Stress and Deformation in Orogenic Wedges with Power-Law Rheology

by

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A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the
degree of Doctor of Philosophy

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Abstract

Within the framework of plate tectonic theory, orogenic belts are interpreted as forming at convergent margins as a result of subduction and collision of two plates. This thesis deals with stress and deformation in orogenic belts from two different perspectives: (1) brittle, dislocation-type propagation of thrust faults along the base of wedge-shaped elastic thrust sheets; (2) stresses and deformation of orogenic wedges with generalized nonlinear creep rheology. Both treatments are analytical and quantitative from a continuum mechanics approach.

On the scale of a single thrust sheet, an Airy stress function solution is derived for the stress field in an elastic wedge-shaped overthrust block subject to horizontal forces at the rear and at the front. From this stress solution the state of stress on the basal thrust (lower boundary of the block) is obtained. Both normal and shear stresses on the basal thrust are affected by the rear pushing and front buttressing forces and by the slopes of the upper and lower boundaries. For nonlinear components of external horizontal forces exerted on the vertical edges of the block, and/or nonzero slopes of the upper and lower boundaries, the normal and shear stresses vary systematically with position along the basal thrust. The ratio of shear to normal stresses along this plane (the “frictional function” of the thrust) can be compared with the friction coefficient to determine where slippage is likely to occur. Since the frictional function is a function of position, the strength is exceeded only on part of the thrust (near the rear for most values of the parameters). The assumption of simultaneous displacement of the whole thrust sheet on the basal plane is therefore not realistic. Even simple static models such as the present one indicate that the motion on thrust faults must be envisaged in terms of progressive, consecutive “dislocation-type” slippage.

On a larger scale, in order to model some important features commonly associated with the development of orogenic wedges, an analytical solution for deformation and stress in a two-dimensional plane strain incompressible block with power-law bulk rheology is developed. This solution involves material rheology, stress boundary conditions at the rear of the block, and a function $C(x,y)$ representing the deviation of vertical normal stress from lithostatic overburden. If the latter is assumed to be zero, a well accepted approximation in modelling of various geodynamic processes, a simplified solution can be obtained for a linear longitudinal strain rate. This solution is expressed in terms of a group of integration constants and a one-dimensional function of depth representing the normal stress in excess of the lithostatic on the rear boundary. Both constants and function are explicitly computable from material rheology and assigned boundary conditions. Analytical comparison and numerical computations show that this linear longitudinal strain rate solution is a two-dimensional extension of a classic one-dimensional solution derived by Nye (1957) for ice sheets. Also, some semi-quantitative models for orogenic wedges (e.g. Platt, 1986) can be shown to be special cases of this solution. Numerical implementation of the solution suggests that: (1) the dynamics of orogenic wedges varies with material rheology, temperature, and boundary conditions; (2) the depth distribution of the nongravitational normal stress at the rear boundary of the wedge required to maintain a prescribed deformation pattern varies mainly with wedge rheology; (3) wedges with viscoplastic rheology (i.e. a top plastic layer overlying a nonlinear viscous layer) are able to deform with different longitudinal patterns, including compression near the front and extension near the back of the wedge, if acted upon by suitable forces.

The models presented in this thesis are necessarily drastic simplifications of reality. However, they yield two results of general interest and applicability. The first is that *slippage along the basal thrust of an overthrust block propagates along the fault*, and is not synchronous along its whole length. The second is that *orogenic wedges with linearly*

viscous, power-law viscous, and plastic rheologies can show stress and deformation patterns in reasonable agreement with observation, under tectonic forces and kinematic boundary conditions simulating a variety of tectonic styles.

Acknowledgments

The completion of this thesis would have been impossible without the help and cooperation of various people and institutions. My very sincere gratitude goes to my supervisor, Professor G. Ranalli, for his support and advice throughout the whole program. His guidance and encouragement have been essential.

Dr. M. K. Paul of the Geological Survey of Canada is thanked for his contribution to the development the graphic processing program used in this thesis, which is a modification of a plotting code kindly provided by Dr. J. Melosh of the University of Arizona. Professor J.-C. De Bremaecker of Rice University and Dr. A. Yin of the University of California at Los Angeles offered valuable suggestions for the improvement of Chapter 3. I have also benefited from numerous discussions with Drs. A. Lambert, T. James, and K. Wang of the Geological Survey of Canada and Drs. Z. Yin, Z. Yue, and Mr. Z. Deng at Carleton University.

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Most importantly I wish to thank my wife Jian Song and my daughter Shirley for their unwavering support, their love and sacrifices. My final thanks are extended to my parents for their persistent encouragement.

Original Contribution

This thesis contains only the results of the research conducted by the candidate, under the guidance of his supervisor. Chapter 3 has been published in a slightly different form (see footnote on page 32). The candidate is the senior author of this paper, and the supervisor's (second author) contribution has consisted of advice and cooperation in the writing of the paper.

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Chapter 1

Introduction

Orogenic belts provide the geological record of mainly compressive plate tectonic activity, including subduction of one plate beneath another, and collision between crustal masses, such as two continents, a continent and an island arc, or a continent and an oceanic plateau. Structurally, orogenic belts display a variety of styles, but they always include roughly wedge-shaped regions (in cross section), with or without bilateral symmetry, verging towards the foreland. These wedge-shaped regions are termed *orogenic wedges*.

Within the framework of plate tectonic theory, orogenic belts are interpreted as forming at convergent margins as a result of subduction and collision of two plates. Thus study of an orogenic belt provides vital information for the understanding of the interaction between two tectonic plates. There are basically two different points of views in studying the dynamics of orogenic wedges: on a smaller scale, foreland thin-skinned thrusts are envisioned as rigid blocks moving on discrete faults; on a larger scale, both thin- and thick- skinned wedges are considered as a single, mechanically continuous, deforming body. Considerable progress has been made in the past three decades, regarding the dynamics of orogenic wedges and their tectonic evolution. However, there are still many problems which are not well understood. This thesis deals with stress and deformation in orogenic wedges from two different perspectives: (1) brittle, dislocation-type propagation of thrusting along the base of a single thrust sheet; (2) dynamics of orogenic wedges as a whole with generalized creep rheology. Both treatments are analytical and quantitative from a continuum mechanics approach.

Chapter 2 provides a review on orogenic wedges, including tectonic background, mechanical and dynamic characteristics, and outlines some of the major problems which are dealt with in this thesis.

An elastic solution is derived in Chapter 3 to study the stresses in an individual wedge-shaped thrust sheet. This solution is used to justify the progressive, consecutive “dislocation-like” propagation of the basal thrust, and consequently provides a quantitative explanation for the mechanical paradox of overthrusting.

Chapters 4, 5, and 6 are devoted to the study of orogenic wedges with a generalized power-law creep rheology. A two-dimensional solution for a wedge-shaped block is derived in Chapter 4. Stress, strain rate and velocity fields are expressed explicitly in terms of the bulk rheology of wedge material, the force boundary condition at the wedge’s rear boundary, and a two-dimensional function representing the deviation of vertical normal stress from the lithostatic pressure. A special case for a wedge with linear Newtonian rheology is discussed in detail to show how to determine deviations from lithostatic conditions from other boundary conditions.

A two-dimensional analytical solution is given in Chapter 5 for an orogenic wedge with nonlinear steady-state rheology and a longitudinal strain rate varying linearly with distance from the rear of the wedge. Stress, strain rate, and velocity components are functions of bulk rheology, temperature distribution, and arbitrary constants determinable from dynamic and/or kinematic boundary conditions. Several cases dealing with parallel-sided blocks are discussed in detail, and the results are compared with one-dimensional solutions.

Chapter 6 presents computational results for several wedge-shaped block cases based on the two-dimensional solution derived in Chapter 5. These cases are classified into two groups based on the rheological characteristics of the wedge: the first group deals with

wedges with viscous and power-law rheologies; the second group considers wedges with top plastic and bottom power-law rheologies.

Final discussion and conclusions about this two-dimensional power-law orogenic wedge model form the main part of Chapter 7. Also included in this chapter is a brief discussion on the possible further extensions of the analytical study of orogenic wedges reported in this thesis.

Chapter 2

Orogenic Wedge Models: A Review

2.1 Introduction

This chapter begins with a brief description of orogenic belts and wedges within the framework of plate tectonics. Then, the major developments in the evolution of our understanding about the dynamics of orogens are reviewed. Next, the rheological characteristics of orogenic wedges are discussed. Finally, the objectives of this thesis are outlined at the end of the chapter.

2.2 Orogenic belts , foreland fold-and-thrust belts , and accretionary prisms

Orogenic belts, coinciding with some of the Earth's great mountain chains, represent the geological record of mainly compressive plate tectonic activity. This activity includes subduction of an oceanic plate beneath a continental or oceanic plate, as well as collision between two continental plates (including an island arc, or an oceanic plateau). Orogenic belts are zones of intense deformation (*cf.* Suppe, 1985 for a general discussion). Active orogenic belts are localized along plate boundaries and commonly along continental margins, apparently reflecting a fundamental instability of continental margins relative to oceanic and continental interiors. Figure 2.1 shows the location of some major Phanerozoic orogenic belts, chiefly the American Cordillera, and the Appalachian, Caledonide, and Alpine-Himalayan systems.

Figure 2.1 World map, showing the location of some major orogenic belts (from Twiss and Moores, 1992).

Figure 2.2 Cross section across a model composite orogenic belt (from Twiss and Moores, 1992).

Although significant variations can be found among cross sections of different orogenic belts (*cf.* Twiss and Moores, 1992), they have a number of features in common: a rough bilateral structural (but not geometrical) symmetry; a foreland or undeformed plate on either side; outer foredeeps; fold and thrust belts; sutures marked by ophiolitic rocks; and an internal crystalline core zone of metamorphosed and deformed sedimentary and volcanic rocks, mafic-ultramafic complexes, and granite plutons (Figure 2.2).

Because of the complex structural character of orogenic belts, it is natural to consider only simplified models, for instance the wedge-shaped area (in cross section) between the foreland and the centre of the belt. This part of the orogenic belt is often referred to as the *orogenic wedge* (Pavlis and Bruhn, 1983; Platt, 1986; Dechesne and Mountjoy, 1992; Royden, 1993; Yin, 1993).

As parts of orogenic belts, *foreland fold-and-thrust belts* are belts in which the tectonic style is dominated by overthrusting, i.e., tectonic emplacement of sheets of allochthonous rocks over autochthonous material (Boyer and Elliott, 1982; Price and Cosgrove, 1990). They are 'foreland' because they usually occupy an exterior position (relative to the direction of tectonic transport) with respect to the structural and metamorphic culmination of the orogen. Foreland fold-and-thrust belts usually lie on the continental side of the orogen. *Accretionary prisms*, forming above oceanic subduction zones, are deformed wedge-shaped sedimentary masses between the front trench line and the rear outer arc high (*cf.* Fowler, 1990 for detailed description on subduction zones and accretionary prism).

In this thesis, the term *orogenic wedge* is used in a general sense for both thin- and thick-skinned wedges, while *thin-skinned orogenic wedge* is reserved for foreland fold-and-thrust belts not affecting the basement, and accretionary prisms.

Typical examples of foreland fold-and-thrust belts and accretionary prisms include:

(a) the Barbados accretionary complex, located to the east of the Lesser Antilles island arc, where 65 Ma oceanic lithosphere of the North American plate is subducting beneath the Caribbean plate (Westbrook, 1982; Behrmann et al., 1988) (Figure 2.3a);

(b) the Taiwan fold-and-thrust belt produced by the subduction of the Eurasian plate beneath the Philippine Sea Plate (Suppe, 1980)(Figure 2.3b);

(c) the southern Canadian Cordillera fold-and-thrust belt formed during the late Jurassic and Cretaceous, and involving several interrelated processes including accretion of outboard terranes to the western margin of North America and protracted compression and crustal thickening (Price, 1981; Brown et al., 1992)(Figure 2.3c).

(d) the southern Appalachians deformed during the late Carboniferous to Permian Alleghenian orogeny (Figure 2.3d).

According to Pavlis and Bruhn (1983), thin-skinned and thick-skinned orogenic wedges may represent different stages of the evolution of orogenic belts in subducting regions. Thin-skinned wedges, such as the Barbados accretionary prism, could be the early product of subduction, when the whole complex is in its initial stages of development and consists mainly of the accretionary prism. With the growth of the subduction complex, the horizontal and vertical dimensions of the wedge increase. Thick-skinned wedges are more ductile and metamorphosed than thin-skinned wedges. Some typical thick-skinned orogenic wedges include:

(a) the Alpine orogen, a consequence of the convergence between Europe and Adria, a small plate that includes the continental lithosphere underlying Italy and the Adriatic Sea (Polino et al., 1990)(Figure 2.4a);

(b) the Himalayan orogen developed during the collision of the Indian plate with Eurasia (Burg and Chen, 1984; Burchfiel and Royden, 1985; Burchfiel et al., 1992; Harrison et al., 1992)(Figure 2.4b).

(a)

(b)

(c)

(d)

Figure 2.3 Cross sections of four foreland fold-and-thrust belts and accretionary wedges: (a) Barbados accretionary wedge; (b) Taiwan foreland fold-and-thrust belt; (c) Canadian Rockies; (d) Southern Appalachians. No vertical exaggeration (from Dahlen, 1990).

Figure 2.4a Three cross sections of the Alpine orogen: (1) the easternmost Austrian Alps, (2) the Tauern window, and (3) the central Alps. AD: Adula nappe; AU: Austroalpine basement and cover nappe system of the eastern Alps; BE/RE/TW: Bernstein, Rechnitz and Tauern windows; CA: Canavese zone; GO/TV/AAR/MB/BE/AG: Gotthard, Tavetsch, Aar, Mont Blanc, Belledonne and Argentera Helvetic-Dauphinois basement units; HE: Ultra-helvetic, Helvetic and Dauphinois cover units; HF: Helmintoid flysch of the French-Italian Alps; LPN: Simplon-Tessin lower Penninic nappes; NCA: northern calcareous Alps, Austroalpine; P: Prealpine décollement nappes; PA/GL/MA/PI/AN: Platta-Arosa, Glockner, Malenco-Avers, Piedmont and Antrona ophiolitic units; UP/TN/SU/MR/GP/DM: Upper Penninic, Tauern, Suretta, Monte Rosa, Gran Paradiso, and Dora Maira basement nappes; SA: southern Alps; TA/SB/PN: middle Penninic Tambò, Grand St. Bernard/Briançonnais and Pinerolese nappes; VA: Valais (North-Penninic) ophiolitic and flysch units; PF: Penninic thrust front; PL: Periadriatic (Insubric) fault system. 1: European foreland; 2a: Eastern Austroalpine Eoalpine very low grade metamorphism; 2b: Eastern Austroalpine Eoalpine greenschist and amphibolite facies metamorphism; 3: ophiolitic units, related metamorphic flysch and mélange; 4: flysch décollement units, mostly Cretaceous; 5a: Tertiary European molasse; 5b: Tertiary Po plain molasse; 6: Periadriatic plutons, mostly Oligocene (from Polino et al., 1990).

Figure 2.4b Geological cross section through the Himalayas, simplified from Gansser (1964) and Lyon-Caen and Molnar (1983). STDS: south Tibetan detachment system of Burchfiel et al. (1992). 1: Tertiary and Quaternary sedimentary rocks; 2: Mesozoic Indus flysch; 3: Paleozoic sedimentary rocks of Greater Himalaya; 4: Paleozoic sedimentary rocks of Lesser Himalaya; 5: upper Precambrian and lower Paleozoic sedimentary rocks; 6: Precambrian basement; 7: Himalaya leucogranites.

2.3 Major characteristics of orogenic wedges

The main characteristics of thin-skinned orogenic wedges are (McClay, 1992; Ranalli, 1995):

- (a) Wedge-shaped geometry, thicker at the rear, which is the direction of provenance of the thrust sheets;
- (b) Deformation beginning earlier at the rear than at the front, with a “piggy-back” mode of transport, in which the lowest thrust plane is the most recent and consequently the highest thrust sheets have travelled the farthest;
- (c) Lateral shortening and vertical thickening as the main deformation characteristics;
- (d) A weak basal layer (e.g., evaporites, high pore fluid pressure) sometimes (but not always) occurring between the thrust belt and the basement.

Thick-skinned orogenic belts may additionally display the following features (Pavlis and Bruhn, 1983; Platt, 1986, 1987, 1993):

- (a) Coeval development of compressional structures at the thin-skinned foreland ‘front’ and extensional structures at the thick-skinned ‘rear’ towards the hinterland;
- (b) Exhumation of high pressure-low temperature metamorphic rocks near the rear of the wedge, while convergence is continuing;
- (c) Increasing ductility of the rock material towards the rear low corner of the orogenic complex, due to higher temperature.

2.4 Previous work

Several models (numerical and analytical) have been proposed to analyze the deformation processes in orogenic wedges. Conceptually, these models can be divided into two groups: the first group focuses on the stress and deformation in individual thrust

sheets; the second group assumes that the entire wedge (thin-skinned and/or thick-skinned) behaves as a single, mechanically continuous, unit, and studies its overall evolution.

2.4.1 Models for individual thrust sheets

One of the most debated questions in modelling the movement of thrust sheets deals with the famous “mechanical paradox of overthrusting” (Reade, 1908; Smoluchowski, 1909). This paradox can be illustrated by studying a rectangular block resting on a horizontal base (*cf.* Suppe, 1985; Ranalli, 1995). Assuming that the block is of uniform density ρ , pore fluid factor (ratio of pore fluid pressure to lithostatic pressure) λ , height h and length l , and that the underlying base has cohesion S_0 , frictional coefficient μ_0 and pore fluid factor λ_0 , consideration of force balance allows estimation of the maximum length of the thrust sheet that can move as a coherent unit. This is given by

$$\frac{l_{\max}}{h} = \frac{2C + [R(1 - \lambda) + \lambda]\rho gh}{2S_0 + 2\mu_0\rho gh(1 - \lambda_0)} \quad (2.1)$$

where R is the stress ratio between the maximum and minimum compressional stresses, and C is the uniaxial compressive strength of the block, related to the block's cohesion S and frictional coefficient μ as $C = 2S[(1 + \mu^2)^{1/2} + \mu]$. If the parameters of the block are the same as those at its base, the right-hand side term in equation (2.1) is about 3 (*cf.* Suppe, 1985). This result strictly limits the maximum length of overthrust sheets (physically, this is a consequence of the fact that the pushing force that can be applied at the rear is limited by the strength of the block). To explain the observations of overthrusts with lengths of the order of tens or hundreds of kilometers and thicknesses of only a few kilometers, therefore, very high pore fluid pressures and/or weak layers are required at

their bases. When evidence for such pressure or weak layers is lacking, the occurrence of large overthrusts is difficult to explain. This is the so-called ‘mechanical paradox’ for overthrusts.

In Chapter 3, following a suggestion by Price (1988), we offer a solution to the paradox based on the idea that the thrust propagates with finite speed and does not slide at the same time throughout its length. An Airy stress function solution is derived for incomplete boundary conditions, and computational results from this solution show that the ratio of basal shear stress to basal normal stress decreases from the rear to the front of thrust plane for wedge-shaped thrust blocks, which means that movement of a large coherent overthrust sheet may be realized by a progressive, dislocation-type thrust faulting along its base.

2.4.2 Continuum models for orogenic wedges

A fundamental concept in these models is that the wedge behaves as a single, mechanically continuous, dynamic unit (Platt, 1986). This was argued explicitly by Price (1973a) who suggested that the wedge as a whole is macroscopically ductile in that it does not lose its overall mechanical integrity during deformation.

Most previous studies have focused on the dynamics of thin-skinned orogenic wedges which are generally less than 100 km in width and 10 to 15 km in depth at the rear (as an order of magnitude). All the models proposed in these studies have been trying to account for the deformation of the wedge in terms of the following parameters:

- (a) the overall geometry of the wedge (i.e., top and basal slopes; see Figure 2.5);
- (b) its internal mechanical strength; and
- (c) the mechanical strength of the material between the wedge and the underlying basement.

Figure 2.5 A simplified accretionary wedge model (Platt, 1986): α and β are the top and basal slopes, respectively; $\theta(= \alpha + \beta)$ is the taper of the wedge; h is the thickness of the wedge and τ_b is the shear stress at the base of the wedge.

Figure 2.6 Schematic diagram illustrating a bulldozer wedge (from Dahlen, 1990). α and β are the top and basal slope angles, respectively; $\theta(= \alpha + \beta)$ is the taper of the wedge; h is the local thickness of the wedge at the location where τ_b is the shear stress at the base of the wedge. (Note that the definition of h varies slightly with the choice of coordinate system.)

Elliott (1976) assumed that the material in the wedge is relatively weak and unable to sustain deviatoric stresses of more than 20 MPa on a geological time scale. Based upon this assumption, the deformation of wedge can be considered analogous to that of a glacier, and at equilibrium the basal resistance to the sliding flow of the orogenic wedge is balanced by the gravitational shear stress in the longitudinal direction

$$\tau_b = \rho g h \alpha \quad (2.2)$$

where τ_b is the shear stress at the base, ρ the density of the wedge material, g the gravitational acceleration, α the surface slope, and h the local thickness of the wedge (see Figure 2.5). This is in perfect analogy with the flow of a glacier or ice-sheet (Paterson, 1981). According to equation (2.2), the wedge material spreads under the action of gravity. This *gravity spreading model*, however, is not by itself applicable to the analysis of foreland fold-and-thrust belts since it does not account for the pervasive shortening observed in thrust belts.

Contrary to the simple gravity spreading model, Chapple (1978) assumed that the wedge material is strong enough to stand a longitudinal stress of up to 200 MPa, and that there exists a relatively weak layer along the base of the wedge. By treating the wedge and the underlying weak basal layer as continua with perfectly plastic rheology, and assuming both wedge and weak layer to be at the yielding point throughout the whole block, the following expression can be derived relating the basal shear stress (τ_b) to the wedge's taper (i.e., surface slope α and basement slope β) and the plastic strength of the material (K)

$$\tau_b = \rho g h \alpha + 2K\beta \quad (2.3)$$

The basal shear stress, τ_b , equals the plastic strength of the weak basal layer and can be expressed in terms of the strength of the wedge material as

$$\tau_b = \chi K \quad (2.4)$$

Thus, χ is equal to the ratio of the yield strength of the weak basal layer to that of the wedge. Equations (2.3) and (2.4) show that the resistance to sliding (τ_b) is balanced by two driving forces: gravity-induced lateral spreading and tectonic compression (the solution is strictly valid only for small surface and basement slopes). The most important conclusion from this model is that the wedge-shaped geometry is a consequence of internal dynamics. If the force applied at the rear of the wedge is counteracted by the resistance to sliding on its base, a forward topographic slope (and therefore a greater cross-section at the rear) is needed to prevent the longitudinal stress from exceeding the strength of the material. If the wedge is insufficiently tapered (that is, the sum of topographic and basal slopes is too small), it shortens internally and thickens at the rear until it is able to transmit the force from behind towards the front. This implies that the wedge material is at the yield point throughout. Equations (2.3) and (2.4) also show that the critical taper $\theta(= \alpha + \beta)$ decreases with decreasing strength of the basal layer, i.e., thin, long wedges require a weak base, while a strong base results in a larger critical taper. As a first approximation this model satisfies the characteristics of thin-skinned foreland belts.

The plastic model described above applies to thin-skinned wedges where the temperature is not sufficiently high to induce steady-state viscous flow. Whereas plastic flow does not begin until the stress level reaches some critical value, viscous flow is produced by any stress, no matter how small.

Besides the perfect plasticity used in Chapple's model, another rheology applicable to low-temperature supracrustal thin-skinned wedges is the frictional Coulomb-Navier criterion (Jaeger and Cook, 1969; Paterson, 1981; Ranalli, 1995). This rheology, modified for the effects of pore fluid pressure, is assumed in the popular critical taper wedge model (Davies et al., 1983; Dahlen, 1984; Dahlen et al., 1984; Zhao et al., 1986; Dahlen, 1990). This model is basically analogous to a wedge of snow or sand in front of a moving bulldozer (Figure 2.6). Analogously to the plastic model, the state of stress within a critically tapered wedge is assumed everywhere on the verge of Coulomb-Navier failure, and an expression for the critical taper is derived in terms of material parameters. The essential premise of the critical taper model implies that (a) a critical wedge grows self-similarly; and (b) the wedge shortens or extends to regain the critical state if the taper becomes subcritical or supercritical. There are several expressions for the critical frictional taper, depending upon whether the effects of cohesion and/or pore fluid pressure are considered or not. In the simplest case, where the cohesion of wedge material is ignored and the internal friction coefficient is assumed to be constant within the wedge, the critical taper can be approximated as

$$\alpha + \beta = \frac{1 - \sin \phi}{1 + \sin \phi} (\mu_b + \beta) \quad (2.5)$$

where α and β are the top and basal slope angles, respectively, ϕ is the angle of internal friction, and μ_b is the coefficient of basal friction. For typical values $\phi \approx 35^\circ$ ($\tan \phi \approx 0.7$) and $\beta \approx 3^\circ - 5^\circ$, the critical taper is $\alpha + \beta \approx 11^\circ - 12^\circ$ if the basal frictional coefficient is equal to that of the material within the wedge. The critical taper decreases to 6° if the angle of basal friction is about half of that of the material within the wedge.

The frictional critical taper model has gained a great deal of popularity because of its simple and concise expression relating the overall geometry of a supracrustal wedge to

material parameters which are in principle measurable (e.g., cohesion, coefficient of friction and pore fluid pressure). This model has been applied to several well-studied thrust-and-fold belts and the predicted and measured observables are in reasonable agreement (Davies et al., 1983; Dahlen, 1984; Dahlen et al., 1984; Zhao et al., 1986).

The overall dynamics of the plastic model and of the frictional model are similar. Both models describe a dynamically stable state: once the critical taper of the wedge is reached, the wedge is everywhere at failure. As a matter of fact, the similarity between these two models is also evident from the expression linking the basal shear stress to the gravitational stress associated with topographic slope and the material properties of the wedge

$$\tau_b = \rho gh\alpha + 2K_1\beta \quad (2.6a)$$

$$\tau_b = \rho gh\alpha + 2K_2(\alpha + \beta) \quad (2.6b)$$

The first of equations (2.6) refers to the plastic model, and the second to the frictional model. The parameter K_1 in the first equation represents the yield strength of the wedge, while the parameter K_2 in the second equation is a function of both the internal strength of the wedge and the ratio of pore-fluid to lithostatic pressure.

The frictional critical taper theory was first proposed by Davis et al. (1983), and since then its theoretical formulation has been modified to take into account the spatial variation in porosity, pore-fluid pressure, and other parameters (Zhao et al., 1986; Fletcher, 1989; Dahlen, 1990). One variation of this model simulates extensional accretionary wedges (Xiao et al., 1991). However, critical taper theory and its variations are only applicable to thin-skinned wedges (because of the assumed rheology) and predict a uniform stress state throughout the wedge. (Even for thin-skinned wedges, this model seems to be more applicable to accretionary prisms than to foreland fold-and-thrust belts

(Woodward, 1987)). They are therefore not applicable to those wedges with syn-convergence extension at the rear, as often observed in thick wedges.

Yin (1993) tried to overcome the second limitation of the critical taper model and derived an elastic Airy stress function solution. His model assumes frictional sliding along the base and a linear variation with depth of shear and normal tractions along the rear of the wedge. Using this model, the roles of basal friction, pore fluid pressure, and wedge configuration in controlling the mechanics of thrust wedges are evaluated. The model predicts that (1) relatively low basal frictional resistance leads to dominantly horizontal compression in the wedge, whereas relatively high basal frictional resistance leads to dominantly horizontal extension; (2) a long thrust wedge (>100 km) may show thrust faulting in its lower part and normal faulting in its upper part, even when a moderate to high compressive horizontal stress (100 MPa) is applied at the front of the wedge, whereas a short thrust wedge (< 10 km) can be entirely compressional under the same boundary conditions; (3) the topographic slope is less important than basal friction in affecting the deformational style.

The result on the relative importance of topographic slope and basal friction in producing extensional structures within compressional wedges is very interesting. However, this solution cannot be directly applied to thick orogenic wedges. First, the rheology of large, thick wedges is much more complex than linear elasticity: the bulk rheology of thick wedges is most likely to be power-law steady-state creep (as argued later in this chapter). Second, the spatial distribution of normal and thrust faults in the same wedge shows significant variations of deformation pattern in the longitudinal direction (compressive at the thin-skinned foreland end and extensional at the thick-skinned hinterland end) while Yin's results only show compression-extension variations in the vertical direction and not in the longitudinal.

Although it is only a semi-quantitative solution, Platt's model (Platt, 1986) is probably the most interesting attempt to understand the dynamic stability of orogenic wedges. After presenting detailed arguments about the complicated nature of the rheology of thick orogenic wedges, Platt concludes that their bulk rheology can be described by some general nonlinear relation $\dot{\epsilon} = f(\sigma)$ between the strain rate, $\dot{\epsilon}$, and some invariant of deviatoric stress, σ . Therefore, material will be unstable and flow (i.e., $\dot{\epsilon} \neq 0$) if the deviatoric stress is not equal to zero. Based upon this concept, Platt derived a stability criterion for two-dimensional plane strain thick orogenic wedges (*cf.* Figure 2.5)

$$\tau_b = \rho g h \alpha - 2 \tau_{xx} \theta - 2 \frac{\partial \tau_{xx}}{\partial x} h \quad (2.7)$$

where $\theta (= \alpha + \beta)$ is the taper of the wedge and $\tau_{xx} = \frac{1}{h} \int_0^h \sigma'_{xx} dy$ is the depth-averaged longitudinal deviatoric stress (h is the local thickness of the wedge and the coordinate system is as shown in Figure 2.5). All other parameters in equation (2.7) (e.g., ρ , g , h , α and τ_b) are identical to those used in previous equations. Equation (2.7) states that the shear traction at the base of the wedge is balanced by three terms. The first is the familiar gravitational term due to the surface slope. The second is the result of the depth-averaged longitudinal deviatoric stress acting in a tapered wedge: because of the progressive change in cross-sectional area, this produces a net longitudinal force. The third term results from the gradient $\partial \tau_{xx} / \partial x$ in the longitudinal stress, which also produces a net force in the longitudinal direction.

The condition for stability is that the longitudinal strain rate $\dot{\epsilon}_{xx} = 0$. For a viscous material with rheology of the type $\dot{\epsilon}_{xx} = f(\tau_{xx})$, this implies that both τ_{xx} and $\partial \tau_{xx} / \partial x$ are zero. Consequently, equation (2.7) reduces to a stability criterion identical to that in the gravity spreading model (see equation (2.2))

$$\alpha = \frac{\tau_b}{\rho gh} \quad (2.8)$$

The role of surface slope α to maintain a critical state is apparent from equation (2.7). A value of α that is too small leads to horizontal compression and shortening in the wedge, while a value of α that is too large leads to extension of the wedge. If α is too small, the gravitational term in equation (2.7) is too small, so that either or both terms in τ_{xx} must be negative. Because τ_{xx} is likely to be zero or negative (compressional) at the front of the wedge, a negative $\partial\tau_{xx}/\partial x$ leads to a negative τ_{xx} throughout the wedge. Conversely, if α is too large, equation (2.7) requires that one or both terms in τ_{xx} be positive. The depth-averaged deviatoric stress towards the wedge front is limited by the strength of the material, and positive values of $\partial\tau_{xx}/\partial x$ will tend to lead to positive values of τ_{xx} away from the front of the wedge, resulting in extension and thereby reducing α . The surface slope of wedge is also affected by material fluxes at the front, basement and surface of the wedge. Frontal accretion tends to lengthen the wedge, and consequently α will be low in the frontal region, which will therefore be in compression. Underplating, on the other hand, increases the surface slope α at the back of the wedge and consequently induces extension in the wedge (*cf.* Platt, 1986).

Two factors have limited further applications of Platt's model:

(1) The depth-averaged longitudinal deviatoric stress τ_{xx} is a good approximation to the state of stress when the thickness of wedge is not too large (and therefore the longitudinal deviatoric stress σ'_{xx} does not vary significantly in the y -direction, that is, σ'_{xx} does not change much between the base $y=0$ and the surface $y=h$). For the thick-skinned part of a wedge, it is likely that the state of stress will be quite different at various depths, implying that a depth-averaged parameter such as τ_{xx} may not be an acceptable approximation for the analysis of the effects of a change in topographic slope on the internal deformation of the wedge.

(2) Because the bulk rheology of the wedge is left unspecified in Platt's model, it is impossible to make quantitative predictions from equation (2.7) on the dynamics of the wedge, including stress and velocity fields which affects the exhumation of high-pressure metamorphic rocks often observed in thick orogenic wedges.

The need for a quantitative description capable of accommodating the nonlinear nature of the bulk rheology of thick-skinned wedges has lead to a search for solutions derived in other disciplines for materials with nonlinear rheologies. One such solution which is of great interest to the dynamical analysis of orogenic wedges is a classic glacier flow solution (Nye, 1957). There are basically two reasons making this classic solution attractive: (1) Despite some important differences, there are obvious similarities between the movement of glaciers and ice sheets and that of nappes and thrust sheets (Elliott, 1976). All are wide bodies that are displaced for large distances over subhorizontal surfaces. In being displaced they become internally deformed and high shear strains are developed at their bases (Hudleston, 1992); (2) Nye's solution is for the flow of an ice sheet with power-law bulk rheology, formally analogous to the most commonly observed high-temperature rheology of rocks. The formulation does not specify *a priori* the stress exponent n for the sheet material, so it is sufficiently general to accommodate a wide range of deformation patterns which might be applicable to thick orogenic wedges: linear Newtonian flow ($n=1$), perfectly plasticity ($n \rightarrow \infty$), and power-law steady-state creep ($n = 2 \sim 4$).

In Nye's treatment, a uniform sheet with planar and parallel top and bottom surfaces flows in response to surface forces acting at its trailing edge and body forces due to its sloping upper surface (Figure 2.7). The solution is based on the following assumptions: (1) the material is incompressible and flows in a plane strain pattern; (2) stress and strain rate are linked by a power-law relationship; (3) the stress tensor is function of depth only (i.e., no longitudinal variations); (4) the vertical velocity component v at any given depth

Figure 2.7 Notation and positive sense of stress components in the ice sheet model (from Nye 1957).

does not vary with longitudinal position, that is, $\partial v / \partial x = 0$. In other words, Nye's is a one-dimensional solution for power-law rheology.

A detailed discussion of Nye's solution for stress, strain rate and velocity will be found in Chapter 5. Shown in the followings are some depth-dependent profiles of dimensionless stress, velocity and displacement from Nye's solutions for a block with power-law relation between strain rate $\dot{\epsilon}$ and corresponding stress deviatoric component σ' (i.e., $\dot{\epsilon} = (\sigma'/A)^n$). Figure 2.8a shows the depth distribution of the second invariant of stress for sheets with different power-law stress exponent n . If parameter A remains unchanged, the stress distributions for Newtonian viscous ($n=1$) and almost plastic ($n=10$) sheets bracket the range of stress intensity with depth. Velocity profiles for laminar flows with different power-law stress exponents n are plotted in Figure 2.8b. The effect of n on the deformation of sheets with laminar flow can also be presented in terms of displacement profiles as given in Figure 2.8c.

Nye's solution is applicable to sheets with uniform shortening or extending flows. It is not applicable, however, to sheets with a continuous variation of longitudinal strain rate, because of its one-dimensional assumption for stress which requires the longitudinal strain rate to be constant.

All the models mentioned above give analytical solutions. Numerical models have also been proposed to understand the complexities of foreland thrust belts and orogenic wedges. Some numerical models are briefly reviewed here.

Critical taper models describe wedges in a state of dynamic equilibrium, but do not explain how wedges deform internally to gain or regain their critical taper after equilibrium is disturbed by a change in boundary conditions. Willett (1992) presents a finite-element model of Coulomb wedges in order to simulate the growth of accretionary prisms and foreland fold and thrust belts. The constitutive behaviour adopted in this

Figure 2.8 Depth distributions of dimensionless stress, velocity, and displacement from Nye's one-dimensional theory for different stress exponent n . (a) Second invariant of stress tensor (T); (b) longitudinal velocity (U); (c) longitudinal displacement for laminar flows (each sequence of curves from left to right represents the successive shapes taken by initially straight lines ; Y is dimensionless depth) ((a) and (b) from Nye, 1957; (c) from Wojtal, 1992).

model is *Coulomb plastic rheology*, a variation of perfectly plastic rheology in which the von Mises yield strength is replaced by a scalar parameter derived from the non-cohesive Coulomb frictional criterion. The model demonstrates the self-similar growth of the critical Coulomb wedge and investigates the kinematics of wedges growing by frontal accretion. Changes in boundary conditions (e.g., a change in the basal friction) change the wedge geometry and thus may permit out of sequence thrusting (increase in basal friction) or extensional collapse of parts of the wedge while in compression (decrease in basal friction). This numerical model provides a good quantitative explanation for the formation of syn-convergence extension, even though the bulk rheology adopted is not very appropriate for thick-skinned orogenic wedges.

Beaumont et al. (1992a) describe a finite-element model of the evolution of the Southern Alps of New Zealand. In this model a rigid-plastic rheology is used to model brittle behaviour for the major part of the orogen and plastic-viscous rheology for the shear zone at the bottom of the orogen. This model incorporates tectonic and erosional data, and demonstrates that orogenic structure and metamorphic style may be controlled by the fluvial erosion and climate on the western flank of Southern Alps.

A similar finite-element model was used by Brown et al. (1993) to study the formation of coeval conjugate folds and thrusts observed in the southern Canadian Cordillera (the 'Selkirk fan structure'). This model demonstrates that a uniform Coulomb rheology layer deforming under basal boundary conditions of asymmetric detachment and underthrusting of the underlying substratum develops a structural fan over the leading edge of the detachment. The deforming layer has a cohesionless Coulomb yield criterion followed by a viscous post-yield flow. Results from this numerical model are similar to a sandbox experiment performed by Malavieille (1984). Similar mechanical models have also been applied to the analysis of other compressional orogens (Beaumont et al., 1992b; Willett et al., 1993; Beaumont and Quinlan, 1994).

Recent developments in both numerical and analog models of orogenic wedges are reviewed in detail in McClay (1992).

2.5 Rheology of orogenic wedges

The fundamental difference between thin-skinned and thick-skinned orogenic wedges lies in the rheology: the rock material of supracrustal thrust belts and accretionary prisms is near the surface and consequently behaves in an elastic and brittle (frictional) fashion, while thick-skinned orogenic wedges with thickness large enough to induce metamorphism are unlikely to exhibit either plastic or Coulomb-type elastic-brittle rheologies throughout their volumes. The rheology applicable to the deeper parts of thick-skinned orogenic wedges is most likely to be power-law steady-state creep.

From the rheological viewpoint, the following observations can be made:

(1) Below depths of about 10 - 20 km in the wedge, the material is probably capable of deforming at very low deviatoric stresses because of (a) the importance of pressure-solution processes in deforming metasedimentary rocks (at temperature between 450 and 800 K), and (b) several softening mechanisms that are likely to act cooperatively at depth. Increasing temperature with depth reduces effective viscosities for all ductile deformation mechanisms, including the most common power-law creep (controlled by dislocation recovery). Metamorphic dehydration reactions in clay-rich sediments yield abundant water which reduces plastic yield strength in quartz by an order of magnitude (Blacic, 1975), facilitates diffusional creep (Rutter, 1976, 1983), favours subcritical corrosion cracking (Atkinson, 1980), and lowers the stress needed for brittle fracture (Etheridge, 1983). The evidence suggests that rocks at temperatures in the range of 600 - 800 K can deform at significant rates ($\geq 10^{-14}$ /sec) under differential stresses of 10 MPa (Rutter, 1976).

(2) If the wedge is subjected to higher stresses, deformation will not become unconstrained (as in a perfect plastic or Coulomb material) but will simply occur at a higher rate. This can be described as general (nonlinear) viscoelastic behavior, for which the concept of a discrete, long-term yield-strength is meaningless. The material will deform, although slowly, under any deviatoric stress, however low. On the other hand, the material can “support” high stresses because of its viscous resistance to a high strain-rate.

Taking these considerations into account, a steady-state power-law creep rheology of the type

$$\dot{\epsilon} = A_0 \sigma^n \exp\left(-\frac{Q}{RT}\right) \quad (2.9)$$

is adopted in this thesis as a first approximation to study the stress and deformation in orogenic wedges. In equation (2.9), A_0 is an empirically determined ‘constant’, n is the stress exponent, Q is the activation energy, R is the universal gas constant, and T is absolute temperature. Equation (2.9) fits very well the empirical steady-state creep for most materials at absolute temperatures above one-third to one-half of the melting point (Ranalli, 1995). The advantage of using a power-law constitutive equation such as (2.9) is that a broad spectrum of deformation processes can be formulated by using the same formal equation. For example, $n=1$ in equation (2.9) represents a linear Newtonian viscous flow while a very large value of n simulates a plastic material. Therefore, the rheology of an orogenic wedge can be formulated by a uniform equation even though the bulk rheology of the wedge varies at different depths. On the other hand, the nonlinear nature of power-law creep rheology ($n>1$) makes analytical solutions very difficult to find.

Another major hurdle in devising quantitative models for orogenic wedges (both thin- and thick-skinned) is that the longitudinal deformation pattern in a wedge often varies significantly from the thin-skinned foreland to the thick-skinned hinterland. Large-scale extension in compressional orogens, not only post-compressional but also syn-compressional, is common (Platt, 1986; Dewey, 1988; Twiss and Moores, 1992; Burchfiel et al. 1992; Gottschalk and Oldow, 1988; Oldow et al., 1993; Yin, 1993). Associated closely with the syn-convergent extension at the rear of an orogenic wedge is the exhumation of large volumes of high-pressure metamorphic rocks.

As mentioned above, extensional structures may develop not only concurrently with compressive tectonic processes but also after the convergent process ends. This post-shortening extension can be caused by lateral variations in thickness and/or density within plates and by lithospheric delamination (Dalmayrac and Molnar, 1981; Artyushkov, 1983; Molnar and Lyon-Caen, 1988; Bott, 1993; Ranalli, 1995).

In conclusion, the popular 'critical taper' theories (including both plastic and Coulomb-type frictional models) are not directly applicable to thick-skinned orogenic wedges with respect to both rheology and style of deformation. In terms of bulk rheology, orogenic wedges are dominated by steady-state power-law creep instead of simple homogeneous frictional brittle deformation. In terms of deformation style, orogenic wedges often show longitudinal differences between the compressive front (characterized by significant thrust faults) and the extensional rear. Elastic models (Yin, 1993) are useful to understand the mechanical conditions for the initiation of normal and/or thrust faults in thrust wedges, but the constitutive equation adopted prevents their application to thick-skinned wedges. Nye's (1957) classic theory for glaciers is able to accommodate nonlinear (power-law) rheology, but is one-dimensional (quantities vary only in the vertical direction), and consequently is not capable of simulating the variation of deformation in the longitudinal direction along a wedge.

Platt's (1986, 1993) recognition of the very complicated nonlinear nature of rheology in thick-skinned orogenic wedges and his explanation of dynamic stability in terms of depth-averaged longitudinal deviatoric normal stress (which is related to the longitudinal strain rate by an unspecified creep law) is probably the most interesting and promising attempt to understand the processes at work during the evolution of orogenic belts (e.g., frontal accretion, underplating, exhumation of high-pressure metamorphic rocks, and the coeval development of extensional and compressional tectonics). Platt's model, however, is qualitative and therefore does not allow computation of stresses and deformation fields.

It follows from the previous considerations that a *quantitative description of thick orogenic wedges in terms of a general nonlinear bulk rheology* would be very useful. However, it is also obvious that the complexities of real wedges effectively preclude their detailed modelling. However, a simplified (e.g., homogeneous, isotropic) model would be an important step forward. The major difficulty in obtaining an explicit solution for stress (σ), strain rate ($\dot{\epsilon}$), and velocity fields (v), from the mathematical point of view, is the nonlinear power-law relation between the deviatoric stresses and corresponding strain rates. The search for such an analytical solution is the main motivation of this thesis, whose objective is to provide closed-form, analytical solutions simulating the stress and deformation of orogenic wedges at variable scales. On the scale of a single thrust, a static elastic approach, by the use of the Airy stress function, is used to analyze the development of an overthrust fault and the movement of individual thrust sheets. On the scale of a whole orogenic wedge, a generalized steady-state power-law rheology is incorporated in the modelling, and an explicit analytical solution for stress, strain rate and velocity fields in orogenic wedges is derived.

Chapter 3

Stresses in An Overthrust Sheet and Propagation of Thrusting Along Its Base⁽¹⁾

3.1 Introduction

Orogenic wedge modelling can basically be done on two different scales: on the scale of the whole wedge, where the material deforms as a single, mechanically continuous unit; on the scale of individual thrust sheets, where deformation occurs mainly by relative displacement along thrust faults. In this chapter, attention is focused on the stress and deformation within a single wedge-shaped thrust block and along its base. In particular, an effort is made to improve our understanding of one of the most debated questions in modelling the movement of thrust sheets, that is, the so-called *mechanical paradox of overthrusting*.

The mechanical paradox of overthrusting originates with the obvious difference between the geological observation of large overthrust sheets and the theoretical estimation of the maximum length of an overthrust block that may be displaced along a thrust fault. This theoretical value is determined by using the Coulomb-Navier criterion to compare the strength of the rocks with the stress required to overcome the total frictional resistance to slip (*cf.* Smoluchowski, 1909). In his analysis, Smoluchowski took the coefficient of friction $\mu_0 = 0.15$ (equal to that of iron sliding on iron) and inferred that a 100-km-long block would require a stress far greater than any rock would sustain.

⁽¹⁾ This chapter, with minor modifications, has been published as:
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Since a value of 0.15 is less than the coefficient of friction for most rocks (*cf.* Jaeger and Cook, 1969; Byerlee, 1978; Ranalli, 1995), a larger but more unrealistic stress will result if the coefficient of friction for rocks is taken into consideration (see equation 2.1).

To overcome this conceptual difficulty, a variety of attempts have been made (for a brief review, see Suppe, 1985). For instance, Hubbert and Rubey (1959) considered the possible existence of a layer of high pore fluid pressure to reduce the frictional resistance to sliding. The importance of the cohesive strength of the rocks was further studied by Birch (1961) and Hsü (1969). On the basis of the observation that the total displacement due to earthquake faulting and fault creep clearly varies from place to place over a thrust fault, Price (1973b, 1988) argued that in all mechanical models of overthrusting there is an unstated and untenable assumption that displacement occurs simultaneously over the entire fault surface. This assumption is demonstrably fallacious when applied to seismic slip (De Bremaecker, 1987). Both Price and De Bremaecker concluded that, instead of simultaneous displacement along the thrust fault, a “dislocation-type motion” may be a more reasonable mechanism able to overcome the mechanical paradox. The displacement is achieved by slip episodes that nucleate at a given point on the thrust surface and propagate with finite velocity. The total displacement of the thrust sheet is therefore the resultant of several small consecutive slips, each requiring a force much less than the estimations based on the assumption of simultaneous slippage.

One of the implications of the “dislocation” model is that the state of stress varies from place to place along the thrust, contrary to what is assumed in the “classical” models. Naturally the frictional strength may vary with position too, but in this chapter the focus is on variations of the state of stress. Its variation on the thrust surface is caused by two reasons: statically, the shear and normal stresses are not necessarily constant but depend on the external force system and the model geometry; dynamically, the propagation of the dislocation modifies the stress field (*cf.* Eshelby, 1973; Savage, 1980).

The static inhomogeneous stress state plays a key role in triggering the “dislocation” motion of the entire thrust fault, since slip will occur only where the strength is exceeded.

This chapter investigates the static “initial” conditions of thrusting, that is, the problem of obtaining stress distributions on the thrust plane that are compatible with the initiation of progressive slip. A thrust sheet is modeled as a two-dimensional wedge-shaped block of elastic-brittle rocks, with compressive boundary conditions on both rear and front edges (see Figure 3.1). The assumption of linear elasticity (at stresses below the strength) is of course an approximation, but it is not unrealistic at shallow crustal levels ($\leq 10\text{--}15$ km; see Ranalli, 1995). The model, consequently, applies to thin-skinned thrust sheets but not necessarily to thick (basement) thrusts. The truncated wedge geometry allows derivation for both rectangular and triangular shapes as particular cases. The contact plane between the thrust sheet and the underlying basement is a surface along which displacement may occur. In this sense, the model is different from both the Coulomb wedge model (see, for example, Davis et al., 1983; Dahlen and Suppe, 1988), which considers failure within the wedge, and models where the bottom of the block is taken as fixed or coupled with the underlying basement (see, for example, Hafner, 1951; Mandl and Shippam, 1981; Mandl, 1988; Yin, 1989). The problem is solved by means of Airy stress functions to derive the stress state within the wedge, from which the state of stress along the dipping lower boundary of the sheet can be determined. Airy stress functions have been previously applied to elastic wedges by Yin (1986, 1988, 1993). In the present approach, however, the state of stress on the basal thrust is not predetermined by a frictional law but is a consequence of boundary conditions and of the shape of the wedge. In other words, the solution provided in this chapter is for a model with “incompletely-prescribed boundary conditions” (*cf.* Timoshenko and Goodier, 1970).

The occurrence of displacement along a failure plane is governed by the Coulomb-Navier fracture criterion (*cf.* Jaeger and Cook, 1969)

$$\tau_n = S_0 + \mu_0 \sigma_n \quad (3.1)$$

in which there are two parameters (cohesive strength S_0 and coefficient of friction μ_0) representing the mechanical properties of the rock and two components of stress (normal stress σ_n , corrected for pore fluid pressure if necessary, and shear stress τ_n) describing the state of stress along the failure surface. The solution presented in this chapter shows that the state of stress along the basal thrust fault varies with position if (1) the slopes of the upper and lower boundaries of the thrust sheet are not zero, and/or (2) the horizontal compressional driving forces exerted on the vertical boundaries vary nonlinearly with depth rather than linearly as assumed by Hafner (1951), Hubbert and Rubey (1959), Mandl (1988), and Yin (1989). Although the second contribution is hypothetical, the first is certainly satisfied in most thrust sheets. The results provide some theoretical support for the “dislocation” model for overthrust faults, despite being obtained from simple static considerations.

3.2 State of stress within the thrust sheet

The geometry of the model is shown in Figure 3.1. The origin of x and y coordinates is set at the upper left corner of the block, whose length and rear thickness are L and H . The slopes of the upper free surface and of the lower thrust fault are α and β , respectively (both positive when the thickness of the wedge decreases toward the front).

Engineering stress sign convention is adopted in this thesis and therefore for the two-dimensional case σ_1 and σ_2 represent the minimum and maximum compression, respectively.

Figure 3.1 Geometry of wedge-shaped thrust sheet under horizontal compression. The coordinate origin is set at the upper left corner. L and H are the horizontal length and rear thickness of the block; α and β the slopes of the upper and lower boundaries.

The horizontal compressional forces on both rear and front vertical edges are taken in the general case to be parabolic functions of depth (which includes linearity as a special case), so that the boundary conditions are

$$\begin{aligned} F_x^{OC} &= ay^2 + by + c \\ F_x^{AB} &= a_1y^2 + b_1y + c_1 \\ F_x^{OA} &= F_y^{OA} = 0 \end{aligned} \quad (3.2)$$

where the superscripts OC, AB, and OA refer to the boundary planes on which the forces are applied. The third equation represents conditions on the free surface, where $y = n x$, with $n = \tan \alpha$. The parameters a, a_1, b, b_1, c , and c_1 are taken as positive when acting in the positive x direction. The force (and consequent stress) at the base of the block is not prescribed in advance but is a consequence of the horizontal boundary forces and the shape of the block. According to Kirchhoff's theorem (*cf.* Fung, 1965), the solution is therefore not unique, since the boundary conditions are not fully prescribed. However, it is acceptable, in the sense that it satisfies compatibility of strain (through the biharmonic equation) and the equilibrium conditions. (Solutions for incomplete boundary conditions are not uncommon in elastostatics; see, for example, Fung (1965) and Timoshenko and Goodier (1970).)

The relation between boundary forces and internal stress components can be expressed in terms of the Airy stress function (*cf.* Fung, 1965).

$$\begin{aligned} F_x &= \frac{\partial^2 \Phi}{\partial y^2} \cos \theta - \frac{\partial^2 \Phi}{\partial x \partial y} \sin \theta \\ F_y &= -\frac{\partial^2 \Phi}{\partial x \partial y} \cos \theta + \left(\frac{\partial^2 \Phi}{\partial x^2} - \rho g y \right) \sin \theta \end{aligned} \quad (3.3a)$$

where F_x, F_y are the boundary forces applied in the x and y directions, respectively; $\Phi(x, y)$ is the stress function; θ is the angle between the normal to a given boundary and

the positive x direction, and ρ and g are density and gravity, respectively. The stress function is related to the stress components as

$$\begin{aligned}\sigma_{xx} &= \frac{\partial^2 \Phi}{\partial y^2} \\ \sigma_{yy} &= \frac{\partial^2 \Phi}{\partial x^2} - \rho g y \\ \sigma_{xy} &= -\frac{\partial^2 \Phi}{\partial x \partial y}\end{aligned}\tag{3.3b}$$

where σ_{xx} , σ_{yy} , and σ_{xy} are the three stress components.

Because of the general understanding about the state of stress within the upper crust and the stress boundary conditions applied at the front and at the back of the thrust sheet, it is not likely that the highest order of the polynomial expression for the stress field, with respect to x or y , will exceed three. Without losing generality, the stress function $\Phi(x, y)$ can be assumed to be of the form

$$\begin{aligned}\Phi(x, y) &= \sum_{i=1}^4 k_i x^{i+1} + y \sum_{i=5}^9 k_i x^{i-4} + y^2 \sum_{i=10}^{15} k_i x^{i-10} + y^3 \sum_{i=6}^{21} k_i x^{i-16} \\ &\quad + y^4 \sum_{i=22}^{27} k_i x^{i-22} + y^5 \sum_{i=28}^{33} k_i x^{i-28}\end{aligned}\tag{3.4}$$

where $k_i (i = 1 \text{ to } 33)$ are arbitrary constants to be determined from the boundary conditions and the requirement that $\nabla^4 \Phi \equiv (\partial^4 / \partial x^4 + 2 \partial^4 / \partial x^2 \partial y^2 + \partial^4 / \partial y^4) \Phi = 0$. The latter leads to

$$\begin{aligned}3k_3 + k_{12} + 3k_{22} &= 0 \\ 5k_4 + k_{13} + k_{23} &= 0 \\ k_8 + k_{18} + 5k_{28} &= 0 \\ 5k_9 + 3k_{19} + 5k_{29} &= 0 \\ k_{14} = k_{15} = k_{20} = k_{21} = k_{24} = k_{25} = k_{26} = k_{27} = k_{30} = k_{31} = k_{32} = k_{33} &= 0\end{aligned}\tag{3.5}$$

By combining equations (3.2), (3.3), and (3.4), the condition for traction-free upper surface can be expressed as

$$\begin{aligned}
5n^4k_{29} + 3n^2k_{19} + k_9 &= 0 \\
5n^4k_{28} + 4n^3k_{23} + 3n^2k_{18} + 2nk_{13} + k_8 &= 0 \\
4n^3k_{22} + 3n^2k_{17} + 2nk_{12} + k_7 &= 0 \\
3n^2k_{16} + 2nk_{11} + k_6 &= 0 \\
2nk_{10} + k_5 &= 0
\end{aligned} \tag{3.6a}$$

and

$$\begin{aligned}
n^4k_{29} + 3n^2k_{19} + 5k_9 &= 0 \\
n^4k_{23} + 2n^3k_{18} + 3n^2k_{13} + 4nk_8 + 5k_4 &= 0 \\
n^3k_{17} + 2n^2k_{12} + 3nk_7 + 4k_3 &= 0 \\
n^2k_{11} + 2nk_6 + 3k_2 &= \frac{n}{2}\rho g \\
nk_5 + 2k_1 &= 0
\end{aligned} \tag{3.6b}$$

Equations (3.6a, b), combined with equations (3.2) and (3.3), lead to the determination of the remaining arbitrary constants

$$\begin{aligned}
k_1 &= -\frac{n^2}{2}c \\
k_2 &= \frac{n}{6}\rho g - \frac{n^2t_3}{4t_1}aL + \frac{n^2t_6}{12t_1t_4}(a+a_1)L - \frac{n^3}{3}b + \frac{n^5}{t_1}(b+b_1) + \frac{n^2}{2L}(c+c_1) \\
k_3 &= \frac{n^2t_2}{4t_1}a + \frac{n^4t_5}{3t_1t_4}(a+a_1) + \frac{n^3}{3t_1}\frac{b+b_1}{L} \\
k_4 &= -\frac{n^2t_2^2}{12t_4}\frac{a+a_1}{L} \\
k_5 &= nc \\
k_6 &= \frac{nt_3}{2t_1}aL - \frac{nt_6}{6t_1t_4}(a+a_1)L + \frac{n^2}{2}b - \frac{2n^4}{t_1}(b+b_1) - \frac{n}{L}(c+c_1) \\
k_7 &= -\frac{n(2t_2+t_3)}{6t_1}a - \frac{n^3t_2t_5}{2t_1t_4}(a+a_1) - \frac{n^2t_2}{2t_1}\frac{b+b_1}{L}
\end{aligned}$$

$$\begin{aligned}
k_8 &= \frac{nt_5}{6t_4} \frac{a+a_1}{L} \\
k_9 &= 0 \\
k_{10} &= -\frac{c}{2} \\
k_{11} &= -\frac{t_3}{4t_1} aL + \frac{t_6}{12t_1t_4} (a+a_1)L + \frac{n^3}{t_1} (b+b_1) + \frac{c+c_1}{2L} \\
k_{12} &= -\frac{t_3}{4t_1} a - \frac{n^4t_5}{t_1t_4} (a+a_1) - \frac{n^3}{t_1} \frac{b+b_1}{L} \\
k_{13} &= -\frac{t_2t_5+8n^4}{12t_4} \frac{a+a_1}{L} \\
k_{16} &= -\frac{b}{6} \\
k_{17} &= \frac{nt_5}{6t_4} (a+a_1) + \frac{b+b_1}{6L} \\
k_{18} &= -\frac{nt_5}{6t_4} \frac{a+a_1}{L} \\
k_{19} &= 0 \\
k_{22} &= -\frac{a}{12} \\
k_{23} &= -\frac{a+a_1}{12L} \\
k_{28} &= k_{29} = 0
\end{aligned} \tag{3.7}$$

In equations (3.7), $t_1 = 1 + 3n^2$, $t_2 = 1 - n^2$, $t_3 = 1 + 3n^4$, $t_4 = 1 + 2n^2 + 5n^4$, $t_5 = 1 - 2n^2 + 5n^4$ and $t_6 = 1 + 18n^4 + 16n^6 + 45n^8$.

Recalling equations (3.3), the external forces on rear and front vertical edges and the resulting force on the bottom surface can be written as

$$\begin{aligned}
F_x^{OC} &= ay^2 + by + c \\
F_y^{OC} &= k_5 + 2k_{11}y + 3k_{17}y^2 + 4k_{23}y^3 \\
F_x^{AB} &= a_1y^2 + b_1y + c_1 \\
F_y^{AB} &= -(k_5 + 2Lk_6 + 3L^2k_7 + 4L^3k_8) - 2(k_{11} + 2Lk_{12} + 3L^2k_{13})y \\
&\quad - 3(k_{17} + 2Lk_{18})y^2 - 4k_{23}y^3 \\
F_x^{BC} &= M_0 + M_1x + M_2x^2 + M_3x^3 \\
F_y^{BC} &= N_0 + N_1x + N_2x^2 + N_3x^3
\end{aligned} \tag{3.8}$$

where $m = \tan \beta$, $w = (1 + m^2)^{-1/2}$ and the parameters M_i, N_i ($i = 0$ to 3) are given by

$$\begin{aligned}
M_0 &= w(-k_5 + 2mk_{10} - 2Hk_{11} + 6Hmk_{16} - 3H^2k_{17} + 12H^2mk_{22} - 4H^3k_{23}) \\
M_1 &= 2w(-k_6 + 2mk_{11} - 2Hk_{12} - 3m^2k_{16} + 6Hmk_{17} - 3H^2k_{18} \\
&\quad - 12Hm^2k_{22} + 12H^2mk_{23}) \\
M_2 &= 3w(-k_7 + 2mk_{12} - 2Hk_{13} - 3m^2k_{17} + 6Hmk_{18} + 4m^3k_{22} - 12Hm^2k_{23}) \\
M_3 &= 4w(-k_8 + 2mk_{13} - 3m^2k_{18} + 4m^3k_{23}) \\
N_0 &= w(2k_1 - mk_5 + 2Hk_6 - 2Hmk_{11} + 2H^2k_{12} - 3H^2mk_{17} + 2H^3k_{18} \\
&\quad - 4H^3mk_{23} - \rho gH) \\
N_1 &= 2w(3k_2 - 2mk_6 + 3Hk_7 + m^2k_{11} - 4Hmk_{12} + 3H^2k_{13} + 2Hm^2k_{17} \\
&\quad - 6H^2mk_{18} + 6H^2m^2k_{23} + \frac{m\rho g}{2}) \\
N_2 &= 3w(4k_3 - 3mk_7 + 4Hk_8 + 2m^2k_{12} - 6Hmk_{13} - m^3k_{17} + 6Hm^2k_{18} - 4Hm^3k_{23}) \\
N_3 &= 4w(5k_4 - 4mk_8 + 3m^2k_{13} - 2m^3k_{18} + m^4k_{23})
\end{aligned} \tag{3.9}$$

Equations (3.8) can be proven to satisfy the resulting force and moment equilibria.

The constants k_i can be subdivided into two sets, only one of which is dependent on the upper slope n (see equation (3.7)). Consequently, the stresses can be written as

$$\begin{aligned}
\sigma_{xx} &= \sigma'_{xx} + \sigma''_{xx} \\
\sigma_{yy} &= \sigma'_{yy} + \sigma''_{yy} \\
\sigma_{xy} &= \sigma'_{xy} + \sigma''_{xy}
\end{aligned} \tag{3.10}$$

where a prime denotes the components independent of the upper (and also the lower) slope (that is, the components of the state of stress in a rectangular block of length L and thickness H subject to the same horizontal external forces). The double prime denotes the components that are dependent on the slope n , i.e., the part of the stress field resulting from the effect of the wedge-shaped geometry.

A concise approximate expression for the stress solution can be obtained by neglecting individual terms that constitute $< 1\%$ of the value of any given k_i in equation

(3.7) for reasonable parameter ranges ($\alpha, \beta \leq 6^\circ$, and $L/H \geq 10$; cf. Boyer and Elliott, 1982; Davis et al., 1983; Suppe, 1985). The nonlinear parts are chosen in such way that shear stresses are realistic (i.e., $\tau_n \leq 50$ MPa) and deviations of the vertical stress from lithostatic values are minor (see discussion at the end of this section).

Using the above simplifications and equation (3.3b), the stress components can be written as

$$\begin{aligned}\sigma'_{xx} &= \frac{a+a_1}{6L}x^3 + \frac{a}{2}x^2 + \left(\frac{c+c_1}{L} - \frac{2a-a_1}{6}L\right)x + \frac{b+b_1}{L}xy + \frac{a+a_1}{L}xy^2 - ay^2 - by - c \\ \sigma'_{yy} &= -\frac{a+a_1}{2L}xy^2 + \frac{a}{2}y^2 - \rho gy \\ \sigma'_{xy} &= \frac{a+a_1}{2L}x^2y - axy - \frac{a+a_1}{3L}y^3 - \frac{b+b_1}{2L}y^2 - \left(\frac{c+c_1}{L} - \frac{2a-a_1}{6}L\right)y\end{aligned}\quad (3.11a)$$

and

$$\begin{aligned}\sigma''_{xx} &= n \left[-\frac{a+a_1}{L}x^2y + (a+a_1)xy \right] \\ \sigma''_{yy} &= n \left[2\frac{a+a_1}{L}x^2y - 3axy + \rho gx - \frac{a+a_1}{3L}y^3 - \left(2\frac{c+c_1}{L} - \frac{2a-a_1}{3}L\right)y \right] \\ &\quad + n^2 \left[-\frac{5(a+a_1)}{3L}x^3 + 3ax^2 + \left(3\frac{c+c_1}{L} - \frac{2a-a_1}{2}L\right)x - 3\frac{b+b_1}{L}xy + by - c \right] \\ &\quad + n^3 \frac{4(b+b_1)}{L}x^2 \\ \sigma''_{xy} &= n \left[-\frac{2(a+a_1)}{3L}x^3 + \frac{3a}{2}x^2 + \left(2\frac{c+c_1}{L} - \frac{2a-a_1}{3}L\right)x + \frac{a+a_1}{L}xy^2 - \frac{a+a_1}{2}y^2 - c \right] \\ &\quad + n^2 \left[\frac{3(b+b_1)}{2L}x^2 - bx \right]\end{aligned}\quad (3.11b)$$

Although the slope of the basal surface does not appear explicitly in the stress solution, the thickness of the frontal edge is determined by the slopes of both upper and

lower boundaries, that is, $H' = H - (n + m)L$, and the frontal buttressing force is in fact affected by the lower boundary geometry.

The case of a triangular wedge pushed from the rear can be derived from the general case for vanishing frontal edge thickness. In this case, $a_1 = b_1 = c_1 = 0$ and $(m + n)L = H$. Denoting the stresses in the triangular wedge by τ and adopting the same notation as in equations (3.10), the stress components are

$$\begin{aligned}\tau'_{xx} &= \frac{a}{6L}x^3 + \frac{a}{2}x^2 + \left(\frac{c}{L} - \frac{aL}{3}\right)x + \frac{b}{L}xy + \frac{a}{L}xy^2 - ay^2 - by - c \\ \tau'_{yy} &= -\frac{a}{2L}xy^2 + \frac{a}{2}y^2 - \rho gy \\ \tau'_{xy} &= \frac{a}{2L}x^2y - axy - \frac{a}{3L}y^3 - \frac{b}{2L}y^2 - \left(\frac{c}{L} - \frac{aL}{3}\right)y\end{aligned}\tag{3.12a}$$

and

$$\begin{aligned}\tau''_{xx} &= n \left[-\frac{a}{L}x^2y + axy \right] \\ \tau''_{yy} &= n \left[\frac{2a}{L}x^2y - 3axy + \rho gx - \frac{a}{3L}y^3 - \left(\frac{2c}{L} - \frac{2aL}{3}\right)y \right] \\ &\quad + n^2 \left[-\frac{5a}{3L}x^3 + 3ax^2 + \left(\frac{3c}{L} - aL\right)x - \frac{3b}{L}xy + by - c \right] + n^3 \frac{4b}{L}x^2 \\ \tau''_{xy} &= n \left[-\frac{2a}{3L}x^3 + \frac{3a}{2}x^2 + \left(\frac{2c}{L} - \frac{2aL}{3}\right)x + \frac{a}{L}xy^2 - \frac{a}{2}y^2 - c \right] + n^2 \left[\frac{3b}{2L}x^2 - bx \right]\end{aligned}\tag{3.12b}$$

This stress solution may be applied to the determination of the internal state of stress and the stress along the basal thrust in thin-skinned wedges of approximately triangular cross section.

The principal stress trajectories within the thrust sheet are plotted in Figure 3.2 for the parameters given in Tables 3.1 and 3.2. The trajectories of maximum shear stress are

compatible with the upwardly concave secondary thrusts commonly observed in thrust belts.

Table 3.1 Values of model parameters for rectangular block with dimensions $L = 10$ km, $H = 1$ km and density $\rho = 2800 \text{ kg/m}^3$ subject to nonlinear boundary forces.

Parameters	Case*			
	A	B	C	D
$a \text{ (MPa/km}^2\text{)}$	-0.8	-2.0	-2.0	0.8
$a_1 \text{ (MPa/km}^2\text{)}$	0	0	0	0
$b \text{ (MPa/km)}$	32.9	32.9	35.7	35.7
$b_1 \text{ (MPa/km)}$	-22.0	-22.0	-22.0	-22.0
$c \text{ (MPa)}$	110	110	120	120
$c_1 \text{ (MPa)}$	0	0	0	0

* Cases A, B, C, and D refer to curves in Figures 3.4a, 3.4b, and 3.7a.

Table 3.2 Values of model parameters for wedge-shaped block with dimensions $L = 10$ km, $H = 1$ km subject to linear boundary forces

Parameters	Case*			
	A	B	C	D
$b \text{ (MPa/km)}$	32.9	32.9	35.7	35.7
$b_1 \text{ (MPa/km)}$	-22.0	-22.0	-22.0	-22.0
$c \text{ (MPa)}$	110	110	110	120
$c_1 \text{ (MPa)}$	0	0	0	0
α	2°	2°	2°	2.5°
β	1°	2°	2°	2°

* Cases A, B, C, and D refer to curves in Figures 3.5a, 3.5b, and 3.7b.

A consequence of nonlinear depth dependence of horizontal normal force at the rear of the thrust sheet and nonzero slopes is that the vertical stress within the block deviates from lithostatic values. Conditions causing nonlinear depth distributions of horizontal and vertical normal stresses include lateral density inhomogeneities in the lithosphere (Bott, 1990). Deviations of the vertical stresses from lithostatic values, however, must be

Figure 3.2 Principal stress trajectories within the wedge-shaped thrust sheet subject to nonlinear horizontal compression on the rear vertical edge for the model parameters given in Tables 3.1 and 3.2 combined: (a) case B and (b) case D. Minimum and maximum compressions are represented by the solid and the dashed lines.

Figure 3.3 Deviation of vertical normal stress from lithostatic values for selected model parameters: (a) case B in Table 3.1, (b) case B in Table 3.2, (c) case B in Tables 3.1 and 3.2 combined, and (d) case B in Table 3.3. Numbers beside lines represent the horizontal distance between the rear edge of the block (0.0L) and the vertical profile where the deviation is calculated.

small, since in situ stress measurements show approximately lithostatic conditions in the uppermost few kilometers of the crust (McGarr and Gay, 1978). The deviations of the vertical normal stress from the lithostatic case for several values of the parameters are shown in Figure 3.3: for both thin- and thick- skinned thrust sheets these deviations are always relatively minor ($< 5\text{-}10\%$) and therefore are compatible with observations.

3.3 State of stress on the basal thrust

The normal and shear stresses on the basal surface can be derived from the last two of equations (3.8)

$$\begin{aligned}\sigma_n &= F_y^{BC} \cos \beta + F_x^{BC} \sin \beta \\ &= A_0 + A_1 x + A_2 x^2 + A_3 x^3 \\ \tau_n &= F_x^{BC} \cos \beta - F_y^{BC} \sin \beta \\ &= B_0 + B_1 x + B_2 x^2 + B_3 x^3\end{aligned}\tag{3.13a}$$

where the parameters A_i and B_i ($i = 0$ to 3) are given by

$$\begin{aligned}A_0 &= -\rho g H + \frac{a}{2} H^2 + \frac{m+n}{3} (2a - a_1) H L + (n^2 - m^2) b H - 2(m+n) \frac{c+c_1}{L} H - (m+n)^2 c \\ A_1 &= (m+n) \rho g - \frac{a+a_1}{2L} H^2 - (m+n)^2 \frac{2a-a_1}{2} L - 3(m+n) a H + 3(m^2 - n^2) \frac{b+b_1}{L} H \\ &\quad + 3mn^2 b + 3(m+n)^2 \frac{c+c_1}{L} \\ A_2 &= 3(m+n)^2 a + 2(m+n) \frac{a+a_1}{L} H + 2(2n^3 + 3mn^2 - m^3) \frac{b+b_1}{L} \\ A_3 &= \frac{5(m+n)^2}{3} \frac{a+a_1}{L} \\ B_0 &= m \rho g H + \frac{2a-a_1}{6} H L - \frac{b+b_1}{2L} H^2 - m b H - \frac{c+c_1}{L} H - (m+n) c\end{aligned}$$

$$\begin{aligned}
B_1 &= -m(m+n)\rho g - aH - (m+n)\frac{2a-a_1}{3}L + (m^2-n^2)b + 2m\frac{b+b_1}{L}H \\
&\quad + 2(m+n)\frac{c+c_1}{L} \\
B_2 &= \frac{a+a_1}{2L}H + \frac{3}{2}(m+n)a + \frac{3}{2}(n^2-m^2)\frac{b+b_1}{L} \\
B_3 &= -\frac{2}{3}(m+n)\frac{a+a_1}{L}
\end{aligned} \tag{3.13b}$$

The expressions for A_i and B_i have been simplified following the same procedure and assumptions as described above for the constants k_i . Also, the fact that the basal slope is small has been taken into account.

Equations (3.13a, b) show explicitly the effects of nonlinear boundary forces and of upper and lower slopes on the state of stress along the thrust fault. These two effects are discussed separately.

3.3.1 Nonlinear component of horizontal compression

In order to isolate the role of nonlinear horizontal compression in determining the state of stress on the thrust fault, a simple rectangular block is considered (i.e., $n = m = 0$). Then equations (3.13a), using equations (3.13b), can be written as

$$\begin{aligned}
\sigma_n &= -\rho gH + \frac{a}{2}H^2 - \frac{a+a_1}{2L}H^2x \\
\tau_n &= \frac{2a-a_1}{6}HL - \frac{b+b_1}{2L}H^2 - \frac{c+c_1}{L}H - aHx + \frac{a+a_1}{2L}Hx^2
\end{aligned} \tag{3.14}$$

Therefore the presence of the terms ay^2 and/or a_1y^2 in the external driving forces has the effect of making both shear and normal stresses vary with position along the base of the thrust block. Figure 3.4 shows σ_n and τ_n at the base of a rectangular block for the

Figure 3.4 (a) Normal and (b) shear stresses along the lower boundary of a rectangular block for four different sets of force boundary conditions (given in Table 3.1).

model parameters listed in Table 3.1, that is, nonlinear compression at the rear, and linear resistance at the front less in magnitude than the compression at the rear.

3.3.2 Upper and lower slopes

The upper and lower slopes of the wedge-shaped block also contribute to the variation of the state of stress on the thrust fault. In order to show this, assume $a = a_1 = 0$ in equations (3.13a); it follows that, using equations (3.13b),

$$\begin{aligned}
 \sigma_n &= -\rho g H + (m^2 - n^2) b H - 2(m+n) \frac{c+c_1}{L} H - (m+n)^2 c \\
 &\quad + \left[(m+n) \rho g + 3(m^2 - n^2) \frac{b+b_1}{L} H + 3mn^2 b + 3(m+n)^2 \frac{c+c_1}{L} \right] x \\
 &\quad + 2 \left[2n^3 + 3mn^2 - m^3 \right] \frac{b+b_1}{L} x^2 \\
 \tau_n &= m \rho g H - m b H - \frac{b+b_1}{2L} H^2 - (m+n) c - \frac{c+c_1}{L} H \\
 &\quad + \left[-m(m+n) \rho g + 2m \frac{b+b_1}{L} H + (m^2 - n^2) b + 2(m+n) \frac{c+c_1}{L} \right] x \\
 &\quad + \frac{3(n^2 - m^2)}{2} \frac{b+b_1}{L} x^2
 \end{aligned} \tag{3.15}$$

It can be seen from the above equations that the effects of nonzero slopes ($n \neq 0, m \neq 0$) on the state of stress along the wedge's basement are coupled with the force boundary conditions at the front and at the rear. In the case that the front and rear boundaries are subject to constant stresses (i.e., $a = a_1 = b = b_1 = 0$), the basal normal stress (σ_n) is a simple function of the wedge's taper ($m+n$).

Some examples of the variations of normal and shear stresses on the basal thrust fault, caused by the finite slopes of the wedge, are shown in Figure 3.5 for the model parameters listed in Table 3.2. It can be seen that even for very small slopes the state of stress on the thrust fault varies significantly with position.

Figure 3.5 (a) Normal and (b) shear stresses along the lower boundary of a wedge-shaped block for linear boundary forces and four different sets of parameters (given in Table 3.2).

Figure 3.6 (a) Normal and (b) shear stresses along the lower boundary of a wedge-shaped block subject to nonlinear boundary forces for four different sets of parameters (given in Tables 3.1 and 3.2 combined).

Finally, Figure 3.6 shows the combined effects of nonlinear boundary forces and of sloping upper and lower boundaries on the state of stress along the thrust fault.

3.4 Discussion: initiation and development of thrusting

The analytical solutions obtained in this chapter are relevant to investigate the initiation and development of slip along the basal thrust. A frictional function $\mu(x)$ along the fault can be defined as

$$\mu(x) = \frac{\tau_n(x)}{\sigma_n(x)} \quad (3.16)$$

If μ_0 represents the frictional coefficient of the material, it can be assumed that the thrust fault will be activated only where $\mu(x) \geq \mu_0$. The inclusion of constant finite cohesion would change the value $\mu(x)$ and, consequently, decrease the length of the potentially slipped part of the fault. However, cohesion along preexisting faults is usually low (1 - 10 MPa; *cf.* Suppe, 1985; Ranalli, 1995), and therefore the frictional function defined above can be considered as a reasonable first-order approximation.

The distribution of the frictional function over the basal thrust for different boundary conditions and shapes of the block is shown in Figure 3.7. Slippage can be assumed to occur for values of the frictional function $\mu(x) \geq 0.60$ (a fair approximation for some typical rocks such as granite and sandstone; *cf.* Jaeger and Cook, 1969). Assuming negligible cohesion, failure along the thrust, if $\mu(x)$ decreases toward the front, should begin at the rear and propagate up to the distance x^* , such that $\mu(x^*) \approx 0.60$. The total length of the potentially slipped part varies with the values of the external forces and the slopes of the upper and lower boundaries.

Figure 3.7 Variation of frictional function along the basal thrust for (a) rectangular block, nonlinear boundary forces, parameters as in Table 3.1; (b) wedge-shaped block, linear boundary forces, parameters as in Table 3.2; and (c) wedge-shaped block, nonlinear boundary forces, parameters as in Tables 3.1 and 3.2 combined. In each case, x^* denotes the distance from the rear where the frictional function equals a friction coefficient $\mu_0 = 0.60$ (represented by the horizontal line). See discussion in the text.

In the above discussion it is implicitly assumed that the dynamic effect of dislocations is negligible, i.e., the stresses generated by a dislocation are not considered in our modelling. We are concerned mainly about the initiation and the development of thrusting from a macroscopic point of view.

Several calculations have been performed for different values of force parameters and boundary slopes. In general, combinations with $a < 0$ (gradient of rear pushing force decreasing with increasing depth) result in a frictional function that decreases from the rear to the front of the thrust sheet. A nonlinear front edge force does not affect significantly this conclusion. The larger the difference between horizontal rear and front forces, the higher the frictional function and, consequently, the likelihood and extent of slip. Upper and lower slopes of the order of 2° or larger, even with linear boundary forces (larger at the rear) are favourable to slip, whose likelihood decreases as the frontal resistance increases. The slip proceeds from the rear to the front, as is observed in thrust sheets. This is also the case when the gradient of the rear pushing force increases with depth.

The choice of boundary force parameters is constrained by two requirements: (1) there should be no failure within the block, and (2) the shear stress on the basal thrust should be realistic (i.e., $\tau_n \leq 50$ MPa). The second requirement is satisfied in the present model. To check if the first requirement is met, the critical stress S_c within the block according to the Coulomb-Navier criterion is computed as (Jaeger and Cook, 1969)

$$S_c = \frac{1}{\cos \phi} \left[\frac{(\sigma_{xx} - \sigma_{yy})^2}{4} + \sigma_{xy}^2 \right]^{\frac{1}{2}} + \frac{\sigma_{xx} + \sigma_{yy}}{2} \tan \phi \quad (3.17)$$

where $\phi = \tan^{-1}(\mu_0)$ is the internal friction angle ($\approx 30^\circ$ for $\mu_0 \approx 0.577$). The critical stress maximum within the block is generally less than 50 MPa, that is, below estimated values of rock strength.

3.5 Very large wedges and the effect of fluid pressure

The above results are for a block with $L = 10$ km and $H = 1$ km. For very large thrust blocks (e.g., $L \approx 50 - 100$ km and $H \approx 5 - 10$ km) the shear stress on the basal boundary is caused in large part by the term $(2a - a_1)HL/6$ in the parameter B_0 (see equation (3.13b)), and this puts a constraint on the nonlinear component of the boundary force. Furthermore, since the maximum basal normal stress is mainly caused by the overburden pressure ($\approx \rho gH$), and the maximum realistic value for shear stress in the crust is $\tau_n \approx 50 - 100$ MPa, it is immediately clear that in the absence of pore fluid pressure on the basal thrust the maximum possible rear thickness of a cohesionless thrust sheet should not exceed $H = \tau_n/(\mu_0 \rho g) \approx 6$ km, assuming density $\rho = 2800 \text{ kg m}^{-3}$ and friction coefficient $\mu_0 \approx 0.60$. The presence of pore fluids along the basal thrust increases the maximum thickness of the thrust sheet and also the total slipped length because the normal stress on the fault plane is reduced to $\sigma_n^* = \sigma_n(1 - \lambda)$, where λ is the pore fluid factor (ratio of pressure of water column to overburden pressure). Effective normal stress, shear stress, and frictional function at the base of a 100-km-long thrust sheet are shown in Figure 3.8 for the model parameters given in Table 3.3 and a pore fluid factor $\lambda = 0.75$. These parameters result in critical stresses within the block $S_c \leq 35$ MPa for cases A, B, and C and $S_c \leq 70$ MPa for case D, which is considered geologically unrealistic, as it results in a frictional function increasing towards the front of the thrust sheet. The maximum deviation of vertical normal stress from lithostatic values is generally less than 6%.

Table 3.3 Values of model parameters for wedge-shaped block with dimensions $L = 100$ km, $H = 10$ km subject to nonlinear boundary forces and $\lambda = 0.75$ on the basal thrust.

Parameters	Case*			
	A	B	C	D
a (MPa/km ²)	-0.1	-0.2	-0.2	0.1
a_1 (MPa/km ²)	0	0	0	0
b (MPa/km)	32.9	32.9	35.7	35.7
b_1 (MPa/km)	-22.0	-22.0	-22.0	-22.0
c (MPa)	110	110	110	120
c_1 (MPa)	0	0	0	0
α	2°	2°	2°	2.5°
β	1°	2°	2°	2°

* Cases A, B, C, and D refer to curves in Figures 3.8a, 3.8b, and 3.8c.

In cases where $\mu(x)$ decreases toward the front of the thrust sheet, the minimum pore fluid factor (assumed constant) required to begin slippage at the rear of the basal fault can be calculated from equation (3.16) by using the effective normal stress

$$\begin{aligned}\lambda &= 1 - \frac{\mu(x=0)}{\mu_0} \\ &= 1 - \frac{1}{\mu_0} \frac{B_0}{A_0}\end{aligned}\tag{3.18}$$

where A_0 and B_0 are constant terms of normal and shear stresses, respectively, on the basal fault (see equation (3.13b)). Figure 3.9 shows the minimum λ as a function of the rear thickness of the wedge. It can be seen that (1) increasing the magnitude of the nonlinear part of the rear pushing force has the effect of decreasing the required minimum λ ; (2) if the magnitude of a is large enough (such as $a = -0.1$ MPa/km² in the calculation), λ will be maximum for a thrust sheet with intermediate thickness H and

Figure 3.8 (a) Effective normal stress, (b) shear stress, and (c) frictional function along the basal boundary of a 100-km long wedge-shaped thrust sheet for model parameters listed in Table 3.3. A pore fluid factor $\lambda = 0.75$ is assumed.

Figure 3.9 Minimum pore fluid factor λ (as defined by equation (3.18)) as a function of rear thickness of the thrust sheet for (a) rectangular block and (b) wedge-shaped block. Parameter values, $\mu_0 = 0.60$; $a_1 = 0$, $b = 32.9$ MPa/km, $b_1 = -22.0$ MPa/km, $c = 110$ MPa, and $c_1 = 0$. In both diagrams, squares denote the case when $a = -0.1$, $r = 0.1$; triangles, $a = -0.01$, $r = 0.1$; X, $a = -0.1$, $r = 0.08$; crosses, $a = -0.01$, $r = 0.08$. Units of a are MPa/km²; $r = H/L$ is the aspect ratio of the thrust sheet.

length L (in the calculation, $H \approx 3-6$ km and $L \approx 40-75$ km), which shows that the likelihood of initiation of slip on the basal fault in the presence of nonlinear rear pushing force does not always decrease with the increasing size of the thrust sheet (This is a consequence of the fact that, for some choices of boundary forces parameters, the shear stress on the basal fault, compared with the normal stress, will increase faster with H and L (for the same ratio of $r = H/L$) when $H > 6$ km and $L > 75$ km); (3) the larger the upper and lower boundary slopes, the smaller the required minimum pore fluid factor; and (4) the shape of the relation between λ and rear thickness is determined mainly by the nonlinear part of rear compression and is not affected significantly by the aspect ratio of the thrust sheet.

In general, the extent of the critical length ($\mu(x) \geq \mu_0$) depends on the boundary forces and on the shape of the wedge: increasing the rear compression and/or increasing the upper and lower slopes have the effect of increasing the critical length. For a given length L of the thrust sheet, increasing the rear thickness H decreases the length along which slippage occurs. In all cases, only part of the fault is at stress levels compatible with failure, which supports Price's (1988, 1990) argument (see also Washington, 1990) that initiation of slip does not imply that the whole fault is at failure. Although simple static considerations cannot fully solve Smoluchowski's paradox, they indicate that the stress state at the base of a wedge-shaped block is compatible with slippage initiating at the rear of the block and affecting only a part of the fault at any given time.

Chapter 4

Two-Dimensional Incompressible Power-law Flow

4.1 Introduction

This chapter presents a general solution for a plane strain incompressible material with steady-state power-law rheology. Stress, strain rate and velocity fields are expressed explicitly in terms of bulk rheology, force boundary conditions at the rear of a triangular wedge, and a function $C(x,y)$ which is related to the deviation of vertical normal stress from the lithostatic overburden. When the power-law stress exponent equals unity (that is, the material becomes a linear viscous Newtonian body), the function $C(x,y)$ can be determined by kinematic and/or force boundary conditions and the problem can be solved analytically.

4.2 Model geometry and rheology

The simplified two-dimensional geometry of an orogenic wedge and the associated coordinate system are shown in Figure 4.1. The upper boundary is a traction-free surface with a small constant slope α dipping toward the front. The lower boundary has a slope β that may slightly increase from the front to the rear of the wedge. The origin of the coordinate system is fixed on the upper surface, with positive x axis parallel to the surface towards the front and positive y axis perpendicular to the surface and pointing downwards.

Figure 4.1 Simplified two-dimensional orogenic wedge model: α is the surface slope, β the basal slope, u , v the velocity components, and g the acceleration of gravity. Positive stress components are shown.

The bulk rheology of the wedge material is assumed to be steady state power-law creep, in which strain rate is related to the n th power of the stress ($n \geq 1$). This non-Newtonian flow behaviour is very common in silicate polycrystals above the brittle-ductile transition (*cf.* Ranalli, 1995).

As mentioned in Chapter 2, the bulk rheology of orogenic wedges varies in a complex way from the front to the rear and from the top to the bottom (thick wedges can be as deep as 30 - 40 km). The major advantage of assuming power-law rheology is that, by specifying different values for the parameter n , this general strain rate-stress relation can represent a broad spectrum of deformation styles which may exist in different parts of the orogenic wedge (see Table 4.1 and discussion in Ranalli, 1995).

Table 4.1 Steady state power-law rheology and deformation patterns

stress exponent n	deformation style	applicable to
∞	Perfectly plastic body	near surface (depth < 10 - 15 km)
2 ~ 4	Power-law body	below 10 - 15 km
1	Newtonian body	shear zones with sub-millimetric grain size

The wedge material, as a first-order approximation, is taken to be incompressible. This assumption greatly simplifies the problem, and is also supported by some geological observation (*cf.* Dahlen and Barr, 1989 for details). However, it is not valid when there is large volume reduction due to loss of fluids.

The basic equations of power-law rheology for incompressible flow are as follows (details of derivation and discussion can be found in Ranalli, 1995):

- (1) Strain rate $\dot{\epsilon}_{ij}$ is linearly proportional to deviatoric stress σ'_{ij}

$$\dot{\epsilon}_{ij} = \lambda \sigma'_{ij} \quad (4.1)$$

where the parameter λ is a function of the stress and therefore of position;

(2) There exists a power-law dependence of effective strain rate $\dot{\epsilon}_E$ on effective shear stress τ'_E

$$\dot{\epsilon}_E = A \tau'^n_E \quad (4.2)$$

where the parameter A is a function of material properties, pressure and temperature, and effective stress and strain rate are the second invariant of their respective tensors

$$\tau'^2_E = \frac{1}{2} \sigma'_{ij} \sigma'_{ij} \quad \dot{\epsilon}_E^2 = \frac{1}{2} \dot{\epsilon}_{ij} \dot{\epsilon}_{ij}$$

Combining the above equations with equations (4.1) and (4.2), it follows that

$$\begin{aligned} \dot{\epsilon}_E &= \lambda \tau'_E \\ \lambda &= A \tau'^{(n-1)}_E \end{aligned} \quad (4.3)$$

So that

$$\dot{\epsilon}_{ij} = A \tau'^{(n-1)}_E \sigma'_{ij} \quad (4.4)$$

The parameter A can be expressed as (see e.g. Kirby, 1983; Ranalli, 1995)

$$A = A_0 \exp\left(-\frac{Q + PV}{RT}\right) \quad (4.5)$$

where T is the absolute temperature, R the universal gas constant, P the pressure, and the parameters Q , V and A_0 are material coefficients (Q and V are the creep activation energy and activation volume, respectively; the term PV is negligible when $P \ll k$ (bulk modulus) as is the case in the crust). The parameter A_0 is slightly temperature-dependent, but this is usually neglected in comparison with the exponential term.

4.3 Basic equations

There are three sets of equations involved in the analysis of stress, strain rate, and velocity fields in orogenic wedges (incompressibility and plane strain are assumed). They are:

(1) equations of equilibrium

$$\begin{aligned}\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \rho g_x &= 0 \\ \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \rho g_y &= 0\end{aligned}\tag{4.6}$$

(2) strain rate - velocity relations

$$\begin{aligned}\dot{\epsilon}_{xx} &= \frac{\partial u}{\partial x} \\ \dot{\epsilon}_{yy} &= \frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} \\ \dot{\epsilon}_{xy} &= \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)\end{aligned}\tag{4.7}$$

(3) constitutive equations

$$\begin{aligned}
\dot{\epsilon}_{xx} &= \lambda \sigma'_{xx} \\
\dot{\epsilon}_{yy} &= \lambda \sigma'_{yy} \\
\dot{\epsilon}_{xy} &= \lambda \sigma'_{xy}
\end{aligned} \tag{4.8}$$

where g_x and g_y are the gravitational components in x and y directions; σ_{xx} , σ_{yy} , and σ_{xy} the stress tensor components; ρ the density; and u and v the velocity components in the x and y directions. The parameter λ is defined according to equation (4.3).

The components of the gravitational acceleration are related to surface slope as

$$g_x = g \sin \alpha \quad g_y = g \cos \alpha \tag{4.9}$$

and the stress deviator components are

$$\begin{aligned}
\sigma'_{xx} &= \frac{1}{2}(\sigma_{xx} - \sigma_{yy}) \\
\sigma'_{yy} &= \frac{1}{2}(\sigma_{yy} - \sigma_{xx}) = -\sigma'_{xx} \\
\sigma'_{xy} &= \sigma_{xy}
\end{aligned} \tag{4.10}$$

4.4 Derivation of solutions

There are eight unknowns (i.e., three stress components, two velocity components and three strain rate components) linked by eight equations (i.e., equations (4.6) to (4.8)). Finding an analytical solution for a set of nonlinear partial differential equations has been found to be extremely difficult, and very few closed-form solutions of this problem exist (*cf.* Lliboutry, 1987). The approach adopted in this chapter is to express stress, strain rate, and velocity in terms of one single function (denoted by $C(x,y)$). If this single function can be determined or approximated, then the whole problem becomes solvable.

The choice of $C(x, y)$ is determined by the nature of the problem. For the purpose of modelling orogenic wedges in this thesis, $C(x, y)$ is defined as the deviation of normal stress from the lithostatic overburden in the direction perpendicular to the surface of the wedge (i.e., y -direction in Fig. 4.1). The normal stress in the y -direction does not differ much from the lithostatic pressure (Elliott, 1976; Boyer and Elliott, 1982; Davis et al., 1983; Dahlen, 1984; Suppe, 1985; Platt, 1986; Liu and Ranalli, 1992, 1994; Yin, 1986, 1988, 1989, 1993) since the wedge's topographic slope is relatively small ($\alpha < 6^\circ$). Also, the gradient in shear stress in x -direction is likely to be small compared to the gradient in normal stress in the y -direction.

4.4.1 Stress solution

On the base of the above discussion, it is convenient to express the normal stress σ_{yy} as a sum of lithostatic overburden pressure and a function $C(x, y)$

$$\sigma_{yy} = -\bar{\rho}_y g_y y + C(x, y) \quad (4.11)$$

where g_y is the gravity component defined in equation (4.9) and $\bar{\rho}_y$ is the density averaged in the y -direction between the top surface ($y = 0$) and the depth y , so that $\bar{\rho}_y(x, y) = \frac{1}{y} \int_0^y \rho(x, \xi) d\xi$. The function $C(x, y)$ has the dimension of stress and represents the non-lithostatic part of normal stress in the y -direction.

Substituting equation (4.11) into the second of equations (4.6), one has

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial C(x, y)}{\partial y} = 0 \quad (4.12)$$

from which the shear stress σ_{xy} is readily obtained by integration

$$\sigma_{xy} = T(y) - \int_0^x \frac{\partial C(\tau, y)}{\partial y} d\tau \quad (4.13)$$

where $T(y)$ is an arbitrary integration function.

To determine the normal stress component in the x -direction (σ_{xx}), we combine equation (4.13) with the first of equations (4.6)

$$\frac{\partial \sigma_{xx}}{\partial x} = -T'(y) - \rho g_x + \int_0^x \frac{\partial^2 C(\tau, y)}{\partial y^2} d\tau \quad (4.14)$$

Integration equation (4.14) with respect to x gives

$$\sigma_{xx} = S(y) - x T'(y) - \bar{\rho}_x g_x x + \int_0^x \int_0^\xi \frac{\partial^2 C(\tau, y)}{\partial y^2} d\tau d\zeta \quad (4.15)$$

where $S(y)$ is another arbitrary integration function and $\bar{\rho}_x$ is the density averaged in the x -direction between 0 and x , that is, $\bar{\rho}_x(x, y) = \frac{1}{x} \int_0^x \rho(\tau, y) d\tau$.

Combination of equations (4.11), (4.13) and (4.15) provides a general solution to the equilibrium equations

$$\begin{aligned} \sigma_{xx} &= S(y) - x T'(y) - \bar{\rho}_x g_x x + \int_0^x \int_0^\xi \frac{\partial^2 C(\tau, y)}{\partial y^2} d\tau d\zeta \\ \sigma_{yy} &= -\bar{\rho}_y g_y y + C(x, y) \\ \sigma_{xy} &= T(y) - \int_0^x \frac{\partial C(\tau, y)}{\partial y} d\tau \end{aligned} \quad (4.16)$$

The solution is greatly simplified if the function $C(x,y)$ can be neglected when compared with the lithostatic overburden. In this case, equations (4.16) become

$$\begin{aligned}\sigma_{xx} &= S(y) - x T'(y) - \bar{\rho}_x g_x x \\ \sigma_{yy} &= -\bar{\rho}_y g_y y \\ \sigma_{xy} &= T(y)\end{aligned}\tag{4.17}$$

(which could be further simplified if $\alpha \approx 0^\circ$, $g_x \approx 0$).

The normal stress in the x -direction and the shear stress on the rear “backstop” of the wedge ($x=0$) can be easily found from equations (4.16)

$$\begin{aligned}\sigma_{xx}(0,y) &= S(y) \\ \sigma_{xy}(0,y) &= T(y)\end{aligned}\tag{4.18}$$

Equations (4.18) show that the functions $S(y)$ and $T(y)$ have a precise physical meaning.

Additional constraints on the functions $T(y)$ and $C(x,y)$ are imposed from the condition that the upper boundary of the wedge is traction-free ($\sigma_{yy} = \sigma_{xy} = 0$), i.e.,

$$\begin{aligned}T(0) &= 0 \\ C(x,0) &= 0 \\ [\partial C / \partial y]_{y=0} &= 0\end{aligned}\tag{4.19}$$

Equations (4.16) - or, in simplified form, equations (4.17) when $C(x,y) = 0$ - give the general solution for stress in the wedge, in terms of functions $S(y)$, $T(y)$, and $C(x,y)$, each of which with a precise physical meaning. Their role will be discussed further in the remainder of this chapter.

4.4.2 Strain rate solution

Strain rate components are related to the corresponding deviatoric stress components by equations (4.8) which, combined with equations (4.10) and (4.16) lead to the following expressions

$$\begin{aligned}\dot{\epsilon}_{xx} &= \lambda \sigma'_{xx} = \frac{\lambda}{2} \left[S(y) - x T'(y) + \bar{\rho}_y g_y y - \bar{\rho}_x g_x x + \int_0^x \int_0^\zeta \frac{\partial^2 C(\tau, y)}{\partial y^2} d\tau d\zeta - C(x, y) \right] \\ \dot{\epsilon}_{yy} &= \lambda \sigma'_{yy} = -\dot{\epsilon}_{xx} \\ \dot{\epsilon}_{xy} &= \lambda \sigma'_{xy} = \lambda \left[T(y) - \int_0^x \frac{\partial C(\tau, y)}{\partial y} d\tau \right]\end{aligned}\quad (4.20)$$

The parameter λ can be expressed in terms of either the second invariant of stress (τ'_E) or the second invariant of strain rate ($\dot{\epsilon}_E$) as (see equations (4.2) and (4.3))

$$\lambda = A \tau'^{(n-1)}_E = A^{\frac{1}{n}} \dot{\epsilon}_E^{\frac{n-1}{n}} \quad (4.21a)$$

where

$$\begin{aligned}\tau'_E &= \left(\sigma'^2_{xx} + \sigma'^2_{xy} \right)^{\frac{1}{2}} = \left[\frac{1}{4} (\sigma_{xx} - \sigma_{yy})^2 + \sigma_{xy}^2 \right]^{\frac{1}{2}} \\ \dot{\epsilon}_E &= \left(\dot{\epsilon}_{xx}^2 + \dot{\epsilon}_{xy}^2 \right)^{\frac{1}{2}}\end{aligned}\quad (4.21b)$$

Again, the expression for λ is rather complicated (see equations (4.16) and (4.20)), but it is simplified if $C(x, y) \approx 0$. Even in this latter case, however, it is a nonlinear function of position unless functions S and T and material parameters (i.e., A and n) are constant.

4.4.3 Velocity solution

The velocity components can also be expressed as functions of the stress components. From the first two of equations (4.7), one obtains

$$\begin{aligned} u(x, y) &= \int_0^x \dot{\epsilon}_{xx}(\xi, y) d\xi + H(y) \\ v(x, y) &= -\int_0^y \dot{\epsilon}_{xx}(x, \tau) d\tau + I(x) \end{aligned} \quad (4.22)$$

where $I(x)$ and $H(y)$ are two integration functions. Physically, $H(y)$ is the vertical distribution of the longitudinal velocity component at the backstop since $H(y) = u(x=0, y)$; and $I(x)$ is the longitudinal distribution of vertical velocity component on the top surface since $I(x) = v(x, y=0)$

Substituting equations (4.22) into the third of equations (4.7) gives

$$\int_0^x \frac{\partial \dot{\epsilon}_{xx}(\tau, y)}{\partial y} d\tau - \int_0^y \frac{\partial \dot{\epsilon}_{xx}(x, \xi)}{\partial x} d\xi - 2\dot{\epsilon}_{xy}(x, y) + H'(y) + I'(x) = 0 \quad (4.23)$$

This is the compatibility equation which the strain rate and velocity fields have to satisfy. Note that also the function $C(x, y)$ is required to satisfy this equation, because strain rate components are functions of the stress solution (see equations (4.16)). Combination of equations (4.20), (4.21) and (4.23) gives

$$\begin{aligned} \int_0^x \frac{\partial}{\partial y} \left[A(\sigma'_{xx}{}^2 + \sigma'_{xy}{}^2)^{\frac{n-1}{2}} \sigma'_{xx} \right] dx - \int_0^y \frac{\partial}{\partial x} \left[A(\sigma'_{xx}{}^2 + \sigma'_{xy}{}^2)^{\frac{n-1}{2}} \sigma'_{xx} \right] dy \\ - 2A(\sigma'_{xx}{}^2 + \sigma'_{xy}{}^2)^{\frac{n-1}{2}} \sigma'_{xy} + H'(y) + I'(x) = 0 \end{aligned} \quad (4.24a)$$

where

$$\begin{aligned}\sigma'_{xx} &= \frac{1}{2} \left[S(y) - x T'(y) + \int_0^x \int_0^\xi \frac{\partial^2 C(\tau, y)}{\partial y^2} d\tau d\zeta - \bar{\rho}_x g_x x + \bar{\rho}_y g_y y - C(x, y) \right] \\ \sigma'_{xy} &= T(y) - \int_0^x \frac{\partial C(\tau, y)}{\partial y} d\tau\end{aligned}\tag{4.24b}$$

Although the set of equations (4.22 - 4.24) formally give a velocity solution, this solution (as in the case of strain rate) cannot be given explicitly unless the unknown functions are determined. This problem is considered in the next section.

4.5 Determination of functions $I(x)$ and $H(y)$

There are in total five unknown functions involved in the expressions for stress, strain rate and velocity fields (*cf.* equations (4.16), (4.20) and (4.22)): $S(y)$, $T(y)$, $H(y)$, $I(x)$, and $C(x, y)$. Function $I(x)$ cannot be specified as part of the boundary conditions on the wedge's top surface since a stress boundary condition (i.e., traction-free) has been assumed on the same surface (see section 4.4.1). Functions $S(y)$ and $H(y)$ represent the normal stress and x -direction velocity component along wedge's rear boundary, respectively, and therefore cannot be specified independently from each other. In this section, therefore, the functions $S(y)$ and $T(y)$ are taken as assigned stress boundary conditions along the wedge's rear boundary, and consequently $H(y)$, $I(x)$, and $C(x, y)$ are the three functions to be determined from the other boundary conditions. First, it will be shown that $H(y)$ and $I(x)$ can be expressed as functions of the material bulk rheology (e.g. parameter A and stress exponent n) and the function $C(x, y)$. To simplify the derivation, define a function R such as (*cf.* equation (4.24a))

$$\begin{aligned}
R(x, y) = & \int_0^x \frac{\partial}{\partial y} \left[A \left(\sigma'_{xx}{}^2 + \sigma'_{xy}{}^2 \right)^{\frac{n-1}{2}} \sigma'_{xx} \right] dx - \int_0^y \frac{\partial}{\partial x} \left[A \left(\sigma'_{xx}{}^2 + \sigma'_{xy}{}^2 \right)^{\frac{n-1}{2}} \sigma'_{xx} \right] dy \\
& - 2A \left(\sigma'_{xx}{}^2 + \sigma'_{xy}{}^2 \right)^{\frac{n-1}{2}} \sigma'_{xy}
\end{aligned} \tag{4.25}$$

where the deviatoric components of stress $(\sigma'_{xx}, \sigma'_{xy})$ are given by equation (4.24b). Obviously, the function $R(x, y)$ can be determined once the function $C(x, y)$ is known. Also, $R(x, y)$ is independent of both $I(x)$ and $H(y)$.

Now equation (4.24a) can be rewritten as

$$R(x, y) + H'(y) + I'(x) = 0 \tag{4.26}$$

From which it follows

$$\begin{aligned}
R(x_0, y_0) + H'(y_0) + I'(x_0) &= 0 \\
R(x, y_0) + H'(y_0) + I'(x) &= 0 \\
R(x_0, y) + H'(y) + I'(x_0) &= 0
\end{aligned} \tag{4.27}$$

where (x_0, y_0) are the coordinates of a point within the wedge where the velocity components $u_0 = u(x_0, y_0)$, $v_0 = v(x_0, y_0)$ are specified as the kinematic condition necessary to determine the velocity field.

Equations (4.27) imply that

$$\begin{aligned}
I'(x) &= I'(x_0) + R(x_0, y_0) - R(x, y_0) \\
H'(y) &= -I'(x_0) - R(x_0, y)
\end{aligned} \tag{4.28}$$

Integrating the first and the second of equations (4.28) with respect to x and y , respectively, we have

$$\begin{aligned} I(x) &= I(x_0) + [I'(x_0) + R(x_0, y_0)](x - x_0) - \int_{x_0}^x R(\xi, y_0) d\xi \\ H(y) &= H(y_0) - I'(x_0)(y - y_0) - \int_{y_0}^y R(x_0, \tau) d\tau \end{aligned} \quad (4.29)$$

Obviously, the functions $I(x)$ and $H(y)$ are determined once three constants $I(x_0)$, $H(y_0)$ and $I'(x_0)$ become known. The values of $I(x_0)$ and $H(y_0)$ can be determined as a function of $C(x, y)$ (through the longitudinal strain rate $\dot{\epsilon}_{xx}$) by equation (4.22) since

$$\begin{aligned} u_0 = u(x_0, y_0) &= \int_0^{x_0} \dot{\epsilon}_{xx}(\xi, y_0) d\xi + H(y_0) \\ v_0 = v(x_0, y_0) &= - \int_0^{y_0} \dot{\epsilon}_{xx}(x_0, \tau) d\tau + I(x_0) \end{aligned} \quad (4.30a)$$

where the longitudinal strain rate $\dot{\epsilon}_{xx}(x, y)$ is given by equations (4.20) and (4.21). It follows from equation (4.30a) that

$$\begin{aligned} H(y_0) &= u_0 - \int_0^{x_0} \dot{\epsilon}_{xx}(\xi, y_0) d\xi \\ I(x_0) &= v_0 + \int_0^{y_0} \dot{\epsilon}_{xx}(x_0, \tau) d\tau \end{aligned} \quad (4.30b)$$

Therefore, from equations (4.22), (4.29), and (4.30b) the velocity expression now can be written as

$$\begin{aligned}
u(x, y) &= u_0 - I'(x_0)(y - y_0) - \int_0^{x_0} \dot{\epsilon}_{xx}(\xi, y_0) d\xi + \int_0^x \dot{\epsilon}_{xx}(\xi, y) d\xi - \int_{y_0}^y R(x_0, \tau) d\tau \\
v(x, y) &= v_0 + [I'(x_0) + R(x_0, y_0)](x - x_0) + \int_0^{y_0} \dot{\epsilon}_{xx}(x_0, \tau) d\tau - \int_0^y \dot{\epsilon}_{xx}(x, \tau) d\tau - \int_{x_0}^x R(\xi, y_0) d\xi
\end{aligned} \tag{4.31}$$

Now the only unknown term in the velocity expression, beside the function $C(x, y)$, is $I'(x_0)$, which can be determined in several ways. One such way is to specify the velocity (u_1, v_1) at another point (x_1, y_1) within the block, that is, $u_1 = u(x_1, y_1)$ and $v_1 = v(x_1, y_1)$. Thus, applying equations (4.31) gives

$$\begin{aligned}
u_1 &= u_0 - I'(x_0)(y_1 - y_0) - \int_0^{x_0} \dot{\epsilon}_{xx}(\xi, y_0) d\xi + \int_0^{x_1} \dot{\epsilon}_{xx}(\xi, y_1) d\xi - \int_{y_0}^{y_1} R(x_0, \tau) d\tau \\
v_1 &= v_0 + [I'(x_0) + R(x_0, y_0)](x_1 - x_0) + \int_0^{y_0} \dot{\epsilon}_{xx}(x_0, \tau) d\tau - \int_0^{y_1} \dot{\epsilon}_{xx}(x_1, \tau) d\tau - \int_{x_0}^{x_1} R(\xi, y_0) d\xi
\end{aligned} \tag{4.32}$$

which leads to

$$I'(x_0) = -\frac{1}{y_1 - y_0} \left[u_1 - u_0 + \int_0^{x_0} \dot{\epsilon}_{xx}(\xi, y_0) d\xi - \int_0^{x_1} \dot{\epsilon}_{xx}(\xi, y_1) d\xi + \int_{y_0}^{y_1} R(x_0, \tau) d\tau \right] \tag{4.33a}$$

and

$$\begin{aligned}
R(x_0, y_0) &= \frac{1}{x_1 - x_0} \left[v_1 - v_0 - \int_0^{y_0} \dot{\epsilon}_{xx}(x_0, \tau) d\tau + \int_0^{y_1} \dot{\epsilon}_{xx}(x_1, \tau) d\tau + \int_{x_0}^{x_1} R(\xi, y_0) d\xi \right] \\
&\quad + \frac{1}{y_1 - y_0} \left[u_1 - u_0 + \int_0^{x_0} \dot{\epsilon}_{xx}(\xi, y_0) d\xi - \int_0^{x_1} \dot{\epsilon}_{xx}(\xi, y_1) d\xi + \int_{y_0}^{y_1} R(x_0, \tau) d\tau \right]
\end{aligned} \tag{4.33b}$$

Note that, since the term $I'(x_0)$ is found in both equations (4.32), the two velocity components u_1 , v_1 cannot be specified independently of $C(x, y)$ (and therefore $R(x, y)$). The relation between u_1 , v_1 and $R(x, y)$ is represented by equation (4.33b). Conversely, if both u_1 and v_1 are specified independently from each other, equation (4.33b) is the additional constraint on the determination of function $C(x, y)$.

In the expression for the velocity field, it is always assumed that equation (4.33b) is satisfied by the function $R(x, y)$ and by the velocity specification at point (x_1, y_1) . Consequently, from equations (4.31) and (4.33a) the velocity solution can be expressed as

$$\begin{aligned}
 u(x, y) &= u_0 + \frac{y - y_0}{y_1 - y_0} \left[u_1 - u_0 + \int_0^{x_0} \dot{\epsilon}_{xx}(\xi, y_0) d\xi - \int_0^{x_1} \dot{\epsilon}_{xx}(\xi, y_1) d\xi + \int_{y_0}^{y_1} R(x_0, \tau) d\tau \right] \\
 &\quad - \int_0^{x_0} \dot{\epsilon}_{xx}(\xi, y_0) d\xi + \int_0^x \dot{\epsilon}_{xx}(\xi, y) d\xi - \int_{y_0}^y R(x_0, \tau) d\tau \\
 v(x, y) &= v_0 + \frac{x - x_0}{x_1 - x_0} \left[v_1 - v_0 - \int_0^{y_0} \dot{\epsilon}_{xx}(x_0, \tau) d\tau + \int_0^{y_1} \dot{\epsilon}_{xx}(x_1, \tau) d\tau + \int_{x_0}^{x_1} R(\xi, y_0) d\xi \right] \\
 &\quad + \int_0^{y_0} \dot{\epsilon}_{xx}(x_0, \tau) d\tau - \int_0^y \dot{\epsilon}_{xx}(x, \tau) d\tau - \int_{x_0}^x R(\xi, y_0) d\xi
 \end{aligned} \tag{4.34}$$

Equations (4.34) give a general solution for the velocity field in terms of $C(x, y)$ and the bulk rheology of the wedge (i.e., parameters A and n as in equation (4.3)), since both the function $R(x, y)$ and the longitudinal strain rate $\dot{\epsilon}_{xx}(x, y)$ are functions of $C(x, y)$ and A and n (see equations (4.20) and (4.25)). As discussed in section 4.4, once the function $C(x, y)$ is determined by some means (e.g., boundary condition along the wedge base), equations (4.16), (4.20), and (4.34) represent a complete closed-form solution for a plane strain incompressible material with steady-state power-law rheology. In the next section it is shown how the function $C(x, y)$ can be determined by force and/or velocity boundary conditions in the case of Newtonian rheology.

4.6 The function $C(x, y)$ in the case of Newtonian rheology

It has been proven in the previous sections that a complete analytical solution for two-dimensional power-law flow in an orogenic wedge can be explicitly expressed in terms of the function $C(x, y)$, which is constrained by the compatibility requirements expressed by equations (4.24) and/or by the kinematic requirements expressed by equation (4.33b). However, the derivation of an analytical expression for $C(x, y)$ from equations (4.24) is extremely difficult. Numerical approaches have to be applied at this stage in order to make the problem solvable. Analytical solutions, however, are obtainable for some special cases (e.g., power-law rheology with $C(x, y) = 0$, which is considered in the next chapter). In this section, a Newtonian rheology is considered. A general power-law flow with linear longitudinal strain rate will be considered in the next chapter.

If the wedge has linear Newtonian rheology, i.e., stress exponent $n = 1$, the parameter λ becomes independent of the state of stress (see, equation (4.3)), although it can still be a function of position (because of changes in temperature and/or material). Formally, λ is related to the inverse of the viscosity of a Newtonian material (η).

Assume that the function $C(x, y)$ can be expressed in polynomial form, i.e., by the following two-dimensional series

$$C(x, y) = \sum_{i=1}^m \sum_{j=1}^n k_{ij} x^{i-1} y^{j-1} \quad (4.35)$$

where m and n are positive integers and k_{ij} ($i = 1$ to m , $j = 1$ to n) are arbitrary constants and are to be determined by equations (4.24) with $n = 1$ and appropriate boundary conditions. The following derivation will show that compatibility requirements and boundary conditions can be expressed as linear systems of equations in terms of the

coefficients k_{ij} . This means that the function $C(x, y)$ becomes determinable by numerical procedures.

Compatibility Requirements

The compatibility conditions (equations (4.23) and (4.24)) can be expressed in another form by differentiating equation (4.24a) with respect to x and y , which gives

$$\frac{\partial^2 R}{\partial x \partial y} = \frac{\partial^2 \dot{\epsilon}_{xx}}{\partial y^2} - \frac{\partial^2 \dot{\epsilon}_{xx}}{\partial x^2} - 2 \frac{\partial^2 \dot{\epsilon}_{xy}}{\partial x \partial y} = 0 \quad (4.36)$$

where R is given by equation (4.25) and the strain rate components are determined by equation (4.20). The above is the usual form of the two-dimensional compatibility relation in terms of strain rate components (see e.g. Ranalli, 1995).

In order to transform equation (4.36) into a linear system of equations in terms of k_{ij} , the strain rate components are expressed in terms of k_{ij} first. Combination of equations (4.35) and (4.20) leads to

$$\begin{aligned} \dot{\epsilon}_{xx} &= \frac{S(y) - x T'(y) + \bar{\rho}_y g_y y - \bar{\rho}_x g_x x + \sum_{i=1}^m \sum_{j=1}^n \left[\frac{(j-1)(j-2)}{i(i+1)} x^2 - y^2 \right] x^{i-1} y^{j-3} k_{ij}}{4 \eta(x, y)} \\ &= X(x, y) + \sum_{i=1}^m \sum_{j=1}^n a_{ij}(x, y) k_{ij} \\ \dot{\epsilon}_{yy} &= -\dot{\epsilon}_{xx} \end{aligned} \quad (4.37a)$$

$$\begin{aligned} \dot{\epsilon}_{xy} &= \frac{T(y) - \sum_{i=1}^m \sum_{j=1}^n \frac{j-1}{i} x^i y^{j-2} k_{ij}}{2 \eta(x, y)} \\ &= Y(x, y) + \sum_{i=1}^m \sum_{j=1}^n b_{ij}(x, y) k_{ij} \end{aligned}$$

where the Newtonian viscosity is $\eta = 1/(2\lambda)$ and

$$\begin{aligned}
X(x, y) &= \frac{S(y) - x T'(y) + \bar{\rho}_y g_y y - \bar{\rho}_x g_x x}{4\eta(x, y)} \\
Y(x, y) &= \frac{T(y)}{2\eta(x, y)} \\
a_{ij}(x, y) &= \frac{x^{i-1} y^{j-3}}{4\eta(x, y)} \left[\frac{(j-1)(j-2)}{i(i+1)} x^2 - y^2 \right] \\
b_{ij}(x, y) &= -\frac{j-1}{2i\eta(x, y)} x^i y^{j-2}
\end{aligned} \tag{4.37b}$$

The function $R(x, y)$ can also be expressed as a linear function of the coefficients k_{ij} by combining equations (4.25) and (4.37), which leads to

$$R(x, y) = K(x, y) + \sum_{i=1}^m \sum_{j=1}^n c_{ij}(x, y) k_{ij} \tag{4.38a}$$

where

$$\begin{aligned}
K(x, y) &= \int_0^x \frac{\partial X(\xi, y)}{\partial y} d\xi - \int_0^y \frac{\partial X(x, \tau)}{\partial x} d\tau - 2Y(x, y) \\
c_{ij}(x, y) &= \int_0^x \frac{\partial a_{ij}(\xi, y)}{\partial y} d\xi - \int_0^y \frac{\partial a_{ij}(x, \tau)}{\partial x} d\tau - 2b_{ij}(x, y)
\end{aligned} \tag{4.38b}$$

Combining equations (4.38) and (4.36), one has

$$\frac{\partial^2 K(x, y)}{\partial x \partial y} + \sum_{i=1}^m \sum_{j=1}^n \frac{\partial^2 c_{ij}(x, y)}{\partial x \partial y} k_{ij} = 0 \tag{4.39a}$$

which can be rewritten as

$$\sum_{i=1}^m \sum_{j=1}^n \frac{\partial^2 c_{ij}(x,y)}{\partial x \partial y} k_{ij} = - \frac{\partial^2 K(x,y)}{\partial x \partial y} \quad (4.39b)$$

This is a linear system with respect to the unknown constants k_{ij} , and therefore is numerically computable given the viscosity and the stress boundary conditions applied on the rear boundary of the wedge. Equations (4.39) can be applied to estimate numerically the spatial distribution of the function $C(x,y)$ for an orogenic wedge with Newtonian rheology. This can be done by the following procedure: first, assign reasonable large values to parameters m and n in equation (4.35); second, select a total of $m \times n$ points P_l ($l = 1, \dots, m \times n$) evenly distributed throughout the wedge; third, use the coordinates of points $P_l(x_l, y_l)$ into equations (4.39), which forms a linear system of $m \times n$ equations with respect to a total of $m \times n$ coefficients k_{ij} ; finally, determine k_{ij} by solving this linear system of equations. When a sufficient number of points $P_l(x_l, y_l)$ is selected, the computed polynomial function $C(x,y)$ represents a first-degree approximation to the solution for the compatibility requirements (i.e., equations (4.24)). Once the function $C(x,y)$ is determined, stress, strain rate and velocity fields can be computed by equations (4.16), (4.20), and (4.34).

A function $C(x,y)$ computed by the above procedure is a *general* one because only the state of stress along the rear and the surface of the wedge have been specified (in terms of the functions $S(y)$ and $T(y)$ associated with equation (4.19)). Therefore, it may not necessarily match additional force and/or velocity boundary conditions along the wedge's basement. To find a *special* function $C(x,y)$ which matches these additional boundary conditions, additional constraints must be added to $C(x,y)$. In the following sub-section, these additional equations are derived; they also are linear systems with respect to the coefficients k_{ij} .

Shear stress along the wedge basement

Shear resistance along the wedge basement is related to the state of stress through the following equation (*cf.* Ranalli, 1995)

$$\begin{aligned}
 \tau_b &= \frac{1}{2}(\sigma_{yy} - \sigma_{xx})\sin 2\delta + \sigma_{xy}\cos 2\delta \\
 &= \frac{1}{2}(\sigma_{yy} - \sigma_{xx})\sin 2(90^\circ - \theta) + \sigma_{xy}\cos 2(90^\circ - \theta) \\
 &= \frac{1}{2}(\sigma_{yy} - \sigma_{xx})\sin 2\theta - \sigma_{xy}\cos 2\theta
 \end{aligned} \tag{4.40}$$

where $\theta = \alpha + \beta$ is the sum of upper and lower slopes, and τ_b is the basal shear stress (see Fig. 4.2).

Substituting (4.35) and (4.16) into (4.40), one has

$$\begin{aligned}
 \tau_b(x, y) &= \frac{\sin 2\theta}{2} [xT'(y) - S(y) + \bar{\rho}_x g_x x - \bar{\rho}_y g_y y] - T(y)\cos 2\theta \\
 &\quad - \frac{\sin 2\theta}{2} \sum_{i=1}^m \sum_{j=1}^n \left(\frac{(j-1)(j-2)}{i(i+1)} x^2 - y^2 \right) x^{i-1} y^{j-3} k_{ij} \\
 &\quad + \cos 2\theta \sum_{i=1}^m \sum_{j=1}^n \frac{j-1}{i} x^i y^{j-2} k_{ij}
 \end{aligned} \tag{4.41}$$

where (x, y) are the coordinates along the wedge basement $y = b(x)$. Equation (4.41) can be written as

$$\tau_b(x, y) = Z(x, y) + \sum_{i=1}^m \sum_{j=1}^n d_{ij}(x, y) k_{ij} \tag{4.42a}$$

where

Figure 4.2 Boundary conditions along the basement AB . α and β are the surface and basal slopes of the wedge, respectively; δ is the angle between the normal to the basement and the positive x -direction. In the given coordinate system, the geometry of the basement is represented by a function $y = b(x)$. The velocity at any basal point is represented by its two component (u_b and v_b). τ_b is the basal shear stress.

$$\begin{aligned}
d_{ij}(x, y) &= \frac{j-1}{i} x^i y^{j-2} \cos 2\theta - \frac{\sin 2\theta}{2} \left[\frac{(j-1)(j-2)}{i(i+1)} x^2 - y^2 \right] x^{i-1} y^{j-3} \\
Z(x, y) &= \frac{\sin 2\theta}{2} [xT'(y) - S(y) + \bar{\rho}_x g_x x - \bar{\rho}_y g_y y] - T(y) \cos 2\theta
\end{aligned} \tag{4.42b}$$

Equations (4.42) represent the basal shear stress boundary conditions along the basement expressed as a linear system of equations with respect to the coefficients k_{ij} . Other kinds of force boundary conditions along the wedge basement, such as the Coulomb-Navier fracture criterion (*cf.* Jaeger and Cook, 1969), can also be used and expressed as a linear system of equations with respect to k_{ij} .

Velocity field

To rewrite the velocity field in terms of the series expression for the function $C(x, y)$, equations (4.34) and (4.37) can be combined to give

$$\begin{aligned}
u(x, y) &= u_0 + \frac{y-y_0}{y_1-y_0} (u_1 - u_0) + \frac{y-y_1}{y_1-y_0} \left[\int_0^{x_0} X(\xi, y_0) d\xi + \sum_{i=1}^m \sum_{j=1}^n \left(\int_0^{x_0} a_{ij}(\xi, y_0) d\xi \right) k_{ij} \right] \\
&\quad - \frac{y-y_0}{y_1-y_0} \left[\int_0^{x_1} X(\xi, y_1) d\xi + \sum_{i=1}^m \sum_{j=1}^n \left(\int_0^{x_1} a_{ij}(\xi, y_1) d\xi \right) k_{ij} \right] \\
&\quad + \frac{y-y_0}{y_1-y_0} \left[\int_{y_0}^{y_1} K(x_0, \tau) d\tau + \sum_{i=1}^m \sum_{j=1}^n \left(\int_{y_0}^{y_1} c_{ij}(x_0, \tau) d\tau \right) k_{ij} \right] \\
&\quad + \int_0^x X(\xi, y) d\xi + \sum_{i=1}^m \sum_{j=1}^n \left(\int_0^x a_{ij}(\xi, y) d\xi \right) k_{ij} - \int_{y_0}^y K(x_0, \tau) d\tau - \sum_{i=1}^m \sum_{j=1}^n \left(\int_{y_0}^y c_{ij}(x_0, \tau) d\tau \right) k_{ij}
\end{aligned}$$

$$\begin{aligned}
v(x, y) = & v_0 + \frac{x - x_0}{x_1 - x_0} (v_1 - v_0) - \frac{x - x_1}{x_1 - x_0} \left[\int_0^{y_0} X(x_0, \tau) d\tau + \sum_{i=1}^m \sum_{j=1}^n \left(\int_0^{y_0} a_{ij}(x_0, \tau) d\tau \right) k_{ij} \right] \\
& + \frac{x - x_0}{x_1 - x_0} \left[\int_0^{y_1} X(x_1, \tau) d\tau + \sum_{i=1}^m \sum_{j=1}^n \left(\int_0^{y_1} a_{ij}(x_1, \tau) d\tau \right) k_{ij} \right] \\
& + \frac{x - x_0}{x_1 - x_0} \left[\int_{x_0}^{x_1} K(\xi, y_0) d\xi + \sum_{i=1}^m \sum_{j=1}^n \left(\int_{x_0}^{x_1} c_{ij}(\xi, y_0) d\xi \right) k_{ij} \right] \\
& - \int_0^y X(x, \tau) d\tau - \sum_{i=1}^m \sum_{j=1}^n \left(\int_0^y a_{ij}(x, \tau) d\tau \right) k_{ij} - \int_{x_0}^x K(\xi, y_0) d\xi - \sum_{i=1}^m \sum_{j=1}^n \left(\int_{x_0}^x c_{ij}(\xi, y_0) d\xi \right) k_{ij}
\end{aligned} \tag{4.43}$$

These expressions for the velocity components can be rewritten in the following shorter form

$$\begin{aligned}
u(x, y) &= U(x, y) + \sum_{i=1}^m \sum_{j=1}^n s_{ij}(x, y) k_{ij} \\
v(x, y) &= V(x, y) + \sum_{i=1}^m \sum_{j=1}^n t_{ij}(x, y) k_{ij}
\end{aligned} \tag{4.44a}$$

where

$$\begin{aligned}
U(x, y) &= u_0 + \frac{y - y_0}{y_1 - y_0} (u_1 - u_0) + \frac{y - y_1}{y_1 - y_0} \int_0^{x_0} X(\xi, y_0) d\xi \\
&+ \frac{y - y_0}{y_1 - y_0} \left[- \int_0^{x_1} X(\xi, y_1) d\xi + \int_{y_0}^{y_1} K(x_0, \tau) d\tau \right] + \int_0^x X(\xi, y) d\xi - \int_{y_0}^y K(x_0, \tau) d\tau \\
V(x, y) &= v_0 + \frac{x - x_0}{x_1 - x_0} (v_1 - v_0) - \frac{x - x_1}{x_1 - x_0} \int_0^{y_0} X(x_0, \tau) d\tau \\
&+ \frac{x - x_0}{x_1 - x_0} \left[\int_0^{y_1} X(x_1, \tau) d\tau + \int_{x_0}^{x_1} K(\xi, y_0) d\xi \right] - \int_0^y X(x, \tau) d\tau - \int_{x_0}^x K(\xi, y_0) d\xi
\end{aligned}$$

$$\begin{aligned}
s_{ij}(x, y) &= \frac{y - y_1}{y_1 - y_0} \int_0^{x_0} a_{ij}(\xi, y_0) d\xi + \frac{y - y_0}{y_1 - y_0} \left[- \int_0^{x_1} a_{ij}(\xi, y_1) d\xi + \int_{y_0}^{y_1} c_{ij}(x_0, \tau) d\tau \right] \\
&\quad + \int_0^x a_{ij}(\xi, y) d\xi - \int_{y_0}^y c_{ij}(x_0, \tau) d\tau \\
t_{ij}(x, y) &= - \frac{x - x_1}{x_1 - x_0} \int_0^{y_0} a_{ij}(x_0, \tau) d\tau + \frac{x - x_0}{x_1 - x_0} \left[\int_0^{y_1} a_{ij}(x_1, \tau) d\tau + \int_{x_0}^{x_1} c_{ij}(\xi, y_0) d\xi \right] \\
&\quad - \int_0^y a_{ij}(x, \tau) d\tau - \int_{x_0}^x c_{ij}(\xi, y_0) d\xi
\end{aligned} \tag{4.44b}$$

Therefore, the components of the velocity field can also be expressed as linear functions of the coefficients k_{ij} .

The three important relations derived in this section are equations (4.39), (4.42), and (4.44). Given a Newtonian orogenic wedge with force boundary conditions at its rear “backstop” and force or velocity boundary conditions along its basement, the function $C(x, y)$ can be determined in principle, by numerically solving these three linear systems of equations. Once $C(x, y)$ is known, stresses, strain rates, and velocities can be determined. The focus in this thesis is on analytical solutions. Therefore, the following chapter considers another case which can be solved, namely, non-Newtonian rheology with some constraints on the deformation pattern of the wedge.

Chapter 5

A Linear Longitudinal Strain Rate Solution For Power-Law Rheology

5.1 Introduction

As discussed in Chapter 4, a general analytical solution for stress, strain rate, and velocity for a two-dimensional power-law incompressible wedge can be obtained, but is extremely difficult to implement unless there are further simplifications and assumptions. In this chapter, two basic assumptions relating to the dynamics of orogenic wedges are used in order to derive a complete analytical solution for a wedge with general power-law rheology, namely, (1) the longitudinal strain rate varies linearly in the x -direction at any depth; and (2) the gradient in shear stress in the x -direction is negligible compared to the gradient of the vertical normal stress in the y -direction.

While it greatly simplifies both the derivation and the final expression of the solution, the first assumption allows the solution to simulate one of the most important features of thick-skinned orogenic wedge, i.e., syn-convergence extension (*cf.* sections 2.6 and 5.1). The second assumption is in agreement with in situ stress measurements (McGarr and Gay, 1978) and has been adopted in numerous studies (Elliott, 1976; Boyer and Elliott, 1982; Davis et al., 1983; Dahlen, 1984; Suppe, 1985; Platt, 1986; Liu and Ranalli, 1992, 1994; Yin, 1986, 1988, 1989, 1993).

The linear longitudinal strain rate solution is then compared analytically and numerically to the classic one-dimensional solution derived by Nye (1957) for the movement of ice sheets, and it is shown to be a two-dimensional extension of the one-dimensional case.

Some simple computational results are used to discuss the role of basal force and/or velocity boundary conditions in the stress state and deformation of a rectangular block with power-law rheology.

5.2 Model geometry and rheology

The two-dimensional geometry of the wedge and the associated coordinate system are identical to those used in Chapter 4 (*cf.* Figure 4.1). The upper surface is a traction-free plane with topographic slope α dipping toward the front, while the basement dips from the front to the rear with slope angle β . The coordinate origin O is fixed on the upper surface at the upper corner of the back of the wedge, with positive x -axis parallel to the surface and pointing towards the front, and positive y -axis perpendicular to the surface and pointing downwards.

As in Chapter 4, steady-state creep power-law rheology is assumed for the bulk rheology of the rock material in the wedge. All rheological parameters, such as the pre-exponential term A , the stress exponent n , and the activation energy Q , can vary with position, since they are functions of rock type and temperature.

Because of the identity of this chapter and Chapter 4 in terms of model geometry and bulk rheology, all the basic relations between strain rate and deviatoric stress and between velocity and strain rate remain unchanged (*cf.* equations (4.1) to (4.10)).

There are basically four assumptions adopted in the derivation of the linear longitudinal power-law solution in this chapter. Among them, two fundamental assumptions have been mentioned in Section 5.1; the remaining two are related to the compressibility and the density of wedge material. These four assumptions are expressed by the following equations:

(1) the longitudinal strain rate ($\dot{\epsilon}_{xx}$) varies linearly in the longitudinal direction at any point within the wedge, i.e.,

$$\frac{\partial^2 \dot{\epsilon}_{xx}}{\partial x^2} = 0 \quad (5.1)$$

(2) the gradient of shear stress in the x -direction is negligible compared with the gradient of vertical normal stress in the y -direction

$$\left| \frac{\partial \sigma_{xy}}{\partial x} \right| \ll \left| \frac{\partial \sigma_{yy}}{\partial y} \right| \quad (5.2)$$

(3) the wedge material is incompressible, i.e., for plane strain

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (5.3)$$

(4) the material density varies with depth only

$$\frac{\partial \rho}{\partial x} = 0 \quad (5.4)$$

The third assumption (equation 5.3) has been used in Chapter 4, and the fourth assumption (equation 5.4) is simply a first approximation to the usual density distribution in the upper crust of the Earth. Although it is greatly affected by the abundance of pore fluid (for a detailed discussion see Moore and Vrolijk, 1992), the density in orogenic wedges still varies mainly in the vertical direction because the porosity of the material does not change significantly in the longitudinal direction but varies mainly in the vertical direction (*cf.* Zhao et al., 1986).

5.3 Derivation of the solution

5.3.1 Stress solution

As derived in Chapter 4, a general stress solution to the equations of equilibrium (4.6) can be expressed as

$$\begin{aligned}\sigma_{xx} &= S(y) - x T'(y) - \bar{\rho}_x g_x x + \int_0^x \int_0^\zeta \frac{\partial^2 C(\tau, y)}{\partial y^2} d\tau d\zeta \\ \sigma_{yy} &= -\bar{\rho}_y g_y y + C(x, y) \\ \sigma_{xy} &= T(y) - \int_0^x \frac{\partial C(\tau, y)}{\partial y} d\tau\end{aligned}\tag{5.5}$$

Using equation (5.2), the function $C(x, y)$ can be easily proven to be zero. In other words, the assumption of negligible gradient of shear stress in the x -direction implies that the vertical normal stress σ_{yy} is approximately lithostatic everywhere within the wedge. Because of the assumption about the density (equation (5.4)), we have $\bar{\rho}_x = \rho$ and the stress solution then becomes

$$\begin{aligned}\sigma_{xx} &= S(y) - x T'(y) - \rho g_x x \\ \sigma_{yy} &= -\bar{\rho} g_y y \\ \sigma_{xx} &= T(y)\end{aligned}\tag{5.6}$$

where $\bar{\rho}$ is the depth-averaged density. In order to separate the stress due to gravity from that originating from non-gravitational tectonic forces, two new functions are introduced

$$\begin{aligned}F(y) &= S(y) + \bar{\rho} g_y y \\ G(y) &= T(y) + \bar{\rho} g_x y\end{aligned}\tag{5.7}$$

Combination of equations (5.6) and (5.7) leads to

$$\begin{aligned}\sigma_{xx} &= -\bar{\rho} g_y y + F(y) - x G'(y) \\ \sigma_{yy} &= -\bar{\rho} g_y y \\ \sigma_{xy} &= -\bar{\rho} g_x y + G(y)\end{aligned}\tag{5.8}$$

which is the general stress solution to the equations of equilibrium (4.6), under the assumptions expressed by equations (5.2) and (5.4).

The state of stress along the rear vertical plane is obtained by setting $x=0$ in equations (5.8)

$$\begin{aligned}\sigma_{xx}(0, y) &= -\bar{\rho} g_y y + F(y) \\ \sigma_{yy}(0, y) &= -\bar{\rho} g_y y \\ \sigma_{xy}(0, y) &= -\bar{\rho} g_x y + G(y)\end{aligned}\tag{5.9}$$

Equations (5.9) show that the functions $F(y)$ and $G(y)$ are the non-gravitational (i.e., tectonic) normal and shear stress components applied longitudinally and vertically on the rear of the wedge (Figure 5.1).

5.3.2 Strain rate solution

As in Chapter 4, the strain rate solution can be derived from the relation between strain rate components and corresponding deviatoric stress components, i.e.,

$$\begin{aligned}\dot{\epsilon}_{xx} &= \lambda \sigma'_{xx} = \frac{\lambda}{2} [F(y) - x G'(y)] \\ \dot{\epsilon}_{yy} &= \lambda \sigma'_{yy} = -\frac{\lambda}{2} [F(y) - x G'(y)] \\ \dot{\epsilon}_{xy} &= \lambda \sigma'_{xy} = \lambda [G(y) - \bar{\rho} g_x y]\end{aligned}\tag{5.10}$$

Figure 5.1 State of stress along the wedge rear boundary ($x=0$). The stress axis refers to magnitude. According to the adopted sign convention, $F(y) < 0$ when compressive; $G(y) < 0$ when shearing downwards.

where σ'_{xx} , σ'_{yy} and σ'_{xy} are the deviatoric stress tensor components and the parameter λ , as defined in equation (4.4), can be expressed as in terms of the effective shear stress

$$\begin{aligned}\tau_E &= \left\{ \frac{1}{4} [F(y) - x G'(y)]^2 + [G(y) - \bar{\rho} g_x y]^2 \right\}^{\frac{1}{2}} \\ \lambda &= A \tau_E^{n-1} = A \left\{ \frac{1}{4} [F(y) - x G'(y)]^2 + [G(y) - \bar{\rho} g_x y]^2 \right\}^{\frac{n-1}{2}}\end{aligned}\tag{5.11}$$

5.3.3 Velocity solution

Since the wedge material is incompressible (i.e., equation (5.3)), a scalar stream function $\varphi(x, y)$ can be introduced to represent the longitudinal and vertical velocity components (cf. e.g. Turcotte and Schubert, 1982; Lliboutry, 1987)

$$u = \frac{\partial \varphi}{\partial y} \quad v = -\frac{\partial \varphi}{\partial x}\tag{5.12}$$

where u and v are the velocity components in the x - and y -directions. Thus, the determination of the velocity vector field is reduced to that of the scalar function $\varphi(x, y)$.

Combination of equations (5.10), (5.11), (5.12) and (4.7) gives

$$\begin{aligned}\frac{\partial^2 \varphi}{\partial x \partial y} &= \frac{\lambda}{2} [F(y) - x G'(y)] \\ \frac{\partial^2 \varphi}{\partial y^2} - \frac{\partial^2 \varphi}{\partial x^2} &= 2\lambda [-\bar{\rho} g_x y + G(y)]\end{aligned}\tag{5.13}$$

This is the strain rate - stress relation expressed in terms of the stream function φ .

To simplify the discussion, the assumption of linearity of longitudinal strain rate (equation 5.1) is expressed in terms of the scalar stream function φ , using equations (4.7) and (5.12), as

$$\frac{\partial^3 \varphi}{\partial x^3} = -p(x) \quad (5.14)$$

where $p(x)$ is an arbitrary function of x . The minus sign before the function $p(x)$ is added to harmonize signs in the final strain rate and velocity equations (i.e., equations (5.29)) and therefore bears no substantial significance.

Differentiating the first of equations (5.13) twice with respect to x and combining it with equation (5.14) leads to

$$\frac{F(y) - x G'(y)}{2} \frac{\partial^2 \lambda}{\partial x^2} - G'(y) \frac{\partial \lambda}{\partial x} = 0 \quad (5.15)$$

Equation (5.15) is a second-order partial differential equation which can be transformed into a first-order equation and then solved by standard procedures (*cf.* Spiegel, 1971). A general solution to equation (5.15) is

$$\lambda = \frac{k(y)}{F(y) - x G'(y)} + f_0(y) \quad (5.16)$$

where $k(y)$ and $f_0(y)$ are two integration functions.

To determine these two integration functions, equations (5.16) and the second of equations (5.13) are combined to give

$$\frac{\partial^2 \varphi}{\partial y^2} - \frac{\partial^2 \varphi}{\partial x^2} = 2 \left[\frac{k(y)}{F(y) - x G'(y)} + f_0(y) \right] [-\bar{\rho} g_x y + G(y)] \quad (5.17)$$

Differentiating equation (5.17) with respect to x and combining it with equation (5.14) yields

$$\frac{\partial^3 \phi}{\partial x \partial y^2} = -p(x) + \frac{2 k(y) G'(y)}{[F(y) - x G'(y)]^2} [-\bar{\rho} g_x y + G(y)] \quad (5.18)$$

Again, differentiating equation (5.18) twice with respect to x and applying equation (5.14) to the left-side of equation (5.18) gives

$$-p''(x) + 12 \frac{k(y)[G'(y)]^3}{[F(y) - x G'(y)]^4} [-\bar{\rho} g_x y + G(y)] = 0 \quad (5.19)$$

The left side of equation (5.19) consists of two terms. The first term (i.e., $p''(x)$) is a function of x only while the second term is a function of both x and y . This means that both terms on the left side of the equation have to be two equal constants with opposite sign. Furthermore, it can be proven that these two constants are zero because the only x variable of the second term appears explicitly in its denominator. Consequently, three cases can be derived from equation (5.19), that is,

$$k(y) = 0 \quad p''(x) = 0 \quad (5.20a)$$

$$G'(y) = 0 \quad p''(x) = 0 \quad (5.20b)$$

$$G(y) = \bar{\rho} g_x y \quad p''(x) = 0 \quad (5.20c)$$

The second solution (equation 5.20b) represents a stress field corresponding to flow of a material compressed between two parallel plates (Nye, 1957), and is not applicable to an orogenic wedge with variable longitudinal deformation pattern. The third solution (equation 5.20c) is not applicable either because it is a stress field with zero shear stress ($\sigma_{xy} = 0$) everywhere in the wedge, which is not able to produce any shear deformation. Therefore, only the first solution (equation 5.20a) is relevant.

Equations (5.19) and (5.20a) imply that

$$p(x) = b_1 x + b_2 \quad (5.21)$$

where b_1 and b_2 are two integration constants. Combining equations (5.21) and (5.14) it follows that

$$\frac{\partial^3 \varphi}{\partial x^3} = -b_1 x - b_2 \quad (5.22)$$

that is, by integrating thrice with respect to x ,

$$\varphi(x, y) = -\frac{b_1}{24} x^4 - \frac{b_2}{6} x^3 + \frac{x^2}{2} f_1(y) + x f_2(y) + f_3(y) \quad (5.23)$$

where $f_i(y)$ ($i=1, 2, 3$) are three integration functions. These can be determined by substituting equations (5.16) and (5.23) into the second of equations (5.13)

$$\begin{aligned}
\lambda &= f_0(y) \\
&= \frac{1}{2[-\bar{\rho}g_{xy} + G(y)]} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \\
&= \frac{1}{2[-\bar{\rho}g_{xy} + G(y)]} \left(\frac{\partial^2 \varphi}{\partial y^2} - \frac{\partial^2 \varphi}{\partial x^2} \right) \\
&= \frac{1}{2[-\bar{\rho}g_{xy} + G(y)]} \left\{ \frac{1}{2} [f_1''(y) + b_1] x^2 + [f_2''(y) + b_2] x - f_1(y) + f_3''(y) \right\}
\end{aligned} \tag{5.24}$$

which implies that

$$\begin{aligned}
f_1''(y) &= -b_1 \\
f_2''(y) &= -b_2 \\
f_3''(y) &= 2[-\bar{\rho}g_{xy} + G(y)]f_0(y) + f_1(y)
\end{aligned} \tag{5.25}$$

Consequently the functions $f_i(y)$ ($i=1, 2, 3$) are given by

$$\begin{aligned}
f_1(y) &= -\frac{b_1}{2} y^2 - a_1 y - b_3 \\
f_2(y) &= -\frac{b_2}{2} y^2 - a_2 y - b_4 \\
f_3(y) &= 2 \int_0^y \int_0^\zeta [-\bar{\rho}g_x \tau + G(\tau)] f_0(\tau) d\tau d\zeta - \frac{b_1}{24} y^4 - \frac{a_1}{6} y^3 - \frac{b_3}{2} y^2 - a_3 y - a_4
\end{aligned} \tag{5.26}$$

where a_1, a_2, a_3, a_4, b_3 and b_4 are integration constants.

The functions $f_0(y)$, $F(y)$ and $G(y)$ are not independent of each other. Their relation can be derived by applying equations (5.16), (5.23), and (5.26) into the first of equations (5.12)

$$\begin{aligned}
\frac{\partial u}{\partial x} &= -x(a_1 + b_1 y) - a_2 - b_2 y \\
&= \frac{F(y) - x G'(y)}{2} f_0(y)
\end{aligned}
\tag{5.27}$$

from which it follows that

$$\begin{aligned}
f_0(y) &= -\frac{2(a_2 + b_2 y)}{F(y)} \\
G'(y) &= -\frac{a_1 + b_1 y}{a_2 + b_2 y} F(y)
\end{aligned}
\tag{5.28}$$

Integrating the second of equations (5.28) with respect to y gives

$$G(y) = -\int_0^y \frac{a_1 + b_1 \tau}{a_2 + b_2 \tau} F(\tau) d\tau
\tag{5.29}$$

where the traction-free boundary condition on the surface of the wedge (i.e., $G(0) = 0$) has been used to determine the integration constant. Equation (5.29) implies that, in the case of linear longitudinal strain rate, the normal and shear stress components of nongravitational stress along the rear boundary of the wedge are not independent of each other.

All unknown functions (i.e., λ , φ , G , and $f_i(y)$ ($i=1, 2, 3$)) have been determined or expressed in terms of function F and integration constants. Using equations (5.23), (5.26), (5.28), and (5.29) in combination with equations (5.6), (5.10), and (5.12), the complete general solution for stress, strain rate, and velocity in the case of linear longitudinal strain rate can be expressed as

$$\begin{aligned}\sigma_{xx} &= -\bar{\rho} g_y y + \left(1 + \frac{a_1 + b_1 y}{a_2 + b_2 y} x\right) F(y) \\ \sigma_{yy} &= -\bar{\rho} g_y y\end{aligned}\tag{5.30a}$$

$$\sigma_{xy} = -\int_0^y \left[\rho g_x + \frac{a_1 + b_1 \tau}{a_2 + b_2 \tau} F(\tau) \right] d\tau$$

$$\begin{aligned}\dot{\epsilon}_{xx} &= -(a_1 + b_1 y)x - (a_2 + b_2 y) \\ \dot{\epsilon}_{yy} &= -\dot{\epsilon}_{xx}\end{aligned}\tag{5.30b}$$

$$\dot{\epsilon}_{xy} = \frac{2(a_2 + b_2 y)}{F(y)} \int_0^y \left[\rho g_x + \frac{a_1 + b_1 \tau}{a_2 + b_2 \tau} F(\tau) \right] d\tau$$

$$\begin{aligned}u &= -\frac{x^2}{2}(a_1 + b_1 y) - x(a_2 + b_2 y) - \frac{b_1}{6}y^3 - \frac{a_1}{2}y^2 - b_3 y - a_3 \\ &\quad + 4 \int_0^y \left\{ \frac{a_2 + b_2 \zeta}{F(\zeta)} \int_0^\zeta \left[\rho g_x + \frac{a_1 + b_1 \tau}{a_2 + b_2 \tau} F(\tau) \right] d\tau \right\} d\zeta\end{aligned}\tag{5.30c}$$

$$v = \frac{b_1}{6}x^3 + \frac{b_2}{2}x^2 + x\left(\frac{b_1}{2}y^2 + a_1 y + b_3\right) + \frac{b_2}{2}y^2 + a_2 y + b_4$$

The function $F(y)$ in the above equations is related to the rheology of the wedge material. Substituting equations (5.30a, b) into equations (4.3) and (4.4) gives

$$-\frac{2(a_2 + b_2 y)}{F(y)} = A \left\{ \frac{1}{4} \left[\left(1 + \frac{a_1 + b_1 y}{a_2 + b_2 y} x\right) F(y) \right]^2 + \left[\bar{\rho} g_x y + \int_0^y \frac{a_1 + b_1 \tau}{a_2 + b_2 \tau} F(\tau) d\tau \right]^2 \right\}^{\frac{n-1}{2}} \tag{5.30d}$$

where the parameter $A = A_0 \exp(-Q/RT)$ is a function of temperature and bulk rheology parameters (A_0, Q) (*cf.* equation 2.9). Equation (5.30d), together with equations (5.30a - c), constitute a complete solution for a two-dimensional wedge with power-law rheology and linear longitudinal strain rate.

5.4 Implementation of analytical solution

5.4.1 General considerations

Several observations about equations (5.30) should be noted here:

(1) There are two kinds of unknowns which are involved in equations (5.30), and need to be determined from boundary conditions: the function $F(y)$, and seven constants of integration (i.e., $a_1, a_2, \dots, b_4$). The function $F(y)$ is unknown since it is related to some of the unknown integration constants (*cf.* equation 5.30d). According to their dynamical implications, these seven constants can be classified into two groups: those related to longitudinal strain rate (a_1, a_2, b_1 , and b_2), and those related to rigid-body motion corresponding to the velocity and rotation rate at the origin (a_3, b_3 , and b_4). These unknown function and constants are listed in Table 5.1.

Table 5.1 Physical significance of function and constants appearing in equations (5.30)

$F(y)$	tectonic normal stress at the rear of the wedge ($x=0$)
a_1, a_2	longitudinal strain rate on top surface ($y=0$)
b_1, b_2	vertical gradient of longitudinal strain rate
a_3, b_4	velocity at the origin ($x=y=0$)
b_3	rigid-body rotation rate

(2) The state of stress at the rear of the wedge ($x=0$), represented by the functions $F(y)$ and $G(y)$, is related to the rheology and the strain rate of the wedge. This is indicated by two factors: (a) $G(y)$ is related to $F(y)$ and the strain rate by equation (5.29); and (b) $F(y)$ is a function of the rheology, the temperature distribution, and the longitudinal strain rate of the wedge (i.e., equation 5.30d).

(3) There are two possible ways to deal with equation (5.30d): (a) computation of the strain rate for a given wedge rheology and independently specified $F(y)$; and (b)

computation of $F(y)$ for a given wedge rheology and strain rate (independently specified or determined by other means). The first way is practically not implementable, since it is very likely that the specified $F(y)$ would not be compatible with the given rheology and temperature distribution (because the four constants a_1 , a_2 , b_1 , and b_2 can be determined by applying the coordinates of any four points within the wedge into equation (5.30d) which results in a system of four equations; but equation (5.30d) with these constants may not be satisfied throughout the whole wedge). The second way, on the other hand, allows determination of *the tectonic normal stress required at the rear of a wedge with given material rheology and temperature distribution to maintain a given longitudinal strain rate*. This approach is used in all the computations reported in this thesis.

(4) Additional constraints on the strain rate and the rheology of the wedge are introduced by equation (5.30d): the left side of the equation is a function of y only while the right side of the equation may vary with both x and y . Obviously this kind of additional constraint is caused by the assumption of linear longitudinal strain rate (i.e., equation 5.1) which eventually requires the parameter λ to be a function of y only (see equation 5.24). The effect of this additional constraint, however, can be reasonably minimized if the equations (5.30) are applied to *a short segment of the wedge* so that the right side of equation (5.30d) does not vary significantly in x direction. (It has been observed that, for a typical wedge segment with a longitudinal width less than 10 km, the longitudinal variation of the computed function $F(y)$ is less than 5%.)

5.4.2 Wedge segmentation

The last observation mentioned above emphasizes the need to subdivide the wedge into a number of segments, with two neighbouring segments linked by a thin boundary layer parallel to the y -direction (see Figure 5.2). Within each segment, a set of constants (i.e., a_1 , a_2 ..., b_4) and the function $F(y)$ can be computed from appropriate force and/or

velocity boundary conditions, and consequently the stress, strain rate, and velocity fields can be found. The stress, strain rate, and velocity fields in the thin boundary layer between two neighbouring segments can be found by linear interpolation (of course, boundary conditions specified for the two neighbouring segments must be compatible). Stress, strain rate, and velocity solutions computed for a segmented wedge are a good first approximation to a continuous wedge if the wedge is subdivided into a sufficient number of short segments.

5.4.3 Mutual dependence of the function $F(y)$ and the strain rate field

Within each wedge segment, where longitudinal strain rate is a linear function in the x -direction, the main difficulty in applying the solution given by equations (5.30) derives from the interdependence of function $F(y)$ and longitudinal strain rate (i.e., a_1 , a_2 , b_1 , and b_2). Computation of $F(y)$ from equation (5.30d) requires known values of longitudinal strain rate (a_1 , a_2 , b_1 , and b_2); on the other hand, the computation of the constants of integration requires knowledge of the function $F(y)$ (cf. equations 5.30a - c). This is, of course, only a computational difficulty. Physically, the interdependence of tectonic stress and longitudinal strain rate is simply a consequence of the rheological properties of the wedge.

In order to overcome this computational deadlock between the function $F(y)$ and the longitudinal strain rate field, other conditions must be specified in advance. Specification of the function $F(y)$ is not possible as explained in the previous subsection. Therefore, one must assign sufficient constraints on the spatial distribution of longitudinal strain rate and then compute $F(y)$. Once $F(y)$ is determined, the remaining constants a_3 , b_3 , and b_4

Figure 5.2 Subdivision of the wedge into a series of short segments. See text for details.

(the second group of constants of Table 5.1) can be determined from velocity boundary conditions.

The reasonableness of the assigned spatial constraints on longitudinal strain rate can be tested by the smoothness of solution (for stress, strain rate and velocity) in the whole wedge, and by considering the state of stress along the wedge basement, i.e. the basal shear stress τ_b . This should be reasonably smooth, increase from the front to the rear, and be in the 0 - 100 MPa range (*cf.* section 2.4 for detailed discussion).

5.4.4 Solution implementation

For each wedge segment i ($i=1, \dots, m$), the following two sets of boundary conditions are specified (Figure 5.3): three points on the top boundary with assigned velocity (u, v) and one point on the basement with assigned basal shear stress (τ_b). Within the segment a local coordinate system is defined with origin at the top left corner and local x and y axes parallel to the global axes as defined in Figure 4.1. Applying the velocity boundary conditions to equation (5.30c), one has

$$\begin{aligned} -\frac{x_k^2}{2}a_1 - x_k a_2 - a_3 &= u_k \\ \frac{x_k^3}{6}b_1 + \frac{x_k^2}{2}b_2 + b_3 x_k + b_4 &= v_k \end{aligned} \quad (k = 1, 2, 3) \quad (5.31)$$

where (u_k, v_k) are the velocity components assigned at points $(x_k, 0)$ ($k = 1, 2, 3$).

The constants a_1 , a_2 , and a_3 are computed from the first of equations (5.31). This constrains the longitudinal strain rate along the top surface. An additional equation is needed to compute b_j ($j=1, 2, 3, 4$) from the second of equations (5.31). This additional relation is provided by the specification of shear stress at one point along the lower boundary of the wedge segment (see Figure 5.3)

Figure 5.3 A short segment of wedge with assigned velocity boundary conditions on top and basal shear stress on bottom. Within the segment a local coordinate system is defined with origin at the top left corner. See text for details.

$$\begin{aligned}
\tau_b &= \frac{1}{2}(\sigma_{yy} - \sigma_{xx})\sin 2\theta_b - \sigma_{xy}\cos 2\theta_b \\
&= -\frac{1}{2}\left(1 + \frac{a_1 + b_1 y_b}{a_2 + b_2 y_b} x_b\right) F(y_b) \sin 2\theta_b + \cos 2\theta_b \int_0^{y_b} \left[\rho g_x + \frac{a_1 + b_1 \tau}{a_2 + b_2 \tau} F(\tau) \right] d\tau
\end{aligned} \tag{5.32}$$

where the basal shear stress τ_b is defined at the point (x_b, y_b) , and θ_b is the sum of top and basal slope angles at that point. Unfortunately, equation (5.32) can not be used directly since the function $F(y)$ is invoked in this equation. To solve this dilemma, a simple linear relation between the constant b_1 and the top longitudinal strain rate at the central point of the segment is assumed

$$b_1 = \gamma(a_2 + a_1 w) \tag{5.33}$$

where w is half the width of the wedge segment and the parameter γ is a constant. Together with the second of equations (5.31), equation (5.33) is initially used with a trial value for the parameter γ to compute b_j ($j=1, 2, 3, 4$). The correct value for γ is found by examining the convergence of the computed basal shear stress (τ_b^*) and the assigned basal shear stress (τ_b).

5.4.5 Flowchart and numerical considerations

A flowchart describing the main steps in the solution of equations (5.30) is shown in Figure 5.4.

Equation (5.30d) is a complicated nonlinear function of $F(y)$, and computation of $F(y)$ is very sluggish if equation (5.30d) is used directly. Also, there exists a singular point at depth $y_s = -a_2/b_2$. To avoid these problems, the following modifications of equation (5.30d) are implemented:

Figure 5.4 A simplified flowchart for the implementation of the solution to equations (5.30) for each individual wedge segment (ϵ is a pre-assigned small positive value).

(1) Introducing an effective viscosity function $\eta(y)$, equation (5.30d) is rewritten as

$$\frac{1}{2\eta(y)} = \tilde{A} \left\{ 4[a_2 + b_2 y + (a_1 + b_1 y)w]^2 \eta(y)^2 + \left[\int_0^y [\rho(\tau)g_x - 4(a_1 + b_1 \tau)\eta(\tau)] d\tau \right]^2 \right\}^{\frac{\tilde{n}-1}{2}} \quad (5.34)$$

where w is half the width of each segment and $\tilde{A}(y) = A(w, y)$ and $\tilde{n}(y) = n(w, y)$. The function $\eta(y) = -F(y)/4(a_2 + b_2 y)$ is the effective viscosity of the wedge segment and it becomes the real Newtonian viscosity when the stress exponent of bulk rheology equals unity.

(2) Differentiation of equation (5.34) with respect to y yields

$$\frac{d\eta(y)}{dy} = \frac{h_0(y) + h_1(y)s(y)}{h_2(y)} \quad (5.35a)$$

where

$$\begin{aligned} h_0(y) &= 4(b_2 + b_1 w)[a_2 + b_2 y + (a_1 + b_1 y)w] \eta^2 - \frac{(2\tilde{A}\eta)^{\frac{2}{1-\tilde{n}}}}{1-\tilde{n}} \left[\frac{1}{\tilde{A}} \frac{d\tilde{A}}{dy} + \frac{\ln(2\tilde{A}\eta)}{1-\tilde{n}} \frac{d\tilde{n}}{dy} \right] \\ h_1(y) &= \rho g_x - 4(a_1 + b_1 y)\eta \\ h_2(y) &= \frac{(2\tilde{A}\eta)^{\frac{2}{1-\tilde{n}}}}{(1-\tilde{n})\eta} - 4[a_2 + b_2 y + (a_1 + b_1 y)w]^2 \eta \\ s(y) &= \int_0^y [\rho(\tau)g_x - 4(a_1 + b_1 \tau)\eta(\tau)] d\tau \end{aligned} \quad (5.35b)$$

Equation (5.35a) can be integrated numerically (using the *Runge-Kutta integration method with adaptive step-size control* ; Press et al., 1992). The numerical computation of the

function $s(y)$ is done by dividing the total integration range into a series of small sub-ranges within which integration is approximated by the *trapezoidal rule* (Kreyszig, 1988).

(3) The boundary value $\eta_0(= \eta(0))$ of the viscosity function is computed by setting $y=0$ in equation (5.34)

$$\eta(0) = \frac{|a_2 + a_1 w|^{\frac{1}{\bar{n}(0)} - 1}}{2\tilde{A}(0)^{\frac{1}{\bar{n}(0)}}} \quad (5.36)$$

Equation (5.30d) (see Figure 5.4) is therefore replaced by the combination of equations (5.34), (5.35), and (5.36) to compute the function $F(y)$ through the effective viscosity function $\eta(y)$.

The solution given by equations (5.30a - c) can be conveniently expressed in terms of the effective viscosity function $\eta(y)$ as

$$\begin{aligned} \sigma_{xx} &= -g_y \int_0^y \rho(\tau) d\tau - 4[a_2 + b_2 y + (a_1 + b_1 y)x] \eta(y) \\ \sigma_{yy} &= -g_y \int_0^y \rho(\tau) d\tau \\ \sigma_{xy} &= -\int_0^y [\rho(\tau) g_x - 4(a_1 + b_1 \tau) \eta(\tau)] d\tau \end{aligned} \quad (5.37a)$$

$$\begin{aligned} \dot{\epsilon}_{xx} &= -(a_1 + b_1 y)x - (a_2 + b_2 y) \\ \dot{\epsilon}_{yy} &= -\dot{\epsilon}_{xx} \\ \dot{\epsilon}_{xy} &= -\frac{1}{2\eta(y)} \int_0^y [\rho(\tau) g_x - 4(a_1 + b_1 \tau) \eta(\tau)] d\tau \end{aligned} \quad (5.37b)$$

$$\begin{aligned}
u &= -\frac{x^2}{2}(a_1 + b_1 y) - x(a_2 + b_2 y) - \frac{b_1}{6}y^3 - \frac{a_1}{2}y^2 - b_3 y - a_3 \\
&\quad - \int_0^y \left\{ \frac{1}{\eta(\varsigma)} \int_0^\varsigma [\rho(\tau) g_x - 4(a_1 + b_1 \tau) \eta(\tau)] d\tau \right\} d\varsigma \\
v &= \frac{b_1}{6}x^3 + \frac{b_2}{2}x^2 + x\left(\frac{b_1}{2}y^2 + a_1 y + b_3\right) + \frac{b_2}{2}y^2 + a_2 y + b_4
\end{aligned} \tag{5.37c}$$

5.5 Comparison between two-dimensional and one-dimensional solutions

The above linear longitudinal strain rate solution is a two-dimensional extension of the classic one-dimensional solution for glacier flow (Nye, 1957). In this section this statement is proven by a detailed comparison between the two solutions, including computational results.

It is obvious from Table 5.2 that the solution obtained in this chapter is a two-dimensional extension of Nye's classic one-dimensional solution. The present solution reduces to Nye's solution for $a_1 = b_1 = b_2 = b_3 = 0$, which is equivalent to the fourth assumption used in Nye's analysis (i.e., $\partial v / \partial x = 0$). In order to obtain the solution in the one-dimensional case, gravity was introduced into a solution given by Prandtl in 1923 (*cf.* review by Nádai, 1931) for the compression of a rigid-plastic weightless layer (Nye, 1951), which results in a stress tensor independent of longitudinal location. The less restrictive assumption that the longitudinal gradient in horizontal shear stress is negligible compared with the vertical gradient in vertical normal stress results in a horizontal normal deviatoric stress which is a linear function of the longitudinal location. Through segmentation of the wedge, this allows to accommodate a much more complex velocity field.

Table 5.2 Comparison between Nye's one-dimensional solution and the present two-dimensional solution

1-D theory* (Nye, 1957)	present 2-D solution
<p style="text-align: center;">model geometry and rheology</p> <div style="display: flex; justify-content: space-around;"> </div> <p>The basement is parallel to the top surface ($\alpha = \beta$); power-law rheology is postulated as a function of depth y only</p>	
<p style="text-align: center;">model assumptions</p> <div style="display: flex; justify-content: space-between;"> <div style="width: 48%;"> <p>(1) incompressible plane strain flow; (2) stress tensor is function of depth only; (3) density is function of depth only; (4) $\partial v / \partial x = 0$</p> </div> <div style="width: 48%;"> <p>(1) incompressible plane strain flow; (2) $\partial \sigma_{xy} / \partial x \ll \partial \sigma_{yy} / \partial y$; (3) density is function of depth only; (4) $\partial^2 \dot{\epsilon}_{xx} / \partial x^2 = 0$</p> </div> </div>	
<p style="text-align: center;">stress solution</p> <div style="display: flex; justify-content: space-between;"> <div style="width: 48%;"> $\sigma_{xx} = -\bar{\rho} g_y y + F(y)$ $\sigma_{yy} = -\bar{\rho} g_y y$ $\sigma_{xy} = -\bar{\rho} g_x y$ $-\frac{2a_2}{F(y)} = A^{-n} \tau_E'^{n-1}$ </div> <div style="width: 48%;"> $\sigma_{xx} = -\bar{\rho} g_y y + \left[1 + \frac{a_1 + b_1 y}{a_2 + b_2 y} x \right] F(y)$ $\sigma_{yy} = -\bar{\rho} g_y y$ $\sigma_{xy} = -\bar{\rho} g_x y - \int_0^y \frac{a_1 + b_1 \tau}{a_2 + b_2 \tau} F(\tau) d\tau$ $-\frac{2(a_2 + b_2 y)}{F(y)} = A \tau_E'^{n-1}$ </div> </div>	
<p style="text-align: center;">strain rate solution</p> <div style="display: flex; justify-content: space-between;"> <div style="width: 48%;"> $\dot{\epsilon}_{xx} = -a_2$ $\dot{\epsilon}_{yy} = a_2$ $\dot{\epsilon}_{xy} = \frac{2a_2 \bar{\rho} g_x y}{F(y)}$ </div> <div style="width: 48%;"> $\dot{\epsilon}_{xx} = -(a_1 + b_1 y)x - (a_2 + b_2 y)$ $\dot{\epsilon}_{yy} = (a_1 + b_1 y)x + (a_2 + b_2 y)$ $\dot{\epsilon}_{xy} = \frac{2(a_2 + b_2 y)}{F(y)} \left[\bar{\rho} g_x y + \int_0^y \frac{a_1 + b_1 \tau}{a_2 + b_2 \tau} F(\tau) d\tau \right]$ </div> </div>	
<p style="text-align: center;">velocity solution</p> <div style="display: flex; justify-content: space-between;"> <div style="width: 48%;"> $u = -a_2 x + 4a_2 g_x \int_0^y \frac{\bar{\rho} \tau}{F(\tau)} d\tau - a_3$ $v = a_2 y + b_4$ </div> <div style="width: 48%;"> $u = -\frac{a_1}{2} (x^2 + y^2) - a_2 x - a_3 - \frac{b_1}{2} y \left(x^2 + \frac{y^2}{3} \right) - b_2 x y$ $-b_3 y + 4 \int_0^y \left\{ \frac{a_2 + b_2 \zeta}{F(\zeta)} \left[\bar{\rho} g_x \zeta + \int_0^\zeta \frac{a_1 + b_1 \tau}{a_2 + b_2 \tau} F(\tau) d\tau \right] \right\} d\zeta$ $v = a_1 x y + a_2 y + \frac{b_1}{2} x \left(\frac{x^2}{3} + y^2 \right) + \frac{b_2}{2} (x^2 + y^2) + b_3 x + b_4$ </div> </div>	

* Minor modifications to Nye's original expressions have been made to facilitate comparison.

Figure 5.5 Parallel-sided rectangular block with power-law rheology and basement slope α .

Table 5.3 Values of model parameters for rectangular block with length $L = 250$ km, depth $H = 40$ km and constant density $\rho = 2800$ kg/m³ over a basement with slope angle $\alpha = 3^\circ$

Parameter	Case*					
	A	B	C	D	E	F
$u_0 (\times 10^{-10} \text{ m s}^{-1})$	31.71 **	31.71	31.71	31.71	31.71	31.71
$u_1 (\times 10^{-15} \text{ s}^{-1})$	-1	-1	-1	-1	-1	-1
$u_2 (\times 10^{-20} \text{ m}^{-1} \text{ s}^{-1})$	0	0	-1.8	0	0	0
$v_0 (\times 10^{-10} \text{ m s}^{-1})$	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4
$v_1 (\times 10^{-15} \text{ s}^{-1})$	0	0	0	-4	0	0
τ_0 (MPa)	-57.44	-57.44	-57.44	-57.44	-57.44	-57.44
τ_1 (MPa km ⁻¹)	0	0	0.068	0	0	0
T_0 (K)	§				750	750
T_1 (K km ⁻¹)					0	2.5
$A_0^* (\times 10^{-23} \text{ Pa}^{-1} \text{ s}^{-1})$	5	5	5	5		
$A_1^* (\times 10^{-23} \text{ Pa}^{-1} \text{ s}^{-1} \text{ km}^{-1})$	0	1.125	0	0		
power-law rheology	anorthosite†					

* Cases A, B, C, D, E, and F refer to Figures 5.6, 5.7, 5.8, 5.9, 5.10, and 5.11

A: Newtonian, constant η ($=10^{22}$ Pa s), constant $\dot{\epsilon}_{xx}$ and constant v on surface, constant τ_b

B: Newtonian, depth-dependent η , constant $\dot{\epsilon}_{xx}$ and constant v on surface, constant τ_b

C: Newtonian, constant η ($=10^{22}$ Pa s), variable $\dot{\epsilon}_{xx}$ and constant v on surface, variable τ_b

D: Newtonian, constant η ($=10^{22}$ Pa s), constant $\dot{\epsilon}_{xx}$ and variable v on surface, constant τ_b

E: Power-law, constant T , constant $\dot{\epsilon}_{xx}$ and constant v on surface, constant τ_b

F: Power-law, depth-dependent T , constant $\dot{\epsilon}_{xx}$ and constant v on surface, constant τ_b

** $100 \text{ mm a}^{-1} \approx 31.71 \times 10^{-10} \text{ m s}^{-1}$

§ empty entry indicates that parameter is not applicable

† creep parameters: $A_0 = 2.06 \times 10^{-23} (\text{Pa}^{-3.2} \text{ s}^{-1})$, $n = 3.2$, $Q = 238 (\text{kJ mol}^{-1})$

In order to illustrate this point, the solution given by equations (5.30) is applied to the analysis of a rectangular block with various rheological parameters and boundary conditions (see Figure 5.5). The parameter values used in the computations are listed in Table 5.3. All cases, for simplicity, are parameterized in the following way:

(a) The velocity boundary conditions along the top free surface are parabolic and linear functions of position, respectively, that is

$$\begin{aligned} u_{top} &= u_0 + u_1 x + u_2 x^2 \\ v_{top} &= v_0 + v_1 x \end{aligned} \quad (5.38a)$$

where u_0 , u_1 , u_2 , v_0 , and v_1 are constants.

(b) The basal shear stress along the block basement is a linear function of position

$$\tau_b = \tau_0 + \tau_1 x \quad (5.38b)$$

where τ_0 and τ_1 are constants.

(c) The temperature within the block varies with depth only and is represented by a linear function

$$T = T_0 + T_1 y \quad (5.38c)$$

where T_0 and T_1 are constants.

(d) The rheology of the block can be nonlinear (stress exponent $n > 1$) or linear ($n = 1$). In case of linear rheology, a depth-dependent decreasing function is assumed for the Newtonian viscosity

$$\eta = \frac{1}{2(A_0^* + A_1^* y)} \quad (5.38d)$$

where A_0^* and A_1^* are constants. This expression is extended to the effective viscosity in nonlinear rheology, and a computer program has been written to implement both linear and nonlinear rheologies in a uniform format.

Case A (Figure 5.6) serves as a reference case with constant Newtonian viscosity ($\eta = 10^{22}$ Pa s), and constant basal shear stress ($\tau_b = 57.44$ MPa). The top velocity boundary condition is specified such that the u component varies linearly with x (resulting in a constant longitudinal strain rate of $\dot{\epsilon}_{xx} = -10^{-15} \text{ s}^{-1}$ on the upper surface of the block) while the v component is constant. These velocity boundary conditions are compatible with Nye's one-dimensional solution (Nye, 1957). Consequently, computational results for case A yield simply a two-dimensional representation of Nye's one-dimensional solution in the case of Newtonian viscosity. (Comparisons between the one-dimensional and two-dimensional solutions in the case of nonlinear rheology will be provided by cases E and F.)

The results for case A are presented in Figure 5.6, showing (a) the geometry of the originally-rectangular block after 1 Ma of deformation; (b) the velocity field within the block; (c) the maximum shear stress trajectories ($\tau_{\max} = (\sigma_1 - \sigma_2)/2$), which coincide with potential faulting surfaces in a frictionless (plastic) material; (d) the depth distribution of the “nongravitational” normal and shear stress components (i.e., the functions $F(y)$ and $G(y)$) along the rear vertical edge of the block (which in this case are constant; note that a tectonic compression is required to maintain the deformation regime); (e) contours of effective strain rate; and (f) contours of effective shear stress. Both effective strain rate and effective shear stress in this case vary with depth only. Also, the longitudinal normal deviatoric stress σ'_{xx} ($= F(y)/2$) does not vary with x and is negative at both the front and the back edges of the block, resulting in a state of longitudinal compression.

Results for case B are shown in Figure 5.7. Comparison between cases A and B shows the effect of a depth-dependent Newtonian viscosity on the stress, strain rate and velocity fields. The viscosity in case B decreases from 10^{22} Pa s on the top to 10^{21} Pa s on the bottom of the block (see Table 5.3); consequently the material becomes softer with increasing depth, increasing the relative deformation between the lower and upper parts of the block (Figure 5.7a; note that the scaling factors used in plotting results for the various

Figure 5.6 Stress and deformation within the block for case A (Table 5.3). (a) deformed shape of the block after 1 Ma; (b) velocity field; (c) maximum shear stress trajectories and type of potential faults; (continued)

Figure 5.6 (f) second invariant of deviatoric stress contour map.

Figure 5.7 Stress and deformation within the block for case B (Table 5.3). (a) deformed shape of the block after 1 Ma; (b) velocity field; (c) maximum shear stress trajectories and type of potential faults; (continued)

Figure 5.7 (d) tectonic-induced normal and shear stresses along the rear of the block; (e) second invariant of strain rate contour map; (continued)

Figure 5.7 (f) second invariant of deviatoric stress contour map; (g) Newtonian viscosity contour map (logarithm to base 10)

cases are not the same). This is reflected also in the velocity field (Figure 5.7b), which shows a marked decrease with increasing depth. The maximum shear stress trajectories (Figure 5.7c) tend to become parallel to the base of the block with increasing depth, indicating an increasingly laminar flow. The nongravitational normal stress at the rear of the block necessary to maintain this deformation regime (Figure 5.7d) decreases in magnitude from -40 MPa on the top to -4 MPa at the bottom of the block. The spatial distribution of the effective shear strain rate $\dot{\epsilon}'_E$ (Figure 5.7e) shows that the magnitude of deformation in the lower part of the block in case B is approximately one order larger than that in case A. The effective shear stress σ'_E (Figure 5.7f) in the lower part of the block, however, is roughly the same as that in case A, reflecting the fact that same basal shear stress boundary condition is assigned in both cases A and B. Note that, for incompressible material, the effective shear stress σ'_E is the same as the maximum shear stress $\tau_{\max} = (s_1 - s_2)/2$. It is also noted from Figure 5.7f that σ'_E first decreases towards a minimum (at a depth of about 5 km) and then increases with depth. Because in this case $\sigma'_E = [F(y)^2/4 + (\bar{\rho}g_x y)^2]^{1/2}$ is a function of $F(y)$ and the gravitational shear stress, such a distribution of σ'_E as shown in Figure 5.7f is a direct consequence of the specified depth distribution of the Newtonian viscosity which largely determines the function $F(y)$ (see Figure 5.7d). The spatial variation of the Newtonian viscosity of the block is shown in Figure 5.7g. Similar to case A, normal deviatoric stresses are negative at both the front and the back edges of the block.

Case C (Figure 5.8) differs from case A in two respects: (1) the u component of the velocity boundary condition on the top surface results in a longitudinal strain rate varying linearly in the x -direction (decreasing shortening from the front towards the rear of the block); and (2) the magnitude of basal shear stress is assumed to decrease from the rear toward the front (from 57.44 MPa to 40.44 MPa). The deformed configuration and the velocity field are shown in Figures 5.8a and b, respectively. Both are quite different from those of case A. The velocity decreases from the rear towards the front much faster in case C than in case A. Also the maximum shear stress trajectories (Figure 5.8c) dip more steeply

Figure 5.8 Stress and deformation within the block for case C (Table 5.3). (a) deformed shape of the block after 1 Ma; (b) velocity field; (c) maximum shear stress trajectories; (continued)

Figure 5.8 (d) tectonic-induced normal and shear stresses along the rear boundary of the block; (e) second invariant of strain rate contour map; (continued)

Figure 5.8 (f) second invariant of deviatoric stress contour map.

in case C than in case A. The nongravitational normal stress at the rear of the block necessary to maintain this deformation regime (Figure 5.8d) increases slightly with depth while that in case A remains constant. The spatial distribution of the effective shear strain rate $\dot{\epsilon}'_E$ (Figure 5.8e) shows that the deformation in the block varies mainly in the x -direction in case C rather than in the y -direction as in case A. Similar comparisons can also be made for the spatial distribution of the effective shear stress σ'_E (Figure 5.8f).

Case D (Figure 5.9) can be used to assess the effect on the solution of the v -component of the velocity boundary condition specified on top of the block. Here, the (upward) v -component along the top surface is assumed to increase in magnitude from 0.126 cm a⁻¹ at rear to 3.28 cm a⁻¹ at the front. Figure 5.9a shows the deformation of the block after 1 Ma. The velocity field is plotted in Figure 5.9b. The velocity distribution is greatly affected by the kinematic boundary conditions on the top surface. Trajectories of maximum shear stress are shown in Figure 5.9c; the depth distribution of tectonic stresses along the rear vertical plane is plotted in Figure 5.9d. A change of the v component of the top velocity boundary condition has no effect on the state of stress within the block (Figure 5.9f). On the other hand, an identical stress field and identical rheology result in an identical strain rate field, as can be seen by comparing effective shear strain rates in Figures 5.6e and 5.9e.

Cases E and F consider nonlinear rheology, using the power-law creep parameters of anorthosite (*cf.* Ranalli, 1995). Results for case E are shown in Figure 5.10 in the usual way, with the addition of a further diagram showing the variation of effective viscosity with position as in case B. Comparison between Figures 5.6 and 5.10 reveals a great deal of similarity between cases A and E. This is the result of a near-constant effective viscosity throughout the block in case E (Figure 5.10g); the average value of the effective viscosity is very close to the constant Newtonian viscosity specified in case A. However, the difference between linear rheology and power-law rheology becomes obvious from a comparison of the “tectonic” stress along the rear vertical plane necessary to drive the deformation (Figures 5.6d and 5.10d). The normal stress $F(y)$ in case A has a constant value of -40

Figure 5.9 Stress and deformation within the block for case D (Table 5.3). (a) deformed shape of the block after 1 Ma; (b) velocity field; (c) maximum shear stress trajectories; (continued)

Figure 5.9 (d) tectonic-induced normal and shear stresses along the rear boundary of the block; (e) second invariant of strain rate contour map; (continued)

Figure 5.9 (f) second invariant of deviatoric stress contour map.

Figure 5.10 Stress and deformation within the block for case E (Table 5.3). (a) deformed shape of the block after 1 Ma; (b) velocity field; (c) maximum shear stress trajectories; (continued)

Figure 5.10 (d) tectonic-induced normal and shear stresses along the rear boundary of the block; (e) second invariant of strain rate contour map; (continued)

Figure 5.10 (f) second invariant of deviatoric stress contour map; (g) effective viscosity contour map (logarithm to base 10).

Figure 5.11 Stress and deformation within the block for case F (Table 5.3). (a) deformed shape of the block after 1 Ma; (b) velocity field; (c) maximum shear stress trajectories; (continued)

Figure 5.11 (d) tectonic-induced normal and shear stresses along the rear boundary of the block; (e) second invariant of strain rate contour map; (continued)

Figure 5.11 (f) second invariant of deviatoric stress contour map; (g) effective viscosity contour map (logarithm to base 10).

MPa, while in case E the required compression decreases in magnitude nonlinearly from about -76 MPa at the top to about -30 MPa at the bottom of the block.

A constant temperature field of 750 K has been assumed in case E. In case F, an increase of temperature from the top to the bottom of the block is examined, for the same power-law rheology. The material becomes softer with increasing depth, as the temperature increases from 750 to 850 K across the thickness of the block. Results for this case are shown in Figure 5.11. Comparing them with Figure 5.10, it is seen that even a rather low temperature gradient increases considerably the ductility of the block material. The effect of temperature is also visible in the stress and strain rate fields (Figures 5.11c, e): predominant laminar flow and high concentration of strain rate are found near the base of the block. The increase in ductility with depth is also reflected in the profile of effective viscosity (Figure 5.11g), which decreases from approximately 1.9×10^{22} to 8.9×10^{19} Pa s from the top to the base of the block. The non-zero temperature gradient also affects the depth distribution of the maximum shear stress which, different from that in case E, first decreases towards a minimum (at a depth of about 15 km) and then increases with depth (Figure 5.11f). A similar depth distribution of the maximum shear stress is also found in case B (see Figure 5.7f). As in case B, a combination of temperature, rheology, and top longitudinal deformation pattern as specified in case F leads to such a distribution of $F(y)$ that, when combined with the gravitational shear stress ($\bar{\rho}g_x y$), results in the depth distribution of the maximum shear stress as shown in Figure 5.11f.

Cases A, B, D, E, and F are actually two-dimensional extensions of Nye's (1957) one-dimensional model (with linear and nonlinear rheology) and, as discussed previously, reduce to Nye's solution in the one-dimensional case.

5.6 Comparison between the two-dimensional power-law solution and Platt's model

It is interesting to compare the present solution with Platt's dynamic model for orogenic wedges (Platt, 1986), which has been briefly reviewed in section 2.4 of this thesis. It will be shown that Platt's model is a particular case of the present solution for stress.

It should be pointed out that an additional assumption is needed in Platt's model in order to derive equation (2.7). Such an additional assumption, not mentioned in Platt's paper (Platt, 1986), should be that the longitudinal deviatoric stress τ_{xx} is always equal to zero on the upper surface of the wedge. This assumption is implicitly used when integration with respect to y is performed on the stress equilibrium equation in x -direction (see equations 11 and 12 in Platt, 1986). This additional condition means that the near-surface part of the wedge is always under a lithostatic state of stress, implying that no ductile or brittle deformation occurs in the near-surface area.

In the following, the solution given in equations (5.30) is recast in a format that is directly comparable with Platt's model (i.e., equation 2.7; see Platt, 1986).

Let τ_{xx} and $\bar{\tau}_{xx}$ be the longitudinal deviatoric stress and the depth-averaged longitudinal deviatoric stress, respectively. It follows from equation (5.8) that

$$\tau_{xx} = \frac{F(y) - x G'(y)}{2} \quad (5.39)$$

$$\frac{\partial \bar{\tau}_{xx}}{\partial x} = \frac{\partial}{\partial x} \left[\frac{1}{h} \int_0^h \tau_{xx} dy \right] = -\frac{1}{2h} \int_0^h G'(y) dy = -\frac{G(y=h)}{2h}$$

where $h(x)$ is the thickness of the wedge at the point under consideration (see Figure 5.12). The basal traction (shear stress) τ_b is related to the state of stress within the wedge at that point by the following equation (*cf.* Ranalli, 1995)

Figure 5.12 Simplified wedge model. α is the surface slope and β the basal slope. δ is the angle between the positive x axis and the normal of the base to the wedge; τ_b is the basal shear stress exerted on the wedge by the subducting basement.

$$\begin{aligned}
\tau_b &= \frac{1}{2}(\sigma_{yy} - \sigma_{xx})_{y=h} \sin 2\delta + (\sigma_{xy})_{y=h} \cos 2\delta \\
&= \frac{1}{2}(\sigma_{yy} - \sigma_{xx})_{y=h} \sin 2\theta - (\sigma_{xy})_{y=h} \cos 2\theta
\end{aligned} \tag{5.40}$$

Combining equations (5.40) and (5.39) leads to

$$\tau_b \approx \bar{\rho}gh\alpha - 2\theta(\tau_{xx})_{y=h} + 2h\frac{\partial \bar{\tau}_{xx}}{\partial x} \tag{5.41}$$

where small-angle approximations for α and θ are used.

Comparison of equations (5.41) and (2.7) shows two differences: (1) the signs of the third term on the right side are different; and (2) the stress component in the second term on the right side of equation (5.41) is the longitudinal deviatoric stress on the basement (i.e., $y=h$) while the corresponding term in equation (2.7) is the depth-averaged longitudinal deviatoric stress. The sign difference is a trivial consequence of the difference in coordinate systems adopted in this thesis and in Platt's model (compare Figures 5.12 and 2.5). The second difference is caused by the aforementioned assumption about the stress state along the wedge upper surface.

Therefore, Platt's (1986) stability criterion is compatible with - indeed, it is a particular case of - the stress solution derived in this chapter, subject to the assumption that the longitudinal deviatoric stress, τ_{xx} , does not vary significantly along any depth profile. This condition most likely holds in thin-skinned wedges but may not be satisfied in thick-skinned wedges, where deformation patterns may vary significantly in the vertical direction.

Substituting equations (5.37a) into (5.41), one obtains

$$\tau_b \approx \bar{\rho}gh\alpha - 4\theta[a_2 + b_2h + (a_1 + b_1h)x]\eta(h) - 4h\int_0^h (a_1 + b_1y)\eta(y)dy \tag{5.42}$$

Following the same line of reasoning as in Platt (1986), equation (5.42) implies that a wedge is stable if the following conditions are satisfied

$$a_1 = a_2 = b_1 = b_2 = 0 \quad (5.43)$$

Combination of equations (5.43) and (5.30) or (5.37) shows that a wedge in a *standard state of stress* (where normal stresses are hydrostatic everywhere (Anderson, 1951)) suffers no internal deformation, but can be subject to rigid-body motion (that is, either in translation if the constant $b_3 = 0$ or in rotation if the constants $a_3 = b_4 = 0$. In general, the motion is a combination of translation and rotation). On the other hand, if conditions (5.43) are not satisfied, the longitudinal deviatoric stress and the corresponding strain rate will not be zero everywhere within the wedge, and the four constants (a_1 , a_2 , b_1 , b_2) will vary in a manner related to the average upper slope α of the wedge, if this is different from the critical taper defined by Platt in equation (2.7).

All the previous discussion is based on the assumption that the basal shear stress τ_b remains unchanged during the tectonic evolution of the wedge. This is particularly important, in view of the fact that a change in τ_b has important consequences on the deformational style of orogenic wedges (*cf.* Willett 1992 and section 2.4 of this thesis).

Chapter 6

Orogenic Wedges With Viscous, Power-law, and Plastic Rheologies

6.1 Introduction

Both analytical comparison and numerical computation in Chapter 5 support the following statements: (1) the linear longitudinal strain rate solution (i.e., equations 5.30) is a two-dimensional extension of the classic one-dimensional solution derived by Nye (1957) for ice sheets; (2) some popular qualitative discussions such as Platt's model for orogenic wedges (Platt, 1986) can also be formulated from the two-dimensional solution; (3) two-dimensional blocks (not necessarily rectangular) with generalized power-law rheology and complicated deformation styles can be analyzed by applying the two-dimensional linear longitudinal strain rate solution into a "segmentation approximation model" of the block.

In this chapter, this two-dimensional linear longitudinal strain rate solution is applied to study the stress and deformation in orogenic wedges with rheological parameters and temperature values derived from conditions typically found in orogenic belts.

6.2 Models: geometry, composition and temperature

The geometry of an orogenic wedge as modeled in this chapter is described by three parameters (Figure 6.1): upper and lower slopes (α and β) and the width of the wedge (L). For simplicity, the slope α is assumed to be constant throughout the whole wedge in all the computations, although a variable α can be accommodated through segmentation

of a large wedge block (see Figure 5.2). Similarly a constant basal slope β is assumed in all computations. All computation results are presented within a global coordinate system which is defined with its origin O fixed at the rear top corner of wedge in its initial position and with positive x axis parallel to the wedge top surface and directed towards the front, while the positive y axis is perpendicular to the top surface and pointing to the basement. This coordinate system is termed *global* as against those *local* ones defined in each wedge segment (see Figure 5.3). The basal shear stress τ_b is specified as part of the boundary conditions.

The petrological composition of orogenic wedges is variable. In general, orogenic wedges should have a bulk composition approximating that of continental crust (Pavlis and Bruhn, 1983). All model computations presented in this chapter assume orogenic wedges to consist of rocks with the rheology of quartzite (soft end of the spectrum) and anorthosite (hard end of the spectrum).

Fluid movement within orogenic wedges plays an important role in determining the overall dynamic behaviour during tectonic evolution (von Huene, 1984; Vrolijk, 1987; Fisher and Byrne, 1990; Bebout, 1991). The importance of fluid for rock deformation in the brittle regime is well known (Jaeger and Cook, 1969; Dahlen, 1990; Ranalli, 1995), and it can also affect the bulk ductile rheology: wet and dry rocks of the same type have different values of bulk rheological parameters (Table 6.1). The role of fluid on the dynamics of orogenic wedges is studied in this chapter by choosing both wet and dry rocks as constituents of the wedges.

The magnitude of strain rate in orogenic complexes is generally estimated to be between 10^{-13} and 10^{-15} s^{-1} (Pfiffner and Ramsay, 1982; Ranalli and Murphy, 1987; Twiss and Moores, 1992, p. 382; Crespi et al., 1996).

Figure 6.1 An orogenic wedge model parameterized by surface slope (α), basal slope (β) and width (L). Basal shear stress and gravity are denoted by τ_b and g , respectively.

Table 6.1 Steady-state power-law creep parameters for crustal materials (Ranalli & Murphy, 1987; data modified from Kirby, 1983)

Rock Material	$A_0 (\text{Pa}^{-n} \text{s}^{-1})$	n	$Q (\text{kJ mol}^{-1})$
rocksalt	9.976×10^{-32}	5.3	102
granite	7.924×10^{-29}	3.2	123
granite (wet)	7.943×10^{-16}	1.9	137
quartzite	2.512×10^{-20}	2.4	156
quartzite (wet)	3.990×10^{-18}	2.3	154
albite rock	1.033×10^{-29}	3.9	234
anorthosite	2.060×10^{-23}	3.2	238
quartz diorite	5.024×10^{-18}	2.4	219
diabase	8.038×10^{-25}	3.4	260

Another important factor affecting the rheological behaviour is the thermal regime of the wedges. The importance of temperature in rock deformation can be seen from equations (4.4) and (4.5) which shows explicitly the dependence of the power-law relation between effective shear strain rate and effective shear stress on the temperature. The temperature distribution in the lithosphere can be formulated by the heat conduction equation (*cf.* Ranalli, 1995)

$$K \nabla^2 T = c \rho \frac{\partial T}{\partial t} - H_G \quad (6.1)$$

where c and ρ are the specific heat and density, respectively; K is the thermal conductivity and H_G the heat generation rate per unit volume. Assuming, as a first approximation, a steady-state thermal regime, a constant heat-production function H_G and depth-dependent-only temperature distribution, the solution to equation (6.1) is

$$T(z) = T_0 + \frac{q_0}{K} z - \frac{H_G}{2K} z^2 \quad (6.2)$$

where T_0 and q_0 are surface temperature and heat flow, respectively, and z is depth.

In a ‘typical’ accretionary wedge, observed heat flow and heat-production are about $30 - 40 \text{ mW m}^{-2}$ and $1 \text{ } \mu\text{W m}^{-3}$, respectively (Pavlis and Bruhn, 1983). Heat flow observations and knowledge of thermal properties can be used to calculate geotherms in orogenic wedges (*cf.* e.g. Hyndman and Wang, 1993). In most of the computations presented in this chapter, a linear depth distribution of temperature is assumed, and the effects of temperature on deformation are studied by considering both ‘cool’ and ‘hot’ wedges (i.e., wedges with low and high geothermal gradients).

In the following sections of this chapter, models are presented in two groups: (a) the rheology of the whole wedge is viscous power-law or linear creep; and (b) the rheology of the wedge, controlled mainly by the temperature and rock type, is perfectly plastic in the upper part and power-law in the lower part.

6.3 Orogenic wedges with linearly viscous and power-law rheologies

Following the same procedure used in Section 5.5, the linear longitudinal strain rate solution (i.e., equation 5.30) is applied to investigate the stress and deformation of orogenic wedges subject to certain force and velocity boundary conditions which are specified the same way as in Chapter 5 (see equations 5.38). The three cases considered in this section differ from those studied in Section 5.5 in that shear stress is specified along the wedge basement, and the latter is not parallel to the top surface. The parameter values of these three cases are listed in Table 6.2. The basal shear stress is specified such that for all three cases it decreases in magnitude linearly from -80 MPa at the rear to zero at the front of wedge. A constant density of 2700 kg m^{-3} is assumed, given the minor effects of variations of density on the dynamics of orogenic wedges. The specification of

the v component of the top velocity boundary condition is also the same in all three cases, which is -5 mm a^{-1} (upward) at the rear and zero at the front of the wedge. Attention is focused on the stress and deformational characteristics of orogenic wedges with different homogeneous rheologies and different longitudinal deformation patterns on their top surface.

Table 6.2 Model parameters for orogenic wedges with width $L = 200 \text{ km}$ and top and basal slopes $\alpha = 3^\circ$ and $\beta = 6^\circ$ (constant density $\rho = 2700 \text{ kg m}^{-3}$)

Parameter	Case*		
	A1	B1	C1
$u_0 (\times 10^{-10} \text{ m s}^{-1})$	0	0	0
$u_1 (\times 10^{-15} \text{ s}^{-1})$	-1	-1	1
$u_2 (\times 10^{-20} \text{ m}^{-1} \text{ s}^{-1})$	-2.25	-2.25	-1
$v_0 (\times 10^{-10} \text{ m s}^{-1})$	-1.5855	-1.5855	-1.5855
$v_1 (\times 10^{-15} \text{ s}^{-1})$	0.7927	0.7927	0.7927
$\tau_0 (\text{MPa})$	-80	-80	-80
$\tau_1 (\text{MPa km}^{-1})$	0.4	0.4	0.4
$T_0 (\text{K})$	§	750	750
$T_1 (\text{K km}^{-1})$		0	0
$A_0^* (\times 10^{-23} \text{ Pa}^{-1} \text{ s})$	5		
$A_1^* (\times 10^{-23} \text{ Pa}^{-1} \text{ s}^{-1} \text{ km}^{-1})$	0		
power-law rheology	anorthosite†		

* Cases A1, B1, and C1 refer to Figures 6.2, 6.3, and 6.4

A1: Newtonian, constant $\eta (=10^{22} \text{ Pa s})$, variable $\dot{\epsilon}_{xx}$ and variable v on surface, variable τ_b

B1: Power-law, constant T , variable $\dot{\epsilon}_{xx}$ (always negative) and variable v on surface, variable τ_b

C1: Power-law, constant T , variable $\dot{\epsilon}_{xx}$ (positive and negative) and variable v on surface, variable τ_b

§ empty entry indicates that parameter is not applicable

† rheological parameters: $A_0 = 2.06 \times 10^{-23} (\text{Pa}^{-3.2} \text{ s}^{-1})$, $n = 3.2$, $Q = 238 (\text{kJ mol}^{-1})$

Case A1 (Figure 6.2) is a wedge with constant Newtonian viscosity ($\eta = 10^{22} \text{ Pa s}$) and such a longitudinal component of the top velocity boundary condition that the

longitudinal strain rate $\dot{\epsilon}_{xx}$ varies from -10^{-15} s^{-1} at the rear to -10^{-14} s^{-1} at the front of the wedge. The dominant movement pattern of the wedge material is longitudinal at the front and the vertical movement becomes more significant when approaching the rear of the wedge (Figure 6.2a). The maximum shear stress trajectories (Figure 6.2b) indicate a predominant thrusting deformation regime throughout the wedge. The effective shear strain rate $\dot{\epsilon}'_E$ (Figure 6.2c) varies mainly in the longitudinal direction. The spatial distribution of the effective shear stress σ'_E (Figure 6.2d) is identical to that of the effective strain rate, increasing from 20 MPa at the rear to 200 MPa at the front of the wedge. $F(y)$, the nongravitational normal stress at the rear of the wedge necessary to maintain the deformation regime (Figure 6.2e) decreases from -40 MPa at the top to -50 MPa at the rear bottom of the wedge. $G(y)$, the nongravitational shear stress at the rear of the wedge, varies roughly in a parabolic way with the depth.

Results for case B1 are shown in Figure 6.3. Since cases A1 and B1 have the same force and velocity boundary condition, comparison between the two shows the difference, in terms of stress, strain rate and velocity fields, between wedges with constant Newtonian viscosity and wedges with power-law creep rheology. The rheology of the wedge material in case B1 is that of anorthosite under constant temperature of 750 K (see Table 6.2). The velocity field (Figure 6.3a) is similar to that of case A1. The slopes of the maximum shear stress trajectories (Figure 6.3b) are slightly flatter in case B1 than in case A1, especially in the rear lower part of the wedge. The effective shear strain rate $\dot{\epsilon}'_E$ (Figure 6.3c) varies mainly in the longitudinal direction in the upper part of the wedge and varies significantly in the vertical direction in the rear lower part of the wedge. The spatial distributions of the effective shear stress σ'_E (Figure 6.3d) and the effective viscosity (Figure 6.3e) are similar to that of the effective shear strain rate. At the rear of the wedge, the effective viscosity decreases from $1.8 \times 10^{22} \text{ Pa s}$ at the top to

Figure 6.2 Stress and deformation within the wedge for case A1 of Table 6.2. (a) Velocity field; (b) maximum shear stress trajectories; (c) second invariant of strain rate tensor; (continued)

Figure 6.2 (d) second invariant of deviatoric stress tensor; (e) tectonic-induced normal and shear stresses along the rear boundary of the wedge necessary to maintain the deformation.

3.8×10^{21} Pa s at the bottom of the wedge. The nongravitational normal stress at the rear of the wedge necessary to maintain this deformation regime (Figure 6.3f) decreases nonlinearly from about -72 MPa at the top to about -15 MPa at the bottom of the wedge, differing significantly from that in case A1. The depth distribution of the nongravitational shear stress at the rear of the wedge is similar in both cases.

Case C1 (Figure 6.4) differs from case B1 by the longitudinal component of the velocity boundary condition specified on the top of the wedge. The longitudinal strain rate $\dot{\epsilon}_{xx}$ on the top surface of the wedge varies linearly from 10^{-15} s^{-1} at the rear to $-3 \times 10^{-15} \text{ s}^{-1}$ at the front. The velocity field (Figure 6.4a) is significantly different from that in case B1. The magnitude of velocity vector increases with depth in the rear part of the wedge. The maximum shear stress trajectories (Figure 6.4b) show coeval development of an extensional regime (normal faulting) at the rear and a compressional regime (thrusting) at the front of the wedge. The effective shear strain rate $\dot{\epsilon}'_E$ (Figure 6.4c) varies mainly in the vertical direction and is highly concentrated in the rear lower part of the wedge. The spatial distributions of the effective shear stress σ'_E (Figure 6.4d) and the effective viscosity (Figure 6.4e) are similar in that both show a wedge having an upper region with mainly lateral variation and a lower region with dominantly vertical variation. The nongravitational normal stress at the rear of the wedge necessary to maintain this deformation regime is extensional; $F(y)$ (Figure 6.4f) decreases nonlinearly from about 80 MPa on the top to about 10 MPa at the bottom of the wedge. The depth distribution of the nongravitational shear stress at the rear of the wedge, $G(y)$, is similar to both cases A1 and B1.

Figure 6.3 Stress and deformation within the wedge for case B1 of Table 6.2. (a) Velocity field; (b) maximum shear stress trajectories; (c) second invariant of strain rate tensor; (continued)

Figure 6.3 (d) second invariant of deviatoric stress tensor; (e) effective viscosity;
(continued)

Figure 6.3 (f) tectonic-induced normal and shear stresses along the rear boundary of the wedge necessary to maintain the deformation.

Figure 6.4 Stress and deformation within the wedge for case C1 of Table 6.2. (a) Velocity field; (b) maximum shear stress trajectories; (c) second invariant of strain rate tensor; (continued)

Figure 6.4 (d) second invariant of deviatoric stress tensor; (e) effective viscosity;
(continued)

Figure 6.4 (f) tectonic-induced normal and shear stresses along the rear boundary of the wedge necessary to maintain the deformation.

6.4 Orogenic wedges with viscoplastic rheology

6.4.1 Low-temperature plastic upper layer

Deformation within orogenic wedges varies not only in the longitudinal direction as discussed in Section 2.6, but also in the vertical direction (mainly due to the change in rheology of rock material). Crustal materials behave differently under various pressure and temperature conditions. The stress difference ($\sigma_1 - \sigma_2$) of geodynamic interest is in the range 1- 100 MPa (Ranalli, 1995). At this stress level, bulk rheology of crustal rocks is mainly controlled by temperature which, as a first approximation, increases linearly with depth. Deformation of crustal rock is mainly frictional-plastic at shallow depth (where temperature is low); power-law rheology becomes more dominant when temperature increases (i.e., deeper in crust). Therefore, the whole wedge can be roughly divided into two regions: upper plastic and lower power-law creep domains (Figure 6.5). The critical temperature (T_c) boundary separating frictional-plastic and power-law creep regimes is roughly half of the solidus temperature (T_m) of the material, which is itself pressure-dependent. As a first approximation, we take T_c to be 600 K for quartzite (550 K if the material is wet), and 700 K for anorthosite. (Constraints on these values come from the depth distribution of seismicity; *cf.* Ranalli, 1993.)

A general power-law rheology as formulated in equations (4.2) - (4.5) provides a uniform expression which, when the stress exponent n varies, may represent both plastic and viscous rheology (see Table 4.1). This is also shown by the stress-strain rate curves in Figure 6.6. Within the framework of general power-law rheology as adopted in this thesis, the upper frictional-plastic region can be approximated as perfectly plastic (that is, no deformation if the effective shear stress σ'_E is less than the plastic strength σ_Y ; plastic deformation when σ'_E equals σ_Y).

Recalling that the effect of rheology on the dynamics of the wedge is realized through the function $F(y)$ (equation 5.34), there are two different ways to simulate the plastic behaviour in upper part of the wedge, namely, (a) by replacing the stress exponent n in the upper part of the wedge with a larger value and then calculate $F(y)$; or (b) by replacing equation (5.34) with another relation (i.e., Mises-Hencky criterion, *cf.* Reiner, 1960) which models the perfectly plastic flow. The latter approach is adopted in the computations to be discussed in this chapter. The effective shear stress σ'_E can be expressed in terms of the effective viscosity and other parameters by the following equation (see equation 5.34)

$$\sigma'_E = \left\{ 4[a_2 + b_2 y + (a_1 + b_1 y)w]^2 \eta(y)^2 + \left[\int_0^y [\rho(\tau)g_x - 4(a_1 + b_1 \tau)\eta(\tau)] d\tau \right]^2 \right\}^{\frac{1}{2}} \quad (6.3)$$

where all the symbols have the same meanings as in equation (5.34). Assuming that the strength of rock material in the upper part of the wedge can be expressed as a function of depth such as $s(y)$, the Mises-Hencky criterion for perfectly plastic flow as a function of depth becomes

$$\begin{aligned} [s(y)]^2 &= \sigma'^2_E \\ &= 4[a_2 + b_2 y + (a_1 + b_1 y)w]^2 \eta(y)^2 + \left[\int_0^y [\rho(\tau)g_x - 4(a_1 + b_1 \tau)\eta(\tau)] d\tau \right]^2 \end{aligned} \quad (6.4)$$

Figure 6.5 An orogenic wedge subdivided into upper perfectly plastic and lower power-law creep layers.

Figure 6.6 A general power-law creep model ($\dot{\epsilon}_E = \lambda \sigma_E'^n$) with variable stress exponent n and corresponding stress-strain rate graphs. The parameter λ is adjusted to give the same strain rate for each curve at a stress of 5 (values of stress and strain-rate are arbitrary).

As a first approximation, a linear function of depth y has been assumed for the plastic strength function $s(y)$

$$s(y) = s_0 + k \rho(y) g y \cos \alpha \quad (6.5)$$

where s_0 is the plastic strength at the top of each wedge segment and $\rho(y)$ is the density; g is gravitational acceleration and α the topographic slope. A value of 0.1 has been applied to the constant k to specify the slope at which plastic strength increases with depth. This is less than the rate of increase of frictional strength at low pressure in dry rocks, but this rate decreases with increasing pressure and pore fluid pressure (*cf.* Ranalli, 1995).

Within each wedge segment, the top plastic strength s_0 is required to meet two conditions:

(1) The computed effective viscosity distribution $\eta(y)$ must be continuous at the depth h_p , the thickness of the top plastic layer of the concerned wedge segment (determination of h_p will be briefly discussed at the end of this section). This continuity requirement can be easily derived from equations (5.34) and (6.5) as

$$\frac{1}{2\eta(h_p)} = \tilde{A}(h_p) \left[s_0 + k \rho(h_p) g h_p \cos \alpha \right]^{\frac{\tilde{n}(h_p)-1}{2}} \quad (6.6)$$

where the functions $\tilde{A}(y)$ and $\tilde{n}(y)$ have been defined in the context of equation (5.34).

(2) The computed effective viscosity distribution $\eta(y)$ must satisfy the basal shear stress boundary conditions (see equation 5.32 and Figure 5.4).

In other words, for a given wedge segment with upper plastic and lower power-law layers, the effective viscosity $\eta(y)$ is determined by the following steps: firstly, assign

initial values to both parameters s_0 and γ (see equation 5.33); secondly, adjust s_0 to satisfy the following two equations

$$\begin{aligned} \sigma_E'^2 &= [s_0 + k \rho(y) g y \cos \alpha]^2 & 0 \leq y \leq h_p \\ \frac{1}{2\eta(y)} &= \tilde{A}(y) \sigma_E'^{\frac{\tilde{n}(y)-1}{2}} & h_p \leq y \leq \bar{h} \end{aligned} \quad (6.7)$$

where $\bar{h}$ is thickness of the wedge segment and σ_E' the effective shear stress; thirdly, check if the resulting distribution of $\eta(y)$ is in agreement with the basal shear stress boundary conditions (i.e., equation 5.32). This three-step procedure is repeated until two values for s_0 and γ are found that satisfy all the constraints.

For those wedge segments (such as in the thin-skinned part of the wedge) where the temperature throughout the whole depth is below the critical temperature (and therefore $h_p \geq \bar{h}$), plastic flow is the only deformation pattern. In this case only the first and the third steps are required to take to compute the acceptable values for parameters s_0 and γ .

The process of calculating s_0 and γ described above is a *two point boundary value problem* and can be solved by several numerical techniques (for detailed discussion see Press et al. , 1992). One of these techniques, the so-called *shooting method* is adopted in this thesis.

The thickness of the top plastic layer, h_p , depends on both the geothermal gradient and the material composition of the wedge. The critical temperature $T_c(y)$ is about one half of $T_m(y)$, which is here assumed constant for a given material (*cf.* subsection 6.4.1). Comparison between $T_c(y)$ and the temperature distribution $T(y)$ gives h_p (Figure 6.7).

Figure 6.7 Determination of the thickness of the top plastic layer. T_c is the depth distribution of the critical plastic temperature, assuming uniform composition; T_1 , T_2 , and T_3 are three geotherms with different geothermal gradients. Depths at points a , b , and c are the thickness of the top plastic layers for these three geotherms.

6.4.2 Effects of model parameters on the dynamics of orogenic wedges with plastic upper layer

The model parameters investigated in this section include (a) upper and lower slopes of the wedge (i.e., α and β); (b) material composition and dry/wet conditions; (c) temperature; (d) density of the wedge material, and (e) basal shear resistance. A temperature of 0 °C (≈ 273 K) on the top of the wedge is used in all computations.

Boundary conditions for all models in this chapter are specified in a way identical to that used in Section 5.5 (i.e., equations 5.38a - d). The variation of density of the wedge material is parameterized by the following linear function of depth

$$\rho = \rho_0 + \rho_1 y \quad (6.8)$$

where ρ_0 and ρ_1 are constants. (In most calculations, the density has been taken constant for simplicity, but this does not entail any loss of generality.)

Four cases with associated model parameters are listed in Table 6.3. The top velocity boundary conditions are specified such that the longitudinal strain rate along the top of the wedge varies linearly in the x -direction from -10^{-15} s^{-1} at the rear to -10^{-14} s^{-1} at the front of the wedge. It should be noted that, during the discussion, reference will be made to some cases not listed in Table 6.3.

Case A2 (Figures 6.8) serves as a reference case with the typical power-law rheology of anorthosite and a low geothermal gradient. The thickness of the top plastic layer is about 28.5 km as represented by the dashed line in the velocity field diagram (Figure 6.8a). The wedge material moves in a pattern roughly comparable to the flow of a fluid jetting out a diverging channel. The maximum shear stress trajectories (Figure 6.8b)

Table 6.3 Parameters for viscoplastic wedges with $L = 200$ km and top shortening rate increasing towards the front.

Parameter	Case*			
	A2	B2	C2	D2
α	3°	3°	3°	3°
β	6°	6°	6°	6°
$u_0 (\times 10^{-10} \text{ m s}^{-1})$	0	0	0	0
$u_1 (\times 10^{-15} \text{ s}^{-1})$	-1	-1	-1	-1
$u_2 (\times 10^{-20} \text{ m}^{-1} \text{ s}^{-1})$	-2.25	-2.25	-2.25	-2.25
$v_0 (\times 10^{-10} \text{ m s}^{-1})$	-0.4	-0.4	-0.4	-0.4
$v_1 (\times 10^{-15} \text{ s}^{-1})$	0.2	0.2	0.2	0.2
τ_0 (MPa)	-80	-80	-80	-160
τ_1 (MPa km ⁻¹)	0.35	0.35	0.35	0.75
T_0 (K)	273	273	273	273
T_1 (K km ⁻¹)	15	15	25	15
ρ_0 (kg m ⁻³)	2700§	2700	2700§	2700§
ρ_1 (kg m ⁻³ km ⁻¹)	0	0	0	0
rock material	anorthosite†	quartzite‡	anorthosite†	anorthosite†

* Cases A2, B2, C2, and D2 refer to Figures 6.8, 6.9, 6.10, and 6.11.

A2: Power-law of anorthosite, low thermal gradient and small basal shear stress

B2: Power-law of quartzite, low thermal gradient and small basal shear stress

C2: Power-law of anorthosite, high thermal gradient and small basal shear stress

D2: Power-law of anorthosite, low thermal gradient and large basal shear stress

§ a constant density is assumed for both anorthosite and quartzite given the trivial effect of variations of density on the dynamics of orogenic wedges

† rheological parameters for anorthosite: $A_0 = 2.06 \times 10^{-23} (\text{Pa}^{-3.2} \text{ s}^{-1})$, $n = 3.2$, $Q = 238 (\text{kJ mol}^{-1})$

‡ rheological parameters for quartzite: $A_0 = 2.512 \times 10^{-20} (\text{Pa}^{-2.4} \text{ s}^{-1})$, $n = 2.4$, $Q = 156 (\text{kJ mol}^{-1})$

indicate a homogenous thrust deformation regime throughout the wedge. The effective shear strain rate $\dot{\epsilon}'_E$ (Figure 6.8c) varies mainly in the longitudinal direction within the upper plastic layer, increasing from 10^{-15} s^{-1} at the rear to 10^{-14} s^{-1} at the front of the wedge; $\dot{\epsilon}'_E$ becomes a increasing function of the depth only within the rear lower power-law region of the wedge. The effective shear stress σ'_E (Figure 6.8d) varies mainly in the y-direction, firstly increasing with depth within the upper plastic layer and then

decreasing with depth in the lower power-law region (Note that the small bump in the effective shear stress contour map when the power-law creep zone appears is basically an artifact of calculations, due to the iteration being stopped to speed up the computation process. Similar observations also apply to the effective shear stress contour maps for all the following cases.) The effective viscosity (Figure 6.8e) varies in both the x - and the y -directions in the top plastic layer, increasing with depth and decreasing from the rear towards the front of the wedge; within the rear lower power-law region of the wedge the effective viscosity decreases rapidly with depth. (Note that the effective viscosity in the plastic zone of a viscoplastic wedge can be understood, based on its formal definition, as the ratio between the deviatoric stress and the corresponding strain rate which is a result of the assigned boundary conditions.) The nongravitational normal compressive stress $F(y)$ at the rear of the wedge responsible for the deformation regime (Figure 6.8f) first increases in magnitude from -45 MPa on the top to -160 MPa at the bottom of the top plastic layer and then decreases rapidly at the bottom of the wedge. The nongravitational shear stress $G(y)$ at the rear of the wedge does not show sharp changes.

To study the effect of top and basal slopes on the dynamics of wedges, a separate case was designed (not listed in Table 6.3). All model parameters are identical to those of case A1 except that its top and basal slopes are $\alpha = \beta = 4.5^\circ$. Comparison of solutions between these two cases reveals a great deal of similarity in terms of deformation and longitudinal strain rate. The major difference is observed in the nongravitational normal stress required to maintain the deformation regime at the rear of the wedge: the magnitude of $F(y)$ in the new case is about 3-5 MPa less than that of case A2.

Wedges with variable density have also been considered, with a density of 2700 kg m^{-3} at the top linearly increasing to 2900 kg m^{-3} at the depth of 30 km (not listed

Figure 6.8 Stress and deformation within the wedge for case A2 of Table 6.3. (a) Velocity field; (b) maximum shear stress trajectories; (c) second invariant of strain rate tensor; (continued)

Figure 6.8 (d) second invariant of deviatoric stress tensor; (e) effective viscosity;
(continued)

Figure 6.8 (f) tectonic-induced normal and shear stresses along the rear boundary of the wedge necessary to maintain the deformation.

in Table 6.3). Compared to case A2, the only significant effect of the increased density is observed in the state of stress at the rear of the wedge: for the same boundary conditions, the nongravitational normal stress required to maintain the deformation regime at the rear of the wedge with increasing material density is smaller by an amount of about 5 MPa than that with constant material density. This also justifies our assumption of constant density independent of rock type.

The rock material (quartzite) parameterized in case B2 (Figure 6.9) is much softer than the anorthosite of case A2 (see Tables 6.1 and 6.2). Since boundary conditions for cases A2 and B2 are identical (see Table 6.3), comparison between these two cases shows the effects of rheology on the deformation and stress pattern. Most of the deformation is concentrated in the rear lower power-law region of the wedge (Figure 6.9a) where the dominant flow pattern is laminar (Figure 6.9b). The strong negative horizontal component of velocity appearing in the rear lower power-law region is a consequence of the rapid decrease in the effective viscosity combined with the basal shear stress. The spatial distribution of the effective shear strain rate $\dot{\epsilon}'_E$ (Figure 6.9c) shows a similar pattern. The effective shear stress σ'_E (Figure 6.9d) varies mainly in the y -direction, increasing from about 6 MPa at the top to about 79 MPa at the bottom of the wedge. The effective viscosity (Figure 6.9e) does not vary significantly in the top plastic layer but decreases mainly in the y -direction from 7.8×10^{21} Pa s at the bottom of the top plastic layer to 1.4×10^{19} Pa s at the bottom of the wedge. Similar to case A2 (Figure 6.8f), the nongravitational normal stress at the rear of the wedge (Figure 6.9f) first increases in magnitude from -12 MPa at the top to -80 MPa at the bottom of the top plastic layer, then decreases rapidly towards the bottom of the wedge; the nongravitational shear stress at the rear of the wedge does not show a significant change when crossing the bottom of the plastic layer.

Figure 6.9 Stress and deformation within the wedge for case B2 of Table 6.3. (a) Velocity field; (b) maximum shear stress trajectories; (c) second invariant of strain rate tensor; (continued)

Figure 6.9 (d) second invariant of deviatoric stress tensor; (e) effective viscosity;
(continued)

Figure 6.9 (f) tectonic-induced normal and shear stresses along the rear boundary of the wedge necessary to maintain the deformation.

The effect of hydration within the wedge was examined by comparing case B2 (with dry quartzite material) to a case with wet quartzite material (not listed in Table 6.3). All the boundary conditions are the same for this new case and case B2. Comparison shows that (a) wedges with wet material show a larger region of predominant laminar flow; (b) the depth variation of longitudinal strain rate is larger in wet wedges than in dry wedges; and (c) the nongravitational normal stress at the rear of the wedge is also smaller for the wet than that for the dry wedge.

Results for case C2 are shown in Figure 6.10. Comparison between cases A2 and C2 shows the effect of the thermal gradient on the stress, strain rate and velocity fields within a wedge with a top plastic layer. The thickness of the plastic layer in case C2 (with high thermal gradient of 25 K km^{-1}) is 17 km compared to 28.5 km in case A2. Similar to case B2, most of the deformation within the wedge in case C2 is concentrated in the rear lower power-law region (Figure 6.10a) where the predominant flow pattern is laminar (Figure 6.10b), as shown also in the spatial distribution of the effective shear strain rate $\dot{\epsilon}'_E$ (Figure 6.10c). The effective shear stress σ'_E (Figure 6.10d) varies in both the x - and y -directions in the top plastic layer, but varies mainly in the y -direction within the lower power-law region. The effective viscosity (Figure 6.10e) does not vary significantly in the top plastic layer, but decreases mainly in the y -direction in the viscous layer from $8.7 \times 10^{21} \text{ Pa s}$ to $4.9 \times 10^{16} \text{ Pa s}$ at the bottom of the wedge. The nongravitational normal stress at the rear of the wedge (Figure 6.10f) first increases in magnitude from -20 MPa at the top to -75 MPa at the bottom of the top plastic layer and then decreases rapidly towards the bottom of the wedge.

The effect of temperature on the dynamics of wedge deformation is very similar to that of hydrated minerals within the wedge: both increasing temperature and H_2O -content make the wedge material softer.

Figure 6.10 Stress and deformation within the wedge for case C2 of Table 6.3. (a) Velocity field; (b) maximum shear stress trajectories; (c) second invariant of strain rate tensor;
(continued)

Figure 6.10 (d) second invariant of deviatoric stress tensor; (e) effective viscosity;
(continued)

Figure 6.10 (f) tectonic-induced normal and shear stresses along the rear boundary of the wedge.

The significance of basal shear stress was investigated by the comparison between cases A2 and D2. In both cases, the basal shear stress τ_b is -10 MPa at the front of the wedge; τ_b in case D2 increases linearly in magnitude to -160 MPa at the rear of the wedge, compared to -80 MPa in case A2. Results for case D2 are shown in Figure 6.11. A larger τ_b increases the shear strain rate and therefore the longitudinal flow in the lower rear power-law region of the wedge (Figure 6.11a), decreases the slopes of potential thrust planes (Figure 6.11b), and increases the effective shear strain rate $\dot{\epsilon}'_E$ (Figure 6.11c) and the effective shear stress σ'_E (Figure 6.11d). The effective viscosity (Figure 6.11e), as usual, decreases significantly within the rear lower power-law region. The nongravitational normal stress at the rear of the wedge (Figure 6.11f) first increases in magnitude from -160 MPa on the top to -280 MPa and then gradually decreases towards the bottom of the wedge; the nongravitational shear stress varies nonlinearly with depth, reaching a maximum (absolute) value of about -110 MPa at the bottom of the wedge.

6.4.3 Orogenic wedges with various deformation patterns

All the cases computed in previous subsection (see Table 6.3) had the same velocity boundary condition at the top of the wedge, resulting in longitudinal compression. In this subsection, attention is focused on the effects of velocity boundary conditions: wedges in extension, wedges in compression, and wedges with frontal compression and rear extension. Three cases are studied (Table 6.4). Common assumptions are that (a) the v component of the velocity on the top of the wedge decreases linearly in the x -direction from -5 mm a⁻¹ (upward) at the rear to zero at the front of the wedge; (b) the basal shear stress decreases linearly with position from -80 MPa at the rear to zero at the front of the wedge; and (c) the density of rock material remains constant throughout the wedge with $\rho = 2700 \text{ kg m}^{-3}$.

Figure 6.11 Stress and deformation within the wedge for case D2 of Table 6.3. (a) Velocity field; (b) maximum shear stress trajectories; (c) second invariant of strain rate tensor; (continued)

Figure 6.11 (d) second invariant of deviatoric stress tensor; (e) effective viscosity;

(continued)

Figure 6.11 (f) tectonic-induced normal and shear stresses along the rear boundary of the wedge.

Table 6.4 Parameters for an anorthosite wedge[†] with $L = 200$ km, $\alpha = 3^\circ$ and $\beta = 6^\circ$, subject to a geothermal gradient of 15 K km^{-1} .

Parameter [‡]	Case*		
	A3	B3	C3
$u_0 (\times 10^{-10} \text{ m s}^{-1})$	0	0	0
$u_1 (\times 10^{-15} \text{ s}^{-1})$	1	-1	1
$u_2 (\times 10^{-20} \text{ m}^{-1} \text{ s}^{-1})$	-0.225	-2.25	-1

* Cases A3, B3, and C3 refer to Figures 6.12, 6.13, and 6.14.

A3: Extensional wedge

B3: Compressional wedge

C3: Wedge with extension at the rear and compression at the front

† rheological parameters for anorthosite: $A_0 = 2.06 \times 10^{-23} (\text{Pa}^{-3.2} \text{ s}^{-1})$, $n = 3.2$, $Q = 238 (\text{kJ mol}^{-1})$

‡ other boundary conditions: $\tau_0 = -80 \text{ MPa}$, $\tau_1 = 0.4 \text{ MPa km}^{-1}$, $v_0 = -5 \text{ mm a}^{-1}$, $v_1 = 2.5 \times 10^{-8} \text{ a}^{-1}$.

6.4.3.1 Extensional wedge

The whole wedge in case A3 (Figure 6.12) is in extension with decreasing longitudinal extension from 10^{-15} s^{-1} at the rear to 10^{-16} s^{-1} at the front of the wedge. The predominant flow direction is from the rear towards the front of the wedge, with maximum velocity magnitude at the rear lower power-law corner region (Figure 6.12a). The trajectories of the maximum shear stress (Figure 6.12b) show a series of potential normal faults concave upward and dipping towards the front. The effective shear strain rate $\dot{\epsilon}'_E$ (Figure 6.12c) varies mainly with depth with most of the deformation concentrated at the rear lower power-law region. The effective shear stress σ'_E (Figure 6.12d) varies mainly in the y -direction (the small staircase variations are introduced by the computation algorithm as explained in the context of case A2). The effective viscosity (Figure 6.12e) decreases significantly within the rear lower power-law region. The nongravitational normal stress at the rear of the wedge required to maintain the deformation regime is extensional (Figure 6.12f), increasing with depth in the top plastic

Figure 6.12 Stress and deformation within the wedge for case A3 of Table 6.4. (a) Velocity field; (b) maximum shear stress trajectories; (c) second invariant of strain rate tensor; (continued)

Figure 6.12 d) second invariant of deviatoric stress tensor; (e) effective viscosity;
(continued)

Figure 6.12 (f) tectonic-induced normal and shear stresses along the rear boundary of the wedge.

layer and then rapidly decreasing; the nongravitational shear stress at the rear of the wedge increases nonlinearly with depth, reaching a maximum magnitude of about -40 MPa at the bottom of the wedge.

6.4.3.2 Compressional wedge

The whole wedge in case B3 (Figure 6.13) is in compression with increasing longitudinal compression from -10^{-15} s^{-1} at the rear to -10^{-14} s^{-1} at the front of the wedge. The longitudinal component of the velocity decreases gradually from about -3.5 cm a^{-1} at the front to zero at the rear while the vertical component increases from zero at the front to -0.5 cm a^{-1} at the rear (Figure 6.13a). The maximum shear stress trajectories (Figure 6.13b) indicate a compressive deformation regime throughout the wedge, represented by a series of upwardly concave thrusts dipping towards the hinterland of the wedge. The effective shear strain rate $\dot{\epsilon}'_E$ (Figure 6.13c) varies mainly in the longitudinal direction within the upper plastic layer, increasing from 10^{-15} s^{-1} at the rear to 10^{-14} s^{-1} at the front of the wedge; $\dot{\epsilon}'_E$ becomes an increasing function of depth only within the rear lower power-law region. The effective shear stress σ'_E (Figure 6.13d) varies mainly in the y -direction. The effective viscosity η (Figure 6.13e) varies in both the x - and the y -directions in the top plastic layer, increasing with depth and decreasing from the rear towards the front of the wedge; η decreases with depth within the rear lower power-law region. The nongravitational normal stress at the rear of the wedge is compressional (Figure 6.13f), increasing within the top plastic layer and decreasing rapidly in the nonlinearly viscous layer; the nongravitational shear stress at the rear of the wedge varies nonlinearly with depth.

Figure 6.13 Stress and deformation within the wedge for case B3 of Table 6.4. (a) Velocity field; (b) maximum shear stress trajectories; (c) second invariant of strain rate tensor; (continued)

Figure 6.13 (d) second invariant of deviatoric stress tensor; (e) effective viscosity;

(continued)

Figure 6.13 (f) tectonic-induced normal and shear stresses along the rear boundary of the wedge.

6.4.3.3 Wedge with rear extension and frontal compression

The wedge in case C3 (Figure 6.14) is given a velocity boundary condition resulting in a longitudinal deformation pattern which is positive (extensional) at the rear and negative (compressional) at the front of the wedge. Changes of the velocity field within the wedge are much more significant in direction than in magnitude (Figure 6.14a): the movement of wedge material at the front is basically longitudinal while that at the rear becomes mainly vertical. The maximum shear stress trajectories (Figure 6.14b) reflect the existence of a compressional and an extensional deformation regime at the front and at the rear of the wedge, respectively. The effective shear strain rate $\dot{\epsilon}'_E$ (Figure 6.14c) is highly concentrated in the rear lower power-law region. The effective shear stress σ'_E (Figure 6.14d) varies mainly in the y -direction. The effective viscosity η (Figure 6.14e) varies in both the x - and the y - directions with maximum η (corresponding to minimum strain rate) at the boundary region separating the rear extension from the frontal compression. The nongravitational normal stress at the rear of the wedge (Figure 6.14f) is tensional, increasing with depth, as usual, in the top plastic layer and decreasing with depth in the lower viscous region. The nongravitational shear stress at the rear of the wedge varies nonlinearly with depth.

The above three cases share a low geothermal gradient. If the geothermal gradient is higher, the size of the nonlinear viscous region increases. By analogy with the cases in the previous section, a laminar flow pattern is expected to become more predominant in the lower viscous part of the wedge; the effective strain rate is also expected to be increasingly concentrated in the viscous area.

Figure 6.14 Stress and deformation within the wedge for case C3 of Table 6.4. (a) Velocity field; (b) maximum shear stress trajectories; (c) second invariant of strain rate tensor; (continued)

Figure 6.14 (d) second invariant of deviatoric stress tensor; (e) effective viscosity;
(continued)

Figure 6.14 (f) tectonic-induced normal and shear stresses along the rear boundary of the wedge.

6.5 Some observations

The cases studied in this chapter, as well as those in the previous chapter, show that wedges with linear, nonlinear, and a combination of rheologies, submitted to prescribed longitudinal strain rates on their top surfaces and prescribed shear stresses on their bases, can deform in compression, extension, or a combination of the two (compression in the frontal part and extension in the back). The detailed patterns of velocity, maximum shear stress trajectories, effective deviatoric stress and strain rate, and effective viscosity depend on the bulk rheology, geothermal gradient, hydration, and boundary conditions. Some solutions include relatively important vertical velocity components throughout the rear of the wedges. In nature, these could be due to underplating, with consequent exhumation of rocks (assuming that erosion keeps pace with uplift).

The conditions at the "backstop" of the wedge which result in these stress and deformation patterns are expressed in terms of the function $F(y)$, which depends mainly on bulk rheology plus the prescribed longitudinal strain rates on the top and the prescribed shear stresses on the base of the wedge (the other function $G(y)$ is related to $F(y)$). $F(y)$ represents the nongravitational horizontal normal stress on the rear boundary, i.e., the normal stress in excess of the lithostatic. As a rule, the absolute value of $F(y)$ (which is compressive in some cases and tensile in others, but never changes sign in a given case) decreases with increasing ductility (i.e., depth) for depth-dependent viscous rheology. For plastic rheology overlying a viscous layer, $F(y)$ increases in magnitude with depth within the top plastic layer, and then decreases with increasing depth in the viscous layer. These general trends are in agreement with usual measurements and analyses of stresses in the crust (*cf.* e.g. Ranalli, 1995), although the details of the depth variation may not be entirely corresponding to natural conditions.

Chapter 7

Conclusion

7.1 Main results

(1) On the scale of an individual thrust sheet, a solution for sheer and normal stress along, the basal thrust shows that, for wedge-shaped overthrust blocks and/or nonlinear boundary conditions, static (frictional) stability conditions are exceeded only towards the use of the block. This results confirms the observation by Price (1988, 1990) that the solution to the paradox of large overthrusts lies in the consecutive - rather than synchronous - nature of slip (*cf.* chapter 3).

(2) On the scale of a whole orogenic wedge, an analytical solution for velocity and stress in a two-dimensional plane strain incompressible block with power-law creep rheology has been derived and explicitly expressed in terms of bulk rheology, state of stress at the rear boundary of the block, and a function $C(x,y)$. representing the deviation of the normal stress in y -direction from the lithostatic overburden. Although a general analytical expression for the function $C(x,y)$ is difficult to derive in the general case, such a systematic description of the dynamics of a two-dimensional block provides the necessary theoretical background on which solutions for two special cases of relevance to orogenic wedges are derived, namely,

(i) In the particular case in which the viscous rheology of the material is linear (i.e., Newtonian flow), the function $C(x,y)$ becomes determinable from the stress and/or velocity boundary conditions, making the problem analytically solvable (*cf.* Chapter 4);

(ii) When the top slope of the block is small and the vertical gradient of the vertical normal stress is negligible compared to the longitudinal gradient of the shear stress, the

function $C(x,y)$ becomes zero, and a closed form for the velocity and stress solution is derived in terms of a generalized power-law creep rheology and stress/velocity boundary conditions (*cf.* Chapter 5).

(3) The solution for power-law creep rheology and linearly varying longitudinal strain rate (case 2(ii) above) is a two-dimensional extension of a classic one-dimensional solution for ice sheets (Nye, 1957). Also, some semi-qualitative tectonic models for orogenic wedges (Platt, 1986) can be derived from the two-dimensional solution.

(4) Stress and velocity fields within orogenic wedges vary with material rheology, temperature, and boundary conditions. In all cases, computational results match reasonably well geological observables such as stress and strain rate intensities, maximum shear trajectories, etc. Orogenic wedges are deformed by the nongravitational normal stress at the rear of the wedge (i.e., $F(y)$), the longitudinal compressive stress due to the top slope of the wedge (i.e., $\bar{\rho}g_x y$), and the basal shear stress (i.e., τ_b). Large basal shear stress favours compression and low basal shear stress favours extension within the wedge.

(5) The depth distribution of the nongravitational normal stress required to maintain the deformation regime (i.e., $F(y)$) varies with wedge rheology and basal shear stress. In general, it decreases with increasing ductility. In rheologically layered wedges, it increases in magnitude with depth when bulk rheology is plastic, but decreases when the rheology is power-law viscous.

(6) Wedges under appropriate stress and velocity boundary conditions are able to deform with different patterns in their longitudinal direction, that is, compression near the front and extension near the rear of the wedge (*cf.* chapter 6). However, in order to have a normal faulting regime at the rear, the nongravitational stress $F(y)$ must be tensional. This does not preclude topography-induced extension in a compressional regime, as the present models do not take into account the effects of changing topography.

(7) Although velocity boundary conditions are assigned on the surface of the wedges, rather than on its basement, computational results suggest that the uplift rates at the surface and along the basement do not vary significantly at the rear of the wedges. This implies that assigned vertical components of velocity can be taken to simulate underplating. In an analogous manner, basement-parallel components of velocity can be taken to simulate dragging by underthrusting.

7.2 Discussion

The nonlinear viscous solution provides a comprehensive, analytical means to represent the stress and deformation of an orogenic wedge composed of material lacking a long-term yield strength. Under any nonzero deviatoric stress, such an orogenic wedge tends to flow internally (and therefore the surface slope α is modified), until it reaches a stable configuration, in which the gravitational forces generated by its surface slope balance the given basal shear stress (say, exerted by the subduction). Therefore, the surface slope of the wedge should be envisioned as a dynamic variable which is adjusted throughout the whole deformation process of the wedge.

This solution can also be readily applied to investigate the exhumation of high pressure-low temperature metamorphic rocks near the rear of an unstable orogenic wedge in terms of the pressure-temperature conditions at various times (Rubie, 1984; Platt and Lister, 1985; Platt, 1993; Ranalli, 1995). Combined with a given erosion rate at the top surface of the wedge, the velocity expression of the solution (i.e., equation 5.30c) provides an estimation for the exhumation rate of the material throughout the wedge. Integration of the exhumation rate with depth leads to the time variation of the pressure conditions of rocks within the wedge. As a first approximation, the depth variation of temperature can be independently assumed, neglecting viscous shear heating.

Consideration of viscous shear heating will lead to *thermomechanical coupling*, which significantly increases the complexity of the problem (for a detailed discussion, see Ranalli, 1995).

Various mechanisms for the uplift of high-pressure metamorphic rocks near the rear of an orogenic wedge have been proposed, including (a) buoyant rise (Ernst, 1984); (b) entrainment of high-pressure blocks in relatively low-density diapirs (England and Holland, 1979; Carlson, 1981; Moore, 1984); and (c) regional uplift coupled with erosion at the surface (Platt, 1975; Rubie, 1984). Computational results in this thesis indicate that uplift rates at the top and underplating rates along the basement do not differ significantly at the rear of the wedge, suggesting that erosion rate at the surface, coupled with tectonic extension, could be an essential factor in determining the P-T-t path for high-pressure metamorphic rocks in large orogenic wedges.

In order to test the applicability of the nonlinear viscous solution, computational results should be compared with relevant geological observations (such as strain rate at the surface and P-T-t paths of metamorphic rocks near the rear of the wedge), which also provide necessary boundary constraints to the dynamic modelling of each individual orogenic wedge. For instance, precise strain rate data can be obtained using Global Positioning System technology in various parts of orogenic belts (Crespi et al., 1996). Such surface strain rate data can be readily used as boundary conditions in modelling the dynamics of orogenic wedges.

There are two major problems that have to be dealt with in the study of the stress and deformation fields in orogenic wedges. The first is to derive a closed form of a general solution capable of describing orogenic wedges with complex nonlinear rheologies and complex deformation patterns which may vary in the longitudinal and the vertical directions. The second is to implement particular cases of the solution in a way suitable to the modelling of wedge deformation under acceptably realistic conditions.

One of the basic assumptions adopted in this thesis in the derivation of the analytical solution is that the gradient of shear stress in the x -direction is negligible compared with the gradient of vertical normal stress in the y -direction, which results in $C(x,y) = 0$, that is, the normal stress in y -direction is equal to the lithostatic overburden at any depth. This approximation may hold well in the shallower part of the wedge, but not necessarily in the rear deeper parts where deformation is often concentrated. Therefore, future studies of orogenic dynamics should include, if possible, estimation of the spatial distribution of the function $C(x,y)$ and its effect on the evolution of orogenic wedges. This pattern is of general tectonic significance, since the “zero $C(x,y)$ ” assumptions has long been taken as one of the basic approximation in numerous geological and tectonic models (*cf.* Elliott, 1976; Davis et al., 1983; Platt, 1986).

Another basic assumption in this thesis is that the longitudinal strain rate, at any depth, is a linear function of the distance from the backstop. this allows the accommodation of strain rates and theologies varying in the longitudinal direction, by segmentation of the wedge. In theory, it should be possible to accommodate nonlinearly-varying longitudinal strain rates, which is another problem awaiting solution.

In the present analytical solution, the nongravitational stresses along the rear vertical boundary plane (i.e., $F(y)$ and $G(y)$) are derived quantities, i.e., those necessary to maintain a linear longitudinal strain rate under specified top and bottom boundary conditions. Ideally, they should be assigned as part of prescribed boundary conditions, but this is not possible in the framework of the present solution. Also, the velocity boundary condition should be specified on the wedge basement instead of on the top surface. Such improvements may be realized by adopting finite element method formalism, which in the present context involves a further subdivision of the wedge, not only in the longitudinal direction but also in the vertical direction. Values of $F(y)$ and

$G(y)$ could be assigned at a number of points along the rear vertical boundary. However, implementation of such boundary conditions may prove to be difficult.

On the positive side, the analytical solution presented in this thesis has the advantage of *generality*. Since a large number of geological and dynamic parameters have been included, this solution can be applied to model the dynamics of different types of orogenic wedges and other geological processes.

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