto using JWD (Joss-Waldvogel Disdrometer) and POSS (Precipitation Occurrence Sensor System) was emphasised on the effects of inaccuracies in small-sized drops detection by these instruments on the derived relation between specific attenuation and reflectivity factor. The resulting power-law coefficients from this study show some differences compared to those recommended by the ITU-R, which are the smallest for frequencies around 20–30 GHz. A further study showed that the statistics of rain attenuation on a radio link computed using the power-law resolution is resistant to spatial averaging.

Analysis of cell area coverage under rain with uniformly LOS (line-of-sight) links in the cell area indicated that rain attenuation varies spatially and that the coverage statistics are non-linearly dependent on the cell radius. The impact of mixed LOS/NLOS environment was introduced by assuming lognormal shadowing characterised by two parameters, namely, the path loss exponent and excess loss standard deviation. Coverage was computed by adding the mean and excess path losses to radar-derived rain attenuation and integrating probability of coverage over the entire cell area given fade margin at the cell perimeter. A model for predicting the coverage statistics was derived from the simulation result, which consists of estimation of the normalised coverage, i.e. coverage under rain relative to that under clear sky conditions, and multiplication with clear-sky coverage predicted from the knowledge of lognormal shadowing parameters using the Jakes' method.

Evaluation of average angular correlation function of attenuation on two converging links indicated that cell-site diversity is potentially beneficial especially in heavy rains, even when the two hubs involved are not located in the exactly opposite directions with respect to the position of the subscriber terminal. Detailed evaluation of angular diversity gain of two diversity links with selection combining showed dependence on angular separation θ in a general form of $\sin^k(\theta/2)$. The model reduces to an approximately root-sinusoidal form ($k = 0.5$) when both diversity links are of equal length. When the two links have different lengths, the model reduces to a sinusoid ($k = 1$), identical to the ITU-R recommended model. Based on an evaluation of the general model for various combinations of link length and angular separation, a set of criteria for determining the benefit of cell-site diversity for a given subscriber terminal position are presented.

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List of Mathematical Symbols and Notations

a, b	(i) Coefficients of Y - R power-law relation for spherical raindrops; (ii) Parameters of Jakes' cell area coverage prediction in equation (4-6)
$a, b, c, d, e, f, g, h, i, j$	Coefficients of coverage prediction model given by equations (4-20) and (4-21)
$a_{V,H}, b_{V,H}$	Coefficients of Y - R power-law relation for vertically or horizontally polarized radio wave, associated with spheroidal raindrops
A	Attenuation
$A(L, \phi, t)$	Rain attenuation at time t on a L -km link oriented at ϕ azimuth
$A(p)$	p^{th} percentile of attenuation
$A_d(p)$	p^{th} percentile of attenuation in a diversity configuration
A_{fs}	Free space path loss
A_{max}	(i) Highest rain attenuation experienced by a link; (ii) Maximum allowable attenuation
A_n	Total attenuation on the n^{th} link of a two-link diversity configuration
$A_{n,\text{rain}}$	Rain attenuation on the n^{th} link of a two-link diversity configuration

$A_{rain}(l_r, t)$	Rain attenuation on link l_r at observation time t
$A_{shadowed}$	Path attenuation in an environment with shadowing, also referred to as clear-sky attenuation
c, d	Coefficients of Y - Z power law relation for spherical raindrops
$c_{V,H}, d_{V,H}$	Coefficients of Y - Z power-law relation for vertically or horizontally polarized radio wave, associated with spheroidal raindrops
c_1, c_2	Arbitrary constants
C	Cell area coverage
$C(t)$	Cell area coverage at observation t
$C(P_{Th})$	Cell area coverage when receiver threshold P_{Th} is applied
$C_{clear-sky}$	Cell area coverage under clear sky
C_h, C_l	Predicted cell area coverages associated with p_h and p_l
C_{norm}	Cell area coverage normalized to $C_{clear-sky}$
C_p	Predicted p^{th} percentile of cell area coverage
D	Raindrop diameter
D_m	Average raindrop diameter in a size interval of a DSD measuring equipment
$erf(\cdot)$	Error function

$f_n(\cdot, \cdot, \cdot)$	Generic function identifier
$f_{V/H}(D)$	Forward scattering amplitude for vertically or horizontally polarized radio wave as a function of diameter D
$f_X(x)$	Probability density function of random variable X
g_t, g_r	Transmit/receive antenna gain
G	Angular diversity gain
$G(p)$	Angular diversity gain for time percentage of p
$G(\theta, p)$	Diversity gain for angular separation θ and time percentage of p
$G_{180^\circ}(p)$	Diversity gain at 180° angular separation and time percentage of p
$G_{\max}$	Maximum value of angular separation-dependent diversity gain, usually synonymous to G_{180°
$\text{Im}[x]$	Imaginary part of x
k	(i) Average received power at the edge of the cell; (ii) Exponent of $\sin(\theta/2)$ in the diversity gain model in equation (5-6)
k_α, k_σ	Gradients of changes in normalized coverage due to variation in α and σ
K_{ext}	Extinction efficiency factor
l_n	Length of the n^{th} segment of a radio link simulated on a radar image

l_r	Link identifier
L	(i) Link length; (ii) Cell radius
L_n	Length of the n^{th} link in a n -link diversity configuration
$\min(x,y)$	The lower between x and y
M	Power margin available to subscriber terminals at the edge of the cell
$M(D)$	Number of drops having diameter D detected by a DSD measuring equipment on the ground per unit volume per unit range of diameter
N	Number of samples
$N(D)$	Number of drops having diameter D per unit volume per unit range of diameter
$N(\mu, \sigma)$	Gaussian distribution with mean μ and variance σ^2
p	(i) Percentage of time that the parameter in question exceeds a specified value; (ii) Probability that the parameter in question exceeds a specified value
$\bar{p}$	Average received power
p_h, p_l	Higher and lower time percentages, for which coefficients of cell area coverage prediction model are known, with respect to a time percentage of interest when applying expressions (4-20) and (4-21)

P	(i) Probability that the parameter in question exceeds a specified value; (ii) Received power
$P(l_r)$	Power received at a subscriber terminal linked to the hub by link l_r
$\text{Pr}[E]$	Probability of occurrence of event E
P_{Th}	Receiver power threshold
Q_a	Absorption cross-section
Q_{ext}	Extinction cross-section
Q_s	Scattering cross-section
r	(i) Propagation range; (ii) Radial distance from the cell centre
r_0	Cut-off distance from base station, as defined in Tan's cell area coverage prediction model in equation (4-9)
R	Rainfall intensity
$R_L(p)$	p^{th} percentile of rain rate averaged over the area of a cell of radius L
$S(f)$	Doppler power spectrum
$\bar{S}(f, D)$	Volume-averaged Doppler power density at frequency f generated by a drop of diameter D
t	(i) Observation time; (ii) Water temperature

$v(D)$	Terminal fall velocity of a raindrop having diameter D
$V(D)$	Equivalent sampling volume of POSS-type DSD measuring apparatus for drops having diameter D
$x((n+k))_m$	Circular shift operation of k units on an m -element sequence $x(n)$
Y	Specific attenuation
Y_n	Specific attenuation on the n^{th} segment of a radio link simulated on a radar image
$Y_{V/H}$	Specific attenuation for vertically or horizontally polarized radio wave
Z	Radar reflectivity factor
α	(i) Exponent of propagation in mean path loss-distance power-law relation; (ii) Raindrop size parameter
δ_g	Percentage of decrease in diversity gain resulting from changing the angular separation from 180°
$\Delta S(r)$	Incremental area at distance r from the cell centre used in computation of area coverage of a simulated cell
ΔTh	Received power threshold relative to the noise floor
ϕ	Azimuth orientation of a radio link
λ	Wavelength

$\mu(r)$	Mean path loss due to shadowing as a function of propagation range r
π	3.141592654
θ	Angular separation between two converging radio links
$\theta_{\text{cut-off}}$	Angular separation associated with a desired value of δ_g
$\rho(L, \theta)$	Time-average circular autocorrelation function of attenuation on two L -km links with separation angle θ .
$\rho(L, \theta, t)$	Circular autocorrelation function of attenuation on two L -km links with separation angle θ at observation time t .
σ	Standard deviation in dB of lognormal shadowing loss
$\sum_{n=n_1}^{n_2} x(n)$	Summation of elements of sequence $x(n)$ over a range of n from n_1 to n_2
$\int_{y_1}^{y_2} x(y)dy$	Integration of continuous function $x(y)$ over a range of y from y_1 to y_2
$\langle x \rangle_t$	Average of x over parameter t
$*$	Convolution operator

List of Acronyms and Abbreviations

AES	Atmospheric Environment Service
BS	base station
CAPPI	constant-amplitude plane position indicator
CATV	Community Antenna Television Service
CCDF	complementary cumulative distribution function
CDF	cumulative distribution function
CRABS	Cellular Radio Access for Broadband Services
CW	continuous carrier-wave
DSD	drop size distribution
DSL	Digital Subscriber Line
EHF	Extremely High Frequency
ESA	European Space Agency
FFT	Fast Fourier Transform
HFC	Hybrid Fibre Coax
IEEE	Institute of Electrical and Electronics Engineers

ITU-R	International Telecommunications Union - Radiocommunications
JWD	Joss-Waldvogel Disdrometer
LMCS	Local Multipoint Communication Services
LMDS	Local Multipoint Distribution System
LOS	line-of-sight
mm-wave	millimetre-wave
MMDS	Multichannel Multipoint Distribution System
MRO	McGill Radar Observatory
MTPL	maximum tolerable path loss
NLOS	non line-of-sight
pdf	probability density function
POSS	Precipitation Occurrence Sensor System
RF	radio frequency
Rx	receiver
SIR	signal-to-interference ratio
SS	synthetic storm

SSE **sum of squared errors**

ST **subscriber terminal**

Tx **transmitter**

UHF **Ultra High Frequency**

Glossary

absorption cross section: Area of a surface that when multiplied by the power intensity of the incident plane wave equals the power dissipated as heat in a water drop.

angular diversity gain: Diversity gain obtained by employing multiple radio links converging to one node in a diversity mode.

area-averaged rainfall rate: Average of point rainfall rate measurements over an area of interest, usually area of a radio cell.

area-weighted path reduction factor: Form of path reduction factor adopted by Tan's coverage prediction model, which is derived by taking into consideration the cell area size and the area-averaged rain rate statistics.

base station: Node in a cellular radio network which serves as radio interface unit and provides radio coverage to subscribers in a radio cell area. It is often referred to as *hub*.

cell area coverage: Fraction of the area of a cell in which subscriber terminals can receive transmitted radio wave from the hub with power greater than the receiver threshold.

cell-site diversity: Link performance improvement technique for fixed cellular systems in which a subscriber terminal can choose one of multiple hubs associated with the highest received power. This term is used herein to refer to the use of such a scheme for mitigating the impact of rain attenuation and to distinguish it from macrodiversity, which in principle is the synonym, that is aimed at combating the effects of shadowing loss in

mobile cellular systems.

clear-sky attenuation: Radio wave attenuation undergone by a radio link under clear sky conditions, generally due only to distance-dependent propagation loss and shadowing.

clear sky conditions: Conditions of the environment of propagation path in the absence of precipitation. Under this condition, the only phenomena that might decrease the received power on a radio link are distance-dependent loss and shadowing.

clear-sky coverage: Coverage of radio cell under clear sky conditions.

convective rain: Type of rain events characterised in terms of spatial and temporal variabilities by the presence of high intensity rain confined in small areas referred to as rain cells interspersed with low-intensity areas, all of which change rapidly and last for a short duration.

converging links: Multiple radio links of a common end.

distance exponent: Exponent of distance in the power-law relation of mean path loss and distance.

diversity gain: Decrease in attenuation for a specified percentage of time resulting from the application of diversity with respect to single-link cases.

extinction cross section: Sum of absorption and scattering cross sections of a water drop, that is area of a surface that when multiplied by the power intensity of the incident plane wave equals the total power loss due to collision with the water drop. See also

absorption cross section and scattering cross section.

fade margin: Amount of attenuation in excess of a predicted value that the receiver can experience without the received power falling below the receiver threshold.

fading: Condition of a radio link during which the received power is lower than the receiver threshold.

fixed cellular system: Cellular radio systems in which subscriber terminal locations are permanent.

free space path loss: Attenuation under LOS conditions in the absence of multiple paths, characterised by a power-law relation between the attenuation and distance with propagation exponent of two.

hub: Synonym of *base station*.

line-of-sight (LOS): Condition in which the transmitter and receiver of a radio link can see each other, i.e. there is no obstacle along the propagation path.

lognormal shadowing: Shadowing environment characterised by induced loss that is spatially varying according to lognormal distribution.

macrodiversity: A link improvement technique for cellular systems to combat the effects of shadowing loss, in which a subscriber terminal can choose one out of multiple base stations that gives the best received power. See also *cell-site diversity*.

millimetre-wave: Frequency spectrum that, loosely defined in this thesis, ranges from 10 to

60 GHz.

mixed LOS/NLOS: Condition of a radio cell environment in which some subscriber terminals are in LOS while some others are in NLOS with respect to the hub.

non line-of-sight: Condition in which the propagation path between the transmitter and the receiver is obstructed.

normalised coverage: Area coverage of a cell under precipitation compared to that under clear-sky conditions.

point rain rate: Rainfall rate measured at one point in space, usually refers to rain intensity measurements made using a raingauge.

point-to-multipoint: Adjective referring to radio communications configuration in which one node, usually a part of the network such as a hub, communicates with multiple nodes, typically subscriber terminals.

point-to-point: Adjective describing radio communication systems in which a terminal communicates with one and only one other terminal. Example includes backbone radio links connecting hubs in a radio cellular network.

power-law relation: Relation of two quantities in the form of $Y = aX^b$, where X and Y are the quantities, a and b parameters of the relation.

path loss exponent: Synonym of *distance exponent*.

radio cell: Area in a cellular system served by a hub that provides coverage to subscribers

located in the cell area.

rain cell: Small region in an area experiencing convective rain in which high intensity rain is confined, characterised by the presence of rain intensity peak(s) within the cell area and surrounded by a more widespread low-intensity rain.

receiver threshold: Minimum received power detectable by the receiver.

reliability: Fraction of time during which a system functions satisfactorily. In the context of radio communication link, it is the fraction of time during which fading does not occur.

scattering cross section: Area that when multiplied by the power intensity of the incident plane wave equals the total power scattered by the drop in all directions other than that of the incident ray.

selection combining: One of linear combining techniques for diversity in which the output of the diversity system is selected from the branch with the best performance, e.g. the best SNR.

stratiform rain: Type of rain events characterised by the homogeneously low rain intensities over a large extent of space and duration.

subscriber terminal: Radio interface unit of a radio cellular subscriber.

synthetic storm: Method of simulating spatial variation of rain specific attenuation from point rain rate time-series measurements by applying wind velocity that shifts the point measurements along the radio link of interest.

Chapter 1 - Introduction

1.1 Background

The past decade has witnessed the rapid progress of technologies enabling various multimedia communication services. Increasing demand for these new services has driven exploration into broadband access techniques capable of delivering high rate information to a large number of users. A number of such techniques that utilise twisted-pair copper wires with various DSL (digital subscriber line) technologies, HFC (hybrid fibre coaxial) cables using the existing CATV (Community Antenna Television Service, or cable television) network, millimetre-wave radio, or optical fibres have been proposed. Each has its own technical and economical strengths and weaknesses [Gag97, Sam00].

More recently, interest has also been growing steadily in the development of systems that can provide wireless access to broadband services for fixed subscribers over wide coverage areas, involving many buildings. Such terrestrial wireless systems offer a number of advantages, including the low cost and relative ease in deployment and configuration, when compared to their wired counterparts. A number of new systems such as MMDS (Multichannel Multipoint Distribution Service) have been introduced for the aforesaid purpose and were designed to operate in frequency bands around 2–3 GHz [Bie00, Bol01]. A typical MMDS system, when coupled with the application of cellular network architecture with sectored antennas, can support aggregate data rates up to around 37 Mbps per radio channel and a maximum of 10 Mbps per user with typical cell radius as large as 40 km [Bie00].

Much higher data rates, however, are expected out of a family of cellular-based broadband wireless systems operating in the spectral region between 10 GHz and 60 GHz generally termed as LMDS (Local Multipoint Distribution Systems). Projected signalling rates deliverable over LMDS radio channels range from 2 through 155 Mbps, with exact rates dependent on the type of individual subscriber terminals and the desired type of applications [Pet99]. In Canada, a similar system known as LMCS (Local Multipoint Communication Services) is also being developed. In North America, the standardization efforts by IEEE were preceded by auction of licensed EHF bands around 30 GHz intended for such broadband wireless access applications [Mar99, Pet99]. In Europe, different portions of EHF bands, namely those around 40 GHz and 25 GHz, have been allocated for use with LMDS systems [Nor00].

The Extremely High Frequency (EHF) bands offer high spectral bandwidth, which translates to high capacity channels. A combination of these high-capacity RF channels and frequency reuse inherent in cellular networking schemes leads to an enhancement in overall system capacity. This is especially true when many radio cells of smaller dimension are used, which is more possible to happen at EHF as a result of the high power losses imposed by the millimetric wavelength.

The use of extremely high frequencies, while providing greater bandwidth, immediately raises new problems pertinent to propagation [Bul98]. Not only do EHF radio waves undergo immense loss due to the smaller effective aperture of antennas operating at higher frequencies [Siw95], but they are also easily weakened or completely blocked by relatively small obstructions between link terminals. In addition, they can as well be heavily

attenuated by intense rain having large concentrations of drops with a relatively large size compared to the wavelength.

The latter phenomenon is particularly of interest when the influence of rain is put into the context of point-to-multipoint, cellular radio networks, in which spatial statistics of rain attenuation are as important as single-point temporal statistics. In fact, all of the aforesaid factors limit the range of wireless access links in 30 GHz LMCS/LMDS systems to less than 5 kilometers, and in many cases, less than 1 or 2 km. This is especially true if these systems are required to maintain very high reliability. While the use of such redundancy schemes as channel encoding can facilitate to some extent the achievement of such a goal, design of a new EHF wireless access network must thoroughly take into account the impact of shadowing losses and rain attenuation in the link budget. Each of these phenomena can give rise to degraded performance of individual subscriber links or reduced coverage area of serviced cells. The impact of obstruction and diffraction by buildings and trees in a cellular environment that might contribute to the cell coverage reduction has been a central topic of many research projects and was a topic of another doctoral research at Carleton University [DeC00]. However, only a few reports from one research group can be found that addresses the impact of rain on coverage [Tan98, Tan00].

Since area coverage is of great interest in cellular-based systems, the research reported herein has the objective of determining an area coverage analysis technique that can be used to predict the percentage of area covered by radio cells under precipitation on a statistical basis. The area coverage under precipitation is presented as degradation from that under clear-sky conditions.

In addition, the application of cell-site diversity to improve the base-to-subscriber link performance during rain, which is generally thought of as a solution to the coverage degradation issue, requires examination of the diversity gains possible with scenarios in which two base stations can serve each subscriber. These objectives will be restated in the next section, along with more details concerning the problems that were expected to be encountered in their achievement and a succinct description of the research method to be used.

A special feature of the work reported herein is the use of radar measurements of the radar reflectivity factor of rain. Radar measurements, capable of capturing the spatial inhomogeneity of rain intensity, were used as the basis for the link attenuation calculations. To build the link attenuation database, hypothetical links and radio cells were simulated over the radar measurement area. An important issue that was studied in this thesis is the procedure for analysing radar-measured reflectivity factors to yield the equivalent specific attenuation, which is the quantity of interest in the link simulations. The results from this study provided a preliminary step in using the radar records for rain attenuation simulations and cell area coverage analyses to follow.

Although LMDS was mentioned above as a class of systems that might benefit from the results of this study, our investigation is executed without any reference to specific system architecture. Consequently, the results are very general in nature and hence applicable for use in the design of any system that incorporates point-to-multipoint radio accesses at 30 GHz.

1.2 Problem statement, formulation and methodology

Three issues were investigated:

a. The relation between the radar reflectivity factor and the specific attenuation of rain

Study of this was undertaken for a number of reasons. First, it was considered necessary to review the power-law function relating radar-measured reflectivity factor and the specific attenuation of rain at mm-wave by applying raindrop size distribution (DSD) measurements made using modern measuring apparatuses, i.e., the JWD (Joss-Waldvogel Disdrometer) and POSS (Precipitation Occurrence Sensor System). The use of these DSD measurements was intended to improve the accuracy in the power-law coefficients obtained accordingly, in comparison with those computed from DSD models such as Laws-Parson's (L-P) or Marshall-Palmer's (M-P) which were derived from measurements made using older techniques of DSD measurement.

In addition, as the DSD measurements were recorded in Montreal and Toronto areas, the resulting power-law coefficients should give the best approximation for the relation for use with the Montreal radar data. In brief, the problem examined was to determine the most appropriate values of parameters c and d in relation

$$Y = cZ^d \quad (1-1)$$

where Y denotes the specific attenuation of rain (dB/km) at 30 GHz and Z is the radar reflectivity factor (mm^6/m^3). The criteria used for determining the best values were the minimum of the sum of squared errors that result from log-linear fitting of the form given by equation (1-1) to pairs of Y and Z values computed from DSD measurements.

b. Statistics of cell area coverage

The issue of cell area coverage in rain has only recently received attention owing to the fact that for terrestrial systems EHF band frequencies have never before been used to provide area coverage. The problem arises since radio links at EHF are highly attenuated by rain implying a potential temporary obstacle to reception in regions within the area. Therefore the objective, stated properly, is to determine statistically the reduction, with respect to that under clear sky conditions, in the percentage of area within a cell radius that can be covered under rainy conditions. The uncovered areas are those having power reception that is weakened by rain such that reliable operation is not possible. In this regard, unreliable operation simply means power reception below a received power threshold, at which signal power or SNR is too low to guarantee adequate service.

Since the dependence of cell coverage on the variation in rain rate statistics for different regions (*i.e.* change in rain region) is unknown and beyond the scope of this thesis, it would be conservatively assumed that the resulting model applies only for climatic areas similar to that in the vicinity of Montreal region, where the radar measurements were made. Therefore, the problem addressed was to find a model in the form

$$C = f_1(p, M, S) \Big|_{f=30 \text{ GHz, Montreal}} \quad (1-2)$$

where C denotes area coverage, M the specified fade margin, p the percentage of time during which the area coverage should exceed C , and S the service cell area.

It was considered that a more general model should take into consideration both rain attenuation and shadowing loss. Hence, it was necessary to develop a technique that allows

the computation of cell area coverage under the combined effects of shadowing and rain attenuation. In the analysis, a statistical model of cell area coverage based on lognormal shadowing assumptions was adopted to complement the rain attenuation obtained from simulations. Two parameters were used to describe the effects of lognormal shadowing loss in the model, namely the exponent α of distance in the power-law function that describes the increase of mean path loss with distance and the standard deviation σ of the lognormal shadowing loss. Since the simulations assumed circularly-shaped radio cells with the base station or hub located at the centre, the service area is represented by the cell radius L . In addition, a quantity referred to as normalised coverage C_{norm} was used to describe the portion of cell area still covered when it rains compared to that under clear sky conditions. Hence, the desired model of cell area coverage is of the following form.

$$C_{norm} = f_2(p, M, L, \alpha, \sigma) \Big|_{f=30 \text{ GHz, Montreal}} \quad (1-3)$$

While the resulting model has limitations in the region and frequency spectrum of its applicability, the technique of analysing the joint impact of shadowing and rain attenuation developed in the reported work can be applied to studies of similar type for other frequencies, using weather radar data made in other regions.

c. Diversity gain for cell-site diversity

The research reported herein addressed the determination of diversity gain experienced by the combination of two links that converge into one radio station and operate in a selection diversity mode. The results of this study are expected to be useful in

considering the application of cell-site diversity in fixed cellular systems to enhance the subscriber's radio access link reliability in a rainy situation.

The model of angular separation-dependent diversity gain that was sought was conceived as having such inputs as the percentage p of outage time of interest, the lengths L_1 and L_2 of radio links serving as the diversity branches and the angle θ between the branches. It was to be of the form

$$G = f_3(p, L_1, L_2, \theta) \Big|_{f=30 \text{ GHz, Montreal}} \quad (1-4)$$

To arrive at such a model, configurations of converging links with different or equal lengths were simulated on radar images. Cumulative probability distribution functions (CDFs) of attenuation were computed for individual links and for combinations of two links operating in diversity mode with selection combining. From these CDFs, diversity gain as a function of link separation angle in any two-link arrangement could be determined.

1.3 Summary of results

The most important results of the work reported in this thesis are summarised below.

a. On the relation between the radar reflectivity factor and the specific attenuation of rain

- (i) Uncertainties in the measurements of small drops by JWD and POSS measurement systems were not critical to the Y - Z and Y - R relationships at frequencies around 30 GHz or less.

- (ii) The Y - R power-law coefficients, given in Table 3-4 for vertically and horizontally polarised waves, were found to differ slightly from those recommended by ITU-R computed from Laws-Parson DSD measurements.
- (iii) Statistics of path attenuation computed using the power-law relationship were not crucially dependent on spatial averaging, as shown by overlapping ranges of confidence between CDFs of attenuation on links of various lengths derived using the Y - Z relationships and direct computation from the measured DSDs.

b. On the statistics of cell area coverage

- (i) Evaluation of coverage using the radar data and a technique that considers the combined effects of rain attenuation and lognormal shadowing is reported in Chapter 4. Normalised coverage under rain, when expressed as percentage of coverage under clear sky conditions and evaluated for a constant margin at the edge of the cell, was not sensitive to variation in the path loss exponent and lognormal shadowing loss standard deviation. On the other hand, cell dimension and the available fade margin, both of which are system parameters, were found to heavily influence the amount of reduction in coverage under rain when compared to that under clear sky conditions.
- (ii) A model in the general form given by (1-3) was developed using the CDF of simulated cell coverage to predict normalised coverage under rain. This is given by the set of equations (4-20) and (4-21) for various percentages of time, with coefficients presented in Table 4-11.

c. On the diversity gain for cell-site diversity

- (i) Examination of the correlation of attenuation between two links showed that cell-site diversity under rainy conditions would be equally effective within the 100° – 260° range of angular separations between them, and would perform more effectively in heavy storms than in light rains.
- (ii) A new model given by equation (5-6), which is more general in form compared to Tan's model [Tan00], was proposed for prediction of diversity gain expected from a combination of link lengths and separation angle at a given time percentage.
- (iii) Values of the parameters of the new model, $G_{180^{\circ}}$ and k , each a function of link length, angular separation and probability level, were derived by fitting the model to simulation results obtained for different reference links and are given in Table 5-1. Based on these values, a set of criteria for determining subscriber locations for which cell-site diversity would be potentially beneficial was specified.

1.4 Research significance

As stated previously, the analyses of DSD measurements in Chapter 3 were of the purpose of addressing the Y - Z relations to be used in the later parts of this study. The contributions from this part of the study were new Y - Z power-law coefficients derived using modern DSD measurements, as opposed to those based on L-P or M-P DSD models, as well as new statistics of rain attenuation, intensity and radar reflectivity factor collected from Montreal area.

The cell area coverage analysis presented in Chapter 4 provided new contributions in the evaluation of area coverage at EHF under rainy conditions since research in this subject had not been done before, with the exception of Tan's work [Tan98] that we only became aware of after the reported work had been started. This work also introduced a technique to assess the combined effects of rain attenuation and shadowing loss on cell area coverage, by adopting Jakes' approach to coverage prediction for an environment with lognormal shadowing [Jak94]. The impact of the two detrimental factors in combination on coverage had never been studied previously. Tan's work merely dealt with the impact of rain attenuation, with all the subscribers assumed to be uniformly under LOS conditions. The results of the research reported herein, hence, provide a new method for determining mm-wave cellular coverage during precipitation when the environment imposes a non-ideal case of mixed LOS/NLOS conditions for subscriber terminals in a cell. A coverage prediction model was developed subsequently, which is applicable for 30 GHz cellular systems with circular cells.

The study on cell-site diversity in Chapter 5 produced a model for predicting the diversity gain experienced by a subscriber terminal in a cellular radio network that employs cell-site diversity. The model is of a more generic form and shows an improvement in accuracy compared to the one proposed by Tan *et al* [Tan00] that is currently adopted by ITU-R in their recommendation on the design of mm-wave terrestrial broadband radio access systems [ITU99]. Also, various combinations of two diversity links, either identical in length or otherwise, were examined, providing a more thorough examination of the cell-site diversity issue than the previous work reported in this field.

1.5 Thesis organization

This thesis is organised into six chapters. Following this introductory chapter, a general literature review of topics to be discussed in later chapters is given. These include mainly issues of propagation under shadowing conditions and/or precipitation.

Chapter 3 deals primarily with the derivation of power-law coefficients of the Y - Z relation making use of DSD measurements made in Montreal and Toronto. At the end of this chapter, a brief description of the use of Montreal radar measurements for simulating rain attenuation in radio cells is presented.

Chapter 4 starts with reviews of previous work in quantifying cell area coverage under the effects of shadowing loss or rain attenuation. The second half of this chapter discusses the computation of cell coverage statistics from the radar-based simulations while assuming lognormally distributed shadowing losses. It is concluded by the specification of prediction model for use in estimating area coverage of circular cells, given the parameters of the cell and the shadowing environment.

Chapter 5 describes the evaluation of selection diversity systems for two-link configurations using radar measurements. Angular correlation function and diversity gain as a function of separation angle are examined and discussed. Based upon the latter, a new model is presented and compared to the model previously proposed by Tan *et al.* Finally, an example is given of how the spatial characteristics of rain attenuation reflected by the new model determine the locations of potential users of cell-site diversity in a cellular network.

Finally, conclusions and suggestions for future topics are presented in Chapter 6.

Chapter 2 - Relevant Literature Background

2.1 Introduction

This chapter contains no original contribution and is intended to highlight background information and provide the necessary basis for analyses reported in subsequent chapters. The material covers subjects in the field of radio wave propagation relevant to the problem being studied. In particular, those on the impact of rain attenuation on radio wave propagation, in the portion of the millimetre-wave band considered for use in terrestrial point-to-multipoint communication systems are reviewed. This band is loosely defined here as ranging from about 20 to 60 GHz. The background material, which can be a theoretical foundation or an experimental result, is cited from reports of previous work conducted by other researchers or from relevant textbooks.

The review begins with theoretical background for radio attenuation under clear sky conditions. Next, the method for quantifying rain attenuation is described, followed by a discussion on the spatial variability of rain. To conclude the chapter, the issues of mm-wave radio cell coverage and the feasibility of cell-site diversity are discussed, with references to the spatial variation of rain attenuation.

2.2 Clear sky attenuation

Due to spreading, a radio wave propagating through a medium such as the earth's atmosphere experiences power loss. In "free space", where there are no structures interacting with the radio wave during its travel toward a receive antenna, the power loss increases as a

quadratic function of the propagation range. The power P_r received at a distance r from the transmit antenna is given by [Jak94]

$$P_r = \left(\frac{\lambda}{4\pi r} \right)^2 g_t g_r P_t \quad (2-1)$$

where P_t denotes the transmitted power, r the distance in metres, λ the wavelength in metres, and g_t and g_r gains of the transmit and receive antennas, respectively. Hence, the free space path loss A_{fs} is in linear proportion to the square of distance, i.e. $A_{fs} \sim r^2$. If a receiver at distance r_1 from the transmitter experiences a free space path loss of A_{fs1} dB, the path loss A_{fs2} experienced by a receiver at r_2 in free space can be calculated as

$$A_{fs2} = A_{fs1} + 20 \log_{10} \left(\frac{r_2}{r_1} \right). \quad (2-2)$$

In actual terrestrial radio communication systems, such as mobile or outdoor wireless cellular systems, a similar form of expression for the power loss is commonly used, usually with distance exponent larger than two. For instance, a model for line-of-sight (LOS) propagation with a specular reflection from the planar earth's surface suggests the use of a two-segment power-distance curve separated on the distance axis by a breakpoint associated with the edge of the region where there is the first Fresnel zone clearance [Feu94]. In the path segment closer to the transmitter a distance exponent near two can usually be expected, whereas the loss at distances beyond the breakpoint obeys larger exponent, usually close to four. In mobile cellular systems, an exponent of four is actually predicted beforehand for subscriber terminals that are sufficiently distant from the base station to cause a ground reflection at a grazing angle [Jak94].

Deviation from the free-space quadratic form can also happen when a radio wave propagates under non line-of-sight (NLOS) conditions due to obstacles, such as buildings, hills, or trees, encountered along the propagation path. In this scenario, the transmitted signal can arrive at the receiver through several different mechanisms such as diffraction, reflection, or transmission through the obstructing media. This usually results in a steeper increase of power loss with distance compared to free-space, i.e. distance exponent of larger than two [Feu94].

A statistical model that is most commonly used to describe the spatial variation of power loss includes the adoption of lognormally distributed shadowing loss, that comprises the average effects of multipath addition and/or obstruction shadowing described in the previous two paragraphs. The median of the shadowing increases with distance in a power-law manner with a distance exponent higher than two. The model can be expressed mathematically by the following pair of expression:

$$A_{\text{shadowed}} \sim N(\mu(r), \sigma) \quad (2-3)$$

where

$$\mu(r) = c_2 + 10\alpha \log r \quad (2-4)$$

In the above, A_{shadowed} is the power loss in decibels in an environment with shadowing, $N(\cdot, \cdot)$ denotes Gaussian distribution with range-dependent median $\mu(r)$ and standard deviation σ , and α represents distance exponent. It can be seen that at a distance r from the transmitter, the variation of path loss can be adequately described by two parameters, α and σ .

This model was used in our analysis of coverage reported later in Chapter 4, following the method introduced by Jakes [Jak94].

2.3 Rain attenuation at mm-wave frequencies

2.3.1 Quantification of rain attenuation

When electromagnetic waves interact with falling raindrops, a number of phenomena can result. These include attenuation, wave depolarization, and scattering [Bru95, Now77, Ogu83]. The overall effect immediately observable on a radio link is degradation in the quality of communications, which can take a form of weakened signal reception, inter-channel interference in a dual-polarization system, or interference with other systems occupying the same spectral region. Among these rain-induced impairments, attenuation is the most influential on the success or, rather, failure of communications over radio channels, particularly at microwave and millimeter-wave frequencies [Cra80].

Theoretical study into the quantification of rain-induced attenuation starts with an examination of how an electromagnetic wave behaves when encountering a single rain drop. The concepts of absorption and scattering cross sections have been introduced to quantify the extent to which a rain drop of a given size depletes energy from a radio wave impinging upon it [Bat73, Ogu83]. The absorption cross section Q_a is the area of a surface that when multiplied by the power flux density of the incident plane wave will be equal to the power dissipated as heat in the water drop. The scattering cross section Q_s is the area that when multiplied by the incident power flux density will result in the total power scattered by the drop in all directions other than that of the incident ray. These two quantities together

constitute the total cross section tantamount to the total power loss of the radio wave, a quantity commonly known as the extinction cross section Q_{ext} . The value of this quantity depends on the shape and size of the drop, the wavelength and polarization of the radio wave with respect to the drop orientation, and the water temperature.

Raindrop sizes vary up to approximately six millimetres. A radio wave of a certain wavelength behaves differently when interacting with drops of different sizes. For the case of spherical drops, when the drop is relatively small compared to the wavelength, a simple expression for each of the absorption and scattering cross sections is given by Rayleigh's scattering formula [Bat73]. This formula is applicable in a range of size parameter $\alpha = \pi D/\lambda$ that satisfies $\alpha \ll 1$ known as the Rayleigh region, where D denotes the drop diameter and λ the wavelength, both given in the same unit. In this region, the extinction efficiency factor $K_{ext} = Q_{ext}/(\pi D^2/4)$ can be approximated by a linear function of the size parameter, and the absorption component resulting from the interaction dominates over the scattering component [Pen62]. This dominance diminishes, as drops become larger. When drop sizes are comparable to the wavelength, which is the case for wavelengths of 3 centimeters or less, non-linearity in the relationship between the extinction efficiency factor and the size parameter starts to arise, and hence a more complex representation of the extinction cross section is required. Mie's scattering equation provides the exact solution in this region of wavelength known as the Mie region. The equation expands the extinction cross section into a series with terms containing Mie's scattering coefficients. A number of algorithms have been written and encoded for use in fast computation on computers of these coefficients [Dei69, Rog76a, Fla].

At any point in space the specific attenuation, the magnitude of radio wave attenuation in dB per unit distance, can be calculated by taking into account the distribution of drop sizes measured at that point and integrating the extinction cross section over the entire range of possible diameters [Ogu83]. The specific attenuation Y in dB/km is given by

$$Y = \frac{10^{-2}}{\ln(10)} \int_D Q_{ext}(D) N(D) dD \quad (2-5)$$

where $N(D)dD$ in m^{-3} is the number per unit volume of drops having diameter D in millimetres within the $[D-dD/2, D+dD/2]$ range, and the extinction cross section $Q_{ext}(D, \lambda, t)$ expressed in mm^2 is a function of diameter, wavelength λ , and water temperature t .

Equation (2-5) can be expressed in another form, which is preferred for computing specific attenuation experienced by a vertically or horizontally polarised wave when it encounters oblate spheroidal raindrops, can be computed as [Ish78]:

$$Y_{V/H} = \frac{2 \times 10^4 \lambda}{\ln(10)} \int_D \text{Im}[f_{V/H}(D)] N(D) dD \quad (2-6)$$

where λ is the wavelength in metres and $f_{V/H}(D)$ represents the forward scattering amplitude in metres, with $\text{Im}[\cdot]$ denoting the imaginary part and subscript V/H indicating the wave polarization. The forward scattering amplitude is related to the extinction cross section through the expression: $Q_{ext}(D) = (4\pi/k) \text{Im}[f_{V/H}(D)]$, with $k = 2\pi/\lambda$ being the wave number. Once the axial ratio of the oblate raindrop is determined and the diameter D in millimetres of the equivolumic sphere is specified, the forward scattering amplitude can be computed using the point matching technique [Ogu73] for both vertically and horizontally polarised waves.

In the work reported herein, a modified form of a Fortran code that employs the point matching technique originally written by researchers from European Space Agency (ESA)¹ was used. The code, which was originally designed to compute the forward scattering amplitude for a fixed set of drop sizes, was modified so that it could evaluate the discrete values of raindrop diameters employed by raindrop size distribution measuring apparatuses used in the work reported in Chapter 3.

Having the knowledge of drop size distribution, one can as well theoretically derive two other important parameters that will frequently be referred to herein. The first is rainfall rate R , commonly expressed in mm/h, which can be calculated as [Ogu83]:

$$R = 6 \times 10^{-4} \pi \int_D D^3 v(D) N(D) dD \quad (2-7)$$

where $v(D)$ is the terminal fall velocity in m/s of drops of diameter D . The other is the radar reflectivity factor Z , given in mm⁶/m³, which is defined as [Bat73]:

$$Z = \int_D D^6 N(D) dD \quad (2-8)$$

The integrands in equations (2-5) through (2-8) are powers of raindrop diameter multiplied by either the raindrop fall velocity or the extinction cross section, each of which can also be roughly approximated by some power of the drop diameter [Ul83]. Therefore it

¹ The Fortran code "esa_pm.for" was used and modified by courtesy of Dr. J.P.V. Poiarés Baptista of the ESA on behalf of the other co-authors: Dr. A.W. Dissanayake, who is the original author, Dr. M. Schnuber and Dr. W. Lechner. We gratefully thank them for the kind permission.

is quite reasonable to expect the existence of a compact relationship between each pair of these quantities. A power-law form has been adopted to describe these relationships. By examining the results from field experiments or by applying one of several existing models for drop size distributions, researchers have attempted to derive values for coefficients of power law relations in the form of [Bat73, Ols78]

$$\begin{aligned} Y &= aR^b \\ Y &= cZ^d \end{aligned} \quad (2-9)$$

Relations of this form are commonly used in predicting statistics of rain attenuation when only measurements of one or the other of these parameters are available. The values of coefficients a , b , c and d depend upon a number of factors, primarily frequency, orientation of wave polarization with respect to the raindrop shape, and raindrop size distribution (DSD). The most commonly adopted model of DSD is the one derived by Marshall and Palmer from Laws and Parsons' and their own measurements [Bat73]. It is given by

$$N(D) = 8000 \exp(-4.1R^{-0.21}D), \quad (2-10)$$

where $N(D)$ denotes the number of raindrops having equal-volume spherical diameter D mm per m^3 volume per unit size interval, and R denotes rain rate in mm/h. This model was used, for example, to derive the Z - R relationship [Bat73]:

$$Z = 200R^{1.6} \quad (2-11)$$

for stratiform rain events.

2.3.2 Effects of rain attenuation on radio waves

Many publications have provided typical values of rain attenuation at various wavelengths and rain rates for various distributions of raindrop size [Bat73, Fre97]. A typical estimate of specific attenuation computed using an empirically derived relationship that involves the Laws-Parson drop size distribution model at 30 GHz (or equivalently, 1-cm wavelength) is 5 dB/km at rain rate of approximately 25 mm/hr [Fre97]. Now assuming a uniform rainfall along a one-kilometre radio link having that value of specific attenuation, the total power loss due to rain is equal to 5 dB. In reality, however, rain rate and specific attenuation vary with time and space, leading to a random variation of the radio link power loss. An extra margin on the power link budget to account for rain attenuation therefore needs to be specified such that it can accommodate for the rain attenuation for a specified percentage of time. Work in this area has been done in the past and resulted in rain attenuation distributions [Seg82] and prediction models [Bru95, Cra80, Cra96, Fre97]. A brief overview of the spatial variation of rain is given in the next section.

2.4 Spatial variability of rain

2.4.1 The cellular structure of high intensity rain

Studies using radar observations have revealed that within a precipitation region, two distinguishable types of rain can occur. High intensity rain, which induces high attenuation, is concentrated in small areas, commonly referred to by meteorologists as rain cells, and is usually ascribed to convective rain events. Such cells have one or more rain rate maxima within their boundaries and typically last for less than 15 minutes [Cra96]. Regardless of the

definition used to determine their boundary, be it a -3 dB contour in reflectivity factor [Cra96], or a contour of a given rain intensity [Bon98], these cells are irregularly shaped. Important spatial parameters include radius of equal-area circle, distance between nearest cells, maximum rain intensity in the cell and its decay as a function of distance [Cra96]. Various rainstorm models to be used in path attenuation simulation and prediction have been proposed, many of which assume ideal rain cells with circular shape on the horizontal cross-section [Rog76b]. Other field-based studies have emphasised the statistics of rain patches, which are the segment lengths of a radio path that intersects a rain cell, computed on the basis of either radar observations or synthetic storm simulations using rain gauge measurements [Bon98].

The other type of rain area includes the regions outside high-intensity cells, sometimes referred to in the literature as debris or filling areas [Cra96, Bon98]. These areas are characterised by almost uniformly widespread low-intensity, longer-lasting rain.

Naturally, in microwave and mm-wave radio communications, high-intensity rain is of greater interest. Since rain intensity is known to vary in space, the interest becomes even greater when the mm-wave systems in question operate in point-to-multipoint configuration for which area coverage becomes an important factor of its success. In addition, the knowledge that high-intensity rain events tend to be confined in small areas or cells raises an interest in the use of cell-site diversity to overcome the effects of high attenuation due to rain in a mm-wave point-to-multipoint system with cellular radio networking. Hence, subsequent sections will briefly present the statistics that describe such events.

2.4.2 Rain cell statistics

Much work has been done by atmospheric scientists and weather researchers to statistically characterise rain storm cells. The results of two of many projects in this field are described below to give a rough illustration of the spatial variation of rain cells.

a. Crane and Hardy [Cra96]

Crane and Hardy analysed a series of weather radar observations collected in Kansas, Texas during a period equivalent to 25 days of summer, convective storms. In the analyses, a volume cell was defined as an area enclosing a reflectivity maximum, bordered by the -3 dB contour. Multiple volume cells can occur in clusters, forming showers with multiple cells.

The radar study suffered from a bias in the statistics of the number of cells, rain intensity, and cell size and lifetime due partially to the decrease of spatial resolution with distance from the radar system and to the non-zero elevation angle of the antenna. Nevertheless, estimates of the desired rain cell parameters were acquired by applying a couple of different methods to overcome the problems. One method consisted of computing individual averages for different ranges of distance from the radar and applying a linear fit to obtain the zero-distance intercept. The other involved computation of a mathematical model of the joint distribution of square-root area and either reflectivity or rain intensity, from which the average cell area could be derived by taking the expected value of the area over the ranges of both of the quantities involved. From these methods, they found the values in the following table to be the best estimates.

Table 2-1 Best estimates of rain cell parameters from Crane [Cra96]

Average rain cell parameter (unit)	Best estimate
Area (km ²)	1.0
Linear extension (Square-root area, km)	0.9
Spacing (km)	5.5
Rain rate (mm/h)	22.3
Lifetime (min)	9.9

b. Bonati and Binaghi [Bon98]

Bonati and Binaghi examined rain cell storms recorded by an S-band meteorological radar system in 1980 and 1989 in Milan. The evaluated parameters include the dimension of rain cells and patches, obtained respectively by identifying areas confined within closed contours of rain intensity of interest and by simulating 70-km radio links using only the 1989 data, as well as spacing between cells or patches. A rain patch was defined as a segment of a radio link that experiences rain rate higher than a given value. Besides using the radar data, rain patches on a radio link were also simulated by means of synthetic storm (SS) techniques using rain gauge measurements made in 1989. An excerpt from the results for various methods is tabulated below [Bon98].

Table 2-2 Statistics of rain cell parameters from Bonati [Bon98]

Data source	Rain rate contour (mm/h)	Probability [Parameter < x]	Parameter	
			Diameter (km)	Spacing (km)
Rain cell identification, using radar data from 1980	20	0.5 (median)	3.0	
		0.9	7.0	
	40	0.5	2.5	
		0.9	5.8	
Rain cell identification, using radar data from 1989	20	0.5	2.0	
		0.9	5.0	
	40	0.5	1.5	
		0.9	5.0	
Rain patch identification on simulated 70-km links, using radar from 1989	20	0.5	2.0	2.5
		0.9	5.0	10.0
	40	0.5	1.5	2.5
		0.9	4.0	12.5
Rain patch identification on SS simulations, using rain gauge data from 1989	20	0.5	2.0	
		0.9	7.5	
	40	0.5	1.5	
		0.9	5.2	

2.4.3 Discussion

Bonati's findings indicate that rain cells containing rain intensities of at least 20 mm/h, equivalent to a specific attenuation of 3.4 dB/km for a vertically-polarised wave at 30 GHz when the *Y-R* coefficients given in Chapter 3 is applied (see Table 3-4), are generally of 2-3 km median diameter. Higher intensities are confined to yet smaller areas. Crane's linear size estimate for cells enclosing areas with average intensity of around 22 mm/h is 0.9 km. In addition, the spacing between the closest cells could be as large as 5.5 km.

These results show, firstly, that rain intensity tends to vary spatially, especially when showers occur in a small region. This observation implies that in assessing the effect of rain attenuation on the coverage of a radio cell having a radius of 1-5 km (a diameter of 2-10 km), it is not appropriate to assume a spatially uniform rain rate over the entire radio cell area. A better method is to take account of the spatial variability of rain attenuation, either by applying a storm model or by using the results of rain rate measurement over the area of interest. The latter approach can be achieved by using weather radar measurements of reflectivity factors and converting them to the equivalent values of rain specific attenuation.

Secondly, the relatively small size of high-intensity rain cells and the comparatively large spacing among them mean that two different parts of the same region might experience different rain rate. This implies further that there could be an advantage in employing cell-site diversity techniques for subscribers of a fixed cellular system.

2.5 Summary

The objective of writing this chapter was to present a literature overview of subjects that are frequently referred to in the discussions in later chapters. These are summarised below.

- a. Propagation under clear sky conditions can be influenced by shadowing loss, which is commonly modelled as being lognormally distributed with its mean increasing with Tx-Rx distance in a power-law fashion. In applying this model, a specific environment is characterised by two parameters: the distance exponent of the power-law relation and the standard deviation of the lognormal shadowing loss. When there is no obstruction and

multipath, path loss obeys a deterministic power-law relation with distance exponent of two.

- b. Given the raindrop size distribution, such measures of precipitation as rain intensity, radar reflectivity factor and specific attenuation can be derived theoretically using a set of integrals given by equations (2-5) through (2-8). Equation (2-9) presents a general power-law form that is commonly used to convert measurements of rain intensity or radar reflectivity factor into specific attenuation. Chapter 3 reports on the power-law model parameters derived from raindrop size distribution (DSD) measurements made in Montreal and Toronto to be used in later chapters to convert radar reflectivity factor measurements into specific attenuation.
- c. Rain intensities are known to vary both in time and space, especially during convective precipitation when high intensity showers are confined in small areas widely referred to as rain cells. The statistics of rain cells obtained from radar measurements show that, in general, rain of higher intensities is associated with smaller sized and sparsely distributed cells. This is the motivation herein for abandoning spatially uniform rain models in assessing rain attenuation influences on radio cell coverage and, further, for studying the effects of spatially and temporally varying rain attenuation on coverage by using radar measurements as reported in Chapter 4. In addition, it appears that in high intensity rain events, cell-site diversity technique could improve the reliability of radio links in mm-wave radio cellular systems. This stimulated the investigation into this subject reported in Chapter 5.

Chapter 3 - Radar Measurement of Rain Attenuation

3.1 Introduction

As outlined in Chapter 2, in the field of microwave and millimetre-wave radio communications, attenuation is the most pronounced effect of rain. Knowledge of rain attenuation statistics is hence necessary for the design of systems in these radio frequency bands. Since rain intensity and attenuation vary in time and space, it is important to consider both the time and spatial variations in analyzing rain attenuation statistics. Use of weather radar data for such statistical examinations has an advantage in that it can provide the information on both types of variation to the extent allowed by the radar time and spatial resolutions. Consequently, radar data have been extensively used in research activities to study the time and spatial variation of rain.

In comparison to other field measurement methods, such as rainfall rate measurements by a network of rain gauges and direct measurement of attenuation on multiple radio links, use of a radar system offers several advantages:

- a. There is no necessity for multiple measuring apparatuses that are distributed in a large area and must work in synchronism [Blo79].
 - b. A radar system can provide data for a much larger observation area, the dimension of which depends only on the radar transmit power and pulse repetition frequency [Bat73].
- The only factors that can reduce the usable area are ground clutter, the melting layer altitude and the spatial resolution acceptable for the desired analysis.

- c. Radar images allow the simulation of various arbitrary configurations of hypothetical radio links. This makes radar databases useful in the analysis of joint probability of attenuation exceedance on multiple convergent links with various lengths and separation angles [Blo79], as well as of coverage of circular radio cells of various radii [Tan98, Tan00].

By measuring the proportion of transmitted power scattered back by raindrops, a weather radar system can be calibrated to provide information on the spatial distribution of rain attenuation and rainfall rate within an area of hundreds of square kilometres, centred on the radar site. The conversion of data from backscatter measurements of the radar reflectivity factor to rain rates and specific attenuations is accomplished by applying empirically derived relationships between it and either rain intensity or specific attenuation, i.e., the $Y-Z$ and $R-Z$ power-law relationships given by equation (2-9). Operating at bands of centimetric wavelength (S (2–4 GHz) and X (8–12 GHz) bands being the most commonly employed), a weather radar system is capable of measuring reflectivity factors without losing a significant amount of back-scattered power to rain attenuation. Such radar systems are presently being operated by the McGill University Radar Observatory [McG]. A weather radar system can provide complete information on the spatial distribution of rain attenuation, which is impossible to obtain from rain gauge measurements. Consequently, radar data have been frequently used in investigations of the spatial characteristics of rain structure and the spatial variation of rain rate and attenuation it induces [Bon98, Cra82, Dru73a, Dru73b, Fer78, Gol75, Hod76, Rog72, Tan98, Tan00]. The most relevant three of many works in evaluating attenuation statistics for terrestrial paths are briefly described below.

Drufuca conducted a radar-based investigation of rain attenuation statistics over terrestrial links [Dru73a]. He simulated a multiplicity of radio paths over the entire rainy area covered by a radar system operated by McGill University, Montreal and computed the probability of attenuation exceedance on these paths, either individually or jointly. Joint probabilities were deduced in the form of conditional probabilities for two double-path scenarios. In the first, the two paths were parallel with each other but separated by a distance, a scenario usually found in one of multiple segments of a point-to-point radio communication with diversity. In the other scenario, the two paths were oriented collinearly, thus creating a two-segment radio link with the connecting point being a repeater. His work, using radar measurements covering the summer months of 1971, was more concerned with exploring the spatial information of rain intensity provided by the radar data to overcome the limitation in temporal extent of the rain measurement.

Another terrestrial link experiment that was performed using radar data is reported in [Fer78]. The authors of this work examined joint probabilities of mixtures of earth-space and terrestrial links upon which diversity scenarios were imposed. They used time measurements of reflectivity factor over the radar coverage area to compute the statistics of rain attenuation for the simulated configurations. Similar to Drufuca's work, Fergusson's was more focused on exploring the possibilities of simulating a variety of radio link configurations using spatial information of rain intensity available from the radar data, collected in the summer of 1975.

Finally, a series of studies by Rutherford-Appleton Laboratory researchers [God94, God97, Tan98, Tan00] made use of two years of radar measurements in the same way the radar data are treated herein. They simulated links and radio cells over radar images and

derived a model for the prediction of diversity gain and diversity improvement for two converging links of equal length and area coverage of a circular cell under precipitation. A more detailed overview will be given of this work later in Chapter 4.

Prior to using radar data in the analyses to assess attenuation, it is considered necessary to revisit the relationship between the radar-measured reflectivity factor and rain attenuation, as well as other subjects relevant to the application of radar data in the reported work. This is of particular interest since modern measurement techniques make it possible to gain more accurate knowledge of the effects of the raindrop size spectrum on both the radar reflectivity factor and the specific attenuation of rain than older methods such as the flour method. These advances have opened an opportunity to obtain a more accurate expression for converting radar-measured reflectivity factor into specific attenuation. While this was our main concern, in our work we also examined the relationship between rain attenuation and rain intensity, as rain gauge measurements are still being widely used to collect indirect measurements of rain attenuation.

This chapter is divided into several sections. Following this introductory section, a brief review of the Y - Z and Y - R relationships and other relevant subjects is given. Next, the derivation of relationship between specific attenuation and radar reflectivity factor from the measured DSD is presented. The parameters of the power-law expression relating the two quantities were derived from drop size distribution measurements. The resulting power-law coefficients were then applied and used in a synthetic storm simulation using rain intensities derived from the DSD measurements to determine the influence of applying these parameters in the estimation of rain attenuation statistics, especially considering the impact of spatial

averaging. Finally, the manner in which the simulation of radio links and cells for analyses in the subsequent chapters is accomplished using radar images is described. A summary can be found in the final section of this chapter.

3.2 Review of the Y - Z and Y - R relationships and other relevant issues

A commonly employed method to gain statistical information on rain attenuation is by transforming rain gauge measurements of rainfall rates, denoted by R , or radar-measured reflectivity factors, denoted by Z , into specific attenuations Y through the application of Y - R or Y - Z relationships. These relationships are usually expressed in power-law form given by equation (2-9). Coefficients a , b , c and d are calculated from measurements of raindrop size distribution, or by adopting such commonly adopted raindrop size distribution models as Laws and Parsons' (L-P) or Marshall and Palmer's (M-P) ([Bat73, Ols78], equation (2-10)).

However, it was considered that accuracy in the statistics of rain attenuation obtained from the L-P and M-P models can still be improved. The above two commonly used models were established from measurements using the flour method [Ogu83, Rin97], the operation of which requires an accurate knowledge of the size-dependent terminal fall velocity of drops¹. These measurements were made when accurate terminal fall speed measurements such as those made by Gunn and Kinzer were not yet available. To be precise, Laws and

¹ The number of drops per m^3 in size range $[D-dD/2, D+dD/2]$ (mm) that are present in the air is $N(D)dD = \frac{M(D)dD}{v(D)}$ [Rin97], where $M(D)dD$ is the number of drops per m^2 measurement area on the ground per second, detected by the measuring equipment, and $v(D)$ denotes the terminal fall velocity of drops in m/s.

Parsons published their DSD measurement result in 1943, followed by Marshall and Palmer who proposed their DSD model based on Laws and Parsons' and their own measurements in 1948, whereas Gunn and Kinzer's raindrop terminal velocity measurements were published in 1949 [Bat73]. Furthermore, in a recent study on the DSD measurement-based derivation of Z-R relationship [Cam00], Campos and Zawadzki pointed out that there is a great deal of uncertainty in the analysis of drop size distribution measurements that was not taken into account in the past.

Since more sophisticated instruments have been developed over the years, it is desirable to revisit the relationships by examining empirical drop size distributions measured by modern instruments. In particular, two measurement systems were employed in this study, namely the POSS (Precipitation Occurrence Sensor System) and the JWD (Joss-Waldvogel Disdrometer, the last word containing a rather loose reference to the term "drop size distribution"). The use of these two instruments that work on two completely different principles enables evaluation of instrumental uncertainties in the measurements, in particular the known limitation in detecting small-sized drops [Ogu83]. Using these two systems, drop size distribution (DSD) samples were collected independently in Toronto and Montreal as part of another project. The Montreal measurements were collected by McGill Radar Observatory (MRO), while Atmospheric Environment Service (AES) was responsible for the Toronto data².

² All sets of DSD measurements used herein were provided by Prof. I. Zawadzki of MRO, including the Toronto DSDs that were made available by Dr. P.I. Joe of AES through Prof. Zawadzki.

By analyzing the measured DSDs, the coefficients of the power-law expressions for both spherical and oblate raindrops were found. It was also considered important to examine the impact of uncertainties in the sampling of small drops on the resulting Y - Z and Y - R relationships. This is addressed separately for each DSD measurement system.

It is considered of interest also to investigate the effects of grouping reflectivity factor samples into multiple intervals prior to the Z -to- Y conversion for noise reduction purposes. This interest arises from the fact that, due to the raindrops acting as incoherent targets distributed within the radar sample volume, the back-scattered power of radar pulses sensing the same volume varies from one pulse to another [Bat73]. As a consequence, the received power averaged over several echoes as well as the radar reflectivity factor subsequently derived, both expressed in decibels relative to an appropriate reference, exhibit a “noisy” characteristic. This in turn creates uncertainties in the resulting values of rain attenuation calculated from the radar reflectivity factors. The study on the grouping of reflectivity factors into intervals is therefore motivated by two intuitive expectations; one can expect that grouping Z measurements in sufficiently large intervals would mitigate the uncertainties inherent in radar measurements, while simultaneously averaging out the impacts of variability in the Y - Z relationship.

Finally, it is known that radar measurements are subject to inevitable spatial averaging over the finite sampling volume of the radar [Bat73]. The averaging volume is a function of the radar pulse width, the antenna beamwidth, and the distance from the radar. For a fixed set of radar parameters, the averaging volume increases in direct proportion to the distance (range). Thus, the variability of rain rate within the measurement volume increases

with range. It is therefore necessary to assess whether this inherent spatial averaging affects the statistics of rain attenuation.

In brief, the next sections report on studies of:

- (i) The effects of uncertainties in small raindrop count in DSD measurements on the Y - Z and Y - R relationships.
- (ii) The impact of stratification applied to measurements of Z on calculated values of Y and its possible application in deriving Y from “noisy” radar measurements of Z .
- (iii) New coefficients for Y - Z and Y - R power law relationships, and results regarding the effects of their application on the cumulative distribution functions (CDFs) for path attenuation.
- (iv) The impact of spatial averaging of Z on the CDFs for attenuation.

The findings from the above studies are our original contributions to the field of rain attenuation prediction from radar measurements.

3.3 Derivation of rain attenuation and radar reflectivity factor values from raindrop size distribution measurements

3.3.1 Calculation of rain attenuation and radar reflectivity factor

Equations for the calculation of the radar reflectivity factor, rainfall intensity, and rain attenuation from a raindrop size distribution were given in Chapter 2. When raindrop size spectra are obtained from field measurements, integral equations (2-5) through (2-8) are

approximated by discrete summations over the drop size range detected by the measuring instruments. The infinitesimal interval dD is replaced by a finite interval ΔD , while the integrands are evaluated at the average diameter of each interval. The range and average of each diameter interval are dependent on the instrument and, for the instruments used in this work, are given in [She94]. The specific attenuation was computed at a given radio wavelength for a water temperature of 0°C, with the refractive index of water as modelled by Ray [Ray72]. All calculations of the extinction cross-section or forward scattering amplitude were conducted using the modified "esa_pm.for" code described in Chapter 2.

3.3.2 Raindrop size distribution (DSD) measurements

Two types of DSD measuring instruments were used in collecting DSD samples. The Montreal samples were recorded using a POSS system, while in Toronto both the POSS and JWD systems were used. These DSD measurement projects were not part of our work. The measurements, however, were made available for this work by Prof. Isztar Zawadzki of McGill University.

a. POSS (Precipitation Occurrence Sensor System)

POSS is an X-band CW bistatic radar system that measures the radio wave scattered by raindrops falling through the volume commonly illuminated by the transmit and receive antennas [She94]. A drop traversing this volume will induce a voltage signal with a frequency proportional to the drop's velocity and an amplitude that is a function of its size. By sampling the detected CW signal and processing it through an FFT, a Doppler power spectrum $S(f)$ can be obtained. The measured Doppler spectrum is related to the drop size distribution $N(D)dD$ by the integral [She94]:

$$S(f) = \int_{D_{\min}}^{D_{\max}} V(D) \bar{S}(f, D) N(D) dD \quad (3-1)$$

In this equation, $V(D)$ is the equivalent measurement volume for drops of diameter D , and $\bar{S}(f, D)$ is the volume-averaged Doppler power density at frequency f generated by a drop of diameter D . The integral is evaluated for drop diameters ranging from 0.34 mm, the smallest detectable by the system, to the maximum in the distribution. Prior to the measurement the product $V(D)\bar{S}(f, D)$, which is a function of wind speed, is determined through computer simulations that use theoretical far-field calculations and near-field measurements of the radar antenna patterns. By using a one-minute average of Doppler spectra for $S(f)$ and applying an inverse approximation of the discrete version of the above integral, an estimate of $N(D)$ can be obtained.

Estimates of number concentration $N(D)$ are acquired for 34 diameter intervals with their respective means, D_m , each selected such that the frequency at which $\bar{S}(f, D_m)$ reaches a maximum matches the associated discrete frequency in the Doppler spectrum. Sheppard and Joe reported that this method of calculating the volume-averaged spectrum leads to large overestimation of the number of small drops at high rain rates [She94] as compared to results from JWD measurements. This issue is to be briefly examined later in this section. One of the important characteristics that should be mentioned of the POSS system is its large sample volume, which allows the system to produce a better sample than other DSD measuring instruments.

The details of the set-up and calibration of the POSS system operated in Montreal are described in [Cam00], whereas the set-up used in Toronto is explained in [She94].

b. JWD (Joss-Waldvogel Disdrometer)

The JWD operates on a totally different principle. It converts the momentum of each falling drop impacting on a sensor's surface into an electric pulse of commensurate voltage. A power-law relationship with previously calculated coefficients is then used to convert the peak voltage of each pulse into an estimate of the drop diameter. The number concentration measurement is a simple 30-second drop counting process, in which the detectable diameter range is divided into 20 intervals. Due to an unwanted dead-time phenomenon that arises from the temporary nonfunctioning of the sensor head after each drop impact, an underestimation of the number of smaller drops can occur and requires correction [She94]. There are thus two forms of JWD data available, uncorrected and corrected, from the experiment in Toronto. In the subsequent analyses, only the corrected form will be used. The details of the JWD set-up are given in [She94].

The DSD measurements made in Montreal and Toronto are described below:

a. Montreal measurements

The POSS sensor operated in Montreal was installed on the top of a 14-floor building on the downtown campus of McGill University. A total of 6,582 DSD samples were selected from a data pool measured during stratiform rain events in the summers of 1994, 1997 and 1998. DSDs from convective events were not included in the analyses here due to insufficiency in the number of DSD samples from such events observed in the same periods. Stratiform rain was identified by examining vertical profiles of radar reflectivity factor on days corresponding to the occurrence of the bright band. These vertical profiles were

recorded by an UHF profiler, a vertically pointed radar system at 915 MHz, operated by the McGill Radar Observatory.

Table 3-1 presents the list of recognised stratiform rain events involved in the study. A record for one day includes samples from all stratiform events occurring on that day except for a couple of days for which the events have been divided into two records. Figure 3-1 shows the results of computations of Z , Y and R from these DSD samples for a frequency of 30 GHz, assuming spherical drops. These samples are ordered chronologically in the figure, starting with the event on April 13, 1994 and ending with that on August 29, 1998, with each consecutive event separated by a vertical dotted line. Since these measurements came from different events, the curve of each quantity shows large variations, particularly at the boundary between consecutive events. This figure, however, adequately serves to determine the range of variations in Z , Y and R to be used in the following analysis.

Table 3-1 Rain events recorded in Montreal

No.	Date of event	No. of samples	No.	Date of event	No. of samples
1	13 April 1994	490	13	5 May 1998	119
2	16 April 1994	463	14	6 May 1998	165
3	26 April 1994	930	15	2 June 1998	90
4	27 April 1994	161	16	12 June 1998	111
5	1 May 1994	740	17	13 June 1998	266
6	8 May 1994	608	18	14 June 1998	303
7	1 November 1997	235	19	10 August 1998	178
8	9 March 1998	174	20	11 August 1998 01:00 – 03:00	119
9	26 March 1998	142	21	11 August 1998 18:30 – 20:00	90
10	16 April 1998	90	22	20 August 1998 21:30 – 23:30	113
11	2 May 1998	333	23	20 August 1998 17:00 – 20:00	125
12	4 May 1998	198	24	29 August 1998	339

Figure 3-1 Z , R and Y for a wavelength of 1 cm, computed from all DSD samples measured in Montreal.

b. Toronto measurements

The Toronto data sets were independently collected during rain events that included both stratiform and convective types [She94]. A DSD stratification and averaging procedure was applied to these data according to rain rate R . The procedure was intended to obtain an average function $N(D)$ for a segment of the entire range of rain rate of interest, hence producing a set of average DSDs that cover the whole range. The span of each segment of rain rate was chosen such that there were just enough triplets of (Z, R, Y) to perform regression over the entire range of these parameters. Too narrow an interval might yield a statistically inadequate number of DSD samples, while a segment too wide could cause the loss of details in the shape of the Y - Z and Y - R relations due to averaging and insufficiency in number of points to be used in the regression. Accordingly, the DSD samples were classified into intervals of rain rate, the upper limit of which increases with the value of R by a factor of two³. Hence, the rainfall rate bins used were 0–0.5, 0.5–1, 1–2, 2–4, ..., 64–128 mm/h, resulting in a total of 9 segments covering a range of 0–128 mm/h of rain intensity.

To determine how inaccuracies in measuring the quantity of drops with the smallest diameters affect the Y - Z and Y - R relationships, an exercise was performed as follows for both

³ Although not explained in [She94], it is conjectured that this method of determining the intervals was chosen based on the almost log-linear increase of number density $N(D)$ when rain rate R multiplied by a power of a positive integer within the measured range of R . This can be easily tested by assuming, for instance, the M-P model in equation (2-10). The factor of two was selected to obtain an adequately large number of intervals to cover the whole range of rain rate measurements.

the JWD and POSS DSD data from Toronto. Three different values of rain attenuation were computed from each DSD average for spherical raindrops. The first was calculated by including the measurements of drops in all diameter intervals. The second was obtained by setting the measurements for diameter intervals with mean less than 0.5 mm equal to that for the interval having average diameter just above 0.5 mm. The third completely excluded the drops in the intervals with mean diameter less than 0.5 mm. The first and the third methods are expected to give the upper and the lower bounds of the computed attenuation, whereas the outcome of the second method yields an intermediate value that should lie closer to the actual average value of attenuation. In the diagrams that follow, the three results are referred to as “upper”, “lower” and “mid”, respectively. These calculations were made for several radio frequencies for raindrops of spherical shape.

Results from the JWD measurements at 100 GHz, presented in Figure 3-2, show practically no variation in the $Y-Z$ and $Y-R$ curves expected from using the different methods described above. It can be expected that at lower frequencies, even less variation occurs. Moreover, the small variations that can be observed occur only at the lower range of attenuation, which is of little interest for radio communications applications, in which the high attenuation tail of the distribution is usually of more importance. Therefore, it is concluded that in later calculations of rain attenuation, we can safely ignore the uncertain number of drops in intervals with mean diameter less than 0.5 mm for the JWD case.

On the other hand, the POSS DSDs suffer from large overestimation of the number of small drops. The impact can be seen in the results calculated using POSS data for 30 and 100 GHz given in Figures 3-3 and 3-4. At 100 GHz the result of the first method, in which the

heavily overestimated number of small drops is included, departs considerably from the curves resulting from the other two methods, which differ only slightly at all ranges of attenuation.

At 30 GHz, the result shows smaller differences among the three curves as the relative contribution of small drops to the total attenuation diminishes. In the sequel, for the analyses of attenuation at 30 GHz, the number of drops in any interval with mean diameter less than 0.5 mm falsely measured by the POSS will be ignored.

Figure 3-2 Variations in Y - Z and Y - R relationships computed using the Toronto JWD data derived by applying three different methods for a frequency of 100 GHz.

Figure 3-3 Variations in Y - Z and Y - R relationships computed using the Toronto POSS data derived by applying three different methods for a frequency of 30 GHz.

Figure 3-4 Variations in Y - Z and Y - R relationships computed using the Toronto POSS data derived by applying three different methods for a frequency of 100 GHz.

The above observations are also supported by a series of plots shown in Figure 3-5 that demonstrate the contribution in specific attenuation of raindrops detected in diameter bins with average less than 0.5 mm for frequencies of 10, 30 and 60 GHz. Each of the curves is plotted by applying equation (2-6) within the range of individual drop size bins and normalising the results with respect to the total specific attenuation for the entire drop size measurement range. The vertical dashed line on each plot separates the drop bins with average diameter less than 0.5 mm from the rest of the spectrum. It is found that for average reflectivity of around 31–32 dBZ (approximately $1250\text{--}1600\text{ mm}^6/\text{m}^3$) or specific attenuation of 0.5-0.6 dB/km at 30 GHz, the total drops in the bins with average diameters less than 0.5 mm contribute only 4.50%, 8.77% and 1.71% of the attenuation for Montreal POSS, Toronto POSS and JWD measurements, respectively. These percentages are the sums of the pdf's in the intervals less than 0.5 mm in Figure 3-5. The results for 50 dBZ reflectivity, or equivalently, 15 dB/km attenuation at 30 GHz demonstrate 6.51% and 0.06% contribution of the lowest drop ranges to the attenuation for Toronto POSS and JWD, respectively.

Figure 3-5 Specific attenuation contributions of individual drop size bins: (a) Montreal POSS, (b) Toronto POSS measurements.

Figure 3-5 Specific attenuation contributions of individual drop size bins: (c) Toronto JWD measurements. (continued)

3.4 The Y - Z relationship

3.4.1 Grouping of DSD samples

In order to derive the parameters of the Y - Z and Y - R relationships, the procedure described in the following was applied to the Montreal samples. From individual DSDs, the corresponding estimates of radar reflectivity factor Z were calculated using equation (2-8). The resulting set of reflectivity factors was subsequently divided into a number of intervals, the upper limit of which multiplies by a factor of three with the increasing value of Z . This factor was selected in order to obtain interval widths in R as close as possible to those employed by Sheppard and Joe [She94], since an increase in R by a factor of two is roughly

equivalent to an increase in Z by a factor of three (see equation (2-11) obtained by applying the M-P model).

The nine intervals of reflectivity factors that result, each having more than ten samples within, are 0–3, 3–9, 9–27, ..., 6561–19683 mm^6/m^3 . Next, the entire set of the corresponding DSD samples is divided into as many classes, each associated to one of the intervals of Z . A DSD average for each of the nine classes is thereafter calculated over all samples within that class. Figure 3-6 shows the nine DSD averages produced by the above procedure, with vertical axis depicting the number density $N(D)$ of raindrops in ($\text{mm}^{-1}\text{m}^{-3}$) and horizontal axis the drop size D in millimetre. From the resulting DSD averages, nine triplets of (Z, R, Y) can be calculated for any radio wavelength. Table 3-2 lists the details of the nine intervals of Z .

To examine whether the Y - Z and Y - R relationships obtained from the above-described procedure change with the interval multiplication factor, another stratification by a factor of six in Z is accomplished. These intervals are also given in Table 3-2. The specific attenuation Y at one-centimetre wavelength for spherical raindrops and the rain rate R are computed for each resulting DSD average. The relationships are presented in Figure 3-7, together with those produced by the use of factor of three.

As can be seen from Figure 3-7, the Y - Z and Y - R curves produced by the adoption of multiplication factor of 3 do not differ significantly from their counterparts that arise from the use of factor 6. This indicates that Y - Z and Y - R relationships are not very sensitive to variation in the width of the Z intervals. It further implies that in using the “noisy” radar-measured reflectivity factors, a sufficiently large factor for Z intervals can be selected and

applied to the radar data so as to alleviate the effects of the uncertainties in radar measurement without changing the statistics of the rain attenuation derived thereafter. This finding is important in determining intervals to be used in constructing the distribution function of rain-induced attenuation estimates computed from radar reflectivity factor measurements.

Table 3-2 Intervals of Z from Montreal data

Factor = 3			Factor = 6		
Range of Z (mm ⁶ /m ³)	Z average (mm ⁶ /m ³)	Number of samples	Range of Z (mm ⁶ /m ³)	Z average (mm ⁶ /m ³)	Number of samples
0-3	1.22	706	0-6	2.28	1062
3-9	5.61	587	6-36	18.53	1396
9-27	17.09	866	36-216	100.23	2000
27-81	49.67	1195	216-1296	548.45	1693
81-243	146.62	1242	1296-7776	2725.70	421
243-729	437.54	1138	7776-46656	13716.53	10
729-2187	1195.39	622			
2187-6561	3429.55	199			
6561-19683	8551.29	25

Figure 3-6 Average DSDs for different ranges of Z obtained from Montreal measurements. The ranges of Z in mm^6/m^3 are given in the legend.

Figure 3-7 Y - Z and Y - R plots for the averaged DSDs from Montreal with Z interval width multiplication factors of three and six.

3.4.2 Derivation of the Y - Z and Y - R power-law coefficients

Figures 3-8 through 3-11 plot the three curves of Y - Z and Y - R pairs for the three different sets of measurement data from the two cities at four different frequencies: 10, 30, 60 and 100 GHz (equivalent to wavelengths 3, 1, 0.5 and 0.3 cm, respectively). These plots are for the spherical raindrop case. In each figure, “Mon. Pavg” denotes the plot obtained from Montreal POSS measurements with Z interval multiplication factor of 3, “Tor. Pavg” denotes the Toronto POSS, and “Tor. Javg” denotes the Toronto JWD results. The curves for MP’s DSD model are also plotted in the same graphs for comparison.

It is considered that by taking the average from the three sets of average DSDs, a Y - Z relationship can be derived for any radio wavelength (or frequency) of interest. This is because it is assumed that both Montreal and Toronto belong to the same precipitation climate region and hence share the same DSD characteristics, and that differences among the DSD measurements are indicative only of the measurement error of the instruments. However, the difference in drop size intervals employed by different measurement apparatuses preclude the averaging from being performed on the DSD's themselves. As shown in Figures 3-8 through 3-10, there are regions of Z where the Montreal and the Toronto data sets provide different values of Y . For practical purposes, an average value of Y is needed in these regions. Also, the Montreal data encompass the lower ranges of attenuation, whereas the higher ranges are covered by the Toronto measurements that were made during both stratiform and convective events. The following steps are therefore taken. All (Y, Z) and (Y, R) pairs obtained from averaged DSDs measured in Montreal and Toronto are evaluated in a single linear regression in logarithmic scale. The desired mathematical

expression for each relationship is given by equation (2-9). By applying the above procedure, a table of coefficients a , b , c and d can be established for various radio frequencies and for both spherical and oblate spheroidal raindrops.

Table 3-3 provides the values of the coefficients for spherical drops at frequencies in the 10-100 GHz range with 10 GHz spacing, whereas the results for the case of oblate spheroidal raindrops are given in Table 3-4. For oblate spheroidal drops, the drop axial ratio is assumed to obey the relation $s = 1 - 0.41D/9$ used by Oguchi [Ogu73], where s is the ratio of minor to major axis of the spheroid, and D the diameter of the equivolumic sphere in millimetres. Since in the computation of attenuation the number of drops having diameter less than 0.5 mm is ignored, these results might have some inaccuracies for the higher range of frequencies examined herein. In particular, for a frequency of 70 GHz or higher, at a reflectivity factor around 50 dBZ the difference in specific attenuation is more than 5 dB/km between the cases when small drops detected by the POSS system in Toronto are accounted for and when they are ignored. In the relationships given in these tables, Y is given in dB/km, R in mm/h, and Z in mm^6/m^3 .

A comparison of the set of coefficients a and b in Table 3-4 with the values recommended by ITU-R based on Laws-Parson DSD model [ITU92, Fre97] shows slight differences between the two sets, with those for frequencies around 20–30 GHz being the closest. For frequencies below 20 GHz and rainfall rates above 10 mm/h, specific attenuation predicted by the ITU-R parameters is higher than that predicted using ours (e.g., about 0.08 dB difference per kilometer range at 20 mm/h and 0.39 dB/km at 50 mm/h for vertically polarised 10 GHz radio waves). For the frequencies higher than 20 GHz, our predicted

attenuation is higher than the ITU-R at intensities higher than 10 mm/h (e.g., about 0.66 dB/km difference at 20 mm/h and 2.8 dB/km at 50 mm/h for 40 GHz frequency and vertical polarisation). For a vertically polarised wave at 30 GHz, our coefficients yield a higher prediction of specific attenuation compared to the results from using the ITU-R coefficients, e.g. 0.10 dB/km difference at 20 mm/h rain rate and 0.73 dB/km at 50 mm/h. These discrepancies can be ascribed to differences in the method of DSD measurement and processing, i.e. the way the measurements of small drops are treated. For use in calculations of rain attenuation from rain gauge or radar measurements made in Montreal or Toronto, our parameters are recommended.

Table 3-3 Coefficients for Y - R and Y - Z relationships for spherical drops

Freq. (GHz)	$Y = cZ^d$		$Y = aR^b$	
	c	d	a	b
10	1.336E-04	0.802	1.292E-02	1.093
20	6.787E-04	0.798	6.392E-02	1.087
30	1.781E-03	0.786	1.566E-01	1.071
40	3.811E-03	0.757	2.846E-01	1.032
50	7.013E-03	0.722	4.289E-01	0.985
60	1.128E-02	0.689	5.721E-01	0.941
70	1.626E-02	0.662	7.037E-01	0.903
80	2.155E-02	0.639	8.192E-01	0.872
90	2.682E-02	0.620	9.180E-01	0.848
100	3.184E-02	0.606	1.001	0.827

Table 3-4 Coefficients of Y - Z and Y - R relationships for V- and H-polarised waves

Freq. (GHz)	$Y = c_V Z^{d_V}, c_H Z^{d_H}$				$Y = a_V R^{b_V}, a_H R^{b_H}$			
	c_V	d_V	c_H	d_H	a_V	b_V	a_H	b_H
10	1.287E-04	0.798	1.353E-04	0.810	1.213E-02	1.087	1.363E-02	1.103
20	6.651E-04	0.789	6.902E-04	0.804	5.965E-02	1.075	6.730E-02	1.095
30	1.737E-03	0.776	1.827E-03	0.790	1.448E-01	1.058	1.643E-01	1.076
40	3.662E-03	0.751	3.936E-03	0.759	2.641E-01	1.024	2.967E-01	1.034
50	6.688E-03	0.719	7.252E-03	0.722	4.013E-01	0.980	4.442E-01	0.985
60	1.075E-02	0.688	1.164E-02	0.689	5.400E-01	0.938	5.891E-01	0.940
70	1.553E-02	0.661	1.673E-02	0.661	6.695E-01	0.902	7.212E-01	0.902
80	2.066E-02	0.639	2.211E-02	0.638	7.847E-01	0.872	8.364E-01	0.871
90	2.583E-02	0.620	2.744E-02	0.620	8.841E-01	0.848	9.343E-01	0.846
100	3.079E-02	0.606	3.252E-02	0.605	9.686E-01	0.828	1.017	0.826

Figure 3-8 Y - Z and Y - R plots from averaged DSDs measured in Montreal and Toronto at 10 GHz ($\lambda = 3$ cm).

Figure 3-9 Y - Z and Y - R plots from averaged DSDs measured in Montreal and Toronto at 30 GHz ($\lambda = 1$ cm).

Figure 3- 10 Y - Z and Y - R plots from averaged DSDs measured in Montreal and Toronto at 60 GHz ($\lambda = 0.5$ cm).

Figure 3-11 Y - Z and Y - R plots from averaged DSDs measured in Montreal and Toronto at 100 GHz ($\lambda = 0.3$ cm).

3.5 Evaluation of the use of the Y-Z relationship for calculating link attenuation statistics

This section compares the cumulative frequency distributions of attenuation computed directly from the Montreal DSDs with those computed through the relationships derived in the foregoing section. In particular, we will examine the statistics of attenuation at 30 GHz over a radio path of a given length for the case of spherical raindrops. To realise the comparison for various path lengths, an average storm velocity of 54.4 km/h was assumed to shift the rain structure linearly, as well as the associated drop size distribution, along the path being considered⁴. Since the DSD measurements are one-minute average spectra of drop number concentration, each can thus be regarded as a 0.91-kilometre average when the above translation velocity is applied. Computing an estimate of 0.91-kilometre attenuation is thus equivalent to applying the integral equation given by (2-6) to a DSD sample, and multiplying the resulting specific attenuation in dB/km by 0.91.

For comparison, this was also accomplished by computing the rain rate and the radar reflectivity factor through equations (2-7) and (2-8), followed by the application of the power-law relationships using the appropriate values of coefficients. Specifically, relationships for 30 GHz,

⁴ The average velocity of 54.4 km/h was provided by A. Bellon of McGill University and was obtained by averaging the magnitudes of 1310 velocities produced by cross-correlating CAPPI weather radar images recorded successively at McGill University radar facility [Zaw98].

$$\begin{aligned}
 Y &= 1.78 \times 10^{-3} Z^{0.79} \\
 Y &= 0.157 R^{1.07}
 \end{aligned}
 \tag{3-2}$$

are used herein. Using either of these methods, 6,582 0.91-kilometre averages of specific attenuation result. An estimate of attenuation over a 910-metre link is again obtained by multiplying the average specific attenuation by 0.91.

For longer paths, estimates of attenuation over a length $r = 0.91n$ km, where n is an integer, can be calculated by means of one of the following methods. The most straightforward is by summing up n consecutive values of 0.91-kilometre attenuation estimates, an approach commonly used in generating attenuation statistics from point rain rate measurements by invoking the synthetic storm technique (see, e.g., [Dru73a, Dru73b]).

Alternatively, r -kilometre DSD averages can be obtained by averaging n -element groups of successive one-minute (or 0.91-kilometre) DSD averages. From each DSD average that results, r -kilometre averages of radar reflectivity factor and rain rate can be calculated. Again, expression given in (3-2) are used to acquire the equivalent specific attenuation averages, which are in turn multiplied by the path length r to yield the estimates of total attenuation on the path. The execution of DSD averaging over lengths more than 0.91 kilometres just described can also be used to examine the effect of spatial averaging, which is inherent and inevitable in radar measurement, on the statistics of attenuation obtained by the use of relations in (3-2).

In this study, the path lengths are chosen to be 4.55, 9.1 and 18.2 kilometres. These distances correspond to averaging of 5, 10, and 20 DSD samples, respectively. For the 5-sample averaging, an averaging method with disjoint windowing (as opposed to employing a

running window) yields 1316 samples. For the longer two paths, however, a running average procedure must be performed to gain an adequate number of samples. Using windows of 10 and 20 samples long, 6573 and 6563 attenuation samples result, respectively. Figure 3-12 shows the cumulative frequency distributions of attenuation estimates that arise from the above three methods.

Each of the four graphs in the figure demonstrates a comparison among the three for the specified path length. In general, it can be observed that there is no significant difference among the three distributions, which is true for all path lengths under consideration. A Kolmogorov-Smirnov test [Dix69] showed that with 80% confidence the ranges in which the actual curves of individual pdf's might lie are approximately 0.013 for cases with volume samples of 6582, 6573 and 6563, and are around 0.029 for the case with 1316 samples. It can be seen from each of the figures that these confidence ranges overlap each other, which further supports the visual observation that there are no differences that can be claimed among the pdf's. This implies that the relationships given in (3-2) can be used to predict attenuation statistics at 30 GHz from measurements of rain rate or radar reflectivity factor with sufficient accuracy. It also implies that the Y - Z relationship at 30 GHz is not very sensitive to the extent of spatial averaging applied to the radar reflectivity factor.

Figure 3-12 Cumulative frequency distributions of attenuation at 30 GHz computed over various radio path lengths using three different methods: (i) direct computation from DSD measurements, (ii) application of *Y-R* and (iii) *Y-Z* relationships based on the measured DSDs.

3.6 Simulation of radio links and cells

Data analysed during work reported herein were recorded during field measurements of radar reflectivity factors (in mm^6/m^3) over a circular area of 45-km radius centred at the radar antenna site. The S-band horizontally scanning radar system that was used for the measurements is operated by the McGill Radar Observatory and is located near the Macdonald campus of McGill University in Montreal. Measurements were sequentially executed at 24 elevation angles. At each elevation angle, a 2-dimensional circular radar image is recorded with a resolution of $1^\circ \times 150 \text{ m}$; the actual size of each radar bin is therefore dependent on the distance of the point of interest from the radar antenna. The radar system completes the entire set of measurement and produces a single radar image for each elevation every five minutes.

In this study, only images recorded at elevation angles 3.4° and 7.1° were used. Measurements at these elevation angles were used in combination; reflectivity factors at points closer than 15 km from the radar antenna were taken from the image recorded at elevation 7.1° , whereas measurements at longer distances were extracted from the 3.4° -elevation image. This was done to avoid ground clutter effects at shorter distances and to obtain a relatively large radius of coverage without illuminating the highly reflective snow melting layer, which often exists at the maximum height of stratiform rain.

In each radar bin, the detected average radar reflectivity factor Z , in mm^6/m^3 , was converted into the equivalent average specific attenuation Y in dB/km by applying:

$$Y = 1.74 \times 10^{-3} Z^{0.776} \quad (3-3)$$

which uses the constants determined in Section 3.5. The values taken from Table 3-4 are for vertically polarised radio waves at 30 GHz, 0°C water temperature, and raindrop shape as given by Oguchi's oblate spheroid model (see Table 3-4).

In the reported work, two circular radio cells were simulated. In each cell, radio links are simulated radially around the cell centre, with adjacent links being separated by 1°. Attenuation was computed along every link at various distances from the centre, ranging from 100 metres to 5 kilometres with 100-metre resolution. The calculation technique is similar to that used by Drufuca and Fergusson [Dru74, Fer78]. A simulated link therefore consists of several segments that are bounded by its intersections with two sides of the radar bins that it traverses. Attenuation on a segment of the link is the specific attenuation in the corresponding bin multiplied by the length of the segment within that bin. The total attenuation on a link is the sum of attenuations on its constituent segments. Figure 3-13 is a sketch of a possible scenario showing a simulated radio link traversing a section of a radar image. In the figure, the total rain attenuation along the simulated link is $A = Y_1 \cdot l_1 + Y_2 \cdot l_2 + Y_3 \cdot l_3 + Y_4 \cdot l_4$, where Y_n is the specific attenuation in dB/km recorded within radar image bin n , and l_n is the length in km of the link segment in that bin.

A sketch of the circular cells simulated on a radar image is given in Figure 3-14. In the Cartesian coordinate system the two cell centres are positioned at (-13, -12) and (-7, -21) kilometres, respectively, with respect to the radar antenna position near the Macdonald campus of McGill University, Montreal. These are in a region southwest of Montreal. Figure 3-15 depicts a region in a typical radar image on which two concentric cells centred at (-13, -12) with radii of 2 and 4 kilometres are simulated.

Figure 3-13 Sketch of a simulated radio link

Figure 3-14 Sketch of a radar image with the two simulated radio cells.

Figure 3-15 Partial area of a typical radar image with two simulated concentric radio cells of radii 2 and 4 km.

Since the radar scans at non-zero elevation angles, the radar coverage plane is slanted in reality, resembling the lateral surface of an inverted cone with the bottom tip at the radar site. Consequently, the radar reflectivity measurement actually takes place at different elevations above the ground, which is a linear function of range from the radar antenna. Hence, it might be inferred that there could be a difference in the rain attenuation derived from the radar data with that which actually occurs on the ground. However, observations on vertically pointing radar records collected in Montreal [Fab98] show that generally in stratiform rain events, reflectivity is roughly constant with height from the ground to the base of the melting layer or to the ice regime in the case of rain originating from compact ice. For the radar measurements used in subsequent analyses, these upper boundaries approximately correspond to the 0° -isotherm level, which is about 2 km high during winter and even higher during other seasons [Fab98]. Similarly, during showers the reflectivity is generally constant from the ground up to one kilometre above the shower top, located at about the 0° isotherm. In the special case of heavy convective events, there is so much time-varying fine-scale structure that it is considered almost unlikely for anyone to find any systematic deviation from the constant reflectivity below the 0° isotherm [Fab00].

The highest altitude at which a reflectivity measurement is used in the attenuation computation corresponds to the point on the simulated cell circumference most distant from the radar antenna site. For the cell simulated herein, it is equal to 1.35 kilometres above the ground, which can be expected to be below the 0° isotherm at all times. Hence, for practical purposes, the attenuation computed from reflectivity on the slanted radar coverage plane is considered identical to that actually occurring at ground level.

3.7 Summary

Several subjects pertinent to the preparation of simulations and analyses in subsequent chapters have been discussed in this chapter. Objectives of this work are primarily to obtain an appropriate expression for converting radar measurements of reflectivity factor into equivalent values of specific attenuation. Points of importance are summarised below.

- a. An examination was made of the impact on the accuracy of $Y-Z$ and $Y-R$ relationships of uncertainty in the measurement of number density for small drops using either the JWD or POSS system. The JWD's inaccuracy is found to cause only slight variation in the predicted value of attenuation at the lowest ranges only, as demonstrated by the results from three different methods in treating the number of detected drops smaller than 0.5 mm in diameter. Ignoring them or setting their density uniformly equal to that of raindrops with diameter just slightly larger than 0.5 mm would result in almost the same values of attenuation as when the measured density is used in calculating the attenuation. This slight variation at low ranges is considered to be insignificant, especially when one desires to focus merely on large attenuation values, as is the case in the field of radio communications. Meanwhile, the POSS data exhibits large overestimation in the number density of small drops. While its impact on the predicted attenuation is marked at 100 GHz, it is not significant at 30 GHz or lower. These results lead to a conclusion that the uncertainty in the measurements of small drops is not critical to the $Y-Z$ and $Y-R$ relationships at frequencies around 30 GHz or less.

- b. The Y - Z and Y - R relationships were derived from measured DSDs. An intermediate step was taken in so doing, which involved grouping the DSDs into intervals according to their respective Z values and then averaging the results over the width of each interval. These DSD averages were subsequently used in the derivation of the two relationships. The upper limit of each interval in Z was made to multiply by a factor r , such that the ranges of Z become $0-r$, $r-r^2$, r^2-r^3 , Various factors have been tried out, with a result showing that the two relationships are not sensitive to the change in the width of the grouping interval. This implies that different criteria can be adopted to establish the intervals in grouping the measured radar reflectivity factor estimates without changing the statistics of the resulting attenuation estimates. This finding is of great importance when one deals with high resolution radar-measured reflectivity factors, in order to average out noise components in the derivation of estimates of attenuation. In such a case, a factor for the interval width multiplication can be selected such that the reflectivity factor interval widths are larger than the magnitude of the noise. Thus, the effects of noise in radar measurement can be alleviated while keeping the attenuation statistics unchanged.
- c. The availability of DSD averages corresponding to mid to high ranges of attenuation from the Toronto JWD and POSS measurements complements the Montreal POSS DSD averages which cover lower to mid ranges. In mid regions of attenuation, where DSD averages exist from measurements in both sites, the corresponding values of attenuation do not differ much between these sites despite the use of different types of DSD measuring instruments. Hence, within the uncertainty in the measurements, Toronto and

Montreal data show the same average characteristics of attenuation in relation with the other two parameters, rain rate and radar reflectivity factor.

- d. The derivation of the coefficients of the desired power-law relationships was accomplished using the DSD averages from both measurement sites. These coefficients are found to differ slightly from those recommended by ITU-R due to differences in the method of measurement and parameter derivation. Specific attenuation estimates predicted from rain intensities higher than 20 mm/h using the ITU-R coefficients are higher than those predicted using the reported coefficients for a frequency of 20 GHz or lower. For frequencies above 30 GHz and rain rates higher than 20 mm/h, our coefficients yield a higher estimate of the specific attenuation than those recommended by the ITU-R.
- e. When the derived $Y-Z$ and $Y-R$ relationships are used to compute the statistics of attenuation at 30 GHz using the Montreal DSD measurements, the cumulative frequency distribution functions produced do not exhibit significant difference compared to that of the attenuation directly computed from the DSD samples, confirming the accuracy of the power-law parameters obtained.
- f. It was shown that the statistics of path attenuation computed using the power-law relationships are not sensitive to spatial averaging. This result is important when attenuation statistics are to be computed for hypothetical radio links simulated over radar images, the measurements of which inevitably involve inherent spatial averaging processes due to the finite beamwidth of the radar antenna.

- g. The procedure for simulating 30 GHz radio links on the radar images and calculating the rain attenuation along each of them was presented. Circular radio cells can be simulated with radial links spaced at 1° azimuthally. 360 links can therefore be simulated from each cell centre.**

Chapter 4 - Cell Area Coverage

4.1 Introduction

Cell area coverage in the context of fixed wireless systems has been given several definitions. A general definition most commonly used refers to the percentage of locations, which can be the rooftops of residential or office buildings that can be covered, given location optimisation, in the area of the cell [Gra97c, DeC99, DeC00]. In this thesis, it is defined as the fraction of the total area of a radio cell within which any subscriber's terminal receives the signal transmitted from the hub above the minimum acceptable received power, similar to the definition of coverage in a mobile radio system. Coverage is expressed relative to the ideal, circular area that the cell base station or hub would cover under free-space conditions, hereby referred to as the nominal area. Reduction in the covered area occurs when such phenomena as shadowing and rain are present such that the total attenuation includes their effects in addition to the free space loss. Later in this chapter, area coverage under precipitation is expressed as a percentage with respect to the coverage under clear sky conditions, which involve the impact of free space path loss and lognormal shadowing, and referred to as *normalised coverage* for brevity. The term *minimum acceptable received power*, also referred to herein as *receiver threshold*, means the minimum received power, assumed to be fixed and equal for all subscriber terminals, that still allows the receiver to achieve the minimum required performance measure, such as bit error rate in digital systems.

The latter definition of coverage is more commonly associated with mobile cellular systems. In the context of fixed cellular systems it corresponds to a worst-case scenario since

in each subscriber location there is some extent of freedom that allows the selection of the best spot for the subscriber's antenna, e.g., on the building rooftop.

The definition of coverage can also be viewed from the attenuation standpoint. That is, a point in the cell is considered to be covered when a signal transmitted from the hub to that point undergoes a total attenuation that does not exceed a maximum allowable value. The total attenuation in this case includes the distance-dependent mean path loss, shadowing loss, and rain attenuation. Coverage C can be expressed mathematically as

$$C = \frac{\int_{\theta=0}^{2\pi} \int_{r=0}^L f(r, \theta) r dr d\theta}{\pi L^2} \times 100\% \quad (4-1)$$

in the case of circularly shaped cells where L is the nominal radius of the cell, r and θ represent the radial distance and azimuth to a subscriber in the cell, respectively, and

$$f(r, \theta) = \begin{cases} 1, & A_{r,\theta} \leq A_{\max} \\ 0, & \text{elsewhere} \end{cases} \quad (4-2)$$

$A_{r,\theta}$ denotes the total attenuation experienced by a subscriber terminal located at distance r and azimuth θ from the hub or base station, and $A_{\max}$ is the maximum allowable attenuation.

A number of studies to be reviewed in the next section have been conducted by other researchers to investigate radio cell coverage, most of them for mm-wave applications, and to find a method for its prediction. Many of these studies, either theoretical or experimental, were undertaken for clear-sky conditions [Jak94, Bid99, DeC00, Sei95a, Gra97a, Pap97, Sev00]. Some resulted in schemes that can be used to predict cell area coverage [Jak94,

Bid99]. While their findings are applicable only to clear-sky situations, it is considered that reviews of such reports would be useful in understanding the present study of the impact of rain attenuation. That is, it is considered that clear-sky results can be combined with those resulting from the study herein of the influence of rain to yield an estimate of coverage that considers the impact of both shadowing and rain.

Several other studies [Gra97a, DeC00] have been carried out that incorporate the effects of spatially uniform rain intensities over the whole cell area, predicted using well-established path attenuation prediction methods. In using such methods, 2-D spatial details of rain intensities are ignored. This is essentially equivalent to using the statistics of a quantity measured at a single point in space to predict the statistics of the same quantity measured over a two-dimensional space. However, an overview is also made of these efforts, in order to obtain an insight into the problem of coverage and to examine whether some aspects of these studies are applicable to the study at hand.

There has been only one previous study of the impact of rain on mm-wave cell area coverage that took spatial variability into account [Tan98, Tan00]. This was effected by assuming the existence of rain attenuation only, aside from the free space path loss; no shadowing was considered. This will also be discussed in the next section.

The goal of this thesis is to derive cell area coverage statistics using spatial information of rain attenuation provided by radar measurements. Components of attenuation that are considered include mean path loss that increases with range from a hub, lognormally distributed shadowing loss, and radar-derived rain attenuation. The method of computing the

coverage of simulated cells from radar measurements will be described in the third section, immediately followed by a section that reports the radar-based simulation results.

Given complete and accurate knowledge of the built environment, it is considered ultimately possible to predict clear-sky coverage performance of a cell under consideration. It is therefore deemed most useful if the coverage under precipitation is presented in the form of normalised coverage. A simple method for coverage prediction might be constructed by estimating the clear-sky coverage using Jakes' method [Jak94] to be described in the next section. Coverage under precipitation can then be obtained from this clear-sky estimate by applying the normalised coverage factor to the clear-sky estimate. Based on this strategy, a method for cell area coverage prediction can be devised from simulation results and is described in the fourth section of this chapter. Finally, a summary can be found in the last section.

4.2 Millimetre-wave Radio Cell Area Coverage

4.2.1 Clear-sky mixed LOS/NLOS coverage

a. An approach based on statistical modelling

This approach to area coverage prediction is generic in nature and is useful to obtain a rough prediction of coverage at a pre-system deployment stage in a new area. The predicted value is obtained by substituting parameters that characterise the spatial variation of path loss in the local area under consideration into an equation for the statistical distribution of average path loss. These parameters include a path loss exponent and the variance of lognormal shadowing required when Jakes' model is to be used [Jak94], or building height and spatial

density averages when Biddiscombe's method [Bid99] is chosen. The path loss exponent and lognormal shadowing variance can be obtained from a statistically adequate number of field measurements of path loss in the area of interest, while the building height and density statistics might be available from local geographic records collected and maintained by some organizations or government bodies.

- **Jakes [Jak94]**

In the design of mobile cellular communication systems, interest lies in statistically modelling the local mean received power (averaged over any small-scale fading) at a given distance from the base station. An accurate model allows a reliable prediction of cell area coverage, defined as the fractional area that is covered within the nominal area of the cell. In an environment with shadowing, it is common to model the local mean as a lognormally distributed random variable with a mean $\bar{x}$ being dependent on distance, usually through a power-law mechanism, and a standard deviation σ in dB that varies with the shadowing environment [Jak94]. That is, the probability density function for the local mean received power p expressed in dB relative to a power unit, e.g. dBm, is of the following form.

$$f_p(p) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(p - \bar{p})^2}{2\sigma^2}\right) \quad (4-3)$$

The average received power $\bar{p}$ changes with distance r from the hub and is given by

$$\bar{p} = k - 10\alpha \log_{10}\left(\frac{r}{L}\right) \quad (4-4)$$

where, in a cellular network consisting of circular cells, L represents the cell radius, α denotes the path loss exponent, and k , whose value depends on the transmit power and the gains of the antennas, is the mean at the cell perimeter, i.e. at $r = L$.

For millimetre-wave frequencies around 30 GHz, there have been a number of field surveys that indicate the closeness of the above model to real measurements [Bul98a, Bro00a], although only one actual statistical test was reported for the lognormality of excess path loss [Bul98b]. These surveys are described in the papers listed below.

- **Bultitude *et al.* [Bul98a]**

CW measurements at 28.5 GHz were conducted in the Parkwood Hills area of Ottawa in 1996 by Bultitude *et al* [Bul98a]. The clutter environment consisted of residential buildings with trees in the surrounding areas. The transmit antenna was at the top of a 50-m building, whereas the receiving antenna was affixed to a mast mounted at 5m above ground level on a trailer that could be towed throughout the measurement area, with short stops at each roadside measurement location. The 5-m antenna height was selected to simulate subscriber station mounting locations that might be selected near the eaves of bungalows that typified surrounding houses. Both Tx and Rx antennas were directional, with 23° and 12° azimuthal beamwidths, respectively. A power-law distance-versus-received power relation was fit to the measurements. Average propagation loss in the mixed LOS/NLOS environment of 66 measurement locations obeyed a model having a range exponent of 5. An experimentally determined distribution of excess loss with respect to the 5th power model was found both by visual inspection and use of Kolmogorov-Smirnov test with $\pm 13\%$ error

margin at 90% confidence level [Bul98b] to fit well with a lognormal distribution having a standard deviation of 17 dB that was calculated from the data.

Another set of measurements made from a different site in Kanata having similar clutter characteristics also indicated a path loss exponent close to 5. There was, however, a slight, unconfirmed indication that the lognormal shadowing standard deviation changed with distance from the transmitter, being 1.3 and 2.4 dB at 3.25 and 3.5 km, respectively. This latter finding was considered inconclusive because there were only 46 and 24 samples in measurement pools for 3.25 and 3.5 km range, respectively.

- Bronson [Bro00a, Bro00b]

Bronson [Bro00a, Bro00b] made a set of CW measurements at 29.5 GHz in the Glebe area of Ottawa. Both transmitter and receiver used lens antennas, each having an 8° azimuthal beamwidth. The receive antenna was mounted in the trunk of a parked car at 1 m above ground level. The transmit antenna was set up on a 20 metre tall building. The power-law fit to the path loss versus distance data resulted in a distance exponent of 4.01. The variation around the power-law fit was found to have a standard deviation of 13.89 dB. However, neither statistical test nor visual inspection was reported on the goodness of fit of the data to a theoretical lognormal distribution with the same standard deviation.

Other similar millimetre-wave field surveys either did not examine the above statistical properties [Sev00], or were executed in different portions of the millimetre-wave band [Reu73, Ham97].

Using equations (4-3) and (4-4) to model local average path loss, the probability that a subscriber at distance r from the base station is covered can be obtained as follows [Jak94].

$$\Pr[P(r) > P_{Th}] = \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left(\frac{-M + 10\alpha \log_{10}(r/L)}{\sigma\sqrt{2}}\right) \quad (4-5)$$

where $M = k - P_{Th}$ is power fade margin at the cell perimeter, and P_{Th} is the minimum acceptable received power. Finally the cell area coverage $C(P_{Th})$ when threshold P_{Th} is applied is

$$\begin{aligned} C(P_{Th}) &= \frac{1}{\pi L^2} \int_0^{2\pi} \int_0^L \Pr[P(r) > P_{Th}] r dr d\theta \\ &= \frac{1}{2} \left[1 - \operatorname{erf}(a) + \exp\left(\frac{1-2ab}{b^2}\right) \left(1 - \operatorname{erf}\left(\frac{1-ab}{b}\right) \right) \right] \end{aligned} \quad (4-6)$$

where $a = -M/\sigma\sqrt{2}$ and $b = 10\alpha/\sigma\sqrt{2} \ln 10$.

It is considered reasonable to extend the above theoretical analysis of coverage for clear-sky situations to those in which rainy conditions are prevalent. It was attempted to find the probability density function (pdf) of the sum of independent attenuation components due to shadowing and rain. Since the two types of losses are independent of each other, the density function of the overall random variable, expressed in dB, is the result of convolution between them. That is,

$$f_T(x) = f_S(x) * f_R(x) \quad (4-7)$$

where $f_T(x)$ denotes the probability density function of total attenuation on a radio link, whereas $f_S(x)$ and $f_R(x)$ represent the density functions of clear-sky path loss (i.e., lognormal shadowing) and rain attenuation, respectively, and $*$ denotes the convolution operator.

Following the cell coverage derivation procedure previously described, equation (4-6) can be solved by applying the relationship:

$$\Pr[P(r) > P_{Th}] = \int_{-\infty}^{A_{max}} f_T(x) dx \quad (4-8)$$

where A_{max} is the maximum attenuation associated with the received power threshold P_{Th} .

After in-depth consideration of the steps involved, it was concluded unlikely that this approach could be followed to completion for several reasons, including:

- (i) Fitting a theoretical distribution to an empirical distribution of rain attenuation was not attempted due to anticipated difficulties in doing so.
- (ii) When a closed form of the rain attenuation distribution was assumed (lognormal as proposed by Lin [Lin73] or gamma as proposed by Morita *et al* [Mor76]), the pdf of the total attenuation in (4-7) was found to be complex, involving a form of hypergeometric function. The application of this form to integral equations (4-8) and (4-6) was therefore not preferable.
- (iii) The application of the integral expression essentially assumes no azimuthal and radial correlation of rain attenuation, which also means neglect of the two-dimensional spatial knowledge of rain attenuation.

- (iv) It is also recognised that the distributions of rain attenuation in the cited references describe *temporal* fading due to rain attenuation, whereas the clear-sky distribution refers to the *spatial* variation of shadowing loss. The implication of this difference in nature of the variation of attenuation was not investigated further.

Regardless of the above considerations, the lognormal shadowing is considered to be a reasonable model and is incorporated in the computation of coverage at a specified range from the hub. Herein, the mean, instead of comprising only the power-law degradation with distance, will also incorporate the rain attenuation derived from the radar measurements. This will be described in detail in Section 4.3. Gong's work [Gon97] is an example of extension of the above notion of lognormally distributed shadowing loss to the context of fixed cellular systems.

- **Biddiscombe [Bid99, ITU99].**

Biddiscombe took a completely different approach [Bid99, ITU99]. He started by assuming that the effect of building obstruction to a mm-wave radiowave is either nil, i.e. the Line-of-Sight (LOS) condition, or complete blockage. He proceeded with modelling building heights in a coverage area as being Rayleigh distributed and characterising the blockage probability by taking into account three local parameters, namely the Rayleigh distribution parameter, the ratio of land area covered by buildings to the total area, and the average number of buildings per unit area. With the above assumptions and locally gathered information on the statistics of building dimensions and density, the probability of coverage for a subscriber located at a certain distance from the transmitter was estimated as the probability that none of the buildings on any radio path of interest causes obstruction. He

subsequently compared the result with ray-tracing simulation using a local database containing 3-D information of buildings, trees and terrain of Malvern area in England. In the simulation, however, trees were ignored and the terrain was assumed to be of uniform elevation.

The most serious limitations of this approach are summarised below.

1. It assumes that a radio link either has a clear path or is completely blocked by the buildings, thereby not considering diffraction as a potential mechanism for a mm-wave propagating to the receiver site. Millimetre-wave field measurements made by other researchers, e.g. those reported in [DeC00], however, revealed that the actual interaction between radio wave and ground structures (i.e. buildings, trees and terrain) is not that simple. In fact, attenuation at the same distance was found to vary from one location to another depending on the type of environment surrounding the radio link and take on a continuous range of values, rather than indicative of a two-state scenario.
2. It does not take into account the wave interaction with trees, either in the form of diffraction over and below them, or transmission through them.

Considering the above stated points regarding Biddiscombe's approach and the more tractable form of Jakes' method, the latter method is preferable for use in estimating clear sky coverage to be involved later in analyses of coverage under precipitation.

b. Ray tracing methods

Use of ray tracing methods, which apply electromagnetic wave theory in the estimation of losses caused by reflection and diffraction on a 3-D map of terrain, buildings

and vegetation, is one of several approaches that is of considerable current interest in studying area coverage of a cell. The most apparent difficulty in applying this method is the requirement of having as detailed, accurate, and current a 3-D regional map as possible, which often translates to an increase in cost. Nevertheless, this method can still be used to examine statistically the cell area coverage problem, using portions of an available map as individual samples in a Monte-Carlo type simulation. Another major limitation is computer resources and running time; the more accurate the ray tracing, the more computing time it takes and the more memory it needs.

Two coverage studies that employed ray tracing are briefly described in the following, followed by a general discussion on the use of this technique in estimating cell area coverage.

- **Biddiscombe [Bid99]**

Biddiscombe [Bid99] conducted a ray-tracing simulation to validate the result of his coverage model, as described earlier in this section. In particular he used a 3-D map of buildings for the Malvern region in England. Concentric annuli at discrete radii around a simulated hub site were generated, into each of which a set of buildings at approximately the same radius was grouped. Major limitations of this work have been discussed in 4.2.1a.

- **DeCruyenaere [DeC98, DeC99, DeC00]**

A more complete ray-tracing procedure was devised by DeCruyenaere [DeC98, DeC99, DeC00] and applied on 3-D maps of Ottawa area, the primary objective being to evaluate the effectiveness of various coverage enhancement methods. Aside from polyprism-

shaped model proposed for buildings, he also proposed separate models for a tree and a thicket of trees.

Discussion

As stated before, although perfect accuracy can not be realised with currently available techniques, the ray-tracing approach has the potential of producing a reasonable estimate of coverage based on the actual structure of the clutter environment. This could be done using mm-wave ray tracing codes with high accuracy and resolution, and an adequately detailed and up-to-date 3-D database. However, such a digitised, ready-to-use database is either hard to find or very costly, while the simulation result is obviously applicable only to that particular environment and hub site arrangement. When this approach is to be used for statistical analysis through a Monte Carlo simulation, a region in the map can not be reused so as to obtain as high a degree of independence as possible among the samples. Consequently this approach is not suitable when shadowing that varies from one environment to another is to be considered in addition to rain attenuation in a cell area coverage simulation, because a large number of maps of different areas would be needed to obtain an adequate degree of independence.

c. Field measurement

Researchers have reported a number of field measurement projects to quantify cell area coverage at millimetre-wave in the real world. The following is a list of these studies, some aimed at obtaining statistics for initial investigation or for model development, while some others were designed to validate or optimise models of physical structures (buildings, vegetation and terrain) for ray-tracing simulation or models for coverage prediction.

- **Seidel and Arnold [Sei95a, Sei95b]**

Seidel and Arnold [Sei95a, Sei95b] reported a coverage measurement trial at 28 GHz in a built-up area with 2- or 3-story residential and a cluster of multi-story apartment buildings. The transmit antenna was at an elevation of 95 metres, whereas that of the receive antenna was varied from 3.4 metres to 13.3 metres above the ground, on roadways. The coverage measurements made at 77 sites are tabulated below. It should be noted, however, that the antenna environment on a roadway is different from that in a typical subscriber antenna mounting position. Hence, measurement in each of the selected sites might not reflect the point coverage when the measurement was actually made on, say, the rooftop of a residential building.

Table 4-1 Seidel's coverage measurements for various hub antenna heights and ranges of distance

Rx antenna height (metres)	All locations	Locations <3km from Tx	Locations <2km from Tx	Locations <1km from Tx
13.3	68%	68%	72%	86%
7.3	46%	45%	50%	71%
3.4, 4.0	26%	27%	30%	48%

- **Gray [Gra97a, Gra97c]**

Another 28 GHz measurement campaign was executed by Hewlett-Packard, as reported by Gray in [Gra97a, Gra97c]. The project had the objective of compiling path loss data as a function of distance for various hub antenna heights and types of environment. No information is given in the papers as to whether the measurements were made with receive antenna being installed on rooftops of actual buildings or on a vehicle parked near them.

They found that for clear-sky coverage, hub antenna height and number of obstacles, such as vegetation, are key factors in determining the excess path loss exponent. For example, a suburban area with moderate tree density was associated with total path loss exponent of 6. Gray noted that the distance at which the path loss slope starts to deviate from that of free space depends primarily on hub antenna height and secondarily on number of obstacles. On the other hand, the path loss slope itself was found to depend more heavily on the latter than the former.

These results were applied in a coverage prediction using an empirically derived set of equations for rectangular four-sectored cells with multi-hub diversity. The equations, of which the derivation is not reported, were used to predict point coverage at a given range in a multi-hub system in an environment characterised by range-dependent excess loss. The impact of spatially uniform rain attenuation was also incorporated in the area coverage calculation. Tables 4-2 to 4-4 were extracted from the coverage curves given in references [Gra97a, Gra97c], with radius being equivalent to one half of hub spacing and uniform rain attenuation derived for region C of Crane's rain map [Cra96]. The tables each show the effect of variations in hub antenna height, vegetation density and excess link margin upon cell area coverage.

Table 4-2 Gray's coverage measurements for various hub antenna heights in areas with moderate tree density

Cell radius (km)	Hub antenna height (m)		
	15	25	30
1	78%	85%	87%
2	50%	68%	72%
3	23%	45%	53%
4	13%	25%	32%

5	8%	17%	20%
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Table 4-3 Gray's coverage measurements for various vegetation densities (hub antenna height not given)

Cell radius (km)	Tree density		
	Low	Moderate	High
1	85%	78%	64%
2	80%	68%	45%
3	72%	43%	20%
4	67%	32%	16%

Table 4-4 Gray's coverage measurements for various excess link margin at 1-km radius and moderate tree density

Antenna hub height (ft)	Excess link margin (dB)			
	0	10	20	30
50	25%	38%	57%	75%
75	45%	55%	75%	90%

- **Papazian *et al* [Pap97]**

Papazian *et al* carried out a field measurement project, including a series of narrowband measurements at 28.8 GHz in two suburban areas, Northglenn, Colorado and San Jose, California [Pap97]. The Tx and Rx antennas had 90° and 2.5° azimuthal beamwidths, respectively. A number of houses were selected as measurement sites, at each of which the measurement van was parked on the curbside near the house with the antenna mast elevated one meter higher than the rooftop.

Both measurement regions had flat terrain, with populated areas dominated by one- or two-story residences of similar type, height and density. The vegetation in Northglenn was more mature and had a denser canopy than those in San Jose. This was considered to be the primary cause of any difference in coverage measurement between the two sites. Each measurement area was square-shaped, representing a quadrant of a square cell. At each of the selected measurement sites within a cell quadrant, a time series of power measurement was made. An individual site was considered as being covered when the power loss was less than a maximum specified value. The coverage statistics were treated as a function of the maximum tolerable attenuation, which includes the effects of the distance-dependent path loss and other losses, such as those due to ground clutter. The entries in the following tables were visually interpreted from the curves for San Jose in the cited paper, each depicting the influence on coverage of varying cell radius and hub antenna height.

Table 4-5 Papazian's coverage measurements for various hub antenna heights at 1-km radius in San Jose

Hub antenna height (ft)	Tolerable total attenuation (dB)					
	130	135	140	145	150	155
40	26%	28%	34%	44%	62%	74%
80	58%	66%	73%	77%	82%	82%

Table 4-6 Coverage for various cell radii at 80-ft hub antenna height in San Jose

Cell radius (km)	Tolerable total attenuation (dB)					
	130	135	140	145	150	155
1	58%	66%	73%	77%	82%	82%
2	19%	21%	33%	44%	56%	65%

- **Seville [Sev00]**

A series of measurements was made in the UK at frequencies near 40 GHz, the target band for the European CRABS project (Cellular Radio Access for Broadband Services), by Seville *et al* [Sev00]. They employed two transmitters at two different locations 7 kilometres apart. The transmitter Tx₁ nearer to the measurement sites was positioned at a height of 18 metres above ground level, whereas the more distant Tx₂ was installed on a 23-metre mast. No information on the antenna radiation patterns was given. The relative position or approximate angular separation between the two transmitters with respect to the receiving sites was not reported. Measurement points were scattered at distances ranging from about 700 metres to 5 km from Tx₁, and were each approximately 7 km from Tx₂. At each measurement location the receive antenna was raised to a height of 8 metres above the roadway. The results were used to evaluate coverage from the nearer transmitter and the increase in coverage when the best signal from the two transmitters was selected, i.e., a macrodiversity configuration with selection combining. It was found that with the receive threshold set at the noise floor, an increase in coverage of 4% could be achieved with the two transmitters by employing selection diversity. Values in the following table were read from the coverage curve given in [Sev00], parameterised in terms of the received power threshold ΔTh relative to the noise floor.

Table 4-7 Seville's coverage measurements for various receiver thresholds

Transmitter	Relative receiver threshold ΔTh above noise floor (dB)						
	0	2	4	6	8	10	12
Tx ₁ only	63%	56%	50%	45%	43%	41%	37%
Tx ₁ and Tx ₂	67%	61%	57%	52%	50%	47%	42%

- **Bultitude *et al* [Bul98b] and DeCruyenaere [DeC00]**

Bultitude *et al* [Bul98b] performed a coverage survey at 28.7 GHz in Parkwood Hills, Ottawa in October 1996, the results of which were later used by DeCruyenaere [DeC00] to validate the coverage prediction obtained through his ray-tracing procedure. The transmit antenna had a 23° horizontal beamwidth and was positioned on a rooftop at a height of 40 metres. The 12°-beamwidth receive antenna was carried on a vehicle, fixed on a rotatable platform at about 15-foot elevation. Any of the 66 measurement points was considered not to be covered when the path loss was larger than a prescribed MTPL (maximum tolerable path loss). As the measurement in each location was accomplished by rotating the receive antenna platform in a full circle, any specular reflection could be detected. In such a way, the coverage study could be achieved for three different scenarios of reception at each point: (i) Only direct path, whether it was LOS or not, was considered, (ii) The receiver selected the path with the strongest reception, and (iii) The receiver used all the signals it received using maximal ratio combining scheme. The results for the three scenarios are presented in the following table.

Table 4-8 DeCruyenaere's coverage results (%) for various values of maximum tolerable path loss (MTPL)

Scenario	Maximum tolerable path loss (MTPL, in dB)												
	120	125	130	135	140	145	150	155	160	165	170	175	180
1	8	11	12	20	30	41	53	69	83	91	95	99	100
2	9	12	13	26	38	46	57	73	85	93	99	100	100
3	11	12	17	27	39	49	61	77	91	93	99	100	100

Discussion

By observing the experimental results described above, it becomes clear that there are a large number of local factors related to clutter in the environment of interest as well as system-related parameters that influence coverage. The environment-related factors include type of vegetation, foliage density, building density and height distribution, and terrain. Among the system parameters are hub and subscriber terminal antenna height. It is clear that if each of these parameters were to be explicitly included in a coverage analysis, computations would be very time-consuming and results would be cumbersome to interpret. In addition, the model derived from such a process would have too many parameters to be of practical use.

In the coverage studies reported herein, the effects of the above individual parameters are represented by only two well-known parameters, namely the path loss exponent and lognormal shadowing standard deviation.

4.2.2 Non line-of-sight (NLOS) coverage with uniform rain

Generally, the effect of rain attenuation on coverage prediction has been treated in a somewhat *ad hoc* manner. Since the only available, well-established method for statistical prediction of rain attenuation concerns point-to-point radio links, those people evaluating the effect of rain attenuation in a cellular context tend to apply that method to individual radio links within a cell. This is commensurate with assuming spatially uniform rain attenuation. In particular, such a procedure has been suggested by Gray [Gra97a, Gra97c] and later also used by DeCruyenaere [DeC99, DeC00].

- **Gray [Gra97a, Gra97b, Gra97c]**

Gray suggested a method by which the specific attenuation in dB/km for the desired availability, e.g. the 99.9th percentile, was obtained from an available time series database of rain rate recorded in the local region under consideration. In his example, he assumed the rain region C for San Francisco area as given by Crane [Cra96] and used the specific attenuation corresponding with 99.9% availability, that is, an attenuation of 1.62 dB/km. A term was then added when computing attenuation on each path, which was a product of the specific attenuation and the path length.

Using the above method, he made a comparison of coverage under precipitation for three different rain regions, F, C and D2. A clear-sky excess loss factor of 2 (equivalent to total path loss exponent of 4) and 4-km hub spacing (or 2-km cell radius) was assumed. The following are the results.

Table 4-9 Gray's coverage prediction for different rain regions

Rain region [Cra96]	Availability (%)				
	99.9	99.97	99.99	99.997	99.999
F	83%	80%	75%	65%	41%
C	82%	79%	71%	56%	32%
D2	79%	71%	59%	32%	-

- **DeCruyenaere [DeC98, DeC99, DeC00]**

DeCruyenaere followed Gray's method in his ray-tracing simulation. His simulation was so designed that any rain region of interest as well as any desired rain rate percentile could be chosen. For instance, in the simulations for Ottawa, located in rain region D1, a

99.99% rain rate of 37 mm/hr or, equivalently, specific attenuation of 8.0 dB/km was chosen [DeC00]. To each segment of a simulated ray, rain attenuation was applied, the amount of which was obtained by multiplying the above specific attenuation with the length of the segment.

Discussion

The assumption of spatially uniform rain might be justifiable when the rain is of widespread type in which intensities tend to be uniformly low over a large area, or when the area under consideration is small compared to rain cell dimension under convective rain. In mm-wave fixed cellular systems, radio cell dimensions are at least on the same order as those of typical convective rain cells, as has been discussed in Chapter 2. A uniform rain assumption should not be retained in this situation.

4.2.3 Line-of-sight (LOS) coverage with non-uniform rain

Another, considerably different approach in treating cell area coverage under precipitation was introduced by a group of researchers at Rutherford Appleton Laboratory, U.K. [God97, ITU99, Tan98, Tan00]. In terms of research methodology, their approach is similar to the one reported here. They used weather radar measurements over a two-dimensional space to take into account the spatial variability of rain attenuation in examining coverage statistics. They subsequently proposed a cell coverage prediction model based on the use of an area-averaged rain rate, fit to the radar-derived coverage statistics. However, they assume only free space path loss, i.e., a path loss exponent of two for the distance-dependent clear-sky losses, as described below.

Using two-year S-band radar records over a 160-km² area, they derived the path attenuation at 42 GHz and used it to compute cell area coverage statistics at the aforesaid frequency as well as area-averaged rain intensity. The area-averaged rainfall rate was subsequently used in a prediction model for 42 GHz cell coverage as follows.

The predicted coverage as a function of cell radius L (km), fade margin M (dB) at the edge of the cell, and time percentage p was obtained from:

$$C(L, M, p) = \left(\frac{r_0}{L} \right)^2 \times 100\% \quad (4-9)$$

The cut-off distance r_0 (km) is the radial distance at which the received power exactly equal the receive threshold under free space (LOS) propagation condition and uniform rain at intensity of the p^{th} percentile of cell area-averaged rain rate. The value of r_0 is hence obtained by solving the following equation for r ,

$$a[R_L(p)]^b r [1.5 + 1.1(2r^{-0.04} - 2.25)\log_{10} R_L(p)] - M + 20 \log_{10} \left(\frac{r}{L} \right) = 0 \quad (4-10)$$

where a and b are the power-law parameters of the Y - R relation, r (km) represents radial distance from the cell centre, and $R_L(p)$ is the p^{th} percentile of rain rate averaged over the cell area. The entire first term in the above equation constitutes the equivalent attenuation on a r -km link computed using the area-averaged rain rate. It contains a lengthy factor, $[1.5 + 1.1(2r^{-0.04} - 2.25)\log_{10} R_L(p)]$, which is the area-weighted path reduction factor. The factor 20 in the last term reflects the free space exponent of two.

The prediction method developed in the above study appears to be the most attractive for use in 30 GHz fixed cellular context. We chose to develop our own model, however, due to the following considerations.

1. Statistically, rain intensity and its horizontal structure are believed to be significantly climate-dependent [Bru95]. The fashion in which area-averaged rain rate is used to predict area coverage thus needs to be examined against various climatic regions before the above method can be used safely beyond the region for which it was originally developed.
2. In a similar note, the incorporation of a path reduction factor means that the application of this method to climatic regions and frequency bands other than those used in its derivation should be done with caution [Bru95].
3. Aside from the radar-derived rain attenuation, only free space loss is accounted for in the prediction calculation. Difficulties arise when the outcome of such a study is to be used in predicting the actual area coverage of a cell where the effects of shadowing are present, the impact of which on coverage is inseparable from and just as significant as that of rain attenuation. A more logical and useful approach in this regard is by not attempting to separate the effects of both shadowing and rain on cell coverage. Rather, the random component of the total attenuation used in deriving the coverage must already take into account both phenomena.

We use different reasoning in our approach. It is believed that in order to have a statistically adequate accuracy for predicting reliabilities as high as 99.999% without neglecting the year-to-year variation, a sample size on the order of that which can be

compiled over 10 years or so is required [Ols99]. In addition, it is also realised that the DSD measurements used to parameterise the Y - Z power-law relation used in our work were recorded in Montreal and Toronto, whereas the radar system used to record the radar images was located in Montreal. Therefore in our own study a model that explicitly incorporates the dependence on geographical location and frequency and that predicts very high reliabilities will not be attempted due to limitations in the volume and geographical extent of database. Rather, the model obtained herein applies only locally, that is to areas with climatic characteristics similar to Montreal, and tries to predict reliabilities with adequate confidence only down to 99.99%.

It is, however, worthwhile to note that the method we developed to acquire coverage estimates under rain attenuation and shadowing, to be presented later in this chapter, can be used to obtain coverage statistics for any climate region provided the radar data are available.

4.3 Computation of area coverage under precipitation

4.3.1 Method for computation of instantaneous coverage

In the sequel, two cases are considered in the computation of cell area coverage under rainy conditions. The first case assumes an ideal scenario in which all the subscribers in the nominal area of the circular cell are under LOS condition. In this situation, the entire nominal area is covered under clear sky conditions. In the second scenario, the covered area under clear sky conditions degrades from the nominal due to shadowing effects, which create a mix of LOS and NLOS subscribers in the nominal area.

a. Coverage in cells with only LOS subscribers

The IEEE 802.16 committee, one of several international standard committees that work on standardisation of technology for broadband wireless communication systems, defined a model for broadband millimetre-wave point-to-multipoint radio access systems, referred to as 802.16.1 systems, that must satisfy several requirements [IEE99]. It was stated that for such systems that generally operate at frequencies in the range of 10–66 GHz, a line-of-sight (LOS) condition between a hub and a subscriber station is mandatory. An investigation in which LOS conditions are assumed for all locations within the radio cell is therefore of interest.

It is realistically assumed that at least one of the hub or subscriber antennas employ a directional antenna with a very narrow beam in azimuth and elevation in order to reject multiple paths that occur due to reflections and diffraction. Hence, only the LOS path is considered to exist between the transmitter and the receiver. This is a condition that also occurs under a free space scenario, which means that the propagation loss increases quadratically with range between Tx and Rx. The total attenuation experienced by a radio wave is therefore the sum of the free space loss and rain attenuation. If the attenuation is larger than a prescribed threshold that is determined according to the margin available to subscriber terminals on the circumference of the cell, then the subscriber in question is considered not covered.

In the simulation, a subscriber in a cell is considered covered when it experiences the combination of free space path loss and rain attenuation that is less than a threshold specified with respect to the free space loss at the perimeter of the cell. The periphery of the covered

area within each cell is then taken to be the locus of points along each radial at which total attenuation exceeds the specified threshold, and numerical integration is used to estimate the coverage area. This is expressed as a fraction of the circular area that would normally be covered under clear sky conditions at the specified threshold in the absence of obstruction shadowing and multipath effects. Coverage of a cell with a L -km radius can therefore be obtained as:

$$C = \frac{1}{\pi L^2} \sum_{\theta=1^\circ}^{360^\circ} \sum_{r=0}^L f(r, \theta) \Delta S(r) \quad (4-11)$$

where

$$f(r, \theta) = \begin{cases} 1, & A(r, \theta) + 20 \log_{10}(r/L) - M \leq 0 \\ 0, & \text{elsewhere} \end{cases} \quad (4-12)$$

with $A(r, \theta)$ representing the rain attenuation in dB experienced by a subscriber terminal at range r and azimuth θ , M denoting the power margin available at the edge of the cell, and

$$\Delta S(r) = \frac{\pi}{360} [r^2 - (r - 0.1)^2] \text{ (km}^2\text{)}, \quad 10r = 1, 2, \dots, 10L \quad (4-13)$$

representing the area of a bin bordered at the far side by an arc of a r -km circle.

b. Coverage in cells with shadowing

As can be seen from discussions in the preceding section, there are two types of variability in cell coverage. The first type is ascribed to variation in shadowing structures from one location to another. This produces random variation in shadowing loss among different radio links of equal length, triggered by two different phenomena. One is the

multiple paths that add destructively in the shadowed region. With a multiplicity of paths involved, the point of occurrence of shadowing is randomly located. The local mean of received power (or, inversely, path loss) is commonly described as lognormally distributed, with lognormal mean that changes with distance in power-law manner, whereas the finer-scale variation in that local area is referred to as fast fading. The second contributor to shadowing loss is physical obstruction by buildings or other structures, which might result in complete blockage of power, weaker transmission through the obstacle, or diffraction by the edges of the obstacle. The presence of different obstacles in the environment, e.g. in a built-up area, could change the distance exponent in the power-law model and the standard deviation of the lognormal distribution. This causes cell area coverage to vary from one cell to another. The statistics of this coverage variation can be predicted using the models discussed in Section 4.2.1a.

The second type of variation occurs due to time variation of rain structure in a specific location. Not only does this variation happen for different rain events, but it also occurs over time intervals on the order of minutes during a single storm event. Section 4.2.2 described model CDFs that have been proposed by other researchers to describe the attenuation variation due to the time-varying nature of rain, and outlined the difficulties in applying these rain attenuation models without ignoring the lognormal variation of shadowing. These difficulties lead us to the following method of analysis.

Firstly in the absence of rain, we consider that the lognormal shadowing loss model applies in all locations, with its mean increasing with distance from the hub in a power-law fashion. Hence, the prediction of probability (over all locations) that a subscriber location at a

given distance is covered, i.e. point coverage, under clear sky requires only two parameters, namely the distance power-law exponent and the lognormal shadowing standard deviation. These, together with power margin information, are the input to point coverage equation (4-5). Subsequently, the prediction for clear-sky cell area coverage can be obtained by using equation (4-6).

Similarly, in the presence of rain, the point coverage *at a given time* can be computed by still assuming lognormal shadowing. In this case, however, the *instantaneous* rain attenuation is added to the mean. The computed value can be regarded as instantaneous coverage of a site at a particular position relative to the hub and associated with link l_r , mathematically expressed as

$$\Pr[P(l_r) > P_{th}] = \frac{1}{2} - \frac{1}{2} \operatorname{erf} \left(\frac{-M + 10\alpha \log_{10}(r/L) + A_{rain}(l_r, t)}{\sigma\sqrt{2}} \right) \quad (4-14)$$

The probability on the left-hand side reflects the spatial variation of shadowing. All the symbols are defined as for equation (4-5), except for an additional quantity $A_{rain}(l_r, t)$ that denotes rain attenuation on link l_r at observation time t . The diagram in Figure 4-1 describes pictorially the computation of coverage of the subscriber station associated with link l_r under precipitation.

Figure 4-1 Sketch portraying the concept of distance-dependent outage probability

To obtain the instantaneous area coverage prediction for a cell simulated on a radar image multiple radial links are simulated with 1° separation between them, yielding 360 radials of the same length. Along each link, rain attenuation and point coverage are calculated at multiple discrete distances with 100 metre spacings, with the farthest being the cell radius. Hence, for a cell of L -km radius, the number of bins evaluated for point coverage is $360 \times L \times 1000 / 100 = 3600L$. Since the size of each bin depends on its distance from the cell centre, the area coverage computation must include weighting of each bin by its incremental area. The instantaneous area coverage is therefore expressed as:

$$C(t) = \frac{1}{\pi L^2} \sum_{l_r} \Pr[P(l_r) > P_{th}] \Delta S(r) \quad (4-15)$$

where $\Delta S(r)$ is defined in (4-13).

4.3.2 Sample size considerations

Since sequences of radar records can be regarded as time series measurements of rain structures, the instantaneous coverage values obtained from simulations serve as samples in a Monte Carlo simulation. The total number of samples simply equals the number of radar images used in the simulation. If successive samples from such a simulation with a total of N samples are independent from one another, then observing the frequency of occurrence of the event that coverage is less than its $(100p)^{\text{th}}$ percentile is identical to observing the number of failures with probability of occurrence p in N independent Bernoulli trials. Hence, if the probability that coverage is less than C_p is p (with $\pm 0.1p$ confidence interval for a confidence coefficient of 90%), then as many as N simulated cells are required to obtain C_p [Sch90] where

$$N = \left(z_{0.05} \sqrt{p(1-p)} / 0.1p \right)^2 = 1.645^2 \times 100 \times (1-p) / p \quad (4-16)$$

In the above, $z_{x/2}$ is defined by $\Pr[-z_{x/2} \leq Z \leq +z_{x/2}] = 1 - x$, where $Z \sim N(0, 1)$. Table 4-10 shows the number N of cells required to achieve the above condition for various values of probability p . The use of smaller number of cells is possible, but the accuracy of the result in terms of either the confidence interval or the confidence coefficient has to be sacrificed.

With almost two complete years of radar measurements, the number of samples that could be used for our analyses is 139,722 (see Appendix B), covering both clear-sky and rainy periods in which recordings were made. This is merely one half of the required number of samples to obtain $\pm 0.1p$ for $p = 10^{-3}$ or 0.1%. For a 90% confidence coefficient and the

above given number of samples, the confidence intervals for various values of probabilities p (expressed as $100p\%$) can be calculated as follows [Sch90]:

$$p \pm B = p \pm z_{0.05} \sqrt{p(1-p)/N} = p \pm 1.645 \sqrt{p(1-p)/139722} \quad (4-17)$$

The results are given in Table 4-10.

Table 4-10 Required number of simulations for $\pm 0.1p$ confidence, and confidence interval obtained from 139,722 simulations with 90% confidence coefficient

Probability p	Equivalent length of time per 365-day year	Required number of samples (N) for confidence interval $p \pm 0.1p$	Percentage interval $100(p \pm B) \%$ for sample volume $N =$ 139,722
10^{-1}	36.5 days	2435	10 ± 0.1
10^{-2}	87.6 hrs (3.7 days)	26790	1 ± 0.04
10^{-3}	526 mins. (8.8 hrs.)	2.7×10^5	0.1 ± 0.01
10^{-4}	52.6 mins.	2.7×10^6	0.01 ± 0.004
10^{-5}	5.3 mins.	2.7×10^7	0.001 ± 0.001

In calculating the confidence intervals, an underlying assumption that should not be forgotten is that successive samples are independent from each other. With radar sampling period of 5 minutes, this independence is only realised to some degree under high-intensity showers, the type of events with more infrequent occurrence, during which the intensity changes more rapidly than it does under stratiform rain [Cra96]. Table 4-10, therefore, present the ideal case with independent samples, whereas in the real situation the confidence intervals for a 90% coefficient would be larger.

With only 139,722 observations, it is also realised that the number of events associated with the 0.01th percentile is only 13. It is thus concluded that the statistics that are considered reliable are those above the 0.03th percentile, for which 41 events occur, hence assuring the statistical accuracy. Therefore, although the statistics for probabilities down to 0.01% are shown in the CDFs and included in the evaluation, it must be emphasised that the use of results from quantiles less than 0.03% should be made with caution and is not recommended. As the same radar data are used in Chapter 5 in the examination of cell-site diversity, the confidence intervals given in Table 4-10 as well as the consideration above also apply to the statistics of rain attenuation obtained in that chapter. It should be noted that in examining all the CDFs presented in this thesis all the recorded events, either rainy or not, were included in their computation so that all the probability values stated herein are absolute, not conditional.

A Kolmogorov-Smirnov test [Dix69] showed that with 80% confidence the interval in which the actual distribution of coverage and attenuation might be located is $1.07/\sqrt{138939} \approx 0.003$ above and below the observed distribution.

4.4 Statistical results for cells with only LOS subscribers

Instantaneous coverage was estimated by applying equation (4-11). Then, CDFs of coverage were examined for various cell radii and power margins at the cell edge. Due to the ideal LOS conditions for all subscribers, the covered area under clear sky conditions would include the entire cell area.

4.4.1 Variation in cell radius

Cell radius was varied from 1 to 5 kilometres with 1 kilometre spacings. Figures 4-2 and 4-3 show the CDFs when the power margin available at the edge of the cell is set to 5 and 10 dB, respectively. The curves should be interpreted as follows. In Figure 4-2, for instance, with a cell radius of 2 kilometres, the area coverage is less than 82% for only 0.1% of the time, or equivalently, a total of 8.8 hours in one year. Conversely, due to rain attenuation, for 99.9% of the time the area coverage is larger than 82%. In this scenario, coverage is 100% under clear sky conditions and path loss is maximum at the cell rim.

Figure 4-2 CDFs of coverage for a power margin of 5 dB and various radii.

Figure 4-3 CDFs of coverage for a power margin of 10 dB and various radii.

Observations that can be drawn from the figures are:

- a. Comparing the results with those reported in an earlier paper [Hen00] that were obtained from only the first year of the radar data, consistency in the statistics can be observed. The present results show a larger range of probabilities since the amount of data available doubles that which were used in the preliminary examination reported in [Hen00].
- b. For a specified threshold with respect to free space loss at the cell margin it is more difficult to attain the same coverage under precipitation when the cell radius is increased.
- c. Generally, for a radio cell of any radius in the range of 1-5 km, there is no significant degradation in coverage in 99% of the time for fade margins as low as 5 dB at the edge of the cell.

- d. When a fade margin of 5 dB is applied to subscriber terminals at the edge of the cell, the area coverage is not significantly degraded for 99.9% of the time when the cell radius is only one kilometre. However, when the radius is expanded to 2 km, the degradation starts to become significant; coverage is less than 82% of its clear-sky value for 0.1% of the time (8.8 hours in total per year) for 2-km radius cells, and gets even worse for larger cells. When a fade margin of 10 dB is applied, 1- and 2-km radius cells hardly suffer from coverage reduction for 99.9% of the time, but cells having a radius of 3 kilometres or larger might attain only 88% or less of the clear-sky coverage for 0.1% of the time.

- e. For 0.03% of the time, or a total of 16 minutes in one 365-day year, with a power margin of 5 dB a 1-km cell might experience 78% coverage or less with respect to clear-sky. Cells with larger radius have less coverage at the same probability level. However, when the available margin at the edge of the cell is increased to 10 dB, a 1-km cell performs well, having coverage less than 96% of the clear-sky value for only 16 minutes in one year. Cells with a radius of 2 km or larger might experience coverage less than 76% of the clear-sky level for the same total portion of time in a year.

- f. Decreasing cell size could be a means of lowering the probability of having coverage less than a given level for a specified percentage of time. For instance when a power margin of 10 dB is available for terminals at the cell perimeter, a decrease in cell radius from 4 to 2 km would decrease the percentage of time that coverage is less 90% from about 0.15% (~13 hrs in a year) to 0.06% (~5 hrs). A further decrease to 1 km radius would yield 0.02% of time (~105 minutes in a year) for coverage less than 90% with respect to the clear-sky value.

- g. A non-linear relation is found to exist between cell radius and coverage for a given percentage of time, but cell radius and percentage of time for a specified coverage are in approximately linear proportion to each other. In particular, to attain the same coverage level with the same threshold, an increase in cell radius by a factor of 2 would roughly increase the probability of having coverage less than a specified value by a factor of 3. Nevertheless, no attempt was made to determine a mathematical form that estimates the relationship.

4.4.2 Variation in margin at the cell edge

In the simulations, power margin at the edge of the cell was varied from 5 to 25 dB with 5 dB intervals. Figures 4-4 and 4-5 show the CDFs for cell radius of 2 and 4 km, respectively. The interpretation is the same as before.

Figure 4-4 CDFs of coverage for a cell radius of 2 km and various power margins.

Figure 4-5 CDFs of coverage for a cell radius of 4 km and various power margins.

Some remarks that can be made about these figures are:

- a. For cells with a radius of 2 km, coverage might reduce to less than 82% for 0.1% of the time when a margin of 5 dB is applied. The coverage reduction is considerably mitigated when the margin is increased to 10 dB, under which condition the coverage remains above 96% for 99.9% of the time. For 99.97% of the time, however, 90% or more coverage can only be obtained when a margin of 15 dB is given to the subscribers at the edge of the cell. Further improvement is shown for even larger margins, but these margin values might be impractical.
- b. A poorer coverage performance is demonstrated by cells with a 4 km radius, for which the coverage could only be maintained above 90% for 99.9% of time when a margin of 15 dB is available, and above 95% when a margin of 20 dB is applied. However, to maintain coverage of at least 82% for 99.97% of the time, a margin as high as 25 dB is needed.
- c. As in the case of coverage for various radii, the probability of having coverage less than a given percentage can be reduced by enlarging the available power margin at the cell perimeter. For instance, coverage less than 90% can occur in cells with a 2-km radius for 0.15, 0.06 and 0.03% of the time when margins of 5, 10 and 15 dB are applied, respectively.
- d. The effects of variation in available power margin at the cell perimeter either on coverage for a specified percentage of time, or on percentage of time for the same coverage, appear to obey a non-linear relation, the mathematical form of which was not sought in this thesis.

4.4.3 Discussion

The CDFs of coverage presented above generally demonstrate the significance of variation in radius and margin on area coverage in rain. Examination of the CDFs for 4-km radius against similar CDFs reported by Tan for 40 GHz LOS cells with 5-km radius and 10 and 15 dB margins [Tan98] shows reasonable differences, which could be explained by the dissimilarities in cell radius and operating frequency. Unlike Tan, who proceeded with devising a model based on these results, this thesis work attempted further to evaluate the coverage when shadowing loss is taken into account. Hence, the next section reports simulations similar to the previous but with lognormal shadowing loss superimposed on the radar-detected rain attenuation.

4.5 Statistical results for mixed LOS/NLOS environments

For this section, the simulations were performed using the method described in Section 4.3.1b. The individual effects of variation in path loss exponent α , lognormal shadowing standard deviation σ , cell radius L and margin M available at the edge of the cell are first examined. CDFs of coverage and coverage normalised with respect to the clear-sky level are given for each case. The normalised coverage $C_{\text{norm}}(t)$ is obtained as:

$$C_{\text{norm}}(t) = \frac{C(t)}{C_{\text{clear-sky}}} \quad (4-18)$$

where $C(t)$ is the instantaneous coverage calculated using (4-15) and $C_{\text{clear-sky}}$ is the clear sky coverage obtained from equation (4-6). It should also be noted that in this scenario, the margin at the cell rim is, unlike in the case of cells with uniformly LOS subscribers, defined

with respect to the clear-sky mean path loss with path loss exponent of α , rather than free space loss.

The CDF curves should be interpreted in the manner described in Section 4.4. When a phrase like "60% coverage at probability level of 0.01%" is used in the following discussion, it refers to the point on the CDF curve in question where the coverage is higher than 60% for 99.99% of the time.

4.5.1 Variation in path loss exponent

In the simulations, the path loss exponent was varied from 3 to 5. The selection of this range was based on mm-wave measurement results [Bul98a, Bro00] that showed variations in the empirically derived distance exponent around 4. It should be noted that in varying the path loss exponent, the margin at the cell perimeter was maintained at a specified level with respect to the mean path loss. This implies that the transmit power at the hub was adjusted to account for the variation in path loss exponent such that the overall effect caused the margin at the perimeter to remain constant. The interpretation of the presented results, hence, should be made with this kept in mind.

Several combinations of the other three parameters, shadowing standard deviation, cell radius and margin were tried, but the CDFs shown in Figure 4-6 and 4-7 are only those for $\sigma = 14$ dB, $L = 2$ km, and $M = 10$ dB. The curve in Figure 4-6 does not show 100% coverage for the upper end of probability due to the fact that even when there is no rain, coverage is degraded by shadowing effects. In fact, the coverage level at a 100% level of probability exactly equals that for clear sky conditions, which can be computed using (4-6).

For the above values of σ and M , the application of equation (4-6) yields coverage of 86, 88 and 89% when α is 3, 4 and 5, respectively.

Figure 4-6 CDFs of coverage for $L = 2$ km, $M = 10$ dB, $\sigma = 14$ dB and various α

Figure 4-7 CDFs of normalised coverage for $L = 2$ km, $M = 10$ dB, $\sigma = 14$ dB and various α

From Figures 4-6 and 4-7, the following observations can be made.

- a. For a given percentage of time, the coverage for cases with larger path loss exponents is found to be better. This is because in all these cases the margin available at the cell edge was fixed at 10 dB, which means that a steeper increase in path loss with distance requires higher transmitted power from the hub in order to maintain the same receiver threshold at the cell edge.
- b. In environments with the same shadowing variation but different distance exponents, when the same cell radius and margin are applied, a small variation in probability of having coverage less than a given value can be expected for various exponents. To attain 80% coverage in an environment with distance exponent of 3 is slightly more difficult than to do so when the distance exponent is 5, i.e. the former causes the probability of

having less than 80% coverage to be 0.2% while the latter 0.09%. However, this happens only because keeping M at a fixed value implies that the transmitted power must be reduced when the distance exponent is decreased.

- c. When the normalised coverage given in Figure 4-7 is considered, the degradation from clear-sky coverage is only slightly influenced by the magnitude of the range exponent α . As before, this occurs because the margin is kept constant when the distance exponent is varied.

4.5.2 Variation in the standard deviation of lognormal shadowing

In the simulations, the shadowing standard deviation was varied from 11 through 17 dB. This range was selected according to values of standard deviation of excess path loss obtained from measurements reported in [Bul98a, Bro00] (see part (a) of Section 4.2.1). The CDFs of coverage and normalised coverage shown in Figures 4-8 and 4-9 were obtained for parameters α , L and M being set to 4, 2 km and 10 dB, respectively.

Figure 4-8 CDFs of coverage for $L = 2$ km, $M = 10$ dB, $\alpha = 4$ and various σ .

Figure 4-9 CDFs of normalised coverage for $L = 2$ km, $M = 10$ dB, $\alpha = 4$ and various σ .

The following observation can be made from the above figures.

- a. As in previous cases, at the upper end of the probability level the coverage is essentially that under clear-sky conditions, which can be computed by applying equation (4-6).
- b. At the lower end of the probability range, the CDFs of coverage for these different standard deviations converge. This is because lower range of probability levels can be associated with events having intense rain over parts of the radio cell which makes reception in a large portion of the cell area fall under the receiver threshold, regardless of the shadowing conditions of these locations.
- c. From different curves of normalised coverage in Figure 4-9 it is observed that degradation from clear-sky coverage is hardly influenced at all by variations in the shadowing standard deviation.

4.5.3 Variation in cell radius

The cell radius was varied in the range of 1-5 km with 1-km steps, a range considered reasonable for mm-wave fixed cellular applications. The CDFs computed for parameters M , α and σ set to 10 dB, 4 and 14 dB, respectively, are presented in Figures 4-10 and 4-11. The coverage at probabilities above 10%, i.e., those for clear sky cases, should be identical for all radii, as can be predicted by applying the above parameter values to equation (4-6). However, the values for 1- and 2-km cells shown in Figure 4-10 are slightly different from the others due to relatively lower resolution in the area calculation for these two sizes.

Figure 4-10 CDFs of coverage for $M = 10$ dB, $\alpha = 4$, $\sigma = 14$ dB and various L .

Figure 4-11 CDFs of normalised coverage for $M = 10$ dB, $\alpha = 4$, $\sigma = 14$ dB and various L .

A number of observations can be made from the above figures.

- a. While different cell radii result in identical clear-sky coverage, the values for reduced coverage due to rain attenuation are much different from one radius to another, with smaller cells exhibiting less degradation than the larger ones.
- b. Coverage degrades non-linearly with increasing cell radius. For instance, at 0.03% probability, less degradation in area coverage is observed when the cell radius is increased from 4 to 5 km, whereas a more severe degradation occurs when the radius is changed from 1 to 2 km. This implies that for larger radii, the dimension of rain cells during the most intense rain events is similar to or less than that of the radio cells.
- c. Probability of achieving coverage less than a given percentage changes approximately linearly with cell radius. The probability is lower when the cell is of smaller dimension. For parameter settings that lead to the curves in Figure 4-10, coverage of, say, 80% or less is achieved with probability of 0.05, 0.11, 0.2, 0.3 and 0.4% when the cell radius is 1, 2, 3, 4 and 5 km, respectively. Similar observations can be made for normalised coverage.

4.5.4 Variation in margin at the cell edge

The amount of power margin available to a subscriber terminal located at the edge of the cell was varied from 5 to 25 dB in 5 dB steps. Figures 4-12 and 4-13 shows CDFs of coverage and normalised coverage when parameters L , α and σ were fixed at 2 km, 4, and 14 dB, respectively.

Figure 4-12 CDFs of coverage for $L = 2$ km, $\alpha = 4$, $\sigma = 14$ dB and various M .

Figure 4-13 CDFs of normalised coverage for $L = 2$ km, $\alpha = 4$, $\sigma = 14$ dB and various M .

The following can be observed:

- a. Variation in power margin at the edge of the cell with intervals of 5 dB brings on large variation in cell area coverage at any probability level either under clear sky conditions or in precipitation.
- b. For a given probability level, the variation in coverage is non-linearly proportional to the change in margin. When the power margin is increased from 5 to 10 dB, a larger change in coverage occurs compared to the case when the margin is increased from 20 to 25 dB.
- c. For the same cell size and shadowing environment, the probability that coverage is less than a specified value can be reduced by enhancing the margin available at the cell perimeter. For example, for $L = 2$ km, $\alpha = 4$, and $\sigma = 14$ dB, the probability that coverage is less than 75% can be made as low as 0.4, 0.06 and 0.02% by setting M to 5, 10 and 15 dB, respectively.

4.5.5 Discussion

A number of findings that arise from the above observations are analysed below.

- a. From the examination of normalised coverage CDFs, changes in path loss exponent and lognormal shadowing standard deviation are found to have insignificant effects on the degradation in coverage under rain. However, this should be coupled with an understanding that in evaluating the effects of various exponents on normalised coverage, for any change in path loss exponent, an adjustment in the hub transmit power is implied such that the resulting margin at the cell perimeter remains unchanged. This leads to reasoning that regardless of the variety in shadowing environment, coverage degradation

due to rain from its clear-sky value would show a similar cumulative distribution as long as the same margin is maintained at the cell perimeter. Unless neighborhoods with very different shadowing structures are concerned, e.g. a built-up area with high-rise buildings and a suburb with sparse one- or two-story residential houses and low vegetation, the cumulative distribution that describes the amount of fractional area uncovered due only to precipitation is fairly independent of the type of environment.

- b. In contrast to the influence of shadowing, variations in cell size and fade margin exhibit great impact on the coverage degradation due to rain. Changes in either of these two parameters non-linearly influence the degradation in area coverage due to rain at a specified percentage of time. In general, it is obvious from the CDFs that smaller cell size yields better area coverage under rain, although under clear sky conditions coverage does not depend on cell dimension as long as the same level of fade margin available at the cell edge can be maintained. Also, it is apparent and not surprising that larger margins cause larger coverage, with or without the presence of rain.
- c. For any combination of the four simulation parameters, namely cell radius, fade margin available at the cell perimeter, path loss exponent and the standard deviation of lognormal shadowing, it is only for a total time percentage of 1%, or 3.7 days annually, that area coverage is degraded by rain to less than 96% of the value under the clear sky conditions.

4.6 Modelling for coverage prediction at 30 GHz

Modelling of coverage as a function of the four parameters described above is designed to facilitate coverage prediction in regions for which the characteristics of

shadowing are known. Since in the application of an actual cellular system shadowing can not be totally avoided, the model proposed herein is concerned with prediction of coverage under the effects of lognormal shadowing and rain attenuation. Furthermore, since coverage of a circular cell under clear sky conditions can be predicted using equation (4-6) given the values of lognormal distribution parameters associated with the target environment, the model was designed to predict the scaling factor required to obtain coverage under rain from the clear-sky value. That is, the model proposed predicts the normalised coverage C_{norm} , from which cell area coverage C can be obtained as follows:

$$C = C_{\text{norm}} \cdot C_{\text{clear-sky}} \quad (4-19)$$

where $C_{\text{clear-sky}}$ is computed using (4-6).

In the next section, the assumptions used and the limitations of the model, the procedure of derivation and the recommended procedure of application are described.

4.6.1 Assumptions and limitations

The following assumptions are used throughout the derivation and application of the model to be derived:

- a. The cell is circularly shaped with the hub located at its centre.
- b. There are no technical difficulties, such as limitations in the hub antenna radiation pattern or transmitted power, in achieving maximum coverage. The only factors that might reduce coverage are shadowing and rain.

- c. The subscriber stations each communicate with the hub through a radio link, which can be under a LOS or NLOS condition, hence the term "mixed LOS/NLOS". LOS subscribers are covered all the time under clear sky conditions but might not be covered under heavy precipitation. A NLOS subscriber can be covered when the signal it receives from the hub is not overly attenuated by an obstacle and rain.
- d. The mean path loss under clear sky conditions increases with distance in a power-law fashion. The excess path loss leading to variation in shadowing is lognormally distributed. The effect of short-term fading is not considered herein.

Considering the manner in which radar data were measured and coverage simulations were performed, the following limitations are imposed on the model:

- a. Because the model was derived from radar measurements made in Montreal, it is applicable for prediction only in regions having the same temporal and spatial characteristics of rain. Since it is currently unknown whether or not rain maps that classify geographical areas according to regional average point rain rate distribution can justifiably be used to identify regions having the same spatial distribution statistics, the use of the reported model is therefore suggested only for areas in the same rain region based upon Crane's or ITU-R's rain map.
- b. The model was derived and is therefore applicable only for cells operating at 30 GHz.
- c. The model is applicable only for the range of parameter values used in its derivation. These are: path loss exponent ranging from 3 to 5, lognormal shadowing standard

deviation from 11 to 17 dB, cell radius of 1 to 5 kilometres, and margin available at the edge of the cell from 5 to 25 dB.

The following points should be clarified as well:

- a. The derivation of the model does not assume the details of shadowing structure. The structure is described statistically by the path loss exponent and standard deviation of lognormally distributed excess path loss.
- b. The model does not predict the performance of individual links within the cell.
- c. The derivation avoids the use of statistics of point rain rate or attenuation derived therefrom. In addition, unlike the prediction model proposed by Tan et al [Tan98, Tan00, ITU99], it does not use the statistics of area-averaged rain rate, for reasons stated in Section 4.2.3.
- d. Since the derivation only involves measurements in Montreal, the model derived avoids the inclusion of any parameter that reflects the effects of differences in regional characteristics of rain, except it is specified that the resulting model is applicable only in regions with similar characteristics.

4.6.2 Modelling steps

a. Variation of normalised coverage with respect to α and σ

The first step in the modelling process involved determining how normalised area coverage obtained from simulations changes when each of the four parameters, namely cell radius L , margin M available at the edge of the cell, path loss exponent α and lognormal

shadowing standard deviation σ , is varied. This was aimed at providing the basis for a set of analytical functions to be nominated as candidate for a model that explains the change in coverage due to variation of each parameter at a specified percentage of time.

Specifically, by fixing three of the parameters the general relation between coverage and the fourth parameter at a given percentage of time can be studied. For instance, setting $L = 2$ km, $M = 10$ dB and $\sigma = 14$ dB produces a set of curves shown in Figure 4-14 that demonstrate slow changes in coverage when α varies from 3 to 5 for various percentages of time. Likewise, the curves depicted in Figure 4-15 exhibit linear increase of coverage when σ is gradually increased from 11 through 17 dB while the other parameters are set as follows: $L = 2$ km, $M = 10$ dB and $\alpha = 4$.

Figure 4-14 Changes in coverage statistics for $\sigma = 14$ dB, $L = 2$ km, $M = 10$ dB and various α .

Figure 4-15 Changes in coverage statistics for $\alpha = 4$, $L = 2$ km, $M = 10$ dB and various σ .

These two examples indicate that the derivative of coverage with respect to each of these two varying parameters when the other three parameters set to the aforementioned values would be a small positive constant. When coverage for different combinations of the other three fixed parameters was examined, an increase in coverage with similar gradual slope over the same ranges of α and σ was observed as well.

A different situation occurs for 99.7% time percentage or less, for which the coverage appears to be constant across the range of these parameters. It was concluded accordingly that in the model, the dependence of coverage on these two parameters must be expressed as two individual terms in the form of $k_{\alpha}\alpha$ and $k_{\sigma}\sigma$, where k_{α} and k_{σ} are the gradients of the slopes whose values depend on the desired percentage of time. For time percentage of 99.7% or less, these gradients are set to zero. For higher time percentages, the two gradients were

examined to identify their linear dependence on any of the other three parameters. It was found that for these time percentages, k_α is highly correlated with L , whereas k_σ shows high correlation with M . From these observations, it was decided that the coverage prediction model for time percentages of 99.9% and up should include two terms to accommodate for the effects of variation in α and σ ,

$$(e + fL)\alpha + (g + hM)\sigma$$

where e, f, g and h are coefficients whose values were to be determined through fitting of the model to the coverage statistics from all combinations of parameter values.

b. Variation of coverage with respect to L

Figure 4-16 shows the change in coverage when cell radius L is varied from 1 to 5 km, while the other parameters being set as follows: $M = 10$ dB, $\alpha = 4$ and $\sigma = 14$ dB. For time percentages of 99, 99.5 and 99.7 %, coverage appears to be linearly proportional to cell radius with a negative gradient. Accordingly, a general linear function having a form of $A + BL$ was assumed to describe the effect of cell radius on coverage, with A and B being functions of M whose forms were to be determined later. When the values obtained for A and B were examined for various M , it was found that both coefficients are a linear function of M and hence can be expressed as $A = a + bM$ and $B = c + dM$.

Figure 4-16 Variation of coverage for different cell radii, with $M = 10$ dB, $\alpha = 4$ and $\sigma = 14$ dB.

For time percentages of 99.9, 99.95, 99.97 and 99.99%, coverage curves decrease non-linearly with cell radius. When Figure 4-16 was redrawn on a semilog paper as shown in Figure 4-17, it was found that the logarithmic change of coverage is approximately linear with respect to cell radius. Consequently, a function in the form of $\exp(A + BL)$ was adopted to describe the change. Another function in the form of $(A + BL)^{-1}$ was actually considered as well, but an examination of residuals resulting from nonlinear regression turned out to be in favour of the exponential model.

Figure 4-17 Semilog plot of coverage for different cell radii, with $M = 10$ dB, $\alpha = 4$ and $\sigma = 14$ dB.

A further examination of coefficients in the $\exp(A + BL)$ showed that A changes in a loglinear fashion with M , while B changes either approximately linearly or quadratically with M . Hence, A was replaced by a function $a + b \ln M$, and B by either $L(c + dM)$ or $L(c + dM^2)$. It was found later from non-linear regression that B is better approximated by a linear function of M than a quadratic one.

The final form of the model is therefore as follows. For time percentages of 99, 99.5 and 99.7%, the model takes the following form:

$$C_{\text{norm}} = a + bM + L(c + dM) \quad (4-20)$$

For time percentages of 99.9, 99.95, 99.97 and 99.99%, the model is as follows:

$$C_{\text{norm}} = \exp[a + b \ln M + L(c + dM)] + \alpha(e + fL) + \sigma(g + hM) \quad (4-21)$$

d. Non-linear regression

The best-fit values of coefficients in equations (4-20) and (4-21) were obtained through non-linear regression using the Gauss-Newton method [Seb89]. The regression was performed for each time percentage, the results of which are given in Table 4-11. The table also presents the 95% confidence range for every coefficient and for each time percentage being examined and the maximum error of the model at the simulation points.

Table 4-11 Values of model coefficients obtained from non-linear regression

	100-p (%)						
	99	99.5	99.7	99.9	99.95	99.97	99.99
<i>a</i>	1.000 ±0.001	1.000 ±0.003	1.000 ±0.004	-0.25 ±0.04	-0.27 ±0.07	-0.38 ±0.06	-0.50 ±0.04
<i>b</i>	0.00002 ±0.00008	0.0001 ±0.0002	0.0002 ±0.0002	0.09 ±0.01	0.08 ±0.02	0.12 ±0.02	0.17 ±0.01
<i>c</i>	-0.0118 ±0.0004	-0.0229 ±0.0008	-0.033 ±0.001	-0.118 ±0.006	-0.20 ±0.01	-0.25 ±0.01	-0.43 ±0.01
<i>d</i>	0.00043 ±0.00003	0.00081 ±0.00005	0.00112 ±0.00007	0.0034 ±0.0002	0.0047 ±0.0005	0.0062 ±0.0004	0.0091 ±0.0004
<i>e</i>				0.006 ±0.003	0.007 ±0.004	0.015 ±0.003	0.009 ±0.002
<i>f</i>				0.0025 ±0.0009	0.005 ±0.001	0.0023 ±0.0009	0.0034 ±0.0005
<i>g</i>				0.009 ±0.001	0.010 ±0.002	0.010 ±0.001	0.0091 ±0.0007
<i>h</i>				-0.00045 ±0.00007	-0.0003 ±0.0001	0.00040 ±0.00009	0.00033 ±0.00004
Max. error (%)	1.46	2.90	4.25	6.73	9.43	7.06	3.88

4.6.3 Procedure of application

To predict the coverage that a cell would be able to attain for a specified percentage of time, the following procedure should be followed:

- a. Determine the distance exponent and excess (shadowing) loss standard deviation according to the type of environment in which the cell would be deployed. These two parameters can be obtained by field measurement.
- b. Compute the clear-sky coverage using equation (4-6).
- c. Compute the scaling factor or normalised coverage using equation (4-20) or (4-21), depending on the percentage of time of interest, with coefficients as given in Table 4-11.
- d. Obtain the coverage by applying (4-19).

If model and coefficients for the desired percentage of time p is not available in the table, it is recommended to compute coverage values, C_h and C_l , for the minimum higher and maximum lower time percentages, p_h and p_l respectively, for which the coefficients are available from Table 4-11. Coverage C_p for the desired percentage of time can thus be estimated by linear interpolation with the time percentage expressed in logarithmic scale:

$$C_p = \frac{\log\left(\frac{p}{p_l}\right)}{\log\left(\frac{p_h}{p_l}\right)}(C_h - C_l) + C_l \quad (4-22)$$

As an example, using the range exponent and excess loss standard deviation from Bultitude's field measurement [Bul98b], the following prediction of coverage can be obtained for a cell with 2 km radius and 10 dB margin available at its edge.

Table 4-12 Exemplary model parameters and clear-sky coverage

Radius, $L = 2$ km
Margin, $M = 10$ dB
Exponent, $\alpha = 5$
Shadowing standard deviation, $\sigma = 17$ dB
Clear-sky coverage, $C_{\text{clear-sky}} = 85.55$ % (eqn. (4-6))

Table 4-13 Predicted coverage for the case with $L = 2$ km, $M = 10$ dB, $\alpha = 5$ and $\sigma = 17$ dB at various time percentages

(100- p) %	Model equation	C_{norm}		C
		Simulation	Prediction	
99.99%	(4-21)	64.53%	63.96%	54.72%
99.97%		81.00%	81.93%	70.09%
99.95%		86.15%	87.63%	74.97%
99.90%		91.62%	92.75%	79.34%
99.70%	(4-20)	96.35%	95.76%	81.92%
99.50%		97.48%	97.13%	83.09%
99.00%		98.70%	98.53%	84.29%

An estimate of normalised coverage at 99.8% ($p = 0.2\%$) can be obtained for the above parameters as follows:

$$\begin{aligned}
 C_{0.2} &= \frac{\log\left(\frac{0.2}{0.1}\right)}{\log\left(\frac{0.3}{0.1}\right)} (95.76 - 92.75) + 92.75 \\
 &= 94.65
 \end{aligned}$$

Hence, the predicted coverage in question is 94.65% relative to clear-sky conditions and the absolute coverage is 80.97% for 99.8% of the time.

4.7 Summary

The objective of this chapter was to examine cell area coverage in a 30 GHz cellular system under precipitation if shadowing losses are imposed on the links connecting subscribers to the hub. The examination was performed by first studying past research projects that investigated mm-wave cell coverage due to shadowing, and then picking the method appropriate for simulations to account for the effects of shadowing on coverage under rainy conditions. CDFs of coverage and normalised coverage were obtained from these simulations, from which a prediction model was devised. The emphasis of the contribution is the adoption of lognormally distributed shadowing loss and the application of a slightly modified version of Jakes' integral equation to compute coverage under shadowing and rain. The important findings of each section of the chapter are summarised below.

- a. Examination of field measurement results and ray-tracing simulations at around 30 GHz reported by other researchers suggested that there are a large number of parameters associated with structures of shadowing in a local area, such as distribution of building heights and vegetation. To avoid having too many parameters in the simulation of the effects of shadowing on area coverage, it was decided to use the lognormal shadowing

model to describe the location-varying shadowing loss. According to the model, the impact of shadowing loss on radio propagation was characterised by only two parameters, namely the distance exponent of a mean path loss-distance power-law relation and standard deviation of lognormal excess path loss. The computation of cell area coverage followed the technique suggested by Jakes [Jak94].

- b. In the past, the impact of rain attenuation was usually accounted for in assessing cell area coverage by assuming uniform rain rate over the area of the cell and applying the rain rate-attenuation model commonly available for use in point-to-point communication link design. In order to take rain spatial and time variation into consideration, time series measurements of radar reflectivity factors over areas illuminated by McGill weather radar system were used for simulating 30-GHz radio cells.
- c. Evaluation of cell area coverage degradation due only to rain, by assuming line-of-sight conditions for all subscribers in the cell area, was indicative of the spatial non-uniformity of rain attenuation and its effect on coverage. Furthermore, the relation between cell radius and coverage at a given percentage of time exhibited non-linearity.
- d. When the impact of shadowing loss and rain attenuation were combined, a simulation method was developed to compute cell area coverage that involves a slight modification to Jakes' technique. The modification included the addition of radar-measured rain attenuation along a radio link to the distance-dependent mean path loss predicted from the power-law relation using the prescribed distance exponent. The sum was then used together with the lognormal standard deviation to determine probability of coverage at any point in the cell. An estimate of the coverage could be subsequently acquired by

applying the discrete version of Jakes' integral equation. This method of computing cell area coverage under the shadowing loss and rain attenuation using radar data is the primary contribution of the work reported in this section. Although the coverage statistics reported as well as the coverage model accordingly developed were derived for Montreal area, similar statistics and model can be obtained for other areas using the developed method given the required radar measurements.

- e. Analysis of normalised coverage, i.e. coverage under rain normalised to the clear-sky value, was considered useful since coverage under clear-sky conditions can be predicted simply by applying Jakes' equation. It was found that coverage under rain when expressed as percentage of coverage under clear sky was not sensitive to variation in the shadowing environment as characterised by the path loss exponent and shadowing loss standard deviation, as long as the margin at the cell perimeter was kept constant. The latter implies that any increase in the path loss exponent is accompanied by an enhancement of hub transmit power to maintain the same margin at the cell perimeter. Cell dimension and fade margin made available to the subscribers, however, heavily influenced the amount of reduction in coverage under rain when compared to that under clear sky conditions.
- f. Based on the computed CDFs, a model was proposed to predict normalised coverage under rainy conditions for a given percentage of time and a combination of values of the four parameters: cell radius, margin at the edge of the cell, path loss exponent and shadowing loss standard deviation. Since the model was a result of a straightforward derivation from CDFs of coverage obtained for Montreal area, it is intended only for

prediction in Montreal areas or others that show similarities in the spatial characteristics of rain. A procedure for application of the model was presented, coupled with a practical example.

Chapter 5 - Cell Site Diversity

5.1 Introduction

The discussion in Chapter 2 pointed out that at high rainfall intensities, the horizontal structure of rain is highly variable. It is frequently observed that during a shower high intensity rain is localised in a very small area surrounded by a region of more uniform, low intensity rain. Hence, in a cellular system under rainy conditions with very localised storms, there is a potential that a subscriber terminal receiving a heavily attenuated signal from one hub can obtain less attenuated, acceptable reception from another hub. This potential advantage makes cell-site diversity very attractive since it could improve area coverage, which would otherwise degrade due to rain attenuation. This is similar in principle to macrodiversity schemes employed in mobile cellular systems that make use of partially correlated shadowing among links that connect a mobile terminal to different base stations [Jak94].

It is realised, however, that at the early stages of mm-wave fixed cellular network development, cell-site diversity might not be easily provided. Hence, diversity analyses presented in this chapter serve merely as an investigation into potential advantages of its application under rainy conditions. Its actual implementation must wait until the mm-wave fixed cellular network is equipped with the necessary technologies. These include technologies that, along with other features, allow the subscriber antenna main beam to be rotated, either electrically or physically, so that the subscriber terminal can switch from one hub to another. In addition, the network architecture should allow for such hub switching

procedures and all the necessary reconfiguration at the higher communication layers that follows, a mechanism known as *handover* in mobile cellular systems, to be performed when required. Schematically, such a scenario can be illustrated as in Figure 5-1 for circular cells with overlapping areas. When the link between a subscriber terminal (ST) and Hub 1, which is the closest and hence the default hub for ST, experiences performance impairments due to a high-intensity shower, there remains a possibility that Hub 2 or 3 can take over the service delivery. Which of the two hubs is selected to receive the handover again depends on which one can produce a better signal reception at ST.

Figure 5-1 Typical scenario for cell-site diversity

In determining the potential effectiveness of cell-site diversity, several quantities or functions can be adopted as indicators. The most simple is the angular correlation function of rain attenuation. This function reflects the correlation of rain attenuation on two links that converge to one common end where the subscriber terminal is located, making a certain

angle with respect to each other. Hence, analysis of correlation is also a good means for examining the range of separation angles that can be used to achieve effective diversity. Another indicator is the diversity gain, which is the difference in attenuation at the same attenuation exceedance probability between a single link and two links in a diversity configuration. This can also be examined as a function of separation angle in the diversity configuration.

A receive diversity system in general seeks to produce the maximum output signal, in terms of power or signal-to-noise ratio, out of a multi-channel system by combining the individual signals from the antennas. Among various linear combining methods [Jak94], the two most practical for cell-site diversity or macrodiversity in a fixed cellular system are:

- (i) Selection combining, in which only the hub that gives the best received signal at every sampling time is selected.
- (ii) Switched combining, in which after the current connection drops below a specified performance threshold, the system is switched to an alternate hub and remains unchanged until performance drops back below the threshold, at which point the connection must be switched again to other possible hubs.

Selection combining is less practical due to the need for a mechanism to monitor simultaneously the quality of multiple channels. It has been chosen for this study, as it is more straightforward to examine analytically because the output signal is simply taken from the channel with the best signal or, equivalently, the least attenuation. Expressed mathematically, the attenuation A to a subscriber terminal in a two-link diversity system with selection combining at any time is:

$$A = \min(A_1, A_2) \quad (5-1)$$

where A_1 and A_2 are the attenuation values on the individual links. It should be noted that this treatment only considers the effect of rain attenuation and free space path loss. The impact of shadowing is not taken into account mainly because of the difficulty in applying the knowledge of spatial correlation of lognormal shadowing to the simulation problem at hand.

Following this introductory section, a literature review of investigations into the use of cell-site diversity in cellular systems is given. Next, the measures of diversity performance outlined above are discussed, starting with angular correlation. Then diversity gain is examined for various separation angles. A simple model is proposed to predict diversity gain under rainy conditions given the length of each leg in a two-link diversity system and the angular separation. Finally, a summary is provided in the final section.

5.2 Literature Review

5.2.1 Macrodiversity in mobile cellular systems

Macroscopic diversity or macrodiversity has been widely adopted in mobile/portable cellular communication systems to combat shadowing effects, to improve SIR performance and, finally, to increase system capacity [Jak94, Ber87, Arn88]. A subscriber terminal is usually covered by multiple base stations and picks up the one that gives the best signal. This scenario is successful since the links to these different base stations experience only partially correlated shadow fading, as was found experimentally by Arnold *et al* [Arn88]. They conjectured that shadowing effects on a link can be described as consisting of a location-specific component, which is the source of correlation among different links from different

base stations to the same subscriber terminal, and a path-specific component unique to each of the links in question.

Discussion

A similar reasoning becomes the basis of cell-site diversity to combat rain attenuation, provided that the rain structure is so spatially localised that attenuation on one leg of a two-link diversity is only partially correlated with that on the other leg. This latter consideration implies that cell-site diversity is expected to work most effectively under convective rain events with very localised showers.

5.2.2 Site diversity in point-to-point microwave systems

In the field of point-to-point microwave radio communications, some analytical work has been done by others to derive the reliability, under precipitation, of multi-segment terrestrial and earth-satellite radio links [Kan91, Kan96, Kan99]. In such a multi-segment terrestrial configuration, the first and the last segment and possibly some others along the link are in the form of two diversity links converging to one point, which can be either a terminal or a repeater.

Two mathematical models of joint probability of attenuation on a two-link diversity structure have been derived. The two models differ primarily in the adopted theoretical distribution of time-varying rain attenuation (in dB), with lognormal and gamma distributions being considered in particular [Lin73, Mor76]. The joint probabilities were initially derived for two-link terrestrial configurations [Mor76, Mor77, Kan87, Kan91] but later extended for

application in satellite communication systems employing dual and triple-site diversity with interference coming from a neighboring earth station [Mor78, Kan96, Kan99].

Discussion

In addressing the advantage of diversity, the performance measure regarded to be of greatest interest is diversity gain. While this measure can be derived analytically using the approach in the foregoing references, an intermediate step is required to do so. This step includes determining an appropriate analytical model for the spatio-temporal distribution of rain attenuation and the spatial characteristics of rain in the form of rain rate correlation coefficient. The derivation of the latter would necessitate another massive effort consisting of evaluation of radar data, and hence, was not attempted in this study.

5.2.3 Cell-site diversity for fixed cellular systems under precipitation

Researchers from Rutherford Appleton Laboratory have been engaged in a series of projects that examined the performance of cell-site diversity at 40 GHz for fixed wireless systems under precipitation in Europe [Tan00, ITU99]. They used radar images collected over a period of two years covering a 160-km² area [Tan98], over which they simulated arrangements of diversity systems with two identical-length links and selection combining (see eqn. (5-1)). The performance improvement over a system without diversity was given in terms of diversity gain and improvement. The gain $G(p)$ at probability or percentage of time p can be obtained from the radar data as

$$G(p) = A(p) - A_d(p). \quad (5-2)$$

In equation (5-2), $A(p)$ and $A_d(p)$ are the p^{th} percentile of attenuation in decibels in a system without and with diversity, respectively. They discovered that the gain changed with separation angle roughly in accordance with a half sinusoid. Accordingly, they proposed the following relationship for prediction of diversity gain for separation angle θ ,

$$G(\theta, p) = G_{\max}(p) \sin(\theta / 2). \quad (5-3)$$

In this equation, $G_{\max}(p)$ is the predicted gain of a diversity system with two links of exactly opposite directions. For configurations of two 4-km links, its value was found from simulations using two years of radar data collected in the United Kingdom to range from 1 to 7 dB for time percentage p ranging from 1 to .01 %.

Discussion

The method used in the aforementioned study was adopted in the study reported herein. However, the results and the model derived therefrom are different from those described in the work discussed above. In particular, the gain at larger angular separations was discovered from our diversity study to be relatively insensitive to the angle. Thus a separation of, say, 110° between the two branches of a diversity system is adequate to yield a near maximum diversity gain. It is consequently considered that the half-sinusoid model of gain variation with angular separation can be improved by modifying the sinusoidal shape to accommodate for this insensitivity to separations around 180° .

5.3 Angular correlation

To gain preliminary insight into the potential advantage offered by cell site diversity, the correlation of attenuations on two converging links was investigated. The correlation measure considered herein is the normalised angular autocorrelation function of rain attenuation (expressed in linear terms) averaged over all simulation samples. For each set of commonly centred radial links simulated on a radar record for time t , the angular autocorrelation function $\rho(L, \theta, t)$ between attenuation levels on two links of identical length L kilometres separated in azimuth by θ degrees was evaluated by applying the circular correlation procedure. The resulting functions from all simulations were subsequently averaged over the whole radar measurement period. That is,

$$\rho(L, \theta) = \left\langle \frac{\sum_{\phi=0^\circ}^{359^\circ} 10^{A(L, \phi, t)/10} 10^{A(L, ((\phi+\theta))_{360}, t)/10}}{\sum_{\phi=0^\circ}^{359^\circ} (10^{A(L, \phi, t)/10})^2} \right\rangle, \quad (5-4)$$

where $A(L, \phi, t)$ denotes the rain attenuation in dB on a radial link at azimuth ϕ degrees at measurement time t , $((\bullet))_n$ indicates that the summation inside the double brackets is done modulo n , and $\langle \bullet \rangle_t$ indicates averaging over time. This function is computed for a specific value of L and is used to examine the variation in attenuation correlation for different separation angles, regardless of the azimuthal orientation of the paired links.

To study the effect of rain intensity on the correlation of attenuation, a number of curves of the average correlation function $\rho(L, \theta)$ were computed, each corresponding to a different attenuation (X dB). The correlation function of a set of all radial links separated by

angles spanning 360° , whose maximum attenuation was $A_{\max}$, was included in the averaging to obtain the curve associated with attenuation X only if $A_{\max} > X$. The result for two 2-kilometre links is shown in Figure 5-2. The ten curves from top to bottom represent the correlation for $X = 1, 2, 3, \dots, 10$ dB.

A number of observations can be made from Figure 5-2.

- a. The magnitude of correlation decreases when the limit for maximum attenuation in a radio cell is greater. This indicates that correlation between attenuations on pairs of links with different azimuthal orientations is less when the radio cell area contains high-intensity rain cells, as anticipated.
- b. For lower X , correlation is high and exhibits relative flatness over a large range of separation angle, e.g. about 20° – 340° for $X = 1$ dB, in which the correlation varies little in the 0.90–0.95 range. A possible physical explanation that can be offered for this is that the majority of recorded rain events are of low intensity and widespread. In these situations, attenuations on two converging links are more likely to be highly correlated, regardless of the separation angle.
- c. However, for greater X , e.g. 10 dB, correlation decreases steeply when the separation angle increases from 0° to around 100° , is relatively flat for separation between 100° and 260° with a minimum of 0.27 at 180° , then increases rapidly again toward 360° . This is well anticipated considering high-intensity rain tends to be confined in small areas such that as small an angular separation as 100° is sufficient to create nearly as low a correlation of attenuation between the two links as it would with a separation of 180° .

These observations raise an expectation that cell-site diversity would be most effective under high-intensity rain and almost equally so for any angular separation between 100° and 260° .

Figure 5-2 Average attenuation correlation function for two, 2 km long converging links with different angular separations at the point of convergence.

5.4 Computation of diversity gain

Since it is apparent that for a range of angular separations at moderate and high rain rates, correlation of attenuations on two converging links is low, it is of interest to proceed with an examination of potential diversity gains. Diversity gain is defined here as in (5-2).

In addition to the adoption of selection combining for this study, it is also assumed that both links of the diversity structure are under LOS conditions at all times and experience

clear-sky attenuation that obeys free-space quadratic increase with range. The latter is achieved when an antenna with a very narrow beam both in azimuth and elevation is employed at both ends of the links, such that the effects of multipath propagation can be ignored, and it is assured that there is no obstruction along any of the radio links.

By the above assumptions, for a configuration with two links identical in length, equation (5-1) for selection diversity can be applied directly with A_1 and A_2 denoting rain attenuation on the first and second link, respectively. The link selected at any time is the one with the lower rain attenuation. However, for a configuration in which one of the links is shorter than the other, A_1 denotes rain attenuation on the shorter link, while A_2 represents the sum of rain attenuation on the other link and an extra term to account for the difference in free space loss between them. Hence,

$$\begin{aligned} A_1 &= A_{1,\text{rain}} \\ A_2 &= A_{2,\text{rain}} + 20\log(L_2/L_1) \end{aligned} \quad (5-5)$$

where $A_{n,\text{rain}}$ and L_n represent rain attenuation (dB) and length (km) of the n^{th} link, respectively, with the first link being the shorter.

In the next section, CCDFs of attenuation for a configuration of two links working in a selection diversity mode and subtending various angles are shown, with one of the links being a reference and fixed for the various angles. It was discovered that when the reference link was varied in azimuth with respect to the common node, the resulting CCDF of attenuation, and consequently, the diversity gain would vary as well. Hence, in addressing the statistical dependence of diversity gain on angular separation, CCDFs for configurations

with various reference links at different azimuths were used to fit a model, a function of angular separation with symmetry around 180° , through non-linear regression.

5.5 Results from radar measurement

All radar images recorded during the two years of measurement were included in the simulations. Several scenarios were considered for the simulated diversity configurations. Figure 5-3 shows the CCDFs for cases in which the two diversity branches are both links of 2-km length with separation angles of 90° , 180° and 270° . Figure 5-4 shows the CCDFs for configurations with the same three angular separations, but with the reference link 2 km long and the alternate 3 km in length. In this case, the effect of the difference in propagation range is taken into account in the computation of diversity gain by applying equation (5-16). The two results presented here were obtained by orienting the 2-km reference link at an azimuth of 0° with respect to the converging point, which is the centre of the simulated cell used in the study reported in Chapter 4.

Figure 5-3 CCDFs for single-link and two-branch diversity configurations, in which the link length for both branches is 2 km.

Figure 5-4 CCDFs for single-link and diversity configurations for main branch and alternate link lengths of 2 and 3 km, respectively.

A number of observations can be made from these figures:

- a. Figure 5-3 indicates that for this particular arrangement, for percentages of time larger than 1% there is little potential gain offered by the diversity scheme with two 2-km links. However, the diversity gain is higher for smaller time percentages. It can be seen that for two 2-km links with 180° separation, the gain is approximately 2 dB at 0.1% and 5 dB at 0.03%. Thus, diversity would serve as protection against rare occurrences of heavy convective rain, exactly when it is most needed.
- b. When the attenuation CCDFs for various separation angles are compared, it is observed that for fading time of 1% the difference in the diversity gain is negligible. At 0.1%, the scheme with 180° separation is better by less than one dB compared to the 90° and 270° separations. At 0.03% this difference becomes approximately one decibel. One could conclude accordingly that for this diversity configuration there is not much to be gained when the angular separation is increased from 90° to 180° . A final conclusion on this, however, is reserved until results from using multiple references at different azimuths are examined statistically.
- c. Diversity gain appears to diminish when the second link is 3 km in length, as shown in Figure 5-4. There is only negligible gain at 0.1%, whereas at 0.03% the gain becomes approximately 3 dB for 180° separation. This phenomenon is ascribed to the difference in path length that leads to the clear-sky path loss on the second link being larger by about 3.5 dB than that on the main link according to equation (5-5). Larger attenuation due to rain is also expected on the 3-km link compared to the 2-km one, hence reducing the diversity gain even further.

- d. From Figure 5-4 for the case of 2- and 3-km links, it is observed that diversity gain for 90° separation is greater than that for 270° , giving an impression of asymmetry in the spatial distribution of rain. However, later when results for different references are examined individually, it will be apparent that each of them shows high variation of gain as a function of angular separation. This might be caused by the two-year limitation of radar data used in the simulation. However, further examination reported below on the average of this function over configurations with reference links pointing at different azimuths shows near-symmetrical variation of diversity gain around 180° separation.

To clarify the observation above, the solid line in Figure 5-5 depicts average diversity gain at 0.1% probability as a function of angular separation computed from multiple simulations with reference links oriented at different azimuths. Specifically, the reference azimuths ranged from 0° to 330° with 30° steps. The figure also shows at every separation angle a multiple of 30° the range of $\pm\sigma$ around the gain average, where σ is the standard deviation of gain for that particular separation angle. A curve in the form of $G_{180^\circ}\sin^k(\theta/2)$ is given by the dashed line, with parameters G_{180° and k computed by non-linearly fitting the expression to the simulation results.

Figure 5-5 Average diversity gain for 2-km link arrangements.

It is observed from the figure:

- a. The diversity gain shows some amount of variation as indicated by standard deviation of around 0.3 dB, say, for the 180° angular separation, when cases with different reference azimuths are examined. Asymmetry of the gain around angular separation of 180° in individual cases is noticeable from the asymmetry of the standard deviation values around 180°. However, the average gain appears to be symmetrical around 180°.
- b. The $G_{180^\circ} \sin^k(\theta/2)$ curve well approximates the diversity gain average.

These observations indicate that the development of a model to be used in diversity gain prediction should involve outcomes obtained from diversity arrangements with different reference links, as in the next section, and take the form of function introduced above.

5.6 Modelling for prediction of cell-site diversity gain

5.6.1 Considerations

Considerations from previous observations are summarised below:

- a. The shape of the attenuation correlation curves shown in Figure 5-2 for links having identical lengths separated by an angle shows relative flatness for angular separation ranging from 100° to 260° . This observation is supported by the findings from diversity gain CCDFs for different angular separation as described in the previous section. In fact, the diversity gain curves reported in Tan's paper [Tan00] also demonstrate relative insensitivity to separation angle approximately in the 90° - 270° range.
- b. The angular asymmetry of diversity gain around 180° is found to vary for different lengths of the branch links and for different reference links. With a realistic assumption that rain intensity has equal probability of occurrence anywhere in the area including and surrounding the radio cell, it was deduced that a model to be fit to the angular separation-diversity gain values obtained from the simulation must be symmetrical around 180° .

Tan's sinusoidal model given by equation (5-3) already takes the symmetry into account in predicting diversity gain for a specific angular separation. However, the model shape around its peak at 180° appears too sharp when compared to the shape of the diversity gain variation obtained from radar simulations. In addition, Tan's study [Tan00] also lacked treatment for diversity configurations with two links having different lengths. It is therefore considered necessary to propose a new model to eliminate these problems.

5.6.2 Assumptions and limitations

The following assumptions are the bases for the model proposed and evaluated in the following sub-section.

- a. The relation that describes the variation of diversity gain as a function of angular separation is symmetrical around 180° . On average, diversity gain is maximum when the angle separating the two links is 180° , that is, when the two diversity branches are oriented in opposite directions to one another.
- b. The model proposed is applicable only for LOS links operating at 30 GHz in the rain region into which the areas surrounding Montreal have been classified and for the range of link lengths used in the simulation study. The range of link lengths considered herein, i.e. 1–4 km, suggests the application of this model for cellular systems consisting of small cells only, with typical spacing between adjacent hubs of 2–8 km.

5.6.3 Prediction model

With all the above points being considered, a new model for separation angle-dependent diversity gain is proposed in the form of:

$$G(\theta, p) = G_{180^\circ}(p) \sin^{k(p)}(\theta/2), \quad 0^\circ < \theta < 360^\circ \quad (5-6)$$

where $G(\theta, p)$ is the diversity gain obtained for angular separation of θ at fading time percentage of $p\%$, $G_{180^\circ}(p)$ and $k(p)$ are parameters of the model that vary for different combinations of link lengths.

This model was fit through non-linear regression using the Gauss-Newton method [Seb89] to diversity gain values calculated from simulations with the reference links pointing at 12 different azimuths. These reference azimuths ranged from 0° to 330° with 30° steps. In the case of two links of unequal lengths, the shorter link was taken as the reference, that is, $L_1 \leq L_2$. The fitting procedure produced pairs of parameters G_{180° and k for various combinations of link lengths and percentages of time.

Some of the fitted curves are presented in Figures 5-6 through 5-9, on each of which the scatter plot of the simulation results and the fitted curves of sinusoid and root sinusoid form are superimposed. These curves are for two diversity scenarios: Figures 5-6 and 5-7 show the results for configurations with both links being of 2 km length, whereas Figures 5-8 and 5-9 exhibit the fit for configurations in which the reference link is 2 km long while the alternate is 3 km in length. For each scenario, the gain values at fading time percentages of 0.1% and 0.03% (or equivalently, link reliabilities of 99.9% and 99.97%) are shown. A more complete list of the results for lengths varying between 1 and 4 kilometres and for reliabilities of 99.9%, 99.95%, 99.97% and 99.99% is shown in Table 5-1. Combinations for which G_{180° yields a value less than 0.5 dB are not shown, since gain levels as low as this are considered to have little use in practical systems. The two coefficients of the model, G_{180° and k , are shown in the table together with their respective confidence ranges for a 95% confidence coefficient [Seb89, Chapter 5].

Figure 5-6 Diversity gain for two 2-km links at 99.9% reliability.

Figure 5-7 Diversity gain for two 2-km links at 99.97% reliability.

Figure 5-8 Diversity gain for 2 and 3-km links at 99.9% reliability.

Figure 5-9 Diversity gain for 2 and 3-km links at 99.97% reliability.

Table 5-1 Parameters of the proposed new model for various combinations of lengths and link reliabilities.

L_1 (km)	L_2 (km)	Link reliability, 100- p (%)	G_{180° (dB)	k	Sum of squared errors		
					$\sin^k(\theta/2)$	$\sin^{0.5}(\theta/2)$	$\sin(\theta/2)$
1	1	99.9	0.78 ± 0.03	0.41 ± 0.09	2.3	2.4	4.4
		99.95	1.21 ± 0.08	0.4 ± 0.1	10.8	11.0	16.0
		99.97	1.57 ± 0.09	0.4 ± 0.1	15.7	16.1	25.2
		99.99	3.0 ± 0.2	0.4 ± 0.2	108.4	109.3	138.5
	2	99.99	0.7 ± 0.2	0.7 ± 0.6	42.5	42.7	42.7
2	2	99.9	1.99 ± 0.06	0.46 ± 0.07	7.7	7.7	18.3
		99.95	3.0 ± 0.1	0.43 ± 0.09	33.3	33.9	60.8
		99.97	4.5 ± 0.2	0.43 ± 0.09	75.4	76.9	138.6
		99.99	7.4 ± 0.4	0.5 ± 0.1	263.0	263.8	406.4
	3	99.9	0.63 ± 0.04	1.3 ± 0.2	2.2	3.6	2.3
		99.95	1.31 ± 0.09	1.0 ± 0.2	12.6	15.9	12.6
		99.97	2.4 ± 0.2	0.9 ± 0.2	40.3	46.6	41.0
		99.99	4.4 ± 0.4	0.8 ± 0.2	260.9	276.6	265.8
	4	99.95	0.74 ± 0.08	1.4 ± 0.4	9.2	11.3	9.5
		99.97	1.7 ± 0.1	1.2 ± 0.3	30.5	39.1	31.1
		99.99	3.2 ± 0.4	1.1 ± 0.4	206.1	227.6	206.2
3	3	99.9	3.4 ± 0.1	0.49 ± 0.07	25.6	25.6	52.4
		99.95	5.6 ± 0.2	0.54 ± 0.09	89.6	90.2	144.2
		99.97	7.5 ± 0.2	0.44 ± 0.06	91.1	93.4	254.5
		99.99	13.4 ± 0.7	0.5 ± 0.1	1000.5	1001.4	1439.5
	4	99.9	2.0 ± 0.1	1.0 ± 0.2	20.8	26.7	20.8
		99.95	3.8 ± 0.2	0.9 ± 0.1	60.9	78.0	62.0
		99.97	5.6 ± 0.2	0.8 ± 0.1	81.1	103.9	90.6
		99.99	10.4 ± 0.8	0.7 ± 0.2	1147.6	1180.3	1228.5
4	4	99.9	4.9 ± 0.2	0.47 ± 0.08	66.4	66.6	126.7
		99.95	8.5 ± 0.3	0.49 ± 0.07	147.8	147.8	310.4
		99.97	11.9 ± 0.4	0.53 ± 0.07	231.5	233.2	493.8
		99.99	18.6 ± 0.9	0.5 ± 0.1	1592.2	1601.4	2176.2

From the foregoing figures and table, it can be observed that:

- a. For arrangements having two links of equal length ($L_1 = L_2$), exponent k is in the range of 0.4-0.6. A consistent relation between k and p was not observed. The values of k are lower for shorter links ($L_1 = L_2 = 1, 2$ km) than for longer ones ($L_1 = L_2 = 3, 4$ km). For these configurations, a function of root-sinusoidal form $G_{180^\circ} \sin^{0.5}(\theta/2)$ produced sums of squared errors only slightly larger than those of the proposed model.
- b. For two links of unequal length ($L_1 \neq L_2$), k ranges from approximately 0.7-1.4, and tends to be lower for higher ratios of L_1/L_2 and higher reliabilities. For these arrangements, the sinusoidal function of angular separation $G_{180^\circ} \sin(\theta/2)$ shows slightly poorer fit compared to the more general form of the proposed model.
- c. An examination of the values of G_{180° for various combinations of L_1 and L_2 shows that $L_1/L_2 \geq 0.75$ is necessary to avoid conditions wherein the difference in path loss from overcoming the advantage of diversity, i.e. to maintain the diversity gain at 2 dB or larger at 99.9%, or 3 dB or larger at 99.95% reliability. This is true at least for the range of link lengths considered in the simulation, i.e. 1–4 km.
- d. When $L_1 = L_2$, the link length should be at least 2 km in order for G_{180° to reach 2 dB or more for reliability of 99.9%, and 3 dB or more for 99.95% reliability.

5.6.4 Application of the model

When using the model given by (5-6), it is also of interest to determine the range of separation angles for which the decrease in diversity gain from its peak at 180° is expected to

be less than a specified fraction δ_g at a given percentage of time. Equation (5-6) can be modified to consider this and rewritten as:

$$\begin{aligned}\theta_{\text{cut-off}} &= 2 \sin^{-1} \left[(1 - \delta_g)^{1/k} \right] \quad (\text{rad}) \\ &= \frac{360}{\pi} \sin^{-1} \left[(1 - \delta_g)^{1/k} \right] \quad (^\circ)\end{aligned}\tag{5-7}$$

where $\theta_{\text{cut-off}}$ is the separation angle for which the diversity gain (dB) diminishes by a fraction δ_g from the maximum value at 180° . Although not explicitly stated, the evaluation of the expression also depends on the percentage of outage time or reliability level being considered. Using the case of two 4-km links as an example (see Table 5-1) for which $G_{180^\circ} \approx 5$ dB to attain 0.1% outage time due to rain attenuation, a value $\delta_g = 10\%$ refers to a gain of 4.5 dB for angular separation of $\theta_{\text{cut-off}}$. The ranges of separation angles computed using (5-7) for various values of k and δ_g are given in Table 5-2.

Table 5-2 Ranges of separation angles for various values of k and δ_g

k	δ_g		
	5%	10%	20%
0.5	129° - 231°	108° - 252°	80° - 280°
0.7	137° - 223°	119° - 241°	93° - 267°
1.0	144° - 216°	128° - 232°	106° - 254°
1.4	149° - 211°	136° - 224°	117° - 243°

An observation can be made as follows. Consider only diversity systems with links of almost identical length, for which the value of k is approximately 0.5 at any probability level as demonstrated by the results presented in Table 5-1. Also consider that only base stations

with angular separations that result in a reduction of 10% or less in diversity gain relative to G_{180° are deemed acceptable. A subscriber located at approximately the same distance from two hubs qualifies as user of cell-site diversity when the two hubs and the subscriber subtend an angle in the 108° – 252° range.

Based on the foregoing observation, a set of criteria can be proposed to determine whether a subscriber station might benefit from cell-site diversity during rain in a 30 GHz fixed cellular system that employs small cells:

- a. The two hubs under consideration must involve links with approximately equal levels of clear sky attenuation to the subscriber terminal.
- b. The hubs must be at almost the same distance, with a minimum of 2 km, from the subscriber. The distance ratio between the closer and the more distant hub should not be less than 0.75.
- c. The positions of the hubs should be such that they make an angle close to 180° with respect to the subscriber terminal. However, if a reduction by 10% in diversity gain (expressed in dB) is tolerable, a separation angle as small as 108° is acceptable.

To provide an example, a scenario involving multiple base stations that operate in diversity are considered. This means the areas of the corresponding radio cells would overlap one another, causing the cell shape to follow the shape of a polygon, such as a rectangle or a hexagon, rather than a circle. Consider a square area covered by 4 corner base stations, BS_0 through BS_3 , which is a part of a larger cellular network with 4-sectored rectangular cells. It is assumed that all subscribers in the area satisfy criterion (a), whereas both (b) and (c) are

required for the cell-site diversity scheme to work effectively during rain. It is also assumed that the spacing between two closest hubs is 4 km or more, such that subscribers located exactly on the cell border will satisfy criterion (b). The portions of the area in which subscriber terminals satisfy these criteria and benefit most from the use of the diversity scheme are illustrated in Figure 5-10.

The shaded regions are those that can be covered by two or more base stations under the constraints given above. The total area of these regions is found to be approximately 28%. It is also observed that these regions are located near the borders among these cells, which are also the locations of subscribers that suffer the most from rain attenuation. Subscribers in areas closer to the base stations, conversely, would not realise much advantage from cell-site diversity but at the same time are less likely to experience outage due to rain attenuation.

Figure 5-10 Areas in a 4-sectored rectangular cell that could benefit from cell-site diversity, as predicted by the proposed model with 10% gain reduction allowed.

When Tan's model, given by equation (5-3) which is equivalent to expression (5-6) with $k = 1$, is used, the range of acceptable separation angles is $128^\circ - 232^\circ$. The total area of the regions where cell-site diversity would be effective, based on the application of the same constraints as those in the previous example, is found to be around 19%, as depicted in Figure 5-11. This underestimates the portion of the service area that could benefit from cell-site diversity, as previously predicted using $k = 0.5$.

Figure 5-11 Areas in a 4-sectored rectangular cell that could benefit from cell-site diversity, as predicted by Tan's model with 10% gain reduction allowed.

When the model with $k = 0.5$ is again used, but this time allowing reduction in diversity gain of as much as 20%, corresponding with $\theta_{\text{cut-off}} = 80^\circ$, the regions that are effectively covered by more than one hub in a diversity system is 32%. As can be seen in Figure 5-12, there are even some portions of the area around the centre of the square where more than two hubs are available to serve a subscriber station in a diversity configuration.

Figure 5-12 Areas in a 4-sectored rectangular cell that could benefit from cell-site diversity, as predicted by the proposed model with 20% gain reduction allowed.

5.7 Summary

The objective of the work reported in this chapter was to assess the potential advantage of applying cell-site diversity to improve the performance of a subscriber link under rainy conditions in a fixed cellular system operating at 30 GHz. The indicator of performance improvement being considered was diversity gain, which was evaluated over a range of percentiles of CCDFs for rain attenuation for a number of combinations of link lengths and various separation angles between the links. Prior to examining diversity gain, a

study was also conducted on the angular correlation function of attenuation on two converging links. The results of all these studies are summarised below.

- a. The angular correlation function was obtained by computing the circular autocorrelation function of attenuation, given in linear units, with respect to separation angle. The outcomes, each normalised to its maximum, from a large number of simulations using radar data were averaged to produce an average correlation function. The resulting function indicates that correlation between links for angular separation ranging from 100° and 260° is not very sensitive to separation angle, which raises an expectation that cell-site diversity under rainy conditions would be equally effective within the aforementioned range of angular separations.
- b. Examination of the correlation function also shows that periods of higher-intensity rain yield lower correlation of attenuations on converging links with angular separation in the 100° – 260° range than those of lower-intensity rain. This indicates that cell-site diversity would perform better under a heavy storm than it would in a light rain.
- c. The diversity configurations were simulated for LOS links by adopting the selection combining method, in which at any given time, the one of the two links that experiences smaller attenuation is selected. In simulations for configurations consisting of two links with different lengths, a term accounting for the difference in path loss was added to the total attenuation experienced by the longer link. A profile of diversity gain as a function of separation angle was obtained by fixing one link as the reference and changing the azimuthal orientation of the other link with respect to the converging point. The shape of the curve, i.e. the symmetry, the maximum gain and the sharpness/flatness around the

maximum near 180° , was found to vary slightly from one profile to another, each corresponding to a unique reference link. This is considered to be a result of limitations in the period of radar measurement, the effects of which were averaged out by evaluating two-link configurations with various azimuthal orientation.

- d. A new model in the form given by equation (5-6) was proposed for prediction of diversity gain expected for a combination of link lengths and separation angle at a given time percentage. Two parameters of the model G_{180° and k , each of which varies with link length, angular separation and probability level, were derived by fitting the model to simulation results obtained for different reference links.
- e. The values of k and the sums of squared errors presented in Table 5-1 indicate that for arrangements in which the two links are of identical length, a model of root sinusoid can produce better prediction of the variation of diversity gain with angular separation when compared to Tan's sinusoidal model. When the two links are different in length, k varies around unity depending on the ratio of the lengths. In this situation, a sinusoidal shape (which is used in Tan's model, see equation (5-3)) can be used to obtain a rough estimate of the gain variation with respect to separation angle.
- f. When the values of G_{180° are examined for links of unequal lengths, it was found that a minimum L_1/L_2 ratio of 0.75, where $L_1 \leq L_2$ are the lengths of the links, is necessary for the diversity system to remain beneficial, at least for 99.9% link reliability and for link lengths ranging from 1 to 4 km.

- g. When results from the modelling of diversity gain were applied in a cellular scenario consisting of 4-sectored rectangular cells with each square region served by 4 corner hubs, it was found that 28% of the area can potentially gain from the application of cell-site diversity in rain. These regions were found to be concentrated near the borders among adjacent cells.**
- h. When considering the use of cell-site diversity for a subscriber station in a fixed cellular system with small cells operating at 30 GHz, it is recommended that all of these criteria are fulfilled:**

 - (i) The subscriber terminal must be situated such that the clear sky losses are approximately equal from both main and alternative hubs.**
 - (ii) The subscriber terminal must be at least 2 km away and at as nearly an equal distance as possible from both hubs. The ratio of distance from the subscriber between the closer and the more distant hubs must be 0.75 at the minimum.**
 - (iii) The two hubs must be in as nearly opposite directions as possible from each other with respect to the subscriber terminal. However, when the two hubs make an angle with respect to the subscriber position in the 110°-250° range, cell-site diversity is still considered advantageous for the subscriber, although diversity gain will be diminished.**

Chapter 6 - Conclusions

6.1 Summary of motivation and problems

The thesis work was motivated by the adoption of mm-wave frequencies, particularly those around 30 GHz, for carrier waves in broadband fixed cellular systems in North America. Interest has grown in studying the influence of rain attenuation on cell area coverage in these systems due specifically to the fact that mm-wave frequencies are more prone to rain attenuation and that only recently, with the introduction of mm-wave carriers, has the spatial variation of rain attenuation become an important factor in the success of the operation of such systems. Three issues, including one that was more pertinent with the study on the expression relating radar-measured reflectivity factor to rain attenuation used later in the analyses of the other two problems, were identified:

- (i) *Derivation of coefficients of the power-law relationships between the specific attenuation of rain and each of radar reflectivity factor and rain intensity using new measurements collected by means of JWD and POSS systems. This was in part targeted at obtaining the most appropriate power-law coefficients for Montreal areas, where the radar measurements used in the subsequent stages of this work were made. The analysis involved was also intended to obtain higher accuracy in the derived coefficient values by adopting modern DSD measurement techniques, as opposed to the flour methods employed by Laws-Parson and Marshall-Palmer.*
- (ii) *Statistics of cell area coverage that consider the effect of rain attenuation and lognormal shadowing. The central issue in this regard was the development of a*

technique that can be used to analyze the combined effects of rain attenuation and shadowing loss on coverage. Parameters that were considered as the source of variability in area coverage include two system-related parameters, namely nominal cell radius and margin at the cell perimeter, and two others that are dictated by the environment, namely propagation exponent and shadowing standard deviation.

- (iii) *The use of cell-site diversity under the impact of rain attenuation.* One of the main goals was to develop a more accurate model for the prediction of diversity gain under rainy conditions compared to the ITU-R recommended model. The other objective was to devise a set of criteria based on the developed model to determine the feasibility of using cell-site diversity for a specific subscriber terminal to combat the impact of fading due to rain attenuation.

6.2 Radar measurements of rain attenuation

6.2.1 Method of analysis

The analysis of the first-mentioned issue was centred upon the treatment of DSD measurements recorded separately in Montreal and Toronto using JWD and POSS. An average DSD was computed for each of disjoint intervals that together cover the entire range of rain intensity values of interest. Triplets of specific attenuation, radar reflectivity factor and rain intensity (Y , Z , R) were computed from these average DSDs using (2-6), (2-7) and (2-8), the results of which were used in loglinear fitting of Y - Z and Y - R power-law relationships. By applying the coefficients obtained from the regression, the impact of spatial

averaging of specific attenuation on the distribution function of path attenuation was examined by invoking synthetic storm simulation technique.

6.2.2 Summary of results

- a. As far as JWD and POSS are concerned, the uncertainties in the detection of small sized raindrops in DSD measurements do not significantly influence the relationships at frequencies around 30 GHz or less.
- b. An evaluation of Y - Z relationship based on measured DSDs shows that a variety of criteria for determining intervals in grouping the measured reflectivity can be adopted without significantly influencing the attenuation estimates obtained from the relationships. This implies that the reflectivity interval width can be made larger than the magnitude of radar measurement uncertainties such that the noisy effects of these uncertainties can be alleviated in using radar measurements.
- c. The coefficients of the Y - Z and Y - R power-law relationships were derived from DSD measurements made in Toronto and Montreal, and are presented in Tables 3-3 and 3-4. Comparison with the ITU-R coefficients for linearly polarised waves exhibits slight differences. This is considered to be a result of differences in the DSD used in the derivation process. The coefficients reported in this thesis were calculated from new DSD measurements using JWD and POSS systems, as opposed to Laws and Parson's measurements that were made using the flour method and used to derive the ITU-R coefficients.

- d. Evaluation using synthetic storm simulation shows that the statistics of path attenuation computed using the power-law relationship are not sensitive to variation in the degree of spatial averaging.

6.3 Cell area coverage

6.3.1 Method of analysis

Radio cells were simulated over images of radar reflectivity factor measurements. Rain attenuation on each radial link simulated in the cell was computed by numerically integrating the specific attenuation measured by the radar system.

In the ideal case where all subscribers in the nominal area of the cell are uniformly LOS, the length along each radial link in a cell that was considered covered was that over which the combination of free space loss and rain attenuation was less than a fade margin that had been specified with respect to the free space loss at the perimeter of the cell. The periphery of the covered area within each cell was taken to be the locus of points along each radial at which total attenuation exceeds the specified threshold, and numerical integration was used to estimate the coverage area.

In a more realistic case, where the subscribers in the nominal area of the cell can be in either LOS or NLOS conditions, shadowing loss was modelled as lognormally distributed with a specified standard deviation and a mean that was a power-law function of distance from the hub. In the simulation, rain attenuation obtained from radar measurements was added to the shadowing mean and the coverage was computed using Jakes' method of integrating the probability of a subscriber being covered over the entire nominal area.

Coverage under rainy conditions was examined in this regard as a fraction of clear-sky coverage, the latter easily predicted using Jakes' method given the standard deviation of the shadowing loss and the distance exponent.

6.3.2 Summary of results

- a. When a LOS assumption is applied to all points in the cell area, radio cell simulation results indicated the impact of non-homogeneity of rain attenuation on cell area coverage. Specifically, cell area coverage was found to be non-linearly dependent on the cell radius.
- b. Due to its simple, dual-parameter nature, lognormal shadowing model was found to be the most convenient to adopt among various models for shadowing loss when the effects of shadowing was to be incorporated in the analyses of cell area coverage. Attenuation experienced by a subscriber terminal at any point in the cell area in a radar-based simulation can be predicted by adding a path loss with a specified distance exponent and excess loss having a given lognormal standard deviation on top of the rain attenuation estimated from the radar measurement. Cell area coverage for a mixed LOS/NLOS environment can be estimated by computing the discrete integral form of Jakes' prediction method.
- c. The extent of degradation in coverage of a radio cell under precipitation with respect to coverage under clear-sky conditions was found to be nearly independent from the variation in shadowing environment characterised by the propagation exponent and shadowing loss standard deviation. However, the various propagation exponents were evaluated while maintaining a constant margin at the cell perimeter, implying that a change in propagation exponent was compensated for by adjusting the transmit power

accordingly. On the other hand, system parameters such as cell dimension and fade margin provided to subscriber terminals at the cell perimeter heavily influence the degradation of coverage under rainy conditions.

- d. A coverage prediction model was derived from the coverage CDFs obtained from the simulations and was given by equations (4-20) and (4-21). The model takes as inputs values for four parameters: cell radius, margin at the edge of the cell, propagation exponent and shadowing loss standard deviation.

6.4 Cell-site diversity

6.4.1 Method of analysis

Multiple links converging to a common node were simulated over radar images. Attenuation on each of these links was calculated assuming free space path loss. Average angular correlation function of rain attenuation on these links was computed to gain an initial insight into the feasibility of employing cell-site diversity in 30 GHz fixed cellular systems under rainy conditions. Subsequently, diversity gain for two-link configurations with selection combining was evaluated as a function of lengths of the branch links, angular separation and percentage of outage time. For cases where the branch links were of different lengths, a term (in dB) was added to the rain attenuation on the longer link to account for the difference in clear-sky path loss.

6.4.2 Summary of results

- a. The angular correlation function of attenuation on links separated by an angle exhibits insensitivity to the angular separation in the 100° – 260° range. It was also found that

periods that contain higher-intensity events yield lower correlation for the same angular separation compared to periods with uniformly low intensity rain.

- b. When cell-site diversity with selection combining was simulated, the diversity gain was found on average to vary with the angular separation θ according to a function in the general form of $\sin^k(\theta/2)$, as given by equation (5-6). The values of k are dependent on the lengths of individual links and probability level of interest. For configurations with two links of identical length, a root-sinusoid model can be used to roughly predict the diversity gain given the separation angle. When the two links are of different length, the sinusoidal model used by ITU-R is sufficient to yield a rough estimate of the diversity gain.
- c. Cell-site diversity was found to be most advantageous when the length ratio between the shorter and the longer links is 0.75 at the minimum, and the links are oriented in nearly opposite directions with respect to the subscriber terminal. A relatively large deviation from ideally opposite directions can be used, however, when a slight reduction in the diversity gain is acceptable. A set of steps is recommended for use in deciding on the usefulness of cell-site diversity for a subscriber terminal, as described in section 5.6.4.

6.5 Recommendations for further study

Various portions of the research reported in this thesis can be extended further into more elaborate or more advanced studies, involving a larger number or different sets of factors that might affect cell area coverage yet haven't been thoroughly studied in this work. This can lead to the following list of relevant subjects for future research:

- a. Refinement of the Y - Z and Y - R power-law coefficients, using measurements made over longer periods and from multiple locations.
- b. Betterment of the model accuracy, either for the coverage or the cell-site diversity study, by employing long-term radar measurement such that the effects of year-to-year variation can be innately incorporated in the statistics.
- c. Evaluation of cell area coverage for other operating frequencies useful for design consideration of other cellular radio systems. A study on the area-averaged rain rate might also be helpful in producing a more universally applicable prediction model since radar data are not commonly available for other areas in the world. The best result would ideally be achieved when similar evaluations, possibly using high-resolution radar records, can be conducted in areas of different climate regions all over the world.
- d. Inclusion of multi-hub arrangement with cell-site diversity in the simulation and analysis of cell-area coverage.
- e. Examination of cell-site diversity with multi-hub (more than two) configuration, possibly employing a more practical combining method, such as scanned combining. This can be paralleled with an analytical attempt at predicting the results by adopting Kanellopoulos' model of joint probability of rain attenuations on multiple, converging links [Kan96, Kan99].
- f. Study of cell-site diversity that takes into account the influence of shadowing, hence producing a more realistic model for separation angle-dependent diversity gain.

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Appendix A - Radar System Specification and Calibration

A.1 System specification

The following description of the McGill Radar Observatory's S-band radar system was kindly provided by Dr. Aldo Bellon and Dr. Frederic Fabry. A brief technical summary can also be found on the Radar Observatory's website at <http://www.radar.mcgill.ca>.

The radar system is located near the Macdonald campus of McGill University, Montreal. It collects reflectivity measurements at 24 elevation angles, ranging from 0.5° to 34.4°, with total scanning period of 5 minutes. The 24 elevation angles are divided into two interleaving subsets of 12 elevation angles so that each set is associated with a scanning cycle of 2.5 minutes. The system specification is listed in Table A-1.

Table A-1 Radar system specification

Wavelength	10.4 cm
Peak power	800 kW
Minimum Detectable Signal (MDS), Doppler mode	-105 dBm
Minimum Detectable Signal (MDS), reflectivity mode	-100 dBm
Antenna Diameter	9.15 m
Beam width	0.86 °
Half pulse length	150 m
Pulse Repetition Frequency (PRF), Doppler mode	1200 Hz
Pulse Repetition Frequency (PRF), reflectivity mode	300 Hz
Maximum unambiguous range, Doppler mode	125 km
Maximum unambiguous range, reflectivity mode	480 km
Nyquist Velocity	31.5 m/s
Time of volume scan of 12 elevation angles	2.5 min

A.2 Calibration

The following is a summary of the procedure taken to calibrate the McGill S-band radar system. The descriptions are due to Dr. Aldo Bellon and Prof. Isztar Zawadzki of McGill Radar Observatory.

Calibration of the radar system is conducted by making use of measurements from two raingauges: one operated at the radar antenna site, the other located 20 km away. Reflectivity factor estimates are obtained from rainfall rate measurements by applying $Z = 200R^{1.6}$ or $300R^{1.5}$ relationship, dependent on the type of precipitation, which are subsequently compared with the corresponding radar measurements at the same sites. The radar measurements used in comparison are derived from a CAPPI computed at around 2–2.5 km height. The gauge-to-radar measurement ratio can then be used as a calibration factor.

When a bias is found to occur persistently during a series of radar measurements of an event, the calibration factor is updated and applied on subsequent measurements. There is no change made, however, on the calibration factor of preceding measurements already archived, thereby introducing an inherent time delay on the application of the proper calibration factor. Recent evaluations using a third data source in the form of disdrometer DSD measurements have shown that the calibrated radar data obtained by applying the raingauge measurements differ only by ± 2 dB.

All radar data files used in the study reported in this thesis incorporate two parameters, namely *dbz_offset* and *dbz_slope*, a single pair of which are associated with each radar record. The values of *dbz_offset* from a series of radar measurements indicate the

variation of calibration factor over time. These parameters are used later to obtain the correct estimate of radar reflectivity factor in dBZ from a recorded datum as follows:

$$dBZ = dBZ_{rec} \times dbz_slope + dbz_offset \quad (A-1)$$

where dBZ_{rec} represents the recorded value of radar reflectivity factor.

Appendix B - Periods with Available Radar Measurements

Table B-1 presents a list of periods of radar measurements that are actually available and were used in the radio cell simulation. For the purpose of this research, radar measurements were collected roughly over a two-year period, spanning from October 16, 1998 to October 5, 2000. However, throughout this total period there were a number of records missing due to various causes, including radar inactivity during maintenance or system upgrade as well as accidental loss of data files during recording phases or file transferring processes. The cause of such data unavailability, whenever known, is stated in the last column of the table.

It should be noted, however, that even during the actual measurement periods failures in data acquisition and recording occurred from time to time. Hence, the actual number of available records from a one-month period with exact start and end dates given is not necessarily equal to what is predicted by calculation that takes the 5-minute sampling period into account. It was found in the end that the total number of usable radar records is 139,722. The number for each month is also given in Table B-1.

Table B-1 Periods with available radar measurements

Year	Month	Days	Number of records	Remark
1998	Oct	16 – 31	4102	
	Nov	1 – 30	8516	
	Dec	1 – 30	6435	
1999	Jan	3 – 31	5738	
	Feb	1 – 28	6273	
	Mar	1 – 31	8884	
	Apr	1 – 6	1574	Throughout April-May 1999, the radar was not operating due to the installation of new dual-polarization radar system.
	May	—	0	
	Jun	1 – 30	6464	
	Jul	1 – 31	7402	
	Aug	1 – 31	5608	
	Sep	1 – 30	7833	
	Oct	1 – 31	8745	
	Nov	1 – 25	7113	Records from the period of 25 Nov–11 Dec 2000 are missing due to accidental data file loss.
	Dec	11 – 31	5205	
2000	Jan	4 – 18	3370	Radar data are not available for the period of 18 Jan – 18 Mar 2000 due to a combination of hardware and software failures.
	Feb	—	0	
	Mar	18 – 31	1144	
	Apr	1 – 30	6585	
	May	1 – 31	7194	
	Jun	1 – 30	5299	
	Jul	1 – 31	7742	
	Aug	1 – 31	8493	
	Sep	1 – 30	8635	
	Oct	1 – 5	1368	

Appendix C - Radar Data File Format

C.1 File format

This appendix describes the radar data file format and the program code required to decode the records in each file, the latter to be explained in the next section. The descriptions presented in the following are all taken from documents accompanying the radar data files from McGill Radar Observatory. Acknowledgement should therefore be given to the people at MRO for preparing the documents.

The McGill S-band radar antenna system rotates at a rate of 6 rpm, creating 24 different radar images in each 5-minute cycle at 24 elevation angles. Only images from two elevation angles, namely 3.4° and 7.1° , were recorded and used in this study. The scanned image for the 3.4° elevation is of 45-km radius, whereas the one for 7.1° has a radius of 15 km; both are recorded at a resolution of 1° in azimuth and 150 metres radially.

The data files were each stored as a sequence of radar records and named *scandata.n*, where *n* is the file index. A single radar record in the sequence contains the following blocks of information:

a. Header (4096 bytes), arranged in the following format:

Length	Type of data	Content	Remark
56	2-byte integer	Unused	
1	4-byte integer	Hour	Time of recording
1	4-byte integer	Minute	
1	4-byte integer	Second	
1	4-byte integer	Day	
1	4-byte integer	Month	
1	4-byte integer	Year	
1	4-byte integer	Unused	
1	4-byte floating point	Radar latitude	
1	4-byte floating point	Radar longitude	
1	4-byte integer	Number of elevation angles	
24	4-byte floating point	Elevation angles	
1	2-byte integer	Azimuth offset	
317	2-byte integer	Unused	
1	2-byte integer	Metprm	Indicator of type of radar measurements: reflectivity or velocity (Doppler). Unused in this study.
1	2-byte integer	Unused	
1	4-byte floating point	dBZ offset	See equation (A-1)
1	4-byte floating point	dBZ slope	
1600	2-byte integer	Unused	

b. Raw reflectivity data (145408 bytes)

Length	Type of data	Content	Remark
108000	1-byte integer	360×300 matrix of reflectivity at 3.4° elevation	360° azimuth span with 1° resolution; 45 km radius with 150 m resolution
36000	1-byte integer	360×100 matrix of reflectivity at 7.1° elevation	360° azimuth span with 1° resolution; 15 km radius with 150 m resolution
1408	1-byte integer	Unused	

Note: The MIPS microprocessor that powered the workstation controlling the radar data acquisition and recording treats numerical data according to the Little Endian convention, i.e. multiple-byte data are written with the last byte being the least significant byte. In order to process the recorded data on Intel-powered machines that obey the Big Endian convention (the last byte is the most significant byte), a byte swap procedure must be effected on each multiple-byte variable. That is,

- For 2-byte data, swap byte 0 and 1.
- For 4-byte data, swap byte 0 and 3, swap byte 1 and 2.

C.2 File decoding

The following is the source code in C language for data file decoding, authored by researchers at McGill Radar Observatory, except for the part for byte swap procedure that was added later for the simulations reported in this thesis.

```

/*****
A program to read CRC scan records from the disk file and print out
- the time stamp in the headers
- reflectivity in dBZ for the required pixels
*****/

#include <fcntl.h>
#include <stdio.h>
#include <stdlib.h>
#include <errno.h>
#include <iomanip.h>
#include <iostream.h>
#include <io.h>
#include <sys\stat.h>
#include <math.h>

/*****/
/***** structure for the header of the CRC scan records
/**  emminute,esecond,day,month,year : GMT time of data collection
/**  bridge_az_offset : offset to be added to get the azimuth of the beam
/**  dbz_offset,dbz_slop : slope and offset to be used to convert the
/**          : level stored to reflectivity in dBz unis.
/*****/
typedef struct {
    short int    dum[56];
    unsigned int ehour;
    unsigned int eminute;
    unsigned int esecound;
    unsigned int day;
    unsigned int month;
    unsigned int year;
    unsigned int dum1;
    float        radlat,radlon;
    unsigned int nelves;
    float        elvang[24];
    short int    bridge_az_offset;
    short int    dum3[317];
    short int    metprm;
    short int    dum4;
    float        dbz_offset,dbz_slope;
    short int    dum5[1600];
} Vscan_Header ;

typedef struct {
    short int    dum[56];
    unsigned int hour2elev[34];
    short int    bridge_az_offset;
    short int    dum3[317];
    short int    metprm;
    short int    dum4;
    unsigned int dbz_offset,dbz_slope;
    short int    dum5[1600];
} Preswap_Header ;

short unsigned int swap_short_int(short int input)
{

```

```

    unsigned char upper, lower;
    short unsigned int output, dummy;

    if (input<0) dummy = (pow(2,15) + input) + pow(2,15);
    else dummy = input;
        upper = dummy/256;
    lower = dummy - int(upper*256);
    output = int(lower*256) + upper;
    return output;
}

unsigned int swap_int(int input)
{
    unsigned char char1MSB, char2, char3, char4LSB;
    unsigned int output, dummy;

    if (input<0) dummy = (pow(2,31) + input) + pow(2,31);
    else dummy = input;
    char1MSB = dummy/16777216;
    char2 = (dummy - int(char1MSB*16777216))/65536;
    char3 = (dummy - (int(char1MSB*16777216) + int(char2*65536)))/256;
    char4LSB = dummy - (int(char1MSB*16777216) + int(char2*65536) +
        int(char3*256));
    output = char4LSB*16777216 + char3*65536 + char2*256 + char1MSB;
    return output;
}

/*****/
/***** structure for the CRC record *****/
/***** Length of the record = 73*2048 Bytes *****/
/***** Length of Vscan_header = 2*2048 Bytes *****/
/***** Length of Scan info = 71*2048 Bytes *****/
/*****/
/***** declarations for CRC data collection *****/
/**** for CRC    collect 150m gates
    - for elev 10 (3.4 deg) : 0 to 45 km ( gates 0 - 299)
    - for elev 14 (7.1 deg) : 0 to 15 km ( gates 0 -99 ) *****/
#define GATES_ELV1  300
#define GATES_ELV2  100
typedef struct {
    Preswap_Header dumhed;
    unsigned char z_elv1[360][GATES_ELV1];
    unsigned char z_elv2[360][GATES_ELV2];
    char dum[1408];
} Crc;

Vscan_Header hed;
int crc_gates[2] = {GATES_ELV1,GATES_ELV2};
Crc  zcrc;  /**** structure for crc ref gates *****/

int main(int argc, char **argv)
{
    unsigned int ip,istat,iqf,ics,reclen,icount,ipl,n,I,
        REQ_ELV,ielv,iaz,igate,
        unused,dum,temphandle,written,handle;
    float  dbm,dbm_coff,diff,vel,velconst,dslope,doffset,

```



```

        BEG_RANGE,END_RANGE,BEG_AZIMUTH,END_AZIMUTH,
        elvang[2] = {3.4,7.1}, azimuth,range;
char fname[100];
double refz;

char dummy1, dummy2, iyy,
    strdummy1[10], strdummy2[10];
long int newdigit;
int noitems;

strcpy(name, dBZ);

/***** ascertain that the record lengths are as defined above *****/
printf("\n---- Size of Vscan_header : %d  Size of Crc : %d ----\n",
    sizeof(Vscan_Header),sizeof(Crc));
printf("\n---- Size of Preswap_header : %d \n",
    sizeof(Preswap_Header));

/***** set defaults *****/
sprintf(fname,"../crc_archive_dir/crc_archive");

/**** get arguments from the command line if available ****/
for (i=1; i<argc; i++) {
    if ( !strcmp(argv[i], "-f") && ( (i+1) < argc) ) {
        i++;
        strcpy(fname, argv[i]);}

    else {
        printf("Invalid command-line arg ... : %s\n", argv[i]);
        goto end; }
} /**** loop to get all command line arguments ***/

/**** open the archive file ****/
printf("\n----- file name : %s ----- \n",fname);
iqf = open(fname,O_RDONLY|O_BINARY);
if(iqf < 0) printf("open %s  error : %d\n",fname,errno);

/**** read crc scans in a loop and print out the header info *****/
/****

for(;;) {
    icount = read(iqf,&zcrc,sizeof(Crc));
    if(icount != sizeof(Crc)){
        printf("read crc error:%d number of bytes reported:%d\n",
            errno,icount);
        goto end;}

/**** Convert the header format from Little Endian to Big Endian *****/

for(dum=0; dum<34; dum++)
    zcrc.dumhed.hour2elev[dum] = swap_int(zcrc.dumhed.hour2elev[dum]);
    zcrc.dumhed.bridge_az_offset =
        swap_short_int(zcrc.dumhed.bridge_az_offset);
    zcrc.dumhed.metprm = swap_short_int(zcrc.dumhed.metprm);

```

```

zcrc.dumhed.dbz_offset = swap_int(zcrc.dumhed.dbz_offset);
zcrc.dumhed.dbz_slope = swap_int(zcrc.dumhed.dbz_slope);
if ((temphandle = open("c:\\temp\\temp.tmp",
    O_CREAT|O_TRUNC|O_BINARY|O_RDWR, S_IRREAD|S_IWRITE)) == -1) {
    printf("Error opening temp.\n");
    switch(errno) {
        case EACCES : printf("Permission denied.\n"); break;
        case EINVACC : printf("Invalid access code.\n"); break;
        case EMFILE : printf("Too many opened files.\n"); break;
        case ENOENT : printf("No such file or directory.\n");
            break;
        default : printf("Unknown reason.\n"); break;
    }
    goto end; }

if ((written = write(temphandle,&zcrc.dumhed,sizeof(Preswap_Header)))
    != sizeof(Preswap_Header)){
    printf("Bytes written : %d \n",written);
    printf("Error writing preswap header.\n");
    switch(errno) {
        case EACCES : printf("Permission denied.\n"); break;
        case EBADF : printf("Bad file number.\n"); break;
        default : printf("Unknown reason.\n"); break;
    }
    close(temphandle);
    goto end; }
if (!seek(temphandle,0L,SEEK_SET) != 0) {
    printf("Error resetting pointer for reading Vscan header.\n");
    goto end; }
if (read(temphandle,&hed,sizeof(Vscan_Header)) != sizeof(Vscan_Header)) {
    printf("Error reading Vscan header.\n");
    goto end; }
close(temphandle);

/***** print out the header *****/
printf("\n----- %d/%d/%d  %02d:%02d:%02d GMT ----- \n",
    hed.year,hed.month,hed.day,
    hed.ehour,hed.eminute,hed.essecond);

/*****
/***** EXAMPLE OF GETTING THE DATA *****/
/*****
/***** The following code prints out the reflectivity in dBz units
for elevation REQ_ELV, for azimuths BEG_AZIMUTH to END_AZIMUTH and
for ranges BEG_RANGE to END_RANGE *****/

REQ_ELV = 0; /***** 0 : 3.4 degs 1: 7.1 deg *****/
BEG_AZIMUTH = 90; END_AZIMUTH = 91; /*** azimuth range 0 - 359 deg ***/
BEG_RANGE = 11; END_RANGE = 11.25; /* range is 0 to 45 km when REQ_ELV=0 */
/* range is 0 to 15 km when REQ_ELV=1 */

for ( ielv = 0; ielv < ? ; ielv++){
    for ( iaz = 0; iaz < 360; iaz++) {
        for ( igate= 0; igate < crc_gates[ielv]; igate++) {
            if(ielv == REQ_ELV) {
                azimuth = iaz + hed.bridge_az_offset + 0.5;

```

```

        /*** azimuth in deg of radial iaz ***/
        range = (igate * 0.15) + 0.075;
        /*** range in km of gate igate ***/

/***** print out info if required *****/
if(azimuth >= BEG_AZIMUTH && azimuth <= END_AZIMUTH &&
    range >= BEG_RANGE && range <= END_RANGE) {
    if ( ielv == 0 ) refz = zcrc.z_elv1[iaz][igate] ;
    if ( ielv == 1 ) refz = zcrc.z_elv2[iaz][igate] ;
/***** a value of 0 indicates signal below detectable threshold.
Hence convert to dBZ units only if refz is not 0 *****/
    if(refz != 0)
        refz = refz * hed.dbz_slope + hed.dbz_offset;
    printf("el:%4.1f deg, az:%5.1f deg, range:%6.3f km, Z:%5.1f dBZ\n",
        elvang[ielv],azimuth,range,refz);
    } /**** check for req azimuth and range ****/
} /**** check for required elevation scan ****/
}}} /**** loops on gate,beam and elevation ****/

} /**** infinite loop for printing the headers ****/
end:

printf ("end of the file reached\n");
} /**** end of read_crc_scans *****/

```

An example of the displayed result from the above code is as follows:

```

----- Size of Vscan_header : 4096  Size of Crc : 149504 -----
----- Size of Preswap_header : 4096

----- file name : scandata.x -----

----- 1999/1/15  23:48:49 GMT -----
el: 3.4 deg  az: 90.5 deg  range:11.025 km  Z:  0.0 dBZ
el: 3.4 deg  az: 90.5 deg  range:11.175 km  Z:  0.2 dBZ

----- 1999/1/15  23:53:49 GMT -----
el: 3.4 deg  az: 90.5 deg  range:11.025 km  Z:  0.0 dBZ
el: 3.4 deg  az: 90.5 deg  range:11.175 km  Z:  0.0 dBZ

----- 1999/1/15  23:58:49 GMT -----
el: 3.4 deg  az: 90.5 deg  range:11.025 km  Z:  0.0 dBZ
el: 3.4 deg  az: 90.5 deg  range:11.175 km  Z: -3.4 dBZ

----- 1999/1/16  00:03:49 GMT -----
el: 3.4 deg  az: 90.5 deg  range:11.025 km  Z: -2.2 dBZ
el: 3.4 deg  az: 90.5 deg  range:11.175 km  Z: -3.4 dBZ

```