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Adaptive Equalization and Diversity Combining for a Mobile Radio Channel

by

Norm W.K. Lo, B.A.Sc.

**A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of
Master of Engineering**

**Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
Department of Systems and Computer Engineering
Carleton University**

May, 1990

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Combining for a Mobile Radio Channel”**

submitted by Norm W.K. Lo, B.A.Sc.
in partial fulfilment of the requirements for
the degree of Master of Engineering

Thesis Co-Supervisor

Thesis Co-Supervisor

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May, 1990

Abstract

Digital mobile radio (DMR) communications occur over a fading multipath channel. In the case of low bit-rate, narrowband communications, some form of adaptive equalization and diversity combining is required to mitigate the intersymbol interference and rapid time variations in the DMR channel. In this thesis, we study the performance of a DMR system which employs a jointly adaptive decision-feedback equalizer and diversity combiner whose parameters are periodically computed with current estimates of the channel impulse response (CIR) at each diversity branch. We employ a novel block-adaptive strategy which computes the time-varying CIR by interpolating a set of CIR estimates obtained through periodic training. This interpolation strategy has the advantage of immunity to decision errors which would quite likely occur during a deep fade. We show that, despite incurring some inherent processing delay and throughput reduction, CIR interpolation is feasible for tracking a rapidly fading channel. We found via computer simulation that reliable digital communications can be achieved with our proposed receiver and block-adaptive scheme. Furthermore, we discovered that the system performance is limited, in the form of an irreducible bit error rate at high signal-to-noise ratios, by the CIR estimation of the rapidly fading channel.

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List of Symbols

α	– attenuation factor for a continuous channel
α_m	– rms gain of the m th tap in the FIR multipath channel model
β	– number of symbols per transmitted frame
γ	– instantaneous channel SNR
Γ_s	– average channel SNR per symbol
Γ	– average channel SNR per diversity branch
$\delta(\ )$	– delta function
δ_n	– relative frequency scaling factor for the n th Doppler frequency
$\Delta_a(kT)$	– transmitted $\frac{\pi}{4}$ –shifted, DQPSK data symbol
$\hat{\Delta}_a(kT)$	– received $\frac{\pi}{4}$ –shifted, DQPSK data symbol
$\Delta_t(kT)$	– transmitted $\frac{\pi}{4}$ –shifted, DQPSK training symbol
ϵ	– excess transmission bandwidth parameter
η	– interpolator window parameter (e.g., rolloff factor)
θ	– phase of a DMR CIR
λ	– wavelength
λ_a	– average arrival rate of a Poisson process
μ	– number of symbol periods between interpolated CIR samples
π	– “pi” constant = 3.14159
$\Pi(r; \eta)$	– windowing function of the interpolator
$\rho(d)$	– normalized spatial correlation of antennas
σ_g^2	– mean rms power (i.e., variance) of a DMR CIR
σ_n^2	– AWGN variance
τ	– propagation delay in the channel
τ_{rms}	– rms delay spread of the channel
τ_{ave}	– average delay spread of the channel

ϕ	– incident angle of signal reception
$\Phi(\Delta f)$	– spaced-frequency correlation function
$\Phi(\Delta t)$	– spaced-time correlation function
$\Phi(\tau; \delta t)$	– autocorrelation function of the actual baseband CIR
$\Phi(\tau)$	– multipath rms power delay profile (MPDP)
Φ	– autocorrelation matrix of the OCDFE
Φ_a	– BPSK autocorrelation matrix for BLS training
Φ_{Δ}	– $\frac{\pi}{4}$ -shifted, DQPSK autocorrelation matrix for BLS training
$\Phi_n((k-l)\frac{T}{2})$	– autocorrelation function of the received-filtered additive noise
ω_d	– digital frequency of an estimated CIR
ω'_d	– digital frequency of an interpolated CIR
$a(kT)$	– input QPSK data symbol
$\hat{a}(kT)$	– output QPSK data symbol
$a_t(kT)$	– input BPSK training symbol
B	– signal transmission bandwidth in real passband
B_c	– coherence bandwidth of a fading multipath channel
B_{rms}	– rms Doppler spread of channel
c	– speed of light (2.99×10^8 meters/second)
d	– distance between antennas
d_m	– normalized Doppler fading component of the m th multipath
D_t	– interpolation processing delay
DT	– processing delay in symbol periods of the OCDFE
e	– natural exponent constant = 2.71828
e_s	– number of symbol errors
$E[\ ]$	– expectation operator
E_b	– energy per bit
E_z, H_x, H_y	– field components of electromagnetic waves

f	– frequency (Fourier transform of the delay τ)
f_c	– carrier frequency
f_d	– Doppler frequency
f_{dm}	– maximum Doppler frequency
$f(r)$	– unit sample response of a realistic reconstruction filter/interpolator
$\tilde{f}(r)$	– unit sample response of an ideal reconstruction filter/interpolator
$F(e^{j\omega'})$	– frequency response of a realistic reconstruction filter/interpolator
$\tilde{F}(e^{j\omega'})$	– frequency response of an ideal reconstruction filter/interpolator
$\tilde{g}_m$	– signal response at the m th multipath channel output
$g(\tau; t)$	– actual baseband CIR
G	– envelope of a DMR CIR
$G(f; t)$	– channel transfer function
$h_c(t)$	– continuous-time CIR signal
$\underline{h}(qT_s)$	– estimated CIR from training
$\hat{\underline{h}}(rT'_s)$	– interpolated CIR sample
$\bar{\underline{h}}(rT'_s)$	– oversampled estimated CIR from training
$H(e^{j\omega})$	– frequency response of each component of an estimated CIR
$\bar{H}(e^{j\omega'})$	– frequency response of each oversampled estimated CIR component
$H(f)$	– frequency response of transmit/receive filters
j	– imaginary number constant = $\sqrt{-1}$
$J_0(x)$	– zeroth order Bessel function
k	– time index for the data symbol interval
L	– number of diversity channels
m	– symbol alphabet size (e.g., $m = 4$ for QPSK signaling)
$\overline{mse}$	– time averaged interpolation MSE
$n_l(t)$	– receive-filtered additive noise in the l th diversity channel

N	– total number of transmitted symbols in an ensemble of data frames
N_c	– T-spaced overall CIR length
N_d	– number of symbols per data sequence
N_o	– noise spectral density
N_p	– length in symbol periods of the unit sample response of the interpolator
$N_s + 1$	– number of weighted sinusoids in the “sum of sines” channel model
N_t	– number of symbols per training sequence
$\underline{p}$	– cross-correlation vector of the OCDFE
$\underline{p}_t$	– cross-correlation vector for BLS training
$\overline{P}_e$	– average probability of bit error (BER)
$\overline{P}_s$	– average probability of symbol error
q	– time index for the CIR training interval
Q	– number of CIR samples used for each interpolation
r	– time index for the CIR interpolation interval
$r(t)$	– received complex baseband signal
$r_l(kT)$	– received signal at the l th diversity branch
R	– interpolation factor
$s_l(kT)$	– desired signal
s_n	– n th Doppler rms component in the “sum of sines” model
$S(\tau; f_d)$	– scattering function
$S(f_d)$	– Doppler power spectrum
t	– time dependence of a fading channel
$\overline{tmse}$	– time averaged total interpolation MSE
T	– symbol period
T_c	– coherence time of a fading multipath channel
%T	– percentage data throughput
T_s	– CIR sampling period before interpolation

- T'_s – CIR sampling period after interpolation
- $u(t)$ – transmitted complex baseband signal
- $v(t)$ – output signal of a complex baseband channel
- v_m – mobile speed
- $\underline{w}$ – weight coefficient vector of the overall OCDFE
- $\underline{w}_l$ – OCDFE weight coefficient vector of the l th diversity branch
- $\underline{w}_b$ – OCDFE weight coefficient vector of the feedback branch
- $W = \frac{B}{2}$ – signal transmission bandwidth in complex baseband
- $\underline{x}(kT)$ – input signal vector of the overall OCDFE
- $y(kT)$ – $\frac{1}{T}$ –sampled OCDFE output signal (input to the decision device)

List of Acronyms

ACI	– Adjacent Channel Interference
AWGN	– Additive White Gaussian Noise
BER	– Probability of Bit Error
BLS	– Block Least Squares
BPSK	– Binary Phase Shift Keying
CCI	– Co-channel Interference
CDMA	– Code Division Multiple Access
CELP	– Code Excited Linear Prediction
CIR	– Channel Impulse Response
CTF	– Channel Transfer Function
dB	– decibel
DCR	– Digital Cellular Radio
DFE	– Decision-Feedback Equalizer
DMR	– Digital Mobile Radio
DPSK	– Differentially Encoded Phase Shift Keying
DQPSK	– Differentially Encoded Quadrature Phase Shift Keying
DSP	– Digital Signal Processing
FDMA	– Frequency Division Multiple Access
FIR	– Finite Impulse Response
FSE	– Fractionally Spaced Equalizer
HF	– High Frequency
Hz	– Hertz (cycles per second)
IEEE	– Institute of Electrical and Electronics Engineers
IIR	– Infinite Impulse Response
ISDN	– Integrated Services Digital Network

ISI	– Intersymbol Interference
kbps	– 10^3 bits per second
kHz	– 10^3 Hertz
kph	– kilometers per hour
ksps	– 10^3 symbols per second
LE	– Linear Equalizer
LMS	– Least Mean Square
LS	– Least Squares
MHz	– 10^6 Hertz
MLSE	– Maximum Length Sequence Estimator
MMSE	– Minimum Mean Squared Error
MPDP	– Multipath Power Delay Profile
ms	– 10^{-3} seconds
MSE	– Mean Squared Error
OCDFE	– Optimum Combiner and Decision-Feedback Equalizer
PAM	– Pulse Amplitude Modulation
pdf	– probability density function
PLL	– Phase-Locked Loop
PSK	– Phase Shift Keying
RC	– Raised Cosine
RELDP	– Regular-Pulse Excited Linear Prediction
RLS	– Recursive Least Squares
RMS	– Root Mean Squared
SER	– Signal to Estimation Noise Ratio
SFCF	– Spaced-Frequency Correlation Function
SHF	– Super-High Frequency
SNR	– Signal to Noise Ratio

SRRC   - Square Root Raised Cosine
STCF   - Spaced-Time Correlation Function
TDL   - Tapped Delay Line
TDMA   - Time Division Multiple Access
UHF   - Ultra-High Frequency
VLSI   - Very Large Scale Integration
WSSUS   - Wide-Sense Stationary Uncorrelated Scattering
ZF   - Zero Forcing

Chapter 1 Introduction

1.1 Digital Mobile Communications

With the current popular demand for mobile communications, cellular mobile radio continues to be a rapidly growing industry which requires much strategic planning, research and development. The future trend is to replace the existing analogue technology of cellular mobile radio systems with digital technology. Some of the advantages of digital technology include the availability of powerful and relatively inexpensive VLSI/DSP hardware, superior speech quality, reduced transmission noise sensitivity, greater data security, increasing user demand for data communications and a natural extension to future ISDN implementation [1, 2].

One important objective in the design of a digital mobile radio (DMR) communication system is to maximize its user capacity. Thus, efficient utilization of the limited available bandwidth is of paramount importance. To accomplish this, researchers are developing low bit rate speech coders (e.g., RELP and CELP coders) [3, 4], bandwidth efficient modulation techniques (e.g., minimum phase and Trellis coding), multiple access schemes (e.g., spread spectrum/CDMA, TDMA and FDMA) and optimum strategies for frequency reuse and cell splitting in cellular mobile radio systems [5, 6].

At the same time, one must address the more fundamental issue of providing reliable digital communications over a mobile radio channel. This objective is quite challenging, given the difficult nature of such a communications channel. Specifically, the mobile radio transmission medium is an inherently Rayleigh fading multipath channel. Thus, mobile radio communications is subject to signal dispersion (due to frequency-selective multipath fading) and time-variant signal propagation (due to vehicular motion). These two channel properties can introduce intersymbol interference (ISI) and random Doppler fading of the received signal, respectively [5, 6, 7].

In addition to ISI, Doppler fading and the ubiquitous additive white Gaussian noise (AWGN), the impairments of adjacent channel and co-channel interference (ACI and CCI) can also be present in mobile radio communications. ACI and CCI arise

consequentially from channel spacing reduction (in frequency) and cell splitting (in space) [6]. Furthermore, the requirement of stringent transmit and receive filtering, to minimize bandwidth requirements and to limit the adverse effects of ACI and CCI, can severely aggravate the ISI problem [8].

In general, ACI and CCI limit the system user capacity per unit area, ISI limits the maximum data rate, while Doppler fading limits the coverage area [9, 10]. Ultimately, all of these impairments can lead to a severe degradation in the performance of the communications system. Thus, the proper communications technique and receiver structure are required to overcome these impairments and, thereby, to ensure reliable digital communications over a mobile radio channel.

1.2 Thesis Motivation

The North American standard for digital cellular radio (DCR) communications is presently being formulated by planning groups from the communications industry and research institutions. The currently proposed system incorporates narrowband TDMA with three users occupying a frequency channel of 30 kHz bandwidth and providing an overall bit rate of 48 $kbps$. The modulation scheme is $\pi/4$ -shifted differentially encoded quadrature phase shift keying (DQPSK) with a symbol rate, $\frac{1}{T}$, of 24 ksp/s .¹ Thus, the minimum transmission bandwidth is 24 kHz . Furthermore, the proposed transmit and receive filters have small excess bandwidths corresponding to tight rolloffs of 35% [11].

With the tight filtering that is proposed for the DCR system, data symbol dispersion can span several symbol periods [8]. Consequently, ISI can be a significant impairment. In addition, when mobiles are at highway speeds, the radio channel exhibits Doppler fading rates of up to 100 Hz . Consequently, for the relatively low symbol rate of 24 ksp/s , a DMR signal amplitude can experience fast Doppler fading. Given the worst-case scenario of severe ISI and rapid Doppler fading, some form of adaptive equalization and diversity combining is required to overcome these channel impairments.

---

¹ The corresponding symbol period, T , is 41.7 μs .

1.3 Thesis Description

In this thesis research, we investigate the performance of a DMR reception technique employing adaptive equalization, antenna diversity and optimum (in the mean squared sense) combining. The essential objective of the reception strategy is channel amelioration with the desired end result being a minimally distorted channel. Specifically, the aim is to use an adaptive reception strategy and structure to minimize the effects of the channel impairments. The channel impairments of frequency-selectivity, Doppler fading and additive noise are considered in this thesis research. The reduction of ISI is accomplished with a decision-feedback equalizer (DFE), while the detrimental effect of Doppler fading is alleviated by block adaptation, explicit diversity combining and the implicit diversity which is inherent in a frequency-selective, fading channel [7, 12]. We do not investigate the effects of ACI and CCI in our present research. However, the tight filtering and diversity combining employed in the receiver would tend to minimize these interference effects. Furthermore, we assume perfect carrier recovery and timing synchronization.

For a relatively slow-fading channel, the equalization problem is quite tractable. A typical solution employing a DFE of sufficient length (to span most of the overall channel impulse response (CIR)) can reduce the ISI to an acceptable level. However, for the case of the DMR channel, the rapid Doppler fading can severely hamper the channel equalization task. Furthermore, even if ISI were not a problem (as in the case of "flat" fading), the rapid time variations in the CIR can still cause frequent deep fades in the received signal. To overcome this problem of rapid fading, one can increase the signaling rate at the expense of introducing more ISI [13]. However, given the constraints of a relatively low signaling rate of 24 *kbps* for the North American DCR standard, one must resort to other solutions like the use of diversity combining and adaptive techniques.

The block adaptation strategy is based on a signaling format which partitions the transmitted data symbols into contiguous frames, with each frame comprising of an initial, known training sequence followed by a data sequence [14, 15]. Each known training sequence is used to estimate the CIR of each diversity channel. Thus, the

OCDFE² receiver can be periodically computed at the beginning of each data frame from the updated CIR estimates.

However, the random time variation of a DMR channel can be quite severe during a typical data block sequence. Consequently, learning the channel at just the start of each data sequence is not sufficient to follow the time variations of the channel. Therefore, one must either increase the frequency of training (at the expense of a reduction in the system throughput) and/or track the fast fading channel with an adaptive algorithm. As an example of the latter strategy, a continuous, symbol-by-symbol adaptive algorithm such as the Least Mean Square (LMS) algorithm may be used to track the randomly changing CIR during a data block sequence. However, it has been found that, due to its sensitivity to DFE error propagation, the decision-directed LMS adaptive algorithm cannot properly track a worst-case (i.e., maximum Doppler fade rate of 100 Hz) DMR channel. A more complex adaptive algorithm such as the Recursive Least Squares (RLS) algorithm would not improve the CIR tracking, since it has been shown that LMS can perform CIR tracking at least as well as RLS [16, 17].

In this thesis research, we investigate a novel method of block adaptation for tracking a time-varying CIR during a data block sequence. Instead of explicitly tracking the time-varying CIR by using a continuous, decision-directed adaptive algorithm like LMS, we employ a block adaptation strategy which computes the time-varying CIR by interpolating a set of estimated CIR values (obtained through periodic training) from adjacent block sequences. This adaptation strategy does not continuously track the channel and it does not require receiver decisions. Consequently, it has the advantage of immunity to decision errors which can quite likely occur during a deep fade. However, it has the disadvantage of inherent processing delay and possible reduction in system throughput. With this new approach, both the training and tracking of the rapidly fading DMR channel employ block adaptation.

---

² The acronym, OCDFE, stands for Optimum Combiner and Decision-Feedback Equalizer.

1.4 Related Research

The current interest in digital communications over fading multipath media such as the HF and mobile radio channels is typified by the recent special issues in IEEE journals [18, 19, 20]. Specifically, adaptive equalization and diversity combining for these channels have been and are being intensely researched. Since Monsen first analyzed the performance of a DFE for a fading dispersive channel [12], many other researchers have investigated equalization techniques for such a channel. For instance, Hsu and Tront et.al. studied the performance of a Kalman DFE for the HF channel while Davidson et.al. investigated the performance of block Kalman and LMS DFEs for the HF channel [14, 21, 22, 23]. In addition, Ling and Proakis studied the LS and LMS lattice DFE for the HF channel [24]. For the wideband TDMA Pan-European DMR system, D'Avella et.al. and Uesugi et.al. recently proposed an MLSE detector and an LMS DFE, respectively [13, 25]. On the other hand, Hattori et.al. investigated a Kalman DFE with two-branch post-detection diversity for the DMR channel [26].

Bello and Nelin were among the first researchers to study diversity combining for time-dispersive fading channels [27]. Later, Glance and Greenstein investigated diversity combining for a quasi-static frequency-selective DMR channel [28]. Winters also investigated optimum combining to mitigate CCI in a quasi-static flat fading DMR channel [29]. Meanwhile, Monsen studied MMSE equalization for quasi-static diversity channels with ACI [30]. Recently, Adachi et.al., Hattori et.al. and Yamao et.al. investigated two-branch post-detection diversity receivers for the DMR channel [26, 31, 32].

The above referenced research either assumed negligible time variation in the channel (i.e., quasi-static) or direct adaptation of the equalizer/combiner parameters. However, an alternative adaptation approach is to compute the equalizer/combiner coefficients from current channel estimates [8]. McLaughlin et.al. investigated continuously adaptive LS and LMS CIR estimation for HF channels [17] while Crozier et.al. investigated a block LS adaptive algorithm for CIR estimation for a DMR channel [15, 33]. Crozier used this block LS algorithm for periodic training while Davidson used an RLS algorithm

for block-by-block training [14, 15, 23, 33]. Furthermore, Cavers studied a flat fading DMR system employing MMSE interpolation to provide amplitude and phase reference for detection [34]. Cavers' interpolation of a single complex channel component (i.e., essentially a one-tap CIR) differs from our approach of estimating and interpolating the CIR of a time-dispersive fading channel.

1.5 Thesis Organization

This thesis report is organized as follows. Initially in chapter two, we review the salient properties and characteristics of the mobile radio channel. In particular, we discuss the time dispersion due to multipath and frequency dispersion due to Doppler fading. Furthermore, we describe the FIR and "sum of sines" models of a multipath Rayleigh fading channel. In chapter three, we present a brief overview of adaptive equalization and diversity combining techniques used to mitigate intersymbol interference and fading. Specifically, we concentrate on block-adaptive strategies, decision-feedback equalization and optimum diversity combining. In chapter four, we describe the DMR communications system which we investigated in our research. In addition to describing the channel model, signaling format, modulation technique and transmit/receive filtering, we also discuss, in detail, the channel estimation and interpolation approaches used in our DMR system. Furthermore, we derive the Wiener solution for the computation of the optimum OCDFE coefficients in terms of the channel impulse responses and noise statistics. In chapter five, we present and discuss results of our computer simulation experiments of the fixed interpolation and system performance. In addition, we describe the system simulation methodology and the interpolation and system performance criteria. Finally in chapter six, we complete the thesis with a concluding discussion and recommendations for further research.

Chapter 2 The Mobile Radio Channel

2.1 Properties of the Mobile Radio Channel

The mobile radio channel is characterized by dispersive signal propagation (due to multipath delay spreads) and spatial variations in the received signal strength (due to multipath fading) [35].³ The severity with which these channel distortions impair a transmitted signal depends on the ratio of the signaling rate, $\frac{1}{T}$, (or, equivalently, the signal transmission bandwidth, B)⁴ to the channel coherence bandwidth, B_c , and (RMS) Doppler spread, B_{rms} , respectively [7, 36]. For mobile radio communications, both time dispersion and rapid fading of the received signal are significant problems which require adaptive signal processing at the receiver. Moreover, in addition to the thermal noise (AWGN) that is intrinsic to any communication system, ACI and CCI can also be significant impairments in a cellular mobile radio system [5, 6].

Multipath fading is due to signal propagation via multiple, randomly time-varying paths resulting from terrain and building reflection and scattering of the transmitted signal. In general, for a given transmitted signal, there are a (possibly infinite) number of potential received signals with random time-varying amplitudes and phases which add with varying constructive and destructive interference to produce a net received signal. For the mobile radio channel, the random nature of this multipath fading is characterized by Rayleigh statistics. Specifically, the received signal is modelled with a Rayleigh distributed envelope and a uniformly distributed phase. This follows from the assumption that all scatterers and reflectors are in random motion relative to a moving

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³ We are not considering the spectrally flat and lognormally distributed shadow fading that is caused by 'macroscopic' signal blockage and attenuation by large obstacles such as buildings [5, 6].

⁴ For a complex baseband modulation scheme like QPSK, its real passband signal bandwidth, B , is twice that of its one-sided, complex baseband bandwidth. Thus, the Nyquist criterion for zero ISI requires that $\frac{1}{T} \leq 2(\frac{B}{2}) = B$, with $B = \frac{1}{T}$ being the minimum bandwidth.

mobile so that the received power is uniformly distributed in angle. Thus, a mobile radio transmission medium can be described by a zero mean channel impulse response⁵ [5, 6].

In general, a transmitted signal over a fading multipath channel will result in a multiple number of received signals all having different propagation delays. Consequently, the sum of all these multipath signals will produce a net received signal which can be dispersed in time. For this situation, the channel is frequency-selective [7]. However, if the propagation delay is the same for all frequency components of the transmitted signal (i.e., linear phase), then the channel exhibits frequency-nonselectivity or "flat" fading.⁶ This frequency-nonselectivity implies that there is no time dispersion in the received signal.

The degree of time dispersion in a received signal depends on factors such as antenna height and the ratio of the channel multipath (RMS) spread, τ_{rms} , to the signaling period, T .⁷ For example, some urban mobile radio environments can have multipath delay spreads of up to tens of microseconds [37, 38]. Thus, for a typical DMR system with a sufficiently high signaling rate, the channel can indeed be frequency-selective. This is especially true if the transmit and receive filters of the DMR system have long impulse responses which effectively increases the span of the overall channel impulse response [8].

In mobile radio communications, temporal fading occurs mainly because of vehicular motion which can produce Doppler shifts of up to 100 Hz in the received signal [36]. As a mobile is in motion, the positions of buildings and terrain are randomly changing relative to the receiver at the mobile. Thus, the multipath channel is, in effect, randomly changing with time due to the vehicular motion. Furthermore, the various multipath signals interfere with each other to generate a spatial, standing wave signal pattern. As

⁵ If there are both fixed (e.g., line of sight propagation) and randomly moving scatterers, e.g., in nonurban DMR environments or in satellite communications, then the channel exhibits Ricean fading with a nonzero mean impulse response [7, 36].

⁶ If the signal amplitude is not randomly fading, then this frequency-nonselective channel reduces to an ideal, distortionless transmission medium.

⁷ Or, equivalently, the ratio of the signaling rate, $\frac{1}{T}$, to the channel coherence bandwidth, B_c .

a mobile moves through this standing wave pattern, its received signal strength fluctuates randomly, with successive minima (i.e., deep fades) at about every half a wavelength of the transmitted signal [5, 6]. This phenomenon is a result of both the dispersive nature of multipath propagation and the random nature of the fading channel due to vehicular motion. Thus, even when the channel is frequency-nonselective, the received signal can still experience deep fades.

The effect of this fading rate depends on factors such as the ratio of the signal transmission bandwidth, B , to the (RMS) Doppler spread of the channel, B_{rms} ,⁸ carrier frequency, f_c , and mobile velocity, v_m . For example, typical DMR communications at a signaling rate of 24 *kps* and a mobile velocity of 120 *kph*, has a channel impulse response which can vary up to 2.6% of its peak amplitude in a symbol period (see Appendix A). This may be considered slow fading for symbol detection, but rather fast fading for a block-adaptive receiver which tracks the time-varying channel by processing blocks of tens or hundreds of symbols at a time.

2.2 Characteristics of a Fading Multipath Channel

In general, a fading multipath transmission medium such as the Rayleigh fading mobile radio channel can be sufficiently characterized by second order statistics of its complex baseband impulse response, $g(\tau; t)$. Specifically, these relevant second order statistics are the autocorrelation functions and power spectral densities of the channel. With the assumption that the channel is wide sense stationary with uncorrelated scattering (WSSUS)⁹, the fading multipath channel can be characterized by its autocorrelation function¹⁰, $\Phi(\tau; \Delta t)$ [7, 36]:

$$\frac{1}{2} E[g^*(\tau; t)g(\tau + \Delta\tau; t + \Delta t)] = \Phi(\tau; \Delta t)\delta(\Delta\tau) \quad (2.1)$$

where the time difference, Δt , simplification follows from the WSS assumption and the delta function of the delay, $\delta(\Delta\tau)$, simplification follows from the US assumption. The

⁸ Or equivalently, the ratio of the channel coherence time, T_c , to the signaling period, T .

⁹ The uncorrelated scattering (US) assumption is valid since the Doppler spectrum of a mobile radio channel is different at different delays.

¹⁰ As a matter of notation, the scaling factor of $\frac{1}{2}$ is used for complex baseband signals [7].

expectation operation, $E[\ ]$, is with respect to the random process, $g(\tau; t)$, while the complex conjugate operator is denoted by the symbol $*$.

2.2.1 Channel Characterization of Multipath Time Dispersion

The time dispersion due to a fading multipath channel can be characterized by the multipath (RMS) power delay profile (MPDP), $\Phi(\tau)$. This characterization is basically a measure of the variance of the CIR as a function of the propagation delay, τ . In general, the MPDP depends on the multipath geometry of the channel. However, for simplicity, it is assumed to be the same for all diversity paths [27]. Furthermore, the MPDP is a special case of the autocorrelation function, $\Phi(\tau; \Delta t)$, with a time difference of zero (i.e., $\Delta t = 0$) [7, 36]. This shows that while the MPDP characterizes a channel's time dispersive nature due to multipath, it contains no information about its time variation due to Doppler fading.

The MPDP, $\Phi(\tau)$, is defined as a function of a continuous delay, τ . However, without loss of generality, this definition for a continuous multipath channel model also extends to the discrete case. It can be shown that in the limit of infinite multipaths, the MPDP is identical for both the discrete and continuous multipath channel models [39]. In general, the MPDP is nonzero for only a finite range of propagation delays. This is due to the physical property that the amplitude attenuation of a radio signal is a function of approximately the fourth power of the propagation distance [6]. Consequently, the signal strength would be vanishingly small for large propagation delays. This implies that a radio communication channel has an approximately finite impulse response (FIR). For cellular mobile radio communications in typical urban environments with relatively short multipaths, the MPDP can be approximated by a one-sided exponential distribution of the propagation delay, τ [28]:

$$\Phi(\tau) = \frac{1}{\tau_{rms}} e^{-\frac{\tau}{\tau_{rms}}} \quad \tau \geq 0 \quad (2.2)$$

The degree to which ISI affects the bit error rate (BER) performance of DMR communications is partly dependent on the multipath delay spread of the channel.

A more significant source of ISI is the use of transmit and receive filters with tight bandwidth constraints whose long impulse responses can worsen the time dispersion of the overall channel [8]. The signal dispersion due to channel distortion can be quantified by the root-mean-squared (RMS) delay spread of the MPDP [36]:

$$\tau_{rms} = \sqrt{\frac{\int_0^{\infty} \Phi(\tau)(\tau - \tau_{ave})^2 d\tau}{\int_0^{\infty} \Phi(\tau) d\tau}} \quad (2.3)$$

where the average delay spread is:

$$\tau_{ave} = \frac{\int_0^{\infty} \Phi(\tau)\tau d\tau}{\int_0^{\infty} \Phi(\tau) d\tau} \quad (2.4)$$

The RMS delay spread is defined strictly for the continuous multipath channel model. However, by taking the appropriate limits, the integrations can be replaced with summations and an analogous expression for the RMS delay spread can be obtained for the discrete multipath channel model:

$$\tau_{rms} = \sqrt{\frac{\sum_{m=1}^M (\tau_m - \tau_{ave})^2 \alpha_m^2}{\sum_{m=1}^M \alpha_m^2}} \quad (2.5)$$

where the average delay spread is:

$$\tau_{ave} = \frac{\sum_{m=1}^M \tau_m \alpha_m^2}{\sum_{m=1}^M \alpha_m^2} \quad (2.6)$$

and M is the number of unique propagation paths.

For the discrete channel model, the m th multipath component of the CIR is:

$$g_m = \alpha_m d_m \quad (2.7)$$

where α_m is the RMS gain of the m th multipath and d_m is the normalized Doppler fading component of the m th multipath. The m th multipath CIR component has a variance of:

$$E[g_m^2] = \alpha_m^2 E[d_m^2] \quad (2.8)$$

The ratio of the RMS delay spread of the overall channel (including the transmit and receive filters) to the data symbol period, $\frac{\tau_{\text{RMS}}}{T}$, largely determines the severity of the ISI impairment [7, 36]. Specifically, a $\frac{\tau_{\text{RMS}}}{T}$ ratio which is much less than unity indicates that the time dispersion of the received signal is negligible and that the multipath cannot be resolved in time [7]. On the other hand, a $\frac{\tau_{\text{RMS}}}{T}$ ratio which is much greater than unity indicates severe time dispersion of the received signal. For this latter case, some of the multipaths in the channel can be resolved in time and, as a consequence, the time dispersion in the received signal leads to ISI distortion. Thus, the channel must be equalized to overcome the degradation in BER performance and limitation in the data rate. However, for the case of a time-dispersive channel, despite the increase in ISI distortion, the resolution of distinct, statistically independent¹¹ multipaths also provides a form of implicit diversity which can provide additional immunity to signal fading [12].

A completely equivalent characterization of the time dispersion of a multipath channel can be formulated in the frequency domain by taking the Fourier transform of the MPDP. The result is the spaced-frequency correlation function (SFCF), $\Phi(\Delta f)$, [7, 36]:

$$\Phi(\Delta f) = \int_{-\infty}^{\infty} \Phi(\tau) e^{-j2\pi\Delta f\tau} d\tau \quad (2.9)$$

It is noted that this SFCF can also be obtained by reversing the order of the autocorrelation and Fourier transform operations [7]. Specifically, one can first take the Fourier

¹¹ For a fading multipath channel, all propagation paths are uncorrelated with each other (i.e., assumption of uncorrelated scattering) and each path can be characterized as a Gaussian random process (i.e., Rayleigh fading). Consequently, all the paths are statistically independent of each other [7]

transform (with respect to the propagation delay, τ) of the complex baseband CIR, $g(\tau; t)$, to obtain the time-varying channel transfer function (CTF):

$$G(f; t) = \int_{-\infty}^{\infty} g(\tau; t) e^{-j2\pi f\tau} d\tau \quad (2.10)$$

Then one can take the autocorrelation (with respect to the signal frequency, f) of the CTF to yield:

$$\Phi(\Delta f) = \frac{1}{2} E[G^*(f; t) G(f + \Delta f; t)] \quad (2.11)$$

With the time dependence removed (by setting $\Delta t = 0$), this last expression is just the SFCF. From this last equation, it is apparent that the SFCF provides a measure of the autocorrelation of the CTF. This frequency correlation of the CTF, as characterized by the SFCF, is completely analogous to the propagation time correlation of the CIR, as characterized by the MPDP. In fact, they are Fourier transform pairs.

Analogous to the MPDP RMS delay spread, τ_{rms} , the SFCF coherence bandwidth, $B_c \sim \frac{1}{\tau_{rms}}$, can be defined as a measure of the spectral "flatness" of the multipath channel affecting the signal propagating through it. In other words, the channel coherence bandwidth, B_c , can be thought of as a measure of the largest separation in signal frequency, Δf , at which the CTF, $G(f)$, is still essentially correlated. If the signal bandwidth, B , (i.e., the maximum separation in signal frequency), is less than the coherence bandwidth of the channel, B_c , then the channel is frequency-nonselective or "flat" fading. That is, all frequency components of a signal experience the same propagation delay. This corresponds to the case of no time dispersion of a received signal. Otherwise, if B is greater than B_c , then the channel is frequency-selective and the received signal is dispersed in time. Thus, the frequency-selectivity of the multipath channel can be determined in the frequency domain by the ratio of the signal bandwidth to the coherence bandwidth of the channel, $\frac{B}{B_c} \sim \frac{\tau_{rms}}{T}$, where T is the symbol period.

2.2.2 Channel Characterization of Doppler Frequency Dispersion

The characterization of a fading multipath channel can also be formulated with the scattering function, $S(\tau; f_d)$ [7, 36]:

$$S(\tau; f_d) = \int_{-\infty}^{\infty} \Phi(\tau; \Delta t) e^{-j2\pi f_d \Delta t} d(\Delta t) \quad (2.12)$$

The scattering function contains explicit information about both the Doppler frequency spread and the time dispersion of a fading multipath channel.

Specifically, the Doppler power spectrum, $S(f_d)$, characterizes the Doppler frequency dispersion of a fading channel. This spectrum can be obtained from the scattering function by integrating out the time dispersive effect of the fading multipath channel:

$$S(f_d) = \int_{-\infty}^{\infty} S(\tau; f_d) d\tau \quad (2.13)$$

The fading effect on a received signal propagated through a fading multipath channel is dependent on the Doppler bandwidth of the transmission medium. For the case of mobile communications where the mobile travels through the standing wave propagation pattern of the channel, the fading nature of the received signal at the mobile can be quantified by the Doppler frequency:

$$f_d = f_c \left(\frac{v_m}{c} \right) \cos \phi \quad (2.14)$$

where

f_c is the passband signal frequency (essentially the carrier frequency)

v_m is the speed of the mobile

c is the speed of light

ϕ is the incident angle of signal reception

The Doppler frequency is the difference between a source frequency and an observed frequency. A nonzero Doppler frequency is a direct consequence of the relative motion between a transmitter and a receiver [40]. The spectrum bandwidth of such Doppler frequencies provides a measure of how much the spectrum of a signal can vary in frequency as a result of its propagation through a fading channel [5]. As an example, a pure tone signal which undergoes Doppler fading has a resultant signal spectrum which is spread out by the maximum Doppler frequency of the fading channel. This Doppler frequency spread corresponds to a random time variation of the received signal. Thus, one can think of the Doppler frequency as the rate at which each component of the CIR of a fading multipath channel can randomly vary in time. In particular for DMR communications, Doppler fading of a received signal is essentially due to vehicular motion through the multipath standing wave pattern of the propagating signal. The faster one travels through this spatial signal pattern, the higher the Doppler fade rate of the received signal.

Analogous to the RMS delay spread, τ_{rms} , for a frequency-selective channel, the degree of signal fading can be quantified by the root-mean-squared (RMS) Doppler spread, as defined for a continuous multipath channel model [36]:

$$B_{rms} = \sqrt{\frac{\int_{-\infty}^{\infty} S(f_d) f_d^2 df_d}{\int_{-\infty}^{\infty} S(f_d) df_d}} \quad (2.15)$$

This RMS Doppler spread can also be defined for a discrete multipath channel model:

$$B_{rms} = \sqrt{\frac{\sum_{n=-N_s}^{N_s} s_n^2 f_{d,n}^2}{\sum_{n=-N_s}^{N_s} s_n^2}} \quad (2.16)$$

where s_n is the RMS value of the n th discrete Doppler frequency and $N_s + 1$ is the number of discrete frequency components in the one-sided, baseband Doppler spectrum.

This formula for the RMS Doppler spread, B_{rms} , differs from that for the RMS delay spread, τ_{rms} , in that the Doppler spectrum is an even function of the Doppler

frequency, while the MPDP is inherently causal (i.e., no negative delays) with a positive mean delay spread.

The ratio of the RMS Doppler spread of the channel to the signal bandwidth, $\frac{B_{rms}}{B}$, determines the degree of signal fading. Specifically, a $\frac{B_{rms}}{B}$ ratio which is much less than unity indicates slow signal fading, while a $\frac{B_{rms}}{B}$ ratio which is much larger than unity indicates fast signal fading. Of course, the Doppler fade rate can also be quantified equivalently by the maximum Doppler frequency, f_{dm} , instead of by the RMS Doppler spread, B_{rms} .

The time dynamics of a multipath fading mobile radio channel can also be characterized in the time domain via the spaced-time correlation function (STCF), $\Phi(\Delta t)$ [7, 36]:

$$\Phi(\Delta t) = \int_{-\infty}^{\infty} S(f_d) e^{j2\pi\Delta t f_d} df_d \quad (2.17)$$

Analogous to the RMS Doppler spread, B_{rms} , the coherence time, $T_c \sim \frac{1}{B_{rms}}$, can be defined to be a measure of the time variation of the fading channel. Thus, the degree of signal fading in a mobile radio channel can be determined in the time domain by the ratio of the symbol period to the coherence time of the channel, $\frac{T}{T_c} \sim \frac{B_{rms}}{B}$.

2.3 Model of a Fading Multipath Channel

In general, a fading multipath channel can be modelled as a time-varying filter with random attenuation and phase. For a modulated communication system like DMR, it is simpler to model its communications at the complex baseband signal level. Moreover, the channel can assume either a discrete or a continuous multipath model [7]. The discrete channel model is more suitable and natural for channel measurement and computer simulation, while the continuous channel model is more tractable for mathematical analysis [39]. In particular, the discrete multipath channel model is appropriate for channels such as mobile radio (800–900 MHz band) and ionospheric shortwave ("skywave") radio (HF: 3–30 MHz band), while the continuous multipath

channel model is suitable for channels such as tropospheric scatter, "beyond-the-horizon" radio (UHF: 300–3000 MHz band, and SHF: 3000–30,000 MHz band) [36].

The continuous multipath channel model assumes a continuum of propagation paths. Thus, its time-variant complex baseband CIR can be written as [7, 36]:

$$g(\tau; t) = \alpha(\tau; t)e^{-j2\pi f_c \tau} \quad (2.18)$$

where

α is the attenuation factor

τ is the propagation delay

t is the time dependence of the channel

f_c is the carrier frequency of the transmitted signal

On the other hand, the discrete multipath channel model assumes a finite number of statistically independent propagation paths. Furthermore, each multipath has a time-varying attenuation factor and propagation delay. The channel can be characterized in complex baseband by the following time-variant impulse response [7, 36]:

$$g(\tau; t) = \sum_m \alpha_m(t)e^{-j2\pi f_c \tau_m(t)} \delta(\tau - \tau_m(t)) \quad (2.19)$$

where

m is the index of the m th multipath

$\delta(\tau)$ is the transmitted impulse signal

$\alpha_m(t)$ is the attenuation factor of the m th multipath

$\tau_m(t)$ is the propagation delay of the m th multipath

For the case of mobile communications, the discrete multipath channel model is more intuitive and physically meaningful. The amplitude, $\alpha_m(t)$, is Rayleigh distributed, while the phase, $2\pi f_c \tau_m(t)$, is uniformly distributed. The propagation delay (i.e., arrival time), $\tau_m(t)$, is a Poisson process with an average arrival rate, λ_a , (i.e., average number of paths per unit time) [39].

Mobile radio communications over a fading multipath channel with random scatterers can be modelled as a stochastic process. Assuming a sufficiently small signal

bandwidth such that it does not resolve the individual propagation paths, then, by the central limit theorem, as the sum of a large number of multipath signals, each component of the complex baseband channel impulse response, $g(\tau; t)$, can be modelled with its real and imaginary parts, g_I and g_Q , being independent, zero-mean Gaussian random processes [5, 6, 7, 36]:¹²

$$g = g_I + jg_Q = Ge^{j\theta} \quad (2.20)$$

with a first-order probability density function (pdf) of:

$$p(g_I, g_Q) = \frac{1}{2\pi\sigma_g^2} e^{-\frac{(g_I^2 + g_Q^2)}{2\sigma_g^2}} \quad (2.21)$$

where the average power (or variance), σ_g^2 , of the channel impulse response is:

$$\sigma_g^2 = \frac{1}{2} E[g^2] \quad (2.22)$$

Furthermore, it follows that the channel impulse response's envelope, G , phase, θ , and instantaneous power, $g^2 = \frac{G^2}{2}$, are of Rayleigh, uniform and exponential distributions, respectively:

$$p(G) = \frac{G}{\sigma_g^2} e^{-\frac{G^2}{2\sigma_g^2}} \quad 0 \leq G < \infty \quad (2.23)$$

$$p(\theta) = \frac{1}{2\pi} \quad 0 \leq \theta \leq 2\pi \quad (2.24)$$

$$p(g^2) = \frac{1}{\sigma_g^2} e^{-\frac{g^2}{\sigma_g^2}} \quad 0 \leq g^2 < \infty \quad (2.25)$$

2.3.1 FIR Model of a Frequency-Selective Fading Channel

In general, radio communications occur over a fading multipath channel which has a finite impulse response (FIR). In particular, a frequency-selective channel can be modelled as an FIR filter implemented as a tapped delay line (TDL) [7, 22]. The

¹² The time dependence, t , of the channel impulse response, g , is implicit.

TDL model is made up of evenly-spaced tap coefficients, each of which represents one resolved multipath. Moreover, the length of the TDL model should span the multipath delay spread of the channel. To model the Rayleigh fading in the multipath channel, each tap coefficient has a complex-valued gain whose value randomly varies according to Rayleigh statistics. Since the tap coefficients are uncorrelated¹³ Gaussian random processes, they also fade independently of each other.

To demonstrate the FIR model of a frequency-selective channel, we consider a real passband signal, $u(t)$, of bandwidth, B . Equivalently, its complex baseband signal has a one-sided bandwidth of $\frac{B}{2}$. By applying the sampling theorem, the spectrum of this bandlimited signal can be represented as [7]:

$$U(f) = \begin{cases} \frac{1}{B} \sum_{m=-\infty}^{\infty} u\left(\frac{m}{B}\right) e^{-j\frac{2\pi m f}{B}} & |f| \leq \frac{B}{2} \\ 0 & |f| > \frac{B}{2} \end{cases} \quad (2.26)$$

The channel output signal, $r(t)$, is the convolution of the channel, $g(\tau; t)$, with the input signal, $u(t)$:

$$r(t) = \int_{-\infty}^{\infty} g(\tau; t) u(t - \tau) d\tau \quad (2.27)$$

or in terms of the channel transfer function (CTF):

$$r(t) = \int_{-\infty}^{\infty} G(f; t) U(f) e^{j2\pi f t} df \quad (2.28)$$

Substituting the input spectrum, $U(f)$, into this last equation yields:

$$r(t) = \frac{1}{B} \sum_{m=-\infty}^{\infty} u\left(\frac{m}{B}\right) \int_{-\infty}^{\infty} G(f; t) e^{j2\pi f(t - \frac{m}{B})} df \quad (2.29)$$

Recognizing the integral as an inverse Fourier transform and then changing the order of convolution, we obtain the TDL form of the frequency-selective channel, with discrete

¹³ By the uncorrelated scattering (US) assumption.

time variable, $t = kT$:

$$r(kT) = \sum_{m=1}^M g_m(kT) u\left(kT - \frac{m}{B}\right) \quad (2.30)$$

where the randomly varying, $\frac{1}{B}$ -spaced tap coefficients are:

$$g_m(kT) = \frac{1}{B} g\left(\frac{m}{B}; kT\right) \quad (2.31)$$

This TDL model distinguishes multipaths to a time resolution of $\frac{1}{B}$ [7, 22]. For the case of QPSK signaling with a minimum transmission bandwidth, $B = \frac{1}{T}$, the TDL model has a maximum time resolution of T . Therefore, a T -spaced TDL, with a sufficient number of tap coefficients, M , to span the channel multipath spread, would be adequate to model the frequency-selective channel. However, if the bandwidth constraints are relaxed to allow for some excess bandwidth, then the transmission bandwidth would be:

$$B = \frac{1 + \epsilon}{T} \quad (2.32)$$

where ϵ is the excess bandwidth parameter, with $\epsilon = 0$ corresponding to zero excess bandwidth and $\epsilon = 1$ corresponding to 100% excess bandwidth.

In general, for a given excess transmission bandwidth, the multipath resolution of the TDL channel model is $\frac{T}{1+\epsilon}$. Thus, for the case of signal transmission with a nonzero excess bandwidth, e.g., 25%, a more closely spaced, e.g., $\frac{T}{2}$ -spaced TDL channel model would be appropriate.

2.3.2 "Sum of Sines" Model of a Rayleigh Fading Channel

Mobile radio communications exhibit Doppler fading due to vehicular motion. In particular, the random time variation in the received signal strength is characteristically Rayleigh fading. Specifically, the real and imaginary components of the CIR are independent, zero-mean, Gaussian random processes. This, in turn, corresponds to a CIR whose envelope, phase and instantaneous power have Rayleigh, uniform and exponential

statistics, respectively [5, 6, 36]. Furthermore, mobile radio fading is characterized at baseband by an autocorrelation function, $\Phi(\Delta t)$, of the form¹⁴ [5]:

$$\Phi(\Delta t) \propto J_0(2\pi f_{dm} \Delta t) \quad (2.33)$$

where

f_{dm} is the maximum Doppler frequency

Δt is the time difference

$J_0()$ is the zeroth-order Bessel function

and by a baseband power spectrum, $S(f_d)$, of the form [5]:

$$S(f_d) \propto \begin{cases} \left(1 - \left(\frac{f_d}{f_{dm}}\right)^2\right)^{-\frac{1}{2}} & |f_d| \leq f_{dm} \\ 0 & |f_d| > f_{dm} \end{cases} \quad (2.34)$$

where

f_d is the Doppler frequency

f_{dm} is the maximum Doppler frequency

The salient characteristics of Rayleigh fading are represented by:

1. – Rayleigh envelope distribution
2. – uniform phase distribution
3. – Doppler power spectrum
4. – autocorrelation (Bessel) function

If a channel is Rayleigh fading, then each multipath component of its CIR must possess these salient characteristics. Particularly for simulating Rayleigh fading in an FIR channel model, each tap coefficient, $g_m(kT)$, of the TDL should have a random time variation which is consistent with the noted characteristics.

A method which uses a sum of weighted sinusoids with discrete frequencies spanning the Doppler spectrum has been used to simulate Rayleigh fading [5, 41]. For the sake of

¹⁴ This is an autocorrelation of received radio signals averaged over their random phases [5]. This autocorrelation function characterizes the time variation in a fading channel and is essentially the spaced-time correlation function (STCF) as described previously.

reference, we call this method the "sum of sines" model of a Rayleigh fading channel. For the case of an FIR channel model, the "sum of sines" method can be used to simulate the Rayleigh fading of each tap coefficient. Specifically, the m th tap coefficient, $g_m(kT)$ of the complex baseband CIR is:

$$g_m(kT) = \sum_{n=0}^{N_s} g_{m,n} \cos(2\pi \delta_n f_{dm} kT) \quad (2.35)$$

where

kT is the discrete-time variable

$g_{m,n}$ is the complex-valued amplitude for the n th Doppler frequency

f_{dm} is the maximum Doppler frequency

δ_n is the relative frequency scaling factor for the n th Doppler frequency

$N_s + 1$ is the number of weighted sinusoids

The relative frequency scaling factor for the n th Doppler frequency is:

$$\delta_n = \cos\left(2\pi \frac{n}{N_s}\right) \quad 0 < \delta_n \leq 1 \quad (2.36)$$

with $n = 0$ corresponding to the maximum Doppler frequency

The in-phase and quadrature components of the amplitude for the n th Doppler frequency are, respectively:

$$\text{Re}[g_{m,n}] = \begin{cases} \sqrt{2} \cos\left(\frac{\pi}{4}\right) & n = 0 \\ 2 \cos\left(\frac{\pi n}{N_s}\right) & 1 \leq n \leq N_s \end{cases} \quad (2.37)$$

and

$$\text{Im}[g_{m,n}] = \begin{cases} \sqrt{2} \sin\left(\frac{\pi}{4}\right) & n = 0 \\ 2 \sin\left(\frac{\pi n}{N_s}\right) & 1 \leq n \leq N_s \end{cases} \quad (2.38)$$

With this "sum of sines" method, the use of a larger number of weighted sinusoids leads to a more accurate Rayleigh fading model. It has been shown that Rayleigh fading can be accurately simulated with this "sum of sines" method using just nine ($N_s = 8$) discrete sinusoids [5, 41].

Chapter 3 Adaptive Equalization and Diversity Combining Methods

3.1 Intersymbol Interference

Linear distortion in amplitude and phase can cause time dispersion of a transmitted signal in a communications channel.¹⁵ In particular for a DMR channel, this time dispersion is caused by multipath propagation and is aggravated by high signaling rates and bandlimiting filters with relatively long impulse responses. For efficient bandlimited signaling, this time dispersion results in intersymbol interference (ISI) [7, 42].

As a potential impairment in all pulse modulated communications, ISI is characterized by smearing of the transmitted pulses such that multiple adjacent symbols contribute to the received signal at each sampling instant [7, 42]. The phenomenon of ISI can be illustrated with (complex) baseband pulse-amplitude modulated (PAM) signaling. The received signal, $r(t)$, is composed of a superposition of overall CIR¹⁶ translates, $g(t - kT)$, weighted by transmitted PAM symbols, $a(kT)$, and corrupted by received-filtered noise, $n(t)$:

$$r(t) = \sum_k a(kT)g(t - kT) + n(t) \quad (3.1)$$

At the sampling instant, $t = mT + \tau$, the sampled received signal becomes:

$$r(mT + \tau) = a(mT)g(\tau) + \sum_{k \neq m} a(kT)g((m - k)T + \tau) + n(mT + \tau) \quad (3.2)$$

where

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¹⁵ In general, nonlinear distortion and other impairments can also be contributing factors [7, 42].

¹⁶ The overall CIR is the combined impulse response of the actual channel and the transmit and receive filters.

the first ($k = m$) term is the desired received signal
 the second term is the summation of undesired ISI components
 the third term is the sampled received-filtered noise and
 τ is the channel propagation delay

From the above equation, it is evident that reliable detection of the received symbol, $a(mT)g(\tau)$, is possible only if the additive noise and ISI are relatively small at the sampling instants so that the "eye" is sufficiently open. The ISI impairment can be reduced by signaling at a lower rate or by using more bandwidth which, in turn, relaxes the bandlimiting filter requirements. However, this strategy is unacceptable because of its inefficient use of the available bandwidth. An alternative approach is to use appropriately bandlimited signals which do not cause ISI. This zero-ISI condition requires the overall CIR to have zero crossings at all sampling instants other than at the desired sample [7, 42]:

$$g((m - k)T + \tau) = \begin{cases} 1 & m = k - \frac{\tau}{T} \\ 0 & \text{otherwise} \end{cases} \quad (3.3)$$

This zero-ISI condition corresponds to the Nyquist criterion in the frequency domain [7, 42]:

$$\frac{1}{T} \sum_k G\left(f - \frac{k}{T}\right) = 1 \quad |f| \leq \frac{1}{2T} \quad (3.4)$$

where $W = \frac{1}{2T}$ is the Nyquist, or minimum, bandwidth

The Nyquist criterion for zero ISI states that the folded (i.e., aliased) spectrum of the sampled overall CIR has constant gain within the Nyquist bandwidth. As an example, the ideal lowpass filter with minimum bandwidth, $W = \frac{1}{2T}$, satisfies the Nyquist criterion while achieving optimum bandwidth efficiency. However, such an ideal filter is not realizable and is susceptible to an infinite amount of ISI if the sampling is not performed exactly at the desired symbol instant. This divergent infinite sum of ISI components is due to the fact that the impulse response of the ideal lowpass filter has tails which diminish proportionally to $\frac{1}{t}$ [7, 42].

Lowpass filters with excess bandwidth and odd symmetry about $\frac{1}{2T}$ Hz also satisfy the Nyquist criterion [7, 42]. For example, one often uses bandlimiting filters with a raised-cosine frequency response. Such filters with excess bandwidth have impulse responses whose tails decay proportionally to $\frac{1}{t^2}$ and, thus, have convergent infinite sums of ISI components for all sampling phases. Moreover, the use of excess bandwidth filtering produces no ISI at the expense of introducing aliasing distortion. However, this aliasing is acceptable since digital signaling transmits information only at the data sampling instants and not in the actual waveform. Thus, symbol detection does not require the faithful reconstruction of the transmitted waveform but, rather, just the information-bearing component (e.g., the phase in PSK signaling) of the transmitted waveform at the sampling instants.

3.2 Channel Equalization Techniques

In DMR communications, adaptive equalization is required to minimize ISI even if Nyquist filtering is used. This is because the multipath fading channel can still cause time dispersion in the transmitted signal. Moreover, for bandlimited communications, ISI has an infinite span. Thus, in order to remove ISI completely, an equalizer must have an infinite length. However, in practice, an equalizer has a finite length which spans most of the energy of the CIR. Furthermore, an equalizer is usually implemented as an FIR filter in the form of a tapped delay line (TDL).

The coefficients of an equalizer are computed according to performance criteria such as the zero forcing (ZF) or the minimum mean squared error (MMSE) criteria [7, 42]. Specifically, the ZF criterion requires that the equalizer coefficients be optimized so that the ISI is forced to zero (i.e., satisfying the Nyquist criterion), regardless of the noise enhancement. On the other hand, the MMSE criterion chooses the equalizer coefficients which minimize the equalizer output MSE due to both ISI and noise.

The linear equalizer (LE) is the simplest equalizer structure. It essentially attempts to "invert" the folded spectrum of the overall CIR so as to satisfy the Nyquist criterion. It is well known that the linear equalizer suffers from unacceptable noise enhancement

when the channel experiences severe amplitude distortion [7, 42]. Thus, the linear equalizer is not suitable for DMR communications where deep fades can often occur.

Nonlinear equalizers are less susceptible to the noise enhancement problem since, unlike the LE, they do not directly attempt to invert the folded spectrum of the channel. Specifically, the maximum likelihood sequence estimator (MLSE) using the Viterbi algorithm yields optimum performance in the presence of AWGN¹⁷ [7, 42]. At high SNR and as long as the ISI is not too great, MLSE performance approaches that of the matched filter bound (i.e., with an isolated pulse and, thus, no ISI). An MLSE achieves optimum performance by using the information in a received symbol sequence (i.e., a finite-state Markov process) to detect each data symbol. Furthermore, inherent to this strategy is the requirement of a CIR estimate.

The MLSE performance is achieved at a cost in processing delay and complexity. The MLSE processing delay (i.e., its memory) is typically several times the CIR length while the computational complexity per detected symbol is proportionally to the number of channel states, m^{N_c+1} , where m is the symbol alphabet size (e.g., $m = 4$ for QPSK signaling) and N_c is the $\frac{T}{2}$ -spaced CIR length. Since the MLSE computational complexity varies exponentially with the multipath spread, this overhead can be prohibitively high for very dispersive channels. In addition, channel estimation performed with a continuously adaptive algorithm, e.g., decision-directed LMS, requires estimates of the transmitted data at the receiver in order to form an error signal for its adaptation. However, an MLSE detector can provide these data symbols only after a processing delay of its sequence memory. Consequently, the continuously adaptive CIR estimator suffers a lag error due to processing delays from both itself and the MLSE. This lag error in the CIR estimation can be unacceptable for rapidly fading channels. On the other hand, a block-adaptive CIR estimator such as a CIR interpolator does not incur a lag error due to the MLSE delay since it does not require an error signal for

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¹⁷ Strictly speaking, an MLSE minimizes the probability of sequence error and not the probability of symbol/bit error. However, for most cases, the difference between these two performance criteria is not significant.

its adaptation. Thus, a CIR interpolator suffers from a tracking error due to only its processing delay.

3.2.1 Decision-Feedback Equalization

The decision-feedback equalizer (DFE) is another nonlinear equalizer which can perform much better than the LE [7, 42, 43]. In fact, as long as decision error propagation is not severe, the DFE performance is nearly as good as that of the MLSE [44, 45]. Furthermore, the DFE has the advantage of being of similar complexity as the LE but much less complex than the MLSE. The DFE is comprised of a forward transversal filter, w_f , (which is essentially an LE) and a feedback transversal filter, w_b (see Figure 3.1). With both a forward and a feedback filter, a DFE has more degrees of freedom to process the ISI in the received signal. Specifically, the forward filter removes the precursor ISI so that zero crossings occur at all samples of the received signal prior to the main lobe at the current sample instant. In essence, the forward filter accomplishes this by shifting most of the received signal energy in the anticausal part of the received signal response (i.e. precursor) to the causal part of the received signal response (i.e., postcursor). Consequently, the postcursor ISI at the samples after the main lobe of the received signal response can be eliminated by subtracting the feedback-filtered detected symbols from the output of the forward filter. As long as the forward filter spans at least the CIR length, the causal feedback filter only needs to span at most the CIR length.

Assuming that past decisions are correct, then the removal of the postcursor ISI does not incur any noise enhancement. Furthermore, since the forward filter only has to remove the precursor ISI, it is subjected to less noise enhancement than if it had to remove all the ISI (i.e., to "invert" the folded channel response). Thus, the DFE outputs less noise power and generally achieves a lower MSE than the LE. This is especially true for channels with severe amplitude distortion (e.g., the DMR channel). However, the improved MSE performance of the DFE comes with an associated susceptibility to error propagation. That is, a decision error can generate an incorrect postcursor ISI estimate which, in turn, will further increase the probability of decision errors. This DFE

Figure 3.1 T-spaced Decision-Feedback Equalizer

error propagation is especially critical for DMR communications over a rapidly fading channel where frequent deep fades can severely compromise the DFE performance.

The forward and feedback coefficients of the DFE can be jointly computed to minimize the MSE at the equalizer output. This Wiener solution for a finite-length DFE is derived in Appendix F for the more general case of an OCDFE with diversity combining and forward filters which are implemented as fractionally spaced equalizers (FSE), e.g., $\frac{T}{2}$ -spaced. An FSE is used since it gives a finer resolution of the multipath in a DMR channel. Moreover, an FSE processes unaliased received signals so that it can realize the optimum receiver of a matched filter followed by a T -spaced equalizer [42, 43]. In addition, an FSE is more immune to variations in the sampling phase because its lack of aliasing allows the folded CIR spectrum to satisfy the Nyquist criterion without requiring to have odd symmetry about $\frac{1}{2T}$ Hz. This added immunity to the sampling phase is quite important in DMR communications over a fast fading channel where timing recovery can be difficult. However, an FSE does have the disadvantage of requiring more coefficients than a T -spaced equalizer to span the same CIR length. This additional overhead in the FSE can result in more self-noise and slower convergence in the equalizer adaptation.

3.3 Doppler Fading and Diversity Combining

Due to its inherently fading multipath channel, mobile radio communications is subject to time dispersion and Doppler fading of the received signal [5, 6]. When the DMR channel is in a deep fade (i.e., the received signal strength is strongly attenuated), the instantaneous SNR at the channel output can be very low. During such a deep fade, detection errors are more likely to occur. Consequently, the degradation in BER performance of the communications system is commensurate to the reduction in the instantaneous channel SNR. The following BER comparison illustrates the extent to which the Rayleigh fading of a mobile radio channel can degrade communications performance.

For typical digital communications, e.g., using coherent DPSK modulation, over a nonfading channel, the BER¹⁸ is proportional to the decaying exponential of the channel SNR, γ [7]:

$$P_e(\gamma) \propto e^{-\gamma} \quad (3.5)$$

For the case of communications over a fading multipath channel, the BER is averaged over the statistics of the fading SNR, $p(\gamma)$, to yield an average BER¹⁹ which is proportional to only the inverse of the mean channel SNR, Γ [5, 6, 7]:

$$\overline{P_e}(\Gamma) = \int_0^{\infty} P_e(\gamma)p(\gamma)d\gamma \propto \frac{1}{\Gamma} \quad (3.6)$$

Thus, by comparing the last two equations, it is evident that signal fading can drastically reduce BER performance.

The use of diversity²⁰ is perhaps the most effective technique for improving the BER performance of communications in the presence of channel fading [5, 6, 7]. Other techniques include the adaptive strategy of dynamic channel assignment [9], increase of the transmitted power, antenna height and/or antenna size [5, 6]. However, such solutions are relatively expensive and perhaps not feasible. For example, increasing the transmitted power reduces the spectrum efficiency, since the distance between co-frequency transmitters must be increased in order to maintain an acceptable CCI level. Moreover, diversity not only mitigates the effects of multipath fading, but it also suppresses random FM of angle modulated systems and co-channel interference in

¹⁸ For M-ary multilevel signaling with Gray encoding, the BER, P_e , is essentially equivalent to the symbol detection error rate.

¹⁹ For bursty channels such as slowly fading, indoor wireless channels, the performance criterion of outage probability is perhaps more useful than that of average BER [46].

²⁰ We consider only 'microscopic' diversity of independent, 'short-term', Rayleigh fading signals. We will not consider 'macroscopic' diversity of independent, 'long-term', lognormal fading signals. 'Microscopic' diversity is implemented at the radio link level (e.g., between antennas at one mobile site) while 'macroscopic' diversity is implemented at a higher system level (e.g., between different mobile sites) [6].

frequency reuse systems such as cellular mobile radio. Hence, the use of diversity can also lessen the severity of an irreducible BER and increase system capacity [29, 47].

The aim of diversity is to protect a received signal from deep fades by adding redundancy²¹ to the transmitted signal in order to reduce the probability of making a detection error. This is accomplished by sending multiple versions of a signal to the receiver over independently fading channels. Specifically, if the probability of one signal fading below a certain threshold is p , then the probability of all signals over the L independent diversity channels simultaneously fading below that threshold is p^L . Moreover, for such an L th order diversity system, whose individual diversity branches each have an average BER of $\overline{P_e}(\Gamma) \propto \frac{1}{\Gamma}$, the overall average BER is proportional to the inverse L th power of the mean SNR per branch, Γ [5, 6, 7]:

$$\overline{P_{e,L}}(\Gamma) \propto \frac{1}{\Gamma^L} \quad (3.7)$$

Thus, the use of diversity can significantly improve the BER performance under fading conditions.

3.4 Diversity Channel Techniques

Frequency, time and antenna diversity are the three general techniques used to provide independently fading received versions of a transmitted signal over a fading multipath channel [5, 6, 7]. In general, these diversity schemes can be implemented at the transmitter and/or at the receiver. Furthermore, antenna diversity can be achieved by space, field, angle or polarization diversity techniques. Specifically for mobile radio channels, spectrally efficient antenna (i.e., space) diversity is the technique of choice [48]. Moreover, space diversity with optimum combining can mitigate noise and interference, as well as multipath fading of the received signal [29].

All diversity techniques endeavour to provide independently fading paths to the receiver. Frequency diversity achieves this by transmitting a signal over carriers which are separated in frequency by greater than the coherence bandwidth of the channel,

²¹ This is analogous to the (more sophisticated) added redundancy of error control codes.

B_c . Time diversity, on the other hand, transmits signals which are separated in time by greater than the coherence time of the channel, T_c . While time diversity does not have the disadvantage of increased spectrum requirement of frequency diversity, it does have inherent time delays which must be tolerated. Moreover, time diversity requires an infinite time separation between diversity channels if the Doppler bandwidth is zero (i.e., when a mobile is at rest). Therefore, time diversity is clearly not suitable for mobile radio communications [5, 6].

3.4.1 Antenna Diversity

Because of its spectral efficiency and relative simplicity, space diversity is the most practical and common of the antenna diversity techniques. The received signals at the spatially separated antennas have a normalized correlation coefficient, $\rho(d)$, specified in terms of the spatial parameter, d :

$$\rho(d) = J_0^2\left(\frac{2\pi d}{\lambda}\right) \quad (3.8)$$

where λ is the wavelength of the carrier and $J_0()$ is the zeroth-order Bessel function.

Assuming that the received signal has a uniformly distributed angle of arrival, then $\rho(d)$ has its first null at $d = 0.375\lambda$, while $\rho(d) < 0.2$ for $d > 0.375\lambda$. In practice, choosing a minimum antenna spacing of $d = \frac{\lambda}{4}$ usually ensures sufficiently uncorrelated (and independently fading) received signals [5, 6, 49].

In general, for an adaptive, L -antenna phased array under nonfading conditions, signal resolution is in terms of spatial or angular separation [50, 51]. That is, an antenna array can only resolve signals from transmitting mobiles if they are sufficiently spaced apart (i.e., separated in angle relative to the receiving antenna array). In other words, if two signals have phase differences between receive antennas which are sufficiently different, then the antenna array can resolve the signals. For the case of multipath fading in a mobile radio environment, signal resolution for an antenna array is independent of mobile location and spatial or angular separation of the incoming signal [29, 49]. As long as the array antennas are spaced more than a quarter wavelength apart, the signal phases and amplitudes from one antenna to another are independent. Thus, this

independent fading criterion renders the antenna array pattern irrelevant. Conversely, the received signal phases from sufficiently separated transmitter antennas are also independent. Therefore, the resolution for received signals at an antenna array does not depend on the absolute or relative spatial position of the transmitting mobiles. As long as two signals do not have roughly the same phase differences between receive antennas, the antenna array can resolve them. This condition for signal resolution is usually met, since mobiles are generally in motion [29].

Finally, the other antenna diversity techniques are now briefly described. Field diversity exploits the decorrelation of a radio signal's three field components, E_z , H_x and H_y , to provide independently fading versions of the transmitted signal, while angle diversity uses directional antennas at the receiver to point into angularly spaced sectors to achieve the same objective. On the other hand, polarization diversity transmits orthogonal signals that are vertically and horizontally polarized. For example, these polarized signals can be received with loop and whip antennas, respectively.

3.5 Diversity Signal Combining Techniques

The purpose of diversity combining is to process the independently fading signals received from the L diversity channels in such a manner that, for a given receiver complexity, the output decision SNR is maximized to ensure the best possible symbol detection performance [5, 6, 7]. In general, diversity combining produces an output signal which is a linearly weighted (in amplitude and phase) sum of the L received diversity signals (see Figure 3.2):

$$y(t) = \sum_{l=0}^{L-1} w_l(t)r_l(t) \quad (3.9)$$

where

$r_l(t)$ is the received signal from the l th diversity channel

$w_l(t)$ is the adaptive, combiner weight coefficient for the l th diversity channel

L is the number of diversity channels

Figure 3.2 L-Branch Diversity Combiner

Regardless of where in the spectrum the combining is performed (e.g., at RF, IF or baseband), the statistics of the output signal does not change. Thus, demodulation can be performed either before (i.e., predetection) or after (i.e., post-detection) combining [5, 35].

At the l th diversity branch, the received signal, $r_l(t)$, is the sum of the desired signal, $s_l(t)$, and the AWGN, $n_l(t)$.²² Assuming that the phases of all the received signals in the L diversity branches are the same²³, the output desired signal is:

$$s(t) = \sum_{l=0}^{L-1} w_l(t) s_l(t) \quad (3.10)$$

Furthermore, the output AWGN power is:

$$n^2(t) = N_0 \sum_{l=0}^{L-1} w_l^2(t) \quad (3.11)$$

where N_0 is the noise power spectral density.

Thus, the instantaneous SNR at the combiner output is:

$$\gamma = \frac{s^2(t)}{n^2(t)} \quad (3.12)$$

Depending on the algorithm for choosing the weights, $w_l(t)$, there are four different types of combiners: selection, switched, equal-gain and maximal-ratio [5, 6]. Diversity combining at the receiver can be further classified as either predetection (with signal cophasing) or post-detection. For mobile radio, predetection combining is often used. The reason for this is that, while being of similar complexity, predetection combining performs at least as well as post-detection combining [5, 6]. However, post-detection for the case of two-branch selection diversity combining has been shown to improve SNR performance by several decibels [31].

²² In general, the received signal can also contain some interference. However, in this thesis, we will not be considering this impairment.

²³ This can be achieved in the combiner by a cophasing circuit [5, 6].

Selection combining chooses the received signal from the l th diversity branch which has the largest instantaneous SNR. It performs this selection by setting all diversity branch weights to zero except for the l th branch with the largest SNR. Consequently, selection combining has the advantage that it does not require cophasing of the received diversity signals. The resulting SNR at the selection combiner output is:

$$\gamma_{\text{selection}} = \frac{s_l^2(t)}{N_0} \quad (3.13)$$

Selection combining is simple in concept, but because of its variable threshold, it is not easily implementable. Switched combining, on the other hand, is a more practical technique because it has a fixed threshold and can be implemented with just one receiver branch with a commutator switch to the L antenna, diversity branches. The strategy of switched combining is to remain with the currently selected received signal branch, regardless of whether it has the highest instantaneous SNR, until its instantaneous SNR falls below a fixed threshold level. Once the instantaneous SNR of the currently selected received signal falls below the fixed threshold level, the combiner switches to the branch signal with the highest SNR at that instant. In general, selection diversity combining always performs better than switched diversity combining [5, 6].

Equal-gain combining is a relatively simple technique where all the individual phase-locked (i.e., coherent) branch signals are summed with equal unit weighting. The resulting SNR at the equal-gain combiner is:

$$\gamma_{\text{equal-gain}} = \frac{\left(\sum_{l=0}^{L-1} s_l(t) \right)^2}{LN_0} \quad (3.14)$$

3.5.1 Maximal-Ratio Combining

Maximal-ratio combining yields optimum performance in terms of maximum SNR²⁴ (in the mean squared sense) at the receiver output [5, 6, 7]. Such an optimum combining

²⁴ If interference is also present, then this combining strategy will optimize the signal-to-interference-plus-noise ratio (SINR) [29].

strategy maximizes the SNR of the received signal under any fading conditions by tracking the random time variations in the channel and adaptively updating the combiner weights. Specifically, it achieves this optimum performance by multiplying the received signal at each diversity branch with a weight which is the complex conjugate of the channel response of that particular branch. This effectively weights each branch signal proportionally to its branch SNR and compensates for the phase shift (time delay) of each diversity channel. The resulting optimized SNR at the combiner output is:

$$\gamma_{\text{maximal-ratio}} = \frac{\sum_{l=0}^{L-1} s_l^2(t)}{N_0} \quad (3.15)$$

Assuming that the received signals at each diversity branch are uncorrelated and that each branch has the same average SNR, Γ , the pdf (distributed over the Rayleigh fading instantaneous SNR, γ) for an L -branch diversity, maximal-ratio combiner is [5, 6, 7]:

$$p_L(\gamma) = \frac{\gamma^{L-1} e^{-\gamma}}{(L-1)! \Gamma^L} \quad (3.16)$$

Thus, the BER performance of the maximal-ratio combiner can be calculated, for a given signal modulation scheme with stationary BER, $P_e(\gamma)$, by averaging over the fading in the instantaneous received SNR, γ :

$$\overline{P_e}(\Gamma) = \int_0^{\infty} P_e(\gamma) p_L(\gamma) d\gamma \quad (3.17)$$

This general BER expression is a function of the average SNR per branch, Γ . Because of the use of diversity combining, the average SNR per received symbol is actually $L\Gamma$ [7]. This SNR gain can be thought of as the diversity gain. A diversity technique effectively increases the amount of received signal power (relative to the noise power) at the combiner by maximizing its usage of the available transmitted signal power. Since a fixed amount of signal power is transmitted in any case and if there are independently fading channels available, then diversity should be employed to utilize as much of this

signal power as is feasible. Of course, this diversity gain in the SNR of the received signal comes at the expense of either receiver complexity (antenna diversity), bandwidth overhead (frequency diversity) or processing delay (time diversity).

Finally, any type of diversity combiner can be generalized by allowing each of its weight coefficients to be a vector of coefficients. This is analogous to having an FIR filter at each diversity branch. Accordingly, the received signal at each diversity branch must also be a vector of the same length as the branch weights. This generalization of the diversity combiner is useful for situations when the dispersive nature of a channel requires a receiver to process a portion of the CIR at a time. In particular, this is true for the ISI minimization problem. For this case, each diversity branch contains an FIR linear equalizer whose length spans the portion of the CIR corresponding to the "precursor" ISI which it attempts to cancel. Of course, the "postcursor" ISI can be reduced by employing a "feedback" FIR filter which makes use of the detected data. This generalization of a conventional diversity combiner and decision-feedback equalizer (DFE) is the type of receiver structure which we investigate in this thesis. We call this hybrid receiver structure an optimum combiner and decision-feedback equalizer (OCDFE).

3.6 Adaptive Retransmission

Antenna diversity techniques, e.g., maximal-ratio combining, are located at the receiver. Thus, in general, both the mobile and the base station require such a diversity receiver. However, the technique of adaptive retransmission²⁵ requires transmitter diversity combining at only one end of the communication link, e.g., at the base station. This reduction in receiver overhead is a consequence of reciprocity of signal propagation between the base station and mobile.²⁶ Thus, by using adaptive retransmission, one can realize the benefits of diversity combining with only the base station having a diversity antenna array and its associated adaptive signal processing. In this arrangement, all the mobiles simply have a single antenna each. This approach has the advantage of a

²⁵ Also known as transmitting antenna diversity.

²⁶ In general, reciprocity of signal propagation is an acceptable assumption for mobile radio communications [52].

reduction in hardware complexity and space at the mobile receivers. However, such a time-division retransmission technique requires that all transmitters and receivers at the mobiles and base stations be synchronized, so as to avoid interference via time contention [1, 29, 52].

Adaptive retransmission uses a single frequency channel with transmission in both directions time-shared as in a single frequency TDM/TDMA architecture. Furthermore, this adaptive retransmission strategy is still valid if the transmission in both directions uses two frequencies that are spaced sufficiently close together (i.e., less than the coherence bandwidth of the channel). For transmission from a mobile to a base station, the received signals at all diversity branches at the base station are cophased and weighted according to a particular combining technique. On the other hand, for transmission from a base station to a mobile, the phases of the transmitted signals at the base station are adjusted to be the complex conjugate of the received weights (i.e., same amplitude but negative phase) [1, 29, 52].

3.7 Adaptation Strategies

In DMR communications where the channel is rapidly fading, the receiver needs to adapt its parameters, e.g., OCDFE coefficients. In general, there are two distinct tasks in the adaptation process: training and tracking. Initially when the CIR is unknown, the receiver needs to be trained with a known pseudo-random training sequence so that it can learn the channel. This ideal reference training allows the receiver parameters to converge to their optimum values during an initial period when the receiver decisions are unreliable. However, the fading channel can still be rapidly varying in between training exercises when the actual data is transmitted. Thus, the receiver must track the time variations in the channel so that its optimum parameters are updated to reflect the changing CIR.

Receiver adaptation can be achieved with two different strategies. The OCDFE coefficients can be adapted directly or they can be computed (e.g., Wiener solution) from updated channel estimates [8, 14, 23]. In either case, the training and tracking

processes can assume either symbol-by-symbol or block-by-block adaptation. In our DMR system, we compute the OCDFE receiver from current channel estimates which are obtained with block-adaptive training and tracking algorithms.

Symbol-by-symbol adaptation employs an error signal to direct the adaptation process. The error signal is usually derived from the difference between the output signals of the OCDFE and the decision device. This use of the output signal of the decision device to approximate the actual transmitted signal is referred to as decision-directed adaptation and is often used for continuous tracking. On the other hand, the actual transmitted signal is used to determine the error signal in the case of ideal reference training. Common examples of continuously adaptive algorithms are the Least Mean Square (LMS) and Recursive Least Squares (RLS) algorithms [7, 53].

Block adaptation has the advantage of reduced computation since the OCDFE parameters are updated only once per data block.²⁷ However, block adaptation also incurs a penalty in processing delay which can lead to a lag error in the training and tracking of the receiver. For communications over a rapidly fading channel, periodic training is essential. Examples of such block-adaptive methods include windowing techniques like the autocorrelation, covariance and prewindowing methods [53]. In our DMR system, we employ a Block Least Squares (BLS) training algorithm (see Section 4.6.1). Furthermore, block-adaptive tracking strategies are also useful for DMR communications over a rapidly fading channel since decision-directed tracking is quite susceptible to error propagation. In our DMR system, we employ a CIR interpolation strategy to track the rapidly fading channel (see Section 4.6.2).

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²⁷ In the case of block-adaptive tracking with CIR interpolation, the OCDFE parameters are computed at every interpolated CIR sample.

Chapter 4 Description of the DMR Communications System

4.1 System Overview and Specifications

In this thesis research, we investigate a DMR communications system with specifications that are similar to those of the proposed narrowband TDMA, DCR system for North America [11]. A block diagram of the DMR communications system is presented in Figure 4.3. Specifically, a carrier, which could service three users in a TDMA format, is required to transmit an overall bit rate of 48 *kbps* in an allotted channel bandwidth of 30 *kHz*. For a $\frac{\pi}{4}$ -shifted DQPSK modulation scheme, the symbol rate and minimum transmission bandwidth are 24 *ksp*s and 24 *kHz*, respectively. In our investigation, we choose a transmission bandwidth, B , of 30 *kHz*. This corresponds to a use of 25% excess bandwidth and an associated 0.25 Nyquist rolloff in the bandlimiting transmit and receive filters.

Narrowband DMR communications in typical urban environments can experience severe multipath fading [8]. Consequently, the DMR communications system requires some form of adaptive equalization and diversity combining. For the purpose of our investigation of block adaptation, we employ a signaling format which interleaves training sequences with data sequences. These training sequences are used to update the CIR estimates periodically. Furthermore, a contiguous set of these trained CIR estimates are interpolated to yield intermediate CIR estimates during a data sequence. This CIR interpolation, in effect, tracks (with an inherent processing delay) the random time variations of the channel. The interpolated CIR estimates are then used to adapt the receiver. Specifically, these interpolated CIR estimates are used to compute optimum coefficients (in the mean squared sense) of a diversity combiner and decision-feedback equalizer (OCDFE).

Figure 4.3 Complex Baseband Model of a DMR Communications System

4.2 Channel Model

For mobile radio communications in a typical urban environment, the transmission medium is characterized by multipath propagation delays of up to tens of microseconds [37, 38, 54]. In addition, for the proposed narrowband DMR system, the mobile radio channel can exhibit significant Doppler fading at normal vehicular speeds. Specifically, for 900 MHz radio communications at a highway speed of 120 kph, the associated Doppler frequency is approximately 100 Hz. Thus, the multipath fading of the DMR channel can be quite severe.

The DMR channel is modelled in complex baseband by an FIR filter which is implemented as a $\frac{T}{2}$ -spaced TDL [7]. The TDL contains three nonzero complex tap coefficients, $g_m(kT)$, each of which represents a distinct multipath. The multipath profile of the channel is specified by a three-path "mountainous terrain" MPDP in decibels, dB, relative to the first multipath [54]:

$$\begin{aligned} MPDP &= [0 \quad -5 \quad -15 \quad -\infty] \text{ dB} \\ \text{tap delay} &= [0 \quad \frac{T}{2} \quad T \quad \frac{3T}{2}] \end{aligned} \quad (4.1)$$

This multipath channel model is essentially a three-path model, since the fourth path, at a delay of $\frac{3T}{2}$, has zero energy and is not included in the multipath model (see Figure 4.4). Furthermore, this MPDP has a multipath length of approximately one symbol period. In particular, for a symbol period, T , of 41.7 μs , this MPDP has an average delay spread, τ_{ave} , of 5.87 μs and an RMS delay spread, τ_{rms} , of 10.40 μs .

For the TDL channel model, the RMS tap gains are:

$$\begin{aligned} \text{rms gain, } \alpha_m &= [0.861 \quad 0.484 \quad 0.153 \quad 0.0] \\ \text{tap number, } m &= [1 \quad 2 \quad 3 \quad 4] \end{aligned} \quad (4.2)$$

These tap gains are chosen so as to satisfy the "mountainous terrain" MPDP and to yield unit channel energy (see Figure 4.5).

To model the Rayleigh fading in a DMR channel, each complex tap coefficient, $g_m(kT)$, of the TDL channel model contains a normalized factor, $d_m(kT)$, whose time

**Figure 4.4 T/2-spaced Tapped Delay Line
Model of a 3-Ray Multipath Fading Channel**

Figure 4.5 "Mountainous Terrain" MPDP of a 3-Ray Multipath Fading Channel

variation is modelled with the "sum of sines" method [5, 41]. Thus, the m th tap coefficient of the TDL channel model is:

$$g_m(kT) = \alpha_m d_m(kT) \quad (4.3)$$

where

$$d_m = \sqrt{\frac{2}{\sum_{n=0}^{N_s} g_{m,n}^2}} \sum_{n=0}^{N_s} g_{m,n} \cos(2\pi \delta_n f_{dm} kT) \quad (4.4)$$

and α_m is the RMS multipath gain

To simulate Rayleigh fading, this channel model uses nine ($N_s = 8$) sinusoids with distinct Doppler frequencies. Moreover, the maximum Doppler frequency, f_{dm} , can take on values from 0 Hz up to 100 Hz, to reflect the normal vehicular speeds associated with mobile radio communications. For a $\frac{T}{2}$ -spaced channel with a maximum Doppler frequency of 100 Hz, relative to a symbol rate, $\frac{1}{T}$, of 24 ksp/s, the "sum of sines" time variation of a typical channel tap coefficient is shown in Figure 4.6. Furthermore, Rayleigh fading is evident in the time variation of the RMS power of the channel tap coefficient (see Figure 4.7). Also, the "sum of sines" nature of the Rayleigh fading model is explicit in the Doppler spectrum of the channel tap coefficient (see Figure 4.8). Note that the normalized bandwidth of this Doppler spectrum is $\frac{f_{dm}T}{2} = 0.0021$ and that the nine weighted sinusoidal components are more densely distributed towards the maximum Doppler frequency. This nonuniform distribution of the sinusoidal components reproduces the Rayleigh fading power spectrum with its characteristic peak at the maximum Doppler frequency [5].

Complex AWGN of variance, σ_n^2 , is added to the transmitted signal at the output of the TDL channel model.²⁸ Thus, the SNR at the channel output is:

$$\frac{E_b}{N_0} = \frac{E \left[\sum_m |\hat{g}_m(kT)|^2 \right]}{2\sigma_n^2} \quad (4.5)$$

where

²⁸ With this particular definition, each in-phase and quadrature noise component has a variance of $\frac{\sigma_n^2}{2}$.

Figure 4.6 Time Evolution of a Rayleigh Fading Channel Tap Coefficient

Figure 4.7 Time Evolution of the Power of a Rayleigh Fading Channel

Figure 4.8 Rayleigh Fading Power Spectrum using a "Sum of Sines" Model

E_b is the energy per bit

N_0 is the noise spectral density

and the variance of the m th multipath response at the channel output is:

$$E[|\tilde{g}_m(kT)|^2] = E[|a(kT)|^2] |b(kT)|^2 E[|g_m(kT)|^2] \quad (4.6)$$

where

$E[|a(kT)|^2]$ is the variance of the input data

$|b(kT)|^2$ is the transmit filter energy

$E[|g_m(kT)|^2]$ is the variance of the m th multipath

For the sake of facility in a symbol-by-symbol simulation, the total energy per transmitted symbol at the channel output, $E\left[\sum_{m=1}^M |\tilde{g}_m(kT)|^2\right]$, can be normalized to unity. Specifically, the input QPSK data symbols, transmit filter and channel filter each have unit energy.²⁹ This implies that the energy per bit (i.e., each in-phase and quadrature component of a QPSK signal) is $\frac{1}{2}$. In fact, this is the source of the one-half factor in the SNR formula stated above. Of course, if the noise variance or the symbol energy were defined differently, then the SNR formulation could also be different, but equivalent.

4.3 Signaling Format

As discussed previously, the severity of multipath fading in DMR communications requires the use of an adaptation strategy. Specifically, block adaptation using periodic, known training sequences is appropriate for DMR communications. This adaptive strategy constrains a signaling format to consist of contiguous frames of transmitted symbols. Each of these frames is comprised of an initial training sequence of length N_t symbols followed by a data sequence of length N_d symbols [15]. Consequently, the number of symbols in each transmitted frame is:

$$\beta = N_t + N_d \quad (4.7)$$

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²⁹ The QPSK signals are defined to be $\frac{1}{\sqrt{2}}(\pm 1 \pm j)$.

Thus, the progression of transmitted frames presents alternating sequences of known training symbols and uncorrelated data symbols to the receiver (see Figure 4.9). Furthermore, since a training sequence is transmitted every frame; therefore, the CIR estimate can be updated once every β symbols.

This frame-by-frame signaling strategy is basically what is proposed for the narrowband TDMA, DCR system for North America [11]. However, our signaling format of alternating training and data sequences is rather simplified. For instance, we have not accounted for signaling information such as guard and ramp times, synchronization and other control signals which would occur at the beginning of each time slot. Nonetheless, for our purposes of investigating block adaptation, diversity combining and equalization, we can ignore these other issues.

For our simplified signaling format, we can define a measure of system overhead called the percentage data throughput:

$$\%T = \frac{100N_d}{N_d + N_t} \% = \frac{100N_d}{\beta} \% \quad (4.8)$$

4.4 Modulation Technique

For our investigation, the DMR communications system uses a linear phase modulation technique known as $\pi/4$ -shifted DQPSK [55]. This modulation scheme is currently proposed for the North American DCR system [11]. As with all linear modulation methods, this technique provides a high spectrum efficiency relative to FSK techniques [56]. Considering our DMR system as a case in point, a QPSK-type modulation scheme with 25% excess bandwidth yields a spectrum efficiency of $\frac{48 \text{ kbps}}{30 \text{ kHz}} = 1.6 \frac{\text{bps}}{\text{Hz}}$. This relatively high spectrum efficiency is important for high capacity voice or data communications.

Besides its linearity, $\pi/4$ -shifted DQPSK also possesses other useful properties. For instance, due to the $\pi/4$ -shifting of the signal constellation, the dynamic range of its signal envelope is relatively small. Consequently, this linear modulation method is less affected (i.e., less spectrum spreading) by nonlinear, but power-efficient, transmitting amplifiers

Figure 4.9 Signaling Format

[56]. In addition, $\frac{\pi}{4}$ -shifted DQPSK can employ noncoherent detection techniques, e.g., differential detection or frequency discrimination detection, which require differential encoding/decoding. Noncoherent detection is useful for communications in fast fading environments where carrier recovery can be a problem for coherent detection. Since noncoherent detection does not require carrier recovery³⁰, it achieves a lower irreducible BER due to random FM noise than coherent detection [32, 56]. Furthermore, $\frac{\pi}{4}$ -shifted DQPSK demodulation can be easily implemented with a relatively simple limiter-discriminator and an integrate-and-dump filter [56]. However, $\frac{\pi}{4}$ -shifted DQPSK does have the disadvantage that its four-level eye diagram requires a sufficiently accurate and complex modulator [56]. Also, for the case when less than 30% excess bandwidth is used, the amplitude and phase of a $\frac{\pi}{4}$ -shifted DQPSK signal can change quite rapidly. The associated high frequency distortion can adversely affect the performance of linearized power amplifiers [32].

Despite its advantage of not requiring carrier recovery, noncoherent detection does incur the overhead of differential encoding/decoding. In fact, for an ideal AWGN channel, differential decoding of QPSK signals can degrade the BER performance by a factor of two. This BER degradation due to differential decoding is valid only for the ideal AWGN channel where the independence of symbol errors results in two decoding errors for each symbol error. However, for a fading channel where symbol errors tend to occur in bursts, the BER degradation due to differential decoding is less severe than for the ideal AWGN case. This is because e_s consecutive symbol errors in a burst cause only $e_s + 1$, rather than $2e_s$, decoding errors. Nevertheless, if coherent detection is used, as is the case when equalization is implemented, then differential decoding would lead to a degradation in BER performance and thus should not be used in general. Despite this inferior BER performance of differential decoding, a system using coherent detection could still benefit from using differential encoding/decoding to resolve carrier phase ambiguity of $\frac{\pi}{2}$ radians (for QPSK) which can arise due to errors in the phase-locked

³⁰ Differential detection does require the phase information of two consecutive symbol periods.

loop (PLL) carrier recovery circuit or in the channel estimation. This is in fact the reason why our system uses differential encoding/decoding.

The signal constellation of $\frac{\pi}{4}$ -shifted DQPSK consists of two QPSK constellations with one $\frac{\pi}{4}$ -rotated relative to the other (see Figure 4.10) [11, 56]. The $\frac{\pi}{4}$ -shifted DQPSK modulation technique uses differential encoding and decoding. In the differential encoding process, transmitted symbols contain information about the phase difference between successive symbols rather than the symbol phases themselves [55]. Specifically, the $\frac{\pi}{4}$ -shifted DQPSK output signal, $\Delta_a(kT)$, from the differential encoder at time, kT , is:

$$\Delta_a(kT) = a(kT)\Delta_a((k-1)T) \quad (4.9)$$

where $a(kT)$ is the input QPSK data signal

In effect, a phase addition occurs at the differential encoder:

$$\angle[\Delta_a(kT)] = \angle[a(kT)] + \angle[\Delta_a((k-1)T)] \quad (4.10)$$

where $\angle[\ ]$ is the phase of a signal

Since each input signal, $a(kT)$, is QPSK, the phase of the differentially encoded signal, $\Delta_a(kT)$, can only change by $\pm\frac{\pi}{4}$ or $\pm\frac{3\pi}{4}$ relative to the phase of its previous value, $\Delta_a((k-1)T)$. Furthermore, the differentially encoded signal can be mapped into Gray codes before transmit filtering [11, 56].

Differential decoding can be performed after the detection process at the receiver. Specifically, the output QPSK signal, $\hat{a}(kT)$, of the differential decoder at time, kT , is the received QPSK data symbol:

$$\hat{a}(kT) = \hat{\Delta}_a(kT)\hat{\Delta}_a^*((k-1)T) \quad (4.11)$$

where $\hat{\Delta}_a(kT)$ is the $\frac{\pi}{4}$ -shifted DQPSK received and detected signal at the input to the differential decoder and $*$ is the complex conjugate operator.

In effect, a phase subtraction occurs at the differential decoder to get back the input QPSK signal:³¹

$$\angle[\hat{a}(kT)] = \angle[\hat{\Delta}_a(kT)] - \angle[\hat{\Delta}_a((k-1)T)] \quad (4.12)$$

³¹ The correct QPSK signal is recovered only if the symbol decision is correct.

Figure 4.10 Pi/4-Shifted DQPSK Signal Constellation

4.5 Transmit and Receive Filtering

The specification of a QPSK transmission bandwidth, $B = \frac{1+\epsilon}{T}$, of 30 kHz and a symbol rate, $\frac{1}{T}$, of 24 ksp/s requires a rolloff factor, ϵ , of 0.25 in the transmit and receive filters. Since the channel spacing is also 30 kHz, this choice of 25% excess bandwidth allows for the maximum filter rolloff while ensuring zero ACI (assuming perfect filter tolerance). Of course, one could use more excess bandwidth so as to relax the design requirements of the filtering and to reduce the ISI problem associated with the relatively long impulse responses of the transmit and receive filters. However, for a given symbol rate, this facilitation of the filter design incurs the penalty of increased susceptibility to ACI.

Both the transmit and receive filters are designed at baseband to have linear phase and square root raised cosine (SRRC) frequency response (see Figures 4.11, 4.12)[11]:

$$H(f) = \begin{cases} \sqrt{T} & 0 \leq |f| \leq \frac{1-\epsilon}{2T} \\ \sqrt{\frac{T}{2} (1 - \sin(\frac{\pi}{2\epsilon}(2T|f| - 1)))} & \frac{1-\epsilon}{2T} \leq |f| \leq \frac{1+\epsilon}{2T} \\ 0 & |f| > \frac{1+\epsilon}{2T} \end{cases} \quad (4.13)$$

where ϵ is the Nyquist rolloff factor

We choose this SRRC filter design so that the cascaded combination of both the transmit and receive filters has linear phase and a raised cosine (RC) frequency response which satisfies Nyquist's criterion for zero ISI (see Figure 4.14). That is, the impulse response of the transmit and receive filter tandem has zero-crossings at integer multiples of the symbol period (see Figure 4.13). Thus, if the channel were distortionless (which is not the case in DMR!) and if the receiver were to sample correctly at the baud instants, then communications with this system incurs no ISI performance degradation. On the other hand, a possible disadvantage of SRRC filters is their noncausal impulse response, since an inherent delay is required to process the precursor part of the impulse response.

For a cascaded transmit and receive filter pair, with each filter being 25% SRRC, most of the energy in the overall impulse response is contained within a five symbol

Figure 4.11 Impulse Response of an SRRC Filter

Figure 4.12 Frequency Response of an SRRC Filter

Figure 4.13 Impulse Response of a Raised-Cosine Overall Filtering

Figure 4.14 Frequency Response of a Raised-Cosine Overall Filtering

period centered about its peak (see Figure 4.13). Since the actual CIR of our DMR communication system is roughly one symbol period in length, the overall CIR length (including the transmit and receive filters) is about six symbol periods. Finally, the transmit and receive filters are both implemented in our DMR system as $\frac{T}{2}$ -spaced FIR filters with the number of filter taps being 55 and 64, respectively [8].

4.6 Block Adaptation

4.6.1 Periodic Channel Estimation with Training Sequences

The aim of channel estimation is to learn the channel impulse response (CIR) of an unfamiliar channel which can, in general, vary randomly with time. For a relatively static channel, an initial training exercise would be sufficient to allow the receiver to adapt to the channel and to operate for the rest of the communication period without further retraining. However, some form of decision-directed adaptation is usually required to track a channel during the data communication mode. On the other hand, for a fast fading channel, decision-directed tracking may not be adequate to keep up with the channel variations. Therefore, periodic training with known data symbols is required for the receiver to learn the rapidly varying channel on a regular basis. For example, in a narrowband TDMA, DCR system, a user would most likely encounter drastically different channels from one time slot to the next. Thus, the need arises for the use of training at the beginning of each time slot to update the receiver parameters. Furthermore, especially for a rapidly fading channel, the CIR also needs to be tracked during the data sequence. Training sequences with known data symbols are used to learn a channel so that unreliable receiver decisions are avoided. This is due to the fact that, since the CIR is not known ahead of time, the initial operation of the receiver would tend to yield an unacceptably high percentage of decision errors.

Receiver update using channel estimates is a distinctly two-step adaptation process which entails CIR estimation followed by receiver (e.g., OCDFE) coefficient computation. This adaptive approach is fundamentally different from the one-step approach of directly adapting the receiver parameters. For our block adaptation strategy of CIR

interpolation without explicit CIR tracking, the main advantage of the two-step adaptive approach over the more direct one-step method is that time variations in the CIR are less severe than in the receiver parameters. Thus, to achieve the same performance, CIR interpolation would require less frequent training (i.e., a lower "sampling rate") than interpolation of the receiver parameters. Moreover, with CIR interpolation, the maximum rate of time variation in the CIR is upper-bounded by the maximum Doppler frequency. However, with receiver parameter interpolation, the maximum rate of change in the receiver parameters is not obvious and would therefore require some trial and error to achieve optimum performance. Furthermore, when the receiver coefficients fluctuate abruptly³², an interpolator can fail to follow the time variations of the receiver parameters. That is, the interpolation sampling rate may not be sufficiently high to reconstruct the abruptly time-varying receiver coefficients.

It is worth noting that for adaptation strategies other than CIR interpolation, e.g., decision-directed CIR tracking, LMS channel estimation is usually faster and less complex (even with the subsequent receiver computation) than LMS adaptation of the receiver coefficients. Furthermore, if a receiver uses a maximum-likelihood sequence estimator (MLSE) instead of, say, a decision-feedback equalizer (DFE), then channel estimation is the obvious direct approach to receiver adaptation [15].

In the DMR communications system of this thesis investigation, channel estimation is performed with a Block Least Squares (BLS) adaptive algorithm under the assumption that the CIR is fixed during the training period [15].³³ For an overall CIR length of N_c symbol periods, the required minimum length of a training sequence is [15]:

$$N_t \geq 2N_c - 1 \quad (4.14)$$

For the case of our DMR channel, an overall CIR length of six symbol periods is specified. Of this six symbol period length, approximately five symbol periods are due

³² For instance, when the CIR briefly experiences a spectral null (i.e., during a deep fade), an equalizer coefficient, in its attempt to "invert" the folded channel spectrum, could swing quickly from a very large positive value to a very large negative value.

³³ For a rapidly fading channel, this assumption is not valid. Consequently, the training performance would be degraded accordingly.

to the transmit and receive filter impulse responses while about one symbol period is attributed to the actual CIR of the DMR channel. Any remaining energy in the overall CIR which is not accounted for within the six symbol period can be thought of as part of the CIR estimation noise.

For the DMR channel of length $N_c = 6$, an optimum BPSK (Binary Phase Shift Keying) training sequence of length $N_t = 15$ is used to estimate the CIR. It has been shown that channel estimation with a similar training sequence of minimum length suffers about a 3 dB MSE degradation at the equalizer output upon completion of a training period [15]. This performance degradation is measured relative to the MMSE for the case of a known CIR. For our case of a six symbol long CIR, the minimum training sequence length is eleven. Thus, for our training sequence of length $N_t = 15$, the MSE degradation at the end of a training period would probably be somewhat less than 3 dB.

The optimum BPSK training sequence, $\underline{a}_t$, can be converted into a complex $\frac{\pi}{4}$ -shifted DQPSK training sequence, $\underline{\Delta}_t$, by multiplying each k th BPSK symbol in the training sequence by the phase factor, $e^{jk\frac{\pi}{4}}$ [15]:

$$\Delta_{tk} = a_{tk} e^{jk\frac{\pi}{4}} \quad 0 \leq k < N_t \quad (4.15)$$

This new $\frac{\pi}{4}$ -shifted DQPSK training sequence is again an optimum training sequence whose inverse of its autocorrelation matrix is [15]:

$$\Phi_{\Delta}^{-1} = \Lambda \Phi_a^{-1} \Lambda^* \quad (4.16)$$

where Λ is an $N_c \times N_c$ diagonal matrix whose n th diagonal entry is:

$$e^{j(N_c - n - 1)\frac{\pi}{4}} \quad 0 \leq n < N_c \quad (4.17)$$

and Φ_a^{-1} is the inverse of the autocorrelation matrix for the original, BPSK training sequence:

$$\Phi_a^{-1} = \left[\sum_{k=1}^{N_t - N_c + 1} \underline{a}_t(kT) \underline{a}_t^*(kT) \right]^{-1} \quad (4.18)$$

where $N_t - N_c + 1$ is the observation interval of the training process and $*$ denotes complex conjugate transpose.

Given the known BPSK training sequence, $\underline{a}_t$, the inverse of the $\frac{\pi}{4}$ -shifted DQPSK autocorrelation matrix, Φ_{Δ}^{-1} , can be precomputed. Thus, the CIR estimate, $\underline{h}$, can be obtained from the BLS solution:

$$\underline{h} = \Phi_{\Delta}^{-1} \underline{p}_t \quad (4.19)$$

where the cross-correlation vector, $\underline{p}_t$, must be computed from the received data samples, $r(kT)$, during the training observation interval, $N_t - N_c + 1$:

$$\underline{p}_t = \sum_{k=1}^{N_t - N_c + 1} \underline{\Delta}_t(kT) r^*(kT) \quad (4.20)$$

4.6.2 Periodic Channel Tracking with CIR Interpolation

Channel estimation using periodic training with known data symbols is a necessary, but not sufficient, block-adaptive technique for communications over a randomly time-varying transmission medium such as the DMR channel [8, 15]. For such a rapidly fading channel, a reasonably faithful CIR estimate obtained from training at the beginning of a data frame is not likely to remain an accurate representation of the randomly varying CIR during the data frame. In fact, the DMR channel can fluctuate significantly such that the CIR estimate from one training sequence to the next can be drastically different. Thus, a more accurate representation of the randomly varying CIR is required during a data sequence in between training exercises.

There are several strategies to ensure more accurate CIR estimates during the data transmission sequences. For example, since the Doppler frequency is directly proportional to the mobile speed, a reduction in the vehicular speed can lead to a slower time variation in the CIR. However, this is clearly not an acceptable solution. Another method to reduce the effective time variation in the CIR (relative to the symbol rate) is to increase the data symbol rate. This is, in fact, the approach used for the Pan-European DMR (GSM) system [13]. Nevertheless, this option of using a wideband

DMR system is not compatible with the narrowband DMR system of North America [11]. An alternative approach is to use more frequent training at the expense of system throughput. Furthermore, the CIR can be tracked by a continuously adaptive, decision-directed algorithm such as the LMS adaptive algorithm. However, it has been found that such a decision-directed tracking algorithm is quite susceptible to decision errors [8]. For example, in the event of a deep fade in the channel, such a decision-directed algorithm is likely to lose track of the randomly varying CIR and, consequently, cause unreliable detection in the rest of the data frame.

To avoid the error-propagation problem associated with decision-directed tracking, we propose and investigate a novel method which generates interpolated CIR estimates during a data sequence by interpolating trained CIR estimates obtained from adjacent, contiguous training sequences. This block adaptation approach overcomes the error-propagation problem by processing trained CIR estimates that are derived from known training symbols.

Without a loss of generality, the number of trained CIR estimates used for each interpolation, $Q = 2p$, is assumed to be an even number so that the interpolation can be performed for the middle data frame. Consequently, each interpolated CIR estimate in the middle interpolation interval is computed from $\frac{Q}{2}$ past and $\frac{Q}{2}$ future CIR estimates obtained from training. Thus, it is evident that this CIR interpolation approach incurs an inherent processing delay of D_t .

In the worst case when all Q past and future CIR estimates from training are not known at the time of the desired interpolation (e.g., this situation arises at the beginning of a time slot of contiguous data frames), the interpolation delay is:

$$D_t = (Q - 1)\beta T \quad (4.21)$$

where

Q is the number of trained CIR estimates used for each interpolation

β is the number of symbol periods between consecutively trained CIR estimates

T is the symbol period

On the other hand, in the usual, and best, case when the $\frac{Q}{2}$ previously trained CIR estimates are known at the time of interpolation and only $\frac{Q}{2}$ future CIR estimates need to be obtained from training (e.g., this situation occurs during a time slot when at least $\frac{Q}{2}$ trained CIR estimates have been trained previous to the present interpolation data frame), the interpolation delay is:

$$D_t = \frac{Q}{2}\beta T \quad (4.22)$$

The legitimacy of the block-adaptive method of CIR interpolation is a consequence of the sampling theorem (see Appendix B). In particular, to satisfy Nyquist's sampling criterion, the sampling period of the trained CIR estimates is required to be:

$$T_s = \beta T \leq \frac{1}{2f_{dm}} \quad (4.23)$$

where f_{dm} is the maximum Doppler frequency of the equivalent baseband Doppler spectrum of the DMR channel. For the case of a symbol rate of $\frac{1}{T} = 24 \text{ ksp/s}$ and a maximum Doppler frequency of $f_{dm} = 100 \text{ Hz}$, in order for CIR interpolation to generate correct interpolated CIR samples, an individual frame, consisting of one training sequence of length N_t and a data sequence of length N_d , must have a length of $\beta = N_t + N_d \leq 120$ symbol periods.

In general, the sampling theorem is formulated in terms of sampled continuous-time signals. However, for specific applications where the signals are inherently discrete-time sequences, it is simpler to perform the interpolation directly using a digital filter [57, 58]. The complex baseband CIR interpolation in our DMR communications system is an example of such a digital filtering process (see Appendix C).

Specifically, each estimated $\frac{T}{2}$ -spaced CIR sample, $\hat{h}(qT_s)$, has a length, N_c , of six symbol periods.³⁴ Thus, each estimated CIR sample is a vector of length $2N_c = 12$ tap coefficients.³⁵ Since the CIR is modelled in complex baseband, each CIR tap coefficient

³⁴ As mentioned earlier, for a CIR length of six symbol periods, approximately five symbol periods are due to the transmit and receive filters while about one symbol period is attributed to the actual, multipath channel.

³⁵ Equivalently, this $\frac{T}{2}$ -spaced CIR sample can be comprised of two interleaved T -spaced CIR samples of length N_c , which are offset by half a symbol period.

consists of in-phase and quadrature components which are characterized as independent, zero-mean, Gaussian random processes [5, 6, 7, 36]. The CIR interpolation process requires each in-phase and quadrature component of each CIR tap coefficient to be digitally filtered by a real-valued interpolator, $f(rT'_s)$.

In the CIR interpolation strategy, Q consecutively estimated CIR samples, $\underline{h}(qT_s)$, are interpolated by a real-valued digital filter, $f(rT'_s)$, to generate $R - 1$ evenly spaced, interpolated CIR samples, $\hat{\underline{h}}(rT'_s)$, between the two middle estimated CIR samples.³⁶ Given that one estimated CIR sample, $\underline{h}(qT_s)$, is obtained through training at the start of each data sequence; therefore, the Q estimated CIR samples are spaced β symbol periods apart (see Figure 4.15). Furthermore, since $R - 1$ evenly spaced, interpolated CIR samples are generated in a given data sequence between two consecutive training sequences, the $R - 1$ interpolated CIR samples per interpolation interval (i.e., per data sequence) are spaced μ symbol periods apart. Consequently, the interpolation factor, R , can be defined as [57, 58]:

$$R = \frac{T_s}{T'_s} = \frac{\beta T}{\mu T} = \frac{\beta}{\mu} = \frac{N_t + N_d}{\mu} \quad (4.24)$$

where

T_s is the sampling period of the estimated CIR samples

T'_s is the sampling period of the interpolated CIR samples

T is the symbol period

β is the number of symbol periods between estimated CIR samples

μ is the number of symbol periods between interpolated CIR samples

N_t is the number of symbols in a training sequence

N_d is the number of symbols in a data sequence

³⁶ The R th interpolated CIR sample is the same as one of the estimated CIR samples and is, therefore, not counted as a new interpolated CIR sample.

Figure 4.15 CIR Interpolation and Signaling Format

From the above description, it is evident that there are three discrete-time scales involved with the CIR interpolation of Q consecutively estimated CIR samples, $\underline{h}(qT_s)$, to generate $R - 1$ interpolated CIR samples, $\hat{\underline{h}}(rT'_s)$, in a given interpolation interval. At the lowest level, there is the symbol interval, T , with a discrete-time index, $k : 0 \rightarrow (Q - 1)\beta$, spanning all $Q - 1$ contiguous data frames. Next, there is the interpolation interval, $T'_s = \mu T$, with a discrete-time index, $r : 0 \rightarrow R - 1$, running through all R interpolated samples in a given interpolation interval within one data sequence. Finally, there is the training interval, $T_s = \beta T$, with a discrete-time index, $q : 0 \rightarrow Q - 1$, spanning all Q consecutively estimated CIR samples. The interpolation interval, T'_s , determines the spacing between interpolated CIR samples, while the training interval, T_s , determines the spacing between estimated CIR samples. The CIR interpolation is performed at these two time scales which are related by the interpolation factor, $R = \frac{T_s}{T'_s}$.³⁷

The interpolated CIR sequence, $\hat{\underline{h}}(r)$, is the output of the CIR interpolation filter (see Figure 4.16). Thus, it can be represented as the convolution of the oversampled input sequence of Q CIR samples, $\underline{h}(r)$, with the unit sample response of the interpolator, $f(r)$ [57, 58]:

$$\hat{\underline{h}}(r) = \underline{h}(r) * f(r) = \underline{h}\left(\frac{r}{R}\right) * f(r) \quad r = 0, \dots, R - 1 \quad (4.25)$$

where the interpolated output sequence, $\hat{\underline{h}}(r)$, is evaluated over one interpolation interval to generate R interpolated samples (see Appendix D).

For the CIR interpolation in our DMR communications system, we use a practical digital filter, $f(r)$, with a truncated discrete-time window, $\Pi(r; \eta)$, which corresponds to an approximately raised-cosine frequency response of the overall interpolator (see Appendix E):

$$f(r) = \hat{f}(r)\Pi(r; \eta) \quad r = 0, \pm 1, \pm 2, \dots \quad (4.26)$$

³⁷ For the sake of notational simplicity, the sampling periods can be implicit. That is, $qT'_s \rightarrow q$ and $rT'_s \rightarrow r$.

Figure 4.16 Channel Impulse Response (CIR) Interpolator

where the unit sample response of the ideal lowpass filter is:

$$\tilde{f}(r) = \frac{\sin(\frac{\pi r}{R})}{\frac{\pi r}{R}} \quad r = 0, \pm 1, \pm 2, \dots \quad (4.27)$$

and the approximately raised-cosine (in frequency), discrete-time window is:

$$\Pi(r; \eta) = \begin{cases} \frac{\cos(\eta \pi \frac{r}{R})}{1 - (2\eta \frac{r}{R})^2} & -\left(\frac{N_p-1}{2}\right) \leq r \leq \left(\frac{N_p-1}{2}\right) \\ 0 & \text{otherwise} \end{cases} \quad (4.28)$$

where N_p is the width of the discrete-time window (which is just the length of the unit sample response of the interpolation filter). Thus, the unit sample response of the practical interpolator is a truncated Nyquist pulse. Note that since the time response is truncated (i.e., windowed), the frequency response is only approximately raised-cosine. In fact, the frequency response is infinite in duration and exhibits ripple distortion in the passband and stopband. This ripple distortion is especially significant for small values of η . However, for the purposes of description, we refer to such a window function as (approximately) raised-cosine (in frequency). As an example, the unit sample and frequency responses of a rectangular window (i.e., $\eta = 0$) are shown in Figures 4.17 and 4.18.

4.7 Decision-Feedback Equalization and Diversity Combining

The general objective of equalization and diversity combining is to minimize the detrimental effects of distortions in a multipath fading channel. Specifically, equalization can be used to minimize the ISI caused by frequency-selective fading and the relatively long impulse responses of the transmit and receive filters [8, 12]. On the other hand, diversity combining can be used as protection against the frequent deep fading exhibited by the DMR channel [28, 29]. In our DMR communications system, we implement a block-adaptive receiver which jointly incorporates diversity combining and decision-feedback equalization. This OCDFE receiver is adaptively computed using interpolated CIR estimates, $\hat{h}_l(kT)$, from each of the L diversity channels, to yield optimum performance in the MMSE sense.³⁸ The interpolated CIR estimates are generated every

³⁸ OCDFE denotes Optimum Combiner and Decision-Feedback Equalizer.

Figure 4.17 Impulse Response of a Raised-Cosine Interpolator with Zero Rolloff

Figure 4.18 Frequency Response of a Raised-Cosine Interpolator with Zero Rolloff

$T' = \mu T$ symbol periods (i.e., every interpolation interval) and consists of the combined impulse responses of the multipath fading channel and the transmit and receive filters.

The OCDFE receiver has a fractionally spaced forward filter, $\underline{w}_l$, in each of its L diversity branches and a feedback filter, $\underline{w}_b$, in a common feedback loop emanating from the output of the decision device (see Figure 4.19).³⁹ Each forward filter, $\underline{w}_l$, is implemented as a $\frac{T}{2}$ -spaced TDL with N_f tap coefficients (see Figure 4.20). Forward filters with $\frac{T}{2}$ -spaced taps are used because the channel, noise and received signals are all represented by $\frac{T}{2}$ -spaced models. On the other hand, since a detected signal, $\hat{\Delta}_a(kT)$, is generated every symbol period, the feedback filter, $\underline{w}_b$, is implemented as a T -spaced TDL with N_b tap coefficients (see Figure 4.21). At the l th diversity branch, each forward filter essentially minimizes the precursor ISI of the $\frac{2}{T}$ -sampled received signal, $r_l(kT - m\frac{T}{2})$. Furthermore, the remaining postcursor ISI of the $\frac{1}{T}$ -sampled output signal from the L -diversity linear combiner is minimized by subtracting the $\frac{1}{T}$ -sampled output signal from the feedback filter. Thus, assuming proper adaptation, the overall $\frac{1}{T}$ -sampled output signal from the OCDFE, $y(kT)$, should have minimum ISI and noise corresponding to a maximum SNR in the MMSE sense. Moreover, $y(kT)$ is the $\frac{1}{T}$ -sampled input signal to the decision device and can be expressed compactly in vector form as:

$$y(kT) = \underline{w}^* \underline{x}(kT) \quad (4.29)$$

where $*$ denotes the complex conjugate transpose operator and the OCDFE weight vector is comprised of both forward and feedback tap coefficients:

$$\underline{w} = \begin{bmatrix} \underline{w}_0 \\ \underline{w}_1 \\ \vdots \\ \underline{w}_{L-1} \\ \underline{w}_b \end{bmatrix} \quad (4.30)$$

³⁹ For the sake of notational simplicity, the time dependence of the OCDFE weight vectors is implicit. That is, $\underline{w}_l = \underline{w}_l(kT)$ and $\underline{w}_b = \underline{w}_b(kT)$.

while the overall input signal to the OCDFE is comprised of both received signals, $r_l(kT - m\frac{T}{2})$, and detected feedback signals, $\hat{\Delta}_a(kT)$:

$$\underline{x}(kT) = \begin{bmatrix} \underline{r_0}(kT) \\ \underline{r_1}(kT) \\ \vdots \\ \underline{r_{L-1}}(kT) \\ \underline{\hat{\Delta}_a}(kT) \end{bmatrix} \quad (4.31)$$

Using the MMSE criterion to minimize the MSE at the output of the OCDFE:

$$mse = E[|y(kT) - \Delta_a((k-D)T)|^2] \quad (4.32)$$

it can be shown (see Appendix F) that, for a given set of interpolated CIRs and noise statistics in the L diversity channels, the optimum OCDFE coefficients are computed as:

$$\underline{w_{opt}} = \Phi^{-1} \underline{p} \quad (4.33)$$

where the autocorrelation matrix is:

$$\Phi = E[\underline{x}(kT)\underline{x}^*(kT)] \quad (4.34)$$

and the cross-correlation vector is:

$$\underline{p} = E[\underline{x}(kT)\Delta_a^*((k-D)T)] \quad (4.35)$$

with $*$ denoting the complex conjugate transpose operator and DT representing the processing delay in the OCDFE.

The optimum OCDFE weight vector is an $(LN_f + N_b) \times 1$ partitioned column vector:

$$\underline{w_{opt}} = \begin{bmatrix} \underline{w_{0,opt}} \\ \underline{w_{1,opt}} \\ \vdots \\ \underline{w_{L-1,opt}} \\ \underline{w_{b,opt}} \end{bmatrix} \quad (4.36)$$

Figure 4.19 Block Diagram of the OCDFE Receiver

Figure 4.20 Tapped Delay Line Forward Filter in the l th Diversity Branch

Figure 4.21 Tapped Delay Line Feedback Filter

while the OCDFE autocorrelation matrix is an $(LN_f + N_b) \times (LN_f + N_b)$ Hermitian, positive-semidefinite matrix which is partitioned as follows:⁴⁰

$$\Phi = \begin{bmatrix} \Phi_{0,0} & \Phi_{0,1} & \dots & \Phi_{0,L-1} & \Phi_{0,b} \\ \Phi_{1,0} & \Phi_{1,1} & \dots & \Phi_{1,L-1} & \Phi_{1,b} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \Phi_{L-1,0} & \Phi_{L-1,1} & \dots & \Phi_{L-1,L-1} & \Phi_{L-1,b} \\ \Phi_{b,0} & \Phi_{b,1} & \dots & \Phi_{b,L-1} & \Phi_{b,b} \end{bmatrix} \quad (4.37)$$

where all the $N_f \times N_f$ submatrices, $\Phi_{l,q}$, are Hermitian matrices, while the $N_b \times N_b$ diagonal submatrix, $\Phi_{b,b} = I$, is an identity matrix.

Specifically, the Hermitian symmetry of the autocorrelation matrix implies that:

$$\Phi_{l,q} = \Phi_{q,l}^* \quad l, q = 0, 1, \dots, L-1 \quad (4.38)$$

and

$$\Phi_{l,b} = \Phi_{b,l}^* \quad l = 0, 1, \dots, L-1 \quad (4.39)$$

Moreover, the (i, j) th element of a submatrix, $\Phi_{l,q}$, for diversity signal correlation, is:

$$\begin{aligned} \Phi_{l,q}(i, j) &= \sum_k \hat{h}_l \left(kT - i \frac{T}{2} \right) \hat{h}_q^* \left(kT - j \frac{T}{2} \right) + \sigma_n^2 \Phi_n \left((i - j) \frac{T}{2} \right) \\ &\quad i, j = 0, 1, \dots, N_f - 1 \\ &\quad l, q = 0, 1, \dots, L - 1 \end{aligned} \quad (4.40)$$

where

$\hat{h}_l$ and $\hat{h}_q$ are sample coefficients of the interpolated CIRs of the l th and q th diversity channels, respectively

σ_n^2 is the noise variance

Φ_n is the received-filtered, noise correlation function

⁴⁰ The Hermitian symmetry and positive-semidefinite property of the autocorrelation matrix permit the use of the Cholesky Decomposition method to solve for the optimum OCDFE coefficients [7, 59].

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and the (i, j) th element of a submatrix, $\Phi_{l,b}$, for diversity signal and feedback data correlation, is a sample coefficient of the l th interpolated CIR:

$$\begin{aligned}\Phi_{l,b}(i, j) &= \hat{h}_l((D+1)T - i\frac{T}{2} + jT) \quad i = 0, 1, \dots, N_f - 1 \\ &\quad j = 0, 1, \dots, N_b - 1 \\ &\quad l = 0, 1, \dots, L - 1\end{aligned}\quad (4.41)$$

For the case of antenna diversity with independently fading, received branch signals, the ensemble averages (with respect to the fading CIR) of the off-diagonal submatrices are zero-valued:

$$\begin{aligned}E[\Phi_{l,q}(i, j)] &= 0 \quad l \neq q \\ i, j &= 0, 1, \dots, N_f - 1\end{aligned}\quad (4.42)$$

Furthermore, the OCDFE cross-correlation vector is an $(LN_f + N_b) \times 1$ partitioned column vector:

$$\underline{p} = \begin{bmatrix} \underline{p_0} \\ \underline{p_1} \\ \vdots \\ \underline{p_{L-1}} \\ \underline{p_b} \end{bmatrix}\quad (4.43)$$

where the elements of the l th cross-correlation subvector are just $\frac{T}{2}$ -spaced sample coefficients of the l th interpolated CIR:

$$\underline{p_l} = \underline{\hat{h}_l}(kT) = \begin{bmatrix} \hat{h}_l(DT) \\ \hat{h}_l(DT - \frac{T}{2}) \\ \vdots \\ \hat{h}_l(DT - (N_f - 1)\frac{T}{2}) \end{bmatrix} \quad l = 0, 1, \dots, L - 1 \quad (4.44)$$

and

$$\underline{p_b} = \underline{0} \quad (4.45)$$

Chapter 5 Computer Simulation Experiments

5.1 Fixed Interpolator Performance

Our DMR communications system employs CIR interpolation to track the time variations in the rapidly fading channel (see Section 4.6.2). This block-adaptive method requires a digital filter to generate interpolated CIR estimates in a given data frame by processing CIR estimates obtained from periodic training. The optimum interpolator is the unrealizable, ideal lowpass filter (see Appendix C). However, in practice, there is a wide range of interpolator designs which yield varying degrees of performance for a given filter complexity and design objective [57]. For our purposes, we only consider fixed (i.e., nonadaptive) FIR interpolators which employ time-windowing techniques.

Specifically, we investigate the interpolation performance of digital filters which use approximately raised-cosine (in frequency) and generalized Hamming windows (see Appendix E). Both of these discrete-time window functions have a design parameter, $0 \leq \eta \leq 1$, which controls the shape of the time window or, equivalently, the frequency response of the interpolator. As a note, the special case of a rectangular window corresponds to a zero rolloff factor, $\eta = 0$, for the raised-cosine window. On the other hand, for the generalized Hamming window, $\eta = 1$ is equivalent to a rectangular window (see Appendix E).

A C-language program was written to investigate the performance of a digital interpolator. In particular, various interpolators were implemented to track the time variations of a single tap coefficient of a CIR (see Figure 5.22). The Rayleigh fading of the complex-valued tap coefficient is simulated using the "sum of sines" model [5, 41]. Furthermore, the fade rate of the tap coefficient is determined by the maximum Doppler frequency, f_{dm} , which is an input parameter to the simulation. In addition to the window parameter, η , the interpolation is also specified by the number of estimated CIR samples used per interpolation, Q , the number of symbol periods between estimated CIR samples, β , and the interpolation factor, $R = \frac{\beta}{\mu}$. Moreover, a symbol rate, $\frac{1}{T}$, of 24 kbps is assumed throughout the investigation, while each simulation is performed

Figure 5.22 Time Evolution and Interpolation of a Rayleigh Fading Tap Coefficient

over an ensemble of $N = 1000$ trials with independent Rayleigh fading of the tap coefficient.

In our simulation study of interpolator performance, we investigate the effect on the interpolation MSE by the following factors:

1. interpolation sampling period: β , the number of samples between estimated CIR samples used for each interpolation.
2. interpolation filter: optimum choice of the rolloff parameter, η , for the raised-cosine and generalized Hamming windows.
3. interpolation order: Q , the number of estimated CIR samples used for each interpolation.
4. interpolation factor: R , the number of interpolated CIR samples generated in a given interpolation interval.
5. maximum Doppler frequency: f_{dm}
6. estimation noise: σ_e^2 , the variance of the estimation noise of each CIR sample used for an interpolation.

5.1.1 Interpolator Performance Criteria

For a given simulation investigation, the interpolation MSE, averaged over an ensemble of $N = 1000$ trials, of a tap coefficient is computed for each of the R interpolated samples:

$$mse(kT + rT_s') = \frac{1}{N} \sum_{n=0}^{N-1} \left| \hat{h}_n(kT + rT_s') - g_n(kT + rT_s') \right|^2 \quad (5.1)$$

where

$\hat{h}_n$ is the interpolated CIR tap coefficient for the n th simulation trial

g_n is the actual CIR tap coefficient for the n th simulation trial

From this ensemble statistic, a useful measure of interpolation performance is computed as the average MSE. Specifically, the average MSE is the time average of the

MSE over all interpolated samples in a given interval:

$$\overline{mse} = \frac{1}{R} \sum_{r=0}^{R-1} mse(kT + rT_s') \quad (5.2)$$

Furthermore, another measure of interpolation performance is the maximum MSE over all interpolated samples in an interpolation interval. This maximum MSE is always greater than or equal to the average MSE and is an especially meaningful measure for worst-case performance. Typically, the interpolation MSE is smallest near the trained CIR estimates at either ends of the interpolation interval. In fact, the interpolation MSE is zero at the end samples of the interpolation interval if the CIR estimates are exact and if the interpolator possesses orthonormal properties (see Appendix C). However, at the interpolated samples that are further away from the estimated samples at the ends of the interpolation interval, the interpolation MSE tends to increase to a maximum at the middle of the interpolation interval (see Figure 5.23 for the case of $SER = 40$ dB). That is, the maximum MSE occurs at the middle interpolated sample index of:

$$r = \begin{cases} \frac{R}{2} & \text{for even } R \\ \frac{R-1}{2} \text{ and } \frac{R+1}{2} & \text{for odd } R \end{cases} \quad (5.3)$$

However, if the estimated CIR samples are very noisy and, thereby, unreliable, then the opposite is true. In this case, the maximum MSE occurs at the ends of the interpolation interval while the minimum MSE occurs at the middle interpolated sample index (see Figure 5.23 for the case of $SER = 3$ dB). Nevertheless, in general, both the average and the maximum MSE are consistent measures of interpolation performance (see Figure 5.24). Thus, the minimization of either of these performance measures is a valid criterion for the design of an interpolator.

5.1.2 Interpolation Sampling Rate

In order for the CIR interpolation process to satisfy the Nyquist sampling criterion, for a symbol rate of $\frac{1}{T} = 24$ ksp/s and a maximum Doppler frequency of $f_{dm} = 100$ Hz, the spacing between estimated CIR samples must be:

$$\beta \leq \frac{1}{2f_{dm}T} = 120 \text{ symbol periods} \quad (5.4)$$

Figure 5.23 MSE Distribution over an Interpolation Interval

Figure 5.24 Consistency Between the Average MSE and the Maximum MSE

Thus, the normalized sampling rate, $\bar{f}$, is related to β as follows:

$$\bar{f} = \frac{f_s}{f_{min}} = \frac{\beta_{max}}{\beta} = \frac{120}{\beta} \quad (5.5)$$

where $f_{min} = 2f_{dm}$ is the minimum Nyquist frequency. From Equations 4.8 and 4.21, for a given N_t and Q , the relative frequency of training, $\bar{f}$, also determines the system throughput, $\%T$, and worst-case interpolation delay, D_t . Thus, the choice of β not only influences the interpolation performance, but it also affects system overhead. For the specific example of $N_t = 15$ and $Q = 4$, Figure 5.25 illustrates the relationships among these different system parameters. Furthermore, the percentage throughput, $\%T$, is shown on the upper horizontal axis of plots which include the normalized sampling rate, $\bar{f}$.

From the simulation experiments which assume interpolation with noiseless CIR samples, it is found that for a given interpolator, the interpolation MSE performance improves at higher sampling rates (see Figure 5.26). This is a reasonable result since that if the frequency of training is increased, then the tracking of a time varying signal should improve. Furthermore, it is evident from Figure 5.26 that the interpolator performance is worst at the minimum sampling rate of $\bar{f} = 1.0$ and significantly improves for $1.0 < \bar{f} < 2.0$. Specifically, for the case of a 50% rolloff raised-cosine interpolator, one can gain over 22 dB in interpolation MSE performance at a cost of reducing the data throughput from 87.5% to 75%.

The poor performance near $\bar{f} = 1.0$ can be explained by considering the frequency responses of the time-varying signal and interpolator. Near the minimum sampling rate, the Doppler, spectral images of the time-varying tap coefficient are very close together (see Figure C.56). Thus, in order to remove all the spectral images so as to reconstruct the tap coefficient response faithfully at the interpolated samples, the interpolator must have a frequency response which resembles that of an ideal lowpass filter. Such a bandlimited filter has an infinite impulse response which corresponds to the ideal characteristic of the interpolation formula of the sampling theorem (see Appendix B).

However, in practice, the number of samples used for each interpolation, Q , is constrained to a small value so that the processing delay is not prohibitively large. Thus,

interpolation interval length ("beta")	120	100	80	60	40
normalized sampling rate ("fs/(2*fdm)")	1.0	1.2	1.5	2.0	3.0
percentage throughput ("%T")	87.5	85.0	81.3	75.0	62.5
interpolation delay ("Dt") [in milliseconds]	15.0	12.5	10.0	7.5	5.0

**Figure 5.25 Representative Values of System
Parameters; for $N_t=15$, $Q=4$, $1/T=24$ ksps, $fdm=100$ Hz**

Figure 5.26 MSE Performance as a Function of the Sampling Rate

the interpolation performance is compromised by the truncation of the impulse response of the interpolator. This truncation leads to aliasing due to a finite transition bandwidth and ripple distortion in the passband and stopband. This problem is further compounded by the fact that the Rayleigh fading, tap coefficient has a significant portion of its energy near its maximum Doppler frequency. Consequently, interpolation performance is degraded because a practical filter tends to distort these peaks at the edge of the Doppler spectrum.

The filtering problem becomes less severe as the sampling rate is increased. The interpolation performance improves because, as the Doppler spectral images of the fading tap coefficient separate in frequency, the distortion caused by aliasing is reduced. The rate of improvement in the interpolation performance is greatest near the minimum sampling rate since most of the distortion due to aliasing is removed with a spectral separation of a fraction of the Nyquist frequency, e.g., $\Delta f \approx 0.5 \bar{f}$.

Furthermore, it is evident from Figure 5.26 that at and near the Nyquist frequency, the level and reduction rate of aliasing distortion is higher for an interpolation filter with a lower rolloff factor. This result is explained by the fact that a low rolloff filter has a narrow transition bandwidth which is more immune to aliasing. However, a low rolloff factor also generates significant ripples in the passband and stopband. This leads to the phenomenon of local $\overline{mse}$ maxima for the interpolators with rolloffs of 0.0 and 0.5 (see Figure 5.26). These local $\overline{mse}$ maxima are caused by aliasing of the peak of the Doppler, spectral image as it moves through the stopband ripples of the interpolation filter.

It is apparent that at low sampling rates, the interpolation MSE degradation is mainly due to aliasing distortion. Thus, at low sampling rates, an interpolator with a narrow transition bandwidth (e.g., a low rolloff raised-cosine filter) tends to perform better. Furthermore, in general, all interpolators perform better at higher sampling rates since the aliasing problem is reduced. These results are reflected in Figure 5.26.

At a sufficiently high sampling rate, all distortion due to aliasing is removed and the interpolator reaches a limiting MSE caused by its passband distortion. In theory,

if an interpolator were to use $Q = \infty$, exact estimates for its interpolation,⁴¹ then the interpolation MSE would be nonexistent (i.e., $mse = -\infty$ dB). In practice, at high sampling rates, an interpolator with a relatively flat passband response (e.g., a high rolloff raised-cosine filter) tends to perform better (see Figure 5.26). In particular, note that at a sufficiently high sampling rate, the interpolation MSE of the 100% rolloff filter will eventually be than that of the 50% rolloff filter (see Figure 5.26).

A valid conclusion of these simulation results is that high sampling rates yield better interpolation performance. However, a more important conclusion is that at a given sampling rate, depending on the relative contributions of passband and stopband (i.e., aliasing) distortion, there is an optimum fixed interpolator. This is an important result since, in practice, one cannot operate at too high of a sampling rate because of system throughput constraints. Thus, the choice of the window parameter for an interpolator (e.g., rolloff factor, η , for a raised-cosine interpolator) is important to achieving optimum interpolation performance. For example, from Figure 5.26, at a normalized sampling rate of $\bar{f} = 2.0$, a significant difference of over 12 dB in average MSE performance between a raised-cosine interpolator with zero rolloff and 50% rolloff is evident.

5.1.3 Optimum Interpolator Design

In general, a fixed CIR interpolator does not perform optimally because it does not adapt to the random nature of the fading tap coefficients of a CIR. However, as was discussed in the last section, for a given type of fixed interpolator, there is a uniquely optimum filter at each sampling rate. In particular, for the approximately raised-cosine and generalized Hamming windowed interpolators, there is an optimum choice of a windowing parameter (e.g., rolloff factor) that depends on the sampling rate.

For these particular simulations, we assume interpolation which uses noiseless estimates of the tap coefficient (i.e., $SER = \infty$ dB). The simulation results at various sampling rates for the raised-cosine interpolator performance as a function of the rolloff factor are shown in Figure 5.27. Similar results for an interpolator employing a

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⁴¹ That is, to interpolate with an ideal lowpass filter and zero estimation noise.

generalized Hamming window are presented in Figure 5.28. Note that the raised-cosine rolloff factor and the generalized Hamming parameter are inversely related in terms of their window shaping properties. Furthermore, the general results of the two types of windowed interpolators are similar. Thus, to be concise, we concentrate our discussion on the raised-cosine window.

For the raised-cosine interpolator, it is evident from Figure 5.27 that as the sampling rate increases, the optimum value of the rolloff factor also increases. As was discussed in the previous section, the reason for this is that at low sampling rates, the interpolation MSE is dominated by aliasing while as the sampling rate increases, the interpolation MSE becomes relatively less affected by aliasing and progressively more limited by passband distortion.

The simulation results also show that as the sampling rate increases, the MSE gain by selecting the optimum rolloff factor for the interpolator also increases. For example, at the Nyquist rate corresponding to $\bar{f} = 1.0$, the difference in MSE performance between interpolators with the optimum rolloff of $\eta_{opt} = 0.0$ and the worst-case rolloff of $\eta = 1.0$ is less than 2 dB. However, at the normalized sampling rate of $\bar{f} = 3.0$, the difference in MSE performance between interpolators with the optimum rolloff of $\eta_{opt} = 0.7$ and the worst-case rolloff of $\eta = 0.0$ is over 30 dB. This highlights the importance of selecting the proper window parameter for a given interpolator operating at a specific sampling rate.

At their optimum window parameters, the raised-cosine and generalized Hamming interpolators have identical MSE performance for normalized sampling rates below about 1.3. This is mainly because at such low sampling rates where aliasing is the main impairment, the optimum window for both cases is a rectangular window. However, at higher sampling rates where passband distortion is relatively more severe, the window shape or, equivalently, the frequency response of the interpolator becomes important. At higher sampling rates, it is evident from the simulation results shown in Figure 5.29 that the raised-cosine interpolator yields better performance. Thus, we conclude that the raised-cosine interpolator should be used at all sampling rates. Of course, this

Figure 5.27 MSE Performance for a Raised-Cosine Interpolator

Figure 5.28 MSE Performance for a Generalized Hamming Interpolator

Figure 5.29 Optimum Interpolation MSE Performance Comparison

interpolator should use its optimum rolloff factor for a given sampling rate. Specifically, the MSE performance improvement of the raised-cosine interpolator over that of the generalized Hamming interpolator is as much as 5 dB at $\bar{f} = 3.0$.

5.1.4 Optimum Rolloff Factor for a Raised-Cosine Window

The raised-cosine interpolator which we investigated is actually only an approximation to a Nyquist pulse which has an exact raised-cosine frequency response (see Appendix E). For a practical interpolator, a window generates a truncated discrete-time response such that the frequency response is an approximately raised-cosine shape with ripple distortion in the passband and stopband. Furthermore, because of the time truncation, the frequency response of the interpolator has infinite span. Despite these differences, the optimum performance of the raised-cosine interpolator can be analyzed by assuming an exact raised-cosine frequency response. This assumption corresponds to an impractical interpolator which has an infinite time span (i.e., N_p is infinite).

The bandwidth of the ideal raised-cosine interpolator is:

$$W_p = \frac{f_s}{2}(1 + \eta) \quad (5.6)$$

where f_s is the sampling rate of the interpolator

Exact interpolation essentially requires ideal lowpass filtering of the oversampled Doppler spectrum so as to remove its spectral images (see Appendix C). In other words, this undistorted filtering has constant gain in the passband and zero gain in the stopband. An ideal raised-cosine interpolator has a flat response for the frequency range of:

$$f \leq \frac{f_s}{2}(1 - \eta) \quad (5.7)$$

Therefore, to filter a Doppler spectrum of bandwidth f_{dm} without distortion, we require:

$$f_{dm} \leq \frac{f_s}{2}(1 - \eta) \quad (5.8)$$

Furthermore, this can be written in terms of the rolloff factor as a function of the sampling rate of the interpolator:

$$\eta \leq 1 - \frac{1}{\bar{f}} \quad (5.9)$$

where the normalized sampling rate of the interpolator is:

$$\bar{f} = \frac{f_s}{2f_{dm}} = \frac{f_s}{f_{min}} = \frac{\beta_{max}}{\beta} \quad (5.10)$$

In practice, an interpolator has ripple distortion in its passband and stopband. Thus, in order to minimize this ripple distortion, we want to allow for as large a rolloff as possible. In fact, the raised-cosine interpolator has relatively little ripple distortion for $\eta \geq 0.7$. As an example, the frequency response of an interpolator with 70% rolloff is shown in Figure 5.30.

However, we don't want to increase η beyond the point where the flat frequency response of the ideal interpolator no longer spans the Doppler bandwidth of the fading signal. Thus, an approximate expression for the optimum rolloff of a practical interpolator is:

$$\eta_{opt} = 1 - \frac{1}{\bar{f}} \quad (5.11)$$

Since the Doppler spectrum has a peak at its maximum frequency, we generally want the interpolator to sample at higher than the Nyquist rate so as to avoid distortion due to aliasing (see Section 5.1.2). Thus, we can exploit this frequency separation between the spectral images by increasing the filter bandwidth with an associated rolloff, η_{opt} , to achieve a flat response in the passband.

The computed and actual values of the optimum rolloff, η_{opt} , as a function of the sampling rate are plotted in Figure 5.31. The optimum rolloff values obtained from simulation are typically a little larger than the computed values. This is expected since the actual interpolator has ripple distortion which requires more rolloff to smooth out. In contrast, the exact raised-cosine response exhibits no ripple distortion. Thus, the computed result can be thought of as a lower bound on the optimum rolloff factor for a given sampling rate. In practice, the interpolator with an optimum rolloff factor, $\eta_{opt} < 0.7$, has significant ripple distortion (as an example for $\eta = 0.0$, see Figure 4.18).

Figure 5.30 Frequency Response of a 70% Rolloff Raised-Cosine Interpolator

Figure 5.31 Optimum Rolloff Factor for a Raised-Cosine Interpolator

Nevertheless, it is still optimum because if η were higher than η_{opt} , then the distortion caused by the rolloff of the frequency response at the band-edge would further degrade the interpolation MSE.

5.1.5 Interpolation Order

For a given interpolation factor, R , and interpolator, the interpolation MSE performance improves significantly for increasing values of the interpolation order, Q (see Figure 5.32). That is, if more estimated samples are used for each interpolation, then the resulting interpolated samples tend to be more accurate. This result is intuitive, since using more information for each interpolation should yield better performance. Moreover, this is consistent with the sampling theorem which states that exact signal reconstruction occurs for $Q = \infty$. In practice, the processing delay associated with interpolation will determine the limiting value of Q . Furthermore, a diminishing rate of MSE improvement with increasing Q also discourages the use of a large value of Q . It is also noted that the special case of $Q = 2$ corresponds to linear interpolation.

5.1.6 Interpolation Factor

For a given interpolation filter, the effect of R on the tracking performance of the interpolator can be quantified by the measure of the total MSE averaged over all β samples in an interpolation interval:

$$\overline{mse} = \frac{1}{\beta} \sum_{m=0}^{\beta-1} mse((k+m)T) \quad (5.12)$$

In this case, the MSE is defined as:

$$mse((k+m)T) = \frac{1}{N} \sum_{n=0}^{N-1} \left| \tilde{h}_n((k+m)T) - g_n((k+m)T) \right|^2 \quad (5.13)$$

where

$$\begin{aligned} \tilde{h}_n((k+m)T) &= \hat{h}_n(kT + rT_s) & r\mu \leq m < (r+1)\mu \\ & & r = 0, 1, \dots, R-1 \\ & & m = 0, 1, \dots, \beta-1 \end{aligned} \quad (5.14)$$

Figure 5.32 Interpolation MSE Performance as a Function of Q and R

In general, the interpolation factor, R , is less than the number of samples in an interpolation interval, β , such that interpolated CIR samples are not available at all sample instants. The reason for this is that the OCDFE coefficients are computed at each interpolated CIR sample and, thus, for trade-off considerations of computational overhead and tracking performance, we typically only require an interpolated CIR estimate every few samples. We use a “sample-and-hold” method which assumes a constant value of the CIR, $\tilde{h}((k+m)T)$, throughout a time interval of μ samples between interpolated samples, $\hat{h}(kT + rT_s')$. However, since the actual CIR, $g((k+m)T)$, is randomly varying, this use of a constant value of the most recent interpolated CIR sample incurs a lag error. Moreover, this lag error in the CIR tracking increases for decreasing R . On the other hand, in the limit of zero lag error due to the interpolator (i.e., $R = \beta$, $\mu = 1$), the average, total MSE is the same as the average MSE (i.e., $\overline{tmse} = \overline{mse}$).

An improvement in the tracking performance of the interpolator is to modify the “sample-and-hold” approach of specifying the CIR at each sample. An alternative approach is to use the previous interpolated sample for only the first $\frac{\mu}{2}$ samples of the interval between two interpolated samples, and to use the next interpolated sample for the last $\frac{\mu}{2}$ samples in the interval:⁴²

$$\tilde{h}((k+m)T) = \begin{cases} \hat{h}(kT + rT_s') & r\mu \leq m < (r + \frac{1}{2})\mu \\ \hat{h}(kT + (r+1)T_s') & (r + \frac{1}{2})\mu \leq m < (r+1)\mu \end{cases} \quad (5.15)$$

$$r = 0, 1, \dots, R-1$$

$$m = 0, 1, \dots, \beta-1$$

In other words, at a given sample, the estimated CIR is the interpolated CIR that is nearest to this sample. This alternative approach is somewhat more complicated, but it has about half the lag error of the “sample-and-hold” approach.

It is evident from Figure 5.32 that the average interpolation MSE performance is independent of the interpolation factor, R . This result is reasonable since the value of

⁴² Without a loss of generality, we can assume that μ is even.

R shouldn't affect the MSE at each interpolated sample. However, the value of R does affect the amount of recovered information about the fading signal by the interpolation process. In fact, the sampling theorem states that for $Q = \infty$, the interpolated signal can be recovered at all values of R .

An implication of this result is that, for a given sampling period, β symbols, one can arbitrarily increase R to decrease the spacing between the interpolated samples, $\mu = \frac{\beta}{R}$. Even though this increased resolution of the interpolation does not change the average MSE (i.e., $\overline{mse}$) performance at the interpolated samples, it does improve the overall tracking performance of the interpolator for all samples including those between the interpolated samples, as quantified by the total MSE, $\overline{tmse}$. The tracking performance improves with increasing R because more information about the randomly fading signal is available. Another interpretation of the tracking improvement is that more frequently interpolated samples are obtained with a larger value of R . However, there is an associated cost of increased computation to make use of these additional interpolated samples. For example, in our DMR system, we update the OCDFE coefficients every μ symbol periods with the current interpolated CIR. Thus, at a given sampling rate of the interpolation, an increase in R leads to an improvement in the tracking performance, but at a cost of having to compute the OCDFE receiver more often. This performance improvement is shown in Figure 5.33 as a decreasing average, total MSE, $\overline{tmse}$, for increasing R .

In general, assuming a sufficiently high SER, the interpolation MSE gets progressively worse as one moves towards the middle of an interpolation interval (see Figure 5.23 for the case of $SER = 40$ dB). However, this average MSE distribution does not reflect the fact that the interpolation MSE at samples in between interpolated samples is significantly worse since these samples suffer a lag error due to a lack of interpolation information (see Figure 5.34 for the case of $SER = 40$ dB). From Figure 5.34, it is evident that the tracking performance is degraded by the relatively inaccurate samples between interpolated samples. The "sawtooth" shape of the average total MSE distribution is due to the fact that the interpolation performance degradation is better at the

Figure 5.33 Average Total MSE as a Function of the Interpolation Factor

interpolated samples and grows progressively worse towards the samples just previous to each interpolated sample. Thus, for a given computational complexity constraint, the interpolation factor, R , should be set as high as possible.

However, if estimation noise is a significant impairment, then the interpolation MSE improves as one moves towards the middle of an interpolation interval (see Figure 5.23 for the case of $SE R = 3 \text{ dB}$). For this situation when estimation noise is the dominant impairment, the lag error at the samples in between interpolated samples is not that significant any more (see Figure 5.35 for the case of $SE R = 3 \text{ dB}$). Thus, if the estimation noise in the training process is the limiting impairment, then the choice of R becomes less important. Furthermore, in this case when the estimation noise swamps all other sources of error, a smaller value of R would perhaps be preferable to reduce the OCDFE computational complexity while incurring little additional interpolation error.

5.1.7 Maximum Doppler Frequency

In general, the problem of tracking a randomly varying CIR is much easier at lower fade rates. In the context of our investigation, a reduction in the maximum Doppler frequency, f_{dm} , of the tap coefficient significantly improves the MSE performance of a given interpolator (see Figure 5.36). Furthermore, the relative performance improvement due to a decrease in the f_{dm} is greater for lower sampling rates near the Nyquist rate. This result is due to the fact that aliasing of the Doppler spectrum is most pronounced when the spectral images are close together (i.e., when sampling near the Nyquist rate).

A decrease in the maximum Doppler frequency, f_{dm} , for a given sampling rate, f_s , has the same effect⁴³ as an increase in the f_s , for a given f_{dm} . In other words, the effect of both the f_{dm} and the f_s on the interpolation performance is equivalent. Thus, the general results from Section 5.1.2 can apply to the maximum Doppler frequency.

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⁴³ Of course, the decrease in the f_{dm} comes free (neglecting the fact that one would have to drive slower), while the increase in the f_s incurs a penalty in data throughput.

Figure 5.34 Distribution of the Average Total MSE over an Interpolation Interval with Negligible Estimation Noise

Figure 5.35 Distribution of the Average Total MSE over an Interpolation Interval with Significant Estimation Noise

Figure 5.36 Interpolation Performance as a Function of the Maximum Doppler Frequency

5.1.8 Estimation Noise

Up to this point, we have investigated interpolation with exact estimates of the tap coefficients (i.e., $SER = \infty$ dB). However, in practice, imperfect training produces nonexact CIR estimates. In order to simulate this estimation noise, we add random noise, with variance σ_e^2 , to the Rayleigh fading tap coefficient. We can define a measure of the quality of the CIR estimation by the signal-to-estimation-noise-ratio:

$$SER = 10 \log_{10} \left(\frac{\sigma_g^2}{\sigma_e^2} \right) \text{ dB} \quad (5.16)$$

where

$\sigma_g^2 = 1$ is the normalized variance of the CIR tap coefficient

σ_e^2 is the variance of the estimation noise for each CIR tap coefficient

In the simple case of a one-tap fading channel with estimation noise but no AWGN, the SER is just the SNR. For a channel with N_c tap coefficients and assuming a sufficiently long training sequence, the SER is roughly N_c times that of the SER for one tap coefficient.

As can be seen in Figure 5.37, the effect of estimation noise on interpolation performance can be quite significant. In general, estimation noise dominates the tracking problem at low SER, while the interpolation MSE performance is much better at higher values of SER. This is especially true for interpolators operating at high sampling rates. For instance, a 50% rolloff raised-cosine interpolator sampling at twice the Nyquist rate, i.e., $\bar{f} = 2.0$, can lose over 20 dB in MSE performance by going from an SER of 30 dB to 3 dB (see Figure 5.37). This clearly illustrates the strong influence of estimation noise on the interpolator performance.

Moreover, estimation noise can indeed nullify the significant interpolation gains obtained from using optimum rolloff values and a large interpolation factor. Since we have found that interpolation error can be minimal when estimation noise is not present, the degree of SER can determine whether or not a DMR system suffers an irreducible BER at high SNR (assuming ISI is not an issue). In order to achieve interpolation

Figure 5.37 Interpolation Performance as a Function of the Estimation Noise

performance at a negligible⁴⁴ MSE of less than -30 dB, we need to have an SER of over 30 dB.

5.2 Overall DMR Communications System Performance

5.2.1 Description of the DMR System Simulation

The performance of our DMR system is investigated via computer simulation. This simulation of our DMR system employing CIR interpolation and optimum combining and decision-feedback equalization (OCDFE) is performed using a modified and extended version of an existing C-language program [8]. A block-diagrammed flowchart of the DMR system simulation is presented in Figure 5.38.

The DMR communications system is modelled with equivalent, complex baseband signals and simulated using a Monte Carlo approach. Specifically, each computer experiment simulates an ensemble of independent trials of digital communication with a given DMR system. The number of required independent trials in an ensemble should be sufficient for statistically significant computation of system performance measures such as average BER and outage probability. Depending on the number of data symbols used per trial, typically between 2000 and 10,000 independent trials are generated for each ensemble.

Each trial entails the transmission of Q contiguous data frames, each of which consists of an initial known training sequence followed by an uncorrelated data sequence, $\underline{a}(kT)$. The symbols in this set of contiguous data frames are $\frac{\pi}{4}$ -shifted DQPSK-encoded and transmit-filtered using a $\frac{T}{2}$ -spaced, 55 tap, FIR filter with an SRRC impulse response. Specifically, the transmitted symbols, $\underline{\Delta}_a(kT)$, are $\frac{2}{T}$ -sampled such that each data sample is followed by a zero-valued, $\frac{T}{2}$ -spaced sample. Thus, pulse amplitude modulation (e.g., DQPSK) is simulated with the transmitted pulse modulated by the data symbol impulse.

⁴⁴ An interpolation MSE of less than -30 dB is negligible for an SNR of about 20 dB. The interpolation MSE is negligible in the sense that it would not be the cause of an irreducible BER at this operating range.

Figure 5.38 DMR System Simulation Flowchart

Each data frame is transmitted over L independently fading, $\frac{T}{2}$ -spaced diversity channels, $\underline{g}_l(kT)$, each with tap coefficients which exhibit independent Rayleigh fading. The $\frac{T}{2}$ -spaced symbols at the output of each diversity channel are corrupted with $\frac{T}{2}$ -spaced AWGN samples, $n_l(kT)$, and receive-filtered using a $\frac{T}{2}$ -spaced, 64 tap, FIR filter with an SRRC impulse response. In addition, the received signal, $\underline{r}_l(kT - m\frac{T}{2})$, at each diversity branch is represented as two interleaved, T -spaced sequences which are offset by $\frac{T}{2}$ -phase.

It is important to emphasize that the diversity channels fade independently of each other. Specifically, from trial to trial in an ensemble, the fading in each diversity channel is independently simulated by using a random number generator to start the "sum of sines" time variation in a given trial of data frames. However, for each diversity channel, the Rayleigh fading during the Q contiguous data frames of a trial is deterministic and dependent. This dependent fading is required for CIR interpolation over the Q consecutively trained CIR estimates which can be thought of as occurring in a TDMA time slot.

At each diversity channel and for each of Q data frames in a trial, the two $\frac{T}{2}$ -offset received training sequences are used to estimate Q consecutive CIRs, $\underline{h}_l(qT_s)$, which are spaced a data frame, i.e., βT symbols, apart. Subsequent to this training, the trained CIR estimates are filtered by a real-valued, windowed $\frac{\sin x}{x}$ interpolator, $f(rT_s')$, to generate R interpolated CIR estimates, $\hat{\underline{h}}(rT_s')$, which are spaced μT symbols apart within the middle data frame of the trial. Given the interpolated CIR estimates for each of the L diversity channels, the OCDFE coefficients, $\underline{w}_{opt}$, are computed every μT symbols (i.e., at every interpolated CIR sample). Of course, to reduce the computational complexity, this OCDFE coefficient computation can be performed less frequently, but with the associated cost of a degradation in the adaptive performance of the receiver.

Each received data symbol in the middle data frame is processed by the OCDFE to yield an optimally combined and equalized signal, $y(kT)$, which can then be detected by the decision device. Following this, the decision output, $\hat{\Delta}_a((k - D)T)$, is $\frac{\pi}{4}$ -shifted DQPSK-decoded to yield the QPSK decision symbol, $\hat{a}((k - D)T)$.

The simulation trial is complete when all N_d data symbols, $a(kT)$, of the middle data frame are processed. Subsequently, the simulation program proceeds onto the next trial in the ensemble. Upon completion of all the trials, the simulation finishes with the computation of the DMR system performance measures, viz., the average MSE, average BER and outage probability.

In the rest of this chapter, we present some simulation results of a DMR system employing an OCDFE receiver with Block Least-Squares CIR estimation and CIR interpolation with a zero-rolloff raised-cosine interpolator. Unless otherwise stated, we use an OCDFE with $N_f = 3$ taps in the forward filter at each diversity branch and $N_b = 2$ taps in the feedback filter. We denote this configuration as OCDFE(3,2). In addition, we use the following default parameter values: $N_c = 6$, $N_t = 15$, $Q = 4$, $\mu = 5$ and $f_{dm} = 100$ Hz.

In our simulation study, we investigate the DMR system performance (i.e., $\overline{P_e}$ or P_{outage}) as a function of the following factors:

1. frequency of training or, equivalently, normalized interpolation sampling rate: $\bar{f}$
2. maximum Doppler frequency: f_{dm}
3. number of diversity channels: L
4. channel SNR: $\frac{E_b}{N_0}$
5. equalizer length: N_f, N_b
6. interpolator rolloff, order and factor: η, Q, R
7. transmit and receive filter rolloffs: ϵ

5.2.2 System Performance Criteria

5.2.2.1 Mean Squared Error A common and obvious measure of performance for a digital communications system is the average bit error rate (BER) for a given signal-to-noise ratio (SNR) at the receiver. This BER is measured at the output of the receiver decision device. In addition, the mean squared error (MSE) at the output of the OCDFE, but before the decision device, is a good measure of the channel amelioration performance of the OCDFE. Since the reference locations of the MSE

and BER measurements are separated by a nonlinear decision device, the relationship between the MSE and BER is highly nonlinear. Despite their nontrivial relationship, the MSE and BER are sufficiently correlated that the MSE can also provide a somewhat indirect, but meaningful, measure of system performance.

In our investigation of the DMR communications system, the MSE at the OCDFE output is defined as:

$$mse = 10 \log_{10} \left[\frac{1}{2N} \sum_{n=0}^{N-1} |y_n(kT) - \Delta_{a,n}(kT)|^2 \right] \text{ dB} \quad (5.17)$$

where

$y_n(kT)$ is the n th $\frac{\pi}{4}$ -shifted DQPSK symbol at the output of the OCDFE

$\Delta_{a,n}(kT)$ is the n th $\frac{\pi}{4}$ -shifted DQPSK transmitted symbol

N is the total number of transmitted symbols in an ensemble of data frames

For conciseness, the DMR system performance results are presented only in terms of the average bit error rate, $\overline{P_e}$, and the outage probability, P_{outage} .

5.2.2.2 Average Probability of Bit Error The average BER, as a function of the SNR at the channel output, is perhaps the most common measure of performance for a digital communications system. For the case of QPSK signaling where two information bits are carried by one transmitted symbol, the average BER, $\overline{P_e}$, can be assumed to be one half the average symbol error rate, $\overline{P_s}$. For reliable digital transmission, this is a reasonable approximation:

$$\overline{P_s} = 1 - (1 - \overline{P_e})^2 = \overline{P_e}(2 - \overline{P_e}) \approx 2\overline{P_e} \quad (5.18)$$

For digital communications over a multipath fading channel, the BER is averaged over the time variation (due to fading) in the channel. In particular, the average BER can be measured as:

$$\overline{P_e} = \frac{1}{2N} \sum_{n=0}^{N-1} e_n \quad (5.19)$$

with

$$e_n = \begin{cases} 0 & \text{if } a_k = \hat{a}_k \text{ (no bit errors)} \\ 1 & \text{otherwise (at least one bit error)} \end{cases} \quad (5.20)$$

where a_k and $\hat{a}_k$ are QPSK data symbols at the source and sink, respectively.

5.2.2.3 Outage Probability Digital communications over a slowly fading, or quasi-static, channel tends to experience transmission errors in bursts. Even though a quasi-static channel does not exhibit a high frequency of fading, when it is in a deep fade, it takes a relatively longer time to recover to an acceptable SNR level. Consequently, a slowly fading channel tends to cause transmission errors in bursts. Because of this bursty nature, symbol errors are not evenly distributed over a given transmission period. A transmitted data frame tends to have either few symbol errors or many symbol errors. From a systems performance viewpoint, it is useful to know whether a data frame is relatively error-free or relatively error-filled. Because of its inherent averaging over all transmitted symbols, the average BER would not be indicative of the system performance for digital communications (especially with channel coding) over a slowly fading channel. However, the performance measure of outage probability would be more useful in this case.

Outage probability, P_{outage} , is defined as the probability that the BER (or symbol error rate) for a given data frame is above an arbitrary threshold called the outage criterion, P_{oc} [46, 60, 61]. This performance measure can be formulated as:

$$P_{outage} = P\{P_{frame}(\gamma) > P_{oc}\} \quad (5.21)$$

where γ is the instantaneous SNR at a given data frame

In practice, one can measure the outage probability as follows:

$$P_{outage} = \frac{\#data\ frames\ \{P_{frame}(\gamma) > P_{oc}\}}{total\ \#data\ frames\ in\ an\ ensemble} \quad (5.22)$$

where

$$P_{frame}(\gamma) = \frac{1}{2} \left(\frac{\#symbol\ errors/frame}{total\ \#symbols/frame} \right) \quad (5.23)$$

5.2.3 Interpolation Sampling Rate

The normalized sampling rate, $\bar{f}$ (see Equation 5.5), can be interpreted as a relative training frequency or as an interpolation sampling frequency. In either case, CIR estimation and tracking improve with increasing $\bar{f}$. Consequently, the DMR system performance also improves with increasing $\bar{f}$. This superior performance at higher values of $\bar{f}$ is due to more frequent CIR training and improved tracking of the time-varying CIR by the interpolator (see Section 5.1.2).

5.2.3.1 Maximum Doppler Frequency The average BER performance as a function of the normalized sampling rate, $\bar{f}$, for different values of the maximum Doppler frequency, f_{dm} , is illustrated in Figure 5.39 for the case of $\frac{E_b}{N_0} = 20$ dB and $L = 1$. It is evident from this Figure that the average BER is monotonically decreasing for increasing values of $\bar{f}$. Furthermore, this average BER improvement is greater at larger values of f_{dm} . In particular, for a worst-case maximum Doppler frequency of $f_{dm} = 100$ Hz, the average BER decreases from 1.5×10^{-1} to 10^{-2} when $\bar{f}$ goes from 1.0 to 3.0. This improvement is due to more frequent training and improved CIR tracking with the removal of aliasing distortion in the interpolation filter (see Section 5.1.2).

In addition, it is also evident from Figure 5.39 that the system performance is less influenced by the sampling rate, $\bar{f}$, at progressively lower values of the maximum Doppler frequency, f_{dm} (see Section 5.1.7). In fact, at $f_{dm} = 0$ Hz (i.e., nonfading CIR), the average BER is independent of $\bar{f}$, since tracking is not an issue for a time-invariant channel. However, for a fading channel, the value of f_{dm} can strongly affect the system performance. For instance, at values of $\bar{f} \approx 1.0$ near the Nyquist sampling rate, where aliasing distortion is significant in the CIR interpolation process, a reduction in the f_{dm} from a worst-case value of 100 Hz to 50 Hz reduces the average BER by over an order of magnitude from $\sim 10^{-1}$ to less than 10^{-2} . At a more practical value of $\bar{f} = 2.0$ where the aliasing distortion due to the CIR interpolator is negligible and where the higher frequency of training reduces the effect of fading, a reduction in the f_{dm} from

Figure 5.39 Average BER as a Function of Interpolation Sampling Rate and Doppler Fading

a worst-case value of 100 Hz to 50 Hz , nonetheless, reduces the average BER by about five times from 2×10^{-2} to 5×10^{-3} . Thus, vehicular speed can significantly affect the fading problem in a DMR system. In other words, CIR estimation and tracking are crucial to reliable DMR communications.

The system performance gain with increasing $\bar{f}$ incurs an overhead of reduced system throughput, $\%T$ (see Equation 4.8). This associated overhead is displayed on the upper horizontal axis in Figure 5.39. Furthermore, the choice of $\bar{f}$ also affects the interpolation processing delay, D_t (see Equations 4.21 and 4.22). Specifically, at $\bar{f} = 1.0$, the percentage data throughput, $\%T$, is 87.5% and the worst-case interpolation delay, D_t , is 15 ms while at $\bar{f} = 3.0$, $\%T$ is only 62.5% and D_t is 5 ms . Thus, depending on the throughput and processing delay constraints for a particular application, one can select the appropriate sampling rate which will adequately trade-off between system performance and throughput/delay requirements. From the results presented, a value of $\bar{f} = 2.0$ (i.e., twice the Nyquist sampling rate), with an associated $\%T = 75\%$ and $D_t = 7.5\text{ ms}$, is a reasonable choice for DMR communications.

5.2.3.2 Channel Diversity Average BER performance curves as functions of the normalized sampling rate, $\bar{f}$, and the number of diversity channels, L , are shown in Figures 5.40 and 5.41 for the case of $\frac{E_b}{N_0} = 20\text{ dB}$ and $f_{dm} = 100\text{ Hz}$. It is evident from both of these Figures that the system performance improves with increasing $\bar{f}$ and L . As discussed in the previous section, this performance improvement with increasing $\bar{f}$ comes at the expense of decreased system throughput. Furthermore, the use of diversity channels (i.e., $L > 1$) also incurs added hardware complexity in implementing diversity antennas at the mobile receiver.

As shown in Figure 5.40, at high sampling rates, the DMR system does not exhibit an irreducible BER but rather a slowly decreasing BER. On the other hand, at the Nyquist sampling rate (i.e., $\bar{f} = 1.0$) for all diversity cases, the system performance is quite unacceptable with an average BER of over 10^{-1} . This mediocre performance is caused by insufficiently frequent training and by the severe aliasing distortion due to the interpolation filter which leads to poor tracking of the fast fading channel at each

Figure 5.40 Average BER as a Function of Interpolation Sampling Rate and Diversity

Figure 5.41 Average BER as a Function of the Number of Diversity Channels

diversity branch. Thus, adding more independently fading channels only improves the tracking performance marginally. However, for $L = 1$ and $\bar{f} \geq 2.0$, the average BER is on the order of 10^{-2} . For this case, the DMR system performance is acceptable for voice communications. Moreover, the use of diversity can greatly improve the system performance. For example, at a reasonable value of $\bar{f} = 2.0$, the use of $L = 2$ diversity channels can reduce the average BER by an order of magnitude to 2×10^{-3} . Furthermore, the use of $L = 4$ diversity channels at the same value of $\bar{f}$ can achieve an average BER of 4×10^{-5} . Thus, at this operating range, reliable data communications is possible with a relatively small increase in receiver overhead to implement diversity antennas. For this case, the system throughput is 75% and the worst-case interpolation delay is 7.5 ms.

Figure 5.41 displays essentially the same information as Figure 5.40 except that the performance curves for different values of $\bar{f}$ are functions of L . Figure 5.41 highlights again the system performance gains with increasing $\bar{f}$ and L . Furthermore, it is noted that the incremental diversity gain decreases with increasing diversity order, L . In other words, the performance curves exhibit a small positive second derivative which corresponds to a diminishing return with the use of an increasing number of diversity channels.

For system performance comparison to QPSK signaling in an “ideal” (defined here to be slowly fading and nondispersive) AWGN channel, a lower bound performance curve is also displayed in Figure 5.41. Specifically, the stationary BER for coherent QPSK and PSK signaling in a nondistorting AWGN channel is [6, 7]:

$$P_e(\gamma) = \frac{1}{2} \text{erfc}(\sqrt{\gamma}) = Q(\sqrt{2\gamma}) \quad (5.24)$$

where

γ is the instantaneous value of $\frac{E_b}{N_u}$

$\text{erfc}()$ is the complementary error function and

the Q -function is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{y^2}{2}} dy \quad (5.25)$$

By integrating the stationary BER, $P_e(\gamma)$, over the Rayleigh fading of the instantaneous SNR, γ , and the L -diversity combiner statistics (see Equations 3.16 and 3.17), it can be shown that the average BER for QPSK signaling over a slowly fading, nondispersive AWGN channel is [7, 27, 29]:

$$\overline{P_e(\Gamma)} = 2^{-L} \left(1 - \sqrt{\frac{\Gamma}{1+\Gamma}} \right)^L \sum_{l=0}^{L-1} \binom{L-1+l}{l} 2^{-l} \left(1 + \sqrt{\frac{\Gamma}{1+\Gamma}} \right)^l \quad (5.26)$$

where Γ is the average $\frac{E_b}{N_0}$ per diversity channel

For $\Gamma \gg 1$, the use of the binomial expansion can simplify the above expression to [7]:

$$\overline{P_e(\Gamma)} \simeq \left(\frac{1}{4\Gamma} \right)^L \binom{2L-1}{L} \quad \Gamma \gg 1 \quad (5.27)$$

The above expressions for the average BER are clearly lower bounds for an actual DMR system. To obtain a tighter lower bound for the average BER, one must account for the time dispersion, rapid fading and interference that are present in DMR channel. Furthermore, our DMR system employs differential encoding/decoding which can further reduce the average BER performance by up to a factor of two for QPSK (see Section 4.4). In the limit of $\bar{f} \rightarrow \infty$ (i.e., perfect CIR tracking), $N_t \rightarrow \infty$ (i.e., perfect CIR estimation, assuming the channel is properly tracked during the training) and $N_f, N_b \rightarrow \infty$ (i.e., perfect equalization), the average BER curves in Figure 5.41 approach to within a factor of two in BER (due to the differential decoding) of the “ideal” lower bound for coherent QPSK signaling in a slowly fading AWGN channel.

5.2.3.3 Symbol Error Statistics As an indication of symbol error statistics, some representative histograms of symbol errors per frame for a given ensemble are shown in Figures 5.42 to 5.46. When a DMR system has poor performance (e.g., $\bar{f} = 1.0$ and $f_{dm} = 100 \text{ Hz}$), most received data frames contain many symbol errors (see Figures 5.42 and 5.43). Furthermore, in these figures, the bursty nature of symbol errors is indirectly evident in that very few frames have only a few symbol errors – either a

frame has no errors (e.g., with correct CIR tracking) or many errors (e.g., incorrect CIR tracking during a deep fade). With $\bar{f} = 1.0$, the CIR tracking performance is so poor that with $L = 4$ diversity channels, even though the maximum number of symbol errors per frame is lower, the overall system performance is not much better than that of a single channel receiver (see Figure 5.43).

When the normalized sampling rate is increased to $\bar{f} = 2.0$, over two-thirds of the received data frames are error-free and the average BER performance improves by an order of magnitude (see Figure 5.44). Furthermore, very few frames have more than one-third of its symbols in error. System performance with $\bar{f} = 2.0$ improves dramatically for the cases when the maximum Doppler frequency is lowered to $f_{dm} = 50 \text{ Hz}$ and when $L = 4$ diversity channels are used (see Figures 5.45 and 5.46). In the latter case, over 90% of the received frames are error-free while almost none of the frames have more than 10% of its symbols in error. However, even for these cases of relatively good system performance very few data frames have just one symbol error – either a frame has no symbol errors or it has multiple symbol errors. Thus, this is some indication of errors occurring at closely spaced symbols in the same data frame. This was observed in symbol error distributions over a data frame during system simulations.

5.2.3.4 Outage Probability In general, communications over a fading multipath channel can exhibit burst errors [60, 61]. This is particularly true for a slowly fading channel (e.g., indoor wireless) which exhibits a lower zero-crossing frequency and a longer fade duration. Consequently, during a deep fade, error bursts of contiguous symbols tend to occur. On the other hand, a faster fading channel (e.g., DMR) with shorter, but more frequent, fade durations tends to exhibit more symbol errors which are not necessarily grouped in bursts. For the case of burst errors which can effectively render an erroneous frame unreliable, the system performance measure of outage probability, P_{outage} , is more meaningful than the average BER (see Section 5.2.2.3). In simulations of the DMR system, it was found that the average BER and outage probability are both valid measures of system performance.

Figure 5.42 Histogram of Symbol Errors per Frame; for $L=1$, $f_s/(2*f_{dm})=1.0$

Figure 5.43 Histogram of Symbol Errors per Frame; for $L=4$, $f_s/(2*f_{dm})=1.0$

Figure 5.44 Histogram of Symbol Errors per Frame; for $L=1$, $f_s/(2*f_{dm})=2.0$

Figure 5.45 Histogram of Symbol Errors per Frame; for $L=1$, $f_s/(2 \cdot f_{dm})=2.0$, $f_{dm}=50$ Hz

Figure 5.46 Histogram of Symbol Errors per Frame; for $L=4$, $f_s/(2*f_{dm})=2.0$

System performance in terms of the outage probability as a function of $\bar{f}$ and L is shown in Figure 5.47 for $\frac{E_b}{N_0} = 20$ dB and an outage criterion of $P_{oc} = 10^{-2}$ (see Equations 5.22 and 5.23). Comparing the performance curves of this figure with those of Figure 5.40, it is evident that the outage probability and the average BER are consistent measures of the DMR system performance. In this particular case for $P_{oc} = 10^{-2}$, the outage probability is typically about an order of magnitude larger than the average BER. Of course, this difference would vary depending on the value of P_{oc} . In any event, this difference is immaterial since the two performance measures have different interpretations, viz., P_{outage} indicates the probability of a data frame being “unacceptable” (i.e., $P(frame) \geq P_{oc}$) while $\overline{P_e(\Gamma)}$ measures the average probability of a single bit being in error. It is noted that in Figure 5.47 for the case of $\bar{f} = 3.0$ and $L = 4$, $P_{outage} = 0$ and is thus not shown in the logarithmic plot. This example of $P_{outage} = 0$ (for $P_{oc} = 10^{-2}$) is a case where not one data frame in an ensemble has more than $200P_{oc}\%$ of its symbols in error.⁴⁵ Furthermore, near the region of $\bar{f} \simeq 1.0$, the system performance is so bad that practically no data frames meet the outage criterion of $P_{oc} = 10^{-2}$ (i.e., $P_{outage} \simeq 1.0$).

Figure 5.48 displays outage probability curves as functions of the outage criterion for $\frac{E_b}{N_0} = 20$ dB, $L = 1$ and different values of $\bar{f}$. These curves demonstrate again the superior performance at larger values of $\bar{f}$ at all outage criteria. Furthermore, the outage probability decreases greatly for $P_{oc} \geq 10^{-1}$. In fact, the outage probability approaches zero for increasing values of the outage criterion since all transmitted data frames can satisfy an arbitrarily high outage criterion of $P_{oc} = 1.0$. However, at such an operating region, such outage criteria are likely deemed unacceptably high for even voice communications. Thus, outage criteria of 10^{-2} and below are more realistic.

At outage criteria at or below 10^{-2} , the measured outage probability is essentially constant since only error-free data frames can satisfy these outage criteria. This is because the relatively small frame sizes involved does not provide a statistically

⁴⁵ The factor of two arises from the fact that we have arbitrarily defined P_{outage} in terms of bit errors and that we have assumed for QPSK signaling, $Prob(bit\ error) = \frac{1}{2}Prob(symbol\ error)$.

Figure 5.47 Outage Probability as a Function of the Interpolation Sampling Rate and Diversity

Figure 5.48 Outage Probability as a Function of Outage Criterion and Interpolation Sampling Rate

significant outage probability at such low outage criteria. In other words, such small frame sizes does not provide a sufficient bit (or symbol) error resolution. As an example, for the case of $\bar{f} = 2.0$ (i.e., $\beta = 60$), a data frame has a bit error resolution of just:

$$\frac{1}{2(\beta - N_t)} = \frac{1}{90} \quad (5.28)$$

Thus, for a $P_{oc} \leq \frac{1}{90}$, the outage probability is equal to a constant being the percentage of error-free data frames in an ensemble.

Figure 5.49 displays outage probability curves as functions of the outage criterion for $\frac{E_b}{N_0} = 20$ dB, $\bar{f} = 2.0$ and varying diversity order, L . These curves show that the outage probability performance improves significantly for increasing L at all outage criteria. In fact, at sufficiently high outage criteria, an increase in L can cause an outage probability to drop to zero. An example of this, for $P_{oc} = 10^{-1}$, is that $P_{outage} \simeq 10^{-1}$ and 10^{-3} for $L = 1$ and 2, respectively, while $P_{outage} = 0$ for $L = 4$ (which is not shown on the logarithmic plot).

5.2.4 Average Channel SNR

The remaining results in this section illustrate the system performance as a function of the average channel SNR, $\frac{E_b}{N_0}$, for the case of a normalized sampling rate of $\bar{f} = 2.0$ and at varying degrees of channel fading, diversity and equalization. In all cases, the system performance was found to improve with increasing $\frac{E_b}{N_0}$. However, an irreducible BER was also found to occur at high values of $\frac{E_b}{N_0} \geq 20$ dB. This performance floor is likely due to limitations in the ability of the receiver to estimate and track the rapid time variations in the fading channel.

5.2.4.1 Maximum Doppler Frequency The average BER performance as a function of $\frac{E_b}{N_0}$ for different values of the maximum Doppler frequency, f_{dm} , and diversity order, L , is illustrated in Figure 5.50. This figure consists essentially of two sets ($L = 1, 2$) of three curves ($f_{dm} = 10, 50, 100$ Hz). For a given diversity order, L , there is little difference in the system performance for all values of f_{dm} at low values of $\frac{E_b}{N_0}$. This

Figure 5.49 Outage Probability as a Function of Outage Criterion and Diversity

Figure 5.50 Average BER as a Function of Channel SNR, Diversity and Doppler Fading

is primarily because, at this operating range, the channel noise is dominant in the poor system performance, regardless of the CIR estimation and tracking.

As the value of $\frac{E_b}{N_0}$ increases, the degree of fading has a dramatic effect on the system performance. Specifically, for a given $\frac{E_b}{N_0}$ and L , the average BER decreases sharply for decreasing values of f_{dm} . Furthermore, for $f_{dm} \geq 50$ Hz, an irreducible BER is evident for $\frac{E_b}{N_0} \geq 20$ dB. However, an irreducible BER, albeit still evident, manifests itself at higher values of $\frac{E_b}{N_0}$ for slower fading channels, e.g., $\frac{E_b}{N_0} \geq 30$ dB when $f_{dm} = 10$ Hz.

The above result implies that the irreducible BER may be caused by the CIR estimation and tracking. At higher values of f_{dm} , the task of following a rapidly fading CIR can be much more difficult and, thus, leads to a higher estimation and tracking (i.e., interpolation) noise. This increase in the lag error causes the onset of the irreducible BER to occur at a lower value of $\frac{E_b}{N_0}$ where the estimation and tracking noise overtakes the thermal noise as being the limiting factor in the system performance.

5.2.4.2 Channel Diversity As shown in Figure 5.50, the irreducible BER of 2×10^{-3} for the case of $f_{dm} = 100$ Hz and $L = 2$ is actually lower than that of 5×10^{-3} for the case of $f_{dm} = 50$ Hz and $L = 1$. Thus, despite the strong influence of f_{dm} on the irreducible BER, the use of channel diversity can effectively overcome the fast fading in a DMR channel. Even though it does not remove the irreducible BER, the use of diversity does significantly reduce the level of the irreducible BER.

The average BER performance as a function of $\frac{E_b}{N_0}$ for varying numbers of diversity channels, L , is also shown in Figure 5.51, along with lower bound performance curves for coherent QPSK signaling in a slowly fading, nondispersive AWGN channel (see Equation 5.26). The comparison between the lower bound curves and the simulation curves clearly show the drastic effect of the irreducible BER on system performance at high values of $\frac{E_b}{N_0}$. In particular, for a given L at low $\frac{E_b}{N_0}$ where the CIR estimation and tracking noise is less significant relative to the thermal noise, the difference between the simulated performance and the lower performance bound is relatively small, e.g., about an order of magnitude at $\frac{E_b}{N_0} = 5$ dB for $L = 2$. However, at high values of

Figure 5.51 Average BER as a Function of Channel SNR and Diversity

$\frac{E_b}{N_0}$ where the CIR estimation and tracking noise is dominant over the thermal noise, the difference between the simulated performance and the lower performance bound is strikingly large. As an example, for $L = 2$ at an $\frac{E_b}{N_0} = 30$ dB, the difference in average BER performance is over four orders of magnitude. Furthermore, since the theoretical performance improvement with $\frac{E_b}{N_0}$ grows with increasing values of L , the difference in performance between the irreducible BER performance of the simulated results and the theoretical lower bound also grows with increasing values of L . These comparisons further illustrate the severe impact on system performance due to the adaptation lag error.

The system performance as a function of $\frac{E_b}{N_0}$ and L is presented in terms of the outage probability, with an outage criterion of $P_{oc} = 10^{-2}$, in Figure 5.52. These curves show that the outage probability decreases with increasing $\frac{E_b}{N_0}$ and L , in an analogous fashion as those for the average BER in Figure 5.51. Furthermore, Figure 5.53 shows the outage probability performance as a function of the outage criterion for various values of $\frac{E_b}{N_0}$. An irreducible outage probability at high values of $\frac{E_b}{N_0}$ is evident in that the performance curves for $\frac{E_b}{N_0} = 20$ dB and $\frac{E_b}{N_0} = 30$ dB are practically identical.

5.2.4.3 Equalizer Length The effects of ISI and channel fading on the system performance are illustrated in Figure 5.54. Specifically, the average BER performance as a function of $\frac{E_b}{N_0}$ of receivers with and without decision-feedback equalization is shown for different diversity orders, L . When no equalization and no diversity is implemented, i.e., OCDFE(1,0) and $L = 1$, the system performance is severely degraded with an average BER of greater than 10^{-1} at most values of $\frac{E_b}{N_0}$. Clearly, the time dispersion and Doppler fading in a DCR channel need to be ameliorated in order to achieve acceptable system performance. Moreover, despite the significant benefit of explicit diversity combining, a two-channel OCDFE without equalization has only marginally superior performance over a single-channel OCDFE with equalization. This comparison is especially true at high values of $\frac{E_b}{N_0}$ where the thermal noise is negligible. However, at low values of $\frac{E_b}{N_0}$, where there is a significant presence of thermal noise, the performance gain by using an equalizer is less since the equalizer incurs more self-noise in its tap coefficients. Nevertheless, these results indicate that the ISI caused by the multipath channel, in con-

Figure 5.52 Outage Probability as a Function of Channel SNR and Diversity

Figure 5.53 Outage Probability as a Function of Outage Criterion and Channel SNR

junction with the transmit and receive filters, is quite significant if the overall channel is not equalized. In fact, the performance improvement by using equalization can be more than an order of magnitude in average BER when using four diversity channels and operating at $\frac{E_b}{N_0} \geq 15$ dB.

From the results shown in Figure 5.54, it is clear that the use of equalization is important to achieving good system performance. However, there is the possibility that an OCDFE(3,2) receiver might not remove all of the existing ISI. If this were the case, then the residual ISI could account for the irreducible BER at $\frac{E_b}{N_0} \geq 20$ dB. The comparison of receivers with varying degrees of equalization and diversity is shown in Figure 5.55. These results indicate that the OCDFE(3,2) receiver removes most of the ISI, since a longer equalizer with the same diversity order, e.g., OCDFE(5,2), does not improve the system performance at high values of $\frac{E_b}{N_0} \geq 20$ dB. In fact, a longer equalizer performs worse at most values of $\frac{E_b}{N_0}$. This poor performance occurs because a longer equalizer, with more tap coefficients, experiences more self-noise in the computation of its tap coefficients. This argument applies for an equalizer with more feed-forward or feedback taps. Thus, our results indicate that the OCDFE(3,2) receiver is optimum in terms of correcting for ISI.

5.2.5 Effect of CIR Estimation Noise

We also investigated the performance of a system employing transmit and receive filters with higher rolloff factors (e.g., 50%). The simulation results indicate that the use of less stringent filtering improves the system performance marginally. This performance improvement is achieved because higher rolloff filters have lower sidelobes in their impulse responses which lead to a less severe ISI problem. However, this performance improvement incurs a penalty of increased susceptibility to ACI since the filter bandwidth is larger. In addition, we also investigated a system employing interpolation filters with their optimum rolloff values (see Section 5.1.3). We found a negligible difference in performance between using zero rolloff and optimum rolloff interpolators. Furthermore, we also found very little difference in system performance when we varied the spacing, μ , between interpolated samples or the number of CIR estimates used for each interpolation,

Figure 5.54 Average BER as a Function of Channel SNR, with and without Equalization

Figure 5.55 Average BER as a Function of Channel SNR and OCDFE Configuration

Q. These results are consistent with the results of Section 5.1.8 for the case when the estimation noise in the CIR samples is dominant.

In summary, all these results indicate that the system performance is limited by the CIR estimation and not by the CIR interpolation or residual ISI due to suboptimum equalization. Specifically, because the fast fading channel is varying during a training sequence, the CIR estimate obtained at the end of a training sequence lags the actual CIR by about half the observation period of the training. For a training sequence of $N_t = 15$, the observation period is $N_t - N_c + 1 = 10$ (see Section 4.6.1). Thus, for a worst-case maximum Doppler frequency of 100 Hz , the CIR can change by up to about 13% in amplitude during training (see Appendix A). Moreover, because of the limited length of the training sequence, the CIR estimation can have a significant variance. The result of all this is that the CIR estimation noise is a likely source of the irreducible error rate at values of $\frac{E_b}{N_0} \geq 20 \text{ dB}$.

To overcome this CIR estimation error, one can augment the training algorithm with a CIR tracking feature so that it accounts for the time variations of the channel during training. Furthermore, to reduce the estimation noise variance, one can use a longer training sequence at the expense of reduced system throughput. However, a longer training sequence is subjected to more time variations of the fading CIR. Thus, some form of CIR tracking during the CIR training is essential. Another possible improvement to overcome the CIR estimation problem is to account for the lag in the CIR estimates by explicitly shifting the interpolation index by some appropriate time interval, e.g., half the observation length of the training. Moreover, if the CIR estimates used for the CIR interpolation are indeed noisy, then one could use an alternative interpolation filter which, unlike the $\frac{\sin x}{x}$ interpolator, does not preserve the original CIR estimates at the expense of increased error at the intermediate samples of the interpolation. In addition, the tracking performance of the interpolator can be improved by replacing the “sample-and-hold” approach of updating the OCDFE with an alternative approach which computes the OCDFE with the interpolated CIR sample which is nearest to the currently detected sample (see Equation 5.15).

Chapter 6 Conclusions and Recommendations

In the course of this thesis research, we have found that reliable, low bit-rate narrowband communications can be achieved in a DMR system. Specifically, we have demonstrated the use of a jointly adaptive equalizer and diversity combiner which is periodically computed from current estimates of a time-dispersive and rapidly fading channel. In particular, we have shown that a relatively simple fractionally spaced decision-feedback equalizer, with three forward taps in each diversity branch and two feedback taps, can minimize the effect of intersymbol interference. Furthermore, we have found that diversity combining can significantly improve system performance under rapidly fading conditions.

In this thesis, we have proposed and demonstrated a novel block-adaptive method to track the time-varying CIR of a DMR system. Specifically, we have shown that a simple raised-cosine CIR interpolation filter can yield arbitrarily accurate tracking performance as long as the periodically trained CIR estimates have a small variance. In particular, we have found that the optimum performance of the CIR interpolator is dependent on the interpolation sampling rate or, equivalently, the frequency of training. Furthermore, for the case of low CIR estimation noise, the performance of CIR tracking with interpolation improves considerably with decreasing fading rate and with increasing frequency of training, interpolation order and factor. However, we have also found that for the case of high CIR estimation noise, with the exception of the frequency of training, these parameters have a negligible effect on the CIR tracking performance. Thus, we conclude that the adaptation process is limited mainly by the CIR estimation noise due to imperfect training rather than by the lag error due to CIR tracking with interpolation. Moreover, in our system simulation experiments, we have found that this limitation of noisy CIR estimation extends to the system performance. Specifically, we have observed an irreducible bit error rate at high signal-to-noise ratios. However, this system performance limitation is less severe at lower channel fade rates and with the use of explicit channel diversity.

In conclusion, we recommend the following adaptation and receiver parameters for use in the DMR system investigated in this thesis. Assuming a worst-case maximum Doppler frequency of $f_{dm} = 100 \text{ Hz}$, we propose using $Q = 4$ CIR estimates per interpolation with a normalized interpolation sampling frequency of $\bar{f} = 2.0$ (i.e., training sequences spaced every $\beta = 60$ symbol periods). This corresponds to a system overhead of $\%T = 75\%$ data throughput and worst-case interpolation delay of $D_t = 7.5 \text{ ms}$. Furthermore, to reduce receiver computational complexity, we suggest updating the OCDFE receiver every $\mu = 5$ symbol periods. Finally, we recommend employing an OCDFE(3,2) receiver with the choice of the number of diversity channels, L , being application-dependent. Specifically, we have found that using $L = 1, 2, 4$ diversity channels leads to average bit error rates of $\bar{P}_e = 2 \times 10^{-2}, 2 \times 10^{-3}, 4 \times 10^{-5}$, respectively. Thus, an $L = 2$ diversity system yields acceptable performance for digital voice communications while an $L = 4$ diversity system is suitable for data communications.

To conclude our discussion, we recommend the following areas of further research. In addition to the work presented in this thesis, it would be useful to try to improve the CIR estimation and tracking problem. Some suggestions for this task are discussed at the end of the chapter on the simulation results. Moreover, one could perform the simulation experiments with a more accurate model of the fading multipath channel or with actual channel characterization data to represent the DMR channel. Furthermore, one could study the effects of CCI/ACI and carrier recovery/synchronization on the adaptation and system performance. As for alternative reception schemes, one could investigate the performance gains achievable by using forward error correction coding, MLSE detection or other diversity combining techniques to mitigate the DMR channel impairments. Finally, it would be of considerable practical interest to obtain analytical results of the system performance of communications over a rapidly fading and severely time-dispersive channel. For example, useful system performance parameters include a realistic lower bound on average bit error rate performance and a theoretical upper bound on the system capacity.

Appendix A Maximum CIR Variation due to Doppler Fading

Typical DMR communications, at a carrier frequency, f_c , of 900 MHz, exhibits worst-case Doppler fading when the mobile velocity is maximum. This worst-case scenario occurs when a mobile travels at a maximum speed, v_m , of, say, 120 kph, in a direct line with the base station so that the incident angle of signal reception, ϕ , is 0 or π radians. Consequently, the impulse response of the channel experiences a maximum Doppler frequency of:

$$f_{dm} = f_c \left(\frac{v_m}{c} \right) \cos \phi = \frac{(9 \times 10^8)(1.2 \times 10^5)}{(3 \times 10^8)(3.6 \times 10^3)} = 100 \text{ Hz} \quad (\text{A.1})$$

Furthermore, it is assumed that the worst-case Doppler time variation in the fading channel can be characterized as a sinusoid of f_{dm} Hz:

$$g(t) = \alpha \cos(2\pi f_{dm} t) \quad (\text{A.2})$$

which experiences its greatest amplitude change during its zero crossings at odd integer multiples of $\frac{\pi}{2}$ radians.

Thus, a fading channel can experience a maximum amplitude change, $\Delta g_{max}(\Delta t)$, during one symbol period, $\Delta t = T$, centered at one of its zero crossings, e.g., $t = \frac{1}{4f_{dm}}$:

$$\Delta g_{max}(\Delta t=T) = g\left(\frac{1}{4f_{dm}} - \frac{T}{2}\right) - g\left(\frac{1}{4f_{dm}} + \frac{T}{2}\right) = 2\alpha \cos\left(2\pi\left(\frac{1}{4} - \frac{f_{dm}T}{2}\right)\right) \quad (\text{A.3})$$

For typical narrowband DMR communications at a symbol rate, $\frac{1}{T}$, of 24 ksp/s, one symbol period, T , corresponds to a time interval of 41.7 μ s. Therefore,

$$\Delta g_{max}(\Delta t = T) = 2\alpha \cos(1.558) = 0.0262\alpha \quad (\text{A.4})$$

This worst-case scenario results in a channel impulse response which can vary up to approximately 2.6% of its peak amplitude in one symbol period.

Appendix B The Sampling Theorem

For our purposes, since the random time variation of the CIR can be characterized by its equivalent baseband Doppler spectrum, we only need to consider the sampling theorem for the case of lowpass signals. The sampling theorem essentially states that if a finite-energy continuous-time signal, $h_c(t)$, which is bandlimited to W Hz, is sampled uniformly at a sufficiently high rate, $\frac{1}{T_s} \geq 2W$, then this continuous-time signal can be uniquely recovered at every time instant, t . This signal reconstruction can be accomplished with an ideal lowpass filter, $\tilde{F}(f)$, of bandwidth W , which acts as a digital-to-analogue converter. The impulse response of this ideal reconstruction filter, or interpolator, is [62, 63, 64]:

$$\tilde{f}(t) = \frac{\sin(2\pi W t)}{2\pi W t} = \frac{\sin\left(\frac{\pi t}{T_s}\right)}{\frac{\pi t}{T_s}} \quad (\text{B.1})$$

where the second equality assumes the minimum sampling rate of $\frac{1}{T_s} = 2W$.⁴⁶

Therefore, the signal reconstruction of $h_c(t)$ can be expressed by the familiar interpolation formula [57, 64]:

$$h_c(t) = \sum_{k=-\infty}^{\infty} \hat{h}(kT_s) \tilde{f}(t - kT_s) = \sum_{k=-\infty}^{\infty} \hat{h}(kT_s) \frac{\sin\left(\frac{\pi(t-kT_s)}{T_s}\right)}{\frac{\pi(t-kT_s)}{T_s}} \quad (\text{B.2})$$

The criteria for signal reconstruction are implicit in the interpolation formula. Specifically, a continuous-time signal, $h_c(t)$, can be recovered exactly for all t if:

1. the sampling rate, $\frac{1}{T_s}$, is greater than twice the maximum frequency (i.e., bandwidth W) of the signal, $h_c(t)$. This requirement ensures no overlap of the spectral images of $H_c(f)$ (i.e., no aliasing).
2. each interpolated value of $h_c(t)$ requires an infinite number of discrete samples, $\hat{h}(kT_s)$. This infinite impulse response of the ideal interpolator, $\tilde{f}(t)$, corresponds

⁴⁶ This minimum (Nyquist) sampling rate is sufficient for signal reconstruction as long as the signal doesn't have a delta function at the frequency $f = W$ [64].

to an ideal, “brick wall”, lowpass frequency response, $\tilde{F}(f)$, of the reconstruction filter. In practice, a suboptimal, realizable approximation of the ideal interpolator, $f(t)$, is used to interpolate a finite number of samples, $\hat{h}(kT_s)$.

3. the discrete-time samples, $\hat{h}(kT_s)$, are exact (i.e., noiseless sampling).

In practice, if any of the above criteria is not met, then the interpolator performance is suboptimal. Specifically, too low of a sampling rate results in distortion due to aliasing, a nonideal reconstruction filter leads to interpolation noise, while nonexact discrete-time samples manifest sample estimation noise.

Appendix C CIR Interpolation using a Digital Filter

In general, interpolation is essentially a two-step process which results in the increase of the sampling rate and the generation of new interpolated samples [57, 58]. For the specific case of CIR interpolation, the input sequence of Q consecutively estimated CIR samples, $\underline{h}(q)$, initially has its sampling rate, $\frac{1}{T_s}$, increased by the interpolation factor, R , to a higher sampling rate. $\frac{1}{T_s'} = \frac{R}{T_s}$ (see Figure C.56). This can be thought of as adding $R - 1$ evenly spaced, zero-valued samples between every two consecutively estimated CIR samples. This oversampling is done for each of the $2N_c = 12$ complex-valued components of a $\frac{T}{2}$ -spaced estimated CIR sample. The new oversampled CIR sequence is [57, 58]:

$$\underline{\hat{h}}(r) = \begin{cases} \underline{h}\left(\frac{r}{R}\right) & r = 0, \pm R, \dots, \pm \left(\frac{Q-1}{2}\right)R \\ , & \text{otherwise} \end{cases} \quad (\text{C.1})$$

where the index $\frac{r}{R}$ runs through all Q samples of the estimated CIR, $\underline{h}(q)$

This oversampling of the input sequence of estimated CIR samples, $\underline{h}(q)$, also decreases its digital (angular) frequency by a factor of R , from $\omega_d = 2\pi fT_s$, to:

$$\omega_d' = 2\pi fT_s' = 2\pi f \frac{T_s}{R} = \frac{\omega_d}{R} \quad (\text{C.2})$$

so that the Doppler spectrum of each component of the oversampled sequence of CIR estimates, $\underline{\hat{h}}(r)$, is [57, 58]:

$$\bar{H}\left(e^{j\omega_d'}\right) = H\left(e^{jR\omega_d'}\right) \quad (\text{C.3})$$

where

the original Doppler spectrum, $H\left(e^{j\omega_d}\right)$, has a period of 2π , and the oversampled Doppler spectrum, $H\left(e^{j\omega_d'}\right)$, has a period of $\frac{2\pi}{R}$

Figure C.56 Time and Frequency Responses of CIR Interpolation (for a single tap)

The oversampled Doppler spectrum, $\tilde{H}(e^{j\omega'_d})$, contains the baseband Doppler spectrum in the frequency range of $|\omega'_d| \leq \frac{\pi}{R}$ and additional spectral images centered about the harmonics of the original sampling frequency, $\frac{1}{T_s}$, at $\omega'_d = \pm \frac{2\pi}{P}, \pm \frac{4\pi}{R}, \dots$. Thus, the second step in the CIR interpolation process is to recover the baseband Doppler spectrum while filtering out all its undesired spectral images. To achieve this filtering, we can use a realistic approximation, $\tilde{F}(e^{j\omega'_d})$, of the ideal lowpass filter [57, 58]:

$$\tilde{F}(e^{j\omega'_d}) = \begin{cases} R & |\omega'_d| \leq \frac{\pi}{R} \\ 0 & \frac{\pi}{R} < |\omega'_d| < \pi \end{cases} \quad (\text{C.4})$$

which has a period of 2π

This ideal lowpass filter has a corresponding unit sample response of:

$$\tilde{f}(r) = \frac{\sin(\frac{\pi r}{R})}{\frac{\pi r}{R}} \quad r = 0, \pm 1, \pm 2, \dots \quad (\text{C.5})$$

which represents a basis function with the properties of even symmetry:

$$\tilde{f}(r) = \tilde{f}(-r) \quad (\text{C.6})$$

and orthonormality:

$$\tilde{f}(r) = \begin{cases} 1 & r = 0 \\ 0 & r = \pm R, \pm 2R, \dots \end{cases} \quad (\text{C.7})$$

This orthonormal property ensures that there is no interpolation error at the Q original estimated CIR samples.

This unit sample response, $\tilde{f}(r)$, of the ideal, lowpass digital filter is completely analogous to that of the ideal, analogue interpolation filter, $\tilde{f}(t)$, used for reconstructing sampled continuous-time signals (see Appendix B). Consequently, it is apparent that the ideal, digital interpolation filter, $\tilde{f}(r)$, does generate an interpolated CIR sequence, $\tilde{h}(r)$, by replacing the zero-valued samples of the oversampled CIR sequence, $h(r)$, with appropriately weighted and interpolated samples. This ideal interpolator, $\tilde{f}(r)$, is unrealizable since it requires an infinite number of samples for each interpolation. In practice, a realizable, suboptimal interpolator, $f(r)$, can be designed to some arbitrary criterion so as to approximate the ideal interpolator, $\tilde{f}(r)$.

Appendix D CIR Interpolation as a Convolution Process

With an appropriate change of variables, the convolution sum of the interpolation can be explicitly written as [58]:

$$\hat{h}(r) = \sum_{q=\left[\frac{r}{R}-\left(\frac{N_p-1}{2}\right)\frac{1}{R}\right]}^{\left[\frac{r}{R}+\left(\frac{N_p-1}{2}\right)\frac{1}{R}\right]} h(q)f(r-qR) \quad r = 0, \dots, R-1 \quad (\text{D.1})$$

where $[n]$ represents the largest integer that is less than or equal to n

From the convolution sum, it is evident that an interpolation of Q estimated CIR samples requires computation on the order of $\frac{N_p}{R}$. This is because a ratio of $\frac{R-1}{R}$ of the samples of $\hat{h}(r)$ are zero-valued due to R -times oversampling. Consequently, for a given interpolation factor, R , and a given number, Q , of estimated CIR samples, the length, N_p , of the unit sample response of the interpolation filter is [58]:

$$N_p = \begin{cases} QR + 1 & Q \text{ or } R \text{ is even} \\ QR + 2 & \text{otherwise} \end{cases} \quad (\text{D.2})$$

where N_p includes two zero-valued filter coefficients, one at each end of its unit sample response

Without a loss of generality, we stipulate $Q = 2p$ to be an even number so that we can arbitrarily choose the interpolation interval to be the middle data frame which is indexed by $q = p - 1$. Furthermore, by constraining Q to be an even number, we force the length, N_p , of the unit sample response of the interpolator to be an odd number. Forcing N_p to be an odd number ensures that the interpolator exhibits linear phase with a delay of an integer number of samples [57, 58]. By shifting the interpolation to the middle interval, the expression for the convolution sum of the interpolator becomes:

$$\hat{h}((p-1)R + r) = \sum_{q=\left[(p-1)+\frac{r}{R}-\left(\frac{N_p-1}{2}\right)\frac{1}{R}\right]}^{\left[(p-1)+\frac{r}{R}+\left(\frac{N_p-1}{2}\right)\frac{1}{R}\right]} h(q)f((p-1)R + r - qR) \quad (\text{D.3})$$

$$r = 0, \dots, R-1$$

To simplify further the CIR interpolator implementation, we explicitly require all Q estimated CIR samples to occur for $q \geq 0$. This forces the unit sample response of the interpolation filter to be causal, with its peak shifted from $r = 0$ to $r = \frac{N_p-1}{2}$. Therefore, the convolution sum of the interpolation takes the form of:

$$\hat{h}((p-1)R + r) = \sum_{q=0}^{Q-1} \underline{h}(q) f\left((p-1)R + r + \left(\frac{N_p-1}{2}\right) - qR\right) \quad r = 0, \dots, R-1 \quad (\text{D.4})$$

Appendix E CIR Interpolator Design

The ideal interpolator, $\tilde{f}(r)$, is unrealizable because its unit sample response requires an infinite number of samples per interpolation. This corresponds to a “brick wall”, low-pass frequency response, $\tilde{F}(e^{j\omega_d})$, which has a zero transition bandwidth. Furthermore, this ideal interpolator has zero phase or, at worst, linear phase corresponding to a constant group delay of an integer number of samples. This ideal phase characteristic preserves the continuous envelope of the interpolated discrete-time waveform and, thus, is important to the interpolation performance [57, 58].

In the design of a practical interpolator, $f(r)$, the average MSE and the maximum MSE are important performance criteria. Because the time variation in the CIR is random, the expectation over an ensemble of CIRs is required to compute the interpolation MSE. The “average” and “maximum” measures are performed over an interpolation interval of a CIR sequence. Thus, the minimization of either quantity yields a measure of interpolator performance. Furthermore, the length of the unit sample response of an interpolator is also an important design parameter which can be related to the frequency response parameters of passband and stopband ripple and transition bandwidth. In addition, the criterion of zero or linear phase response is also important.

In general, there are two generic types of digital filters: FIR and IIR. Primarily because of its capability to achieve exact linear phase and guaranteed stability, the FIR filter is preferred for interpolator design [57, 58]. Different classes of filter design for interpolators include simple windowing techniques (e.g., Hamming and Kaiser windows), classical methods (e.g., linear and Lagrange interpolators), designs with frequency response specifications (e.g., equal-ripple (Chebyshev) and half-band filters) and relatively complex stochastic filtering which accounts for the signal spectrum to produce an optimum (in the MMSE sense) interpolator [57, 65].

For our specific application of CIR interpolation, we use the simple windowing technique to design and implement a practical CIR interpolator. The main advantages of windowing techniques are that they are easy to implement and have frequency responses

that are well understood [57]. However, their disadvantages include limited control over the frequency response design and their inherently “fixed” design which does not adapt to the statistics of the particular signal (e.g., CIR) to be interpolated [57, 65]. Consequently, these shortcomings in the interpolator design lead to a suboptimal performance. Nevertheless, since our objective is to investigate the performance of an overall DMR communications system, rather than just to design digital filters, the simplicity of windowing techniques is the overriding factor for their use in our application of CIR interpolation.

A windowing technique produces a practical filter response, $f(r)$, by smoothing out the truncation of an ideal filter response, $\tilde{f}(r)$, with a gradually tapered window function, $\Pi(r; \eta)$:

$$f(r) = \tilde{f}(r)\Pi(r; \eta) \quad r = 0, \pm 1, \pm 2, \dots \quad (\text{E.1})$$

where $0 \leq \eta \leq 1$ is a parameter which controls the shape of the window.

We consider two common window functions with approximately raised-cosine profiles in the time and frequency domains. The approximately raised-cosine time function is usually called a “generalized Hamming” window. This window is described in the discrete-time domain as:

$$\Pi(r; \eta) = \begin{cases} \eta + (1 - \eta)\cos\left(\pi r \left(\frac{2}{N_p - 1}\right)\right) & -\left(\frac{N_p - 1}{2}\right) \leq r \leq \left(\frac{N_p - 1}{2}\right) \\ 0 & \text{otherwise} \end{cases} \quad (\text{E.2})$$

with

N_p being the number of coefficients in the unit sample response of the interpolator,

$\eta = 0.54$ yielding the Hamming window,

$\eta = 0.50$ yielding the Hanning window (i.e., raised cosine in time), and

$\eta = 1.0$ yielding the rectangular window, $\Pi(r; 1) = 1$

On the other hand, the second discrete-time window has the form:

$$\Pi(r; \eta) = \begin{cases} \frac{\cos\left(\eta\pi \frac{r}{R}\right)}{1 - (2\eta \frac{r}{R})^2} & -\left(\frac{N_p - 1}{2}\right) \leq r \leq \left(\frac{N_p - 1}{2}\right) \\ 0 & \text{otherwise} \end{cases} \quad (\text{E.3})$$

with $\eta = 0$ yielding the rectangular window, $\Pi(r; 0) = 1$

An interpolator using this second window has a unit sample response which corresponds to a truncated Nyquist pulse. For the sake of description, we call this a raised-cosine window. The reason for this name is that for the ideal case when this raised-cosine window has an infinite time span (i.e., $N_p = \infty$), the corresponding frequency response of the interpolator is raised-cosine:

$$F(e^{j\omega'_d}) = \begin{cases} R & 0 \leq |\omega'_d| < (1 - \eta)\frac{\pi}{R} \\ \frac{R}{2} \left[1 - \sin\left(\frac{R}{2\eta} \left(|\omega'_d| - \frac{\pi}{R} \right) \right) \right] & (1 - \eta)\frac{\pi}{R} \leq |\omega'_d| \leq (1 + \eta)\frac{\pi}{R} \\ 0 & |\omega'_d| > (1 + \eta)\frac{\pi}{R} \end{cases} \quad (\text{E.4})$$

Of course, for practical reasons, a window should have a truncated unit sample response so that the interpolation delay is reasonably small. Thus, a practical interpolator using this raised-cosine window has an approximately raised-cosine frequency response which exhibits ripple distortion in the passband and stopband. This ripple effect is due to the truncation of the discrete-time window. In particular, a zero rolloff, $\eta = 0$, window manifests Gibb's phenomenon of an approximately 9% ripple at the edge of the transition band with $\omega'_d \sim \frac{\pi}{R}$. However, the magnitude of the ripple distortion can be reduced with the use of a higher rolloff window.

The optimum filter depends on the sampling rate of the interpolator (see chapter 5.1). Among the variations of the two classes of discrete-time windows discussed, the raised-cosine window was found to yield slightly better interpolator performance for all sampling rates than the generalized Hamming window.

Appendix F MMSE Computation of Optimum Receiver Coefficients

For the facility of analysis, we assume correct decisions at the receiver:

$$\hat{\Delta}_a(kT) = \Delta_a(kT) \quad (\text{F.1})$$

and uncorrelated data symbols with unit variance:

$$E[\Delta_a(mT)\Delta_a^*(nT)] = \begin{cases} 1 & m = n \\ 0 & \text{otherwise} \end{cases} \quad (\text{F.2})$$

Furthermore, we assume that the noise, $n_l(kT)$, is uncorrelated with the data, $\Delta_a(kT)$.

The global minimum of the MSE at the output of the OCDFE:

$$mse = E\left[|\underline{w}^* \underline{x}(kT) - \Delta_a((k-D)T)|^2\right] \quad (\text{F.3})$$

occurs when its gradient with respect to the OCDFE coefficients is equal to zero. The resulting value of the optimum receiver coefficients is given by the familiar Wiener solution [7, 53]:

$$\underline{w}_{opt} = \Phi^{-1} \underline{p} \quad (\text{F.4})$$

with the corresponding minimum MSE being:

$$mmse = 1 - \underline{p}^* \Phi^{-1} \underline{p} = 1 - \underline{p}^* \underline{w}_{opt} \quad (\text{F.5})$$

The $(LN_f + N_b) \times (LN_f + N_b)$ autocorrelation matrix is:

$$\Phi = E[\underline{x}(kT)\underline{x}^*(kT)] \quad (\text{F.6})$$

and the $(LN_f + N_b) \times 1$ cross-correlation vector is:

$$\underline{p} = E[\underline{x}(kT)\Delta_a^*((k-D)T)] \quad (\text{F.7})$$

where the expectation is with respect to the data symbols, $\Delta_a(kT)$, and noise, $n_l(kT)$, while the channels (i.e., interpolated CIRs, $\hat{h}_l(kT)$) are assumed fixed.

Furthermore, the OCDFE weight vector is expressed as:

$$\underline{w} = \begin{bmatrix} \underline{w_0} \\ \underline{w_1} \\ \vdots \\ \underline{w_{L-1}} \\ \underline{w_b} \end{bmatrix} \quad (\text{F.8})$$

where the forward filter coefficients in the l th diversity branch and the feedback filter coefficients are represented by the respective weight vectors:

$$\underline{w_l} = \begin{bmatrix} w_{lN_f} \\ w_{lN_f+1} \\ \vdots \\ w_{(l+1)N_f-1} \end{bmatrix}, \quad \underline{w_b} = \begin{bmatrix} w_{LN_f} \\ w_{LN_f+1} \\ \vdots \\ w_{LN_f+N_b-1} \end{bmatrix} \quad (\text{F.9})$$

and the overall input signal to the OCDFE is:

$$\underline{r}(kT) = \begin{bmatrix} \underline{r_0}(kT) \\ \underline{r_1}(kT) \\ \vdots \\ \underline{r_{L-1}}(kT) \\ \underline{\hat{\Delta}_a}((k-D)T) \end{bmatrix} \quad (\text{F.10})$$

where the detected $\frac{\pi}{4}$ -shifted DQPSK data vector and the received signal vector in the l th diversity branch are, respectively:

$$\underline{\hat{\Delta}_a}(kT) = \begin{bmatrix} \hat{\Delta}_a((k-D-1)T) \\ \hat{\Delta}_a((k-D-2)T) \\ \vdots \\ \hat{\Delta}_a((k-D-N_b)T) \end{bmatrix}, \quad \underline{r_l}(kT) = \begin{bmatrix} r_l(kT) \\ r_l(kT - \frac{T}{2}) \\ \vdots \\ r_l(kT - (N_f-1)\frac{T}{2}) \end{bmatrix} \quad (\text{F.11})$$

where DT is the processing delay in the OCDFE

In addition, the received signal at the l th diversity branch is:

$$r_l(t) = \sum_k \Delta_a(kT) \hat{h}_l(t - kT) + n_l(t) \quad (\text{F.12})$$

where the $\frac{T}{2}$ -sampled, receive-filtered additive Gaussian noise, $n_l(t - k\frac{T}{2})$, has a variance of σ_n^2 and an autocorrelation function, $\Phi_n((k-l)\frac{T}{2})$, with a 25% excess bandwidth raised-cosine response.

In order to solve for the optimum OCDFE coefficients, we want to express Φ and $\underline{p}$ in terms of the interpolated CIR estimates, $\hat{h}_l(kT)$, and noise autocorrelation function, $\Phi_n((k-l)\frac{T}{2})$.

Specifically, the OCDFE autocorrelation matrix can be expressed as a partitioned matrix:

$$\Phi = E \begin{bmatrix} \underline{r}_0(kT)\underline{r}_0^*(kT) & \dots & \underline{r}_0(kT)\underline{r}_{L-1}^*(kT) & \underline{r}_0(kT)\underline{\Delta}_a^*(kT) \\ \vdots & \ddots & \vdots & \vdots \\ \underline{r}_{L-1}(kT)\underline{r}_0^*(kT) & \dots & \underline{r}_{L-1}(kT)\underline{r}_{L-1}^*(kT) & \underline{r}_{L-1}(kT)\underline{\Delta}_a^*(kT) \\ \underline{\Delta}_a(kT)\underline{r}_0^*(kT) & \dots & \underline{\Delta}_a(kT)\underline{r}_{L-1}^*(kT) & \underline{\Delta}_a(kT)\underline{\Delta}_a^*(kT) \end{bmatrix} \quad (\text{F.13})$$

Each submatrix for diversity signal correlation can be computed as:

$$\Phi_{l,q} = E \left\{ \begin{bmatrix} r_l(kT) \\ \vdots \\ r_l(kT - (N_f - 1)\frac{T}{2}) \end{bmatrix} [r_q^*(kT) \dots r_q^*(kT - (N_f - 1)\frac{T}{2})] \right\} \quad (\text{F.14})$$

where the (i, j) th element of this submatrix can be written as:

$$\begin{aligned} \Phi_{l,q}(i, j) &= E \left[r_l \left(kT - i\frac{T}{2} \right) r_q^* \left(kT - j\frac{T}{2} \right) \right] \\ &= \sum_m \sum_n E [\Delta_a(mT) \Delta_a^*(nT)] \hat{h}_l \left(kT - i\frac{T}{2} - mT \right) \hat{h}_q^* \left(kT - j\frac{T}{2} - nT \right) \\ &\quad + \sum_m E \left[\Delta_a(mT) n_q^* \left(kT - j\frac{T}{2} \right) \right] \hat{h}_l \left(kT - i\frac{T}{2} - mT \right) \\ &\quad + \sum_n E \left[n_l \left(kT - i\frac{T}{2} \right) \Delta_a^*(nT) \right] \hat{h}_q^* \left(kT - j\frac{T}{2} - nT \right) \\ &\quad + E \left[n_l \left(kT - i\frac{T}{2} \right) n_q^* \left(kT - j\frac{T}{2} \right) \right] \end{aligned} \quad (\text{F.15})$$

By the assumption of uncorrelated data symbols, the first term is nonzero if and only if $m = n$, while by the assumption of uncorrelated noise and data samples, the cross-terms

are zero. Thus, the expression simplifies to:

$$\Phi_{l,q}(i,j) = \sum_k \hat{h}_l\left(kT - i\frac{T}{2}\right) \hat{h}_q^*\left(kT - j\frac{T}{2}\right) + \sigma_n^2 \Phi_n\left((i-j)\frac{T}{2}\right) \quad (F.16)$$

$$i, j = 0, 1, \dots, N_f - 1$$

$$l, q = 0, 1, \dots, L - 1$$

From this expression, it is evident that the submatrices possess Hermitian symmetry:

$$\Phi_{l,q} = \Phi_{q,l}^* \quad l, q = 0, 1, \dots, L - 1 \quad (F.17)$$

Each submatrix for the diversity signal and feedback data correlation can be computed as:

$$\Phi_{l,b} = E \left\{ \begin{bmatrix} r_l(kT) \\ \vdots \\ r_l(kT - (N_f - 1)\frac{T}{2}) \end{bmatrix} \begin{bmatrix} \Delta_a((k - D - 1)T) & \dots & \Delta_a((k - D - N_b)T) \end{bmatrix} \right\} \quad (F.18)$$

The (i, j) th element of this submatrix can be written as:

$$\begin{aligned} \Phi_{l,b}(i,j) &= E \left[r_l\left(kT - i\frac{T}{2}\right) \Delta_a^*((k - D - j - 1)T) \right] \\ &= \sum_m E[\Delta_a(mT) \Delta_a^*((k - D - j - 1)T)] \hat{h}_l\left(kT - i\frac{T}{2} - mT\right) \\ &\quad + E \left[n_l\left(kT - i\frac{T}{2}\right) \Delta_a^*((k - D - j - 1)T) \right] \end{aligned} \quad (F.19)$$

By the assumption of uncorrelated data symbols, the first term is nonzero if and only if $m = k - D - j - 1$, while by the assumption of uncorrelated noise and data samples, the second term is zero. Therefore, the expression simplifies to:

$$\Phi_{l,b}(i,j) = \hat{h}_l((D + 1)T - i\frac{T}{2} + jT) \quad \begin{aligned} l &= 0, 1, \dots, L - 1 \\ i &= 0, 1, \dots, N_f - 1 \\ j &= 0, 1, \dots, N_b - 1 \end{aligned} \quad (F.20)$$

Furthermore, these submatrices are also Hermitian symmetric:

$$\Phi_{l,b} = \Phi_{b,l}^* \quad l = 0, 1, \dots, L - 1 \quad (F.21)$$

By the assumption of uncorrelated data symbols, the submatrix for feedback data correlation can be computed to be an $N_b \times N_b$ identity matrix:

$$\Phi_{b,b} = E \left\{ \begin{bmatrix} \Delta_a((k-D-1)T) \\ \vdots \\ \Delta_a((k-D-N_b)T) \end{bmatrix} [\Delta_a^*((k-D-1)T) \dots \Delta_a^*((k-D-N_b)T)] \right\} \quad (F.22)$$

$= I$

The OCDFE cross-correlation vector can be expressed as a partitioned vector:

$$\underline{p} = E \left\{ \begin{bmatrix} \underline{r_0}(kT) \\ \underline{r_1}(kT) \\ \vdots \\ \underline{r_{L-1}}(kT) \\ \underline{\Delta_a}(kT) \end{bmatrix} \Delta_a^*((k-D)T) \right\} \quad (F.23)$$

The subvectors for the diversity signal and data cross-correlation can be computed as:

$$\underline{p_l} = E \left\{ \begin{bmatrix} r_l(kT) \\ r_l(kT - \frac{T}{2}) \\ \vdots \\ r_l(kT - (N_f - 1)\frac{T}{2}) \end{bmatrix} \Delta_a^*((k-D)T) \right\} \quad (F.24)$$

The i th element in the cross-correlation subvector is:

$$\begin{aligned} p_l(i) &= E \left[r_l \left(kT - i\frac{T}{2} \right) \Delta_a^*((k-D)T) \right] \\ &= \sum_m E[\Delta_a(mT) \Delta_a^*((k-D)T)] \hat{h}_l \left(kT - i\frac{T}{2} - mT \right) \\ &\quad + E \left[n_l \left(kT - i\frac{T}{2} \right) \Delta_a^*((k-D)T) \right] \end{aligned} \quad (F.25)$$

From the assumption of uncorrelated data symbols, the first term is nonzero if and only if $m = k - D$, while from the assumption of uncorrelated noise and data samples, the second term is zero. Therefore, the expression simplifies to:

$$\begin{aligned} p_l(i) &= \hat{h}_l(DT - i\frac{T}{2}) & i &= 0, 1, \dots, N_f - 1 \\ & & l &= 0, 1, \dots, L - 1 \end{aligned} \quad (F.26)$$

Similarly, by the assumption of uncorrelated data symbols, the last cross-correlation subvector is a zero-vector:

$$\begin{aligned} \underline{p}_b &= E \left\{ \begin{bmatrix} \Delta_a((k-D-1)T) \\ \Delta_a((k-D-2)T) \\ \vdots \\ \Delta_a((k-D-N_b)T) \end{bmatrix} \Delta_a^*((k-D)T) \right\} \\ &= \underline{0} \end{aligned} \quad (\text{F.27})$$

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