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**NONLINEAR SEISMIC ANALYSIS OF
CONCRETE ARCH DAMS**

by
BAHMAN NORUZIAAN

B.Sc.- Sharif University of Technology, Iran

M.Sc.- Sharif University of Technology, Iran

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

September 1995

Department of Civil and Environmental Engineering*
Carleton University
Ottawa, Canada

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**NONLINEAR SEISMIC ANALYSIS OF
CONCRETE ARCH DAMS**

submitted by

Bahman Noruziaan

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Department of Civil and Environmental Engineering

Thesis Co-supervisor

Thesis Co-supervisor

External Examiner

Carleton University
Ottawa, Canada

Abstract

Concrete arch dams may behave nonlinearly during strong ground motions. The nonlinearity in the dam structure arises from two distinct sources: the relative displacement of the two faces of the contraction joints which are built in the body of arch dams, and possible cracking or compressive over-stress of concrete. The joints may undergo cleavage type deformations or dilatational movements, or a combination thereof.

In this thesis numerical studies are performed to study the significance of the preceding phenomena on the seismic response of arch dams. A nonlinear finite element program called "BANDAAB", was developed with the following two principal attributes: (a) the ability to model the nonlinear behaviour of the joints in the dam and its effect on the overall seismic response of the dam, and (b) the ability to account for concrete nonlinearity due to high compressive stresses and/or cracking of concrete. The program "ADAP88", which was developed at the University of California at Berkeley, was used as the backbone of the program developed here. However, ADAP88 had to be substantially modified to render it capable of dealing with the preceding phenomena.

The proposed joint model can be used to model the opening-closing, free slippage due to large opening, frictional slippage in the closed state, and the strength of the shear keys in the joint. A comprehensive constitutive model which can account for concrete cracking and compressive nonlinearity is implemented in the finite element program BANDAAB. This model is developed by combining a bounding surface model with a three-dimensional tensile cracking model. The bounding surface is intended to model the compressive nonlinear behaviour of concrete while the tensile model can account for the nonlinearities arising from cracking. A procedure is proposed for modelling the combined effect of the two nonlinearities.

Parametric studies are performed to study the effects of the different constitutive parameters of the implemented joint model on the seismic behaviour of a typical arch dam. It is found that shear slippage due to large opening of the joints may have significant effect on the behaviour of the dam. The effect of concrete nonlinearities on the behaviour as well as the progress and distribution of damage in the arch dam are studied. It is found that the

dominant cause of damage is tensile cracking, and this damage is mostly restricted to the upper levels of the dam. The effect of compressive stress on the tensile strength of concrete is found to be important. However, for the particular dam and ground motion considered in the present investigation, the effect of concrete nonlinearity on the overall response of the dam was found to be small. Analysis of other dams subjected to different ground motions would be needed to fully determine the consequences of concrete nonlinearities on the serviceability and safety of arch dams.

*"And so may we be those who rejuvenate this world, O Mazda (Wise One),
grant me support and truth so one can be convinced even when knowledge is in doubt".*
Zarathushtra (1700 B.C.)

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Notation

a	corrective scalar factor
$\mathbf{a}$	$[\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3]^T$
A_L, A_u	behavioural functions for the bounding surface model
α	multiplier factor in the softening model
b_0, b_1	Rayleigh's proportionality constants
$\mathbf{B}$	displacement-strain transformation matrix
β	$\arctan \frac{\tau}{\eta}$
β, γ	parameters of the Newmark time integration method
β_1, β_2	$\cos \beta, \sin \beta$
c	cohesion of the joint surface and/or the strength of the shear keys
$\mathbf{c}$	damping matrix of the linear substructures
$\mathbf{C}_T$	tangential damping matrix of the nonlinear substructure
C_{ijkl}	compliance tensor
D	$= \frac{r}{R}$, normalized distance
$\bar{\mathbf{D}}$	the average value of the rigidity matrix $\mathbf{D}$
$\mathbf{D}_{ep}$	elasto-plastic rigidity matrix
$\mathbf{D}_{in}$	inelastic rigidity matrix
δ	slip margin
δ^*	distance of the point from the bounding lines
δ_c	crack opening
δ_{c0}	stress free crack opening
δ_{ij}	Kronecker delta
δ_m^*	initial distance of the point from the bounding lines
$\mathbf{e}$	deviatoric strain tensor
$\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$	orthogonal unit vectors at a point on the joint surface
$\mathbf{e}^e, \mathbf{e}^p$	elastic and plastic deviatoric strain tensors
E	Young's modulus
ϵ_0	strain corresponding to the uniaxial strength of concrete
ϵ_{cr}	cracking strain

ϵ_f	yield condition tolerance
$\epsilon_{kk}^d, \epsilon_{kk}^o$	hydrostatic strain caused by the hydrostatic stress and the octahedral shear strain
ϵ_p	effective plastic strain
ϵ_v^p	plastic volumetric strain
ϵ_{mar}	stress-free crack strain
ϵ_v	volumetric strain
ϵ	strain tensor
ϵ^p	plastic strain tensor
$\tilde{\epsilon}$	strain sub-increment tensor
$\tilde{\epsilon}^p$	plastic strain sub-increment tensor
ϵ'	local strain tensor
η	cracking parameter
f	loading function
f_c'	uniaxial compressive strength of concrete
f_t'	uniaxial tensile strength of concrete
F_1, F_2	behavioural functions for the bounding surface model
$F_{1,max}$	maximum value of F_1
$\mathbf{f}$	external load vector acting on linear substructures
$\mathbf{F}$	element resisting load vector, external load vector acting on the nonlinear substructure
ϕ	friction angle
g	plastic potential function
G_f	fracture energy release
$G(\cos 3\theta)$	a behavioural function for the bounding surface model
G^*	normalized shear modulus
γ_0^p	plastic octahedral shear strain
H	a constant
H^*	a bounding surface behavioural function
H'	generalized shear modulus
H^p	plastic modulus
H_0^p	plastic modulus corresponding to zero distance

I_1	first invariant of the stress tensor
$I_{1,max}$	maximum value of I_1
J_2	second invariant of the deviatoric stress tensor
J_3	third invariant of the deviatoric stress tensor
J	Jacobian matrix
k_n, k_s	normal and tangential penalty parameters of the joint
$k_{n,n}, k_{n,s}, \dots$	elasto-plastic rigidity coefficients of the joint
k_s^*	reduced tangential penalty parameter
K	damage parameter
K_{max}	maximum value of the damage parameter K
K_R, K_u	associated values of K at the beginning of the most loading and the most unloading process respectively
K_t	tangent bulk modulus
k	stiffness matrix
k_e	elastic stiffness matrix
k_{in}	inelastic stiffness matrix
k^*	effective stiffness matrix
$\bar{k}^*$	condensed effective stiffness matrix
K	stiffness matrix
K_T	tangential stiffness matrix of the nonlinear substructure
K^*	effective stiffness matrix of the nonlinear substructure
κ	hardening parameter
l_c	characteristic length in crack band theory
L	load criterion function
λ	proportionality factor
m	mass matrix of the elastic substructure
m_g	coupling mass matrix between the linear substructures and the foundation
M	mass matrix of the nonlinear substructure
M_g	coupling mass matrix for the non-linear substructure and the foundation
μ	friction coefficient

N	shape function
N	$N \mathbf{a}$
ν	Poisson's ratio
Ω	damage parameter
$\mathbf{p}^*$	effective load vector
$\bar{\mathbf{p}}^*$	condensed effective load vector
$\bar{\bar{\mathbf{p}}}^*$	reduced and condensed effective load vector
$\mathbf{P}$	strain sub-increment-plastic strain sub-increment transformation matrix
$\mathbf{P}$	load vector
	out of balance load vector
$\mathbf{P}_{el}^*$	effective load vector of the linear substructure
$\mathbf{P}_{non}^*$	effective load vector of the nonlinear substructure
$\mathbf{q}$	boundary external load vector
$\mathbf{Q}$	boundary load vector acting on the nonlinear substructure
r	scale factor
r	distance of the stress point from the hydrostatic axis on the deviatoric plane
$\mathbf{r}$	matrix of the direction cosines between ground acceleration and the substructure degrees of freedom
R	distance of the image point from the hydrostatic axis on the deviatoric plane
$\mathbf{R}$	matrix of the direction cosines between ground acceleration and the nonlinear substructure
ρ	shear dilatancy-compaction factor
ρ_1, ρ_2	two contributing parts of the shear dilatancy-compaction factor β
S_{ij}	deviatoric stress tensor
σ	normal stress of the joint
$\sigma_1, \sigma_2, \sigma_3$	three principal stresses
$\sigma'_{11}, \sigma'_{12}, \dots$	local stresses
σ_e	elastic normal stress
σ_h	hydrostatic stress
σ_{tu}	tensile strength

σ	stress tensor
σ_{el}	elastic stress tensor
σ_{in}	inelastic stress tensor
$\dot{\sigma}$	stress sub-increment tensor
σ'	elastic stress tensor
$\dot{\sigma}'$	normal elastic stress sub-increment
$\mathbf{T}$	local to global transformation matrix
τ	resultant shear stress
τ_0	octahedral shear stress
τ_1, τ_2	tangential stresses of the joint
τ_1^e, τ_2^e	elastic tangential stresses
$\dot{\tau}_1^e, \dot{\tau}_2^e$	tangential elastic stress sub-increment
θ	angle between the projection of the position vector of the principal stress and any tensile semi-axis
u_1, u_2	tangential relative displacements
u_1^e, u_2^e	elastic tangential relative displacements
u_1^p, u_2^p	plastic tangential relative displacements
u_1^{bot}, u_2^{bot}	bottom tangential displacements
u_1^{top}, u_2^{top}	top tangential displacements
$\dot{u}_1, \dot{u}_2$	relative tangential displacements sub-increment
$\mathbf{u}$	relative displacement vector nodal displacement vector
$\mathbf{u}^e$	elastic relative displacement vector
$\mathbf{u}^p$	plastic relative displacement vector
$\mathbf{u}^{top}, \mathbf{u}^{bot}$	displacement vectors at the top and the bottom faces of the joint
$\dot{\mathbf{u}}$	velocity vector
$\ddot{\mathbf{u}}$	acceleration vector
$\mathbf{U}$	relative displacement vector of the joint, nodal displacement vector
$\dot{\mathbf{U}}, \mathbf{U}$	nonlinear substructure velocity and displacement vectors
$\mathbf{U}^{top}, \mathbf{U}^{bot}$	displacement vectors at the top and the bottom faces of the joint
$\dot{\mathbf{U}}$	relative displacement vector sub-increment
$\dot{\mathbf{U}}^p$	plastic relative displacement vector sub-increment

v	normal relative displacement
v^e	elastic normal relative displacement
v_p	plastic normal relative displacement
v^{top}, v^{bot}	top and bottom normal displacement
$\tilde{v}$	relative normal displacement sub-increment
$\hat{\mathbf{v}}$	reduced displacement vector
$\mathbf{x}$	global coordinate vector of the points on the joint surface
w_l	weight factor at the integration point
w_p	plastic work

Chapter 1

Introduction

1.1 General

Concrete dams have been and will be built throughout the world, and are among the most important man-made structures. Their importance is due to the ever increasing need for storage of water and for regulating its conveyance to locations where it is needed. Dams are also a major element of the electricity generation infrastructure.

The safety of dams has always been of great concern because failure of these structures can create a great disaster, involving potentially huge human and material losses. This concern is ever increasing because of the increase in the number of people settling in the downstream areas. Considering those dangers, governments have established regulations to monitor the behaviour of these structures and to study their safety based on up-to-date state of knowledge to determine the causes of potential failure before their occurrence and to take remedial steps (NRC, 1990).

One of the most severe loads acting on dams is the inertial loading of the dam body and the impounded water due to strong ground motions. Although only a few concrete gravity and buttress dams have been damaged due to seismic action, e.g. Koyna Dam in India (NRC, 1990), Hsinfengkiang Dam in China (NRC, 1990), Sephid-Rood Dam in Iran (Ahmadi et al 1992), none of them has completely failed to the extent of releasing the upstream stored water. Nevertheless, it is believed that there is an urgent need to study the behaviour of these structures in light of new state of knowledge. It has been shown that previous design assumptions are not conservative, and that a great number of dams located in very active seismic zones still have not experienced the severe seismic action as predicted by seismological considerations, while those that have experienced large magnitude earthquakes were not being operated at their full capacity at the time (NRC, 1990).

Study of the seismic behaviour of dams requires that the effect of a dam's three major components be taken into account, i.e. the dam body, the dam abutments and foundation, and the reservoir and its foundation, as shown in Fig.1.1.

Among these three components, the dam and its foundation may behave in a severely nonlinear fashion during strong ground motions, and this nonlinearity could have a major impact on the safety level of the structure. The present investigation will be confined to the nonlinear seismic behaviour of arch dam body, but in the following sections the importance of the above mentioned components during a ground motion activity will be briefly reviewed.

1.2 Aspects of Non-Linear Behaviour

1.2.1 Nonlinearities in Dam Structure

The stress-strain behaviour of concrete is very complicated; it is path and rate dependent. This behaviour is also affected by such factors as long term sustained loads, temperature, and humidity. Strong ground motions may cause large dynamic stresses in excess of the static stresses in concrete dams.

Concrete has practically negligible tensile strength, and is susceptible to cracking whenever tensile stresses develop. During seismic activities there is a high probability for the development of large tensile stresses in concrete dams, leading to the possible formation of cracks. The development of cracks will cause stress redistribution and transfer of stress to other points in the structure with probable over-stress at those points.

On the other hand, there is a common belief that because concrete behaves linear-elastically up to the level of about half its uniaxial compressive strength, and that concrete dams rarely experience stresses more than this level even during strong ground motions, the nonlinear compressive behaviour of concrete in dams is not a major concern. It will be shown in Chapter 2 of this thesis that the above belief may not be justifiable. It is worthwhile mentioning that experimental and numerical studies on arch dams have indicated the probable development of compressive stresses well above the uniaxial elastic limits of concrete (Niwa and Clough, 1982), (Dowling and Hall, 1989).

The development of tensile cracks can reduce the load bearing area of concrete in bending, resulting in probable increase in compressive stresses on the remaining part. Im-

plementation of a triaxial constitutive model in a concrete dam analysis, with the ability to model the complicated nonlinear cyclic behaviour of concrete, will lead to valuable insight regarding the realistic levels of stress that a concrete arch dam may experience during a strong seismic excitation.

The other mechanism contributing to nonlinearity in arch dams is the movement of the contraction joints which are built within the dam body. Contraction joints are built to release the excess tensile stresses developed because of cooling and shrinkage of concrete. Contraction joints can be keyed or un-keyed. During seismic action these contraction joints will open and close, and the two faces may slide (especially in un-keyed joints) relative to each other, resulting in severe nonlinear behaviour. This mechanism can be a source of energy dissipation and may have a significant effect on the frequency characteristic of the structure, i.e. resulting in change of response due to a shift in the natural period. Furthermore, it has been shown that allowing the joints to open will release the arch direction tensile stresses and will result in the transfer of stresses to the cantilever direction (Fenves et al. 1989).

1.2.2 Nonlinearities in Foundation

The stability of an arch dam depends to a great extent on the stability of its side abutments. It is worthwhile mentioning here that many historical cases of arch dam failure have been identified with instability or failure of canyon walls, e.g. Malpasset Dam in France and Frayle Dam in Peru (Thomas, 1976). Dam designers prefer sites with intact and solid abutment rock, but in many cases such ideal conditions are not available, hence dams have been built on sites which contain discontinuities such as seams and fissures, major cracks and joints, and even faults.

Stress transfer from the dam and the hydraulic pressure in the fissured rock, caused by fluid in the reservoir, will create a new stress field in the rock abutments, which may cause tensile and compressive over-stress resulting in cracking, crushing and sliding of the abutments. The importance of seismic over-stress in this respect is self-evident. Furthermore, the foundation rock is not bounded, and since the finite element mesh can not be extended to infinity, a proper far end boundary condition is generally needed for seismic analysis of these structures.

1.2.3 Nonlinearities Due to Water Action

During severe ground motions, the inertia of the water stored behind the dam will induce large forces on the upstream face of the dam. It has been shown that water compressibility can have a significant effect on the response of the dam (NRC, 1990). To study the effect of compressibility, the geometry of the reservoir, especially its length, should be modeled properly. In view of the problem that the finite element mesh can not be extended to infinity, an appropriate limit or energy transmitting boundary condition at the far end of the reservoir mesh is required in the analyses. Other possibilities may exist for a combined finite element-boundary element analysis in which a portion of the water domain is treated as a linear problem that extends to infinity. The possibility of water sloshing is one source of nonlinearity within the water domain.

Although it has been shown that cavitation in reservoir does not have significant effect on stress distribution in gravity dams, the effect of cavitation on arch dam behaviour is not fully known and therefore needs further investigations (NRC, 1990).

As mentioned earlier, arch dams are not intact because of the built-in contraction joints, and cracks induced by load, temperature, creep and shrinkage of concrete. Water may penetrate inside these discontinuities, changing the stress distribution in the dam, possibly even under static load condition. The significance of these discontinuities to the seismic response of arch dams has not been fully investigated. One significant phenomenon is the closing and reopening of the joints and cracks, leading to possible further extension of the cracks. Closing of the cracks and the joints will compress the water. Because of the small compressibility of water, a larger reaction pressure is induced on the two faces of the joint. On the other hand, sudden opening of the joint may induce suction or negative pressure between the two faces. Theoretically pressure waves travel in water with a very high velocity. During seismic response, a large hydrodynamic pressure may be generated on the upstream face of the dam. This high pressure transmits through the joints and cracks and may change the pressure distribution in the dam.

Both of the above-mentioned phenomena are potential sources of nonlinearity in a dam. The effect of water intrusion in wide cracks can be a major concern (NRC, 1990) and is recommended for further study. In addition, the effect of reservoir sedimentation on energy absorption should also be considered.

1.2.4 Other Factors Affecting Seismic Response

One of the major parameters in any seismic response analysis is the earthquake excitation input. There are two major considerations in seismic response of arch dams:

(a) Spatial variation of ground motion because of the time delay of seismic waves travelling along the height and base of the dam, causing a non-uniform ground motion along the periphery of the dam (NRC, 1990).

(b) The possibility of simultaneous occurrence of the three components of ground motion, i.e., along the stream, cross canyon and in the vertical direction, should be taken into account. In particular, the nonuniform cross canyon motion along with the stream direction motion could induce severe stresses in arch dams.

1.3 Objectives and Scope

Based on the preceding discussion and the future research recommendations on the analytical aspects of arch dams of the NRC report on concrete dams (1990), the need for a comprehensive study which takes into account the effect of the following parameters on the seismic response of a full dam-water-foundation system is evident:

(1) Nonlinear joint behaviour, consisting of opening-closing, relative movement after separation and frictional slippage, and a suitable model for shear key effects.

(2) Non-linear concrete behaviour, considering the triaxial state of stress, especially on tensile strength, with possible over-stress in compression and hysteretic energy dissipation due to nonlinearity and cycling of stresses.

(3) Nonlinear rock abutment behaviour, especially probable instability due to shear failure of major joints and faults and tensile or compressive over-stress due to seismic action. The effect of water migration on the strength of the abutment should be studied rigorously.

In view of the enormous amount of effort needed to study all of the foregoing effects, the objectives of this thesis are limited to:

(1) Development of a finite element code suitable for studying the nonlinear effect of joints and concrete material on the seismic response of arch dams, including the effect of the reservoir.

- (2) Development and implementation of a suitable joint model.
- (3) Selection and implementation of a suitable constitutive model for the nonlinear tensile and compressive behaviour of concrete.
- (4) Study of the effect of the nonlinear joint behaviour and of the nonlinear tensile and compressive behaviour of concrete on the response history of a typical arch dam.

As stated earlier, the effect of the hydrodynamic forces of the reservoir will be considered but the nonlinearities will be limited to the dam body.

1.4 Methodology to be Used

The study of the seismic behaviour of arch dams should consider the effects of dam geometry, foundation material, reservoir, reservoir foundation and the ground motion. The finite element method (FEM) is the most suitable tool for such a study. Between the two alternatives of developing an entirely new FEM program and substantially modifying an existing FEM program to endow it with greater analytical capabilities, it was decided to use the second approach, and the program "ADAP88" (Fenves et al. 1989 and Mojtahedi et al 1992) was adopted as the backbone of the program to be developed here. The reason for the latter was twofold. First, it is essential to build on existing knowledge and avoid duplication; secondly, the development of a completely new FEM program, even for the linear analysis of arch dams, would be an enormous undertaking and would require a great amount of time and effort.

"ADAP88" is a nonlinear finite element program, developed at the University of California at Berkeley. The program has the capability to generate the finite element mesh for a general arch dam and its foundation with a very small amount of input data. Water-structure interaction is taken into account but the liquid is assumed to be incompressible. The program has the capability to perform some nonlinear dynamic analysis by considering the opening and closing of the joints.

Seismic ground motion records can be applied uniformly at the supports and in three directions simultaneously; any non-uniformity in the ground motion along the boundary of the dam is ignored.

Since the program developed in this thesis contains several new features and modifica-

tions in comparison to the original "ADAP88" and to avoid any confusion with regard to the capabilities of these two programs, the new program is called "BANDAAB".

1.5 Layout of the Thesis

A general introduction, a brief description of the problem, the objective of the current research and the methodology used in this thesis are contained in the present Chapter. In Chapter 2 a survey of the relevant literature is presented and important aspects of past researches are discussed. Chapter 3 contains the basic assumptions of the proposed joint element model and its formulation. The numerical implementations of this model is verified and its algorithm is explained in detail. In Chapter 4 the basic characteristics of concrete behaviour are explained. The most important approaches for modelling the nonlinear behaviour of concrete are also introduced and discussed. The selected bounding surface model, as well as the three dimensional tensile model, are explained in detail. The numerical implementations of these models are verified by numerical examples, and the relevant algorithms are explained. The developed combined model, which has the characteristics of both the bounding surface model and the tensile model, is explained and its numerical implementation is verified. In Chapter 5 the fundamental attributes of the adopted computer program "ADAP88" are introduced, and the modifications made to this program are explained. The results of the applied modifications are discussed through the results of the analysis of the Morrow Point Dam subjected to a typical ground motion. In Chapter 6 parametric studies are performed to study the effect of the joint nonlinearity on the seismic behaviour of the Morrow Point dam. Similarly, the effects of concrete nonlinearity on the seismic behaviour of the same dam are also studied. Chapter 7 presents the main conclusions of the present study and the suggestions for future research.

Chapter 2

Literature Survey

2.1 General

During an earthquake, adjacent monoliths in an arch dam, that are separated by contraction joints, may move relative to each other, resulting in the opening and closing of the joint and possible shear movements at the joint surfaces. Under static conditions, the joints are under compression due to the hydrostatic pressure behind the dam. The static compressive force is transmitted along the arch axis to the abutments on the two sides of the dam. The dynamic force during an earthquake generally imposes additional alternating compressive and tensile loads on the joints. If the dynamic tensile load is more than the static compressive force, the joint will open up, which releases a significant amount of the tensile forces in the arch direction, making the structure more flexible, with possible over-stress in the monoliths in the cantilever direction. On the other hand, the compressive face of the joint can be over-stressed due to partial opening of the joint and the resulting reduction of the load bearing area at the joint.

Existing dams, may contain other types of discontinuities beside contraction joints, such as horizontal construction joints, the concrete-rock interface, rock joints in the foundation, and cracks due to loads (e.g. at the heel), thermal or shrinkage effects, or differential settlement. These discontinuities, when acted upon by dynamic forces, may subject a dam to stresses well above its linear limit of behaviour.

In recent years, much research has been carried out by many research groups to study the stability of arch dams under static loads. These studies may be divided into two distinct categories: detailed 2-D analysis of a critical dam section, and sensitivity analysis of load distribution in arch dams in three dimensions (Hohberg, 1992). On the other hand, the number of studies on the dynamic behaviour of arch dams is much smaller. This is because a comprehensive dynamic time history analysis of arch dam structures that takes into consideration all the linear and nonlinear effects, requires significantly more computational effort and time than a static analysis.

This Chapter presents a survey of the previous research on the different nonlinear aspects of a dam and/or its foundation associated with the seismic behaviour of arch dams. In this regard, almost all the previous work done in this area concentrates only on the effects of joint movement on the nonlinear seismic response of arch dams.

2.2 Previous Research

Niwa and Clough (1982) reported the results of a series of experiments conducted on models of a segmented arch rib and on a cantilever monolith. The models were subjected to typical earthquake motion time histories. Results of the arch rib model test showed that allowing the joints to open limits the development of tensile stresses in the arch direction. This has a significant influence on the dam dynamic response characteristics because of the changes in the natural periods of the structure. Concurrently, the compressive stresses in the arch direction between the joints were reported to substantially increase due to a reduction in the load bearing area of the section. The crushing of the joint faces, mainly due to compressive over-stress of the joint face concrete as reported in these experiments, can have a major influence on the stability of the arches.

Row and Schrieker (1984) numerically modelled the contraction joint behaviour using zero length gap elements of linear normal and tangential springs between the dam monoliths to study the seismic behaviour of Xiang Hong Dam. The springs had finite stiffness in compression and zero stiffness in tension when modelling the opening-closing behaviour of the joints. However the sudden drop of stresses in the tangential direction due to joint opening was not modelled in this study.

Hall and Dowling (1985) developed a special method to represent gradual joint opening and closing in two-dimensional slabs by using a single element through the slab thickness in a finite element discretization. The nonlinear compressive behaviour of the joint material and sliding of the faces of the joint were not modelled in their study. They proposed an approximate procedure for extending their method to study the behaviour of arch dams containing vertical contraction joints and pre-existing horizontal planes of weakness. Using this approximate approach, they analyzed a representative horizontal cross section of a concrete arch dam consisting of discrete blocks. They concluded that the pre-compression from hydrostatic pressure provides considerable resistance against the destabilizing effect of

the dynamic loads. They noted that the jointed arch can survive an earthquake significantly stronger than the one predicted from a linear analysis with the assumption of fully closed joints. Because the maximum compressive stress in the dam they obtained in their analyses, which ranged from 8.4 to 21 MPa depending on the acceleration level, was smaller than the uniaxial compressive strength of the material of about 31.5 MPa, they concluded that the assumption of linear stress-strain relation for concrete was acceptable. However, it is noted here that as the calculated stress exceeds 50% of the uniaxial strength, the linear stress-strain relationship assumption for the material may not necessarily be applicable.

In another study, Dowling and Hall (1989) used their aforementioned model to study the effects of the opening-closing of the vertical contraction joints and the horizontal construction joints on the arch dam behaviour. In their study they assumed water as incompressible and the foundation as a mass-less domain. The horizontal construction joints were modeled as pre-determined planes of weakness. The nonlinearities from the nonlinear compressive behaviour of concrete and sliding of the joint faces were neglected in their analysis. Skew cracks were not allowed in the dam, and cracks could only occur at pre-defined horizontal sections. They presented results on the analysis of the Pacoima Dam. The main conclusions of their work are that the compressive stress in the remaining contact area of contraction joints can increase, and that this increase can be large enough in some instances that the assumption of linear elastic behaviour of the dam is no longer valid. Furthermore, the large separations at the contraction joints and at the horizontal joints near the top of the dam can violate the no slip assumption, particularly at un-keyed joints without sufficient frictional resistance. Even at joints with beveled keys, slippage may happen when total separation occurs. There is also the possibility of cracks in the cantilevers because the loads transferred from the arches due to opening of the vertical contraction joints may over-stress the cantilevers. Further, cantilevers of an arch dam may separate from the foundation during earthquakes. But overall, the inability of their model to include the effects of joint slippage and the lack of using a full 3-D material model in the modelling of the concrete material behaviour limit the applicability of the results.

Taskov and Jurukovski (1988) studied the behaviour of the central cantilever monolith of a typical arch dam bounded by two neighbouring monoliths. Friction in the contact area was modelled. Using this model which was stated to have been confirmed by scaled model tests, they concluded that the effects in the seismic response of arch dams due to sliding of

the contact area, crushing of the contact zone, opening of the joints, and friction reduction in the contact area are significant. Significant increase in the cantilever stresses can occur in the upper part of the dam, which may result in cracking of the cantilevers.

Hohberg and Bachmann (1988) analyzed a horizontal slice through an actual arch dam under water pressure. The structure was subjected to displacement patterns that caused all the joints to open, and then was allowed to vibrate freely about its at-rest position. The aim of this work was to understand the numerical problems encountered in joint elements in a finite element analysis. No conclusion was presented about the effect of joints on the behaviour of arch dams.

Weber et al (1990) investigated the nonlinear behaviour of arch dams which included the nonlinear behaviour in joints and pre-determined cracks. In this work the assumption of keyed joints was adopted which implies unlimited shear transfer with full joint shear stiffness across the joints. The dam was assumed to have a rigid foundation and an empty reservoir. The seismic action was applied in the stream direction with the possibility of toppling of the cantilevers. Their results show that the increase in joint opening would cause the cantilevers to undergo bending and torsional motions resulting in a discontinuity angle moving across the crest line of the dam. However the shear key assumption in the model prevents any phase difference between the motions of the cantilevers and sufficiently stabilizes them so as not to develop critical bending stresses.

In another study, Hohberg (1991) analyzed a dam with the same geometry and a similar assumption about the foundation as the one used by Weber et al (1990). He implemented a rather detailed model for the joints which considered frictional slippage and slippage due to separation. Asperities of the joint surfaces were also modelled. The dam was modelled as an assemblage of 3D solid elements, but the water effects were not included. Shear keys were modelled by increasing the height of the asperities. The results of this study confirm the previous observations that the opening of joints reduces tensile stresses in the arch direction.

Fenves et al (1989) developed a joint element to model the opening of contraction joints. The sliding effects at the joints and the nonlinear behaviour of the concrete material of the dam were not considered. The method proposed by Row and Schriker (1984) to model localized nonlinearity was adopted in the modelling of the contraction joints, which greatly

reduced the computational effort in the analysis. In this analysis, the foundation was considered to be flexible, and was modeled as an assemblage of 3D finite elements extended to a selected depth. The water in the reservoir was considered to be incompressible. The effect of sediment energy absorption was not included in the study. The dam structure was modeled as an assemblage of 3D solid elements, thick shell elements and transition elements. The study gave invaluable insight into the behaviour of contraction joints in arch dams, and confirmed some of the observations reported by other researchers.

Bolognini et al (1992) developed a friction type model to account for the sliding behaviour of joints. They used this model to analyze an arch dam with an empty reservoir. In their analysis, the active joints were restricted to the lower part of the dam body. Because of this assumption and the neglect of the effect of impounded water, the influence of the movements of the contraction joints was shown to be insignificant.

The above literature survey reveals the gaps that currently exist in our knowledge of the seismic behaviour of arch dams. These include limited understanding of the effects of joints on the behaviour and performance of arch dams, the lack of a joint model that can simulate both opening-closing and slippage non-linearities as well as the shear key effects. Generally the impounded water and the foundation effects on complete dam behaviour have been neglected, and the significance of the nonlinear concrete material behaviour has not been adequately studied. It is expected here that due to opening of the joints, the adjacent zones will be more susceptible to over-stress either in compression or in tension. This thesis will address some of the preceding issues.

Chapter 3

Non-linear Joint Element

3.1 Joints in Arch Dams

In order to have a better understanding of the behaviour of contraction joints in arch dams during earthquakes, a brief description of the nature and function of contraction joints and their methods of construction is presented here. Concrete is a brittle material and is susceptible to cracking due to its low tensile strength. To avoid the development of tensile stresses due to shrinkage and temperature drop in mass concrete, arch dams are built as assemblages of monoliths separated by contraction joints. Based on past experience, the common practice in the construction of arch dams is to place the contraction joints at 15 to 20 m intervals across the arch dam body. In the case where the dam is thicker than the contraction joint spacing, longitudinal contraction joints are also built within the dam body. It is also a common practice to eliminate every second joint in the upper part of the dam because of reduced dam thickness, resulting in an average distance of 30 to 40 m between the joints (Serafim and Clough 1990).

3.1.1 Shear Boxes or Shear Keys

It is a common practice to use shear boxes or shear keys in dams for the transfer of shear forces across contraction joints. Because of the high compressive stresses normal to the joint plane in static condition, high frictional forces can develop between the joint surfaces where shear keys may not be necessary for the transfer of shear forces across the contraction joints. However, in the case of a dam built in a seismically active region, or if there is a possibility of foundation breakdown, resulting in large relative displacement of the monoliths, the transfer of shear stresses by friction between the dam monoliths can not be relied upon. As a result, the use of shear boxes or shear keys as a supplemental means for shear transfer is a common practice in arch dam construction, as shown in Fig. 3.1.

In order to avoid the very high local stresses that can develop in keyed contraction joints during differential movement of adjacent monoliths, it is suggested that the depth

of the key-way should not exceed 200 mm, and that the sides of the key-way should have a mild slope not more than 30° to the surface of the joint (Thomas, 1976). However, for arch dams, the use of un-beveled keys is also common. In general, the elaborate framework needed for the key-way and the more difficult and complex grouting system of the keyed joints increase the construction cost of the dam.

3.1.2 Grouting

Before the filling of the reservoir and after a major part of the total estimated shrinkage and cooling of the concrete have taken place, all the joints in an arch dam are grouted by a mixture of water and Portland Cement. Although grouting pressure produces a kind of pre-stressing force in dam, it can not be relied upon as a design factor since creep and further shrinkage of concrete can offset this pre-stressing.

Although there is a small tensile strength in a contraction joint which is governed by the adhesion between the grout material and the hardened concrete surfaces, this small tensile strength should not be relied upon for design because the adhesion could deteriorate due to further contraction of the dam and/or possible movements of the monoliths.

3.2 Joint Models

3.2.1 Models for Discontinuities at Joints

Joints, fissures and faults are major components in rock masses, which play an important role in their behaviour. Researchers in geotechnical engineering and rock mechanics have developed models for studying the effects of joints and faults on the distribution of stresses in a rock mass. A survey of the geomechanics literature reveals that generally there are four approaches to modelling discontinuities in rocks, namely interface (joint) elements, distinct elements, the discontinuous deformation method and, the displacement discontinuity method.

The use of interface or joint elements is a common approach to model joint behaviour. This approach is compatible with the finite element modelling of the nonlinear rock mass with joints. A large amount of literature is available on the development of rock joint models (Rossmanith, 1990).

In the distinct element formulation, it is assumed that the displacements due to discontinuities greatly exceed that of the intact rock blocks. Consequently, intact rock is regarded as rigid blocks and only the joint discontinuities may deform in the process of loading (Lorig et al, 1986).

In discontinuous deformation method (Shi and Goodman, 1989), it is assumed that the total deformation of the rock-joint system is the sum of the rock mass deformations and the joint movements. At present, this method does not permit complex nonlinear analysis of the rock blocks, and thus the application of this method is restricted to linear elastic materials. The discontinuous deformation method is essentially a finite element method with the difference that each rock block is treated as a discrete finite element. In comparison, the displacement discontinuity method is essentially a boundary integral technique in which the intact rock behaviour is assumed to be linear elastic (Crawford and Curran, 1983).

In the present study, the joints in an arch dam are modelled using interface elements. The reason is that the concrete in dam monoliths also need to be modeled as a nonlinear material due to possible cracking and over-stress, and none of the other methods mentioned above has the ability to account for such nonlinearity. Furthermore, it is recognized that the commonly used smeared crack approach for modelling brittle materials is not suitable for the modelling of the contraction joints. Because the joints in arch dams are well defined discontinuities, modelling them as interface elements will not cause the difficulties often encountered in discrete crack approach which necessitates mesh refinement and topology modifications when used in modelling brittle materials.

In the selection of a suitable constitutive model for contraction joints, the characteristics of the contraction joints should be taken into account. Firstly, a contraction joint is not a crack and therefore crack models developed for concrete generally do not apply to joints in dams. Furthermore, although the grouting material in the joints can provide some kind of roughness (i.e. asperity), this effect is minimal since the grouting material does not contain large size aggregates. It is generally not necessary to use such sophisticated models in which asperities are taken into account to model joint material in dams.

A constitutive model suitable for the joint elements in arch dams can be developed based on test results of two smooth concrete interfaces filled with a Portland Cement grout

as filler material. However, no such test or experimental data have been reported in the literature. Simple shear tests of concrete blocks will provide the necessary joint material model data for the present study. Unfortunately, experimental data of this kind are still very limited.

Recently Desai and Fishman (1991) have proposed a plasticity based model for rocks. The required parameters of their model are derived from a series of tests conducted on concrete specimens. The asperity effects are not explicitly included in their model. Results on two smooth faces with no asperity effect are reported. Unfortunately, there is no evidence available to ascertain the applicability of their model to joints in arch dams where the compressive stresses can be significantly higher than the range of stresses (350 kPa maximum) used in their studies.

The approach of modelling the shear keys in joints as asperities involves the problem of mixing a characteristic parameter in the joint model with the modelling of a structural element in the dam. Hohberg (1991) has shown this approach to be unsuccessful. In order to maintain the simplicity in the modelling of the joint elements, it is deemed more appropriate to model the effect of the shear key by limiting the relative displacement of the joint faces due to the presence of the shear key. In general a shear key that is not beveled prevents any slippage before the joint is fully opened, whereas for the case of beveled shear keys the two faces of the joint can have some restricted free shear movement when the joint opens as shown in Fig.3.1. The shear and bearing strength of the shear key may have significant effect on the performance of an arch dam.

3.2.2 2D Versus 3D Joint Elements

Since many of the previous studies on constitutive laws for joint elements have been carried out by geotechnical engineers and that many geotechnical problems are modeled as 2-D continua, the majority of the joint elements developed by these researchers are 2-D elements. In the present study, a two dimensional model implies that the shear displacement in the plane of the joint has a single component in the constitutive model. To date, there have been no reported 3-D laboratory tests on joints which include multiple shear displacements along the joint surface and their effect on the joint behaviour. This is important since in the 3-D problem of an arch dam, the tangential shear displacement in the joint plane has two orthogonal components. The magnitude and the direction of the

combined shear displacement continuously change throughout the response. One approach to solve the multi-directional shear displacement problem is to use the vector sum of the two components of the shear displacement and shear stress, and to express the slippage by a resultant vector. Loading and unloading can be represented simply by the change in the magnitude of the shear stress.

Gens et al (1990) developed a 3-D joint element using the vector sum of shear stresses and shear displacements. In another study, Hobbeg (1992) developed a 3-D constitutive model for joint elements. The ability of this model to take into account the anisotropy of the joint surfaces makes it useful in modelling rock joints.

3.2.3 Thin-Layer Versus Zero-Thickness Elements

In general, there are two approaches that can be followed in the modelling of the interface: (1) by zero thickness elements using the relative displacements of the two faces of the joint as the degrees of freedom; (2) by finite thickness elements, that is 3-D elements with very small thickness, and using strains as the modelling parameters in a constitutive formulation. It has been shown that as the thickness of the thin-layer joint element decreases, the converged results are essentially the same as those obtained from the so called zero-thickness element. Hence, if an interface occurs exactly at the junction of the bodies, though it is usually not the case in geomechanics, both formulations will lead to similar normal and shear stresses and displacements (Desai and Nagaraj, 1990). In the case of the discretization of a dam using a coarse mesh, it is generally considered sufficient to model the joints using the zero thickness element approach, because the relative displacements of the joint faces are still very small in comparison with the element size (Hobbeg, 1992).

In the finite thickness element approach, the rigidity matrix is expressed in terms of the Young's modulus and the Poisson's ratio of the interface material. The stresses are calculated from the strains and the rigidity matrix. In the zero thickness approach, the rigidity matrix is expressed in terms of the normal and tangential penalty parameters of the joints, k_n and k_s . The stresses are calculated from the rigidity matrix and the relative displacements, $(\mathbf{u}^{top} - \mathbf{u}^{bot})$, where $\mathbf{u}^{top}$ is the nodal displacements of one face of the element, and $\mathbf{u}^{bot}$ is the nodal displacements of the opposite face. In the penalty formulation, the magnitude of the penalty parameters must be chosen sufficiently large to guarantee that no penetration occurs and that the fully stick condition of the two faces

of the joints is maintained. However, the chosen values should not be too large in order to avoid ill conditioning in the overall stiffness matrix and to prevent the occurrence of very high frequency noises during dynamic response. These are two common difficulties encountered in the modelling of joint interface by joint elements. Hohberg (1992) gives a guideline on choosing the appropriate values for the penalty parameters. The presence of filler material in the joint can be taken into account by using a reduced normal penalty parameter k_n , provided the relative normal displacement of the joint is within a pre-defined limit (Hohberg, 1992). However, this practice is not recommended for dynamic analysis, because any abrupt change in joint stiffness may cause high frequency noises when the relative normal displacement of the joint exceeds the prescribed limit.

In this thesis, the contraction joints are modelled with zero-thickness joint elements. A brief explanation of the element formulation is presented in the following section.

3.2.4 The Zero-Thickness Joint Element

Fig.3.2 shows an eight node joint element. The element consists of two adjacent surfaces which are completely coincident with each other during the initial at rest condition. The kinematics of the element can be expressed in terms of the relative displacement of the two faces of the joint, $\mathbf{u} = \mathbf{u}^{top} - \mathbf{u}^{bot}$, where $\mathbf{u}^{top}$ and $\mathbf{u}^{bot}$ refer to the absolute displacements of the top and the bottom faces of the joints. The relative normal and tangential displacements between the surfaces of the joint produce internal resistant forces or stresses. The relationship between these relative displacements and the surface tractions can be expressed using an element constitutive law.

In the elastic state, this relationship can be expressed as follows

$$\{\sigma\} = [D_e] \{u\} \quad (3.1)$$

or

$$\begin{Bmatrix} \sigma \\ \tau_1 \\ \tau_2 \end{Bmatrix} = \begin{pmatrix} k_n & 0 & 0 \\ 0 & k_s & 0 \\ 0 & 0 & k_s \end{pmatrix} \begin{Bmatrix} v \\ u_1 \\ u_2 \end{Bmatrix} \quad (3.2)$$

where σ , τ_1 and τ_2 are respectively the normal and tangential (shear) stresses, and v , u_1

and u_2 are the relative normal and tangential displacements. Parameters k_n and k_s are the penalty parameters which guarantee the condition of no penetration of the two joint faces, and the condition of no slippage or stick condition. Parameters k_n and k_s have the units of force per volume.

In the penalty formulation, the relative normal and tangential displacements at any point on the surface of the joint are first calculated from the known nodal displacements of the joint element. The stresses at the point are then determined by Eq(3.2) using the appropriate penalty parameters for the considered point. The restoring force vector and the stiffness matrix of the element can also be subsequently determined.

3.3 The Proposed Constitutive Model

The following basic assumptions have been considered in the derivation of the constitutive model for the proposed joint element in this thesis:

- (1) The joint has negligible tensile strength.
- (2) The joint tangential stiffness will be reduced substantially (theoretically to zero) once the joint opening exceeds a pre-defined slip margin δ .
- (3) The normal stress and normal stiffness in an open joint are reduced to zero.
- (4) The joint behaves elastically in sliding when the joint is open, provided the shear stress is less than the shear strength or apparent cohesion of the joint.
- (5) In the open state, when the shear stress exceeds the shear strength limit of the joint, further sliding of the joint is resisted by a constant shear force resulting from its apparent cohesion.
- (6) There is no coupling between the normal and shear displacements when the joint is open. The normal stress remains zero.
- (7) When the joint is closed, the friction between the two faces of the joint becomes effective in resisting the sliding motion. The joint behaves elastically or elasto-plastically, with coupling between the shear and the normal displacements.
- (8) The joint faces are assumed to be isotropic.
- (9) The plasticity model for sliding is assumed to be of the Mohr-Coulomb type, which is a reasonable assumption for joint slippage (NRC, 1990).

3.3.1 Joint Behaviour in Open State

The behaviour of a joint is divided into two basic states, the open state and the closed state. For an open joint, that is when the normal relative displacement v is positive, a threshold slip margin δ is defined to simulate the effect of the shear keys. For joint elements of several meters in size, such as those typically employed in the modelling and discretization of dams, when an integration point indicates opening of the joint, it may not be reasonable to reduce the tangential penalty for the entire tributary area of the integration point. The amount of the reduction effected needs to be studied, and an appropriate slip margin needs to be established. Whenever the slip margin is exceeded, the tangential penalty should be reduced. The slip margin and the amount of the penalty reduction depend on whether the joint is keyed or un-keyed and whether the key is beveled or not.

The behaviour of the joint model in the open state as considered in the present study is shown in Fig.3.3. As shown in Fig.3.3 (a), the normal stiffness of the joint is reduced to zero when it is open and no stress is transferred between the two faces. As soon as the joint is closed with $v \leq 0.0$, the stiffness of the joint recovers. This opening and closing behaviour of the joint is modeled in the original computer program ADAP88.

It can be observed in Fig.3.3 (b) that the sliding behaviour is elastic when the shear stress is less than the limiting value c . For un-keyed joints, this c value is the assumed apparent cohesion, and is much smaller than that of a keyed joint. For a keyed joint, c represents the shear or bearing strength of the keys. Exceeding this limit, the joint slides with constant shear resistance and the unloading is fully elastic.

Depending on the magnitude of v in comparison with the slip margin δ , the magnitude of the tangential penalty k_s changes from a maximum value when $v \leq \delta$, to a much smaller value (theoretically zero) when v exceeds the limit δ in the fully open state.

In the present study, it is found that reducing k_s to zero in a fully open joint causes numerical problem in convergence, especially when the slip margin δ is small and close to zero. A similar numerical problem occurs if the apparent cohesion is assumed to be zero, which in reality implies no resistance to sliding when the joint is open.

The process of tangential loading is illustrated in Fig.3.4. The path 1-2-3-4-5 represents the case which the joint opening is less than the slip margin δ throughout the process of

loading and unloading. Path 1-2-3-4-6-7 represents the case in which the joint is opened at point 4 with the relative normal displacement v greater than δ , thus resulting in the sudden release of stress to point 6, and subsequently remaining fully open to point 7. At point 7 the amount of the joint opening has reduced to less than the slip margin, resulting in the recovery of the tangential penalty.

Hohberg (1992) utilized the concept of apparent cohesion to simulate the shear strength of the key. Cohesional resistance has been found to be effective in plane joints of concrete core samples during direct shear tests (NRC, 1990).

3.3.2 Joint Behaviour in Closed state

As mentioned before, the joint slippage in the closed state is resisted by friction. The two parameters which govern this behaviour are the friction angle and the apparent cohesion. In the present study, a three dimensional model based on the two dimensional Mohr-Coulomb failure criterion is developed to simulate the joint slippage behaviour in the closed state. A similar approach was used by (Hohberg 1992).

In the present formulation, the loading function in the contact stress space can be written as

$$f = \tau + \mu\sigma - c \quad (3.3)$$

where

$$\tau = \sqrt{\tau_1^2 + \tau_2^2} \quad (3.4)$$

and

$$\mu = \tan\phi \quad (3.5)$$

In Eq(3.4), τ_1 and τ_2 are the two components of the shear stress acting on the contact surface in two perpendicular directions, σ is the normal stress and is assumed to be positive in tension, ϕ is the friction angle and c is the apparent cohesion of the joint. The loading surface is a truncated cone along the σ axis. It corresponds to the Drucker-Prager loading surface which resembles a cone along the I_1 axis, as shown in Fig.3.5. Note that I_1 is the first

invariant of the stress tensor. Note also that the stresses in Eq(3.4) can be vectorially added because all stresses act at the same point and have the same tributary area. Substituting the above parameters into Eq(3.3) gives

$$f = \sqrt{(\tau_1^2 + \tau_2^2)} + \sigma \tan \phi - c \quad (3.6)$$

For a joint that behaves elastically, the incremental constitutive relation is

$$\{d\sigma\} = [D_e] \{du'\} \quad (3.7)$$

where

$$\{d\sigma\} = \begin{Bmatrix} d\sigma \\ d\tau_1 \\ d\tau_2 \end{Bmatrix} \quad (3.8)$$

$$[D_e] = \begin{pmatrix} k_n & 0 & 0 \\ 0 & k_s & 0 \\ 0 & 0 & k_s \end{pmatrix} \quad (3.9)$$

$$\{du'\} = \begin{Bmatrix} dv' \\ du'_1 \\ du'_2 \end{Bmatrix} \quad (3.10)$$

in which v is the normal component of the the relative displacement, and u_1, u_2 are the two tangential components of the relative displacement, that is

$$\{u\} = \begin{Bmatrix} v^{top} - v^{bot} \\ u_1^{top} - u_1^{bot} \\ u_2^{top} - u_2^{bot} \end{Bmatrix} \quad (3.11)$$

Subscripts 1 and 2 denote the displacement directions, whereas the superscripts (top) and (bot) refer to the top and bottom faces of the joint.

Once the joint exceeds its elastic limit, further sliding will follow with elasto-plastic behaviour. When the sheared-off element is in a state of sliding, the behaviour is treated as perfectly plastic and the constitutive equation may be written as

$$\begin{Bmatrix} d\sigma \\ d\tau_1 \\ d\tau_2 \end{Bmatrix} = [\mathbf{D}_{ep}] \begin{Bmatrix} dv \\ du_1 \\ du_2 \end{Bmatrix} = \begin{pmatrix} k_{n,n} & k_{n,s1} & k_{n,s2} \\ k_{s1,n} & k_{s1,s1} & k_{s1,s2} \\ k_{s2,n} & k_{s2,s1} & k_{s2,s2} \end{pmatrix} \begin{Bmatrix} dv \\ du_1 \\ du_2 \end{Bmatrix} \quad (3.12)$$

The elasto-plastic rigidity matrix $[\mathbf{D}_{ep}]$ is determined as follows. First the consistency condition is defined as

$$df = \frac{\partial f}{\partial \sigma} d\sigma + \frac{\partial f}{\partial \tau_1} d\tau_1 + \frac{\partial f}{\partial \tau_2} d\tau_2 = 0 \quad (3.13)$$

To calculate the stress increments, the following relationship from the theory of plasticity

$$\{d\mathbf{u}\} = \{d\mathbf{u}'\} + \{d\mathbf{u}^p\} \quad (3.14)$$

is inserted into Eq(3.7). In Eq(3.14), $\{d\mathbf{u}'\}$ is the elastic component of the displacement vector and $\{d\mathbf{u}^p\}$ the plastic component of the displacement vector. Separating the relative displacement into relative elastic and plastic components, the incremental constitutive relationship can be written as follows

$$\begin{Bmatrix} d\sigma \\ d\tau_1 \\ d\tau_2 \end{Bmatrix} = \begin{pmatrix} k_n & 0 & 0 \\ 0 & k_s & 0 \\ 0 & 0 & k_s \end{pmatrix} \begin{Bmatrix} dv - dv^p \\ du_1 - du_1^p \\ du_2 - du_2^p \end{Bmatrix} \quad (3.15)$$

Assuming $g(\sigma, \tau_1, \tau_2)$ to be the plastic potential function and using the flow rule, or the normality rule of plasticity,

$$\{d\mathbf{u}^p\} = \lambda \frac{\partial g}{\partial \{\sigma\}} \quad (3.16)$$

where λ is the proportionality factor. Eq(3.15) may now be written as

$$\begin{Bmatrix} d\sigma \\ d\tau_1 \\ d\tau_2 \end{Bmatrix} = \begin{pmatrix} k_n & 0 & 0 \\ 0 & k_s & 0 \\ 0 & 0 & k_s \end{pmatrix} \begin{Bmatrix} dv - dv^p \\ du_1 - du_1^p \\ du_2 - du_2^p \end{Bmatrix} = \begin{pmatrix} k_n & 0 & 0 \\ 0 & k_s & 0 \\ 0 & 0 & k_s \end{pmatrix} \begin{Bmatrix} dv - \lambda \frac{\partial g}{\partial \sigma} \\ du_1 - \lambda \frac{\partial g}{\partial \tau_1} \\ du_2 - \lambda \frac{\partial g}{\partial \tau_2} \end{Bmatrix} \quad (3.17)$$

Substituting from Eq(3.17) for $d\sigma, d\tau_1, d\tau_2$, Eq(3.13) becomes

$$k_n \frac{\partial f}{\partial \sigma} [dv - \lambda \frac{\partial g}{\partial \sigma}] + k_s \frac{\partial f}{\partial \tau_1} [du_1 - \lambda \frac{\partial g}{\partial \tau_1}] + k_s \frac{\partial f}{\partial \tau_2} [du_2 - \lambda \frac{\partial g}{\partial \tau_2}] = 0 \quad (3.18)$$

After some simple manipulations, λ can be expressed as follows

$$\lambda = \frac{k_n \frac{\partial f}{\partial \sigma} dv + k_s \frac{\partial f}{\partial \tau_1} du_1 + k_s \frac{\partial f}{\partial \tau_2} du_2}{k_n \frac{\partial f}{\partial \sigma} \frac{\partial g}{\partial \sigma} + k_s \frac{\partial f}{\partial \tau_1} \frac{\partial g}{\partial \tau_1} + k_s \frac{\partial f}{\partial \tau_2} \frac{\partial g}{\partial \tau_2}} \quad (3.19)$$

Now by denoting the denominator as

$$H = k_n \frac{\partial f}{\partial \sigma} \frac{\partial g}{\partial \sigma} + k_s \frac{\partial f}{\partial \tau_1} \frac{\partial g}{\partial \tau_1} + k_s \frac{\partial f}{\partial \tau_2} \frac{\partial g}{\partial \tau_2} \quad (3.20)$$

and substituting λ from Eq(3.19) into Eq(3.17), the following constitutive relationship can be obtained.

$$\begin{pmatrix} d\sigma \\ d\tau_1 \\ d\tau_2 \end{pmatrix} = \begin{pmatrix} k_n \left(1 - \frac{1}{H} k_n \frac{\partial f}{\partial \sigma} \frac{\partial g}{\partial \sigma}\right) & -k_s k_n \frac{1}{H} \frac{\partial f}{\partial \tau_1} \frac{\partial g}{\partial \sigma} & -k_s k_n \frac{1}{H} \frac{\partial f}{\partial \tau_2} \frac{\partial g}{\partial \sigma} \\ -k_s k_n \frac{1}{H} \frac{\partial f}{\partial \sigma} \frac{\partial g}{\partial \tau_1} & k_s \left(1 - \frac{1}{H} k_s \frac{\partial f}{\partial \tau_1} \frac{\partial g}{\partial \tau_1}\right) & -k_s^2 \frac{1}{H} \frac{\partial g}{\partial \tau_1} \frac{\partial f}{\partial \tau_2} \\ -k_s k_n \frac{1}{H} \frac{\partial f}{\partial \sigma} \frac{\partial g}{\partial \tau_2} & -k_s^2 \frac{1}{H} \frac{\partial g}{\partial \tau_2} \frac{\partial f}{\partial \tau_1} & k_s \left(1 - \frac{1}{H} k_s \frac{\partial f}{\partial \tau_2} \frac{\partial g}{\partial \tau_2}\right) \end{pmatrix} \begin{pmatrix} dv \\ du_1 \\ du_2 \end{pmatrix} \quad (3.21)$$

Observe that Eq(3.21) has the form of Eq(3.12).

Assuming the flow rule is associative, the functions f and g are the same. The associative flow rule gives symmetric elasto-plastic rigidity matrix $[D_{ep}]$. Using the following notations

$$\tan \beta = \frac{\tau_2}{\tau_1} \quad (3.22)$$

$$\tau_1 = \cos \beta \mid \tau \mid = \beta_1 \mid \tau \mid \quad (3.23)$$

$$\tau_2 = \sin \beta \mid \tau \mid = \beta_2 \mid \tau \mid \quad (3.24)$$

$$\frac{\partial f}{\partial \tau_1} = \frac{\tau_1}{\mid \tau \mid} = \beta_1 \quad (3.25)$$

$$\frac{\partial f}{\partial \tau_2} = \frac{\tau_2}{|\tau|} = \beta_2 \quad (3.26)$$

$$\frac{\partial f}{\partial \sigma} = \mu \quad (3.27)$$

a further simplified form of the elasto-plastic rigidity matrix $[\mathbf{D}_{ep}]$ can be obtained as shown below

$$[\mathbf{D}_{ep}] = \begin{pmatrix} k_n - \frac{1}{H} \mu^2 k_n^2 & -\frac{1}{H} k_n k_s \beta_1 \mu & -\frac{1}{H} k_n k_s \beta_2 \mu \\ -\frac{1}{H} k_n k_s \beta_1 \mu & k_s (1 - \frac{1}{H} k_s \beta_1^2) & -\frac{k_s^2}{H} \beta_1 \beta_2 \\ -\frac{1}{H} k_n k_s \beta_2 \mu & -\frac{k_s^2}{H} \beta_1 \beta_2 & k_s (1 - \frac{1}{H} k_s \beta_2^2) \end{pmatrix} \quad (3.28)$$

where H is a constant given by

$$H = k_n \mu^2 + k_s \beta_1^2 + k_s \beta_2^2 = k_n \mu^2 + k_s \quad (3.29)$$

The off-diagonal terms in matrix $[\mathbf{D}_{ep}]$ reflect the phenomenon of shear dilatancy.

In modelling the deformation behaviour of the joint in the closed state, it is necessary to establish the criteria for loading-unloading of the joint from a plastic state. This can be accomplished using plastic work as the criterion. When the increment of plastic work dw_p is non-negative, the joint is in a state of loading, whereas for the case of a negative increment of plastic work, the joint is unloading. For the joint element, the incremental plastic work is defined as

$$dw_p = \sigma dr^p + \tau_1 du_1^p + \tau_2 du_2^p \quad (3.30)$$

By considering Eq(3.16), Eq(3.30) may be written as follows

$$dw_p = \lambda \left(\sigma \frac{\partial g}{\partial \sigma} + \tau_1 \frac{\partial g}{\partial \tau_1} + \tau_2 \frac{\partial g}{\partial \tau_2} \right) \quad (3.31)$$

And after inserting the value of λ from Eq(3.19), this yields

$$dw_p = \frac{k_n \frac{\partial f}{\partial \sigma} dr + k_s \frac{\partial f}{\partial \tau_1} du_1 + k_s \frac{\partial f}{\partial \tau_2} du_2}{k_n \frac{\partial f}{\partial \sigma} \frac{\partial g}{\partial \sigma} + k_s \frac{\partial f}{\partial \tau_1} \frac{\partial g}{\partial \tau_1} + k_s \frac{\partial f}{\partial \tau_2} \frac{\partial g}{\partial \tau_2}} \left(\sigma \frac{\partial g}{\partial \sigma} + \tau_1 \frac{\partial g}{\partial \tau_1} + \tau_2 \frac{\partial g}{\partial \tau_2} \right) \quad (3.32)$$

If the flow rule is associative, Eq(3.32) becomes

$$dw_p = \frac{k_n \frac{\partial f}{\partial \sigma} dv + k_s \frac{\partial f}{\partial \tau_1} du_1 + k_s \frac{\partial f}{\partial \tau_2} du_2}{k_n \left(\frac{\partial f}{\partial \sigma}\right)^2 + k_s \left(\frac{\partial f}{\partial \tau_1}\right)^2 + k_s \left(\frac{\partial f}{\partial \tau_2}\right)^2} \left(\sigma \frac{\partial f}{\partial \sigma} + \tau_1 \frac{\partial f}{\partial \tau_1} + \tau_2 \frac{\partial f}{\partial \tau_2} \right) \quad (3.33)$$

In Eq(3.33), the denominator is always positive. Hence for a positive plastic work, the following condition must be satisfied.

$$\left(k_n \frac{\partial f}{\partial \sigma} dv + k_s \frac{\partial f}{\partial \tau_1} du_1 + k_s \frac{\partial f}{\partial \tau_2} du_2 \right) \left(\sigma \frac{\partial f}{\partial \sigma} + \tau_1 \frac{\partial f}{\partial \tau_1} + \tau_2 \frac{\partial f}{\partial \tau_2} \right) > 0 \quad (3.34)$$

Using Eqs(3.25) to (3.27), Eq(3.34) becomes

$$(k_n \mu dv + \beta_1 k_s du_1 + \beta_2 k_s du_2) (\sigma \mu + \tau_1 \beta_1 + \tau_2 \beta_2) > 0 \quad (3.35)$$

and since in the plastic state

$$\sigma \mu + \tau_1 \beta_1 + \tau_2 \beta_2 = c > 0 \quad (3.36)$$

the load criterion function, L , which corresponds to a positive plastic work, can be expressed as

$$L = k_n \mu dv + k_s \beta_1 du_1 + k_s \beta_2 du_2 > 0 \quad (3.37)$$

If the condition in Eq(3.37) is satisfied, the joint is in a loading state, otherwise, the joint is in the unloading state and behaves elastically.

3.4 Numerical Implementation of Plasticity Models

In a nonlinear iteration procedure, the stress state and the rigidity matrix are determined from the elasto-plastic constitutive model adopted for the material. The stress tensor $\{\sigma\}_{m+1}^i$ and the material rigidity matrix $[\mathbf{D}]_{m+1}^i$ corresponding to the displacement field $\{\mathbf{U}\}_{m+1}^i$ for the i^{th} iteration in the $(m+1)^{th}$ time step are first calculated. Subsequently the resisting load vector $\{\mathbf{F}\}_{m+1}^i$ and the stiffness of the element $[\mathbf{K}]_{m+1}^i$ are calculated.

In this section, the algorithm for the calculation of the stress tensor for a general elasto-plastic material is explained first. The application of this algorithm to the proposed joint element model is presented in the next section.

3.4.1 Incremental Stresses

Considering a general elasto-plastic material and the corresponding stress and strain tensors, $\{\sigma\}_m$ and $\{\epsilon\}_m$, the effective plastic strain, $(\epsilon_p)_m$ and the strain hardening parameter κ_m , are known at the end of time step m , and taken as the initial conditions for the iteration in time step $(m+1)$. For the i^{th} iteration in time step $(m+1)$, the strain vector can be computed from the displacement vector $\{U\}_{m+1}^i$ as follows

$$\{\epsilon\}_{m+1}^i = [B] \{U\}_{m+1}^i \quad (3.38)$$

where $[B]$ is the displacement-strain transformation matrix. The strain increment can then be calculated as follows

$$\{\Delta\epsilon\}_{m+1}^i = \{\epsilon\}_{m+1}^i - \{\epsilon\}_m \quad (3.39)$$

If the behaviour is elastic, the corresponding elastic stress increment can be determined by the following expression

$$\{\Delta\sigma'\}_{m+1}^i = [D_e] \{\Delta\epsilon\}_{m+1}^i \quad (3.40)$$

To account for the case of over-shoot of the loading surface when the material changes from elastic to plastic state from one time step to the next, a scale factor r is defined and utilized to bring the stress state back onto the loading surface. Assuming the stress state at time step m to be elastic, which satisfies $f(\{\sigma\}_m, \kappa_m) < 0$, and if by adding the elastic stress increment, the resultant stress state moves outside the loading surface for time step $(m+1)$, that is $f(\{\sigma\}_m + \{\Delta\sigma'\}_{m+1}^i, \kappa_m) > 0$, a scale factor r is defined such that $f(\{\sigma\}_m + r \{\Delta\sigma'\}_{m+1}^i, \kappa_m) = 0$.

Correspondingly, the strain vector can be divided into two parts. The first part $r \{\Delta\epsilon\}_{m+1}^i$, corresponds to pure elastic response and the remaining part $(1 - r) \{\Delta\epsilon\}_{m+1}^i$,

represents the elasto-plastic response. Accordingly, the stress increment can be obtained by integration as follows

$$\{\Delta\sigma\}_{m+1}^i = \int_{\{\epsilon\}_m}^{\{\epsilon\}_{m+1}^i} [D_e] (\{d\epsilon\} - \{d\epsilon^p\}) \quad (3.41)$$

or

$$\{\Delta\sigma\}_{m+1}^i = \int_{\{\epsilon\}_m}^{\{\epsilon\}_m + r\{\Delta\epsilon\}_{m+1}^i} [D_e] \{d\epsilon\} + \int_{\{\epsilon\}_m + r\{\Delta\epsilon\}_{m+1}^i}^{\{\epsilon\}_m + \{\Delta\epsilon\}_{m+1}^i} [D_e] (\{d\epsilon\} - \{d\epsilon^p\}) \quad (3.42)$$

or

$$\{\Delta\sigma\}_{m+1}^i = r\{\Delta\sigma'\}_{m+1}^i + \int_{\{\epsilon\}_m + r\{\Delta\epsilon\}_{m+1}^i}^{\{\epsilon\}_m + \{\Delta\epsilon\}_{m+1}^i} [D_e] (\{d\epsilon\} - \{d\epsilon^p\}) \quad (3.43)$$

and finally

$$\{\Delta\sigma\}_{m+1}^i = r\{\Delta\sigma'\}_{m+1}^i + \int_{\{\epsilon\}_m + r\{\Delta\epsilon\}_{m+1}^i}^{\{\epsilon\}_m + \{\Delta\epsilon\}_{m+1}^i} [D_{ep}] \{d\epsilon\} \quad (3.44)$$

The stress tensor corresponding to the displacement field $\{U\}_{m+1}^i$ is then calculated as follows

$$\{\sigma\}_{m+1}^i = \{\sigma\}_m + \{\Delta\sigma\}_{m+1}^i \quad (3.45)$$

In an elasto-plastic analysis, the stress state at all the integration points within a finite element should be checked to determine whether the material at each of the integration points is under elastic loading, plastic loading, or unloading. The elasto-plastic constitutive relation governs only in the case of plastic loading.

To illustrate the implementation of the constitutive model, it is assumed here that one of the integration points is in the elastic state at the end of time step m , that is $f(\{\sigma\}_m, \kappa_m) < 0$. If after adding the elastic stress increment the integration point remains in the elastic state, that is $f(\{\sigma\}_m + \{\Delta\sigma'\}_{m+1}^i, \kappa_m) \leq 0$, the resultant stress increment is then simply the same as the elastic stress increment

$$\{\Delta\sigma\}_{m+1}^i = \{\Delta\sigma'\}_{m+1}^i \quad (3.46)$$

On the other hand, if after adding the elastic stress increment, the stress point moves outside the loading surface, that is $f(\{\sigma\}_m + \{\Delta\sigma'\}_{m+1}^t, \kappa_m) > 0$, the material at that particular integration point is then under an elasto-plastic state. As explained earlier, a scale factor r is defined such that

$$f(\{\sigma\}_m + r\{\Delta\sigma'\}_{m+1}^t, \kappa_m) = 0 \quad (3.47)$$

Expanding Eq(3.47) in its Taylor series, and neglecting the second and higher order terms, Eq(3.47) becomes

$$f(\{\sigma\}_m + r\{\Delta\sigma'\}_{m+1}^t, \kappa_m) = f(\{\sigma\}_m, \kappa_m) + \left\{ \frac{\partial f(\{\sigma\}_m, \kappa_m)}{\partial \{\sigma\}} \right\}^T r\{\Delta\sigma'\}_{m+1}^t = 0 \quad (3.48)$$

This leads to

$$r = \frac{-f(\{\sigma\}_m, \kappa_m)}{\left\{ \frac{\partial f(\{\sigma\}_m, \kappa_m)}{\partial \{\sigma\}} \right\}^T \{\Delta\sigma'\}_{m+1}^t} \quad (3.49)$$

Finally, if the integration point is already in an elasto-plastic state at the end of the time step m , the load criterion function L can be used to determine the loading state (Chen and Hann 1988)

$$L = \left\{ \frac{\partial f}{\partial \{\sigma\}} \right\}^T [D,] \{\Delta\epsilon\} \quad (3.50)$$

If $L \leq 0$, the integration point is in neutral loading or elastic unloading, the point behaves elastically, that is

$$\{\Delta\sigma\}_{m+1}^t = \{\Delta\sigma'\}_{m+1}^t \quad (3.51)$$

But if $L > 0$, the integration point is in a plastic loading state, then Eq(3.44) should be used to calculate the stress state with the scale factor r being zero. In the following section, the evaluation of the integration in Eq(3.44) is presented.

3.4.2 Numerical Integration for Stress Calculation

To calculate the stress state, the integral in Eq(3.44) needs to be evaluated. The numerical procedure for this integration is explained in this section. The integration interval is subdivided into k segments or subintervals. Each of the sub-increments of the remaining part of the strain tensor is defined and calculated as follows

$$\{\Delta\tilde{\epsilon}\}_{m+1}^i = (1 - r) \frac{\{\Delta\epsilon\}_{m+1}^i}{k} \quad (3.52)$$

The stress increment corresponding to the j^{th} subinterval can then be calculated as

$$\{\Delta\sigma\}_{(m+1),j}^i = [D_e] \left(\{\Delta\tilde{\epsilon}\}_{(m+1)}^i - \{\Delta\tilde{\epsilon}^p\}_{(m+1)}^i \right), \quad (3.53)$$

where $\{\Delta\tilde{\epsilon}^p\}$ is the plastic portion of the strain sub-increment. The flow rule is used to calculate $\{\Delta\tilde{\epsilon}^p\}$ as

$$\{\Delta\tilde{\epsilon}^p\} = \lambda \left\{ \frac{\partial g}{\partial \{\sigma\}} \right\} = \frac{L}{H} \left\{ \frac{\partial g}{\partial \{\sigma\}} \right\} \quad (3.54)$$

where

$$H = \left\{ \frac{\partial f}{\partial \{\sigma\}} \right\}^T [D_e] \left\{ \frac{\partial g}{\partial \{\sigma\}} \right\} - n \frac{\partial f}{\partial \kappa} \quad (3.55)$$

with g as the plastic potential function and λ the proportionality factor (Chen and Hann 1988). The load criterion function L is defined in Eq(3.50), but here $\{\Delta\tilde{\epsilon}\}$ should be used instead of $\{\Delta\epsilon\}$. The quantity n is calculated as

$$n = \frac{d\kappa}{d\epsilon_p} [D_e] \sqrt{\left\{ \frac{\partial g}{\partial \{\sigma\}} \right\}^T \left\{ \frac{\partial g}{\partial \{\sigma\}} \right\}} \quad (3.56)$$

Using Eq(3.56) in Eq(3.54), the plastic strain sub-increment can be calculated by the following equation

$$\{\Delta\tilde{\epsilon}^p\}_{(m+1),j}^i = [P]_{(m+1),j}^i \{\Delta\tilde{\epsilon}\}_{m+1}^i \quad (3.57)$$

where $[P]$ is a function of the strain tensor $\{\epsilon\}$, the plastic strain tensor $\{\epsilon^p\}$ and the effective plastic strain ϵ_p , and can be expressed as follows

$$[P] = P(\{\epsilon\}, \{\epsilon^p\}, \epsilon_p) = \frac{1}{H} \left\{ \frac{\partial g}{\partial \{\sigma\}} \right\} \left\{ \frac{\partial f}{\partial \{\sigma\}} \right\}^T [D_e] \quad (3.58)$$

In the evaluation of Eq(3.44), the initial conditions for the numerical integration are

$$\{\epsilon\} = \{\epsilon\}_m + r \{\Delta\epsilon\}_{m+1}^i \quad (3.59)$$

$$\{\sigma\} = \{\sigma\}_m + r \{\Delta\sigma\}_{m+1}^i \quad (3.60)$$

$$\{\epsilon^p\} = \{\epsilon^p\}_m \quad (3.61)$$

$$\epsilon_p = (\epsilon_p)_m \quad (3.62)$$

recognizing that the first term on the right side of Eq(3.44) represents elastic behaviour. Subsequently, for each strain sub-increment, $\{\Delta\tilde{\epsilon}\}_{m+1}^i$, the following steps are followed:

(1) Determine the plastic strain sub-increment, $\{\Delta\tilde{\epsilon}^p\}_{(m+1),j}^i$ by Eq(3.57), and the effective plastic strain sub-increment, $(\Delta\tilde{\epsilon}_p)_{(m+1),j}^i$.

(2) Compute the stress sub-increment, $\{\Delta\tilde{\sigma}\}_{(m+1),j}^i$ as follows

$$\{\Delta\tilde{\sigma}\}_{(m+1),j}^i = [D_e] \left(\{\Delta\tilde{\epsilon}\}_{(m+1)}^i - \{\Delta\tilde{\epsilon}^p\}_{(m+1)}^i \right)_j \quad (3.63)$$

(3) Update the stresses, strains, and hardening parameters

$$\{\sigma\}_{(m+1),j}^i = \{\sigma\}_{(m+1),(j-1)}^i + \{\Delta\tilde{\sigma}\}_{(m+1),j}^i \quad (3.64)$$

$$\{\epsilon\}_{(m+1),j}^i = \{\epsilon\}_{(m+1),(j-1)}^i + \{\Delta\tilde{\epsilon}\}_{(m+1),j}^i \quad (3.65)$$

$$\{\epsilon^p\}_{(m+1),j}^i = \{\epsilon^p\}_{(m+1),(j-1)}^i + \{\Delta\tilde{\epsilon}^p\}_{(m+1),j}^i \quad (3.66)$$

$$(\epsilon_p)_{(m+1),j}^i = (\epsilon_p)_{(m+1),(j-1)}^i + (\Delta \tilde{\epsilon}_p)_{(m+1),j}^i \quad (3.67)$$

$$\kappa_{(m+1),j}^i = \kappa_{(m+1),(j)} (\epsilon_p)^i \quad (3.68)$$

In the process of plastic loading, the stress state should always remain on the loading surface, and the consistency condition $df = 0$ should be satisfied at all times. Due to numerical approximation, this condition may be violated. If that is the case, the final stress state should be brought back to the loading surface. This can be achieved by adding to the stress state a corrective stress increment which is normal to the loading surface. This corrective stress tensor is calculated as follows

$$\{\delta \sigma\} = a \left\{ \frac{\partial f}{\partial \{\sigma\}} \right\} \quad (3.69)$$

where the scalar factor a can be determined by expanding the function f in Taylor's series

$$a = \frac{-f}{\left\{ \frac{\partial f}{\partial \{\sigma\}} \right\}^T \left\{ \frac{\partial f}{\partial \{\sigma\}} \right\}} \quad (3.70)$$

And finally the corrected stress vector $\{\sigma\}_{cor.}$ is obtained as follows

$$\{\sigma\}_{cor.} = \{\sigma\} + a \left\{ \frac{\partial f}{\partial \{\sigma\}} \right\} \quad (3.71)$$

3.4.3 Algorithm for Stress Calculation

In this section, an algorithm is presented for the numerical implementation of the general plasticity model described earlier. An indicator IPEL is assigned to each integration point. When IPEL is zero, the point is in an elastic state, whereas if IPEL is 1, the point is in an elasto-plastic state. The numerical steps to calculate the stresses are summarized as follows

Step 1: Calculate the strain and the elastic stress increments, $\{\Delta \epsilon\}_{m+1}^i$ and $\{\Delta \sigma'\}_{m+1}^i$, respectively.

$$\{\Delta \epsilon\}_{m+1}^i = \{\epsilon\}_{m+1}^i - \{\epsilon\}_m^i \quad (3.72)$$

$$\{\Delta\sigma'\}_{m+1}^t = [D_e] \{\Delta\epsilon\}_{m+1}^t \quad (3.73)$$

Step 2: Depending on the stress state in the previous time step as indicated by the indicator IPEL

(a) If IPEL=1, which implies the integration point is in an elasto-plastic state, calculate the loading criterion L .

(a-1) If $L > 0$, set $r=0$ and go to step 3.

(a-2) If $L \leq 0$, set IPEL=0. The point is in unloading or neutral loading condition, therefore

$$\{\sigma\} = \{\sigma\}_m + \{\Delta\sigma'\}_{m+1}^t \quad (3.74)$$

Go to step 6.

(b) If IPEL=0, the integration point is in an elastic state, calculate the loading function f .

$$f = f\left(\{\sigma\}_m + \{\Delta\sigma'\}_{m+1}^t, \epsilon_p\right) \quad (3.75)$$

(b-1) If $f \leq 0$, the integration point remains in an elastic state.

$$\{\sigma\} = \{\sigma\}_m + \{\Delta\sigma'\}_{m+1}^t \quad (3.76)$$

Go to step 6.

(b-2) If $f > 0$, set IPEL=1. The point becomes elasto-plastic. Calculate the scale factor r such that

$$f\left(\{\sigma\}_m + r \{\Delta\sigma'\}_{m+1}^t, \epsilon_p\right) = 0 \quad (3.77)$$

then update the stresses

$$\{\sigma\}_{m+1}^t = \{\sigma\}_m + r \{\Delta\sigma'\}_{m+1}^t \quad (3.78)$$

Step 3: Calculate the strain sub-increment $\{\Delta\tilde{\epsilon}\}$.

$$\{\Delta\tilde{\epsilon}\} = \frac{(1-r)}{k} \{\Delta\epsilon\}_{m+1}^i \quad (3.79)$$

Step 4: Calculate the incremental stresses by following the numerical procedures discussed in section 3.4.2.

For each subinterval in the integration as discussed earlier, determine the plastic strain sub-increments $\{\Delta\tilde{\epsilon}^p\}_{(m+1),j}^i$, and the effective plastic strain $(\Delta\tilde{\epsilon}_p)_{(m+1),j}^i$ sub-increments for the subinterval j . Calculate the stress sub-increment.

$$\{\Delta\tilde{\sigma}\}_{(m+1),j}^i = [\mathbf{D}_e] \left(\{\Delta\tilde{\epsilon}\}_{(m+1)}^i - \{\Delta\tilde{\epsilon}^p\}_{(m+1)}^i \right)_j \quad (3.80)$$

$$\{\sigma\}_{(m+1),j}^i = \{\sigma\}_{(m+1),(j-1)}^i + \{\Delta\tilde{\sigma}\}_{(m+1),j}^i \quad (3.81)$$

$$(\epsilon_p)_{(m+1),j}^i = (\epsilon_p)_{(m+1),(j-1)}^i + (\Delta\tilde{\epsilon}_p)_{(m+1),j}^i \quad (3.82)$$

Repeat the above evaluations for k times (the number of subintervals for the integration).

Step 5: Based on the calculated stress tensor, calculate the loading function f . If $|f| > \epsilon_f$, where ϵ_f is a prescribed tolerance for yield condition, calculate the corrective scalar factor α of Eq(3.70), and

$$\{\sigma\}_{m+1(cor.)}^i = \{\sigma\}_{m+1}^i + \{\alpha\sigma\}_{m+1}^i \quad (3.83)$$

Step 6: Substitute the calculated stress state as the final stress state for the i^{th} iteration in time step $(m+1)$.

$$\{\sigma\}_{m+1}^i = \{\sigma\}_{m+1(cor.)}^i \quad (3.84)$$

Step 7: Based on the obtained stress state, calculate the elasto-plastic constitutive matrix, $[\mathbf{D}_{ep}]$ if required.

Step 8: Go to the next iteration.

3.5 Numerical Implementation of the Proposed Joint Model

The preceding algorithm explains the calculation of the stress tensor for a general elasto-plastic material. In this section, this algorithm is applied to the proposed joint element model.

3.5.1 Closed State

In the numerical implementation of the proposed zero thickness joint element, the relative displacements at the integration points, instead of the conventional strains, are used in the constitutive formulation. The strain tensor $\{\epsilon\}$ in the general plasticity formulation is thus replaced by the relative displacement vector $\{U\}$ in the present formulation. Also, the proposed joint model is not of the hardening type, so the general effective plastic strain ϵ_p , and the hardening parameter κ which appear in the general formulation are neglected in the following derivation. For easy comparison, the corresponding equation numbers in the general formulation are given in brackets. The algorithm for the proposed joint model is summarized as follows.

Step 1: Calculate the relative displacement increments, [Eq(3.72)]

$$(\Delta r)_{m+1}^i = (r)_{m+1}^i - (r)_m \quad (3.85)$$

$$(\Delta u_1)_{m+1}^i = (u_1)_{m+1}^i - (u_1)_m \quad (3.86)$$

$$(\Delta u_2)_{m+1}^i = (u_2)_{m+1}^i - (u_2)_m \quad (3.87)$$

Step 2: Calculate the elastic stress increments, [Eq(3.73)]

$$(\Delta \sigma')_{m+1}^i = k_n (\Delta r)_{m+1}^i \quad (3.88)$$

$$(\Delta \tau_1')_{m+1}^i = k_s (\Delta u_1)_{m+1}^i \quad (3.89)$$

$$(\Delta \tau_2')_{m+1}^i = k_s (\Delta u_2)_{m+1}^i \quad (3.90)$$

Step 3: Check the elasto-plastic state.

(a) If $IPEL=1$, which implies that the integration point is in an elasto-plastic state at time step m ,

(i) Check the loading criterion function, [Eq(3.50)]

$$L = \left\{ \frac{\partial f}{\partial \{\sigma\}} \right\}^T [\mathbf{D}_e] \{\Delta \mathbf{U}\} \quad (3.91)$$

i.e.

$$L = (\mu \quad \beta_1 \quad \beta_2)_m \begin{pmatrix} k_n & 0 & 0 \\ 0 & k_s & 0 \\ 0 & 0 & k_s \end{pmatrix} \begin{Bmatrix} \Delta v \\ \Delta u_1 \\ \Delta u_2 \end{Bmatrix}_{m+1}^t \quad (3.92)$$

or

$$L = (k_n \mu \quad k_s \beta_1 \quad k_s \beta_2)_m \begin{Bmatrix} \Delta v \\ \Delta u_1 \\ \Delta u_2 \end{Bmatrix}_{m+1}^t \quad (3.93)$$

which is further simplified to

$$L = k_n \mu (\Delta v)_{m+1}^t + k_s \beta_1 (\Delta u_1)_{m+1}^t + k_s \beta_2 (\Delta u_2)_{m+1}^t \quad (3.94)$$

which is the same definition for L as obtained from the plastic work criterion presented in Eq(3.37).

(ii) If $L > 0$, the point is in a loading state. Set IPEL=1 and $r=0$. Go to step 4.

(iii) If $L \leq 0$, it implies that the point is in an unloading state. Set IPEL=0, and calculate the current stress state [Eq(3.74)]

$$(\sigma)_{m+1}^t = (\sigma)_m + (\Delta \sigma')_{m+1}^t \quad (3.95)$$

$$(\tau_1)_{m+1}^t = (\tau_1)_m + (\Delta \tau_1')_{m+1}^t \quad (3.96)$$

$$(\tau_2)_{m+1}^t = (\tau_2)_m + (\Delta \tau_2')_{m+1}^t \quad (3.97)$$

then go to step 11.

(b) If IPEL=0, which implies that the integration point is in an elastic state at time step m,

(i) Calculate the current stress state, assuming full elasticity

$$(\sigma)_{m+1}^i = (\sigma)_m + (\Delta\sigma')_{m+1}^i \quad (3.98)$$

$$(\tau_1)_{m+1}^i = (\tau_1)_m + (\Delta\tau_1')_{m+1}^i \quad (3.99)$$

$$(\tau_2)_{m+1}^i = (\tau_2)_m + (\Delta\tau_2')_{m+1}^i \quad (3.100)$$

(ii) Calculate the value of the loading function f , [Eq(3.75)]

$$f = \sigma_{m+1}^i \mu + \tau_{m+1}^i - c \quad (3.101)$$

where

$$\tau_{m+1}^i = \sqrt{(\tau_1^i)_{m+1}^2 + (\tau_2^i)_{m+1}^2} \quad (3.102)$$

(iii) If $f < 0$, which implies that the integration point remains in the elastic state, the stress state is determined as follows, [Eq(3.76)]

$$(\sigma)_{m+1}^i = (\sigma)_m + (\Delta\sigma')_{m+1}^i \quad (3.103)$$

$$(\tau_1)_{m+1}^i = (\tau_1)_m + (\Delta\tau_1')_{m+1}^i \quad (3.104)$$

$$(\tau_2)_{m+1}^i = (\tau_2)_m + (\Delta\tau_2')_{m+1}^i \quad (3.105)$$

Next, go to step 11.

(iv) If $f \geq 0$, it implies that the stress point is in an elasto-plastic state during this iteration. Set $\text{IPEL} = 1$, and calculate the scale factor r as follows, [Eq(3.49)]

$$r = \frac{-(\sigma)_m \mu + \sqrt{(\tau_1^i)_m^2 + (\tau_2^i)_m^2} - c}{(\mu, \beta_1, \beta_2)_m \left\{ \begin{array}{c} \Delta\sigma' \\ \Delta\tau_1' \\ \Delta\tau_2' \end{array} \right\}_{m+1}^i} \quad (3.106)$$

or

$$r = \frac{-(\sigma)_m \mu + \sqrt{(\tau_1^2)_m + (\tau_2^2)_m - c}}{(\mu(\Delta\sigma^e)_{m+1}^i + (\beta_1)_m(\Delta\tau_1^e)_{m+1}^i + (\beta_2)_m(\Delta\tau_2^e)_{m+1}^i)} \quad (3.107)$$

Step 4: Add the elastic stress contribution to the current stress state, [Eq(3.78)]

$$(\sigma)_{m+1}^i = (\sigma)_m + r(\Delta\sigma^e)_{m+1}^i \quad (3.108)$$

$$(\tau_1)_{m+1}^i = (\tau_1)_m + r(\Delta\tau_1^e)_{m+1}^i \quad (3.109)$$

$$(\tau_2)_{m+1}^i = (\tau_2)_m + r(\Delta\tau_2^e)_{m+1}^i \quad (3.110)$$

Step 5: Calculate the corresponding strain sub-increment, [Eq(3.79)]

$$(\Delta\tilde{v})_{m+1}^i = \frac{1-r}{k}(\Delta v)_{m+1}^i \quad (3.111)$$

$$(\Delta\tilde{u}_1)_{m+1}^i = \frac{1-r}{k}(\Delta u_1)_{m+1}^i \quad (3.112)$$

$$(\Delta\tilde{u}_2)_{m+1}^i = \frac{1-r}{k}(\Delta u_2)_{m+1}^i \quad (3.113)$$

Steps 6 to 8 should be repeated k times for the number of subintervals considered.

Step 6: To calculate the [P] matrix for the subinterval j, the parameter H is calculated first, [Eq(3.55)]

$$H = (\mu \quad (\beta_1) \quad (\beta_2))_{(m+1),(j-1)}^i \begin{pmatrix} k_n & 0 & 0 \\ 0 & k_s & 0 \\ 0 & 0 & k_s \end{pmatrix} \left\{ \begin{matrix} \mu \\ (\beta_1) \\ (\beta_2) \end{matrix} \right\}_{(m+1),(j-1)}^i \quad (3.114)$$

or

$$H = (\mu k_n \quad k_s(\beta_1) \quad k_s(\beta_2))_{(m+1),(j-1)}^i \left\{ \begin{matrix} \mu \\ (\beta_1) \\ (\beta_2) \end{matrix} \right\}_{(m+1),(j-1)}^i \quad (3.115)$$

which leads to

$$H = \left(k_n \mu^2 + k_s (\beta_1)^2 + k_s (\beta_2)^2 \right)_{(m+1),(j-1)}^t = k_n \mu^2 + k_s \quad (3.116)$$

The matrix $[\mathbf{P}]$ is then calculated [Eq(3.58)] as

$$[\mathbf{P}]_{(m+1),j}^t = \frac{1}{k_n \mu^2 + k_s} \left\{ \begin{matrix} \mu \\ \beta_1 \\ \beta_2 \end{matrix} \right\}_{(m+1),(j-1)}^t (\mu \quad \beta_1 \quad \beta_2)_{(m+1),(j-1)}^t \begin{pmatrix} k_n & 0 & 0 \\ 0 & k_s & 0 \\ 0 & 0 & k_s \end{pmatrix} \quad (3.117)$$

$$[\mathbf{P}]_{(m+1),j}^t = \frac{1}{(k_s + k_n \mu^2)} \begin{pmatrix} k_n \mu^2 & k_s \mu \beta_1 & k_s \mu \beta_2 \\ k_n \mu \beta_1 & k_s (\beta_1)^2 & k_s \beta_1 \beta_2 \\ k_n \mu \beta_2 & k_s \beta_1 \beta_2 & k_s (\beta_2)^2 \end{pmatrix}_{(m+1),(j-1)}^t \quad (3.118)$$

Step 7: Calculate the plastic strain sub-increments [Eq(3.57)], where

$$\{\Delta \tilde{\mathbf{U}}^p\}_{(m+1),j}^t = [\mathbf{P}]_{(m+1),j}^t \{\Delta \tilde{\mathbf{U}}\}_{m+1}^t \quad (3.119)$$

and

$$\left\{ \begin{matrix} \Delta \tilde{e}^p \\ \Delta \tilde{u}_1^p \\ \Delta \tilde{u}_2^p \end{matrix} \right\}_{(m+1),j}^t = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{33} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}_{(m+1),j}^t \left\{ \begin{matrix} \Delta \tilde{e} \\ \Delta \tilde{u}_1 \\ \Delta \tilde{u}_2 \end{matrix} \right\}_{(m+1),j}^t \quad (3.120)$$

Step 8: Calculate the elastic stress sub-increments, [Eq(3.63)]

$$(\Delta \tilde{\sigma}^e)_{(m+1),j}^t = k_n (\Delta \tilde{e} - \Delta \tilde{e}^p)_{(m+1),j}^t \quad (3.121)$$

$$(\Delta \tilde{\tau}_1^e)_{(m+1),j}^t = k_s (\Delta \tilde{u}_1 - \Delta \tilde{u}_1^p)_{(m+1),j}^t \quad (3.122)$$

$$(\Delta \tilde{\tau}_2^e)_{(m+1),j}^t = k_s (\Delta \tilde{u}_2 - \Delta \tilde{u}_2^p)_{(m+1),j}^t \quad (3.123)$$

Step 9: Calculate the new stress state, [Eq(3.64)]

$$\sigma_{(m+1)}^t = \sigma_{(m+1)}^t + \sum_{j=1}^k (\Delta \tilde{\sigma}^e)_{(m+1),j}^t \quad (3.124)$$

$$\tau_{1(m+1)}^i = \tau_{1(m+1)}^i + \sum_{j=1}^k (\Delta \bar{\tau}_1^c)_{(m+1),j}^i \quad (3.125)$$

$$\tau_{2(m+1)}^i = \tau_{2(m+1)}^i + \sum_{j=1}^k (\Delta \bar{\tau}_2^c)_{(m+1),j}^i \quad (3.126)$$

Step 10: Check whether the stress departure from the loading surface is within the accepted tolerance or not.

$$f_1 = \sigma_{m+1}^i \mu + \tau_{m+1}^i - c \quad (3.127)$$

where

$$\tau_{m+1}^i = \sqrt{(\tau_1^2)_{m+1}^i + (\tau_2^2)_{m+1}^i} \quad (3.128)$$

If $|\frac{f_1}{c}| < \epsilon_f$, the calculated stress is acceptable. Go to step 11; if not, calculate the corrective scalar factor a , [Eq(3.70)] as follows

$$a = \frac{-f_{m+1}^i}{(\mu^2 + 1)} = \frac{-1}{(\mu^2 + 1)} \left(\mu \sigma + \sqrt{(\tau_1^2 + \tau_2^2)} - c \right)_{m+1}^i \quad (3.129)$$

Add the correction vector, [Eq(3.71)]

$$\begin{Bmatrix} \sigma \\ \tau_1 \\ \tau_2 \end{Bmatrix}_{m+1(cor.)}^i = \begin{Bmatrix} \sigma \\ \tau_1 \\ \tau_2 \end{Bmatrix}_{m+1}^i + a \begin{Bmatrix} \mu \\ \beta_1 \\ \beta_2 \end{Bmatrix}_{m+1}^i \quad (3.130)$$

Step 11: The initial elastic rigidity matrix of the joint is used in the stiffness calculation, following the initial stiffness approach in the Newton-Raphson method, that is

$$[D] = [D_e] \quad (3.131)$$

It was found that this approach results in a more stabilized convergence pattern of the numerical solution than the tangential stiffness approach. However, it should be emphasized here that the stress state of the points are exactly calculated and used in the calculation of the load vector, to satisfy the equilibrium.

Step 12: Go to the next iteration.

3.5.2 Open State

When it is determined that the joint is open at the integration point, that is when $v_{m+1}^i > 0$, the following algorithm is followed.

Step 1: Set the normal stress σ_{m+1}^i equal to zero.

Step 2: If v_{m+1}^i is greater than the slip margin δ , then set

$$(\tau_1)_{m+1}^i = 0 \quad (3.132)$$

$$(\tau_2)_{m+1}^i = 0 \quad (3.133)$$

Go to step 4.

Step 3: But if v_{m+1}^i is less than δ , calculate the tangential displacement increments

$$(\Delta u_1)_{m+1}^i = (u_1)_{m+1}^i - (u_1)_m \quad (3.134)$$

$$(\Delta u_2)_{m+1}^i = (u_2)_{m+1}^i - (u_2)_m \quad (3.135)$$

Calculate the current tangential stresses, assuming full elasticity

$$(\tau_1)_{m+1}^i = (\tau_1)_m + (\Delta \tau_1)_{m+1}^i \quad (3.136)$$

$$(\tau_2)_{m+1}^i = (\tau_2)_m + (\Delta \tau_2)_{m+1}^i \quad (3.137)$$

Next calculate the resultant tangential stress

$$\tau_{m+1}^i = \sqrt{(\tau_1^i)_{m+1}^2 + (\tau_2^i)_{m+1}^2} \quad (3.138)$$

If $\tau_{m+1}^i \geq c$, the shear key is yielding, modify the tangential stresses

$$\tau_1 = \frac{\tau_{1m+1}^i}{\tau_{m+1}^i} c \quad (3.139)$$

$$\tau_2 = \frac{\tau_{2m+1}^i}{\tau_{m+1}^i} c \quad (3.140)$$

Set $IPEL=1$ and go to step 4.

If $\tau_{m+1}^i < c$, the shear key is in an elastic state, no modification of the stresses is needed, set $IPEL=0$.

Step 4: Use the following rigidity matrix to form the stiffness matrix of the element.

$$[D] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & k_s & 0 \\ 0 & 0 & k_s \end{pmatrix} \quad (3.141)$$

Changing the tangential penalties would cause numerical problems and result in nonconvergence and oscillation of the results. For this reason, the elastic tangential penalties are used for stiffness calculations. However as mentioned earlier, the stresses are calculated based on inelastic behaviour and equilibrium is satisfied.

Step 5: Go to the next iteration.

3.6 Stiffness Matrix and Resisting Load Vector

Fig.3.6 shows an eight node joint element. The geometry of the joint surface can be expressed in terms of the nodal coordinates

$$\mathbf{x} = \sum N_j \mathbf{x}_j \quad (3.142)$$

where the vector $\mathbf{x}$ is the global coordinate of a point on the joint surface, vector $\mathbf{x}_j$ is the coordinates of the nodes and N_j is the linear shape function

$$N_j = \frac{1}{4} (1 + r_j r) (1 + s_j s) \quad j = 1, 2, 3, 4 \quad (3.143)$$

in which r_j and s_j are the coordinates of the node j .

Three orthogonal axes with the unit vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ are defined at every point on the joint surface such that $\mathbf{e}_1$ is perpendicular to the surface and $\mathbf{e}_2, \mathbf{e}_3$ are tangential to the surface. The relative displacement at any point in the joint is calculated as

$$\mathbf{U} = \mathbf{U}^{top} - \mathbf{U}^{bot} \quad (3.144)$$

where

$$\mathbf{U}^{top} = \mathbf{a}\mathbf{u}^{top} \quad (3.145)$$

$$\mathbf{U}^{bot} = \mathbf{a}\mathbf{u}^{bot} \quad (3.146)$$

and

$$\mathbf{a} = [\mathbf{e}_1 \quad \mathbf{e}_2 \quad \mathbf{e}_3]^T \quad (3.147)$$

The displacement field on the bottom and top surfaces can be calculated as follows

$$\mathbf{u}^{bot} = N_1\mathbf{u}_1 + N_2\mathbf{u}_2 + N_3\mathbf{u}_3 + N_4\mathbf{u}_4 \quad (3.148)$$

$$\mathbf{u}^{top} = N_5\mathbf{u}_5 + N_6\mathbf{u}_6 + N_7\mathbf{u}_7 + N_8\mathbf{u}_8 \quad (3.149)$$

Substituting Eqs(3.145)to (3.149) into Eq(3.141), the relationship between the relative displacements $\mathbf{U}$ and the nodal displacements $\mathbf{u}$ are established as follows

$$\mathbf{U} = \mathbf{B}\mathbf{u} \quad (3.150)$$

where

$$\mathbf{u} = [\mathbf{u}_1^T \quad \mathbf{u}_2^T \quad \mathbf{u}_3^T \quad \mathbf{u}_4^T \quad \mathbf{u}_5^T \quad \mathbf{u}_6^T \quad \mathbf{u}_7^T \quad \mathbf{u}_8^T]^T \quad (3.151)$$

$$\mathbf{B} = [-N_1 \quad -N_2 \quad -N_3 \quad -N_4 \quad -N_5 \quad -N_6 \quad -N_7 \quad -N_8] \quad (3.152)$$

and,

$$\mathbf{N}_j = N_j\mathbf{a} \quad (3.153)$$

The resisting load vector $\{\mathbf{F}\}$ and the stiffness matrix $[\mathbf{K}]$ are calculated as follows

$$\mathbf{F} = \sum \mathbf{B}^T(r_l, s_l) \sigma_l | \mathbf{J}(r_l, s_l) | w_l \quad (3.154)$$

$$\mathbf{K} = \sum \mathbf{B}^T(r_l, s_l) [\mathbf{D}]_l \mathbf{B}^T(r_l, s_l) | \mathbf{J}(r_l, s_l) | w_l \quad (3.155)$$

where w_l is the weight factor at (r_l, s_l) , and $| \mathbf{J}(r_l, s_l) |$ is the determinant of the Jacobian of the transformation matrix, and the summation is carried out for all the integration points.

3.7 Verification of Numerical Implementation

A program has been written in which the the joint element constitutive law is implemented. Several strain (displacement)-stress paths have been simulated, and some of them are presented in this section.

In Fig.3.7 a typical normal stress-shear stress curve simulated by the implemented joint element model is presented. The early stage of elastic behaviour is represented by path A-B-C' while the plastic flow is simulated by path C'-D-E. Path E-F represents the behaviour when the joint is open.

Fig.3.8 (a) shows the simulated shear displacement-shear stress behaviour. Path A-B-C' represents the elasto-plastic regime in the closed state. Point C' is where the joint has opened. The shear strength in the open state is the apparent cohesion i.e. 20 kPa, represented by path D-E. At point E the joint opening has exceeded the slip margin and as a result there is no shear strength and the shear stress drops to point F.

Fig.3.8 (b) shows the simulated normal displacement-normal stress behaviour. Path A-B-C'-D represents the elasto-plastic behaviour. Although after point C' the joint begins to open, the normal stress still increases due to dilatancy effects. Point D is the critical point after which the joint has fully opened and the normal stress drops to zero. The joint remains open and the ultimate opening is represented by point E. The joint re-closes at point A, and behaves elasto-plastically up to point F.

Unkeyed

Un-beveled key

Beveled key

Fig.3.1: Contraction joints in arch dams

Fig.3.2: The zero thickness joint element

(a)

(b)

Fig.3.3 (a): Joint Normal stress-normal displacement relation
(b): Tangential stress-tangential displacement relation

Fig.3.4: Effect of joint opening on tangential behaviour

Fig.3.5 (a): The joint element loading surface
 (b): The Drucker-Prager loading surface

a) Local and global coordinates

b) Natural coordinate

Fig.3.6: 8 node joint element (After Fenves et al 1989)

Fig.3.7: Normal stress-tangential stress relation, simulated by the joint element model

Fig.3.8 (a): Tangential displacement-shear stress relation
 (b): Normal displacement-normal stress relation
 (simulated by the joint element model)

Chapter 4

Non-linear Behaviour of Concrete

4.1 General

In this chapter, first the available constitutive models for the triaxial behaviour of concrete are briefly surveyed. An important aspect of the behaviour is the tensile cracking which needs to be properly modelled. Based on the literature survey, a model will be chosen which can capture the most important aspects of the behaviour of concrete and which can **efficiently** be implemented in a finite element scheme. The required procedure for implementing the selected model in a finite element program is developed.

The behaviour of concrete under increasing loads can be very complicated. Phenomena such as inelastic strains accumulation due to slippage and crack development, stiffness degradation, shear compaction-dilatancy, strain rate dependancy of strength and stiffness, stiffening effects at the early stages of unloading, hysteretic energy dissipation, hydrostatic pressure sensitivity, post-crack behaviour and shear-normal displacement coupling on crack surfaces are some of the observed characteristics of concrete. Modelling all those characteristics by a single mathematical model is very difficult, if not impossible. Researchers in the field of constitutive modelling have proposed a large number of models, but none of them are complete in the sense that they can handle all the preceding aspects of the behaviour.

Our concern in the following literature survey is to introduce those models which can handle multiaxial stress states and the cyclic behaviour. A model which can cover the tensile behaviour (before and after failure) is also required for a complete analysis of structures.

4.2 Constitutive Models for Concrete

There are several approaches to modelling the complicated stress-strain behaviour of concrete under multiaxial states of stress. These approaches can be divided into the following main groups:

- Linear or nonlinear elasticity theories.
- Perfect or work-hardening plasticity theories.

- Endochronic theory of plasticity.
- Plastic fracturing models.
- Elastic progressive fracturing models.
- Bounding surface models.
- Plastic damage and continuum damage models.

Using an equivalent stress-strain relation, assuming concrete as incrementally elastic and evaluating strain increments in each principal direction based on only the corresponding stress increments can not measure the volumetric dilatancy near maximum stress level, and also the hydrostatic pressure dependency of concrete behaviour. This method is basically called "the equivalent stress method" (Chen and Ting, 1980).

In general two different approaches are employed in the formulation of nonlinear elasticity based constitutive laws. These are: (1) finite (or total) material behaviour characterization in variable secant-modulus form, known as hyperelastic formulation, (2) incremental or differential material description of the hypoelastic type using variable tangent modulus. Hyperelastic formulations model the behaviour as a load-path independent and reversible process with no memory. That is the material response at any stress state is just function of that stress and not the load history. These models can simulate the behaviour of concrete in proportional loading with reasonable accuracy, but they fail to predict the inelastic deformations when the material experiences unloading. The hypoelastic models can trace the behaviour as a path-dependent, irreversible process with limited memory. The response at any instant is a function of the current strain and the history of strain up to that point. These models are mostly suitable for monotonic loading and they fail to represent the path-dependent behaviour and cyclic loading properly (Buyukozturk and Shareef, 1985).

Theory of plasticity which was initially developed for metals has also been used to study the response of concrete. The justification for its use is based on the observation that concrete under very high levels of hydrostatic pressure shows a ductile behaviour before fracturing. Linear or nonlinear elasticity theories have been adopted to represent the behaviour before yield. A generalization of perfect plasticity, i.e. work-hardening plasticity, has been used to represent the evolution of the yield surface due to accumulation of inelastic strains. These models are able to predict the inelastic deformations and dilatancy.

In addition, hydrostatic pressure sensitivity can be modelled by these theories, but they are unable to model the damage accumulation which results in stiffness degradation, and hysteretic energy dissipation within load cycles (Buyukozturk and Shareef, 1985).

Endochronic models were also initially developed for metals. In these models the inelastic strains are not obtained from the loading surface as in plasticity theories, but from the evolution of irreversible damage, namely "intrinsic time", which increases as deformation takes place. The adaptation of this theory to concrete was performed by Bazant and Shieh (1980). The model agrees well with the experimental results, but it involves many functions and parameters which need to be computed by complicated curve fitting procedures. Besides the model is incrementally nonlinear, making its formulation very time consuming in finite element application. (Fardis et al, 1983).

Similar to the endochronic models, plastic-fracturing models are quite powerful and can simulate several important aspects of concrete behaviour with reasonable accuracy (Bazant and Kim, 1979). Among these aspects are the inelastic dilatancy, strain softening after peak and cyclic loading. Unfortunately, the latter theory requires many functions and parameters which need to be determined from several test results for a given concrete mix and subsequent curve fitting procedures. However, the finite element implementation of these models are much easier than the endochronic models since these models are incrementally linear. In the writer's opinion, a suitable model should represent the basic aspects of the behaviour of concrete and should contain as few easily determinable input parameters as possible.

Using fracture surface in strain space, the elastic-progressive fracturing models were developed by Dougill et al (1977). In these models stress increments are decomposed into elastic increments and fracture decrements. The elastic stress-strain relations are obtained using generalized Hook's law and the fracture stress decrements are determined using a fracture surface in strain space with a similar process as calculating plastic strains from loading surface in the theory of plasticity. Unloading in this theory is purely elastic with no energy dissipation in cyclic loading.

The bounding surface models have been used by different researchers to study three-dimensional cyclic behaviour of concrete. This model was initially developed to fill the gap in plasticity theories for modelling cyclic behaviour. Its simplicity, together with its ability to

model the basic characteristics of material behaviour, has made this theory very attractive for implementation in finite element codes. Because of the latter attributes the writer believes that the bounding surface is the most promising approach, and thus a bounding surface model is used in the finite element program BANDAAB. The adopted model is described in the next section.

Damage mechanics, which has been used extensively to model the behaviour of concrete, is based on the real phenomenon of evolution of damage in the material due to load induced strains. This theory can capture important phenomena like stiffness degradation and strain softening after peak, but is still in the development stage and its applications to cyclic loading awaits further investigation.

4.2.1 Bounding Surface Theory

This theory was originally introduced by Dafalias and Popov (1976), and independently by Krieg (1975), to overcome the inability of classical plasticity theory to model the loading-unloading behaviour of materials. The bounding surface is a surface in stress space enclosing the current stress point. For any stress point there is an image point on this surface which is determined by a mapping rule. The bounding surface may expand or shrink to model the hardening or softening behaviour. The plastic modulus which relates the stress increments to plastic strain increments is a function of the distance between the stress point and its image. Plastic deformation can occur at any stress state within the surface. If an elastic nucleus is defined inside the bounding surface, the stress within this elastic surface corresponds to purely elastic strains.

This theory was motivated by the results of extensive tests on different materials. These tests showed that the stress-plastic strain curves converge to specific bounds (lines), and the rate of change of stress with respect to the plastic strain rate, that is the plastic modulus H^p , is a function of both the distance δ^* of the current stress state and the distance δ_m^* of the stress state at the initiation of loading from the bounds, as shown in Fig.4.1. The quantity H^p is an absolutely increasing function of δ^* , starting from H_0^p , corresponding to $\delta^* = 0$, and reaching infinity in the limit when $\delta^* = \delta_m^*$ (Dafalias and Popov, 1976). The distance δ_m^* changes at each stress reversal, and is therefore associated with the most recent loading-unloading case.

The line segment AB in Fig.4.1 represents the elastic region which is within the larger

segment $\overline{A\overline{B}}$, defined by the two bounds. Lines AB and $\overline{A\overline{B}}$ will move and deform in the course of loading, and the magnitude of $A\overline{A}$ is influenced by their movements. Points A and B correspond to the boundaries of the yield surface while $\overline{A}$ and $\overline{B}$ belong to the bounding surface which encloses the yield surface in a general stress space. The two surfaces may translate and deform; they may also come into contact but never intersect, as illustrated by the asymptotic convergence of stress-plastic strain curve in uniaxial stress case.

Proper formulation for the translation and the deformation of the surfaces should be defined for different materials. Also a proper mapping rule, which assigns a mapped point on the bounding surface to the current stress point, should be specified. It is noteworthy to mention that in the multiaxial stress case, the change in the value of δ_m^* is not necessarily from unloading-reloading, but δ_m^* may also change due to drastic changes in the loading direction (Dafalias and Popov, 1976).

4.2.2 Bounding Surface Models

Several researchers have developed constitutive models for concrete based on this theory. Fardis et al (1983) defined their bounding surface model as a function of the stress state and of the maximum principal compressive strain. In their model stress-induced anisotropy was neglected. The bounding surface shrunk in the stress space as a function of the maximum principal strain experienced by the material. In this model, the strains were assumed to be totally plastic and no elastic strains were considered. The model was not able to simulate the softening part of the material behaviour. The assumption of no elastic strain is not in agreement with the real behaviour of concrete, consequently their proposed model has limited capabilities. Also, the model is incrementally non-linear and this is its major deficiency for finite element applications.

A bounding surface model was used by Buyukozturk and Chen (1985) to show the rate independent multiaxial behaviour of concrete. In this model a damage concept was used to characterize the progressive stiffness degradation and to capture the nonlinear material behaviour. In their model the strain is composed of elastic and plastic components. The overall assessment of the damage is based on the evolution of plastic volumetric strain and the plastic octahedral shear strain, which results in a phenomenological approach. It is noteworthy to mention that many aspects of the model are similar to the model developed by Fardis et al (1983).

Meyer and Delgado-Saavedra (1985) used the concept of bounding surface to develop an elasto-viscoplastic material model for plain concrete under arbitrary triaxial stress states and different strain rates. However, they state, "In its present form the model is computationally very expensive and is limited to relatively high strain rates."

Suaris et al (1990) developed a damage model for monotonic and cyclic behaviour of concrete. The model recognizes the tortuous nature of cracks in compression. The damage growth is derived in a way similar to the bounding surface method. In this model a limit fracture surface (which defines the onset of damage), a loading surface, and a bounding surface are defined, where it is assumed that the damage growth will occur whenever the stress state is outside the limiting fracture surface. This model applies to the cases where the principal stress and principal strain axes coincide and do not rotate as the material deforms. The damage model is based on the assumption that the material is perfectly brittle and no permanent strains are predicted when unloaded, an assumption which is not very realistic.

The latest model by Pagnoni et al (1992) is a modification and a more complete form of the work by Buyukozturk and Chen (1985). In their work, Pagnoni et al modified the latter model based on new experimental data, and they added new features for modeling cracking and post cracking behavior. The model seems to be complete compared to the other previously mentioned models and can be used to study the multiaxial behaviour of concrete with reasonable accuracy. However, the energy dissipation in load cycles is underestimated, and the model may generate energy under triaxial cyclic loading conditions. Implementation of the model in finite element codes is reported to be very efficient.

4.3 Selected Bounding Surface Model

In this thesis the bounding surface model proposed by Pagnoni et al (1992) is chosen for implementation as the nonlinear constitutive model in program BANDAAB. Although the model has certain deficiencies as described earlier, nevertheless, considering its overall accuracy and efficiency, it is better than most other models. This model was first developed by Chen and Buyukozturk (1985) and was subsequently improved based on new data.

4.3.1 The Model Definition

As mentioned earlier, concrete undergoes continuous damage in the course of loading. It has been shown that the strain tensor can be a suitable measure of the accumulated damage in concrete. In this model damage is assessed based on the evolution of plastic strain, that is the plastic volumetric strain ϵ_v^p and the plastic octahedral shear strain γ_o^p . The bounding surface is linked with the concept of irreversible damage and damage is accounted for by contracting the bounding surface with increased damage. The damage along the hydrostatic axis is modeled explicitly by the degradation of the tangent bulk modulus during loading. On the deviatoric plane, the damage accumulation is characterized by a parameter K , which traces the amount of damage.

For the deviatoric loading-unloading in the pre-failure range, increments of the damage parameter ΔK are given by

$$\Delta K = \frac{R \Delta D}{H^p F_1(I_1, \theta)} \quad (1.1)$$

whereas for the post-failure range

$$\Delta K = \frac{\Delta \gamma_o^p}{F_1(I_1, \theta)} \quad (1.2)$$

In the above equations, D is the normalized distance of the stress point from the hydrostatic axis in the deviatoric plane as shown in Fig.4.2, which can be expressed in terms of J_2 , the second invariant of the normalized deviatoric stress tensor, H^p is the plastic shear modulus, I_1 is the first invariant of the normalized stress tensor, (positive if compressive), θ is the angle between the projection of the position vector of the principal stress and that of any tensile semi-axis on the deviatoric plane, and γ_o^p is the normalized plastic octahedral shear strain. All stress quantities are normalized with respect to the uniaxial compressive strength of concrete and all strain components are normalized with respect to the strain corresponding to that stress. Stresses and strains are positive when compressive.

It may be noted that this is a pure phenomenological representation of damage. The damage parameter at any instant of time is calculated as:

$$K = \int_{\text{load path}} dK \quad (4.3)$$

But for reasons to be explained later, it is preferred here to name the maximum value of K , K_{max} , the damage parameter.

Function $F_1(I_1, \theta)$ accounts for the dependency of the behaviour on the hydrostatic pressure and stress path dependency on the deviatoric plane. This function is chosen to render $K = 1$ at failure.

Based on experimental observations, the following relationships were proposed by Pagnoni et al (1992).

(i) **For deviatoric loading**

For $I_1 \leq 2$

$$F_1(I_1, \theta) = \frac{0.248I_1^2 + 0.182I_1}{G(\cos 3\theta)} \quad (4.4)$$

For $I_1 > 2$

$$F_1(I_1, \theta) = \frac{1.867I_1^2 - 2.379}{G(\cos 3\theta)} \quad (4.5)$$

(ii) **For deviatoric unloading**

$$F_1(I_1, \theta) = 1.4 \left[0.85 - \frac{I_1 + 0.3}{I_{1,max} + 0.3} \right] \frac{F_{1,max}}{G(\cos 3\theta)} \quad (4.6)$$

where $I_{1,max}$ and $F_{1,max}$ are respectively the maximum values of I_1 and F_1 before the most recent unloading. The function $G(\cos 3\theta)$ was also obtained from test results and curve fitting. It was determined to be

$$G(\cos 3\theta) = 0.25(\cos 3\theta + 5) \quad (4.7)$$

The bounding surface, which is the innermost surface enclosing all the possible stress points for a given damage level, should be a function of the stress and the maximum value of damage K_{max} .

$$F(\sigma_{ij}, K_{max}) = 0 \quad (4.8)$$

Based on regression analysis of the test results of the failure surface, the following relationship was defined for the bounding surface.

$$F(\sigma_{ij}, K_{max}) = \frac{0.25J_2 + 3.10\sqrt{J_2}}{1I_1 + 3.48}(\cos 3\theta + 5) - \frac{40}{K_{max}^2 + 39} \quad (4.9)$$

As K_{max} approaches 1, the above formulation reduces to the expression for the failure surface corresponding to monotonic loading.

The basic assumption in bounding surface theory is the dependency of material moduli on the distance of the stress point from the bounding surface. In the model this distance is measured on the deviatoric plane and along the deviatoric stress (S_{ij}) direction. The normalized distance D is introduced as

$$D = \frac{r}{R} \quad (4.10)$$

where the distances r and R are shown in Fig.4.2. The material is assumed to have failed when $D=1$. Loading-unloading are defined as

- 1) Along the hydrostatic axis, loading is defined when $\Delta I_1 \geq 0$, and unloading when $\Delta I_1 < 0$.
- 2) In the deviatoric plane, loading is defined when $\Delta D \geq 0$ and unloading when $\Delta D < 0$.

4.3.2 Constitutive Law

The strains in this model may be written as

$$de_{ij} = de_{ij}^e + e_{ij} \frac{de_{kk}}{3} \quad (4.11)$$

or

$$de_{ij} = de_{ij}^e + de_{ij}^p + e_{ij} \frac{de_{kk}}{3} \quad (4.12)$$

where dc_{ij} , $d\epsilon_{ij}$, and $d\epsilon_{kk}$ are respectively the increments of the total strain, the deviatoric strain, and the hydrostatic strain, δ_{ij} is the Kronecker delta, and the superscripts e and p denote elastic and plastic conditions.

Assuming the following proposed proportionality relation, (Buyukozturk and Chen 1985)

$$\frac{dc_{ij}^p}{S_{ij}} = \frac{d\gamma_o^p}{\tau_o} \quad (4.13)$$

Eq(4.12) can be written as follows

$$dc_{ij} = \frac{1}{H^e} dS_{ij} + S_{ij} \frac{d\gamma_o^p}{\tau_o} + \frac{\delta_{ij}}{3} (d\epsilon_{kk}^d + d\epsilon_{kk}^o) \quad (4.14)$$

where S_{ij} is the deviatoric stress tensor, τ_o is the octahedral shear stress, and H^e is the generalized shear modulus. The hydrostatic strain is partly due to hydrostatic stress and partly due to octahedral shear strain, and ϵ_{kk}^d is that part of the hydrostatic strain caused by the hydrostatic stress while ϵ_{kk}^o is the part caused by the octahedral shear strain. Accordingly

$$dc_{ij} = \frac{1}{H^e} (d\sigma_{ij} - \delta_{ij} \frac{d\sigma_{kk}}{3}) + S_{ij} \frac{d\gamma_o^p}{\tau_o} + \frac{\delta_{ij}}{3} (\frac{dI_1}{3K_t} + \rho d\gamma_o^p) \quad (4.15)$$

or

$$dc_{ij} = \frac{d\sigma_{ij}}{H^e} + d\gamma_o^p (\frac{S_{ij}}{\tau_o} + \frac{\rho}{3} \delta_{ij}) + \delta_{ij} (\frac{1}{9K_t} - \frac{1}{3H^e}) d\sigma_{kk} \quad (4.16)$$

in which ρ is the shear dilatancy-compaction factor, and K_t is the bulk modulus.

Using the stress-strain relationship

$$d\gamma_o^p = \frac{d\tau_o}{H^p} \quad (4.17)$$

where $d\tau_o$ is the octahedral shear stress increment, and expanding the differential of τ_o as

$$d\tau_o = \frac{\partial \tau_o}{\partial \sigma_{ij}} d\sigma_{ij} \quad (4.18)$$

and knowing that

$$\tau_o = \sqrt{\frac{1}{3}(S_{ij}S_{ij})} \quad (4.19)$$

the final constitutive relationship becomes

$$d\epsilon_{ij} = \frac{d\sigma_{ij}}{H'} + \frac{1}{3H^p} \frac{1}{\tau_o} \left(\frac{S_{ij}}{\tau_o} + \frac{\rho}{3} \delta_{ij} \right) S_{km} d\sigma_{km} + \delta_{ij} \left(\frac{1}{9K_t} - \frac{1}{3H'} \right) d\sigma_{kk} \quad (4.20)$$

One should note that in Eq(4.20) the engineering shear strains $\gamma_{12}, \gamma_{13}, \gamma_{23}$ are calculated and not the conventional shear strains. To write the constitutive relationship in the conventional form

$$d\epsilon_{ij} = C_{ijkl} d\sigma_{kl} \quad (4.21)$$

and after further calculations, the compliance tensor C_{ijkl} , for concrete can be written as

$$C_{ijkl} = \frac{1}{H'} \delta_{ik} \delta_{jl} + \frac{1}{3H^p} \frac{1}{\tau_o} \left(\frac{S_{ij}}{\tau_o} + \frac{\delta_{ij}}{3} \rho \right) S_{kl} + \left(\frac{1}{9K_t} - \frac{1}{3H'} \right) \delta_{ik} \delta_{jl} \quad (4.22)$$

Note that C_{ijkl} is not a symmetric matrix due to the coupling between the deviatoric stresses and the volumetric strains. The parameters used in this model are H', H^p, K_t and ρ . These parameters can be determined from available experimental data.

4.3.3 Model Parameters

The material parameters of the bounding surface model are defined differently for the pre failure and post failure states.

The generalized elastic shear modulus, H' , is taken as twice the initial shear modulus at the beginning of the loading process.

$$H' = 2G^* \quad (4.23)$$

where it may be noted that the rigidity constants are dimensionless. The above relation is used for both the deviatoric loading and the unloading processes. The value of G , for a concrete with Poisson's ratio of $\nu = 0.17$ would be (ACI, 318-83)

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PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

PRECISIONSM RESOLUTION TARGETS

$$G = \frac{E}{2(1+\nu)} = \frac{149600\sqrt{f'_c}}{2.34} \approx 63930\sqrt{f'_c} \quad (1.24)$$

which when normalized with respect to concrete uniaxial compressive strength f'_c and its corresponding strain and ϵ_0 , results in

$$G^* = 63930 \frac{\epsilon_0}{\sqrt{f'_c}} \quad (1.25)$$

where f'_c and G have the unit of kPa.

Generalized plastic shear modulus, H^* , should be related to the accumulated damage and the distance between the current stress point, and its image on the bounding surface. A unique relationship between ΔD and ΔK is adopted by Buyukozturk and Chen (1985),

$$\Delta K = \frac{\Delta D}{H^*} \quad (1.26)$$

and

$$H^p = \frac{R}{F_1} H^* \quad (1.27)$$

where for deviatoric loading H^* is defined as

$$H^* = \frac{2.4(1-D)^{0.65D^2}}{(1+0.7K_{max}^2)A_L} \quad (1.28)$$

with A_L given by

$$A_L = \begin{cases} 1.02 - 0.81 \frac{K_R}{K_{max}} & \text{if } K > K_{max} \\ 1 & \text{if } K \leq K_{max} \end{cases} \quad (1.29)$$

In which K_R is the associated value of K at the beginning of the most recent loading process. Note that F_1 is obtained from Eqs.(1.4) or (1.5). For the unloading case

$$H^* = \frac{2.4}{(1+0.7K_{max}^2)A_u} \quad (1.30)$$

where

$$A_u = \begin{cases} 0 & \text{if } K < 0.20K_{max} \\ \frac{K_u - 0.2K_{max}}{0.8K_{max}} & \text{if } K \geq 0.20K_{max} \end{cases} \quad (4.31)$$

In which K_u is the associated value of K at the beginning of the most recent unloading process.

For the softening branch, H^p should be calculated based on the assumption that after failure the stress point moves on the failure surface. Hence

$$dF = \frac{\partial F}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial F}{\partial K_{max}} dK_{max} = 0 \quad (4.32)$$

but the model assumes a simpler formulation for H^p when concrete undergoes softening. Specifically,

$$H^p = -0.15e^{-0.025(K_{max}-1)^2} F_2(I_{1,max}) \quad (4.33)$$

where, for $I_{1,max} \leq 2.54$

$$F_2(I_{1,max}) = \frac{0.14}{(I_{1,max})^2 - 0.86} \quad (4.34)$$

and for $I_{1,max} > 2.54$

$$F_2(I_{1,max}) = 0.025 \quad (4.35)$$

It is generally accepted that softening and stiffness degradation are both related to damage accumulation, which results in the deterioration of the elastic properties of the material, whereas in this formulation the softening is modeled through changing the plastic shear modulus. On the other hand, the value of K , as defined by Eq(4.3) is not an absolutely increasing function but its maximum value, K_{max} , is always recorded and remembered by the material.

Another important material parameter is the tangent bulk modulus, K_t . Experiments have shown (Buyukozturk and Chen 1985) two stages in K_t variation. The material first

exhibits a decrease in the value of K_t with increase in I_1 , and then a stiffening behaviour occurs with further increase in I_1 . The model considers a relationship which fits the softening part for the entire range of the behaviour.

For hydrostatic loading

$$K_t = \frac{75560}{1 + 0.358I_1^{1.5}} \frac{e_0}{\sqrt{f'_c}} \quad (4.36)$$

while for hydrostatic unloading

$$K_t = 75560 \frac{e_0}{\sqrt{f'_c}} \quad (4.37)$$

For the post-failure range

$$G^* = 50 \quad (4.38)$$

and

$$K_t = 75560 \left(\frac{e_0}{\sqrt{f'_c}} \right) \quad (4.39)$$

It should be mentioned that K_t and G^* are normalized quantities.

Next the shear dilatancy-compaction factor ρ is considered. The relationship between deviatoric deformation and volumetric strain is expressed through the factor ρ . This relationship shows two stages of behaviour, compaction and dilation. Compaction results from crushing and void reduction. Experimental results indicate that compaction occurs during the process of loading, but not during unloading/reloading unless the damage is further increased, that is when K is greater than K_{max} (Buyukozturk and Chen 1985). Shear dilatancy occurs during the loading, unloading and reloading process. To account for these events, ρ is calculated as follows

$$\rho = \rho_1 + \rho_2 \quad (4.40)$$

where

$$\rho_1 = \begin{cases} 1.1e^{-90(K_{max} - 0.6)^2} & \text{for } K = K_{max} \\ 0 & \text{for } K < K_{max} \end{cases} \quad (4.41)$$

and

$$\rho_2 = -1.97 \lambda e^{-2\lambda^2} \quad (4.42)$$

where

$$\lambda = D - 0.2K_{max}^2 \quad (4.43)$$

In the post-peak range

$$\rho = -1.97 \lambda e^{(-2\lambda^2)} \quad (4.44)$$

and

$$\lambda = 1 - 0.2K_{max}^2 \quad (4.45)$$

Note that e is the natural logarithm base. The foregoing expressions complete the numerical description of the bounding surface constitutive model adopted herein. The next step involves the implementation of the model in the finite element code.

4.3.4 Numerical Implementation

Implementing a history-dependent constitutive model in a stress analysis program requires an incremental solution procedure. Because the model is nonlinear, iterations within each step are necessary to reach to a converged solution. Such an iteration scheme is called global iteration.

A subroutine called NUMINT was written to establish the required relationship between stress increments $\{\Delta \sigma\}$ and strain increments $\{\Delta \epsilon\}$ at all the integration points and for the iteration within a load step. As usual the stresses and strains are related by the rigidity matrix $[D]$.

It is noteworthy to mention that in the course of numerical implementation of the model in the finite element program, it was found that the rigidity matrix of the material for the post peak range was unreasonably large. Although the model is theoretically capable of simulating the descending branch of the stress-strain curve, due to the above mentioned observed behaviour, its use in the post peak range in the finite element program was abandoned. Based on the implemented version of the model, as the stress state reaches the failure surface, the material loses its rigidity and can no longer carry any stress.

The main program provides an estimate of the strain tensor $\{\epsilon\}_{m+1}^i$ for iteration i in time step $(m+1)$ as follows

$$\{\epsilon\}_{m+1}^i = [\mathbf{B}]\{\mathbf{u}\}_{m+1}^i \quad (1.16)$$

where $[\mathbf{B}]$ is the displacement strain transformation matrix and $\{\mathbf{u}\}$ is the displacement vector. The strain increment $\{\Delta\epsilon\}$ is calculated by

$$\{\Delta\epsilon\}_{m+1}^i = \{\epsilon\}_{m+1}^i - \{\epsilon\}_m \quad (1.17)$$

For any integration point a flag named FAILURE is specified. If FAILURE=1, indicating that the point has failed, the rigidity matrix $[\mathbf{D}]$ and the stress tensor $\{\sigma\}$ are nullified, and the algorithm continues as will be indicated later in this section. Otherwise the following is adopted.

For a general bounding surface formulation, for each iteration step and for each integration point a sub-stepping technique with a local iteration strategy should be carried out. The sub-stepping technique and local iteration generally increase the rate of convergence and reduce the number of global iterations. However, for the selected model, one sub step has proven to be sufficient (Pagnoni et al, 1992). In the following, the sub stepping concept is explained (Herrmann et al, 1987).

The desired relationship between stress and strain increments can be obtained from the following equation

$$\int_{t_m}^{t_{m+1}} \{d\sigma\} dt = \int_{t_m}^{t_{m+1}} [\mathbf{D}]\{d\epsilon\} dt \quad (1.18)$$

In the above formula the concept of real time is used, in spite of the assumption of rate-independency of the formulation. The strain rate is defined as

$$\{d\epsilon\} = \frac{\{\Delta\epsilon\}}{\Delta t_{m+1}} \quad (4.49)$$

By substituting Eq(4.49) into Eq(4.48) and assuming constant strain rate over the interval, one arrives at

$$\{\Delta\sigma\} = \frac{\{\Delta\epsilon\}}{\Delta t_{m+1}} \int_{t_m}^{t_{m+1}} [\mathbf{D}] dt \quad (4.50)$$

Defining

$$[\bar{\mathbf{D}}] = \frac{1}{\Delta t_{m+1}} \int_{t_m}^{t_{m+1}} [\mathbf{D}] dt \quad (4.51)$$

it becomes

$$\{\Delta\sigma\}_{m+1}^t = [\bar{\mathbf{D}}] \{\Delta\epsilon\}_{m+1}^t \quad (4.52)$$

Thus a good estimate of the average value of $[\mathbf{D}]$ over the time step is needed. To calculate $[\bar{\mathbf{D}}]$ numerically, the current time step is further subdivide into N sub-steps. This results in the evaluation of the integral in Eq(4.51) as follows

$$\int_{t_m}^{t_{m+1}} [\mathbf{D}] dt = \Delta t_n \sum_{n=1}^N [\bar{\mathbf{D}}]_n \quad (4.53)$$

where

$$[\bar{\mathbf{D}}]_n \approx \frac{1}{2} \{[\mathbf{D}]_{n-1} + [\mathbf{D}]_n\} \quad (4.54)$$

The use of sub-steps of equal size, that is

$$\Delta t_n = \frac{\Delta t_{m+1}}{N} \quad (4.55)$$

results in

$$t_{n-1} = t_m + (n-1)\Delta t_n \quad (1.56)$$

$$t_n = t_{n-1} + \Delta t_n \quad (1.57)$$

which in turn leads to

$$\{\epsilon\}_{(m+1),n}^i = \{\epsilon\}_m^i + \frac{n}{N} \{\Delta\epsilon\}_{m+1}^i \quad (1.58)$$

and

$$[\bar{\mathbf{D}}]_{m+1}^i = \frac{1}{2N} \sum_{n=1}^N \{[\mathbf{D}]_{n-1} + [\mathbf{D}]_n\}_{m+1}^i \quad (1.59)$$

The stress estimate for the sub-step n is evaluated by

$$\{\sigma\}_{(m+1),n}^i = \{\sigma\}_m^i + \sum_{j=1}^{n-1} \{\Delta\sigma\}_j^i + \{\Delta\sigma\}_{n-1}^i \quad (1.60)$$

This implies that the initial estimate of stress sub-increment $\{\Delta\sigma\}_n$ is the one calculated in the last sub-step. This estimate is further improved through local iteration, using $\{\Delta\epsilon\}_n$ and $[\bar{\mathbf{D}}]_n$.

If only one sub-step is used, then

$$\{\Delta\sigma\}_{m+1}^i = \frac{1}{2}([\mathbf{D}]_m + [\mathbf{D}]_{m+1}^i) \{\Delta\epsilon\}_{m+1}^i \quad (1.61)$$

where for the first local iteration $[\mathbf{D}]_{m+1}^i$ is a function of

$$\{\epsilon\}_{m+1}^i = \{\epsilon\}_m^i + \{\Delta\epsilon\}_{m+1}^i \quad (1.62)$$

and

$$\{\sigma\}_{m+1}^i = \{\sigma\}_m^i + [\mathbf{D}]_{m+1}^{i-1} \{\Delta\epsilon\}_{m+1}^i \quad (1.63)$$

After calculating an estimate of $\{\sigma\}_{m+1}^i$, the following steps are taken.

Step 1: Calculate I_1 , ΔI_1 , S_{jk} , J_2 , J_3 , θ , τ_0 , $\Delta\tau_0$ and $\Delta\gamma_0^p$, where J_3 is the third invariant of the deviatoric stress tensor.

Step 2: Calculate R , the distance of the image point on the bounding surface from the hydrostatic axis, on the deviatoric plane.

Step 3: Calculate r , the distance of the current stress point from the hydrostatic axis on the deviatoric plane.

Step 4: Calculate the normalized distance D .

Step 5: If $D \geq 0$ the point has failed. Set flag FAILURE equal to one, to indicate that the point has failed. Go to step 16.

Step 6: Calculate

$$(\Delta D)_{m+1}^t = D_{m+1}^t - D_m \quad (4.64)$$

Step 7: Calculate function G from Eq(4.7).

Step 8: Based on the value of ΔD , the point is either in a state of deviatoric loading or unloading. Calculate F_1 appropriately.

Step 9: Based on the value of ΔI_1 , the point is either in the state of hydrostatic loading or unloading. Calculate K_t appropriately.

Step 10: Calculate the plastic shear modulus, H^p .

Step 11: Calculate the shear dilatancy-compaction parameter, ρ .

Step 12: Calculate the damage parameter increment, ΔK , and its total value K .

Step 13: Calculate the components of the compliance tensor, $[C_{ijkl}]$.

Step 14: Calculate the components of the compliance matrix, $[C]$ from C_{ijkl} using the following equations

$$C_{11} = C_{1111} \quad (4.65)$$

$$C_{12} = C_{1122} \quad (4.66)$$

$$C_{13} = C_{1133} \quad (1.67)$$

$$C_{14} = (C_{1112} + C_{1121}) \quad (1.68)$$

$$C_{15} = (C_{1123} + C_{1132}) \quad (1.69)$$

$$C_{16} = (C_{1113} + C_{1131}) \quad (1.70)$$

where the remaining terms may be similarly expressed.

Step 15: Calculate the rigidity matrix $[\mathbf{D}]_{m+1}^i$ by inverting the compliance matrix $[\mathbf{C}]$. Go to step 17.

Step 16: If FAILURE=1, Set $[\mathbf{D}]$ equal to a null matrix.

Step 17: If the local iteration is terminated; go to step 18; and if not, calculate the modified stress increment $\{\Delta\sigma\}_{m+1}^i$ from Eq(1.63). Go to the next local iteration, that is step 1.

Step 18: Calculate the stress state by Eq(1.61). Express the stress tensor and the rigidity matrix $\{\sigma\}_{m+1}^i$ and $[\mathbf{D}]_{m+1}^i$ in non-normalized form.

Step 19: Calculate the resisting load vector of the element by Eqs(1.71).

$$\mathbf{F}_{m+1}^i = \sum_{l=1}^8 [\mathbf{B}]^T(r, s, k)_l \{\sigma\}_{m+1}^i + \mathbf{J}(r, s, k)_l \mid w(r, s, k)_l \quad (1.71)$$

where r, s, k are the coordinates of the integration point, $[\mathbf{J}]$ is the Jacobian matrix and w is the weight factor.

Step 20: Go to the next iteration.

4.3.5 Verification of Numerical Implementation

In this section the results of the model prediction for several stress paths will be presented. It should be emphasized that the purpose here is not so much to verify the degree of precision of the model, but the accuracy of its numerical implementation. The reader

can refer to Buyukozturk and Chen (1985) and Pagnoni et al (1992) to obtain further information about the degree of precision of the model response.

The predictions based on the current implementation are shown in Figs.4.3 to 4.16. Attempts are made to verify the predictions with some well-documented test results. However, in some cases some basic experimental data that are needed in the current model are not available, hence the expected behaviour of the model is presented, without comparison with actual test results.

Figs.4.3 to 4.5 compare the predicted response of concrete under uniaxial loading to its corresponding experimental response. The material properties are shown in the figures. The overall response of the model is satisfactory. It is noteworthy that the Young's modulus for all the predictions is calculated using the relationship introduced in Eq(4.24). The same degree of agreement may be observed in Figs.4.6 to 4.9, where the model predictions are compared with the results of biaxial compression tests. The comparison in this case is acceptable despite some noticeable differences between the experimental and predicted results. In Fig.4.10 the predictions for a triaxial compression loading are presented. Both the axial and the lateral strains are predicted well. The value of the Poisson's ratio in the predictions was assumed to be 0.20. The same material is checked under triaxial extension. Although the trend of the experimental stress strain curve (Figs.4.11 and 4.12) is predicted, the strains, especially the axial strains, are not predicted very well.

In Fig.4.13 the results of model prediction for uniaxial cyclic loading is presented. As one can see the hysteretic energy dissipation is well predicted by the model. In Fig.4.14 the model predictions for a uniaxial cyclic loading is presented in which a full unloading to zero stress does not occur. The prediction shows typical trend of the behaviour reported in Pagnoni et al (1992). Underestimation of the hysteretic energy dissipation is reported here as well. Significant inelastic strain is noted after the loading path crosses the unloading branch. Fig. 4.15 shows the results of the model prediction for biaxial cyclic loading. The specimen is first held under constant lateral stress corresponding to a strain of $0.08 \epsilon_0$, in uniaxial loading. It is then subjected to a cyclic axial loading by varying the stress in one direction. The overall trend shows stiffness degradation and pronounced nonlinearity after the crossing point of unloading-reloading paths is predicted by the model. The hysteretic energy dissipation is however underestimated. The prediction is similar to the one reported in Pagnoni et al (1992), although in the tests with which the predictions were compared in

this reference, the lateral strain remains unchanged.

Fig.4.16 shows the model prediction for a triaxial cyclic loading. The material was first loaded triaxially up to $\sigma_k=9.8$ MPa and then cyclically loaded uniaxially. Energy dissipation is underestimated, but the overall behaviour is in agreement with the results presented in Pagnoni et al (1992).

The preceding simulations illustrate that the models is capable of reproducing the basic response of concrete under different stress regimes. Although the model accuracy can be improved, given the current state of art in constitutive modelling, its performance is satisfactory.

4.4 Tensile Behaviour

4.4.1 Crack Formation

Refined theories for the description of crack development and propagation in concrete are based on continuum damage mechanics or fracture mechanics. Continuum damage mechanics is concerned with the description of progressive weakening of solids due to the development of microcracks. In this theory, the damage of the cracked material is characterized by a damage parameter Ω , related to the cracking parameter η . The expression $\Omega=1-\eta$, which shows how much of the material is undamaged, will affect the elasticity parameters of the material. Three dimensional tensor representations of damage have been developed by researchers which take into account anisotropy in the damage development (Suaris et al, 1990). When a major crack develops due to severe tensile stress or strain in concrete, and how crack extension will affect the material response is not the subject of damage mechanics.

As far as fracture mechanics is concerned, there is still disagreement regarding the applicability of this theory to modelling of concrete behaviour (Yazdani and Schreyer, 1990). Nevertheless, fracture mechanics is one of the popular methods for studying the cracking behaviour of concrete structures.

It has been stated by Yamagouchi and Chen (1990) that linear elastic fracture mechanics is applicable to large structures, while non linear fracture mechanics is more suitable for ordinary structures. For the case where the cracks are meters in length in large structure, and where matrix, bond and aggregates are modeled on a micro scale, linear elastic fracture

mechanics has been successfully applied. The reason for the inapplicability of linear-elastic fracture mechanics in normal size structures is that in the fracture process zone of concrete, the nonlinearity and non homogeneity of concrete limit the use of this theory. Microcracks in the region of cracks and their slow growth prior to unstable crack propagation are the major contributing factors to the limitations of this theory. In fracture mechanics both energy and strength criteria may be used in a rational model. The tensile strain has also been used as a fracture criterion.

For tensile cracking, early models were based on a simple fracture parameter, namely the tensile strength, f_t' . In these theories when this level of stress is reached, a crack is formed perpendicular to the maximum tension direction and the stress abruptly drops to zero. A complete stress-crack opening relation should contain the following four parameters, as illustrated in Fig.4.17, the Young's modulus, the tensile strength, the stress free crack opening and the slope of the descending branch (Reinhardt and Walraven, 1982). It has been shown that using a strain softening relation, especially one which reduces the stress suddenly to zero as the crack forms, regardless of mesh size is un-objective and the results are mesh dependent. As the mesh size reduces, the load required to extend a crack converges to zero, which is not realistic, (Bazant, 1985).

There are two approaches in finite element method to model crack formation and its propagation in concrete structures, the so called smeared crack and discrete crack approaches. In the smeared crack approach, which was first introduced by Rashid (1968) in finite element analysis of concrete structures, cracking is not modeled through a localized crack, but is modeled as a region of material affected by several parallel microcracks, resulting in disintegration of material and discontinuity in stress transfer in that region due to increase in tensile strain. Cracking is modeled by modifying the constitutive relationship for the sampling point in the finite element, and no redefining of the mesh is required. This is the major advantage of the smeared crack approach. Since in the discrete crack method the crack is modeled as a gap between the two adjacent finite elements, once the crack is formed the two sides need to be separated and double node numbering is needed for the two sets of nodes on the two faces of the crack. As the crack propagates, the geometry and node numbering is updated, which is a time-consuming process in a finite element analysis. The discrete crack approach seems to be more suitable when the progress of an existing crack needs to be traced. This approach is approximate in situations where cracking crite-

riterion must be satisfied at different points scattered throughout the structure all at the same time.

In this thesis the smeared crack approach is adopted for crack monitoring because of the simplicity and convenience of its implementation in a finite element code. As mentioned earlier, in smeared crack approach the material moduli are modified so as to satisfy the crack criterion at a sampling point within an element.

There are two procedures for investigating crack propagation in a nonlinear finite element program: (a) allowing the formation of cracks at all locations where the criterion of the onset of cracking is fulfilled; (b) allowing the formation of only one crack at a time at the location where the stress conditions are the most critical, based on a specified criterion. Procedure (a) is more realistic for dynamic problems and for analysis of plain concrete, whereas procedure (b) appears to be more suitable for problems where the crack growth rate is slow.

Tensile strength of concrete is very low and can be affected by many factors. Its value is difficult to measure and can show considerable scatter even in laboratory tests. In the field, uncontrolled environmental and loading conditions cause higher statistical scatter, (Cervera and Hinton, 1986). This is the reason why one generally accepts an approximate method for modelling tensile fracture of concrete. The model adopted should be simple but it should capture the essential properties of cracked concrete and its failure criteria.

4.4.2 Cracking Criterion

For concrete at an un-cracked sampling point, the principal stresses and their directions are evaluated. If the maximum principal stress exceeds a pre-defined limiting value, a crack forms in a plane orthogonal to its direction. In the cracked state, the concrete does not behave isotropically, rather it becomes orthotropic with the axes of orthotropy coinciding with the direction of principal axes.

Once the first crack develops at a point, a record of its orientation, strain normal to the crack plane, and its opening status are made. Subsequent cracks are assumed to form orthogonal to the first crack. The formation of cracks affects the state of stress at a point as follows:

- (a) Stresses normal to open cracks are reduced through an appropriate softening

rule.

- (b) Shear is transferred across open cracks through a shear retention factor.
- (c) Stresses for a closed crack are not reduced in spite of the fact that due to cyclic slippage of the faces and the subsequent abrasion of the asperities, the shear modulus may decrease.
- (d) The Poisson's effect will disappear in the plane of the crack.
- (e) The stresses parallel to the direction of a crack are assumed not to be affected once the crack forms (Chen, 1982).

The limiting value required to define the onset of cracking is established in Fig.4.18. Experiments have shown that the triaxial tensile strength of concrete is independent of the stress ratio (Cervera and Hinton, 1986), hence for triaxial tension case, the tensile strength σ_{tu} is determined by

$$\sigma_{tu}(i) = f'_t \quad i = 1, 2, 3 \quad (4.72)$$

where f'_t is the uniaxial tensile strength of concrete. For tension-tension-compression case, it is assumed that

$$\sigma_{tu}(i) = f'_t \left(1 + \frac{\sigma_j}{f'_c} \right) \quad (4.73)$$

where σ_j is the principal compressive stress and f'_c is the usual uniaxial cylinder strength of concrete. Note that compressive stresses are assumed to be negative. For tension-compression-compression case, the following is adopted

$$\sigma_{tu}(i) = f'_t \left(1 + \frac{\sigma_j}{f'_c} \right) \left(1 + \frac{\sigma_k}{f'_c} \right) \quad (4.74)$$

where σ_j and σ_k are the principal compressive stresses.

For the latter two cases the tensile strength is linearly decreasing due to the orthogonal compressive stress, which recognizes that compression enhances the crack formation in the two other orthogonal tensile directions (Cervera and Hinton 1986, Bathe and Sundberg 1986).

The issue of mesh size sensitivity has been discussed in the previous section. The remedy proposed for it is to modify the tensile strength and the slope of the softening part of the tensile stress-tensile strain curve by a length measure related to the corresponding volume of the integration point, leading to the fracture energy release G_f being constant (Bazant, 1985). Basically the same approach is used by Pagnoni et al (1992) as follows

$$\sigma = f_t' \exp\left(-\frac{(\epsilon - \epsilon_{cr}) f_t' l_c}{G_f}\right) \quad (1.75)$$

where f_t' is the tensile strength of concrete; ϵ is the tensile strain in the direction orthogonal to the crack plane; ϵ_{cr} is $\frac{f_t'}{E}$, G_f is the fracture energy of concrete regarded as a material property, l_c is a characteristic length, here assumed equal to the cube root of the volume associated with the integration point where cracks have occurred. This criterion is valid for the first crack, but no procedure is yet devised for the second and the third crack, (Chrisfield, 1984). As a result in the implemented model, this phenomenon is not considered due to lack of experimental and/or theoretical evidence, and a constant tensile strength with a linear descending branch after cracking is adopted as shown in Fig.1.19. The multiplier α in Fig.1.19 signifies the rate of strain softening and is kept as a variable in the present investigation.

At a cracked point the stresses in the plane of the crack are evaluated and the principal stresses in this plane are calculated, then a check for formation of new cracks is performed. Once a tensile plane of failure has formed, a check is made to detect whether the crack remains active. The strain normal to the crack plane is checked and if it is negative, the crack is assumed to be inactive.

If a tensile failure plane develops, the material stress strain relations are established corresponding to initial principal directions in the plane of crack and normal to this plane. Hence, instead of calculating stresses in the principal direction for the uncracked material, the strains and stresses are calculated in the crack coordinate system. Also when a failure plane is or was active, a subsequent failure plane is assumed to form perpendicular to the first plane if the maximum principal tensile stress in that plane exceeds the limiting tensile failure stress. The probable third failure plane is assumed to be fixed in the adopted procedure.

4.4.3 Rotation of the Principal Axes

After calculating the principal stresses, if the stress state violates the fracture criterion at a point, a crack is assumed to form in the direction of the maximum principal stress. This means that the normal stress will gradually reduce to zero. Retention of some shear stress in the plane of the crack will violate the assumption that the crack plane is a principal plane. Indeed in subsequent stages of loading (unloading), even for the case of an open crack, the principal axes will rotate, unless the crack plane is always assumed to be free of shear stress, i.e. zero shear retention factor. If one adopts the latter assumption, then the other two principal stresses will always be in the plane of the first crack and the subsequent two possible cracks will be orthogonal to each other and to the first crack. This is the concept of fixed unidirectional crack. Many researchers keep the concept of the shear retention factor but ignore the fact that the principal stress axes rotate.

Alternatively, whenever the angle between the crack orientation and the current principal stress direction exceeds some limiting value, a new crack can form in the new principal axes direction. This concept is called the fixed multi-directional crack approach. In this method several cracks can form at the same point.

The third method for dealing with the problem of rotating principal axes is to adopt the rotating crack concept. In this concept the crack orientation is continuously rotating with the rotation of principal axes. To have consistent results, the principal axes of stress and strain should be coaxial and coaxiality can be achieved via an implicit shear term in the rotating principal reference plane (Rotz, 1989). The latter concept can not be used in cyclic loading when a complete reversal of the strain trajectories occurs (Milford and Schnobrich, 1985).

Although the fixed uni-directional crack concept is the simplest method of monitoring crack formation at a point, due to its simplicity, it is still widely used in finite element analysis of concrete structures. Recognizing that the dynamic nonlinear analysis of arch dams has not been previously performed, it is adequate at this stage to use this simple concept of cracking in the present study. It is important to remember that one of the main objectives of this thesis is to assess the effects of concrete cracking on the overall seismic response of concrete arch dams.

The concept of shear retention factor is also adopted to comply with the fact that in a crack plane the shear stress will be transmitted through aggregate interlock action. It is noteworthy to mention that for the fixed crack concept the high shear retention factor may result in erroneous computations (Rotz, 1989), hence the shear retention factor should be kept as low as possible and preferably near zero. However, in practice values between 0.0 and 0.50 have been used for shear retention factor. In reality, the shear transfer should be a function of crack opening.

4.4.4 Algorithm for Tensile Behaviour

The following algorithm is adopted for the crack monitoring procedure and the post cracking behaviour. A flow chart of the numerical implementation is presented in Fig.4.20. The algorithm is described as follows.

Step 1: At each of the integration points, the strain tensor $\{\epsilon\}'_{m+1}$ is calculated via Eq(4.76).

$$\{\epsilon\}'_{m+1} = [\mathbf{B}]\{\mathbf{u}\}'_{m+1} \quad (4.76)$$

where $[\mathbf{B}]$ is the displacement-strain transformation matrix and $\{\mathbf{u}\}$ is the the vector of the nodal displacements. If the point has previously cracked, go to step 8, otherwise

Step 2: Calculate the elastic stress tensor $\{\sigma'\}'_{m+1}$

$$\{\sigma'\}'_{m+1} = [\mathbf{D}]\{\epsilon\}'_{m+1} \quad (4.77)$$

where $[\mathbf{D}]$ is the elastic rigidity matrix as follows

$$[\mathbf{D}] = \frac{E}{(1+\nu)(1-2\nu)} \begin{pmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & \nu & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(1-2\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(1-2\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(1-2\nu) \end{pmatrix} \quad (4.78)$$

in which E and ν are respectively the elastic modulus and the Poisson's ratio of concrete.

In Eq(4.77) the strain tensor comprises the following components

$$\{\epsilon\}^T = (\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}, \gamma_{xy}, \gamma_{yz}, \gamma_{xz}) \quad (1.79)$$

Step 3: Calculate the principal stresses.

Step 4: Check whether the failure criteria is satisfied using Eqs(4.72) to (4.74).

Step 5: If no crack is detected go to step 14, otherwise, calculate and store the direction cosines of the principal axes.

Step 6: Calculate the transformation matrix $[T]$ between the global and the local (crack) coordinates axes.

Step 7: Transform the strain tensor to the new coordinate system by Eq(4.80), where $[T]$ is the transformation matrix.

$$\{\epsilon'\} = [T]\{\epsilon\} \quad (1.80)$$

where $\{\epsilon'\}$ is the strain tensor in the crack coordinate system.

Step 8: Check whether the first crack is active. The crack is assumed to be active if the normal strain across the plane of crack is positive. If this crack is active, calculate the three components of stress $\sigma'_{11}, \sigma'_{12}, \sigma'_{13}$ using the implemented tensile model, considering a shear retention factor of 0.5 and neglecting the Poisson's effect. Next go to step 10.

Step 9: The first crack is inactive, concrete regains its strength in this direction. Calculate the three components of stress $\sigma'_{11}, \sigma'_{12}, \sigma'_{13}$.

Step 10: If the other two cracks have formed, go to step 12; otherwise, if just the second crack has formed, go to step 11; but if no crack has been detected in the plane of the first crack, calculate the remaining stress components, that is $\sigma'_{22}, \sigma'_{33}, \sigma'_{23}$ from the corresponding strains $\epsilon'_{22}, \epsilon'_{33}, \epsilon'_{23}$ using $[D_2]$ defined as follows

$$[D_2] = \frac{1}{1-\nu^2} \begin{pmatrix} E & \nu E & 0 \\ \nu E & E & 0 \\ 0 & 0 & (1-\nu^2)G \end{pmatrix} \quad (1.81)$$

assuming that the state of stress in the plane of crack is plane stress. The next failure plane is always assumed to form perpendicular to the direction of the one that has already developed. The principal stresses and their new direction in this plane are calculated. The direction of the third failure plane is fixed by the condition of orthogonality of the three

crack planes. Check whether the inplane principal stresses satisfy the cracking criteria, and if no other crack is detected go to step 13. But once the second and/or the third crack form, calculate the stress state, considering the implemented tensile model, the shear retention factor and neglecting the Poisson's effect. Calculate and save the direction of new (crack) coordinate axes. Go to step 13.

Step 11: Just the second crack has formed. If it is active, calculate the corresponding stress components, σ'_{22} and σ'_{23} and consider the shear retention factor. But if this crack is not active, no modification of the stress components is required. Check whether the third crack forms. If yes, modify σ'_{33} , applying the tensile model and go to step 13. If not, no modification of this stress components is needed. Go to step 13.

Step 12: All three cracks have formed. If both cracks are active, calculate the two normal stresses based on the applied model, calculate the shear stress considering the shear retention factor and go to step 13. But if both cracks are inactive, concrete regains its strength and the stresses are calculated accordingly, go to step 13. If the second crack is active but the third one is inactive, concrete regains its strength in the third direction. Calculate the stress state and go to step 13. And if the second crack is inactive but the third one is active, concrete regains its strength only in the second direction. Calculate the stress state and go to step 13.

Step 13: Once inelastic stress components at a typical integration point are calculated, they are transformed back to the original coordinate system by the following equation

$$\{\sigma\} = [T]^T \{\sigma'\} \quad (4.82)$$

where $\{\sigma'\}$ is the stress tensor in the new (crack) coordinate system, $\{\sigma\}$ is the stress tensor in the original coordinate system and $[T]$ is the transformation matrix.

Step 14: Calculate the resisting load vector by Eqs(4.71).

Step 15: Go to the next iteration.

4.4.5 Verification of Numerical Implementation

In Fig. 4.21 the uniaxial tensile behaviour of concrete is simulated by the implemented tensile model. The softening behaviour with the unloading-reloading behaviour are represented. Once the tensile strain exceeds the maximum value ϵ_{max} , the crack loses all its

strength. The value of σ_{mar} , (point F) is specified by the user. When the crack re-closes, that is path A-D, the material regains its strength.

4.5 Combined Compressive and Tensile Models

To have a comprehensive understanding of the nonlinear response of a concrete structure, one needs to take both the compressive and the tensile nonlinearity into account. To the writer's knowledge a rigorous three dimensional nonlinear model which is capable of capturing the important aspects of the behaviour of concrete both in compression and in tension is not available yet. The adopted bounding surface model is basically developed to study the compressive behaviour of concrete. Although the tensile modes of failure are also represented by the surface, it is a known fact that post-cracking behaviour of concrete can not be modelled by an isotropic constitutive model. The post-cracking behaviour of concrete is highly orthotropic.

In this thesis the following algorithm is proposed for combining the bounding surface and the cracking models in order to simulate the complete nonlinear response of concrete. The basic idea behind this algorithm is that cracked concrete in the plane of the crack follows the bounding surface model, and that in the direction of a closed crack, the stress components can be evaluated using the bounding surface model. However, when the crack is open, the material follows the post cracking algorithm proposed in Sec.4.4.4. The applied algorithm is illustrated in Fig.4.22, and the basic steps of this algorithm are highlighted in the following.

Step 1: Check whether the point has already failed, and if so, nullify the stress tensor and go to step 7. If not, calculate the stress tensor.

Step 2: If the point has not cracked yet, go to step 3, else if the crack is open, that is if the normal stress is tensile, calculate the post cracking stress. If the crack is closed, no modification of the stress tensor is necessary.

Step 3: Calculate the second trial stress tensor using the bounding surface model. If the stresses indicate failure, nullify the stress tensor and the rigidity matrix and go to step 7.

Step 4: If concrete had cracked before and the crack is open, use the crack algorithm to adjust the stress tensor. Also adjust the rigidity matrix correspondingly and go to step

7. But if the crack is closed, no modification is required, go to step 7.

Step 5: If the concrete had not cracked yet, use the crack monitoring algorithm to check the possible formation of cracks. If no crack occurs, go to step 7.

Step 6: If a crack occurs, use the crack monitoring algorithm to adjust the stress tensor and rigidity matrix.

Step 7: Calculate the resisting force vector of the elements.

Step 8: Go to the next iteration.

4.5.1 Verification of Numerical Implementation

In Fig.4.23 based on the proposed combined model, the simulated stress strain behaviour of concrete under hydrostatic load is presented. Beginning from point A, the material is loaded up to about the uniaxial compressive strength at point B. Then it is unloaded up to point C where it cracks. After point C letting the concrete to unload further, the material follows the softening part of the stress strain curve in tension up to point D, where the material is again loaded. At point E the crack is closed again and the stress is calculated by the bounding surface model. The loading continues up to point F, where the material is again unloaded. At point G it goes into tension again, and follows path GD, with the damaged modulus in tension remembered from the previous history. At point H the stress is again on the softening branch, and follows that path up to point I where the crack is fully open and can carry no more stress.

The simulations presented in Sec.4.3.5, 4.4.5 and this section illustrate some of the characteristics of the constitutive models used for simulating the nonlinear behaviour of concrete in arch dams. Since the objective of this investigation is to determine the effect of material nonlinearities on the overall behaviour of concrete arch dams, these capabilities are considered to be adequate for achieving that objective. The actual accuracy of the results will be difficult to gauge because there are no experimental or analytical results available for the nonlinear response of arch dams caused by concrete nonlinearity.

Fig.4.1: Schematic illustration of the bounding surface concept in two dimensions (After Dafalias and Popov 1976)

Fig.4.2: The bounding surface model, (a) bounding surface, (b) radial mapping.
(After Pagnoni et al 1992)

Fig.4.3: Stress-strain relationship of concrete under uniaxial compression
 (Test data from Kupfer et al 1969)

Fig.4.4: Lateral strain- axial stress relationship of concrete under uniaxial compression
 (Test data from Kupfer et al 1969)

Fig.4.5: Volumetric strain- axial stress relationship of concrete under uniaxial compression

(Test data from Kupfer et al 1969)

Fig.4.6: Axial strain- axial stress relationship of concrete under biaxial compression
 (Test data from Kupfer et al 1969)

Fig.4.7: Lateral strain- axial stress relationship of concrete under biaxial compression
 (Test data from Kupfer et al 1969)

Fig.4.8: Lateral strain- axial stress relationship of concrete under biaxial compression
 (Tset data from Kupfer et al 1969)

Fig.4.9: Volumetric strain- axial stress relationship of concrete under biaxial compression (Tset data from Kupfer et al 1969)

Fig.4.10: Stress-strain relationship of concrete
under triaxial compression
(Test data from Kotsovos and Newman 1979)

Fig.4.11: Axial stress-axial strain relationship of concrete under triaxial extension
 (Test data from Kotsovos and Newman 1979)

Fig.4.12: Axial stress-lateral strain relationship of concrete under triaxial extension

(Test data from Kotsovos and Newman 1979)

Fig.4.13: Typical model prediction for uniaxial cyclic loading

Fig.4.14: Typical model prediction for uniaxial cyclic loading
(not fully unloaded)

Fig.4.15: Typical model prediction for biaxial cyclic loading

Fig.4.16: Typical model prediction for triaxial cyclic loading

**Fig.4.17: Parameters defining the tensile behaviour of concrete
(After Reinhardt and Walraven 1982)**

Fig.4.18: Three-dimensional tensile failure model for concrete
(After Cervera and Hinton 1986)

Fig.4.19: Simplified behaviour of concrete in tension

Fig.4.21: Tensile behaviour of concrete simulated by the implemented model

Fig.4.22: The combined model algorithm

**Fig.4.23: Stress-strain simulation of the combined model
(hydrostatic loading)**

Chapter 5

Computer Implementation

5.1 General

The proposed contraction joint model as discussed in Chapter 3, and the nonlinear material models for concrete as discussed in Chapter 4, are implemented in the developed arch dam finite element computer program BANDAAB. The computer program ADAP88 is used as the backbone of BANDAAB. The original version of ADAP88, that is ADAP, was developed by Clough et al (1973), for linear static and dynamic analysis of arch dams. It was later improved and further developed by Fenves et al (1989) to ADAP88. It is important to mention that the name ADAP was not adopted for the new version of the program developed in this thesis because it was felt that any shortcomings may inadvertently reflect on the original versions of ADAP and ADAP88.

ADAP88 is a nonlinear finite element program for seismic analysis of concrete arch dams in the time domain. The program has the capability to generate the finite element mesh for an arch dam and its foundation requiring only a small amount of input data. Water-structure interaction is taken into account in the analysis, but the compressibility of water is neglected. All three translational components of the seismic ground motion can be applied simultaneously but uniformly at the far field boundary of the foundation.

The capability of the existing joint model in the program is limited to the modelling of the opening and closing behaviour of the contraction joints. The new joint element developed in the present study models the full nonlinear effects in the contraction joint, including the opening and closing behaviour, the free slippage after opening of the joint, the effect of the shear keys, and the frictional slippage when the joint is closed.

In the original program ADAP88, the concrete material is assumed to be linear elastic in both tension and compression. The only source of nonlinearity considered comes from the opening and closing of the contraction joints. In the present study, the nonlinear behaviour of the concrete material is also modelled. A nonlinear bounding surface model is

implemented to model the concrete material prior to failure. In addition, a smeared crack model is adopted to account for the tensile failure of concrete.

In the following section, a general description of the original program ADAP88 is presented. Modifications to the program are discussed.

5.2 Fundamental Aspects of ADAP88

ADAP88 is the result of extensive research on the seismic behaviour of arch dams. In ADAP88 the dam structure is modeled by a combination of 3D solid, thick shell, and 3D thick shell finite elements. The vertical contraction joints in the dam are modeled by zero-thickness joint elements. The foundation is modeled as an assemblage of 3D solid elements. The linear dynamic fluid-structure interaction effect due to the impounded water is considered by the added mass concept. These masses are attached to the upstream nodes of the structure.

Except for the contraction joints, the body of the dam and the foundation are considered to remain linearly elastic in ADAP88. The program uses the sub-structuring technique to model the linear elastic part of the system. The dam body is modeled as a number of linear-elastic substructures separated by the contraction joints. The dam foundation is modelled by another linear elastic substructure, as shown in Fig.5.1.

By the method of static condensation, the contribution to the response of the system from the internal degrees of freedom of the substructures are transferred to the nodal degrees of freedom on the boundaries of the substructures. The stiffness and consistent load vectors corresponding to these boundary degrees of freedom of the substructures are assembled to form the global structure stiffness and load vectors. The boundary degrees of freedom of the linear substructures coincide with the degrees of freedom of the nonlinear joint elements in the model. The resulting general equilibrium can be written as follows

$$[\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{P}\} \quad (5.1)$$

where $[\mathbf{K}]$ is the global stiffness matrix, and $\{\mathbf{P}\}$ is the assembled load vector, including the dynamic loads. Since the stiffness matrix $[\mathbf{K}]$ depends on the deformation of the dam or the displacement vector $\{\mathbf{u}\}$, Eq(5.1) is a nonlinear equation and is thus solved by iteration.

After convergence in the time step, with the boundary displacement vector $\{\mathbf{u}\}$ determined, the response of the internal degrees of freedom can be calculated by the usual procedures of static condensation. For the purpose of this study, extensive modifications have been made to ADAP88, which are described in the following section.

5.3 Modifications of ADAP88 and Further Developments (BANDAAB)

5.3.1 Solution Strategy for Static Analysis

For the implementation of the full nonlinear joint model, the simple iteration scheme in the original program ADAP88 is not adequate as explained below.

In ADAP88 opening of the joint elements is modeled by a sudden drop of stress and reduction of the normal penalty parameter k_n to zero. As long as the joint remains open, k_n remains zero. But as soon as the joint is re-closed, the penalty parameter reverts to a very large number to impose the normal constraint of no penetration between the two joint faces.

For the static analysis, the penalty parameters for shear remains the same for the open and closed conditions. It is assumed that due to the action of the shear key there would not be relative slippage between the two joint faces, even when the joint is open. However, this assumption may not be valid for un-keyed joints. The only criterion for checking the convergence of the solution is convergence in the opening-closing status of the joint elements. Since the only source of nonlinearity in the original program arises from the change in the opening-closing status of the contraction joints, which is clearly an event-to-event type of nonlinear behaviour, it is sufficient to determine convergence of the solution on the basis of the convergence of the opening-closing status at the integration points of the joint elements. When the status remains unchanged for all the integration points from one iteration to the next, the solution is assumed to have converged.

To study the full nonlinear behaviour of the joints, which is clearly no longer an event-to-event phenomenon, the iteration scheme in the modified program is based on the modified Newton-Raphson method. The unbalanced force vector is calculated for each iteration within each time step, which is subsequently used to calculate the displacement increment vector for the next iteration.

The convergence criterion for the iteration is chosen as follows

$$\delta = \frac{\|\Delta P_n^i\|}{\|\Delta P_n\|} < \delta_0 \quad (5.2)$$

where $\|\Delta P_n^i\|$ is the norm of the out-of-balance force at the end of the i -th iteration during time step n , $\|\Delta P_n\|$ is the norm of the out of balance force at the beginning of the time step n , and δ_0 is the convergence tolerance limit. This procedure is applied in computer program BANDAB1, which employs the rigorous constitutive joint model described in Chapter 3. The concrete and foundation materials are assumed to be linear elastic in BANDAB1.

5.3.2 Linear Versus Nonlinear Substructures

As explained in Sec.5.2, in ADAP88, the dam and the foundation material are assumed to behave in a linear elastic manner, and the substructure method is used in order to reduce the number of equilibrium equations. To account for the nonlinear material behaviour and cracking of the concrete material, it is necessary to write the equilibrium equations of the arch dam in terms of all the active degrees of freedom of the system, without the benefit of the substructure formulation. Substantial modifications of ADAP88 were required in order to avoid sub-structuring and to reactivate all the required degrees of freedom. But before describing those modifications, the basic relations governing the behaviour of the structure based on the formulations adopted in the original program ADAP88 are explained.

The equation of motion for a linear substructure at a typical time t_{n+1} is

$$\mathbf{m}\ddot{\mathbf{u}}_{n+1} + \mathbf{c}\dot{\mathbf{u}}_{n+1} + \mathbf{k}\mathbf{u}_{n+1} = \mathbf{f}_{n+1} + \mathbf{q}_{n+1} \quad (5.3)$$

where $\mathbf{u}$ is the displacement vector, and $\mathbf{m}$, $\mathbf{k}$ and $\mathbf{c}$, are respectively, the mass, stiffness and damping matrices of the substructure, $\mathbf{f}$ is the external force acting on the substructure, and $\mathbf{q}$ is the vector of forces acting on the boundary degrees of freedom.

Using the Newmark time integration method, Eq(5.3) for the iteration number k is simplified as

$$\mathbf{k}^* \mathbf{u}_{n+1}^k = \mathbf{p}_{n+1}^* + \mathbf{q}_{n+1}^k \quad (5.4)$$

where the effective stiffness matrix and the effective load vector are

$$\mathbf{k}^* = a_1 \mathbf{m} + a_2 \mathbf{c} + \mathbf{k} \quad (5.5)$$

$$\mathbf{p}_{n+1}^* = \mathbf{f}_{n+1} + \mathbf{m} \{a_1 \dot{\mathbf{u}}_n\} + \mathbf{c} \{a_2 \dot{\mathbf{u}}_n - \ddot{\mathbf{u}}_n\} \quad (5.6)$$

in which

$$\dot{\mathbf{u}}_n = \mathbf{u}_n + \Delta t \dot{\mathbf{u}}_n + \left(\frac{1}{2} - \beta\right) \Delta t^2 \ddot{\mathbf{u}}_n \quad (5.7)$$

$$\ddot{\mathbf{u}}_n = \dot{\mathbf{u}}_n + \Delta t(1 - \gamma) \ddot{\mathbf{u}}_n \quad (5.8)$$

note that $a_1 = 1/(\beta \Delta t^2)$, $a_2 = \gamma/(\beta \Delta t)$ and γ and β are constants of the Newmark β method, and $\dot{\mathbf{u}}$ and $\ddot{\mathbf{u}}$ are the velocity and acceleration vectors, respectively.

Using the method of static condensation, the effective stiffness matrix and the effective load vector are condensed to the boundary degrees of freedom. Eq(5.4) can then be rewritten as follows

$$\bar{\mathbf{k}}^* \mathbf{u}_{b(n+1)}^k = \bar{\mathbf{p}}_{n+1}^* + \mathbf{q}_{n+1}^k \quad (5.9)$$

where $\bar{\mathbf{k}}^*$ is the condensed effective stiffness matrix and $\bar{\mathbf{p}}_{n+1}^*$ is the condensed effective load vector. These equations are written for time step $(n+1)$ and iteration k . It is noted here that the displacement vector $\mathbf{u}_n$ consists of the displacements at the boundary degrees of freedom only.

The equation of motion for the nonlinear substructure, that is the joints, at time step $(n+1)$ is

$$\mathbf{M} \ddot{\mathbf{U}}_{n+1} + \mathbf{P}(\dot{\mathbf{U}}_{n+1}, \mathbf{U}_{n+1}) = \mathbf{F}_{n+1} + \mathbf{Q}_{n+1} \quad (5.10)$$

where $\mathbf{U}$ is the displacement vector for the nonlinear joint elements, $\mathbf{M}$ is the corresponding mass matrix, $\mathbf{P}$ is the resisting load vector, $\mathbf{F}$ is the external load vector, and finally $\mathbf{Q}$ is the vector of forces acting on the boundary of this substructure.

The equilibrium between the linear and nonlinear substructures results in

$$\mathbf{Q} + \sum \mathbf{q} = 0 \quad (5.11)$$

The incremental form of the matrix equilibrium, Eq(5.10), is expressed as follows (Fenves et al, 1989)

$$\mathbf{K}^* \Delta \mathbf{U} = \Delta \mathbf{P}^* \quad (5.12)$$

where

$$\mathbf{K}^* = a_1 \mathbf{M} + a_2 \frac{\partial \mathbf{P}}{\partial \dot{\mathbf{U}}} + \frac{\partial \mathbf{P}}{\partial \mathbf{U}} - \frac{\partial \mathbf{Q}}{\partial \mathbf{U}} \quad (5.13)$$

and

$$\Delta \mathbf{P}^* = \mathbf{F}_{n+1} - \mathbf{M} \ddot{\mathbf{U}}_{n+1}^k - \mathbf{P}(\dot{\mathbf{U}}_{n+1}^k, \mathbf{U}_{n+1}^k) + \mathbf{Q}_{n+1}^k \quad (5.14)$$

Using Eqs(5.9) and (5.11) in Eqs(5.13) and (5.14) leads to

$$\mathbf{K}^* = a_1 \mathbf{M} + a_2 \mathbf{C}_T + \mathbf{K}_T + \sum \bar{\mathbf{k}}^* \quad (5.15)$$

$$\Delta \mathbf{P}^* = \mathbf{F}_{(n+1)} - \mathbf{M} \ddot{\mathbf{U}}_{(n+1)}^k - \mathbf{P}(\dot{\mathbf{U}}_{(n+1)}^k, \mathbf{U}_{(n+1)}^k) + \sum \bar{\mathbf{p}}_{(n+1)}^* - \sum \bar{\mathbf{k}}^* \mathbf{u}_{b(n+1)}^k \quad (5.16)$$

where $\mathbf{C}_T$ is the tangential damping matrix of the nonlinear substructure and is equal to $\frac{\partial \mathbf{P}}{\partial \dot{\mathbf{U}}}$, and $\mathbf{K}_T$ is the tangential stiffness matrix of the nonlinear substructure and is equal to $\frac{\partial \mathbf{P}}{\partial \mathbf{U}}$.

The initial value of acceleration at time t_0 should be evaluated to start the time integration. The dynamic component of displacement and velocity at time t_0 are assumed to be zero. The acceleration for the linear substructure can be evaluated by

$$\mathbf{m} \ddot{\mathbf{u}}_0 = \mathbf{f}_0 + \mathbf{q}_0 \quad (5.17)$$

Applying the method of static condensation leads to

$$\bar{\mathbf{m}}\ddot{\mathbf{u}}_{s(0)} = \bar{\mathbf{f}}_0 + \mathbf{q}_0 \quad (5.18)$$

For the nonlinear substructure, the equation of motion at time zero would be

$$\mathbf{M}\ddot{\mathbf{U}}_0 = \mathbf{F}_0 - \sum \mathbf{q}_0 \quad (5.19)$$

Substituting Eq(5.18) in Eq(5.19),

$$(\mathbf{M} + \sum \bar{\mathbf{m}}) \ddot{\mathbf{U}}_0 = \mathbf{F}_0 + \sum \bar{\mathbf{f}}_0 \quad (5.20)$$

Assuming a uniform ground motion at all the support points, the dynamic load actions on the linear and nonlinear substructures are expressed as follows

$$\mathbf{f} = -(\mathbf{m}\mathbf{r} + \mathbf{m}_g) \ddot{\mathbf{u}}_g \quad (5.21)$$

$$\mathbf{F} = -(\mathbf{M}\mathbf{R} + \mathbf{M}_g) \ddot{\mathbf{U}}_g \quad (5.22)$$

where $\ddot{\mathbf{u}}_g$ is the vector of free-field ground acceleration at the support, $\mathbf{r}$ and $\mathbf{R}$ are the matrices of directional cosines between the components of ground acceleration and the structure degrees of freedom, and $\mathbf{m}_g$ and $\mathbf{M}_g$ are the mass matrices that couple the degrees of freedom in the linear and nonlinear substructures, respectively, with the support points. The notations $\mathbf{r}$ and $\mathbf{R}$ adopted here are not to be confused with those defined for the bounding surface model described in Chapter 4.

At the end of the iteration at a typical time step ($n+1$), the displacements at the boundaries of the linear substructures are calculated. The interior degrees of freedom of the substructure can be calculated through the relations of static condensation.

The response at the next time step depends on the condensed effective load vector, $\bar{\mathbf{p}}_{n+1}^*$, of each linear substructure, which should be calculated from the full load vector $\mathbf{p}_{n+1}^*$. But the calculation of $\mathbf{p}_{n+1}^*$ involves considerable amounts of computation which requires two matrix multiplications. To reduce the amount of calculations further, the following simplification is adopted in ADAP88.

Assuming Rayleigh type damping in the linear substructures

$$\mathbf{c} = b_0 \mathbf{m} + b_1 \mathbf{k} \quad (5.23)$$

where b_0 and b_1 are the proportionality constants, and substituting Eq(5.23) in Eq(5.5), $\mathbf{k}^*$ can be expressed as follows

$$\mathbf{k}^* = (a_1 + a_2 b_0) \mathbf{m} + (1 + a_2 b_1) \mathbf{k} \quad (5.24)$$

and the effective load vector becomes

$$\mathbf{p}_{n+1}^* = \mathbf{f}_{n+1} + \mathbf{m}[c_1 \dot{\mathbf{u}}_n - c_2 \ddot{\mathbf{u}}_n] + \mathbf{k}^*[d_1 \dot{\mathbf{u}}_n - d_2 \ddot{\mathbf{u}}_n] \quad (5.25)$$

where the constants c_1, c_2, d_1, d_2 can be determined from the known values of the constants a_1, a_2, b_0, b_1 (Fenves et al, 1989).

Following the static condensation procedures, the effective load vector can be expressed as follows

$$\sum \bar{\mathbf{p}}_{n+1}^* = \sum \bar{\mathbf{p}}_{b(n+1)}^* + \sum \bar{\mathbf{k}}^* \hat{\mathbf{v}}_n \quad (5.26)$$

where

$$\hat{\mathbf{p}}_{(n+1)}^* = \mathbf{f}_{(n+1)} + \mathbf{m}[c_1 \dot{\mathbf{u}}_n - c_2 \ddot{\mathbf{u}}_n] \quad (5.27)$$

$$\hat{\mathbf{v}} = d_2 (a_2 \dot{\mathbf{u}}_n - \ddot{\mathbf{u}}_n) \quad (5.28)$$

Finally, the out of balance load vector is written as

$$\Delta \mathbf{P}^* = \mathbf{F}_{n+1} - M \ddot{\mathbf{U}}_{n+1}^k - \mathbf{P} \left(\dot{\mathbf{U}}_{(n+1)}^k \cdot \mathbf{U}_{(n+1)}^k \right) + \sum \bar{\mathbf{p}}_{b(n+1)}^* - \sum \bar{\mathbf{k}}^* \mathbf{u}_{b(n+1)}^k + \sum \bar{\mathbf{k}}^* \hat{\mathbf{v}}_n \quad (5.29)$$

The effort to compute the effective load vector from Eq(5.27) is considerably less than to compute this quantity from Eq(5.6) since just one matrix vector multiplication is needed.

The last term in Eq(5.29) comes from the contribution of the proportional damping to the effective load vector. The above mentioned formulation is applicable whenever the dam structure and the foundation behave linearly. However, if nonlinear material models are to be employed for the structure and the foundation, all the degrees of freedom should be taken into account.

In the present formulation, the sub structuring technique and the process of condensing out and reducing the degrees of freedom to those at the boundary are bypassed. The assemblage of the load vector and the stiffness matrices is allowed to proceed in the usual way, as presented in Eqs(5.15) and (5.29). Further, $\bar{\mathbf{k}}^*$ is now replaced by the overall stiffness of the substructures, and the vectors $\mathbf{u}_k$ and $\hat{\mathbf{v}}_k$ are replaced by $\mathbf{u}$ and $\mathbf{v}$ respectively, comprising all the degrees of freedom in the substructures. Also in Eq(5.20), $\bar{\mathbf{m}}$ is replaced by $\mathbf{m}$, and $\bar{\mathbf{f}}_0$ is replaced by $\mathbf{f}_0$. This results in the following fundamental equations governing the behaviour of the entire dam

$$\mathbf{K}^* = a_1 \mathbf{M} + a_2 \mathbf{C}_T + \mathbf{K}_T + \sum \mathbf{k}^* \quad (5.30)$$

$$\Delta \mathbf{P}^* = \mathbf{F}_{n+1} - \mathbf{M} \ddot{\mathbf{U}}_{n+1}^k - \mathbf{P} \left(\dot{\mathbf{U}}_{n+1}^k, \mathbf{U}_{n+1}^k \right) + \sum \mathbf{p}_{n+1}^* - \sum \mathbf{k}^* \mathbf{u}_{n+1}^k \quad (5.31)$$

$$\left(\mathbf{M} + \sum \mathbf{m} \right) \ddot{\mathbf{u}}_0 = \mathbf{F}_0 + \sum \mathbf{f}_0 \quad (5.32)$$

Note that the effect of $\bar{\mathbf{p}}^*$ and the last term in Eq(5.29) are lumped back into $\mathbf{p}^*$ for the sake of clarity. The required modifications are implemented in a new program which will be referred to henceforth as "BANDAAB2". This program has the capability to consider the effects of material nonlinearities as well as the opening closing effects of the contraction joints, as implemented in the original program ADAP88.

In modelling the nonlinear or inelastic concrete material behaviour, an incremental stiffness matrix, $\Delta \mathbf{k}$ is defined by subtracting the inelastic stiffness matrix of the concrete elements from their elastic ones

$$\Delta \mathbf{k} = \mathbf{k}_{el} - \mathbf{k}_i \quad (5.33)$$

where $\mathbf{k}_{in}$ and $\mathbf{k}_e$ are respectively the inelastic and the elastic stiffness matrices of the elements. This procedure obviates the need to form and assemble the entire stiffness and consistent load vector of the structure in each iteration.

By examining Eq(5.5), it is recognized that only $\mathbf{k}$ will continuously change in any substructure which becomes nonlinear. The damping matrix $\mathbf{c}$ is assumed to remain constant and not be affected by the changes in the $\mathbf{k}$ matrix. Clearly, the latter assumption would need further examination in future studies but given the complexity of the problem and the lack of available data regarding the damping properties of cracked un-reinforced concrete, this assumption is considered resonable for the purpose of present investigation.

Substituting the current stiffness, $\mathbf{k}_{in}$, into Eq(5.30) and using Eq(5.33) results in

$$\mathbf{K}^* = a_1 \mathbf{M} + a_2 \mathbf{C}_I + \mathbf{K}_I + \sum (a_1 \mathbf{m} + a_2 \mathbf{c} + \mathbf{k}_e + \Delta \mathbf{k}) \quad (5.34)$$

All the terms in Eq(5.34), with the exception of $\Delta \mathbf{k}$, are calculated in the same manner as in the original program. Only the term $\Delta \mathbf{k}$ needs to be calculated at each iteration for all the affected elements to obtain the global stiffness matrix of the structure.

For each iteration, the nonlinear rigidity matrix, $[\mathbf{D}]_{in}$ should be calculated based on the implemented nonlinear models. The inelastic stiffness matrix is calculated as follows

$$\mathbf{k}_{in} = \int_V \mathbf{B}^T [\mathbf{D}]_{in} \mathbf{B} dV \quad (5.35)$$

where $\mathbf{B}$ is the displacement-strain transformation matrix, and V is the volume of the element. The change in element stiffness is calculated by Eq(5.33). This change is then assembled to form the overall effective stiffness matrix of the structure by Eq(5.34).

After renaming the original non-linear substructure (joint elements) contribution as $\Delta \mathbf{P}_{n+1}^*$, and substituting the proper expression for p_{n+1}^* from Eq(5.6), the out-of-balanced load vector, Eq(5.31) becomes

$$\Delta \mathbf{P}^* = \Delta \mathbf{P}_{n+1}^* + \sum \left\{ \mathbf{f}_{n+1} + \mathbf{m} \{a_1 \dot{\mathbf{u}}_n\} + \mathbf{c} \{a_2 \dot{\mathbf{u}}_n - \dot{\mathbf{u}}_n\} - \mathbf{k}^* \mathbf{u}_{n+1}^A \right\} \quad (5.36)$$

Using the definition of $\mathbf{k}^*$ from Eq(5.5) and Eq(5.33), Eq(5.36) leads to

$$\Delta \mathbf{P}^* = \Delta \mathbf{P}_{out}^* + \sum \left\{ \mathbf{f}_{n+1} + \mathbf{m} \{a_1 \mathbf{u}_n\} + \mathbf{c} \{a_2 \dot{\mathbf{u}}_n - \dot{\mathbf{u}}_n\} - (a_1 \mathbf{m} + a_2 \mathbf{c}) \mathbf{u}_{n+1}^k \right. \\ \left. + (\mathbf{k}_n + \Delta \mathbf{k}) \mathbf{u}_{n+1}^k \right\} \quad (5.37)$$

or finally

$$\Delta \mathbf{P}^* = \Delta \mathbf{P}_{out}^* + \Delta \mathbf{P}_{el}^* - \Delta \mathbf{k} \mathbf{u}_{n+1}^k \quad (5.38)$$

where, $\Delta \mathbf{P}_{el}^*$ is the contribution of the substructure to the out of balanced load vector, considering elastic behaviour. Hence, to take the change in the out of balance load vector into account the last term in Eq(5.38) should be assembled in the out of balance load vector array. In each iteration the inelastic stress tensor $\boldsymbol{\sigma}_{in}$ is calculated from the adopted nonlinear model. The elastic stress tensor $\boldsymbol{\sigma}_{el}$ is also calculated via the elastic rigidity matrix. The difference in resisting (equivalent) load vector is then calculated as follows:

$$\Delta \mathbf{k} \mathbf{u}_{n+1}^k = \int_V [\mathbf{B}]^T (\boldsymbol{\sigma}_{in} - \boldsymbol{\sigma}_{el}) dV \quad (5.39)$$

5.4 Description of the Finite Elements

As mentioned earlier ADAP88 generates a finite element model for the arch dam and its foundation, with a very small amount of input data. In the analysis the dam is modelled by a combination of 8 node zero thickness joint elements, 8 node 3 D elements, 16 node thick shell elements and 16 node 3 D thick shell elements. In the following, these elements are briefly reviewed. For the sake of completeness, in the last part of this section, the modifications applied to the 3D solid elements are explained. The 8 node zero thickness element used in modelling the contraction joints has been described in detail in Chapter 3.

5.4.1 The 8 Node 3D Solid Elements

The 3 D solid elements are eight node isoparametric hexahedron elements. These elements are used to model regions close to the contraction joints (see Fig.5.2), as well as the foundation. They are refined by the use of incompatible deformation mode (Wilson et al. 1973), which greatly improve their effectiveness. The extra incompatible degree of freedom

are condensed out at the element level (Hughes, 1987), with the final stiffness matrix of the element being 21×21 .

5.4.2 Thick Shell Elements

These elements are basically the thick shell element developed by Ahmed et al (1968). However a special integration scheme is used to reduce the high shear energy of the element to improve its bending behaviour. It is an isoparametric hexahedron whose geometry is defined by eight nodes located on its two opposite faces. The deformation of this element is defined through five degrees of freedom at each of the eight points located on its mid-surface. These five degrees of freedom are three translational and two rotational. Wherever this element is connected to a 3D solid element, its stiffness is derived by relating the five degrees of freedom of each mid-surface node to the six degrees of freedom of its corresponding two nodes on the two faces of the element. This element is referred to as a transition element and its stiffness may be 48×48 .

5.4.3 Three Dimensional Thick Shell Elements

These are hexahedron elements with 16 nodal points. Their bending behaviour is improved due to the addition of the incompatible modes. Each node has three degrees of freedom, resulting in a total of 48 degrees of freedom for the element. These elements are used in connecting the dam to its foundation as is shown in Fig.5.2.

5.4.4 Modifications Applied to 3D Solid Elements

As mentioned in Chapter 2, the regions close to the contraction joints are susceptible to severe nonlinear behaviour due to partial opening of the joints, which may result in large overstress of the closed portion of the joints. To study this effect, the 3D solid elements close to the contraction joints were modified such that the nonlinear material models can be implemented in them. For modelling the material nonlinearity, the stresses and strains within the elements should be checked at all the integration points of the element. This enables the program to accurately trace the inelastic behaviour of the material.

The strains at each of the eight Gaussian integration points of the 3D solid element are determined by the displacement-strain transformation matrix $[B]$, which are evaluated once at the beginning of the analysis and stored. The stresses are then evaluated based on

the adopted constitutive laws, and the change in the resisting load vector of the element is calculated using Eq(5.39). Although the inelastic stiffness matrix of the element can be calculated by the use of inelastic rigidity matrix, in the present study the initial linear elastic stiffness of the concrete structure is used in the Newton Raphson strategy. The change in the resisting load vector is then used to calculate the out of balance load vector of the whole structure.

5.5 Verification of Implemented Modifications

The responses of the Morrow Point Dam, subjected to ground motion, have been studied to verify the implemented changes in ADAP88. Morrow Point Dam is a modern arch dam in the United States completed in 1968 (Thomas,1976). Its height is 143 m with a crest length of 226 m. The base thickness of the dam is 21.0 m and its crest thickness is 3.66 m. This dam has been the subject of a number of experimental and numerical studies (NRC, 1990), and is used as a case study for all the analyses in this thesis.

Also in the following analyses, and all subsequent analyses in this thesis, the dam is subjected to the stream and/or vertical component of a typical earthquake. The time histories of these two components are presented in Figs. 5.3 and 5.4. The seismological characteristics of the selected components of ground motions are presented in Appendix I.

In these analyses, the modulus of elasticity of concrete is taken as 23.9 GPa, its Poisson's ratio as 0.20 and its mass density is 2070 kg/m^3 . For the foundation material, the modulus of elasticity is taken as 15.9 GPa and the Poisson's ratio is 0.20. The foundation is assumed as massless. The values of the Rayleigh's proportionality constants b_0 and b_1 are assumed to be 1.37 rad/sec and 0.0017 sec/rad , in order to provide a modal damping ratio of 5% in the first and fifth vibration modes of the monolithic model without the reservoir. For reference, the free vibration properties of the dam structure are presented in Appendix II. For the analyses presented in this chapter, the values of the Newmark's time integration constants i.e. β and γ are assumed to be 0.25 and 0.5, and the time step interval is taken to be 0.01 seconds .

Considering 3 contraction joints in each half of the dam (the dam is almost symmetric) plus one at the center, the general finite element mesh of this dam is shown in Fig. 5.5.

The model was analysed by both the modified programs and the original ADAP88. In

the following sections results of these analyses are presented and discussed.

5.5.1 Effect of Solution Strategy on Static Analysis

The modified program BANDAAAB1 and the original ADAP88 were used to analyse the dam for static as well as dynamic loading, taking the hydrostatic as well as the hydrodynamic loadings into account. Detailed response time histories were obtained, but due to space limitations only a portion of those results is presented here. For the sake of comparison with the results of ADAP88, just the opening-closing effects of the joints were allowed in BANDAAAB1. In order to facilitate the comparison with the results of the original ADAP88, U.S. customary units are used to express the results in this chapter.

In Fig.5.6, the stream component of the dynamic displacement relative to the static deformed state at point D on the dam (see Fig.5.5 for location of points A, B, C, D and E), is presented. It is observed that the results of BANDAAAB1 and ADAP88 are the same. Figs.5.7 to 5.11 show the response histories of the relative displacement for the joint elements located at points A, B, C and D. These elements are all located on the upstream side of the dam. Except for Figs.5.7 and 5.11, the other figures show close agreement between the results obtained by ADAP88 and BANDAAAB1. The magnitude of the responses in Fig.5.11 is so small that the observed differences should be attributed to the computer accuracies. On the other hand, the 20% difference in the static component of the displacement observed in Fig.5.7 is probably due to the differences in the static solution strategies adopted in the two programs.

Figs.5.12 and 5.13 show the stress response time histories of the 3D solid element located on the downstream face at point D. Two components of the stress in the arch direction and in the cantilever direction at the center of the element are presented. The results of ADAP88 and BANDAAAB1 are essentially the same. Figs.5.14 and 5.15, show the stress histories for the thick shell element close to point D. Once again the results of the two program are close to each other.

The reasonable agreement between the results obtained from ADAP88 and BANDAAAB1 show that the event to event strategy used in the original program results in more or less the same response as the modified Newton-Raphson strategy adopted in the modified program.

5.5.2 Effect of Bypassing the Substructure Method

In this section the response time histories obtained from ADAP88 and the modified program BANDAAB2, in which the substructuring technique is bypassed, are compared. The concrete material nonlinearity is not considered in the analyses reported here. Neglecting the gravity effect on the concrete material and considering only the water pressure loading, some of the response time histories are discussed here. It should be mentioned here that in Figs.5.16 to 5.28 the term “hydrodynamic” refers to both hydrostatic and hydrodynamic water pressure.

Figs.5.16 and 5.18 show the three components of the displacement for point D on the upstream face. It is noticed that the results from ADAP88 and BANDAAB2 are very close to each other. Figs.5.19 to 5.21 show the relative normal displacements of the joint elements at points A, C and D, located on the upstream face. The results from the two programs are following the same path and except for Fig.5.19 in which a 20% difference in the static component of displacement is observed, show reasonable agreement. In Figs.5.22 to 5.26 the stress components of the 3D solid elements at points D and E are plotted. Figs.5.27 and 5.28 show the stress time histories for the thick shell element at point D. The close agreement between the results of the two programs is evident.

The minor differences in the above results can be attributed to the increase in the number of the equilibrium equations which will result in an increase in the numerical and truncation errors. Basically the results of the analysis obtained from ADAP88 and BANDAAB2 for the hydrodynamic loading are the same.

The construction sequence may play an important role in static response of an arch dam (NRC, 1990). In practice, the monoliths are built in stages, and they grow in height, leaning on each other as the construction proceeds, and transfer some of the load in the arch direction. To study the construction process precisely, the actual construction procedure should be taken into account, which needs a more detailed study. In ADAP88 individual monoliths are separately analysed for gravity loading, considering their full height and disregarding the adjacent monoliths. The analysis is carried out in two stages, grouping the monoliths in two sets. At each stage, one set of the monoliths are analysed

The modified program BANDAAB2 does not have this capability and the water pressure

loading and the gravity loads are applied to the complete dam. Neither of the two methods represent the actual behaviour, and the real behaviour may be in between. Nevertheless, to see how this phenomenon affects the results of the analysis, the following results are presented. Here, in addition to water pressure loading, the effect of concrete gravity loading on the static stresses is also taken into account. The term “hydrodynamic” refers to both the hydrostatic and hydrodynamic component of water pressure in Figs.5.29 to 5.36.

Figs.5.29 to 5.36 present the stress time histories for the finite elements located in the vicinity of the points D and E on the dam for the gravity and the hydrodynamic loading. As is noted, the figures show rather large differences in the static component of the responses. However, the dynamic components are more or less the same in all these figures, showing close agreement between ADAP88 and BANDAAB2.

Considering that the actual response of the dam maybe somehow in between the two sets of results, and that the writer’s major concern in this thesis is to evaluate the effect of joint and concrete nonlinearities on the seismic response of arch dams, the results of the modified programs are deemed satisfactory for the purpose of this investigation.

Fig.5.1: Substructuring technique in "ADAP88"

Fig.5.2: The element types in the finite element model

Fig.5.3: Accelerograph for the stream component of the applied earthquake

Fig.5.4: Accelerograph for the vertical component of the applied earthquake

Fig.5.5: The finite element mesh for the arch dam analyzed

Fig.5.6: Stream component of the displacement on the upstream face at point D

Fig.5.7: Stream component of the relative displacement on the upstream face at point A (joint element)

Fig.5.8: Stream component of the relative displacement on the upstream face at point B (joint element)

Fig.5.9: Normal component of the relative displacement on the upstream face at point D (joint element)

Fig.5.10: Stream component of the relative displacement on the upstream face at point C (joint element)

Fig.5.11: Stream component of the relative displacement on the upstream face at point D (joint element)

Fig.5.12: Arch component of the stress on the downstream face for the 3D element (point D)

Fig.5.13: Cantilever component of the stress on the downstream face for the 3D element (pointD)

Fig.5.14: Arch component of the stress on the upstream face for the thick-shell element (point E)

Fig.5.15: Arch component of the stress on the downstream face for the thick-shell element (point D)

Fig.5.16: Cross stream component of the displacement on the upstream face at point D (hydrodynamic loading)

Fig.5.17: Stream component of the displacement on the upstream face at point D (hydrodynamic loading)

Fig.5.18: Vertical component of the displacement on the upstream point D (hydrodynamic loading)

Fig. 5.19: Vertical component of the relative displacement on the upstream face at point A (joint element, hydrodynamic loading)

Fig.5.20: Normal component of the relative displacement on the upstream face at point C (joint element, hydrodynamic loading)

Fig.5.21: Normal component of the relative displacement on the upstream face at point D (joint element, hydrodynamic loading)

Fig.5.22: Arch component of the stress on the upstream face for the 3D element (point D, hydrodynamic loading)

Fig.5.23: Arch component of the stress on the downstream face for the 3D element (point D, hydrodynamic loading)

Fig.5.24: Cantilever component of the stress on the downstream face for the 3 D element (point D, hydrodynamic loading)

Fig.5.25: Arch component of the stress on the upstream face for the 3 D element (point E, hydrodynamic loading)

Fig.5.26: Cantilever component of the stress on the upstream face for the 3 D element (point E, hydrodynamic loading)

Fig.5.27: Arch component of the stress on the downstream face for the thick shell element (point D, hydrodynamic loading)

Fig.5.28: Cantilever component of the stress on the downstream face for the thick shell element (point D, hydrodynamic loading)

Fig.5.29: Arch component of the stress on the upstream face for the 3 D element (point D , hydrodynamic+gravity loading)

Fig.5.30: Cantilever component of the stress on the upstream face for the 3 D element (point D, hydrodynamic+gravity loading)

Fig. 5.31: Arch component of the stress on the upstream face for the 3 D element (point E, hydrodynamic+gravity loading)

Fig.5.32: Cantilever component of the stress on the upstream face for the 3 D element (point E, hydrodynamic+gravity loading)

Fig.5.33: Arch component of the stress on the upstream face for the thick shell element (point D, hydrodynamic+gravity loading)

Fig.5.34: Cantilever component of the stress on the upstream face for the thick shell element (point D, hydrodynamic+gravity loading)

Fig.5.35: Arch component of the stress on the downstream face for the thick shell element (point D, hydrodynamic+gravity loading)

Fig.5.36: Cantilever component of the stress on the downstream face for the thick shell element (point D, hydrodynamic+gravity loading)

Chapter 6

Parametric Studies

6.1 Introduction

In this chapter the effects of contraction joints movements and the concrete material nonlinearities on the seismic behaviour of a typical arch dam are discussed. The proposed joint element model alone, has been implemented in the computer program BANDAAB1. To study the effects of the constitutive parameters of this model, the behaviour of the Morrow Point Dam, subjected to a typical ground motions in the stream direction is investigated. To study the effects of concrete nonlinearities, the nonlinear material models for concrete as presented in Chapter 4 are implemented in the computer program BANDAAB2 which also takes the opening-closing behaviour of the joints into account, as implemented in the original program ADAP88. In this part of the study, the preceding dam is subjected to the stream and the vertical components of the same ground motions. The results of the analyses are discussed in this chapter. The time histories of the applied ground motions are presented in Figs. 5.3 and 5.4.

Although a large number of response time histories were obtained, only a few of them are presented and discussed here. However, the conclusions drawn from the results presented here are equally applicable to other response histories. The response at the locations A,B,C,D,E and F of the dam as shown in Fig.6.1, are studied.

The time history responses of the arch dam are determined for the duration of the strong ground shaking of the applied earthquake record (10 seconds). In these analyses, the reservoir is assumed to be full which would produce the maximum inertial effect of the reservoir liquid loading.

For all the analyses of this chapter, the values of the Rayleigh's proportionality constants b_0 and b_1 are assumed to be 1.37 rad/sec and 0.0017 sec/rad , in order to provide a modal damping ratio of 5% in the first and fifth vibration modes of the monolithic model without the reservoir. The values of the Newmark's time integration constants i.e. β and γ are assumed to be 0.36 and 0.7 and the time step interval is taken to be 0.005 seconds .

6.2 Effect of Contraction Joints Movements

The three parameters introduced in the developed model, as discussed earlier, are the slip margin δ , the apparent cohesion c , and the friction angle ϕ . Seven parametric cases have been studied in order to investigate the effects of these parameters on the slippage behaviour of the contraction joints during an earthquake. These seven cases are summarized in Table 6.1.

The seven cases in Table 6.1 simulate the physical properties of different contraction joints. Case 1 simulates the contraction joints with very large and strong un beveled shear keys which are at the same time grouted with very rough grouting material. This case in reality models a no slippage situation. Case 2 represents the contraction joints with very large, but limited strength, un beveled shear keys, where a very rough grouting material is used. In case 3, it is assumed that the shear keys are very large and very strong, but the grouting material has moderate frictional resistance. This case is studied to investigate the possibility of the dominant effect of friction angle on the shear strength of the joints. Case 4 simulates a situation where very large un beveled shear keys are built. However, the strength of the keys and the frictional resistance of the grout material are limited. In Case 5 it is assumed that the joint has no shear keys or has beveled shear keys, but the shear resistance of the joint (cohesion and the friction angle) is very large. By studying the results of this case, the probable significance of free slippage in a joint of this type can be studied. In Case 6 the same assumptions are made for the shear keys as in Case 5. However, the shear resistance of the joint is limited. Case 7 is the same as Case 6 with a reduced value of the friction angle. The latter case intends to simulate the effects of water intrusion into joints and their probable lubricating effects. In the following sections the results of these case studies are discussed.

Before proceeding, it should be mentioned that Case 1 has been chosen as the basis for comparison of the results in the following discussions. The reason for the choice is that, Case 1 represents no shear slippage and comparing the results of other cases with Case 1 will show the significance of the effects of the various parameters of the joints on the response behaviour of the dam.

6.2.1 Un-beveled Shear Keys

By comparing the results from Cases 2 to 4 with Case 1 in the study of the behaviour of un-beveled shear keys of large size across the contraction joints, it is found that the shear strength of the keys and the friction angle of the grouting material have negligible effect on the behaviour of the contraction joints. Since similar responses are obtained in Cases 2 to 4, only the results from case 4 are presented here. In Fig.6.2 the stream component of the relative displacement for the joint element at point B, located on the upstream side, for Cases 1 and 4 is plotted. As can be seen in this figure, the effect of the strength parameters of the joint on the response history is not significant, and the two response histories are the same except at 7.4 second. The same phenomenon is observed in time histories of other response quantities. These observations confirm the notion that as long as the joint faces are engaged, sufficient resistance is provided by the shear keys to prevent shear slippage. It is clear that the assumed shear keys in this case study are strong enough to prevent slippage even if the joints are fully open across the thickness of the dam body, without the help of frictional resistance from the grout material. But as discussed in Chapter 2, it is recommended in the practice of dam engineering to install beveled shear keys in arch dams instead of un-beveled ones so as to avoid stress concentrations at the location of the keys.

6.2.2 Joint Slippage

Cases 5 to 7 are analysed to investigate the effects of free shear slippage in joints with beveled shear keys and joints without shear keys. The results are presented in this section. A slip margin of 3mm has been assumed for all these cases.

Joint Displacement Time Histories

The comparison of the results from Cases 1, 5 and 6 show that the dams with no shear keys, or with beveled shear keys, are very vulnerable to shear slippage across the joints. To show the significance of the effect, the vertical, stream and normal components of the relative displacements of the joint elements located on the upstream face of the dam at point B for Cases 1, 5 and 6 are plotted in Figs.6.3 to 6.5 respectively. The time histories shown in Figs.6.3 and 6.4 clearly indicate that large shear slippage within the joint element occurs at four separate occasions at 4.5 second, 5.2 second, 6.8 second and 8.8 second. At the instants of shear slippage, the joint opening has exceeded the slip margin of 3 mm,

as indicated in Fig. 6.5. The amount of slippage in the stream direction is one order of magnitude larger than the amount of slippage in the vertical direction, which is expected due to the direction of the applied ground motion. Similar behaviour can be observed in the three displacement components of the relative displacement of the contraction joint located at point C on the upstream face of the dam, as plotted in Figs.6.6 to 6.8.

By examining Figs.6.3, 6.4, 6.6 and 6.7 more closely, it can be seen that although the primary cause of slippage in a contraction joint is the loss of shear engagement, as a result of the opening of the joint beyond the slip margin, the strength of the shear key and the friction angle of the grout can also have some effect on slippage. Although the latter effect is substantially less than the effect of free slippage, nevertheless it can make a difference of up to 20% in the amount of maximum slippage as is observed in Fig.6.7. To determine if further reduction in friction angle, due to possible lubricating effect of the water, can have any effect on the slippage of the joints, Figs.6.9 and 6.10 are presented. In these two figures the time histories of the stream and vertical components of the relative displacement of the joint element at point C located on the upstream face of the dam are plotted. As observed in these figures, reduction of the friction angle from 45° to 25° may cause a small change of about 3% in the peak value of slippage. The effect is more pronounced in the stream component.

By considering the stream component of the relative displacement of the joints at points B and C as presented in Figs.6.4 and 6.7, one notices that the amount of slippage can reach up to 90 mm. This observation provides further evidence about the effectiveness of the beveled shear keys. Considering the assumed value of 3 mm for the slip margin, in this case study, and considering a typical shear key with side slopes of 25° , the maximum permissible value of the relative free slippage for such a key would be 6 mm, signifying that if the joint slips more than 6mm, the shear key will become effective as a barrier against free slippage, provided opening does not exceed the specified limit. So after the exceedance of the free slippage limit of 6mm, the shear strength of the key becomes effective against joint slippage and prevents any further free slippage. Even if a value of 10 mm is assumed for the maximum joint opening, the maximum value of free shear slippage is still limited to 20 mm, which is still far less than the 90 mm slippage observed in the stream direction in this investigation.

In conclusion one may state that, the amount of shear slippage can be significantly

reduced by the use of keyed contraction joints in an arch dam. The important role played by the shear keys in controlling the shear slippage across the contraction joints is confirmed by results obtained in the presented study.

In comparing Figs.6.5 and 6.8 where the time histories of the normal component of the relative displacement of the joints at points B and C are plotted, one observes that shear slippage can significantly affect the opening and closing behaviour of the joints. The maximum value of the joint opening can increase to a level up to 30% more than that in the case of no shear slippage, as can be observed in Fig.6.8. It is also observed in these two figures that the maximum penetration of the two faces of the joints can reach 5.8 mm. This is due to the rather smaller value of penalty parameters chosen for this case study to alleviate the problem of noise in the response and oscillations in the results. Part of this penetration can be justified, as it can be argued that the penetration is really the contraction deformation of the joint due to the presence of the grout material in the gap between the two monoliths.

Furthermore, it is observed that the slippage phenomenon has very little effect on the frequency characteristic of the response. This observation is not surprising but expected. The joint slippage is an instantaneous phenomenon, after which the joint comes into full engagement again. The structure regains its normal stiffness, and then vibrates with similar frequency characteristics about the new base-line as established due to the joint slippage.

Nodal Displacement Time Histories

Figs.6.11 to 6.16 show the time histories of the cross stream, stream and vertical components of the displacement at points A and C of Cases 1,5 and 6 on the upstream face of the dam. The results from these figures show that joint slippage affects the displacement field of an arch dam. Of the phenomena considered, joint slippage has a dominant effect on the behaviour of the joint, whereas the strength properties of the joint, as given by cohesion and friction angle, cause negligible differences in the behaviour. This is observed in all 3 components of the displacement. As can be observed in Fig.6.17 where the time histories of the vertical displacement at point B for cases 6 and 7 are plotted, further reduction in the friction angle cause negligible changes in the response.

Examining the results shown in Figs.6.11 to 6.13 more closely, it is concluded that the displacement time histories for Cases 5 and 6, after a sudden jump at about 1.5 second,

follow almost the same pattern of response, oscillating about the new base established by slippage at the time indicated by the sudden jump. A similar behaviour is observed in Figs.6.14 to 6.16, showing the instantaneous effect of joint slippage. The results indicate that slippage results in permanent change to the geometry of the arch dam after the earthquake stops. This phenomenon will affect the stress distribution in the dam and may have significant implications on the short and the long-term behaviour of the structure.

Stress Time Histories

Since the results show that un-beveled shear keys prevent shear slippage at the joints, any difference in the stress distribution of the dam of Cases 2 to 4 from Case 1 can not be attributed to shear slippage. Therefore, in the case of un-beveled shear keys, the nonlinear behaviour in the contraction joint is attributed to that arising from the opening closing behaviour, which has been studied in detail by other researchers in earlier studies. On the other hand, the phenomenon of free slippage may cause very significant changes to the stress distribution in an arch dam structure, which is discussed herein.

The time histories of the stress components in the cantilever and arch direction of the 3D elements located adjacent to points B and D for Cases 1 and 5 are plotted in Figs.6.18, 6.19, 6.22 and 6.24. Looking at these figures, one can observe that the cantilever stress component is more affected by shear slippage than the arch component, with a large permanent residual stress remaining in the dam after the earthquake. It is also observed that free slippage is still the dominant factor that has a major effect on the behaviour of the dam. However, as the results shown in Figs.6.20, 6.21 and 6.23 indicate, where the results from Cases 5 to 7 are compared, the resistance parameters of the joint, which are the cohesion and the friction angle, can also have some influence on the dynamic behaviour of the dam, as also observed earlier in the parametric study. In comparison, the friction angle has the smallest effect in the cases considered, as can be noted from results shown in Figs.6.20 and 6.21.

In Figs.6.25 to 6.31, the time histories for the vertical, stream and normal components of the stresses in joint elements located at points B, C and D are plotted. The results from Cases 1, 5, 6 and 7 are compared in these figures. As observed from these graphs, shear slippage at the contraction joints has a very significant effect on the stress time histories of the joints, particularly in the shear stress response. This phenomenon is observed in

the cases of contraction joints with no shear key and joints with beveled shear keys. In comparison, the normal stress is the component of the joint stress least affected by shear slippage. Comparing results shown in Figs.6.25, 6.26, 6.28 and 6.29, one observes that reducing the strength parameters of the joints, pre-dominantly the cohesion parameter, generally results in changes to the joint shear stresses. The results clearly indicate that the effect of the friction angle is negligible, as also observed previously. This effect can be seen in Fig.6.32 where the vertical component of the shear stress of the joint element at point C' for Cases 6 and 7 is plotted. As can be observed in Fig.6.33 where the vertical component of the shear stress of the joint element at point C' for Cases 1, 2 and 4 is presented, respectively, in the cases of joints with large shear keys, there are minor changes in the shear stresses at some joints due to the reduction of the apparent cohesion.

To investigate how the joint opening affects the shear stresses of the joint elements, the time histories of the vertical and stream components of the shear stresses for Case 5 of the joint element located at point B are plotted in Figs.6.34 and 6.35. Comparing the results in these two figures with that shown in Fig.6.5 where the time history of the normal component of the relative displacement of the same joint element is plotted, one can observe that at the instant 4.5 second, 5.2 second, 6.8 second and 8.8 second, the normal displacement has exceeded the assumed slip margin of 3 mm, resulting in no shear transfer across the joint and the shear stress has dropped to zero at these instants of time. On the other hand, comparing Figs.6.5 and 6.27 where the time histories of the normal relative displacement and the normal stress of the joint element at point B are presented, one can observe that at the instant of time when the joint is opened (e.g. 4.7 second and 8.8 second), there is no normal stress being transferred across the joint element.

The stress in the joint can have significant effects on the integrity of the concrete in the regions adjacent to the contraction joints. These areas have been mentioned in the literature (Hall and Dowling, 1985 and Dowling and Hall, 1989) to be susceptible to failure when the joint faces are partially open. Joint opening usually occurs at the upper levels of an arch dam, therefore shear slippage is generally confined to the upper levels of the dam. Although not presented here due to space limitations, by studying the response time histories at the points located on the lower part of the dam, it is observed that even without shear key, the lower levels of the dam do not experience any slippage even though the shear resistance parameters have only moderate values.

Stress Contours

To investigate the effect of slippage phenomenon on the overall behaviour of the dam, contours of the maximum envelopes for the arch and the cantilever stress components on the upstream and downstream faces of the dam for all the seven cases listed in Table 6.1 are determined. In the following discussion some of the results are presented.

Investigation shows that the stress contours of Cases 1 to 4 are very similar to one another with only minor differences. Therefore only details of the stress contours of Case 1 are presented. In Figs.6.36 to 6.38 the stress contours of the maximum cantilever stress on the downstream face of the dam for Cases 1, 5, and 6 are presented. It is observed that free slippage causes very significant changes in the stress contour pattern. Towards the upper levels of the dam, the effect of shear slippage becomes more pronounced. At some locations as observed in Figs.6.36 and 6.37, the maximum value of the stress component almost doubles. In Figs.6.39 to 6.42 the stress contours of the maximum arch stress on the upstream face of the dam for cases 1, 5, 6 and 7 are plotted. As observed earlier, although the free slippage is the dominant factor in determining the amount of shear slippage in the joints, the strength parameters of the contraction joints still have some minor effects on the behaviour of the dam, as can be observed in Figs.6.40 to 6.42. One can notice that the effect of friction angle is much less than the effect of the apparent cohesion by comparing the results shown in Fig.6.42 and Fig.6.41.

Contours of the cantilever stress component are more affected by shear slippage than those of the arch stress. The reason for this behaviour can be traced back to the excessive displacement at the upper parts of the dam in the stream direction caused by slippage of the joints resulting in over-stress of the dam in the cantilever direction.

Stress contours of the maximum cantilever and arch stresses for case 1 with penalty values of 1.38×10^{11} and 1.38×10^9 N/m³ are presented in Figs.6.43 and 6.44, and Figs.6.36 and 6.39, respectively. By comparing these figures one can observe that reducing the penalty values by two order of magnitudes does not significantly affect the stress distribution in term of the overall response of the dam. The effect is restricted to limited areas within the dam, and the maximum values of the stresses are not changed significantly. Reduction of the penalty values smoothes the convergence pattern of the analyses and solves the problem of oscillation inherent in many joint elements with only minor local effects on the joints.

behaviour.

6.2.3 Conclusion

The following conclusions can be drawn from the results of this study. Shear slippage across the contraction joints can cause very important changes in the displacement and stress field of an arch dam. The primary cause of this phenomenon is the opening of the joints beyond the slip margin. The probable permanent displacements at the joint faces will cause permanent changes in the displacement field of the dam causing permanent changes in the stress pattern of the structure. Since joint opening is most likely to be restricted to the upper parts of the dam, shear slippage will also occur in those areas. Shear keys, whether beveled or un-beveled are very efficient barriers against shear slippage. However, in the case of the beveled shear keys there is still a possibility of free shear slippage. Although the slip margin is the dominant factor affecting the shear slippage behaviour, the bearing or shear strength of the keys and the friction angle of the grouting material can still have some effects on the behaviour of the dams in case of dams with no shear keys or with beveled shear keys. Generally in case of contraction joints with no shear keys, the lower parts of the dam do not experience any slippage and the stress distribution in those areas is not affected. With a tool that can model the nonlinear shear slippage in a contraction joint, dam designers and engineers will have a better understanding of the dynamic behaviour of arch dams.

6.3 Effect of Concrete Nonlinearity

To study the effects of concrete nonlinearity on the seismic response of arch dams, the dam structure is analyzed subjected to both the stream and the vertical components of the ground motion with full reservoir. The three nonlinear concrete models, i.e. the tensile model, the bounding surface model and the combined model, have been implemented in the computer program BANDAB2. Since concrete cracking is the most probable mode of damage in concrete dams (NRC, 1990), a separate set of analyses have been carried out considering only the cracking effects. The tensile strength of concrete is assumed to be $0.15f'_c$ in the analyses. For comparison purpose only, a separate set of analyses have been carried out using only the bounding surface model. Results obtained from combining the

two models, which is considered to be the complete model, considering both compressive and tensile nonlinearity effects, are also presented.

Due to possible over-stress in the regions adjacent to the contraction joints, the material nonlinearity for the concrete is first restricted to the 3D solid elements adjacent to the contraction joints. Subsequently, in order to investigate the effect of this assumption, the zones of material nonlinearity are expanded to cover a larger portion of the dam. In the following, the results of these analyses are presented and discussed.

6.3.1 Restricted Nonlinear Zone

6.3.1.1 Damage Propagation

In order to investigate the propagation of damage within the dam, the damaged elements of the dam based on the implemented models have been identified. In the present study, the term "damage" is not intended to connote any concept of damage defined in the theory of damage mechanics. Here, if an integration point within an element experiences cracking or compressive failure during the response, the element is identified as damaged. The damaged elements identified after 1, 6 and 8 second of the ground motions are shown in Figs.6.45 to 6.53.

Tensile Model

In Figs.6.45 to 6.47, one can observe the progress of damage within the dam based on the tensile model. The damage first occurs mainly close to the heel of the dam and on the upstream face. As the damage spreads, majority of the damaged elements are located at the upper levels. The zones in the proximity of the first contraction joint closest to the abutment remain intact. On the other hand, some areas on the upstream face adjacent to the second contraction joint are damaged. For the areas next to the third and fourth contraction joint the damage is located either on the downstream face or on both the upstream and the downstream faces of the dam. Probably due to the restrictions imposed on the nonlinear concrete area, the damage seems to propagate towards the lower levels of the dam. This behaviour should be verified.

Bounding Surface Model

The damaged elements based on the bounding surface model are presented in Fig.6.48 to 6.50. In this case damage first occurs at the upper level of the dam close to the cen

ter contraction joint on the upstream face. Additional damage occurs at the heel on the upstream face, while at the same time damage of concrete spreads away from the center contraction joint on the downstream face, and also to the areas near to the third contraction joint. Finally, after 8 seconds of ground motions, nearly the entire area next to the joint at the center is damaged, while only a few elements close to the second joint experiences damage. Concrete close to the first contraction joint remains intact.

Combined Model

In Figs.6.51 to 6.53, the damaged elements based on the combined model are presented. In this case, damage to the dam occurs first at the heel, and in a few elements close to the center contraction joint, on the upstream face. Then damage spreads to the areas next to the second and third joints, and as time passes more elements are affected. Concrete next to the first contraction joint stays un-damaged.

One point should be mentioned here. In the implemented bounding surface model, there is no distinction between tensile and compressive modes of failure. As soon as the stress state satisfies the failure function, the point is considered failed. However, study of the damaged points based on the combined model, reveals that almost all of the damaged points (with only a few exceptions) are cracked, while the number of the failure points drops drastically, indicating that those elements have been failed by tensile over-stress and not by compression. In the combined model, the tensile stresses are re-distributed which tends to reduce the occurrence of compressive failure. This shows that the dominant mode of damage is tensile over stress, putting the effect of the compressive stresses in doubt. However as will be shown later, the results from the tensile model and the combined model differ significantly, which shows the effectiveness of the bounding surface model in nonlinear behaviour. On the other hand, a survey of the mode of fracture of the cracked points shows that the majority of the cracked points crack in either a compression-compression-tension or compression-tension-tension mode, which demonstrates the influence of the compressive principal stress(es) on reducing the tensile strength of concrete. Therefore, the understanding of crack propagation in an arch dam requires a model which considers the effect of compression on the tensile strength of concrete, and at the same time is capable of calculating the compressive stresses accurately.

6.3.1.2 Stress Contours

To investigate how the three different models affect the stress distribution in the dam, contours of the envelopes of the stress state on the upstream and downstream faces are studied. Since from the safety point of view, a dam engineer is more interested in the state of tensile stresses, the contours of the maximum arch and cantilever stresses on the upstream face from the elastic, the tensile, and the bounding surface and combined models are presented in Figs.6.54 to 6.61.

Tensile Model

Comparing Figs.6.54 and 6.55 in which the contours of the maximum arch stress on the upstream face for the elastic and tensile models are presented, one can see that both the magnitude and the distribution pattern of the contour lines for the arch stresses are affected due to cracking of concrete. The maximum tensile stress of 3.5 MPa that occurred in the elastic model close to the abutment near to the crest of the dam, and shown in Fig.6.54, reduces in magnitude in the case of the tensile model, as can be observed in Fig.6.55. On the other hand, the maximum value of 2.5 MPa for the elastic model near the crest on the centerline of the dam is reduced to 2 MPa in the tensile model. At the same time, a small region of the stress level 2.5 MPa occurs in the tensile model close to the crest and the second contraction joint as is observed in Fig.6.55. These changes in the stress distribution become more pronounced as one moves towards the upper levels of the dam, where more elements are affected by cracking of concrete. The same kind of observations can be made in Figs.6.58 and 6.59. Here, the contours of the maximum cantilever stresses on the upstream face for the elastic and tensile models are plotted.

Bounding Surface Model

Figs.6.56 and 6.60 present the contours of the maximum arch and cantilever stresses on the upstream face based on the bounding surface model. Comparing these contours with those presented in Figs.6.54 and 6.58, it is seen that the maximum value of the arch stress increases from 3.5 MPa for the elastic model to 4.5 MPa for the bounding surface model. The same trend is observed for the contours of the cantilever stress where the stresses by comparison also increases in the case of the bounding surface model.

Combined Model

Figs.6.57 and 6.61 present the contours of the maximum arch and cantilever stresses on the upstream face based on the combined model. Comparing these contours with their corresponding ones in Figs.6.54 and 6.58, one notices that the maximum value of the arch stress increases for the case of the combined model comparing to the results of the elastic model, but the maximum value of the cantilever stress drops from 4.5 MPa to 4 MPa. The pattern of the contour lines are also different.

6.3.1.3 Displacement Time Histories

In this section the effects of the three implemented models on the displacement time history of the dam are studied. Since the combined model is the most realistic compared with the other two models, the results from this model are first studied in relation to the results of the elastic model to investigate the significance of concrete nonlinearity. To understand how the results can be affected if any of the other two effects, which are the tensile model or the compressive nonlinearity, are ignored, the results from the three implemented models are compared with one another.

Combined Model

A study of the time histories of the displacement at points A to F shows that the full nonlinear behaviour of concrete, as modeled by the combined model, changes the response time histories of the dam. In Figs.6.62 to 6.64, the time histories of the stream component of the displacement at points C, D and E are plotted. These figures show that the consideration of concrete nonlinearity mainly affects the amplitude but not the frequency characteristics of the response. This may be attributed to the assumption of restricting the nonlinearity of concrete to occur within a small volume of the dam, resulting in rather negligible changes on the stiffness of the structure. The effect of nonlinearity becomes more pronounced after 1 seconds of the ground motion, where more and more elements are affected by cracking damage. As can be observed in Fig.6.64, the maximum value of the peak has increased from 15 mm to 19 mm in the nonlinear model.

Tensile Model

A comparison between the responses of the dam obtained from the combined model and the tensile model shows that the two models, give different responses. Comparing Figs.6.65 and 6.66 where the time histories of the stream component of displacements at points B and C from the three models are plotted, one can observe that the results of the tensile model

are very different from those of the combined model. The effect again concentrates on the amplitude of the response with the frequency content being more or less the same. The difference becomes more pronounced after about 5.5 seconds of the applied ground motion.

Bounding Surface Model

Figs.6.65 and 6.66, show that the results from the combined model and the bounding surface model are similar up to about 7 seconds of the applied ground motion. After this instant of time, the results from the two models begin to differ, and as one moves towards the end of the strong ground shaking, the differences become larger. The bounding surface model results in very large displacements in certain areas of the dam when a large number of the elements show failure (see Fig.6.66). This phenomenon is not observed in the cases of the other two models. The reason being that, in the implemented bounding surface model, failure of a point results in zero stress and rigidity. When a large number of points in an area show failure, the structure may then not be able to carry any further load in those areas resulting in large displacements.

6.3.1.4 Displacement Time Histories of the Joint Elements

To see how the three implemented models affect the opening closing response behaviour of the joint elements, the following discussion is presented.

Combined Model

In Figs.6.67 to 6.69 the normal components of the relative displacements for the joint elements on the upstream face of the dam at points B, C and D are presented. As shown in these figures, the concrete nonlinearity can increase the amount of joint opening by 10% at several instants of time. However, the frequency characteristics of the response stay more or less the same. As the joints open more, water can intrude further inside the joint with the possibility of increasing the hydrodynamic effects on the distribution of the load on the structure. This phenomenon should be studied in detail in future research.

Tensile Model

In Figs.6.70 and 6.71, the time histories for the normal component of the relative displacement for the joint elements on the upstream face at points A and D are presented. One can observe that the results from the tensile model differ significantly from those of the combined model. The effect can be an increase or decrease in the amount of the joint

opening. Fig.6.71 shows that the tensile model may result in 25% increase in the maximum opening of the joint element at point D.

Bounding Surface Model

Considering Figs.6.70 and 6.71, one can observe noticeable variations in the time histories of the normal component of the relative displacements from the bounding surface model compared to the combined model. It is observed from Fig.6.71 that after about 6.5 seconds of the ground motion, the results from the bounding surface model show extra softening of the structure, a phenomenon which is not observed in other cases.

The analyses from the bounding surface model and the combined model show a very smooth convergence pattern, whereas in the case of the tensile model some difficulties arise in obtaining convergence. It is not unusual to have convergence problems in dynamic cracking analysis of concrete structures, when a large number of points have cracked (Cervera et al, 1987). Experience of present study shows the stabilizing effect of the modelling of concrete compressive nonlinearity on the convergence behaviour of the analyses. The bounding surface model seems to absorb large amount of energy release due to crack opening, and reduces the possibility of noise in the iterative process of the analysis.

6.3.2 Expanded Nonlinear Zone

In order to investigate how allowing concrete to behave nonlinearly throughout a larger portion of the dam structure will affect the behaviour of the dam, the volume of the dam modeled by the 3D solid elements is extended to cover a larger portion of the dam. The finite element model of the new mesh is shown in Fig.6.72.

It is found that extending these zones too far away from the contraction joints may cause numerical problems if the thick shell elements become too narrow. To alleviate this problem, the thick shell elements should be modified to include a nonlinear material model.

To investigate the effects of the new finite element mesh on the results, the time histories of the stream component of the displacement at points A to E for both the old and the new mesh with the assumption of elastic behaviour for concrete have been studied first. The only source of nonlinearity is the opening-closing effect of the contraction joints. In Figs.6.73 and 6.74, the response time histories for the stream component of displacement at points C and D are compared. Up to about 4 seconds of the applied ground motions,

the two response histories are close to each other, after which they begin to differ. The difference should be attributed to the larger numerical errors creeping into the analyses due to the larger number of the degrees of freedom and considering that the problem is a nonlinear problem due to opening-closing effects of the joint elements.

6.3.2.1 Damage Propagation

To investigate how the damage propagates in the body of the dam if a larger portion of the dam is allowed to behave nonlinearly, the damaged patterns from the three models are studied. It should be mentioned again that, as one integration point within an element is damaged, the whole element is considered to be damaged and is marked in the following figures.

Tensile Model

In Figs.6.75 to 6.77, the damaged elements based on the tensile model, after 1, 6 and 8 seconds of the applied ground motions are shown. The concrete on the upstream face at the heel cracks first before any other location. Further on, more points at the upper levels of the dam on the downstream face are cracked. Although not favourable, the occurrence of cracks on the downstream face reduces the possibility of water penetration into the dam, which may cause some changes in the load distribution of the dam. It should also be noted that in contrast with the results obtained from the analysis based on the restricted nonlinear zones, where damage has been observed to propagate towards the lower levels of the structure, the damage can propagate horizontally in the present case. The results show that in this case the damage is less likely to propagate towards the lower levels of the dam. In the case of the old mesh, the damage is artificially forced to travel vertically to transfer the released stress from the previously damaged elements to the new ones, resulting in their over stress and possible damage.

The results presented here agree with the observation made in the stress contour presented in Figs.6.55 and 6.59. One can see from these figures that the large tensile stresses occur at the upper levels of the dam, and also in some restricted zone close to the abutments. Consequently, cracking is expected to occur in these areas, which is confirmed by the results obtained in this study. However, these observations are made with the fact that the areas close to the abutments modelled by the transition elements have not been modified to include material nonlinearity in the present study.

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Bounding Surface Model

Figs.6.78 to 6-80 show the damaged elements based on the bounding surface model and after 4, 6 and 8 seconds of the applied ground motions. Except some areas close to the center contraction joint, the damaged elements are mostly located on the downstream face. The lower levels of the dam are still safe from any damage.

Combined Model

Figs.6.81 to 6.83 show the damaged elements based on the combined model. The elements are mostly damaged due to cracking. In this case the upstream face of the dam also shows several damaged elements, which is a concern from the safety point of view. Fortunately damage is still restricted to the upper levels of the dam where the hydrodynamic pressure is not very large.

6.3.2.2 The Severity of Damage

Although Figs.6.45 to 6.53 and 6.75 to 6.83 show that a very large number of the 3D solid elements are damaged, a study on the extent of the damage for the three models shows that, in the case of the original mesh, the number of the damaged points are 12%, 8% and 17% of the total integration points in the 3D solid elements for the analyse with the bounding surface, tensile and the combined model, respectively. For the case of the new mesh, the above numbers change to 5%, 10% and 11% respectively. Considering the total number of the 3D solid elements of about 650 in the case of the new mesh, and for the sake of comparison one can say that at most only 65 equivalent 3D elements are completely damaged which cover a very small portion of the structure. On the other hand, in the case of the new finite element model, a rather large part of the dam is still modeled by the shell elements, resulting in the further reduction of the above numbers with respect to the entire dam. Therefore, it is concluded in this particular case study the dam is not severely damaged by the applied ground motion. However, the occurrence of cracks in the body of the dam is not desirable because of: (1) the reduction of the load bearing area of concrete; and (2) water penetration and possible effects on the load distribution within the dam.

6.3.2.3 Dam Response

A study of the time histories of the displacement components of the dam and the stress contours shows that the differences between the elastic model and the other three nonlinear

models are not significant in the case of the new finite element mesh. The four models give similar responses. This shows that, although the nonlinearity is allowed to occur in a larger portion of the dam, the damaged points are more scattered, and the possibility of localized failure is reduced. Considering this possibility and also the fact that the damaged portion of the dam is comparatively small with respect to the whole volume of the dam, this observation is justified.

In Figs.6.84 to 6.87, the time histories of the stream component of the displacements at points A, B, C and D obtained from the combined model for the old and the new finite element mesh and for the combined model are plotted. By the study of these figures one can observe the effect of the extension of the nonlinear zone on the response histories of the dam based on the combined model which is the most realistic one of the three implemented models. It is observed that by letting the nonlinear zone to expand, the peak values of the response history of the dam may increase. For example in Fig.6.86, the peak of the response increases from 62 mm to 78 mm which is an increase of about 25%.

6.3.3 Conclusion

Arch dams may be damaged during strong ground motions. The cause of damage is tensile cracking, forming mostly in the upper levels of the structure. However, the effect of compressive stresses on reducing the tensile strength of concrete was found to be important. Due to this reason, both compressive and tensile stresses should be calculated as precisely as possible. The possibility of concrete crushing due to compressive over-stress in the zones close to the contraction joints was found to be very small. In this particular case study, due to the rather small number of the damaged points in comparison to the overall volume of the dam, the effect of nonlinear behaviour of concrete on the response of the dam was found to be insignificant. However, by restricting the nonlinearity to the zones adjacent to the contraction joints, and due to localization of the damage, the behaviour of the dam was found to be affected significantly. In this situation the damage propagates towards the lower levels of the dam.

Table 6.1: Variation of Parameters for the joint model

Case No.	Slip margin δ (cm)	Apparent cohesion(c)	Friction angle(ϕ)	Remarks
1	320 (∞)	$10f'_c$	90°	Large and strong key, high friction
2	320 (∞)	$0.1f'_c$	90°	Large key with moderate strength, high friction
3	320 (∞)	$10f'_c$	45°	Large and strong key, moderate friction
4	320 (∞)	$0.1f'_c$	45°	Large key with moderate strength, moderate friction
5	0.0	$10f'_c$	90°	No key or beveled key, large shear resistance
6	0.0	$0.1f'_c$	45°	No key or beveled key, moderate shear resistance
7	0.0	$0.1f'_c$	25°	No key or beveled key, low shear resistance

Fig.6.1: The finite element mesh and the sampling points

Fig.6.2: Stream component of the relative displacement at point B on the upstream face (joint element)

Fig.6.3: Vertical component of the relative displacement at point B on the upstream face (joint element)

Fig.6.4: Stream component of the relative displacement at point B on the upstream face (joint element)

Fig.6.5: Normal component of the relative displacement at point B on the upstream face (joint element)

Fig.6.6: Vertical component of the relative displacement at point C on the upstream face (joint element)

Fig.6.7: Stream component of the relative displacement at point C on the upstream face (joint element)

Fig.6.8: Normal component of the relative displacement at point C on the upstream face (joint element)

Fig.6.9: Vertical component of the relative displacement at point C on the upstream face (joint element)

Fig.6.10: Stream component of the relative displacement at point C on the upstream face (joint element)

Fig.6.11: Cross stream component of the displacement on the upstream face at point A

Fig.6.12: Stream component of the displacement on the upstream face at point A

Fig.6.13: Vertical component of the displacement on the upstream face at point A

Fig.6.14: Cross stream component of the displacement on the upstream face at point C

Fig.6.15: Stream component of the displacement on the upstream face at point C

Fig.6.16: Vertical component of the displacement on the upstream face at point C

Fig.6.17: Vertical component of the displacement on the upstream face at point B

Fig.6.18: Cantilever component of the stress on the upstream face for the 3D element (point B)

Fig.6.19: Arch component of the stress on the upstream face for the 3D element (point B)

Fig.6.20: Cantilever component of the stress on the upstream face for the 3D element (point B)

Fig.6.21 Arch component of the stress on the upstream face for the 3D element (point B)

Fig.6.22: Cantilever component of stress on the upstream face for the 3D element (point D)

Fig.6.23 Cantilever component of the stress on the upstream face for the 3D element (point D)

Fig.6.24: Arch component of the stress on the upstream face for the 3D element (point D)

Fig.6.25: Vertical component of the shear stress at point B on the upstream face (joint element)

Fig.6.26 Stream component of the shear stress at point B on the upstream face (joint element)

**Fig.6.27: Normal component of the stress at point B
on the upstream face (joint element)**

Fig.6.28 Vertical component of the shear stress at point C on the upstream face (joint element)

Fig.6.29: Stream component of the shear stress at point C on the upstream face (joint element)

Fig.6.30: Normal component of the stress on the upstream face at point C (joint element)

Fig.6.31: Normal component of the stress on the upstream face at point D (joint element)

Fig.6.32: Vertical component of the shear stress on the upstream face at point C (joint element)

Fig.6.33: Vertical component of the shear stress on the upstream face at point C (joint element)

Fig.6.34: Vertical component of the shear stress on the upstream face at point B (joint element)

Fig.6.35: Stream component of the shear stress on the upstream face at point B (joint element)

Fig.6.36: Contours of maximum cantilever stress (MPa) on the downstream face for case 1

Fig.6.37: Contours of maximum cantilever stress (MPa) on the downstream face for case 5

Fig.6.38: Contours of maximum cantilever stress (MPa) on the downstream face for case 6

Fig.6.39: Contours of maximum arch stress (MPa) on the upstream face for case 1

Fig.6.40: Contours of maximum arch stress (MPa) on the upstream face for case 5

Fig.6.41: Contours of maximum arch stress (MPa) on the upstream face for case 6

Fig.6.42: Contours of maximum arch stress (MPa) on the upstream face for case 7

Fig.6.43: Contours of maximum cantilever stress (MPa) on the down stream face, higher penalty values

Fig.6.44: Contours of maximum arch stress (MPa) on the upstream face for higher penalty values

Fig.6.45: Damaged elements after 4 seconds of the applied ground motion
(based on the tensile model)

Fig.6.46: Damaged elements after 6 seconds of the applied ground motion
(based on the tensile model)

Fig.6.47: Damaged elements after 8 seconds of the applied ground motion
(based on the tensile model)

Fig.6.48: Damaged elements after 4.5 second of the applied ground motion
(based on the bounding surface model)

Fig.6.49: Damaged elements after 6 seconds of the applied ground motion
(based on the bounding surface model)

Fig.6.50: Damaged elements after 8 seconds of the applied ground motion
(based on the bounding surface model)

Fig.6.51: Damaged elements after 4 seconds of the applied ground motion
(based on the combined model)

Fig.6.52: Damaged elements after 6 seconds of the applied ground motion
(based on the combined model)

Fig.6.53: Damaged elements after 8 seconds of the applied ground motion
(based on the combined model)

Fig.6.54: Contours of maximum arch stress (MPa) on the upstream face
(based on the elastic model)

Fig.6.55: Contours of maximum arch stress (MPa) on the upstream face
(based on the tensile model)

Fig. 6.56: Contours of maximum arch stress (MPa) on the upstream face (based on the bounding surface model)

Fig.6.57: Contours of maximum arch stress (MPa) on the upstream face
(based on the combined model)

Fig.6.58: Contours of maximum cantilever stress (MPa) on the upstream face
(based on the elastic model)

Fig.6.59: Contours of maximum cantilever stress (MPa) on the upstream face
(based on the tensile model)

Fig.6.60: Contours of maximum cantilever stress (MPa) on the upstream face
(based on the bounding surface model)

Fig.6.61 : Contours of maximum cantilever stress (MPa) on the upstream face
(based on the combined model)

Fig.6.62: Stream component of the displacement on the upstream face at point C

Fig.6.63: Stream component of the displacement on the upstream face at point D

Fig.6.64: Stream component of the displacement on the upstream face at point E

Fig.6.65: Stream component of the displacement on the upstream face at point B

Fig.6.66: Stream component of the displacement on the upstream face at point C

Fig.6.67: Normal component of the relative displacement at point B on the upstream face (joint element)

Fig.6.68: Normal component of the relative displacement at point C on the upstream face (joint element)

Fig.6.69: Normal component of the relative displacement at point D on the upstream face (joint element)

Fig.6.7C: Normal component of the relative displacement at point A on the upstream face (joint element)

Fig.6.71: Normal component of the relative displacement at point D on the upstream face (joint element)

Fig.6.72: The new finite element mesh

Fig.6.73: Stream component of the displacement on the upstream face at point C

Fig.6.74: Stream component of the displacement on the upstream face at point D

Fig.6.75: Damaged elements after 4 seconds of the applied ground motion for the new mesh (based on the tensile model)

Fig.6.76: Damaged elements after 6 seconds of the applied ground motion for the new mesh (based on the tensile model)

Fig.6.77: Damaged elements after 8 seconds of the applied ground motion for the new mesh (based on the tensile model)

Fig. 6.78: Damaged elements after 4 seconds of the applied ground motion for the new mesh (based on the bounding surface model)

Fig.6.79: Damaged elements after 6 seconds of the applied ground motion for the new mesh (based on the bounding surface model)

Fig.6.80: Damaged elements after 8 seconds of the applied ground motion for the new mesh (based on the bounding surface model)

Fig.6.81: Damaged elements after 4 seconds of the applied ground motion for the new mesh (based on the combined model)

Fig.6.82: Damaged elements after 6 seconds of the applied ground motion for the new mesh (based on the combined model)

Fig.6.83: Damaged elements after 8 seconds of the applied ground motion for the new mesh (based on the combined model)

Fig.6.84: Stream component of the displacement on the upstream face at point (the combined model)

Fig.6.85: Stream component of the displacement on the upstream face at point B (the combined model)

Fig.6.86: Stream component of the displacement on the upstream face at point C (the combined model)

Fig.6.87: Stream Component of the displacement on the upstream face at point D (the combined Model)

Chapter 7

Summary and Conclusions

7.1 Summary

The primary objective of this research was to develop a finite element program which would be capable of analyzing the effects of the contraction joint movements, and concrete nonlinearity on the seismic response of arch dams. To achieve that objective the program ADAP88 was substantially modified and was endowed with nonlinear analysis capabilities as described below. The modified programs are called BAND AAB.

A joint element constitutive law was developed which is capable of modelling the effects of opening-closing, frictional slippage in closed state, free slippage in open state and the presence of the shear keys on the seismic response of arch dams. This model is implemented in the finite element program BAND AAB1. A bounding surface plasticity model and a three dimensional tensile cracking model were selected and implemented in the finite element code BAND AAB2 to study the effect of concrete compressive and tensile nonlinearities on the seismic behaviour of arch dams. A combined model based on the selected bounding surface and tensile models was developed and implemented in the computer program BAND AAB2. The latter model can account for the simultaneous effects of compressive nonlinearity and cracking on the dam response.

The Morrow Point Dam, a typical arch dam, was chosen and its seismic behaviour during typical ground motions was studied. The effects of joint shear slippage and concrete nonlinearities on the seismic behaviour of this arch dam during the applied ground motions were determined. In the following section the main conclusions of the study are presented, with particular reference to the results obtained from the analysis of the Morrow Point Dam.

7.2 Conclusions

At the outset it should be emphasized that the following conclusions mostly apply to this specific case study which involves a dam with a specific geometry and being subjected

to a particular ground motion. Clearly further studies would be needed to fully generalize the conclusions arrived at in the present investigation. However, by carefully observing the results of this study, some generalized conclusions may be arrived at.

7.2.1 Effect of Joint Slippage

- Assuming a value of $0.1f'_c$ for the strength of the shear keys, it was found that, un beveled shear keys do not allow shear slippage to occur across the contraction joints.

- Contraction joints with no shear keys are susceptible to severe shear slippage. the primary cause of slippage is the absence of any shear resistance when the joint is open. The shear slippage is an instantaneous phenomenon after which the joint regains its normal stiffness. Hence, shear slippage does not affect the frequency characteristics of the dam response.

- Although the lack of shear engagement in joints is the dominant cause of shear slippage, it was found that, the resistance parameters of the joint which are the friction angle and the apparent cohesion can change the amount of slippage. The apparent cohesion has the greater effect. Reduction of friction angle due to probable water lubrication effect was found to have minor effects on the seismic response of the dam.

- Shear slippage is restricted to the upper levels of the dam, where opening of the joints is more likely.

- Shear slippage may cause a permanent displacement change, affecting the geometry of the dam.

- Shear slippage may affect the opening of the contraction joints.

- Shear slippage affects the stress distribution within the dam. The upper levels of the dam are more affected.

- Joints with beveled shear keys are also susceptible to shear slippage. The amount of slippage would be much smaller than the cases with no shear keys, and is affected by the slope of the two sides of the key with respect to its surface.

By knowing the amount of slippage and its consequences on the level of stresses and safety of the dam, the dam designer would be able to determine the trade off between the high cost of shear keys installation and the consequences of slippage.

7.2.2 Effect of Concrete Nonlinearity

- During strong ground motions, arch dams are vulnerable to damage. The dominant mode of damage is cracking. Cracking mostly occur in the upper levels of the dam. The regions close to the abutments also experience large tensile stresses and may experience cracking.

- Cracks form on both the upstream and downstream face of the dam, but the downstream face was found to be more susceptible to cracking.

- Concrete crushing due to compressive over-stress, in the zones close to the contraction joints, was found to occur very rarely.

- Cracks mostly occur in a compression-compression tension or compression tension tension state of stress, signifying the effect of compressive stresses on the reduction in the tensile strength of concrete. Hence, the cracking criterion should take this phenomenon into account.

- Due to the small number of damaged points, the response history of the Morrow Point Dam was not affected by the combined effects of cracking and concrete nonlinearity. However, by restricting the nonlinear zone to regions close to the contraction joints, it was found that due to localization of damage, the response of the dam was affected severely. By restricting the nonlinear zone, damage is artificially forced to propagate to the lower levels of the dam, which is not the case in reality. By expanding the nonlinear zones, the damage propagates horizontally in the upper levels of the dam.

7.3 Recommendations for Future Work

- More parametric studies should be performed in order to show the effect of the constitutive parameters of the joint model on the behaviour of different dams subjected to different ground motions.

- The joint element model can be improved by considering the effects of the geometry of the beveled shear keys and parametric studies should be performed to investigate the effect of the enhanced model.

- The concrete nonlinearity should be extended to the whole body of the dam by modifying the thick shell as well as the three dimensional shell elements of the program.

More studies should be conducted about the effects of nonlinearity in other dams subjected to different ground motions.

- The foundation nonlinearity should be taken into account by implementing suitable material models. The developed program is capable of this study with some modifications.

- The joint model and the combined concrete model should be implemented in one program to study the effect of these sources of nonlinearity on the seismic response of arch dams.

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Appendix I

In this thesis the Morrow Point Dam was analyzed with the stream and/or vertical components of a typical earthquake ground motion record. The time histories of these two components are presented in Figs. 5.3 and 5.4. The pseudo-acceleration response spectra for these ground motion components are presented in Fig.I.1. The damping ratio is assumed to be 5% of the critical damping. The first vibration mode period of the monolithic model for the empty and full reservoir cases are identified in the figure. It is observed that the first mode period of the dam is located in the frequency range of the selected ground motion record with significant seismic action and energy.

Fig.1.1: Pseudo-acceleration Response Spectra for Earthquake Ground Motion (5% critical damping)

Appendix II

The vibration frequencies of the Morrow Point Dam with empty and full reservoir are presented in Fig.II.1. Assuming the water is incompressible, the added mass matrix of the reservoir is calculated and then diagonalized through a special procedure explained in the reference (Fenves et al 1989).

**Table II.1: Vibration Frequencies of Morrow Point Dam
(After Fenves et al 1989)**

Vibration Frequency (rad/sec)		
Mode	Empty Reservoir	Full Reservoir
1	20.31	15.73
2	22.34	16.43
3	35.40	22.85
4	37.47	25.00
5	40.40	27.50
6	43.60	31.77
7	48.22	33.50
8	54.12	37.84
9	60.58	42.13
10	60.80	44.13

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